Title
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$\pi^+ n \rightarrow \eta p$: A REGGE FIT OVER A LARGE ENERGY RANGE USING VENEZIANO-TYPE RESIDUE FUNCTIONS[*]

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ABSTRACT

A low-background sample of approximately 350 events from the reaction $\pi^+ n \rightarrow \eta p$ has been obtained from a study of the reaction $\pi^+ d \rightarrow p p \pi^+ \pi^- \pi^0$ in a bubble chamber experiment with incident pion momentum between 1.1 and 2.4 BeV/c. The η production angular distributions and the total cross section are very well fitted by a two-parameter Reggeized A_2- exchange model with Veneziano-type residue functions. The model is also compared with data from other experiments spanning the incident momentum range from near threshold to 18.2 BeV/c, and the agreement is very good.

We report an investigation of the reaction $\pi^+ d \rightarrow p p \pi^+ \pi^- \pi^0$ performed in the fall of 1966 in the deuterium-filled 72-inch bubble chamber at the Lawrence Radiation Laboratory. The momentum of the incident pion beam from the Bevatron encompassed the range from 1.1 to 2.4 BeV/c at the eight momentum settings 1.10, 1.30, 1.53, 1.58, 1.70, 1.86, 2.15, and 2.37 BeV/c; the total exposure size was 14 events/μb. Because of the Fermi motion of the nucleons in the deuteron, a single beam momentum gives rise to a continuum of c.m. energies, and as a result our exposure gives continuous coverage of the c.m. energy range from 1.7 to 2.4 BeV.[1]

The events discussed here all have four visible outgoing tracks; that is, we do not report on events where the "spectator" proton (p) in the reaction

$$\pi^{+} d \rightarrow (p)\, p\, \pi^{+} \pi^{-} \pi^{o} \quad (1)$$

has momentum less than about 85 MeV/c in the laboratory. 128 000 four-pronged events were measured with the Spiral Reader and fitted with the standard LRL fitting programs TVGP and SQUAW. A sample of 8710 events assigned to reaction (1) and having at least one proton in the final state with laboratory momentum less than 300 MeV/c was obtained; the latter requirement is to insure that we are dealing with events in which only the neutron in the deuteron takes part in the interaction.

The $\pi^{+}\pi^{-}\pi^{o}$ mass spectrum from reaction (1) shows prominent η and ω peaks. The η peak contains about 350 events with little background; a maximum-likelihood fit using the program MURTLEBERT[2] was employed to estimate the number of η and ω events and to determine the cross sections for the reactions

$$\pi^{+} n \rightarrow \eta\, p \quad (2a)$$

$$\pi^{+} n \rightarrow \omega\, p \quad (2b)$$

as a function of c.m. energy by normalization to selected final states whose cross section is known from charge-symmetric data.[1] The background fraction in the η mass cut ($530\ \mathrm{MeV}/c^2 < m(\pi^+\pi^-\pi^0) < 570\ \mathrm{MeV}/c^2$) is 13 per cent.

The η production angular distributions and total cross sections have been fitted using a Regge-pole-exchange model involving the simplest allowed meson exchange, the A_2- or R-trajectory. The differential cross section, expressed in terms of the invariant amplitudes A and B, is

$$\frac{d\sigma}{d(\cos\theta)}\ (\mathrm{mb}) = \frac{.3895\ M^2\ q_f}{8\ \pi\ s\ q_i}\left\{\frac{|p|^2}{M^2}|A|^2 + \left[\left(\frac{s-u}{4\ M}\right)^2 + \frac{|k|^2\ t}{4\ M^2}\right]|B|^2 - \left(\frac{s-u}{2\ M}\right)\mathrm{Re}(A^*B)\right\}, \qquad (3)$$

where M is the nucleon mass; s, t, and u are the usual Mandelstam variables; and q_i and q_f are the initial- and final-state c.m. momenta, respectively. k is the t-channel meson momentum, and p is the t-channel baryon momentum. (All momenta and masses are in BeV.) We have chosen to parametrize the invariant amplitudes A and B by the leading order in s of a Veneziano parametrization:

$$A\ (\mathrm{BeV}^{-1}) = a_o\Gamma(1-\alpha(t))(1+e^{-i\pi\alpha(t)})(b's)^{\alpha(t)} \qquad (4a)$$

$$B\ (\mathrm{BeV}^{-2}) = b_o\Gamma(1-\alpha(t))(1+e^{-i\pi\alpha(t)})(b''s)^{\alpha(t)-1} \qquad (4b)$$

