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# STRUCTURAL COHESION AND EMBEDDEDNESS: A HIERARCHICAL CONCEPT OF SOCIAL GROUPS

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Although questions about social cohesion lie at the core of our discipline, definitions are often vague and difficult to operationalize. Here, research on social cohesion and social embeddedness is linked by developing a concept of structural cohesion based on network node connectivity. Structural cohesion is defined as the minimum number of actors who, if removed from a group, would disconnect the group. A structural dimension of embeddedness can then be defined through the hierarchical nesting of these cohesive structures. The empirical applicability of nestedness is demonstrated in two dramatically different substantive settings, and additional theoretical implications with reference to a wide array of substantive fields are discussed.

"[S]ocial solidarity is a wholly moral phenomenon which by itself is not amenable to exact observation and especially not to measurement."

*—Durkheim ([1893] 1984:24)* 

"The social structure [of the dyad] rests immediately on the one and on the other of the two, and the secession of either would destroy the whole.... As soon, however, as there is a sociation of three, a group continues to exist even in case one of the members drops out."

-Simmel ([1908] 1950:123)

**QUESTIONS SURROUNDING** social solidarity are foundational for sociologists and have engaged researchers continuously since Durkheim. Researchers across a wide spectrum of substantive fields employ "cohesion" or "solidarity" as a key element of their work. Social disorganization theorists, for example, tout the importance of "community cohesion" for preventing crime (Sampson and Groves 1989). Political sociologists focus on how a cohesive civil society promotes democracy (Paxton 1999; Putnam 2000). Historical sociologists point to the importance of solidarity for revolutionary action (Bearman 1993; Gould 1991), and that the success of heterodox social movements depends on a cohesive critical mass of true believers (Oliver, Marwell, and Teixeira 1985). Social epidemiologists argue that a cohesive "core" is responsible for the persistence of sexually transmitted diseases (Rothenberg, Potterat, and Woodhouse 1996). Worker solidarity is a key concept in the sociology of work (Hodson 2001). Social psychologists have repeatedly returned to issues surrounding cohesion and solidarity, attempting to understand both its nature (Bollen and Hoyle 1990; Gross and Martin 1952; Roark and Shara 1989) and consequences (Carron 1982; Hansell 1984).

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"structure" Unfortunately, as with (Sewell 1992), the rhetorical power of "cohesion" is both a blessing and a curse. Sociologists are all too familiar with the problem: We study "cohesion" in almost all our substantive domains, and in its ambiguity, it seems to serve as a useful theoretical placeholder. Ubiquity, however, does not equal theoretical consistency. Instead, the exact meaning of cohesion is often left vague, or when specified, done in a particularistic manner that makes it difficult to connect insights from one subfield to another. We identify one generalizable structural dimension of social solidarity. Although the concept we develop is related in certain ways to some, perhaps many, of the meanings of "solidarity" or "cohesion" used in the literature, it is by no means intended to incorporate them all. Instead, we focus on only one dimension. By carefully identifying one aspect of social solidarity, we hope to help clarify one of the multiple meanings contained in this ubiquitous idea.

The social network-based concept we develop is theoretically grounded in insights from Simmel ([1908] 1950) and Durkheim and methodologically ([1893] 1984) grounded in classical graph theory (Harary 1969; Harary, Norman, and Cartwright 1965). D. White and Harary (2001) demonstrate the formal logic by which graph-theoretic measures lend themselves to the study of the structural dimension of social cohesion. Here, we extend a definition of structural cohesion in its most general form, applicable to large-scale analyses in a variety of settings, and provide an algorithm for its use in empirical analyses. The implementation of our algorithm for measuring embedded levels provides an operational specification of one dimension of social embeddedness (Granovetter 1985, 1992), which allows us to specify and explore empirically the unique contribution of this dimension. Here we focus on two empirical settings: friendships among high-school students (Bearman, Jones, and Udry 1996) and the political activity of big businesses (Mizruchi 1992). For adolescent friendships, we show that network position predicts school attachment, using structural cohesion to link the relational to the ideational components of solidarity in a dozen large networks. For the

smaller director-interlock network, we show that joint network embeddedness leads dyads to make similar political contributions, linking network position to coordinated political action. In both cases, we find independent effects for our conception of cohesion net of commonly used alternative measures, substantiating its unique contribution.

# **BACKGROUND AND THEORY**

# SCOPE

Analytically, solidarity can be partitioned into an *ideational* component, referring to members' identification with a collectivity, and a *relational* component (Dorian and Fararo 1998), referring to the observed connections among members of the collectivity. This theoretical distinction, for example, allowed Durkheim to link changes in the common consciousness to the transition from mechanical to organic societies, although he offered no clear measures for these concepts. Research on commitment (Kanter 1968) or perceived cohesion (Bollen and Hoyle 1990) focuses directly on the ideational component of social solidarity. Although often based on an underlying relational theory, much of the national-and community-level work on social cohesion uses ideational indicators of "community cohesion" (Paxton 1999; Sampson and Groves 1989). Distinguishing between the relational and ideational components analytically does not imply a causal precedence of one dimension over the other. Empirically, these two dimensions (and perhaps others) might mutually reinforce each other. Whatever their ultimate causal relation, separating these two dimensions is a prerequisite to identifying the relation between them. Here we leave the wider question of "social solidarity" in the background and focus instead on structural cohesion: a single dimension of the relational component of social solidarity.

Some of the ambiguity surrounding applications of "cohesion" and research on cohesive groups involves differences in scale. Although the theoretical importance of social cohesion is often cast at national levels (Durkheim [1893] 1984; Putnam 2000), most treatments of the relational dimensions of cohesion have focused on small groups. Structural cohesion, however, is no less important at larger scales, although the relational connectivities that might define cohesion cannot be equally dense. An advantage of our concept of structural cohesion is that it applies to groups of any size.<sup>1</sup> In so doing, we add a new dimension to recent literature on large-scale social networks (Barabási and Albert 1999; Newman 2001; Watts 1999) and bridge insights about small-group structure to those at much larger scales.

Identifying cohesive structures is only one part of analyzing structural cohesion, and a more informative approach simultaneously tells us how such groups relate to one another. Our concept of structural cohesion necessarily entails a positional analysis of the resulting groups with respect to their nesting within the population at large. Theoretically, the resulting concept of nestedness captures one dimension of Granovetter's (1985, 1992) concept of social embeddedness. Like "solidarity," "embeddedness" is a multidimensional construct relating generally to the importance of social networks for action. Embeddedness indicates that actors who are integrated in dense clusters or multiplex relations of social networks face different sets of resources and constraints than those who are not embedded in such networks. By specifying an exact structural indicator for one dimension of social embeddedness, we move beyond orienting statements and augment our ability to develop cumulative scientific insights.

Here, we identify an important feature of the relational dimension of social solidarity that is applicable to groups of any size. Following Simmel, that feature is the extent to which a group depends on particular individuals to retain its character as a group. The relevant quantitative measure is the minimum number of individuals whose continued presence is required to retain the group's connectedness (for graph theoretical aspects, see D. White and Harary 2001). For clarity and theoretical consistency, we refer to this relational aspect of social solidarity as *structural cohesion*. Structural cohesion simultaneously defines a *group* property characterizing the collectivity, a *positional* property that situates subgroups relative to each other in a population, and *individual* membership properties. Although we do not claim to capture the full range of either "solidarity" or "embeddedness," structural cohesion provides an exact analytic operationalization of a dimension of each.

## DEFINING STRUCTURAL COHESION

Research on social cohesion has been plagued with contradictory, vague and difficult-to-operationalize definitions. Mizruchi (1992, chap. 3) provides a useful discussion of the conflation of "shared normative sentiments" and "objective characteristics of the social structure" in definitions of cohesion (also see Doreian and Fararo 1998; Mudrack 1989). Many of these definitions share only an intuitive core that rests on how well a group is "held together." What does it mean, for example, that cohesion is defined as a "field of forces that act on members to remain in the group" (Festinger, Schachter, and Back 1950) or "the resistance of a group to disruptive forces" (Gross and Martin 1952)? Dictionary definitions of cohesion rest on similar ambiguities, such as "[t]he action or condition of cohering; cleaving or sticking together" (Oxford English Dictionary 2000). Although we might all agree that cohesive groups should display "connectedness" (O'Reilly and Roberts 1977), what aspects of connectedness should be taken into account?

For concepts of cohesion to be analytically useful, we must differentiate the *relational* togetherness of a group from the *sense* of togetherness that people express. Using only subjective factors, such as a "sense of weness" (Owen 1985) or "attraction-to-group" (Libo 1953), fails to capture the collective nature of a cohesive *group* (Mudrack 1989). Conversely, many treatments that focus exclusively on groups, such as the group's ability to "attract and retain members," com-

<sup>&</sup>lt;sup>1</sup>Because of the long history of small face-toface research on groups, we prefer to avoid the use of the term "groups" altogether, in favor of broader terms such as "collectivity" or "substructure" that carry much less theoretical baggage. Such a substitution, however, results in decidedly awkward writing. We thus maintain the use of "group," but remind readers that our concept is not limited to the small face-to-face primary group structures commonly referred to by the term.

mingle the relational and ideational components of social solidarity. Conflating relational and ideational features of social solidarity in a single measure limits our ability to ask questions about how the relational component of solidarity affects, or is affected by, ideational factors.

