Title
Price-Maker Economic Bidding in Two-Settlement Pool-Based Markets: The Case of Time-Shiftable Loads

Journal
IEEE Transactions on Power Systems, 31(1)

ISSN
0885-8950 1558-0679

Authors
Kohansal, Mahdi
Mohsenian-Rad, Hamed

Publication Date
2016

DOI
10.1109/TPWRS.2015.2405084

Peer reviewed

Price-Maker Economic Bidding in Two-Settlement Pool-Based Markets: The Case of Time-Shiftable Loads

Mahdi Kohansal, *Student Member, IEEE*, and Hamed Mohsenian-Rad, *Senior Member, IEEE*

***Abstract*—In this paper, a new scenario-based stochastic optimization framework is proposed for price-maker economic bidding in day-ahead and real-time markets. The presented methodology is general and can be applied to both demand and supply bids. That is, no restrictive assumptions are made on the characteristics of the pool and its agents. However, our focus is on the operation of time-shiftable loads with deadlines, because they play a central role in creating load flexibility and enhancing demand response and peak-load shaving programs. Both basic and complex time-shiftable load types are addressed, where the latter includes time-shiftable loads that are uninterruptible, have per-time-slot consumption limits or ramp constraints, or comprise several smaller time-shiftable subloads. Four innovative analytical steps are presented in order to transform the originally nonlinear and hard-to-solve price-maker economic bidding optimization problem into a tractable mixed-integer linear program. Accordingly, the global optimal solutions are found for the price and energy bids within a relatively short amount of computational time. A detailed illustrative case study along with multiple case studies based on the California energy market data are presented. It is observed that the proposed optimal price-maker economic bidding approach outperforms optimal price-maker self-scheduling as well as even-load-distribution.**

***Index Terms*—Day-ahead market, demand response, energy bids, price-maker economic bidding, price bids, real-time market, stochastic mixed-integer linear programming, time-shiftable loads.**

NOMENCLATURE

T	Number of daily market intervals.
t	Index of time.
k	Index of random scenario.
l	Index of sub-loads.
s, n, m, o	Indexes of steps in price quota curves.
α, β	Time-shiftable load timing parameters.
e	Time-shiftable load energy usage parameter.
x	Energy bid to day-ahead market.
y	Energy bid to real-time market.
p	Price bid submitted to the day-ahead market.
λ	Cleared price in day-ahead market.
ϕ	Cleared price in real-time market.
q	Dispatched energy for economic bid.
g	Dispatched energy for self-schedule bid.
qth	Maximum day-ahead cleared energy at a price.
$p^{\min}$	Minimum price at each step of function qth
$a^{\max}$	Width of each step in function qth.
$x^{\min}$	Minimum energy at each step in function λ.
$b^{\max}$	Width of each step in function λ.
$y^{\min}$	Minimum energy at each step in function ϕ.
$c^{\max}$	Width of each step in function ϕ.
r	On and off status of a time-shiftable load.
z	Total cleared energy in two-settlement markets.
$Z^{\min}$	Minimum consumption level.
$Z^{\max}$	Maximum consumption level.
$D^{\max}$	Maximum ramp-down rate.
$U^{\max}$	Maximum ramp-up rate.
θ, u, v, w	Binary auxiliary variables.
a, b, c	Continuous auxiliary variables.
C	Energy procurement cost at day-ahead market.

Manuscript received September 25, 2014; revised December 15, 2014 and February 01, 2015; accepted February 08, 2015. Date of publication March 09, 2015; date of current version December 18, 2015. This work was supported by NSF grants ECCS 1253516, ECCS 1307756, and CNS 1319798. Paper no. TPWRS-01321-2014. *(Corresponding author: Hamed Mohsenian-Rad.)*

The authors are with the Department of Electrical Engineering, University of California, Riverside, CA USA (e-mail: mkohansal@ee.ucr.edu; hamed@ee.ucr.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2015.2405084

I. INTRODUCTION

TIME-SHIFTABLE loads, a.k.a. *deferrable loads with deadlines*, play a central role in creating load flexibility and enhancing demand response and peak-load shaving programs. Some examples of time-shiftable loads include: charging plug-in electric vehicles [1], [2], irrigation pumps [3], batch processes in data centers and computer servers [4]–[6], intelligent pools [7], water heaters [8], industrial equipment in process control and manufacturing [9], [10], and various home appliances such as washing machine, dryer, and dish-washer [11]–[16].