Here $\alpha(t)$ is the A_2 trajectory function, which we take to be the straight-line form

$$\alpha(t) = 2 + (t - m_{A_2}^2)b \qquad (5)$$

This parametrization is similar to the standard Regge parametrization of t-channel helicity amplitudes,[3] but the Veneziano model demands that

$b'' = b' = b$ be the universal slope of the linear trajectories. Hence, taking the trajectory slope b from experiment to be 1 BeV^{-2}, each residue function is determined up to an overall constant. The two real numbers a_o and b_o are consequently the only parameters in our fit.[4]

A least-squares fitting procedure to the shape only of the production angular distributions yields the best fit for

$$b_o/a_o = 2.4 \tag{6}$$

This fit is displayed upon the data for the production distributions at each of six c.m. energy intervals in Figures 1 a)-f). The production angular distributions are presented after subtraction of background; the three-pion production cosine distributions for three-pion masses adjacent to the η mass cut are fairly flat at all c.m. energies, so an isotropic background was subtracted. A small upward correction (the shaded events in Figure 1) has been made to the forwardmost two bins in each c.m. energy interval due to the effect of the Pauli exclusion principle in suppressing low-momentum-transfer events in reaction (1).[5]

The curves on the production distributions all satisfy equation (6), but they are normalized separately to have the same area as the respective histogram in Figure 1. It will be seen below, however, that a single choice of scale factor, i.e. a unique choice of a_o and b_o, fits both the shape and absolute scale of all distributions. The secondary zero in the curves of Figure 1 (at $t = -1.3$ BeV^2) occurs because the signature factors in equations (4) go to zero at $\alpha(t) = -1$.

The energy dependence of the total cross section for reaction (2a) and for its charge-symmetric counterpart, $\pi^- p \longrightarrow \eta\, n$, have also been compared to the model; the data points used span the c.m. energy interval from threshold for the reaction up to almost 6 BeV, the highest energy at which it has been studied. Figure 2 is a logarithmic plot of the total cross section for reaction (2a) and its charge-symmetric equivalent vs. c.m. energy, along with the predictions of the Reggeized A_2-exchange model. Figure 2 contains 13 cross-section points from this experiment and one point from another π^+-d experiment[6] for reaction (2a), and 15 points from two more experiments[7,8] done on the charge-symmetric reaction. Curves for three different ratios b_o/a_o are displayed on the data, all normalized to pass through the arbitrarily selected data point at $E_{c.m.} = 3.46$ BeV. For $b_o/a_o = 2.4$ the fit is seen to be very good over the entire range of energies; this is the same ratio which gives the best fit to the shape of the production angular distributions for our experiment.

The parameter values used to obtain the fit to the shape of our production angular distributions and to the total cross section over a wide range of energies are

$$a_o = 28.7, \quad b_o = 68.8 \tag{7}$$

From the two dotted curves in Figure 2 corresponding to $b_o/a_o = 2.2$ and 2.6, it is seen that a variation of only about 5 per cent in the ratio (6) is allowed in order to fit the cross section data. In order to fit the shape of the production cosine distributions, the tolerance is about 10 per cent. Furthermore, in order to fit the width of our experimental production cosine distributions, an A_2 trajectory slope of 1 BeV^{-2} $\pm$ 10 per cent is necessary; it should be noted that this value is obtained under the restriction that b = b' = b", which is not applied in a conventional Regge fit.

Figure 3 shows the differential cross section for

$$\pi^- p \longrightarrow \eta n, \quad \eta \longrightarrow \gamma \gamma \tag{8}$$

from reference 8 at c.m. energies above 2.50 BeV, along with the predictions of the Reggeized A_2-exchange model. The curves are normalized to have the same area as the histograms, which is not actually necessary because of the excellent agreement between the model and the total cross section points of this reference. The agreement between the differential cross sections and the model in Figure 3 is satisfactory.[9]

The model has also been compared with the differential cross section for reaction (2a) measured by Benson[10] at $E_{c.m.}$ = 2.78 BeV and with that for reaction (8) measured by Wahlig and Mannelli[11] at $E_{c.m.}$ = 4.43 BeV. (The comparison is shown in reference 1.) The data of Benson agree very well with the model, even better than the equivalent data of reference 8 shown in Figure 3b. The differential cross section of reference 11 is broader than the model and also broader than the distribution of reference 8 shown in Figure 3d, with which it should be identical. These discrepancies in the production distributions at high energy ($E_{c.m.}$ > 2.5 BeV) should be kept in mind when making detailed comparisons with the model. Cross sections for reaction (8) have also been measured close to threshold by Richards et al.,[12] and at the energies where these measurements overlap with ours, the agreement between the two experiments is excellent. The model fails to describe the differential cross section only for $E_{c.m.}$ values near threshold, around the mass of the N(1550); the model has been compared with the data of reference 12 in this region.