The ability to directly operationalize structural cohesion through social relations is one of the primary strengths of a relational conception of social cohesion. The "forces" and "bonds" that hold the group together are the observed relations among members, and cohesion is an emergent property of the relational pattern.<sup>2</sup>

Based on this prior literature, a preliminary intuitive definition of structural cohesion might read:

Definition 1: A collectivity is structurally cohesive to the extent that the social relations of its members hold it together.

While we will sharpen the terms of definition 1 below, there are five important features of this preliminary definition. (1) It focuses on what appears to be constant in previous definitions of cohesion: A property describing how a collection of actors is united. (2) It is expressed as a group-level property. Individuals may be embedded more or less strongly within a cohesive group, but the group has a unique level of cohesion. (3) This concept is continuous. Some groups will be weakly cohesive (not held together well), while others will be strongly cohesive. (4) Structural cohesion rests on observable social relations among actors. And (5), the definition makes no reference to group size.

What, then, are the relational features that hold collectivities together? Clearly, a collection of individuals with no relations among themselves is not cohesive. If we imagine relations forming among a collection of isolated individuals we might observe a moment when each person in the group is connected to at least one other person in such a way that we could trace a single path from each to the other. Thus, a weak form of structural cohesion starts to emerge as these islands become connected through new relations.<sup>3</sup> This intuition is captured well by Markovsky and Lawler (1994) when they identify "reachability" as an essential feature of relational cohesion. Additionally, as new relations form within this minimally cohesive group, we can trace *multiple paths* through the group. Intuitively, the ability of the group to "hold together" increases with the number of independent ways that group members are linked.

That cohesion seems to increase as we add relations among pairs has prompted many researchers to focus on the volume (or density) of relations within and between groups (Alba 1973; Fershtman 1997; Frank 1995; Richards 1995). There are two problems with using relational volume to capture structural cohesion in a collective. First, consider again our group with one path connecting all members. We can imagine moving a single relation from one pair to another. In so doing, the ability to trace a path between actors may be lost, but the number of relations remains the same. Because volume does not change but reachability does, volume alone cannot account for structural cohesion.

Second, the initial (and weakest) moment of structural cohesion occurs when we can trace only one path from each actor in the network to every other actor in the network. Now imagine that our ability to trace a chain from any one person to another always passes through a single person. This might occur, for example, if all relations revolved around a charismatic leader: Each person might have ties to the leader, and be connected only through the leader to every other member of the group. While connected, such groups are notoriously fragile. As Weber ([1922] 1978:1114) points out, the loss of a charismatic leader will destroy a group whose structure is based on an all-to-one relational pattern. Thus, increasing relational volume but focusing it through a single individual does not necessarily increase the ability of the group to hold together, and in-

<sup>&</sup>lt;sup>2</sup> This concept assumes that the dyadic relation is a positive connection.

<sup>&</sup>lt;sup>3</sup> See Hage and Harary (1996) for a discussion of this process among islands in Oceania. We recognize that social groups can form from the dissolution of past groups; the above discussion is useful only in understanding the character of structural cohesion.

stead makes the network vulnerable to targeted attack.<sup>4</sup>

Markovsky and Lawler (1994; Markovsky 1998) make a related point when they argue that a uniform distribution of ties is needed to prevent a network from splitting into multiple subgroups:

[T]he organization of [cohesive] group ties should be distributed throughout the group in a relatively uniform manner. This implies the absence of any substructures that might be vulnerable, such as via a small number of "cut-points" to calving away from the rest of the structure. (Markovsky 1998:345)

Such vulnerable substructures form when network relations are focused through a small number of actors. If pairs of actors are linked to each other through multiple others, the structure as a whole is less vulnerable to this type of split.

Given the above, we amend our preliminary definition of structural cohesion to make explicit the importance of *multiple independent paths* linking actors together:

Definition 2: A group is structurally cohesive to the extent that multiple independent relational paths among all pairs of members hold it together.

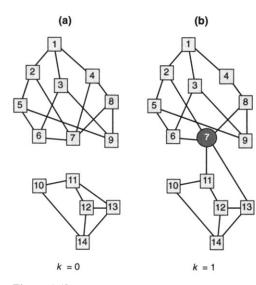
While still preliminary, this new definition provides a metric for structural cohesion that reflects Simmel's ([1908] 1950:135) discussion of the supra-individual status of triads over dyads. In a dyad, the existence of the group rests entirely in the actions of each member, as either member acting unilaterally could destroy the dyad by leaving. Once we have an association of three, however, a connected group remains even if one of the members leaves. In triads, the social unit is not dependent on a single individual, and thus the social unit takes on new uniquely

social characteristics. Groups of any size that depend on connections through a single actor are at one end of definition 2 (weakly cohesive), while those that rest on connections through two actors are stronger, and those depending on connections through many actors are stronger yet. The strongest cohesive groups are those in which every person is directly connected to every other person (cliques), though this level of cohesion is rarely observed except in small primary groups.<sup>5</sup> Intuitively similar ideas motivated earlier measures of social cohesion in networks, such as Seidman and Foster's (1978) treatment of k-plexes (Seidman 1983). They argue that a key feature of cliques that needs to be preserved in any measure of group cohesion "is the robustness of the structure. This property ... is best characterized with reference to the degree to which the structure is vulnerable to the removal of any given individual" (p. 142). The k-plex characterization, however, cannot ensure that this lack of vulnerability is achieved.

To specify our concept of structural cohesion, we need a language capable of accurately expressing relational patterns in a group. The language of graph theory provides this clarity. Because graph-theoretical terms are unfamiliar to many, however, we illustrate the definitions below with reference to Figure 1 following the definition. A network is composed of actors, represented as nodes in the graph, and the relations among them, represented as edges. Structural cohesion depends on how pairs of actors can be linked through chains of relations, or paths. A path in the network is defined as an alternating sequence of distinct nodes and edges, beginning and ending with nodes, in which each edge is incident with its preceding and following nodes (Figure 1a,  $\{1 \rightarrow 2 \rightarrow 5 \rightarrow 6\}$ ). We say that actor *i* can reach actor j if there is a path in the graph starting with *i* and ending with *j* (Figure 1a, 1 can reach 7  $\{1 \rightarrow 3 \rightarrow 6 \rightarrow 7\}$ , but 1 cannot reach 11). Two paths from *i* to *j* are nodeindependent if they have only nodes i and j

<sup>&</sup>lt;sup>4</sup>Recent research on large networks such as the World-Wide Web finds that an extremely small number of nodes are connected to an extremely large number of partners. These networks depend on high-volume actors to remain connected, and targeted interventions (virus attacks in computer networks, education and treatment effects in sexually transmitted disease networks) will disconnect the network and disrupt flow (Barabási and Albert 1999; Pastor-Satorras and Vespignani 2001).

<sup>&</sup>lt;sup>5</sup> It is important that our measure of structural cohesion reaches a maximum with fully connected cliques, linking us to previous concepts of network cohesion.



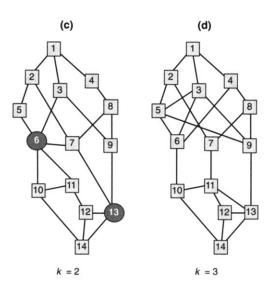


Figure 1. Examples of Connectivity Levels

in common (Figure  $1a, \{1 \rightarrow 3 \rightarrow 6\}$  and  $\{1 \rightarrow 2 \rightarrow 5 \rightarrow 6\}$  are node independent but  $\{1\rightarrow 2\rightarrow 5\rightarrow 6\}$  and  $\{1\rightarrow 2\rightarrow 7\rightarrow 6\}$  are not node independent). If there is a path linking all pairs of actors in the network, then the graph is connected (Figures 1b through 1d). A component of a network consists of all nodes that can be connected to each other by at least one path. Components are the minimum setting for a cohesive structure. A clique is a maximally connected subgraph in which every member is directly connected to every other member (Figure 1a, {12, 13, 14}). A cutset of a graph is a collection of specific nodes that, if removed, would break the component into two or more pieces (Figure 1b, node 7; Figure 1c, nodes 6 and 13). A graph is k-connected (i.e., has node con*nectivity k*) and is called a *k*-component if it has no cutset of fewer than k nodes (Harary 1969:45-46). In graph-theoretic terminology, a 2- or biconnected component is called a bicomponent (Figure 1c) and a 3-connected component a tricomponent (Figure 1d).<sup>6</sup> Any k-component is either a clique or

must have at least two nonadjacent nodes connected by paths, all of which must pass through a cutset of k other nodes (in Figure 1c there are two such paths connecting 3 and 13:  $\{3\rightarrow 6\rightarrow 11\rightarrow 12\rightarrow 13\}$  and  $\{3\rightarrow 9\rightarrow 13\}$ ). What is not so obvious, constituting one of the deepest theorems about graphs, is that a k-connected graph (i.e., having a cutset with exactly k members) also has at least k nodeindependent paths connecting every pair of nodes, and vice versa (see Harary 1969 for Menger's proof).<sup>7</sup>

not the variant usage, that lends itself to discussion of cohesion in terms of k-components.

