ABSTRACT. R versions of the array manipulation functions of APL are given. We do not translate the system functions or other parts of the runtime. Also, the current version has no nested arrays.

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References
APL IN R

1. INTRODUCTION

APL was introduced by Iverson [1962]. It is an array language, with many functions to manipulate multidimensional arrays. R also has multidimensional arrays, but far fewer functions to work with them.

In R there are no scalars, there are vectors of length one. For a vector \( x \) in R we have \( \text{dim}(x) \) equal to NULL and \( \text{length}(x) > 0 \). For an array, including a matrix, we have \( \text{length}(\text{dim}(x)) > 0 \). APL is an array language, which means everything is an array. For each array both the shape \( \rho A \) and the rank \( \rho A \) are defined. Scalars are arrays with shape equal to one, vectors are arrays with rank equal to one.

In 1994 I coded most APL array operations in XLISP-STAT. The code is still available at http://idisk.mac.com/jdeleeuw-Public/utilities/apl/array.lsp. There are some important differences between the R and Lisp versions, because Lisp and APL both have C's row-major ordering, while R (like Matlab) has Fortran's column-major ordering in the array layout. The R version of APL uses column-major ordering. By slightly changing the two basic building blocks of our code, the \text{aplDecode()} and \text{aplEncode()} functions, it would be easy to choose between row-major and column-major layouts. But this would make it more complicated to use the code with the rest of R.

Because of layout, the two arrays \( A \cdot 3 \cdot 3 \cdot 3 \cdot 27 \) and \text{array}(1:27,\text{rep}(3,3)) are different. But what is really helpful in linking the two environments is that \( A \cdot 3 \cdot 3 \cdot 27 \) and \text{as.vector}(\text{array}(1:27,\text{rep}(3,3)))\), which both ravel the array to a vector, give the same result, the vector \( 1:27 \) or \( 1:27 \). This is, of course, because ravelling an array is the inverse of reshaping a vector.

Most of the functions in R are written with arrays of numbers in mind. Most of them will work for array with elements of type logical, and quite a few of them will also work for arrays of type character. We have to keep in mind, however, that APL and R treat character arrays quite differently. In R we have \( \text{length}("aa") \) equal to 1, because "aa" is a vector with as its single element the string "aa". R has no primitive character type, characters are just strings which happen to have only one character in them. In APL strings themselves are vectors of characters, and \( \rho "aa" \) is 2. In R we can say \text{a<-array}("aa",c(2,2,2)), but in APL this gives a domain error. In APL we can say \( 2 \cdot 2 \cdot 2 \cdot "aa" \), which gives the same result as \( 2 \cdot 2 \cdot 2 \cdot "a" \) or \( 2 \cdot 2 \cdot 2 \cdot "a" \).
In this version of the code we have not implemented the nested arrays of APL-2. Nesting gives every array A not just a shape $\rho A$ and a rank $\rho^2 A$, but also a depth. The depth of an array of numbers of characters is one, the depth of a nested array is the maximum depth of its elements.

There are many dialects of APL, and quite a few languages derived from APL, such as $A+$ and $J$. For APL-I we use Helzer [1989] and for APL2 we use IBM [1988] and MicroAPL [2007].

2. Functions


2.2. Decode.


If $L$ is scalar and $R$ is a vector, then $L \cdot R$ is the polynomial $r_1 x^{m-1} + r_2 x^{m-2} + \cdots + r_m$ evaluated at $L$. This means that if the $r_i$ are nonnegative integers less than $L$, then $L \cdot R$ gives the base-$10$ equivalent of the base-$L$ number $R$.

If the arguments $L$ and $R$ are vectors of the same length. In the array context, decode returns the index of element $x[b]$ in an array $x$ with \texttt{dim}($x$) = $a$. Obviously the $R$ implementation, which uses column-major ordering, will give results different from the APL implementation. In APL the expression $3 \cdot 3 \cdot 3 \cdot 1 \cdot 2 \cdot 3$ evaluates to $18$, while \texttt{aplDecode(1:3, rep(3,3))} gives $22$.