The simple two-parameter Regge exchange model described here is thus sufficient to predict accurately both the production angular distributions and the total cross section for reaction (2a) over a range of c.m. energies from 1.7 to 6 BeV (this corresponds to incident beam momentum between 1.1 and 18.2 BeV/c); this indicates that η production in reaction (2a) exhibits duality in the sense that the A_2 exchange which dominates the reaction for high energy and small t also gives a surprisingly good description at lower energy and at large angle.

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FOOTNOTES AND REFERENCES

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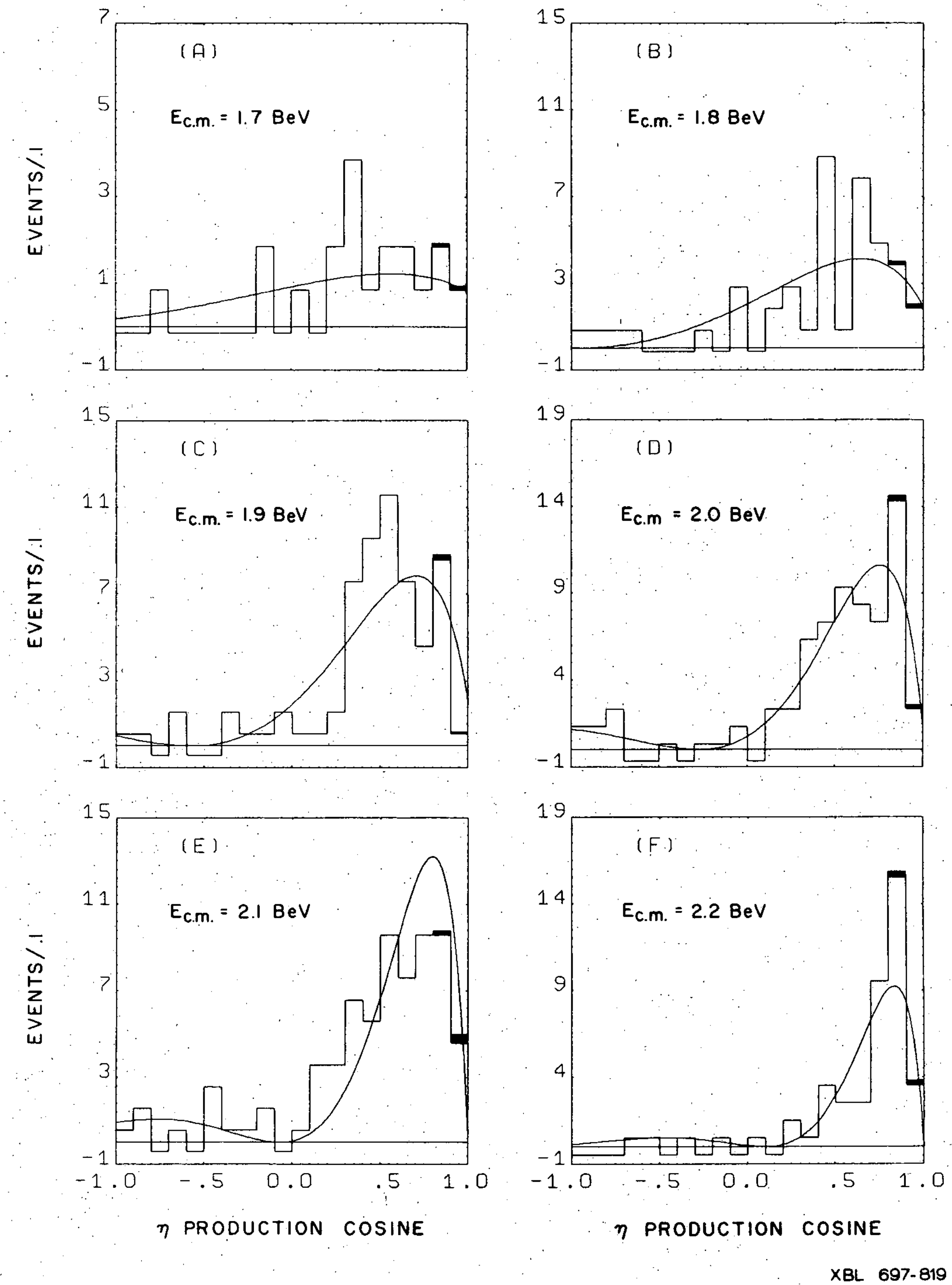
FIGURE CAPTIONS