<sup>7</sup> D. White and Harary (2001) formalize the definition of structural cohesion, review the critiques of alternate measures of cohesive subgroups, and then discuss the relation between connectivity and density. They also examine a second but weaker dimension upon which such groups could be arranged that relates to edge connectivity (also see Borgatti, Everett, and Shirey 1990; Wasserman and Faust 1994), measured by the minimum number of edges that must be removed in a connected group that will result in its disconnection. It can be shown that a graph of any level of edge connectivity may still be separable by removal of a single actor, which means that the unilateral power of actors can be high even when there are many relations connecting people. We differ from D. White and Harary (2001) by generalizing the theoretical links to social solidarity, developing the link between nestedness and embeddedness, providing an algorithm to facilitate empirical research using co-

<sup>&</sup>lt;sup>6</sup>In Harary's terminology, for example, an isolated pair of connected individuals is not 2-connected and do not constitute a bicomponent. The algorithmic and computer science literature constitute a variant usage in which a k-component is any graph that cannot be disconnected by removal of fewer than k nodes, hence a bicomponent, for example, includes an isolated pair of connected individuals. It is Harary's usage and

Based on the intuitive notions captured in Definition 2 and the formal graph properties presented above, we can now provide a final definition of structural cohesion:

Definition 3a: A group's structural cohesion is equal to the minimum number of actors who, if removed from the group, would disconnect the group.

A group is cohesive to the extent that it is robust to disruption, which is captured by node connectivity. For each connectivity value (k) observed in a given network, there is a unique set of subgroups with this level of structural cohesion. Because of the formal equality between the size of the cutset and the number of node-independent paths, the "disconnect" version of Definition 3a can be restated without any loss of meaning in "held together" terms as:

Definition 3b: A group's structural cohesion is equal to the minimum number of independent paths linking each pair of actors in the group.

This pair of equivalent definitions of structural cohesion retains all five aspects of our original intuitive definition of structural cohesion. A collection of actors is united through relational paths that bind nodes together. Node connectivity is a group-level property (a network or k-component as a whole is k-connected), but individuals can be more or less strongly embedded within the group (as the network may admit to nested (k + l)-connected subgroups). This conception scales, ranging from 0 (not connected) to n-1 (a complete clique), and applies to networks of any size.

Structural cohesion is weaker to the degree that connectivity depends on a small number of actors. Such graphs are vulnerable to the activity of fewer and fewer members. As node connectivity increases, vulnerability to unilateral action decreases. Based on Simmel's discussion of the dyad, we argue that a connectivity of 2 (a bicomponent) is the minimum distinction between weak and stronger structurally cohesive groups, which are ranked by their *k*-connectedness. Figure 1 presents examples of networks with differing levels of structural cohesion. Note that in each of these three groups the *number* of relations is held constant, but the edges are arranged such that structural cohesion increases from left to right.

## Cohesive Blocking

An algorithm for identifying structurally cohesive groups is described in Appendix A. Identification involves a recursive process: One first identifies the k-connectivity of an input graph, then removes the k-cutset(s) that hold(s) the network together. One then repeats this procedure on the resulting subgraphs, until no further cutting can be done. As such, any (k+l)-connected set embedded within the network will be identified. Moreover, each iteration of the procedure takes us deeper into the network, as weakly connected nodes are removed first, leaving stronger and stronger connected sets, uncovering the nested structure of cohesion in a network.

This search procedure can result in two types of subgroups. On one hand, we may identify groups that "calve away" from the rest of the population, such as those discussed by Markovsky and Lawler (1994). In such cases, cohesive groups rest "side-byside" in the social structure, one distinct from the other. This is the kind of description commonly used for primary social groups (Cooley 1912), which we expect to exhibit high levels of structural cohesion. Alternatively, structurally cohesive groups could be related like Russian dolls - with increasingly cohesive groups nested inside each other. The most common such example would be a group with a highly cohesive core, surrounded by a somewhat less cohesive periphery, as has been described in widely ranging contexts (Borgatti 1999). A common structural pattern for large systems might be that of hierarchical nesting at low connectivity levels and nonoverlapping groups at high connectivity.

To gain an intuitive sense for the cohesive group detection procedure, consider the example given in Figure 2. This network has a single component inclusive of all nodes. Embedded within this network are two biconnected components: nodes  $\{1-7, 17-$ 23 $\}$  and  $\{7-16\}$ , with node 7 involved in both. Within the first bicomponent, however,

hesive blocking, and expanding the empirical application of these measures to new settings.

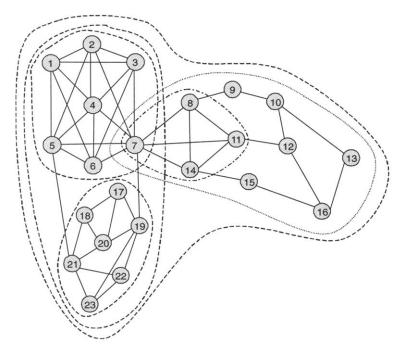


Figure 2. Nested Connectivity Sets

members  $\{1-7\}$  form a 5-component and members  $\{17-23\}$  form a 3-component. Similarly, nodes  $\{7, 8, 11, and 14\}$  form another 3-component (a four-person clique) within the second bicomponent, while the remainder of the group contains no sets more strongly connected than the bicomponent. Thus, the group structure of this network contains a 3-level hierarchy, which is presented in Figure 3.

Because connectivity sets can overlap, group members can belong to multiple groups. Although observed overlaps at high levels of connectivity may be rare, any observed overlaps are likely substantively significant.<sup>8</sup> If an individual belongs to more than one maximal k-cohesive group, that individual is part of a unique subset of k-l or fewer individuals whose removal will disconnect the two groups. Members of such bridging sets are *positionally equivalent* with respect to the larger cohesive sets that they bridge.<sup>9</sup> As such, a *positional* and *relational* structure comes out of the analysis of cohesive groups. These groups are much larger, fewer, and easier to distinguish than are traditional sociological cliques. This procedure provides some of the same theoretical purchase that blockmodels were de-

tionship patterns. Arguments that relative density groups (cf. Frank 1995) solve this problem by assigning each actor to their preferred group (based on number of nominations) fail to account for people who have ties across many subgroups, such that the total number of ties to people in other groups is higher than the number of ties to people in the group they have been assigned to.

<sup>9</sup>In Figure 2, for example, only the removal of node 7 will disconnect the 1-component, removal of  $\{5, 7\}, \{21, 19\}, \{5, 19\}, \{21, 7\}$  will disconnect the bicomponent along similar lines, while removal of only  $\{8, 10\}$  will disconnect 9 from the rest, only  $\{14, 16\}$  will disconnect 15 from the rest, and only  $\{10, 16\}$  will disconnect 13 from the rest, and so forth (see Appendix A).

<sup>&</sup>lt;sup>8</sup> Some researchers consider overlapping subgroups too empirically vexing to provide useful analysis. It is important to point out that (1) kcomponents are strictly limited in the size of such overlaps, making the substantive number of such intermediate positions small—especially compared with cliques; (2) that each such position, because of its known relation to the potential flow paths and cycle structure of the network, can be theoretically articulated in ways that are impossible for clique overlaps; and (3) even when overlaps are empirically difficult to handle, they may well be an accurate description of rela-

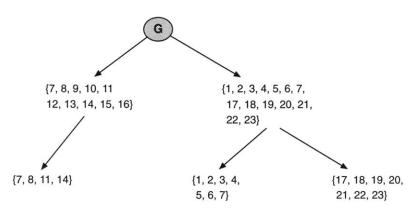


Figure 3. Cohesive Blocking for the Network in Figure 2

signed to provide (Burt 1990; Lorrain and White 1971; H. White, Boorman, and Breiger 1976), but focuses on subgraphs that may overlap rather than on partitions of nodes.<sup>10</sup> Because this method provides the ability to both identify cohesive groups and identify the position of each group in the overall structure, we call the method cohesive blocking. It is important to note the flexibility of this approach. The concept of cohesion presented here provides a way of ordering groups within hierarchically nested trees, with traditional segmented groups occupying separate branches of the cohesion structure (recall Figure 3), but allowing overlap between groups in different branches (e.g., node 7 in Figure 3). The ability to accommodate both nested and segmented structures within a common frame is a strength of our model.

#### RELATION OF COHESION TO SOCIAL EMBEDDEDNESS

A nested concept of cohesion provides a direct link between structural cohesion and an element of social embeddedness (Granovetter 1985, 1992). The general concept of embeddedness has had a significant influence in current sociological research and theory. Although used most often in economic sociology (Baum and Oliver 1992; Portes and Sensenbrenner 1993; Uzzi 1996, 1999) or stratification (Brinton 1988), embeddedness has been used to describe social support (Pescosolido 1992), processes in health and health policy (Healy 1999; Ruef 1999), family demography (Astone et al. 1999), and the analysis of criminal networks (Baker and Faulkner 1993; McCarthy, Hagen, and Cohen. 1998). Most treatments of embeddedness refer to the constraining effects of social relations, contrasting "armslength relations" (Uzzi 1996, 1999) or "atomized individuals" (Granovetter 1985, 1992) to action that is embedded in social relations.<sup>11</sup> Embeddedness is often used to claim a broad orientation to theories of social action, delineating a space for action between "undersocialized" perspectives that treat actors as completely independent utility maximizers and "oversocialized" perspectives that treat actors as cultural dupes.