Recent efforts have been made to incorporate time-shiftable loads into the wholesale electricity markets [17], [18]–[20]. For example, the problem of aggregating residential or other time-shiftable loads using utility-driven incentives is discussed in [17], [20]. [21]. Optimal demand bidding for time-shiftable loads using dynamic programming is presented in [22]–[24], where the time shiftable load of interest is assumed to be *price-taker*. That is, they are assumed to be relatively small so that their operation does *not* have impact on the cleared market price in the day-ahead or real-time markets. In [25], [26], the authors address wholesale electricity market participation of *large* and *price-maker* consumers with time-shiftable and dispatchable loads. However, the focus is on *self-scheduling* operation. That is, the load entity is assumed to submit only energy bids, *but not price bids*. As a result, power procurement is *not* subject to any condition on the clearing market price.

A. Comparison With Related Work

In this paper, our focus is on *price-maker economic bidding* for time-shiftable loads. Unlike in [22]–[24], we *do* consider the size of the load and hence the impact of demand bids on the cleared market price. Also, unlike in [25], [26], we address the more general case where the time-shiftable load submits not only energy bids *but also price bids*. Accordingly, since an economic demand bid is not cleared in the day-ahead market if its price component is lower than the cleared market price, our focus is on *two-settlement* markets, where energy is procured from both day-ahead and real-time markets [27]–[29].

Besides the literature on time-shiftable loads, this paper is also comparable with the literature on price-maker electricity market participation, e.g., for generating companies. In particular, on the methodology side, this paper is a direct and major extension of the analysis in [30], where the authors presented a model for *price-maker self-scheduling* of generating companies in a pool-based electricity market. As a result, in principle, our proposed price-maker economic bidding approach is applicable to not just time-shiftable loads but also generating companies or other types of market participants such as in the examples in [30], and its related studies in [31]–[34].

B. Summary of Technical Contributions

We can summarize the contributions in this paper as follows:

- A new scenario-based stochastic optimization framework is proposed for price-maker economic bidding in *coupled* day-ahead and real-time markets. Without loss of generality, the focus is on time-shiftable loads with deadlines. Besides their core time-flexibility features, other characteristics of time-shiftable loads are also considered, e.g., whether the time-shiftable load is uninterruptible, has per-time-slot consumption limits or ramp constraints, or comprises several smaller time-shiftable subloads.
- The analysis in this paper advances the existing *price-taker* results in [22], because here we consider *price-maker* market participation of time-shiftable loads. This study also advances the existing *self-scheduling* results in [30], because here we consider *economic* bidding.
- Four innovative analytical steps are presented in order to *transform* the originally nonlinear and hard-to-solve price-maker economic bidding optimization problem into a tractable mixed-integer linear program. Accordingly, we find the *global optimal solutions* for the price and energy bids to day-ahead market and the energy bids to real-time market within a short amount of computational time.
- A detailed illustrative example and also multiple case studies based on the California energy market data are presented. The impact of different market scenarios are assessed. The performance is compared with optimal price-maker self-scheduling and even-load-distribution. The proposed design outperforms both of these method.