2.2.2. $R$ code.

```r
1 aplDecode <- function (ind, base) {
  2    if (length (base) == 1)
  3      base <- array (base, aplShape(ind))
  4    b <- c(1, butLast (cumprod (base)))
  5    return (1 + sum (b * (ind - 1)))
  6 }
```

2.2.3.
2.3. Drop.

```r
aplDrop <- function (a, x, drop = FALSE) {
  sa <- aplShape(a); ra <- aplRank(a)
  y <- as.list(rep(0, ra))
  for (i in 1:ra) {
    ss <- sa[i]; xx <- x[i]; sx <- ss - xx
    if (xx >= 0) y[[i]] <- (xx + 1):ss
    if (xx < 0) y[[i]] <- 1:sx
  }
  return (aplSelect(a, y, drop))
}
```

2.4. Encode. Encode $AÎB$ is the inverse of Decode. In APL-I it is known as Representation [Helzer, 1989, 17–21]. It takes a radix vector $A$ and a number $B$ and returns the array indices corresponding to cell $B$ in an array with a $Ð$ of $A$.

```r
aplEncode <- function (rep, base) {
  b <- c(1, butLast(cumprod(base)))
  r <- rep(0, length(b)); s <- rep(-1)
  for (j in length(base):1) {
    r[j] <- s %/% b[j]
    s <- s - r[j] * b[j]
  }
  return (1 + r)
}
```

2.5. Expand.

```r
aplEXV <- function (x, y) {
  z <- rep(0, length(y))
  m <- which(y)
  if (length(m) != length(x))
    stop("Incorrect vector length in aplEXV")
  z[which(y)] <- x
  return (z)
}
```
aplExpand<-function(x,y,axis=1) {
  if (is.vector(x)) return(aplEXV(x,y))
  d<-dim(x); mx<-which(y); e<-d; e[axis]<-m
  if (m != d[axis])
    stop("Incorrect dimension length in aplEX")
  z<-array(0,e)
  apply(z,(1:n)[-axis],function(i) z[i]<-x[i])
}

2.6. Inner Product.

aplIPV<-function(x,y,f="*",g="+"){
  if (length(x) != length(y))
    stop("Incorrect vector length in aplIPV")
  if (length(x) == 0) return(x)
  z<-match.fun(f)(x,y)
  return(aplRDV(z,g))
}

aplInnerProduct<-function(a,b,f="*",g="+") {
  sa<-aplShape(a); sb<-aplShape(b)
  ra<-aplRank(a); rb<-aplRank(b)
  ia<-1:(ra-1); ib<-1:(rb-1)+1:(ra-1)
  ff<-match.fun(f); gg<-match.fun(g)
  ns<-last(sa); nt<-first(sb)
  if (ns != nt)
    stop("non-compatible array dimensions in aplInner")
  sz<-c(butLast(sa),butFirst(sb)); nz<-prod(sz)
  z<-array(0,sz)
  for (i in 1:nz) {
    ivec<-aplEncode(i,sz)
    for (j in 1:ns) {
      aa<-a[aplDecode(c(ivec[ia],j),sa)]
      bb<-b[aplDecode(c(j,ivec[ib]),sb)]
      z[i]<-gg(z[i],ff(aa,bb))
    }
  }
}
2.7. Join.

```r
aplJN<-function(a,b,k) {
  if (is.vector(a) && is.vector(b)) return(c(a,b))
  sa<-aplShape(a); sb<-aplShape(b); ra<-aplRank(a); rb <-aplRank(b)
  if (ra != rb)
    stop("Rank error in aplJN")
  if (!identical(sa[-k],sb[-k]))
    stop("Shape error in aplJN")
  sz<sa; sz[k]<sz[k]+sb[k]; nz<-prod(sz); u<-unit(k, ra)
  z<-array(0,sz)
  for (i in 1:nz) {
    ivec<aplEncode(i,sz)
    if (ivec[k] <= sa[k]) z[i]<-a[aplDecode(ivec,sa)]
    else z[i]<-b[aplDecode(ivec-sa[k]*u,sb)]
  }
  return(z)
}
```

2.8. Member Of.

```r
aplMemberOf<-function(a,b) {
  if (!identical(typeof(a),typeof(b)))
    warning("Arguments of different types in aplMO")
  arrTest(a); arrTest(b)
  sa<-aplShape(a); sb<-aplShape(b)
  na<-prod(sa); nb<-prod(sb)
  z<-array(0,sa)
  for (i in 1:na) {
    z[i]<-0; aa<a[i]
    for (j in 1:nb)
      if (identical(aa,b[j])) z[i]<-1
  }
  return(z)
```

2.9. Outer Product.

\[
\text{aplOP<} \text{function} (x, y, f = "\times") \text{ return } (\text{outer}(x, y, f))
\]