<sup>&</sup>lt;sup>10</sup> Overlaps are crucial to cohesive structures. Ordinarily we think of social groups as designations for sets of individuals. Structural cohesion identifies groups in terms of sets of relationships, as represented by edges in the graph. For example, bicomponents may result in a partition of edges (not individuals) allowing people to be in multiple cohesive groups. It is for this reason that cohesive blocking cannot generally be subsumed as a form of blockmodeling: Cohesive blocks may overlap and do not form partitions.

<sup>&</sup>lt;sup>11</sup> "Embeddedness" suffers from some of the same ambiguity evident in discussions of solidarity and cohesion, with the term being used to denote many different aspects of the importance of relationships for social action. While some researchers have keyed their definition of embeddedness directly to a given type of action outcome, we prefer a concept of embeddedness that lets us *test* whether a particular pattern of relations shapes action, instead of defining embeddedness *in terms of* the resulting constraint.

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To move from orienting statement to empirical investigation, we must identify clear dimensions of embeddedness that would admit to empirical operationalization. Granovetter (1992) points to a key division between "local" and "structural" embeddedness:

"Embeddedness" refers to the fact that economic action and outcomes, like all social action and outcomes, are affected by actors' dyadic (pairwise) relations *and* by the structure of the overall network of relations. As a shorthand, I will refer to these as the relational and the structural aspects of embeddedness. The structural aspect is especially crucial to keep in mind because it is easy to slip into "dyadic atomization," a type of reductionism. (P. 33, italics in original)

Granovetter (1992) further specifies his understanding of structural embeddedness as the degree to which actors are involved in cohesive groups:

[T]o the extent that a dyad's mutual contacts are connected to one another, there is more efficient information spread about what members of the pair are doing, and thus better ability to shape behavior. Such cohesive groups are better not only at spreading information, but also at generating normative, symbolic, and cultural structures that affect our behavior." (P. 35)

Granovetter's concept invokes transitivity (Davis 1963; Holland and Leinhardt 1971; Watts 1999), focusing on the pattern of relations among a focal actor's contacts. One need not limit structural embeddedness to an actor's direct neighborhood, however, but can extend the notion of embeddedness in a cohesive group to the wider social network (Frank and Yasumoto 1998). The concept of k-connected groups provides a clear operationalization of a structural aspect of embeddedness through the degree to which actors' partners (or their partners' partners) are connected to one another through multiple independent paths. As such, because cohesive groups are nested within one another, then each successive k-connected set is more deeply embedded within the network. This deep connectivity nicely captures the intuitive sense of being involved in relations that are, in direct contrast to "armslength" relations, structurally embedded in a social network (Uzzi 1996). As such, one aspect of structural embeddedness—the depth of involvement in a cohesive structure—is captured by this nesting. We define an actor's nestedness in a social network as *the deepest cutset level within which the actor resides.*<sup>12</sup>

#### ALTERNATIVE APPROACHES TO STRUCTURAL COHESION

Node connectivity differs markedly from other approaches to identifying cohesive groups in social networks.<sup>13</sup> While previous work on structural cohesion was motivated by questions of graph vulnerability (Seidman 1983; Seidman and Foster 1978), the resulting empirical measures could not ensure a nonvulnerable graph. Group identification methods based on number of interaction partners (k-plexes, k-cores), minimum withingroup distance (N-cliques), or relative ingroup density, may be structurally cohesive, but are not necessarily so. In every case, the method used to identify groups cannot distinguish multiconnected groups from those vulnerable to the removal of a single actor. As such, any empirical application of these methods to a theoretical problem of structural cohesion risks ambiguous findings. By distinguishing structural cohesion from factors such as density or distance, we can isolate the relative importance of connectivity in social relations from these other factors. Distance between members, the number of common ties, and so forth might affect outcomes of interest, but our ability to extend social theory in formal network terms depends on our ability to unambiguously attribute social mechanisms to network features. Connectivity provides researchers with the ability to disentangle the effects of structural cohesion from other network features.

Using connectivity to capture a key dimension of social cohesion is not a new idea,

<sup>&</sup>lt;sup>12</sup> Similarly, an actor's nestedness in a cohesive group is defined as *the deepest cutset level within that group in which the actor resides.* 

<sup>&</sup>lt;sup>13</sup> D. White and Harary (2001) compare node connectivity approaches to many others, using data on Zachary's Karate Club as an exemplar. A detailed comparison of each alternative method for measuring cohesion that expands on those D. White and Harary (2001), with multiple examples, is available from the authors on request.

though most previous approaches have focused on edge connectivity (Borgatti, Everett, and Shirey 1990; Wasserman and Faust 1994). D. White and Harary (2001) discuss the formal links between node and edge connectivity in detail. Briefly, a graph has edge connectivity k if it has no cutset of k-1 edges, and, by Menger's Theorem, there are k edge-independent paths (as opposed to node-independent paths) connecting every pair of nodes in the graph. Although the two concepts might seem intuitively similar, they can result in radically different assessments of group cohesion. Consider as an example Figure 1b, which is 2-edge-connected. By simply adding ties from node 7, one could increase edge connectivity dramatically, but the graph as a whole would still depend entirely on node 7 to remain connected. As we discuss in more detail below, this kind of dominating central node would increase power inequality in the network and likely highlight divisions within the network.

These substantive weaknesses in the edge connectivity notion may explain why so few people have used it empirically, or have found significant results with this method. Given the formal similarity between node and edge connectivity, why hasn't node connectivity been used before now? Although many reasons are possible, including the discipline's general focus on small primary groups, the technical ability to identify highconnectivity sets may be largely responsible for its lack of use. Harary et al. (1965:25) were the first to propose node connectivity as a measure of cohesion. A fast algorithm to identify tricomponents was developed by computer scientists in 1973 (Hopcroft and Tarjan 1973), though it was never implemented by social scientists, and the ability to identify bicomponents and pairwise node connectivity is only now implemented in the most popular network software (Borgatti et al. 2002). The ability to identify the full connectivity of a graph as well as all cutsets is a recent phenomenon, however. The algorithm presented in Appendix A combines all the necessary elements for a full cohesive blocking, and in addition, provides a measure of structurally cohesive embedding. Thus, while the graph-theoretic ideas surrounding our approach to structural cohesion were introduced in the literature more than 35 years ago, the ability to empirically employ these ideas has only recently become available.

Given the historical focus on small groups, is it reasonable to argue for "cohesion" in aggregates of many thousands of nodes? One might argue, for example, that a single loop connecting 1,000 people is not very cohesive. Why and when would such a graph be considered cohesive? The answer, as Markovsky and Lawler (1994) suggest, depends on the implicit comparison network. Clearly, the substantive social character of a 10-person group differs from that of a 1,000person group. Comparison with a small primary group will always give the impression, if it has high density, that a large group with lower density is less cohesive. We argue, however, that this is the wrong comparison to make-that it conflates analytically distinct dimensions of social structure, such as density or mean path distance, and the number of independent connections. Holding the number of nodes and the density of a network constant, the effect of greater node connectivity is always to increase social cohesion. Structural cohesion unites networks, independent of other factors such as size, with "independence" having the same meaning implied by most statistical models. Thus, the correct comparison to make for a 1,000person bicomponent is with a 1,000-person spanning tree (less cohesive) or 1,000 people divided into 250 four-person groups (less cohesive yet). Other implicit comparisons, of course, are various baseline models of randomness. In a network of 1 million nodes and 2 million edges, bicomponents in the range of 1,000 persons will be common, while a clique of 10 is an extremely rare event in a random network of 30 nodes and 50 edges. For a structurally cohesive group to be substantively significant within a network,<sup>14</sup> whatever its number of nodes and

<sup>&</sup>lt;sup>14</sup> We do not take up here the evaluation of statistical significance. D. White and Harary (2001) show that adding conditional density to node connectivity provides a continuous measure of connectivity that allows for comparability across networks of different sizes. An alternative approach to standardizing structural cohesion involves developing implicit comparison networks (also see Markovsky and Lawler 1994), such as a

number of edges, it must stand out against the background of a relevant baseline model of randomness.

How does nestedness relate to other common network measures? To the extent that nestedness captures the general location of actors and differentiates prominent actors, an actor's nestedness might be thought of as a type of centrality (Freeman 1977; Harary et al. 1965; Wasserman and Faust 1994). However, depth in the network is a group-level property that distinguishes it from centrality measures. Second, because connectivity is related to degree (each member of a k-component must have at least k ties), nestedness is necessarily correlated with degree. As we show in the empirical examples below, however, nestedness is not equivalent to any of these measures, either singly or in combination, and measures something very different.

Although there are multiple dimensions upon which to compare a node connectivity concept of cohesion to alternatives, the real test of the idea is whether it adds anything substantive to our understanding of empirical cases or gives rise to new theoretical hypotheses about social structure. Below we demonstrate the empirical value-added of our hierarchical concept of the relational component of solidarity in two radically different settings and then discuss further theoretical implications of structural cohesion.

#### TWO EMPIRICAL EXAMPLES: HIGH SCHOOLS AND INTERLOCKING DIRECTORATES

To demonstrate the empirical relevance of cohesive blocking, we use data from two different types of networks. First, we use data on friendships among high school students taken from the National Longitudinal Study of Adolescent Health (Add Health).<sup>15</sup> This

random network with similar degree distribution or transitivity levels, which might be determined mathematically in simple cases (Newman, Strogatz, and Watts 2001), or through Monte Carlo simulations as conditioning becomes more complex.