FIG. 1. η production cosine distributions for six 100-MeV-wide c.m. energy intervals centered at the values indicated. Shaded events are added to account for the effect of the Pauli exclusion principle. Some bins have negative contents because of the background subtraction. The curves are the predictions of the Reggeized A_2-exchange model with $b_o/a_o = 2.4$; they are normalized to have the same area as the histograms.

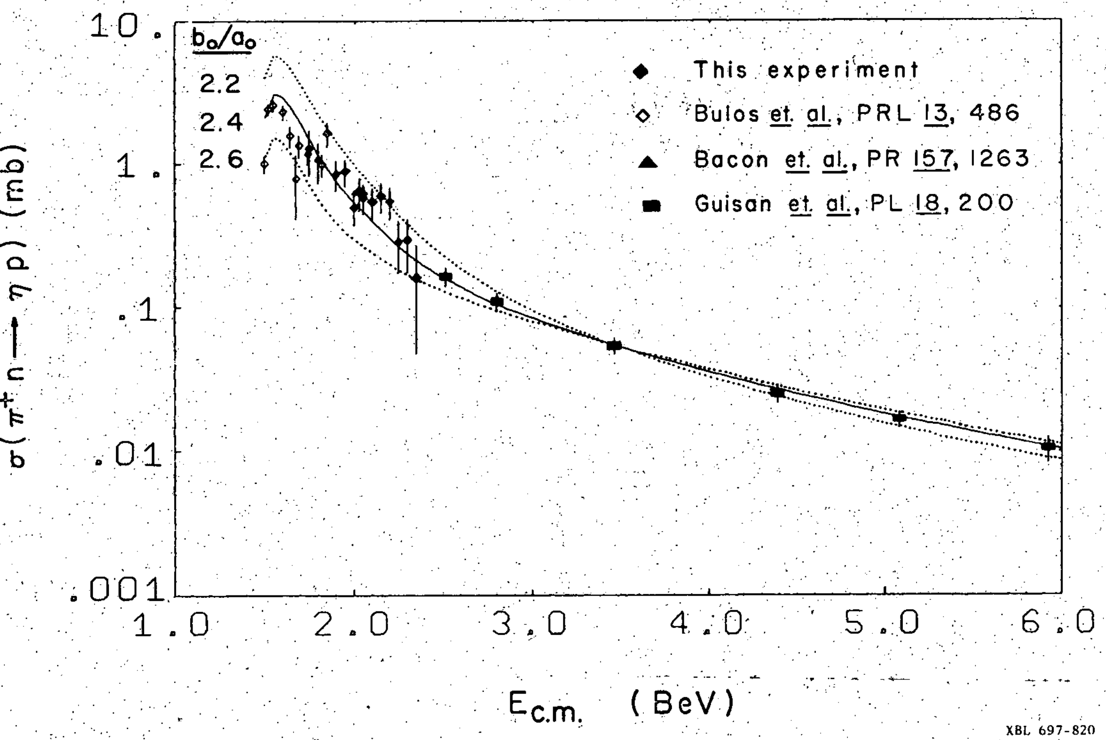
FIG. 2. Cross section for $\pi^+ n \to \eta p$ or $\pi^- p \to \eta n$ vs. c.m. energy. Three predictions of the Reggeized A_2-exchange model are plotted; all curves are normalized to pass through the data point at $E_{c.m.} = 3.46$ BeV.

FIG. 3. Differential cross sections for $\pi^- p \to \eta n$ from reference 8. The curves are the Reggeized A_2-exchange model predictions with $b_o/a_o = 2.4$ and are normalized to have the same area as the histograms.