II. Problem Statement

A. Two-Settlement Electricity Market

In a two-settlement wholesale electricity market, e.g., in California [27], Pennsylvania-Jersey-Maryland [29], and Texas [28], energy is traded in both day-ahead and real-time markets. Buyers and sellers participate in these markets by submitting demand and supply bids, respectively. The bids that are submitted to the day-ahead market indicate an *energy* quantity and possibly also a *price* quantity. Following the terminology that is used in the California energy market, we refer to a bid without price quantity as Self-Schedule and a bid with price quantity as Economic [27]. An Economic bid indicates that the buyer (seller) is willing to purchase (sell) the given quantity of energy *only if* the price is less (more) than or equal to the price bid. A Self-Schedule bid indicates that the buyer (seller) is willing to purchase (sell) the given quantity of energy, regardless of the price. For instance, 51% of the total submitted supply bid capacity and 11% of the total submitted demand bid capacity in California during January 2014 was of type Economic [35]. Mathematically, a Self-Schedule bid is a special case of an Economic bid with an infinite price quantity. As a result, there are more energy cost minimization opportunities for loads in Economic bidding than Self-Schedule bidding. The demand bids that are submitted (or metered) to the real-time market only indicate energy quantities. That is, they are always of type Self-Schedule [27].

B. Day-Ahead Market

Let T denote the number of daily market intervals. For example, in an hourly market, we have $T = 24$. At each time slot t, let x_t and p_t denote the energy bid and the price bid that are submitted to the day-ahead market, respectively. The market outcome depends on not only the bids, but also the market *price quota curve*[1] [30], [36], as shown in Fig. 1. There are one self-schedule and two economic bids shown in this figure. The self-schedule bid does not include a price component; therefore, it is a straight vertical line. Moving this line towards left or right can affect the price, as explained in the price-maker self-scheduling analysis in [30]. In contrast, price-maker economic bids have both energy and price components. As a result, they appear in

[1]For a given hour, the *quota* of a price maker generator or load is defined as the amount of power that it generates or consumes in that hour. The curve that expresses how the market-clearing price changes as this quota changes is called residual generation/demand curve [36] or simply price quota curve [30]. While the price quota curve is *step-wise monotonically decreasing* with respect to generation level for a price maker generator, it is *step-wise monotonically increasing* with respect to consumption level for a price maker consumer. Price quota curves are stepwise because the supply/demand bids are assumed to be blocks of generation/load at given prices [30]. These curves embody the effects of all interactions with competitors and the market rules [30], [36]. The price quota curves of a price maker generator or load can be obtained by either market simulation or using forecasting procedures [36], [37].

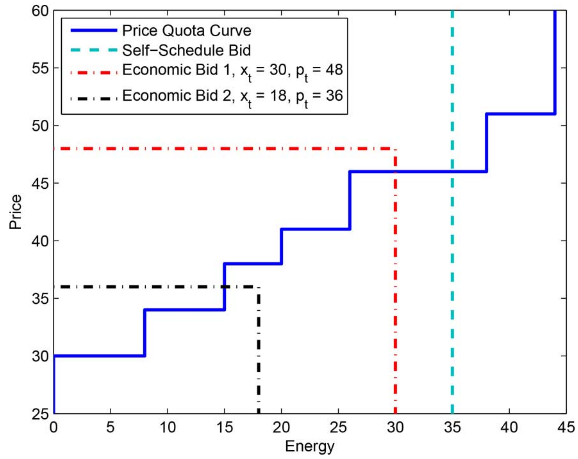

Fig. 1. Example for price-maker self-scheduling and price-maker economic bidding for a given price quota curve in a pool-based market.