<sup>15</sup> This research is based on data from the Add Health project, a program project designed by J. Richard Udry (PI) and Peter Bearman, and funded by grant P01-HD31921 from the National Institute of Child Health and Human Developexample illustrates how cohesive groups can be identified in large settings based on friendship, one of the most commonly studied network relations. The second example uses data on the interlocking directorate networks of 57 large firms in the United States (Mizruchi 1992). Because business solidarity has been an important topic of research on interlocks, we apply our method to this network and show how structural cohesion relates to similar political activity behaviors. Of course, there is not space here to treat the subtle theoretical issues surrounding each of these substantive areas. Instead, the analyses below are designed to highlight how structural cohesion can add to empirical research in widely differing research settings.

## STRUCTURAL COHESION IN ADOLESCENT FRIENDSHIP NETWORKS

Add Health is a school-based study of adolescents in grades 7 through 12. A stratified nationally representative sample of all public and private high schools (defined as schools with an  $11^{\text{th}}$  grade) in the United States with a minimum enrollment of 30 students was drawn from the Quality Education Database (QED) in April 1994 (Bearman et al. 1996). Network data were collected by providing each student with a copy of the roster of all students for their school. Students identified up to 5 male and 5 female (10 total) friends from this roster.<sup>16</sup> Here we

ment to the Carolina Population Center, University of North Carolina at Chapel Hill, with cooperative funding from 17 other agencies. Persons interested in obtaining data files from the National Longitudinal Study of Adolescent Health should contact Add Health, Carolina Population Center, 123 West Franklin Street, Chapel Hill, NC 27516-2524 (addhealth@unc.edu).

<sup>16</sup> The maximum number of nominations allowed was 10, but this restriction affected few students. For all students, mean out-degree is 4.15 with a standard deviation of 3.02, 3 percent of students nominated 10 in-school friends, 23 percent nominated 5 in-school male friends, and 25 percent named 5 in-school female friends. Previous research suggests that close friendship groups have about 5 members (Cotterell 1996; Dunphy 1963). Relations are likely to fall within gender. For the Add Health data as a whole (net of other dyad attributes such as race, joint activities, and transitivity) same-sex ties are about 1.6

use data on more than 4,000 students taken from a dozen schools with between 200 and 500 students (mean = 349), providing a diverse collection of public (83 percent) and private schools from across the United States.<sup>17</sup>

NESTEDNESS AND SCHOOL ATTACH-**MENT.** For each school, we employed the cohesive blocking procedure described in Appendix A to identify all connectivity sets for each school friendship network. At the first level, we have the entire graph, which is usually unconnected (because of the presence of a small number of isolates). Most of the students in every school are contained within the largest bicomponent, and often within the largest tricomponent. As the procedure continues, smaller and more tightly connected groups are identified. In these data, at high levels of connectivity (k > 5), subgroups do not overlap. This implies settings with multiple cores, differentially embedded in the overall school networks. When no further cuts can be made within a group, we have reached the end of the nesting structure for that set of nodes. The level at which this cutting ceases describes the nestedness for each member of that group.

Nestedness within the community should be reflected in a student's perception of his or her place in the school. The Add Health in-school survey asks students to report on how much they like their school, how close they feel to others in the school and how much they feel a part of the school.<sup>18</sup> Here,

times more likely than cross-sex ties (Moody 2001:712). For purposes of identifying connectivity sets, we treat the graph as undirected. The algorithms needed for identifying connectivity can be modified to handle asymmetric ties. It was for directed graphs that Harary et al. (1965) developed their concept of cohesion as connectivity, although they offered no computational algorithms.

<sup>17</sup> This sample represents all schools in the data set of this size. The selected size provides a nice balance between computational complexity and social complexity, as the schools are large enough to be socially differentiated and small enough for group identification to be carried out in a reasonable amount of computer time.

<sup>18</sup> These are three items from the Perceived Cohesion Scale (Bollen and Hoyle 1990), and have a Chronbach's alpha reliability of .82. The confirmatory factor loadings for the three variwe use the mean score of the three items as a measure of school attachment. Building on Markovsky and Lawler's (1994) discussion of solidarity and cohesion, there ought to be a significant positive relation between nestedness in the network and school attachment, net of any other factor that might be associated with school attachment (Markovsky 1998; Markovsky and Chaffee 1995; Markovsky and Lawler 1994; Paxton and Moody forthcoming).

We control for other variables that might affect a student's school attachment. Because gender differences in school performance and school climate are well known (Stockard and Mayberry 1992), we expect female students to have lower school attachment than males. As students age we expect the school to become a less salient focus of their activities, and grade in school is also controlled.<sup>19</sup> Students who perform well in school or who are involved in many extracurricular activities should feel more comfortable in schools. Because students from small schools might feel more attached than students from large schools, we test a school-level effect of size on mean school attachment.

A significant feature of our approach is that we can differentiate the unique effects of network features that are often used to measure "cohesion" in empirical work. First, the number of contacts a person has (degree centrality) reflects their level of involvement in the network. Substantively, we expect that those people with many friends in school are more likely to feel an integrated part of the school. Second, having friends who are all friends with one another is an important feature of network involvement. As such, the density of one's personal (local) network is tested. Third, we expect that those people who are most central in the network should have a greater sense of school attachment. Hence, betweenness centrality is tested. Fi-

ables are all positive, significant, and close in magnitude. For similar work, see Resnick et al. (1997). The other three items used for Bollen and Hoyle's scale were not included in the Add Health school survey.

<sup>19</sup> Because school friendships tend to form within grade, controlling for grade in school captures an important focal feature of the in-school network. nally, it may be the case that the lived community of interest for any student is that set of students with whom they interact most often. We used NEGOPY (Richards 1995) to identify density-based interaction groups within the school, and use the relative group density to measure this effect.<sup>20</sup> If our structural aspect of nestedness captures a unique dimension of network embeddedness, as our discussion above implies, then, controlling for each of these features, we would expect to find an independent effect of structurally cohesive nestedness on school attachment.<sup>21</sup>

Table 1 presents HLM coefficients for models predicting school attachment from nestedness level, school activity, demographic, and other network factors. Model 1 is a baseline model containing only attribute and school variables. As expected, females and students in higher grades tend to have lower school attachment, while students who are involved in many extracurricular activities or who get good grades feel more attached to the school. The coefficient for school size, while negative, is not statistically significant. In Model 2, our measure of network nestedness is added to the model.<sup>22</sup> We see that there is a strong positive relation between nestedness and school attachment. (Note that the size of the standardized coefficient for nestedness is the largest in this model.) Testing the difference in the deviance scores between Model 1 and Model 2 suggests that including nestedness improves the fit of the overall model. In Models 3

<sup>21</sup> We thank an anonymous reviewer for suggesting a 2-level hierarchical linear model to test for these relations, with students nested in schools. The model was specified to allow coefficients to vary randomly across schools, with the school-level intercept (substantively, mean school attachment) regressed on school size.

 $^{22}$  In addition to the nestedness level, we also tested a model using the largest *k*-connectivity value for each student. The results are very similar. Students involved in high-cohesion groups had higher levels of school attachment than others.

through 6, we test the specification including our measure and each of the four alternative network measures. In each case, nestedness remains positive, significant, and strong, while inclusion of the alternative measures adds little explanatory power (as seen by testing against Model 2). In Model 7, we include all potentially confounding network variables, and the relation between nestedness and attachment remains. The largest change in the coefficient for nestedness comes with the addition of degree, which is likely due to collinearity, because every member of a k-component must have degree  $\geq k$ .

These findings suggest that individuals are differentially attached to the school as a whole, and thus the school is differentially united, through structural cohesion. This finding holds net of school-level differences in school attachment, the number of friends people have, the interaction densities among their immediate friends or of their larger density-based interaction group, and their betweenness centrality level. That these other factors do not continue to contribute additionally to school attachment implies a unique effect of structural cohesion that other methods would wrongly have attributed to the other measures of network structure.

COHESION AMONG LARGE AMERICAN BUSINESSES. A long-standing research tradition has focused on the interlocking directorates of large firms (Mizruchi 1982, 1992; Palmer, Friedlan, and Singh 1986; Roy 1983; Roy and Bonacich 1988; Useem 1984). An important question in this literature, "at the core of the debate over the extent to which American society is democratic" (Mizruchi 1992:32), is to what extent business in the United States is unified and, if so, whether it is collusive. If businesses collude in the political sphere, then democracy is threatened. Yet much of the literature has been vague in defining exactly what constitutes business unity, and thus empirical determination of the extent and effect of business unity (and possible collusion) is hard to identify.

Without treading on the issue of collusion per se, we approach the question of business unity as a problem of structural cohesion. Because structural cohesion facilitates the

<sup>&</sup>lt;sup>20</sup> We thank an anonymous reviewer for suggesting this specification, which uses fewer degrees of freedom than an alternative test that uses a dummy variable for each identified subgroup in the school. Tables with the alternative specification are available on request and show no substantive difference in the nestedness effect.