(A)
Ec.m. = 1.7 BeV
(B)
Ec.m. = 1.8 BeV
(C)
Ec.m. = 1.9 BeV
(D)
Ec.m = 2.0 BeV
(E)
Ec.m. = 2.1 BeV
(F)
Ec.m. = 2.2 BeV
EVENTS/.1
η PRODUCTION COSINE
XBL 697-819

FIGURE 1



10.
1.
.1
.01
.001
1.0
2.0
3.0
4.0
5.0
6.0
$\sigma(\pi^+ n \longrightarrow \eta p)$ (mb)
$E_{c.m.}$ (BeV)
b_o/a_o
2.2
2.4
2.6
This experiment
Bulos et. al., PRL 13, 486
Bacon et. al., PR 157, 1263
Guisan et. al., PL 18, 200
XBL 697-820

FIGURE 2

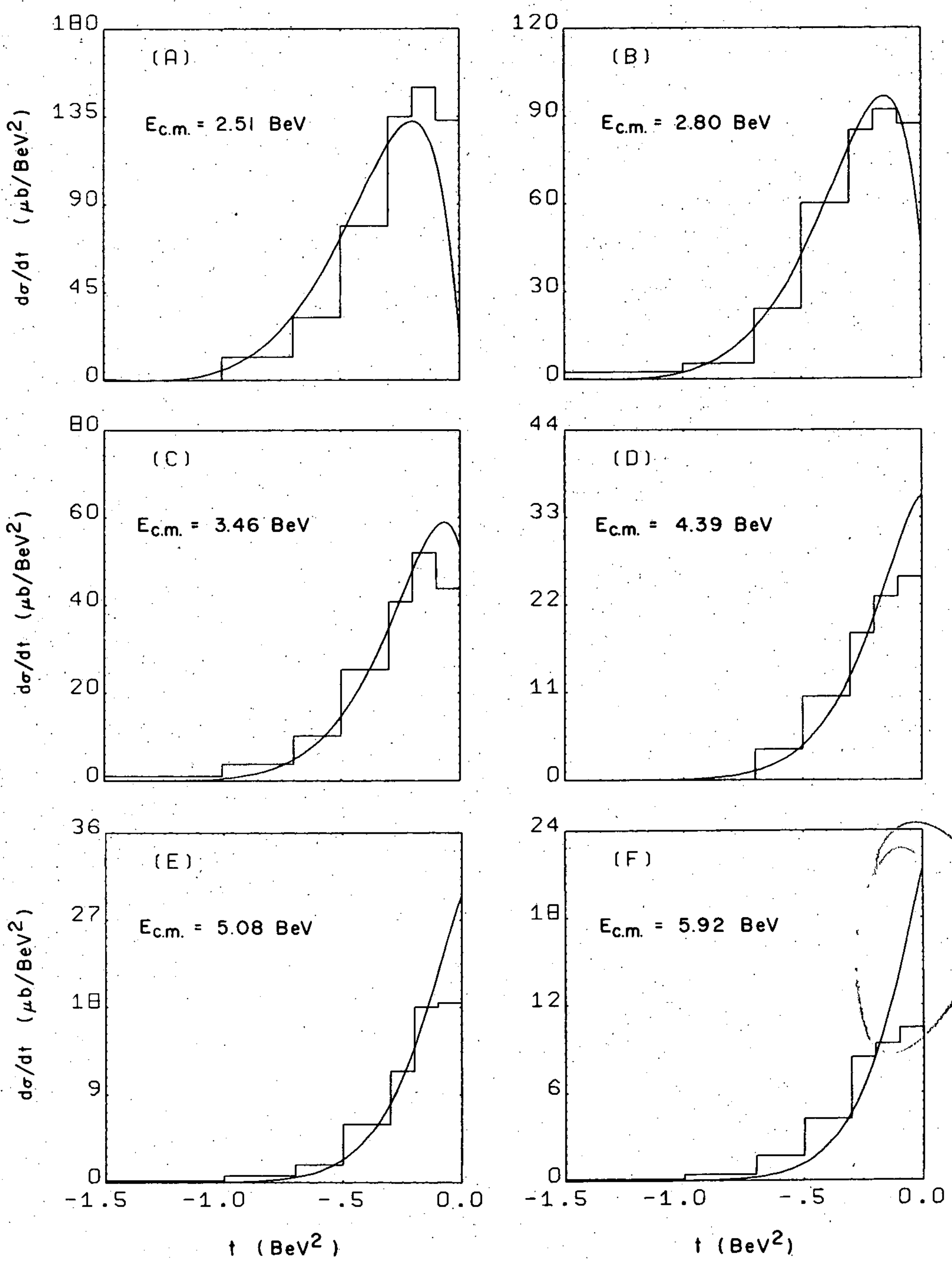


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FIGURE 3

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