form of step functions and they affect not only the price but also the amount of cleared energy quality, as shown in Fig. 2(a) and (b), respectively. In fact, under price-maker economic bidding, the cleared market price and the cleared energy quantity are *two-dimensional* functions $\lambda_t(x_t, p_t)$ and $q_t(x_t, p_t)$, respectively. For each price bid p_t, the cleared market price $\lambda_t(x_t, p_t)$ is a step-wise increasing function of energy bid x_t. Also, for each price bid p_t, the cleared energy quantity $q_t(x_t, p_t)$ is a straight identity line that is *saturated* beyond a certain threshold. Such threshold increases as the price bid p_t increases, allowing larger energy bids to be cleared in the day-ahead market. For the example in Fig. 2, if the energy bid is $x_t = 20$ MW and the price bid is $p_t = 36$ \$/MW, then we have $\lambda_t(x_t, p_t) = 36$ \$/MW and $q_t(x_t, p_t) = 15$ MW. If $x_t = 20$ MW and $p_t = 48$ \$/MW, then we have $\lambda_t(x_t, p_t) = 38$ \$/MW and $q_t(x_t, p_t) = 20$ MW. These numbers are marked on Fig. 2 for clarification.

C. Real-Time Market

Recall from Section II-A that the demand bids in real-time markets do not indicate any price quantity. In fact, in practice, the demand is only metered and then the payments corresponding to the real-time markets are calculated accordingly [38]. At each time slot t, let y_t denote the energy bid that is submitted (or metered) to the real-time market. The cleared market price and the cleared energy quantity are modeled as *one-dimensional* functions $\phi_t(y_t)$ and $g_t(y_t)$, respectively. The former is a step-wise increasing function of energy bid y_t. The latter is simply a straight identity line, i.e., $g_t(y_t) = y_t$.

D. Time-Shiftable Load

A time-shiftable load is a task that requires a certain total energy to finish, but its operation can be scheduled any time within a pre-determined tolerable time frame, where the end of such time frame is the *deadline* to finish operation. As we explained in Section I, time-shiftable loads have recently received a great deal of attention due to their role in demand response programs, e.g., see [1]–[15]. A time-shiftable load is modeled with at least three parameters α, β, and e. Parameters α and β indicate the

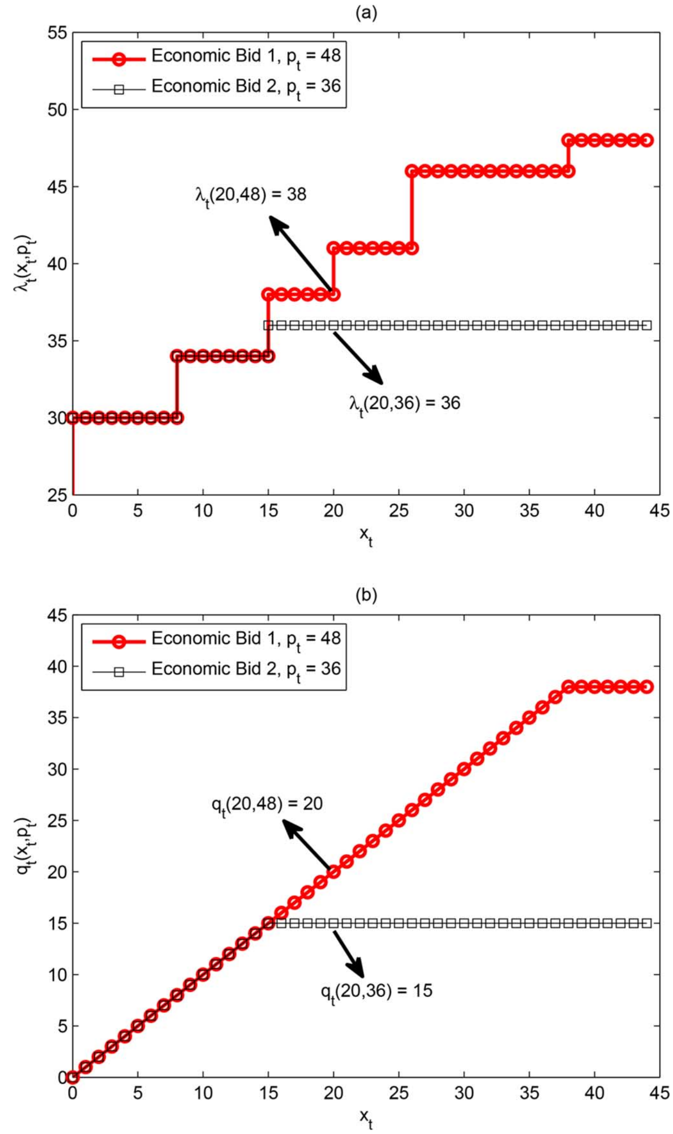

Fig. 2. Example for the market outcome under price-maker economic bidding for two different price bids: (a) cleared price of electricity; (b) cleared energy quantity. Here, the price quota curve is the same as the one in Fig. 1.