Independent Variab	le Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Intercept	3.850 <sup>***</sup> (.285)	3.429*** (.332)	3.361 <sup>***</sup> (.327)	3.467*** (.328)	3.354 <sup>***</sup> (.325)	3.322*** (.327)	3.379 <sup>***</sup> (.062)
School size <sup>a</sup>	021 (.063) [015]	050 (.068) [037]	038 (.067) [027]	064 (.068) [045]	043 (.070) [030]	020 (.066) [015]	058 (.062) [041]
Female	189 <sup>***</sup> (.04) [100]	148 <sup>****</sup> (.001) [075]	155** (.041) [078]	149* (.042) [076]	153** (.042) [078]	149** (.043) [076]	161 <sup>**</sup> (.044) [082]
Grade in school	077 <sup>***</sup> (.018) [097]	048* (.022) [059]	048* (.021) [060]	079* (.021) [060]	050* (.021) [062]	046 (.022) [059]	051* (.022) [063]
Grade-point average	.132*** (.025) [.102]	.099**** (.022) [.077]	.095** (.023) [.074]	.102** (.022) [.079]	.104*** (.022) [.081]	.099** (.023) [.077]	.099** (.023) [.077]
Extracurricular activities	.115**** (.016) [.229]	.076 <sup>***</sup> (.014) [.152]	.078 <sup>***</sup> (.014) [.156]	.076 <sup>***</sup> (.014) [.152]	.075 <sup>***</sup> (.014) [.149]	.076 <sup>***</sup> (.014) [.151]	.076 <sup>***</sup> (.015) [.152]
Nestedness	_	.016 <sup>****</sup> (.001) [.279]	.016*** (.001) [.277]	.016**** (.001) [.268]	.011**** (.002) [.201]	.017*** (.001) [.283]	.011** (.003) [.181]
Local density	_	_	.002 (.001) [.038]	_	_	_	.002 (.001) [.046]
Betweenness centrality	_	—	—	.030 (.027) [.023]	—	_	021 (.043) [016]
Number of friends (degree)	_	—	—	—	.021* (.010) [.087]	_	.029 (.014) [.118]
Relative density groups	_	—	—	—	—	.001 (.002) [.006]	001 (.002) [014]
Deviance <sup>b</sup>	9,842.31	9,577.57***	9,578.29	9,581.13	9,571.06	9,587.60	9,574.31

 Table 1. Coefficients Predicting School Attachment from Network Embeddedness and Other Independent Variables: Add Health, 1994

*Note:* Numbers in parentheses are standard errors; numbers in brackets are standardized coefficients. <sup>a</sup> School-level coefficient.

<sup>b</sup> Number of cases for level 1 = 3,606; number of cases for level 2 = 12.

\* $p \le .05$  \*\* $p \le .01$  \*\*\* $p \le .001$  (two-tailed tests)

flow of information and influence, coordinated action, and thus political activity, ought to be more similar among pairs of firms that are similarly embedded in a structurally cohesive group. Mizruchi (1992) makes this argument well and highlights the importance of financial institutions for unifying business activity. He identifies the number of indirect interlocks between two firms as "the number of banks and insurance companies that have direct interlocks with both manufacturing firms in the dyad" (p. 108). Using data on large manufacturing firms, we identify the cohesive group structure based on indirect interlocks and relate this structure to similarities in political action.

The sample Mizruchi constructed consists of 57 of the largest manufacturing firms drawn from "the twenty major manufacturing industries in the U.S. Census Bureau's Standard Industrial Classification Scheme" in 1980 (Mizruchi 1992:91). In addition to data on directorship structure, he collected data on industry, common stockholding,

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governmental regulations, and political activity. The question of interest is whether the structure of relations among firms affects the similarity of their behavior. To explore whether firms that are similarly embedded also make similar political contributions, Mizruchi constructed a dyad-level political contribution similarity score as a function of the number of common campaign contributions.<sup>23</sup> He modeled this pair-level similarity as a function of geographic proximity, industry, financial interdependence, government regulations, and interlock structure.

A cohesive blocking of this network reveals that most firms are involved in a strongly cohesive group, with 51 of the 57 firms members of the largest bicomponent. The nestedness structure consists of a single hierarchy that is 19 layers deep, and at the lowest level (at which no further minimum cuts can be made that would not isolate all nodes), 28 firms are members of a 14-connected component (the strongest k-component in the graph).

Does joint membership in more deeply nested subsets lead to greater similarity in political contributions? To answer this question, we add an indicator for the deepest layer within which both firms in a dyad are nested. Thus, if firm i is a member of the second layer but not the third, and firm j is a member of the fourth layer but not the fifth, the dyad is coded as being nested in the second layer. As with the school example, we control for other network features. Table 2 presents the results of this model.

Model 1 replicates the analysis presented in Mizruchi (1992), and in the remaining models we add additional network indicators.<sup>24</sup> In the baseline model, we find that the

<sup>23</sup> The score is calculated as

$$S_{ij} = \frac{n_{ij}}{\sqrt{n_i n_j}},$$

where  $S_{ij}$  = the similarity score,  $n_{ij}$  equals the number of common campaign contributions, and  $n_i$  and  $n_j$  equal the number of contributions firm *i* and *j* make, respectively. The dyad-level analysis is based on 1,596 firm dyads.

more financial stockholders two firms have in common, the greater the similarity of their political contributions. Additionally, indirect interlocks through financial institutions or jointly receiving defense contracts leads to similarity of political action. In Model 2, we add the nestedness measure.25 Net of the effects identified in Model 1, we find a strong positive impact of cohesion within the indirect interlock network. As in the school networks, we test for the potentially confounding effects of degree and centrality.<sup>26</sup> No effect of network degree is evident, but betweenness centrality does evidence a moderate association with political similarity. When both variables are entered into the model, the statistical significance of nestedness drops slightly, but the magnitude of the effect remains constant. Based on the standardized coefficients, nestedness exhibits the strongest effect in each of the Models 2 through 5. The more deeply nested a given dyad is in the overall network structure, the more similar their political contributions. The nestedness measure of structural cohesion is a significant predictor of political similarity, in addition to the effect of direct adjacency created through financial interlocks.

Mizruchi (1992) identifies two potential explanations for the importance of financial interlocking on political behavior. Following Mintz and Schwartz (1985), banks and financial institutions may exercise control of firms by seating representatives on their boards. As such, two firms that share many such financial ties face many of the same influencing pressures and therefore behave similarly. A second argument, building on the debate surrounding structural equivalence and cohesion (Burt 1978, 1982), is that actors in similar network positions (i.e., with similar patterns of ties to similar third parties) ought to behave similarly. As in our argument for structural cohesion, Friedkin (1984) argues that influence travels through

<sup>&</sup>lt;sup>24</sup> Following Mizruchi (1992:121) we use the nonparametric quadratic assignment procedure (QAP) to assess the significance level of the regression coefficients. See Mizruchi (1992) for measurement details.

 $<sup>^{25}</sup>$  If instead of the joint nestedness level, we use the connectivity level (k) for the highest k both members are involved in, we find substantively similar results.

<sup>&</sup>lt;sup>26</sup> We cannot test for density-based subgroup effects because NEGOPY assigns all members to the same group. This is a result of the high average degree within this network.

