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DISCLAIMER

A TEST OF THE OPTICAL THEOREM

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ABSTRACT

Forward differential cross sections for $\pi^- p$ elastic scattering at 1.0, 1.5 and 2.0 GeV/c show that the square of the imaginary parts of the nuclear scattering agrees with the optical theorem prediction within ±3%, when averaged over the three momenta.

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In spite of the common belief that the optical theorem is true, it has been suggested that it be tested experimentally [1-3], to check quantum theory at high energy. Here, we report an analysis of $\pi^- p$ elastic scattering at 1.015, 1.527, and 2.004 GeV/c in the forward direction and the comparison of the results with the optical theorem predictions.

Before this publication, tests have been performed using the data from various experiments [4-8]. For $\pi^- p$, the measured forward elastic cross section agrees with the optical theorem prediction within 5 or 10%, but the residual discrepancy is too large to be explained by the experimental errors quoted by the original publications [9]. However, there might be other unidentified errors [9,10]. In other reactions, discrepancies have been noticed [3], but usually correspond to doubtful extrapolations to 0° [11]. Similar uncertainties affect other violations of unitarity [12].

This experiment was performed at the CERN Proton Synchroton, in a π^- separated beam, using the setup of Fig. 1 [13, 14]. The incident particle was defined by the C_1 C_2 coincidence in anticoincidence with a Cerenkov counter $\check{C}$, set to count particles with velocities larger than the beam pions, eliminating most of the muon background. The wire chambers W_1 and W_2, upstream from the target, defined the direction of the incident particle and the downstream wire chambers, W_3 to W_7, defined the direction of the outgoing particle. The data used here concern events with the counter C_5 not firing, i.e. with the scattered particle absorbed in the iron wall, therefore events not associated with a pion decay in the apparatus. A box of counters, C_6, surrounding the target, detected additional particles produced in interactions other than forward elastic scattering and a single counter, C_4, detected most of the unscattered particles. Most of the time, anticoincidences with C_4 and C_6 were required in the trigger so as to decrease the number of uninteresting events recorded. However, for about 1/4 of the events, a special trigger was set that did not require the C_4 and C_6 anticoincidences, in order to study beam efficiencies and biases. More details are given in refs. [13] and [14].

The information from the wire chambers, the counters C_4, C_5, and C_6 and the number of incident particles ($C_1\ C_2\ \overline{\overset{v}{C}}$ counts) were recorded for each trigger on magnetic tapes that were later submitted to two independent analyses*.

In both analyses, events were rejected if there was no track or more than one track in the upstream or the downstream chambers or if the counter C_5 or C_6 had fired. Using bubble chamber data concerning $\pi^- p$ inelastic scattering**, it was possible to show that the contribution of inelastic events to the remaining category of events was essentially negligible†. Then, cross sections were computed from the ratio of the number of accepted events to the number of incident particles counted by the coincidence $C_1\ C_2\ \overline{\overset{v}{C}}$. Corrections were then applied. In analysis I, the loss of particles by the reconstruction in the upstream chambers and the pion transmission through the iron wall were measured using the particles recorded during the special trigger. The other corrections were computed theoretically for each identifiable process that could lead to a loss or to an excess of events. Analysis I is described in detail in refs. [13] and [14].

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*Analysis I was performed at CERN and analysis II at L.B.L.

**We are indebted to R. Longacre for making the data from the $\pi^- p$ LBL-SLAC compilation available to us for this particular purpose.

†0.4%, 0.6%, and 0.7% at 1.0, 1.5 and 2.0 GeV/c respectively.

In analysis II, full advantage is taken of the similarity between the elastically scattered particle and the unscattered beam particle. The differences between the two were supposed to vanish in any extrapolation to 0° . An overall efficiency correction was determined by comparing the number of "unscattered" beam particles reconstructed by our geometry program with the total number of incident particles counted as $C_1 C_2 \overline{\check{C}}$. For this purpose, particles associated with the special trigger and with a measured momentum transfer smaller than about 40 MeV/c were used. In this procedure, the physical processes responsible for the losses do not need to be identified. The correction for the events lost by the C_4 anticoincidence was also measured using the special trigger events, while it was computed by Monte Carlo in analysis I. In analysis II there were only two corrections that were not measured but computed as in analysis I: the losses due to the finite size of the chambers and the excess due to the backward elastic events (with the forward proton misinterpreted as a scattered pion). However, the momentum transfer interval of the data accepted for the fits was restricted in analysis II in such a way that the correction for the finite size of the chambers was never larger than 5%, in order to minimize the dependence upon the Monte Carlo computations. Altogether, analysis II is less sensitive to systematic errors; however, essentially because of the measurement error on the C_4 correction, the resulting statistical errors are larger than in analysis I.