2.10. Ravel.

\[
\text{aplRV<} \text{function}(a) \text{ as.vector}(a)
\]

2.11. Rank.

\[
\text{aplRank<} \text{function}(a) \text{ aplShape(aplShape(a))}
\]

2.12. Reduce.

\[
\text{aplRDV<} \text{function}(x, f = "+") \{ \\
\quad \text{if} \ (\text{length}(x) == 0) \text{ return } (x) \\
\quad s<-x[1] \\
\quad \text{if} \ (\text{length}(x) == 1) \text{ return } (s) \\
\quad \text{for } (i \ in 2:\text{length}(x)) \\
\quad \quad s<-\text{match.fun}(f)(s, x[i]) \\
\quad \text{return } (s) \\
\}
\]

\[
\text{aplReduce<} \text{function}(a, k, f = "+") \{ \\
\quad \text{if} \ (\text{is.vector}(a)) \\
\quad \quad \text{return } (\text{aplRDV}(a, f)) \\
\quad \text{sa<} \text{aplShape}(a); \text{ ra<} \text{aplRank}(a); \text{ na<} \text{prod}(sa) \\
\quad \text{ia<} (1:ra)[-k] \\
\quad \text{sz<} \text{sa}[ia] \\
\quad \text{ff<} \text{match.fun}(f) \\
\quad z<-\text{array}(0, sz); \text{ nz<} \text{prod}(sz) \\
\quad \text{for } (i \ in 1:na) \{ \\
\quad \quad \text{ivec<} \text{aplEncode}(i, sa) \\
\quad \quad \text{jind<} \text{aplDecode}(ivec[-k], sz) \\
\quad \quad z[jind]<-ff(z[jind], a[i])
\}
\]
2.13. Replicate.

```r
aplRPV <- function(x, y) {
  n <- aplShape(x); m <- aplShape(y)
  if (m == 1) y <- rep(y, n)
  if (length(y) != n)
    stop("Length Error in aplCRV")
  z <- vector()
  for (i in 1:n)
    z <- c(z, rep(x[i], y[i]))
  return(z)
}
```

```r
aplReplicate <- function(x, y, k) {
  if (is.vector(x)) return(aplRPV(x, y))
  sx <- aplShape(x); sy <- aplShape(y); sk <- sx[k]
  if (sy == 1) y <- rep(y, sk)
  if (length(y) != sk)
    stop("Length Error in aplRPV")
  sz <- sx; sz[k] <- sum(y); nz <- prod(sz)
  gg <- aplCRV(1:sk, y)
  z <- array(0, sz)
  for (i in 1:nz) {
    jvec <- aplEncode(i, sz)
    jvec[k] <- gg[jvec[k]]
    z[i] <- x[aplDecode(jvec, sx)]
  }
  return(z)
}
```


```r
aplReshape <- function(a, d) return(array(a, d))
```
2.15. **Rotate.** Rotate [Helzer 1989, p. 191–193] shifts the elements of a vector or array dimension. In APL we write \( \text{A} \Phi \text{B} \).

```
1 aplRTV <- function (a, k) {
2   n <- aplShape(a)
3   if (k == 0) return (a)
4   if (k > 0)
5     return (c(a[-(1:k)],a[1:k]))
6   if (k < 0)
7     return (c(a[(n+k+1):n],a[1:(n+k)]))
8 }
```

```
1 aplRotate <- function (a, x, k) {
2   if (is.vector(a)) return (aplRTV(a, k))
3   sa <- aplShape(a); sx <- aplShape(x)
4   if (sx == 1) x <- array(x, sa[-k])
5   if (! identical(sa[-k], aplShape(x)))
6     stop("Index Error in aplRotate")
7   z <- array(0, sa); sz <- sa; nz <- prod(sz); sk <- sz[k]
8   for (i in 1:nz) {
9     ivec <- aplEncode(i, sz)
10    xx <- x[aplDecode(ivec[-k], sx)]
11    ak <- rep(0, sk)
12    for (j in 1:sk) {
13      jvec <- ivec; jvec[k] <- j
14      ak[j] <- a[aplDecode(jvec, sz)]
15    }
16    bk <- aplRTV(ak, xx)
17    for (j in 1:sk) {
18      jvec <- ivec; jvec[k] <- j
19      z[aplDecode(jvec, sz)] <- bk[j]
20    }
21   }
22   return (z)
23 }
```