Variable Description	Model 1	Model 2	Model 3	Model 4	Model 5
Headquarters located in same state	.017	.013	.013	.015	.015
	[.043]	[.032]	[.034]	[.039]	[.039]
Same primary two-digit industry	.012	.017	.017	.016	.016
	[.024]	[.034]	[.036]	[.034]	[.034]
Number of common two-digit industries	004	007	007	003	003
	[008]	[015]	[015]	[006]	[006]
Interdependence based on transactions and concentration	.011 <sup>a</sup>	.009 <sup>a</sup>	.009 ª	.009 <sup>a</sup>	.009 ª
	[.098]	[.080]	[.082]	[.083]	[.083]
Financial institutions that hold stock in both firms	.034*	.029*	.028*	.029*	.029*
	[.213]	[.182]	[.174]	[.181]	[.181]
Board of directors overlaps between firms	.021 ª	.016	.017 ª	.018ª	.018ª
	[.047]	[.036]	[.037]	[.041]	[.041]
Primary membership in regulated industry	.036 ª	.034 ª	.032	.030	.030
	[.115]	[.107]	[.102]	[.096]	[.096]
Common recipient of defense contracts	.084**	.083 <sup>**</sup>	.082*	.082*	.082
	[.170]	[.166]	[.165]	[.166]	[.165]
Financial institutions with which firms interlock	.026 <sup>**</sup>	.010**	.009	.007	.007
	[.178]	[.070]	[.060]	[.050]	[.051]
Level of embeddedness in the indirect interlock network	_	.004* [.201]	.005* [.257]	.004* [.203]	.004 ª [.202]
Absolute difference in degree	—	—	.001 [.087]	—	00 [00]
Absolute difference in between- ness centrality	_	—	_	.010ª [.111]	.010 [.111]
	.171**	.156**	.137**	.144**	.144**
	.195	.217	.222	.229	.228
	Headquarters located in same state Same primary two-digit industry Number of common two-digit industries Interdependence based on transactions and concentration Financial institutions that hold stock in both firms Board of directors overlaps between firms Primary membership in regulated industry Common recipient of defense contracts Financial institutions with which firms interlock Level of embeddedness in the indirect interlock network Absolute difference in between-	Headquarters located in same state.017 [.043]Same primary two-digit industry.012 [.024]Number of common two-digit industries004 [008]Interdependence based on transactions and concentration.011 a [.098]Financial institutions that hold stock in both firms.034* [.213]Board of directors overlaps between firms.021 a [.047]Primary membership in regulated industry.036 a [.115]Common recipient of defense contracts.084** [.170]Financial institutions with which firms interlock.026** [.178]Level of embeddedness in the indirect interlock network-Absolute difference in degree-Absolute difference in between- ness centrality.171**	Headquarters located in same state.017.013 [.032]Same primary two-digit industry.012.017 [.024][.034]Number of common two-digit industries $004$ $007$ [.008][ $015$ ]Interdependence based on transactions and concentration $0.011^{a}$ $.009^{a}$ [.080]Financial institutions that hold stock in both firms $0.34^{*}$ $.029^{*}$ [.213]Board of directors overlaps contracts $.021^{a}$ $.016$ [.047]Primary membership in common recipient of defense contracts $.084^{**}$ $.083^{**}$ [.170]Financial institutions with which firms interlock $.026^{**}$ $.010^{**}$ (.178]Headquarter (.178] $.004^{*}$ $.026^{**}$ Absolute difference in degree $ -$ Absolute difference in between- ness centrality $.171^{**}$ $.156^{**}$	Headquarters located in same state.017.013.013Same primary two-digit industry.012.017.017Interdependence based on transactions and concentration.011 a.009 a.009 aFinancial institutions that hold stock in both firms.021 a.016.017 aBoard of directors overlaps contracts.021 a.016.017 aPrimary membership in regulated industry.036 a.034 a.032Financial institutions with oth firms interlock.036 a.034 a.032Common recipient of defense contracts.084**.083**.082*Financial institutions with oth firms interlock.026**.010**.009Level of embeddedness in the indirect interlock network.004*.005*.001Level of embeddedness in the indirect interlock network.001*.001.087]Absolute difference in degree001.087]Absolute difference in between- ness centrality.171**.156**.137**	Headquarters located in same state.017.013.013.013.015Same primary two-digit industry industries.012.017.017.016[.034][.039]Same primary two-digit industry industries.012.017.017.016[.034][.036][.034]Number of common two-digit industries $004$ $007$ $007$ $003$ [.0.034]Interdependence based on transactions and concentration $.011^{a}$ .009 a.009 a.009 aInterdependence based on transactions and concentration $[.098]$ $[.080]$ $[.082]$ $[.083]$ Financial institutions that hold stock in both firms $.034^{*}$ $.029^{*}$ $.029^{*}$ Board of directors overlaps between firms $.021^{a}$ .016 $.017^{a}$ $.018^{a}$ Primary membership in regulated industry $.036^{a}$ $.034^{a}$ $.032$ .030Common recipient of defense contracts $.084^{**}$ $.083^{**}$ $.082^{*}$ $.082^{*}$ Level of embeddedness in the indirect interlock $004^{*}$ $.005^{*}$ $.004^{*}$ Level of embeddedness in the indirect interlock network $004^{*}$ $.005^{*}$ $.004^{*}$ Absolute difference in degree $$ $$ $.001^{a}$ $.114^{**}$ $.171^{**}$ $.156^{**}$ $.137^{**}$ $.144^{**}$

 
 Table 2. QAP Coefficients Predicting Political Action Similarity from Network Embeddedness and Other Dyad Attributes: 57 U.S. Manufacturing Firms, 1980

Source: Mizruchi (1992).

*Note*: Quadratic Assignment Procedure (QAP) determines significance levels based on permutation tests and does not produce normal standard errors. Numbers in brackets are standardized coefficients.

<sup>a</sup> Coefficient is significant at p < .10. We report this significance level in keeping with Mizruchi [1992]. \* $p \le .05$  \*\* $p \le .01$  (two-tailed tests)

multiple paths and thus has an effect beyond the direct link between two actors. His argument is supported by our finding that the multiple, independent paths that link pairs of structurally cohesive actors help transfer information among firms in a way that is able to coordinate politically similar activity.

# THEORETICAL IMPLICATIONS OF STRUCTURAL COHESION

The above two empirical examples demonstrate the empirical validity of a structural conception of social cohesion. Because we have created a formal specification for structural cohesion, we can link network structure

to actor mechanisms (such as information flow) to derive further theoretical consequences of structural cohesion. A defining property of a k-component (by Menger's Theorem) is that every pair of actors in the collectivity is connected by at least k independent paths. The presence of multiple paths, passing through different actors, implies that if any one actor is removed, alternative links among members remain to maintain social solidarity. Information and resources can flow through multiple paths, making control of resources within the group by a small (< k) number of people difficult. Although many potential implications likely follow in particular substantive areas, we focus below on three broad types of sociological questions: resource and risk flow, community and class formation, and power.

#### **RESOURCE AND RISK FLOW**

A focus on structural cohesion provides new insights into diffusion, augmenting current approaches that focus largely on network distance. The length of a path (number of edges) is often considered critical for the flow of goods through a network, as flow may degrade with relational distance. That is, the probability that a resource flows between two nonadjacent actors is equal to the product of each dyadic transition probability along the path(s) connecting them. When multiplied over long distances, the efficacy of the information diminishes, even if the pairwise transmission probability is high. For example, the probability that a message will arrive intact over a six-step chain<sup>27</sup> when each dyadic transmission probability is .9 will be .53. The fragility of long-distance communication rests on the fact that at any step in the communication chain, one person's failure to pass the information will disrupt the flow.

For a structurally cohesive group, however, expected information degradation decreases with each additional *independent* path in the network. For example, the comparable probability of a six-step communication arriving given two independent paths is .78.<sup>28</sup> As the number of independent paths increases, the likelihood of the information transmission increases. When the flow is not subject to degradation, but only to interruption, increasing connectivity will provide faster and more reliable transmission throughout the network.<sup>29</sup> In a high-connectivity network, even if many people stop transmission (effectively removing them-

<sup>29</sup> Computer viruses are an excellent example of such flows.

selves from the network), alternate paths provide an opportunity for spread.

Nonoverlapping (k+l)-cohesive subgroups within a larger k-connected population have important implications for the long-distance carrying capacity of the network. Local pockets of high connectivity act as amplifying substations for information (or resource, or viral) flow that comes into the more highly connected group, boosting a signal's strength,<sup>30</sup> and sending it back out into the wider population. This pattern directly reflects the core concept of sexually transmitted diseases (Rothenberg et al. 1996), which may account for the high prevalence of many STDs in the face of quite low pair-wise transmission probabilities. The observed patterns typical in smallworld graphs (Milgram 1969; Watts 1999) are a natural result of local relational action nested within a larger network setting. Thus, processes based on the formal properties of connectivity may account for many of the observed substantive features of small-world networks.

Social network researchers have traditionally focused on small, highly connected groups. Identifying connectivity as a central element of cohesion frees us from focusing on small groups by identifying patterns through which influence or information can travel long distances. The rise of electronic communication and distributed information systems suggest that distance will become less salient as information can travel through channels that are robust to degradation. By extending our vision of cohesion from small local groups to large extended relations, we are able to capture essential elements of large-scale social organization that have only been hinted at by previous social network research, providing an empirical tool for understanding realistically sized lived communities.

## COMMUNITY AND CLASS FORMATION

Structural cohesion provides us with a useful tool for understanding processes related

<sup>&</sup>lt;sup>27</sup> This is the purported average acquaintance distance among all people in the United States (Milgram 1969).

<sup>&</sup>lt;sup>28</sup> We calculate this as the product of the dyadic probabilities for each path, minus the probability of transmission through both paths. Thus, for two paths of length d, the formula is  $2(p_{ij})^d - (p_{ij})^{2d}$ , where d is the distance. This is a simplification, as dyadic transmission rates are often variable and highly context-specific.

<sup>&</sup>lt;sup>30</sup> Signal amplification might depend on averaging or combining degraded copies of the same signal or message so as to filter noise, thus increasing reliability.

to the formation of social classes, ethnicity, and social institutions. Although a longstanding promise of network research (Emirbayer 1997; Rapoport and Horvath 1961; H. White et al. 1976), the conceptual tools needed to identify the empirical traces of such processes have been sorely lacking. In contrast, Brudner and White (1997) showed that membership in a structurally cohesive group based on marital and close kinship ties among households in an Austrian farming village was correlated with stratified class membership, defined by single-heir succession to ownership of the productive resources of farmsteads and farmlands.

Linking structurally cohesive subgroup membership to institutions that provide formal access to power suggests a new approach to the study of social stratification and the state. D. White et al. (2002), for example, identify an informally organized "invisible state" created by the intersections of structurally cohesive groups across multiple administrative levels. They show that those who share administrative offices during overlapping time spans build dense cliquelike social ties within a political nucleus while maintaining sparse locally tree-like ties with structurally cohesive groups (globally multiconnected) in the larger region and community. The locally dense and the globally sparse multiconnected ties act as different kinds of amplifiers for the feedback relations between larger cohesive groups and their government representatives.

In his classic statement on the development of social capital, Coleman (1988) argued that a closed-loop structure connecting adolescents' friends' parents increases effective normative regulation in a community. The key structural feature responsible for this increased ability is that biconnected components (loops) allow information to flow freely throughout the community, allowing normative ideas to be exchanged and reinforced. Communities in which parents are connected to each other only indirectly through adolescents will likely have weaker normative regulation. Adolescents in such communities occupy a powerful position, controlling the flow of information. This fact is recognized by any teen that successfully dupes parents into thinking they are at a friend's house while the friend similarly claimed to be at theirs. In general, the emergence of community through exchange occurs when goods and information cycle through the community, as evidenced clearly in work on generalized exchange (Bearman 1997).