In either analysis, the theoretical expression of the elastic differential cross sections was assumed to be

$$d\sigma/dt = (d\sigma/dt)_C + (d\sigma/dt)_N + (d\sigma/dt)_I \tag{1}$$

$$(d\sigma/dt)_N = 0.0511\, \sigma^2(1 + \alpha^2)\, e^{-bt} \tag{2}$$

$$(d\sigma/dt)_C = 2.61 \times 10^{-4} \times \beta^{-2}\, t^{-2}(1 + t/0.71)^{-8}$$

$$(d\sigma/dt)_I = 2\sqrt{(d\sigma/dt)_C \cdot (d\sigma/dt)_N}\,(\alpha\cos\delta - \sin\delta)/\sqrt{1+\alpha^2} \qquad (3)$$

$$\delta = [\ln(9.5t) + 0.577]/137\,\beta. \quad \text{(see ref. [15])} \qquad (4)$$

where b is the slope parameter, α is the ratio of the real to the imaginary part of the nuclear scattering and σ a parameter proportional to the imaginary part of the nuclear scattering in the forward direction. The cross sections are in mb, the momentum transfers are in $(GeV/c)^2$.

For the fits, expression (1) was slightly modified to take into account the perturbation due to multiple scattering and geometrical resolution. Fits were performed to the measured differential cross sections, adjusting the parameters σ, α, and b with a minimum -χ^2 technique.

The results of these fits and the corresponding statistical errors are shown in Table 1. The discrepancies between the two analyses are of the order of 2 to 3% for σ and between 0 and 0.08 in absolute value for α. They are due to: different t ranges used for the fits, different C_4 corrections, and different normalizations resulting from differences of 1% or so in the value of several correction factors. This probably shows the effect of many small systematic errors in the Monte Carlo computations of analysis I or in the measurements of analysis II. Since systematic errors are expected to act sometimes in the same and sometimes in the opposite direction in the two analyses, we considered the differences in the results as an estimation of the systematic errors multiplied by the factor $\sqrt{2}$. This estimation was then combined quadratically with the smallest of the two statistical errors to give the errors on the combined results in Table 1. As to the central values, we used the average of the results obtained in the two analyses.

For testing the optical theorem, we used the most precise measurements of the total cross section σ_t for $\pi^- p$ in this range of momentum [4]. Table 1 shows: values of σ_t interpolated between the momenta measured in ref. [4], the value of a parameter η defined as in ref. [2],

$$\eta = \left(\frac{\sigma_t}{\sigma}\right)^2 , \tag{5}$$

and the number of standard deviations between η and unity.

The optical theorem predicts that η should be 1. That prediction fits the data, but deviations in η of about 5% cannot be ruled out for individual momenta. In particular, the point at 1.0 GeV/c is worth further investigation. For the three momenta together, the overall χ^2 is 3.7 for three degrees of freedom and the average value of η is 0.993. Since most of the errors on η are systematic, they are not considered as independent and the error on the average is still the typical 3% of the individual systematic errors*.

In conclusion, this experiment does not show any violation of the optical theorem and the sensitivity of the test is about 3% for the average value of the parameter η of eq. (5) over the three momenta. This represents an improvement of a factor of about 3 over previous tests [9].

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* In a previous publication [14], our values of α, based on analysis I, were found to be consistent with the predictions based on the dispersion relations computed in ref. [16]. Analysis II agrees with this statement also. The $\pi^+ p$ data of ref. 14 was not used here because its Coulomb nuclear interference is constructive and it follows that the errors on the parameter σ are much larger.

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Table 1. Results of the two analyses of the data and the combined results.

Momentum		1.015	1.527	2.004
Analysis I	σ	61.0 ± 0.5	34.9 ± 0.2	35.4 ± 0.2
	α	-0.04± 0.025	-0.14± 0.03	-0.10± 0.02
	χ^2	26.5	53.3	44.5
degrees of freedom		30	51	53
range of t (GeV/c)		0.0009 - 0.038	0.0012 - 0.090	0.0014 - 0.096
Analysis II	σ	62.1 ± 0.8	35.6 ±0.3	36.6 ± 0.2
	α	0.00± 0.04	-0.22± 0.05	-0.09± 0.04
	χ^2	30.1	37.9	27.4
degrees of freedom		25	26	24
range of t $(\mathrm{GeV}/c)^2$		0.0009 - 0.029	0.0015 - 0.067	0.0016 - 0.096
Combined results	σ	61.5 ± 0.9	35.3 ± 0.5	36.0 ± 0.9
	α	-0.02± 0.03	-0.18± 0.05	-0.10± 0.02
Total cross section σ_t (mb) (from ref. 4)		59.8 ± 0.18	35.6 ± 0.18	36.3 ± 0.27
$\eta \pm \delta\eta$		0.945±0.030	1.017±0.030	1.017±0.052
$\lvert 1-\eta \rvert / \delta\eta$		1.8	0.6	0.3