2.16. **Scan.**
aplSCV <- function(x, f = "+") {
    if (length(x) <= 1) return(x)
    return(sapply(1:length(x), function(i) aplRDV(x[1:i], f))}
}

aplSC <- function(a, k, f = "+") {
    if (is.vector(a)) return(aplSCV(a, f))
    sa <- aplShape(a); ra <- aplRank(a); sk <- sa[k]; u <- unit(k, ra)
    ff <- match.fun(f)
    na <- prod(sa); z <- a
    for (i in 1:na) {
        ivec <- aplEncode(i, sa)
        sk <- ivec[k]
        if (sk == 1) z[i] <- a[i]
            else z[i] <- ff(z[aplDecode(ivec-u, sa)], a[i])
    }
    return(z)
}

2.17. Select.

aplSelect <- function(a, x, drop = FALSE) {
    sa <- aplShape(a); ra <- aplRank(a)
    sz <- sapply(x, length)
    z <- array(0, sz); nz <- prod(sz)
    for (i in 1:nz) {
        ivec <- aplEncode(i, sz)
        jvec <- vector()
        for (j in 1:ra)
            jvec <- c(jvec, x[[j]][ivec[j]])
        z[i] <- a[aplDecode(jvec, sa)]
    }
    if (drop) return(drop(z)) else return(z)
}
2.18. **Shape.** Shape is the monadic version of \( \mathcal{P} \), while reshape is the dyadic version. Shape gives the dimensions of an array, an Reshape modifies them. `aplRank()` is not really a standard APL function, but we use it as shorthand for \( \mathcal{P} \mathcal{R} \mathcal{A} \).

```apl
aplShape<-function(a) {
  if (is.vector(a)) return(length(a))
  return(dim(a))
}
```

2.19. **Take.**

```apl
aplTK<-function(a,x,drop=FALSE) {
  sa<-aplShape(a); ra<-aplRank(a)
  y<-as.list(rep(0,ra))
  for (i in 1:ra) {
    ss<-sa[i]; xx<-x[i]; sx<-ss-xx
    if (xx > 0) y[[i]]<-1:xx
    if (xx < 0) y[[i]]<-(sx+1):ss
  }
  return(aplSelect(a,y,drop))
}
```

2.20. **Transpose.** APL has both a monadic \( \mathcal{A} \) and a dyadic \( \mathcal{B} \mathcal{A} \) transpose. This APL transpose has a somewhat tortuous relationship with R’s `aperm()`.

The monadic `aplTranspose(a)` and `aperm(a)` are always the same, they reverse the order of the dimensions.

If \( x \) is a permutation of \( 1:aplRank(a) \), then `aperm(a,x)` is actually equal to `aplTranspose(a, order(x))`. For permutations we could consequently define `aplTranspose(a,x)` simply as `aperm(a, order(x))` (which would undoubtedly be more efficient as well).

If \( x \) is not a permutation, then `aperm(a,x)` is undefined, but `aplTranspose(a, x)` can still be defined in some cases.

If \( x \) has `aplRank(a)` elements equal to one of \( 1:m \), with each of \( 1:m \) occurring at least once, then `aplTranspose(a,x)` has rank \( m \). For obvious reasons dyadic transpose is not used a great deal.
aplTranspose <- function (a, x = rev(1:aplRank(a))) {
  sa <- aplShape(a); ra <- aplRank(a)
  if (length(x) != ra)
    stop("Length Error in aplTranspose")
  rz <- max(x); sz <- rep(0, rz)
  for (i in 1:rz)
    sz[i] <- min(sa[which(x == i)])
  nz <- prod(sz)
  z <- array(0, sz)
  for (i in 1:nz)
    z[i] <- a[aplDecode(aplEncode(i, sz)[x], sa)]
  return(z)
}

3. Utilities

first <- function (x) return (x[1])
butFirst <- function (x) return (x[-1])
last <- function (x) return (x[length(x)])
butLast <- function (x) return (x[-length(x)])
unit <- function (i, n) ifelse (i == (1:n), 1, 0)

References

DEPARTMENT OF STATISTICS, UNIVERSITY OF CALIFORNIA, LOS ANGELES, CA 90095-1554

E-mail address, Jan de Leeuw: deleeuw@stat.ucla.edu

URL, Jan de Leeuw: http://gifi.stat.ucla.edu