## Power

The substantive character of groups that are vulnerable to unilateral action differs significantly from that expected of groups with multiple independent connections. The group as a whole is vulnerable to the will and activities of those who can destroy the group by leaving. Moreover, actors that can disconnect the group are also actors that can control the flow of resources in the network. As has long been known from Network Exchange Theory, networks with structural features leading to control of resource flows generate power inequality (Willer 1999).

In contrast to weak structurally cohesive groups, however, collectivities that do not depend on individual actors are less easily segmented. The presence of multiple paths, passing through different actors, implies that if any one actor is removed, alternative links among members still exist to maintain social solidarity. Information and resources can flow through multiple paths, making minority control of resources within the group difficult. As such, the inequality of power implicit in weakly cohesive structures is not so pronounced in stronger structures. In general, structurally cohesive networks are characterized by a reduction in the power provided by structural holes (Burt 1982), as local holes are closed at longer distances, uniting the entire group.

The development of "just-in-time" inventory systems provides a compelling example. When viewed as a network of resource flows, the most efficient production systems resemble spanning trees, with tight couplings among plants. Under this structure, labor has accentuated power because strikes, which effectively remove the struck factory from the production network, disconnect the entire production line. Recent trends toward "just-in-time" production processes are not new, but were used extensively early in the auto industry. It became clear, however, that this production structure gave labor power. To counter, management expanded the production network to include alternative sources (other factories and storehouses), building redundancy (i.e., structural cohesion) into the system (Schwartz 2001).

# **CONCLUSION AND DISCUSSION**

Social solidarity is a central concept in sociology. We have argued that solidarity can be analytically divided into an ideational component and a relational component (and perhaps others). We have defined *structural cohesion* as a measure of the relational component. The essential substantive feature of a strongly cohesive group is that it has a status beyond any individual group member. We operationalize this concept of social cohesion through the graph-theoretic property of connectivity (Harary 1969; Harary et al. 1965), showing that structural cohesion increases with each additional independent path in a network.

When does cohesion start? Following authors such as Markovsky and Lawler (1994), we argue that cohesion starts (weakly) when every actor can reach every other actor through at least one relational path—the paths that link actors are the social glue holding them together. We show that structural cohesion *scales* in that it is weakest when there is one path connecting actors, stronger when there are two node-independent paths, stronger yet with three node-independent paths, and finally when, for *n* actors, there are almost as many (n - 1) independent paths between each pair.

Our conceptualization of structural cohesion simultaneously provides an operationalization of one of the structural dimensions of network embeddedness. Cohesive sets in a network are nested, such that highly cohesive groups are nested within less cohesive groups. Because the process for identifying the nested connectivity sets is based on identifying the most fragile points in a network, those actors who are involved in the most highly connected portions of the network are often deeply insulated from perturbations in the overall network. Given the theoretical importance of the generalized concept of embeddedness in sociology, a measure of structural embeddedness is an important asset to help provide clear-cut empirical studies of embeddedness.

Our analysis of structural cohesion has focused on the basic network features of social cohesion, without regard to the particular features that might be relevant in any given case. We suspect that researchers could modify aspects of our structural conception of cohesion as theory dictates. Thus, in settings where flows degrade quickly, one could account for the level of cohesion by incorporating a measure of path length, tie strength, or the ratio of connectivity to group size. We caution, however, that much of the theoretical power of our concept of cohesion rests on the idea that multiple indirect paths (perhaps routed through strongly cohesive subgroups) can magnify signals such that long distances can be united through social connections. Additionally, although we expect that structurally cohesive groups will also be stable groups, this expectation must be tested empirically.

The qualitative relational feature we focus on, grounded in Simmel ([1908] 1950), is whether a group depends on particular individuals for its group status. The relevant quantitative measure is the number of individuals whose involvement is required to keep the group connected. Here we have applied our method in an effort to show how cohesion might be profitably used in different types of empirical settings. The settings tested here are clearly only a small subset of the types of settings in which cohesion might be important, and our tests have focused on only one dimension of a decidedly complex concept. As with any single-dimensional network measure, our concept of structural cohesion filters out a particular aspect of the network. Previous literature has focused on alternative elements, such as path distance or density. However, there are some aspects of networks that might conceivably be important for wider questions about solidarity that are not captured in our measure. For example, all current measures treat networks as static, although the realization of any given network is time-dependent (Moody 2002). Future work might benefit by building time explicitly into models of social cohesion. Second, while our network focuses on the vulnerability of the network to node removal (also see Borgatti 2002), we do not examine

the probability that a given node will, in fact, be removed. For any given context, some nodes may be more strongly entrenched in the setting than others, which might provide a contextual corollary to the ideas developed here. Finally, a direct link between the relational structure and the ideational structure could be identified by layering observed social relations with ideational similarity measures, as are derived through shared membership in groups or identification with particular ideas (Breiger 1974; Ennis 1992).

Further theory and research are required to understand how relation type or strength affects the importance of cohesive structures for substantive outcomes. Of particular interest will be work that, as with Durkheim's Division of Labor ([1893] 1984), specifies the relation between structural cohesion and ideational components of social solidarity (Paxton and Moody forthcoming). The connection will require a sustained treatment of the ideational components of solidarity, as might build from treatments of individual attachment to a group (Bollen and Hoyle 1990; Lawler 1992), or questions about identity (Hogg 1992). Our hope is that by providing a clear and concise definition and operationalization of structural cohesion, and a methodological tool for analysis, researchers in many fields will be better able to conduct their work.

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# APPENDIX A

**Cohesive Blocking Procedure for Identifying Connectivity Sets** 

Combining algorithms from computer science (Ball and Provan 1983; Even and Tarjan 1975; Kanevsky 1990, 1993), we can identify cutsets in a network as follows:

- (1) Identify the connectivity, k, of the input graph.
- (2) Identify all k-cutsets at the current level of connectivity.
- (3) Generate new graph components based on the removal of these cutsets (nodes in the cutset belong to both sides of the induced cut).
- (4) If the graph is neither complete nor trivial, return to 1; else end.

This procedure is repeated until all nested connectivity sets have been enumerated.<sup>a</sup> Walking through the example in Figure 2, we would first identify the component (Step 1), and identify the cutnode {7} (Step 2).<sup>b</sup> Separating the two subgraphs at node 7 (Step 3) induces two new components: {7-16} and  $\{1-7, 17-23\}$  that are neither complete nor trivial. Within each induced subgraph we repeat the process, starting by identifying the subgraph connectivity. Within the {7-16} bicomponent, we identify {8, 10}, {10, 16}, and {14, 16} as the 2-cuts for this subgraph, each of which leads to a single minimum degree cut-we call these types of cuts singleton cuts (e.g., of 9, 13, or 15. The graph remaining after the singleton cuts have been removed is {7, 8, 11, 14, 10, 12, and 16}, which is 1-connected, with {7, 8, 11, and 14 the largest included tricomponent). Because {7, 8, 11, and 14} form a completely connected clique, we stop here and return to the other graph induced by removing node 7,  $(\{1-7,17-23\})$ .

<sup>&</sup>lt;sup>a</sup> SAS IML programs for identifying the full connectivity sets of a network are available (Moody 1999).

<sup>&</sup>lt;sup>b</sup> An efficient algorithm for doing so can be found in Gibbons (1985).

Again, this graph is a bicomponent. Cutsets  $\{5, 7\}$  and  $\{21, 19\}$ ,  $\{21, 7\}$ , and  $\{5, 19\}$  induce two graphs of higher cohesion:  $\{1-7\}$  and  $\{17-23\}$  that are of maximal connectivity, as further cuts will induce only singleton partitions.

One can represent the hierarchical nesting of connectivity groups as a directed tree, with the total graph as the root, and each subgraph that derives from it a new node. The *cohesive blocking* of a network consists of identifying all cohesive substructures within the network and relating them to each other in terms of the nested branching of the subgroups. The blocking for the example above is given in Figure 3 (with singleton cuts not represented).

Testing for k-connectivity (Step 1) can be accomplished with a network maximum flow algorithm developed by Even and Tarjan (1975).<sup>c</sup> An algorithm for identifying all k-cutsets of the graph (Step 2) was developed by Ball and Provan (1983), and was extended by Kanevsky (1990, 1993) to find all minimum-size separating vertex sets.<sup>d</sup> One must ap-

° This is an extension of Dinic's algorithm and runs in  $O(V^{1/2}E^2)$  time.

<sup>d</sup> Which runs in  $O(2^k V^3)$  time.

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ply these two procedures for every induced subgraph, and thus the total running time of the algorithm can be substantial. Two steps can be taken to reduce the computation time. First, there are lineartime algorithms for identifying k-connected components for  $k \leq 3$ , and one can start searching with these algorithms, limiting the number of levels at which one has to run the full connectivity algorithms (Fussell, Ramachandran, and Thurimella 1993; Hopcroft and Tarjan 1973). Second, in many empirical networks the most common cutset occurs for singleton cuts. Because the procedure is nested, one can search for nodes with degree less than or equal to the connectivity of the parent graph (the graph from which the current graph was derived), remove them from the network, and thus apply the network flow search only after the singleton cuts have been removed.e

<sup>e</sup> Additionally, there are approximation approaches (Auletta et al. 1999; Khuller and Raghavachari 1995) that could be used to identify graph connectivity within a certain amount of error, which would be faster.

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