beginning and the end of the time interval at which the operation of the load can be scheduled, where $1 \le \alpha < \beta \le T$. A higher $\beta - \alpha$ indicates *more time flexibility*. Parameter e denotes the total energy that must be consumed in order to finish the time-shiftable task of interest.

The above model describes a time-shiftable load in its most generic form. However, in general, a time-shiftable load may have other characteristics, including inter-temporal dynamics, or other types of constraints. Several of such additional characteristics will be discussed in details later in Section IV.

E. Optimization Problem

In practice, market participation is prone to *uncertainty*. Let K denote the number of random market scenarios. At each time slot t and for each scenario $k = 1, \ldots, K$, the multiplications $q_{t,k}(x_t, p_t)\lambda_{t,k}(x_t, p_t)$ and $y_{t,k}\phi_{t,k}(y_{t,k})$ indicate the cost of power procurement from the day-ahead market and the real-time market, respectively. The price-maker economic bidding problem for time-shiftable loads can be formulated as

$$\begin{aligned}\mathbf{min} \quad & \frac{1}{K}\sum_{k=1}^{K}\sum_{t=\alpha}^{\beta} q_{t,k}(x_t,p_t)\lambda_{t,k}(x_t,p_t) + y_{t,k}\phi_{t,k}(y_{t,k}) \\ \mathbf{s.t.} \quad & \sum_{t=\alpha}^{\beta} q_{t,k}(x_t,p_t) + y_{t,k} = e, \quad \forall\, k \end{aligned} \tag{1}$$

where the optimization variables are x_t, p_t,, and $y_{t,k}$ for any time slot t and any market scenario k. The objective in (1) is to minimize the expected value of the total energy expenditure to finish the task. The equality constraints assure that for all scenarios, the total energy purchased matches the target energy level e. Note that, the energy bids to real-time market act as *recourse variables* [39], [40]. As a result, they are specific to each scenario k to purchase a total of $e - \sum_{t=\alpha}^{\beta} q_{t,k}(x_t,p_t)$ MWh energy from the real-time market under scenario k.

The nonlinear mixed-integer stochastic optimization problem in (1) is difficult to solve. In fact, it is recently shown in [22] that even if the time-shiftable load is small and price-taker, i.e., $\lambda_{t,k}$ and $\phi_{t,k}$ are independent of the bids, then solving problem (1) is still a challenging task due to the nonlinearity in $q_{t,k}(x_t,p_t)$. Nevertheless, we will next present an innovative method to find the global optimal solution of problem (1) within a short amount of computational time.

III. Proposed Solution Method

In this section, we explain how we can reformulate problem (1) as a mixed-integer linear program. This is done by taking four key steps. The reformulated optimization problem is then solved efficiently using various mixed-integer linear programming solvers, such as CPLEX [41] or MOSEK [42].