FIGURE CAPTION

Fig. 1. Experimental layout. C_1, C_2, C_3, C_4, and C_5 are scintillators. C_6 is a box composed of scintillator counters and of lead γ converters. $\check{C}$ is a gas filled Cerenkov counter, and W_1 to W_7 are wire chambers. The counter C_3 was used to measure the total cross section for monitoring purpose during the time of data taking. The shaded area is an iron wall used in conjunction with C_5 as a muon identifier.

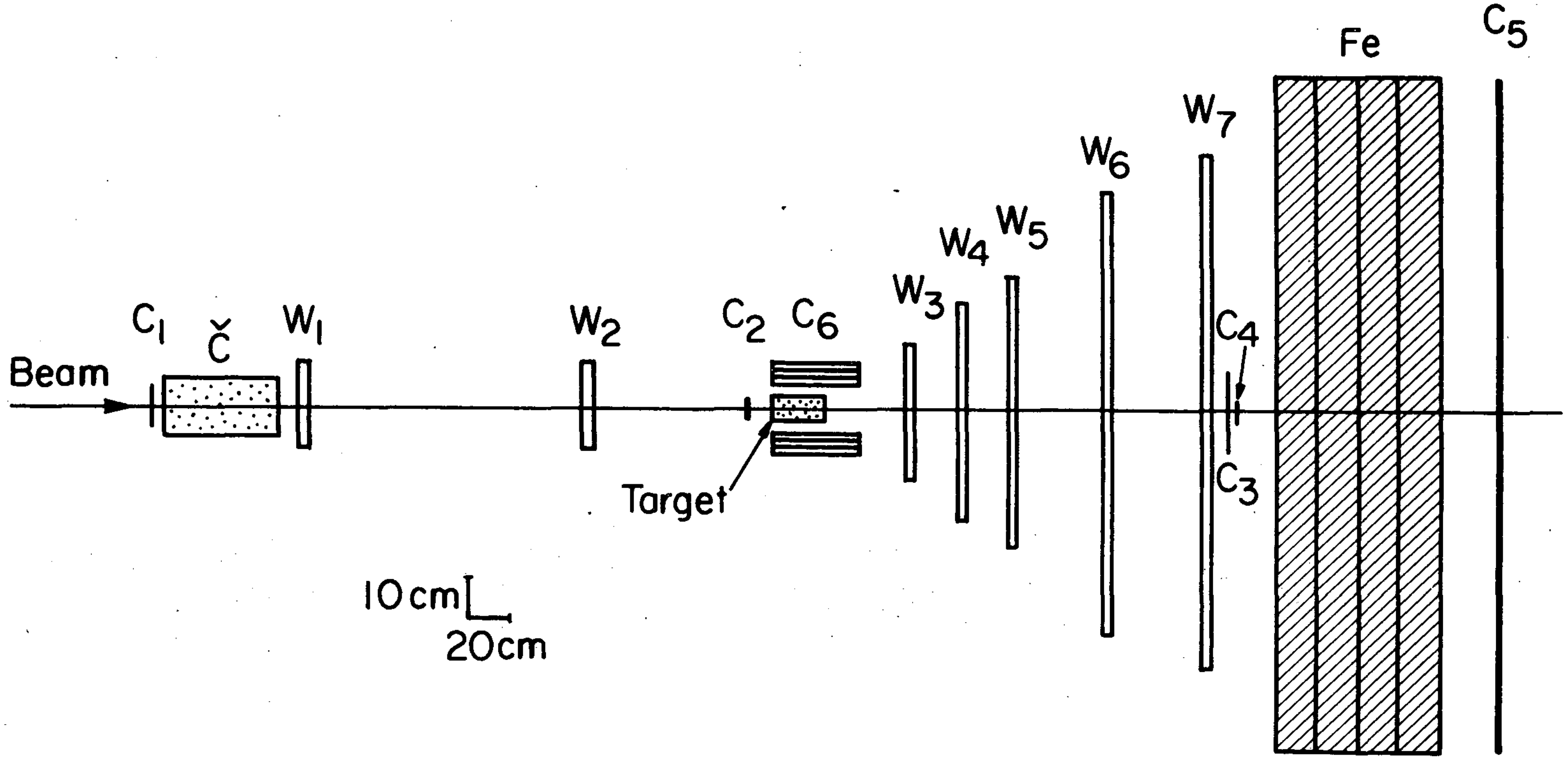

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Fig. 1.

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