A. Problem Reformulation Steps

Step 1: At each time slot t and for each scenario k, we define $q^{\rm th}_{t,k}(p_t)$ as the maximum energy quantity that can be cleared in the day-ahead market when the price bid is p_t. For example, from Fig. 2(b), we have $q^{\rm th}_{t,k}(36) = 15$ MWh. This is because the cleared energy curve for any bid with $p_t = \$36$ is bounded by 15 MWh. In other words, if the time-shiftable load seeks to procure more than 15 MWh, then it must submit a price bid that is higher than \$36. As another example, we have $q^{\rm th}_{t,k}(48) = 38$ MWh. A method to model $q^{\rm th}_{t,k}(p_t)$ will be provided later in Step 3. However, for now, assume that the value of $q^{\rm th}_{t,k}(p_t)$ is given for each price bid p_t. We can write

$$q_{t,k}(x_t,p_t) = \begin{cases} x_t, & \text{if } x_t \le q^{\rm th}_{t,k}(p_t), \\ q^{\rm th}_{t,k}(p_t), & \text{otherwise.}\end{cases} \tag{2}$$

In other words, we have

$$q_{t,k}(x_t,p_t) = \min\left\{x_t,\, q^{\rm th}_{t,k}(p_t)\right\}. \tag{3}$$

Next, we define a new auxiliary variable as

$$\theta_{t,k} = \begin{cases} 0, & \text{if } x_t \le q^{\rm th}_{t,k}(p_t), \\ 1, & \text{otherwise.}\end{cases} \tag{4}$$

From (2)–(4), we have

$$\theta_{t,k} = 0 \Leftrightarrow q_{t,k}(x_t,p_t) = x_t \le q^{\rm th}_{t,k}(p_t) \tag{5}$$

and

$$\theta_{t,k} = 1 \Leftrightarrow q_{t,k}(x_t,p_t) = q^{\rm th}_{t,k}(p_t) \le x_t. \tag{6}$$

Interestingly, for any time slot t and any market scenario k, the relationships in (5) and (6) are equivalent to

$$\theta_{t,k} \in \{0,1\}, \tag{7}$$
$$x_t - \theta_{t,k}L \le q_{t,k}(x_t,p_t) \le x_t, \tag{8}$$
$$q^{\rm th}_{t,k}(p_t) - (1-\theta_{t,k})L \le q_{t,k}(x_t,p_t) \le q^{\rm th}_{t,k}(p_t) \tag{9}$$

where L is a large number compared to load size e. To show the above, we note that, if $\theta_{t,k} = 0$, then (8) and (9) become

$$x_t \le q_{t,k}(x_t,p_t) \le x_t, \tag{10}$$
$$q^{\rm th}_{t,k}(p_t) - L \le q_{t,k}(x_t,p_t) \le q^{\rm th}_{t,k}(p_t). \tag{11}$$

The lower bound and the upper bound in (10) are equal. Also, since L is a large number, the lower bound constraint in (11) is not binding. Therefore, we can conclude that the relationship in (5) holds. If $\theta_{t,k} = 1$, then (8) and (9) become

$$x_t - L \le q_{t,k}(x_t,p_t) \le x_t, \tag{12}$$
$$q^{\rm th}_{t,k}(p_t) \le q_{t,k}(x_t,p_t) \le q^{\rm th}_{t,k}(p_t) \tag{13}$$

The lower bound constraint in (11) is not binding. From this and because the lower bound and the upper bound in (10) are equal, we can conclude the relationship in (6).

Step 2: At each time slot t and for each scenario k, the cleared market price in the day-ahead market is obtained as

$$\lambda_{t,k}(x_t,p_t) = \begin{cases} \lambda_{t,k}(x_t,\infty) & \text{if } x_t \le q^{\rm th}_{t,k}, \\ p_t & \text{otherwise}\end{cases} \tag{14}$$

where $\lambda_{t,k}(x_t,\infty)$ is the price quota curve for an infinite price bid, i.e., the price curve under Self-Scheduling [30]. Next, we note that the cost of power procurement from the day-ahead market at time slot t and under scenario k is modeled as

$$C_{t,k}(x_t,p_t) = q_{t,k}(x_t,p_t)\,\lambda_{t,k}(x_t,p_t). \tag{15}$$

From (2) and (14), we can rewrite the above expression as

$$C_{t,k}(x_t,p_t) = \begin{cases} x_t\lambda_{t,k}(x_t,\infty) & \text{if } x_t \le q^{\rm th}_{t,k}, \\ q^{\rm th}_{t,k}(p_t)p_t & \text{otherwise.}\end{cases} \tag{16}$$

If $x_t \le q^{\rm th}_{t,k}(p_t)$, then $\lambda_{t,k}(x_t,\infty) \le p_t$. Accordingly, we have $x_t\lambda_{t,k}(x_t,\infty) \le q^{\rm th}_t(p_t)p_t$. Also, if $x_t > q^{\rm th}_t(p_t)$, then $\lambda_{t,k}(x_t,\infty) \ge p_t$. Accordingly, we have $x_t\lambda_{t,k}(x_t,\infty) \ge q^{\rm th}_t(p_t)p_t$. Therefore, we can rewrite (16) as

$$C_{t,k}(x_t,p_t) = \min\left\{x_t\lambda_{t,k}(x_t,\infty),\, q^{\rm th}_{t,k}(p_t)p_t\right\}. \tag{17}$$

From (4), (16), and (17), we have

$$\theta_{t,k} = 0 \Leftrightarrow C_{t,k}(x_t,p_t) = x_t\lambda_{t,k}(x_t,\infty) \le q^{\rm th}_{t,k}(p_t)p_t \tag{18}$$

and

$$\theta_{t,k} = 1 \Leftrightarrow C_{t,k}(x_t,p_t) = q^{\rm th}_{t,k}(p_t)p_t \le x_t\lambda_{t,k}(x_t,\infty). \tag{19}$$

Again, for any time slot t and any market scenario k, the relationships in (18) and (19) are equivalent to (7) and

$$x_t \lambda_{t,k}(x_t, \infty) - \theta_{t,k} L \leq C_{t,k}(x_t, p_t) \leq x_t \lambda_{t,k}(x_t, \infty), \tag{20}$$

$$q_t^{\text{th}}(p_t) p_t - (1 - \theta_{t,k}) L \leq C_{t,k}(x_t, p_t) \leq q_t^{\text{th}}(p_t) p_t \tag{21}$$

where L is again a large number. To show the above equivalence, we note that if $\theta_{t,k} = 0$, then (20) and (21) become

$$x_t \lambda_{t,k}(x_t, \infty) \leq C_{t,k}(x_t, p_t) \leq x_t \lambda_{t,k}(x_t, \infty), \tag{22}$$

$$q_t^{\text{th}}(p_t) p_t - L \leq C_{t,k}(x_t, p_t) \leq q_t^{\text{th}}(p_t) p_t. \tag{23}$$

The lower bound and the upper bound in (22) are equal. Also, since L is a large number, the lower bound constraint in (23) is not binding. Therefore, we can conclude that the relationship in (18) holds. If $\theta_{t,k} = 1$, then (20) and (21) become

$$x_t \lambda_{t,k}(x_t, \infty) - L \leq C_{t,k}(x_t, p_t) \leq x_t \lambda_{t,k}(x_t, \infty), \tag{24}$$

$$q_t^{\text{th}}(p_t) p_t \leq C_{t,k}(x_t, p_t) \leq q_t^{\text{th}}(p_t) p_t. \tag{25}$$

The lower bound constraint in (24) is not binding. From this and because the lower bound and the upper bound in (25) are equal, we can conclude the relationship in (19).

Step 3: At each time slot t and for each scenario k, the threshold $q_{t,k}^{\text{th}}(p_t)$ is a step-wise linear function of price bid p_t. For example, in Fig. 1, $q_{t,k}^{\text{th}}(p_t)$ is 0 for any $p_t < 30$, it is 8 for any $30 \leq p_t < 34$, it is 15 for any $34 \leq p_t < 38$, and so on and so forth. Following the general methodology in [30] for modeling step-wise linear functions, we can write

$$q_{t,k}^{\text{th}}(p_t) = \sum_{i=1}^{n_{t,k}} x_{t,k,s}^{\min} u_{t,k,s}, \tag{26}$$

$$p_t = \sum_{i=1}^{n_{t,k}} \left(a_{t,k,s} + u_{t,k,s} p_{t,k,s}^{\min} \right) \tag{27}$$

where

$$u_{t,k,s} \in \{0, 1\}, \tag{28}$$

$$0 \leq a_{t,k,s} \leq u_{t,k,s} a_{t,k,s}^{\max}, \tag{29}$$

$$\sum_{s=1}^{n_{t,k}} u_{t,k,s} = 1. \tag{30}$$

Here, $n_{t,k}$ is the number of price steps in the step-wise linear function $q_{t,k}^{\text{th}}(p_t)$, parameter $p_{t,k,s}^{\min}$ is the minimum price in step number s, parameter $x_{t,k,s}^{\min}$ is the cleared energy in step number s, parameter $a_{t,k,s}^{\max}$ is the width of step number s, $v_{t,k,s}$ and $a_{t,k,s}$ are auxiliary variables. For example, in Fig. 1, we have $p_{t,k,1}^{\min} = 0$, $p_{t,k,2}^{\min} = 30$, $p_{t,k,3}^{\min} = 34$, $p_{t,k,4}^{\min} = 38$, $x_{t,k,1}^{\min} = 0$, $x_{t,k,2}^{\min} = 8$, $x_{t,k,3}^{\min} = 15$, $x_{t,k,4}^{\min} = 20$, $a_{t,k,1}^{\max} = 30$, $a_{t,k,2}^{\max} = 4$, $a_{t,k,3}^{\max} = 4$, $a_{t,k,4}^{\max} = 3$, etc. We can also write

$$q_{t,k}^{\text{th}}(p_t)\, p_t = \sum_{s=1}^{n_{t,k}} x_{t,k,s}^{\min} \left(c_{t,k,s} + v_{t,k,s} p_{t,k,s}^{\min} \right). \tag{31}$$

Step 4: Finally, at each time slot t and for each scenario k, we can again adjust the modeling approach in [30] and write

$$x_t \lambda_{t,k}(x_t, \infty) = \sum_{s=1}^{m_{t,k}} \lambda_{t,k,s} \left(b_{t,k,s} + v_{t,k,s} x_{t,k,s}^{\min} \right), \tag{32}$$

$$x_t = \sum_{s=1}^{m_{t,k}} \left(b_{t,k,s} + v_{t,k,s} x_{t,k,s}^{\min} \right), \tag{33}$$

and

$$y_{t,k} \phi_{t,k}(y_{t,k}) = \sum_{s=1}^{o_{t,k}} \phi_{t,k,s} \left(c_{t,k,s} + w_{t,k,s} y_{t,k,s}^{\min} \right), \tag{34}$$

$$y_{t,k} = \sum_{s=1}^{o_{t,k}} \left(c_{t,k,s} + w_{t,k,s} y_{t,k,s}^{\min} \right) \tag{35}$$

where

$$v_{t,k,s} \in \{0, 1\}, \tag{36}$$

$$w_{t,k,s} \in \{0, 1\}, \tag{37}$$

$$0 \leq b_{t,k,s} \leq v_{t,k,s} b_{t,k,s}^{\max}, \tag{38}$$

$$0 \leq c_{t,k,s} \leq w_{t,k,s} c_{t,k,s}^{\max}, \tag{39}$$

$$\sum_{s=1}^{m_{t,k}} v_{t,k,s} = 1, \tag{40}$$

$$\sum_{s=1}^{o_{t,k}} w_{t,k,s} = 1. \tag{41}$$

Here, parameters $m_{t,k}$, $x_{t,k,s}^{\min}$, and $b_{t,k,s}^{\max}$ characterize the step-wise linear day-ahead price quota curve $\lambda_{t,k}(x_t, \infty)$ under Self-Schedule bidding; and $o_{t,k}$, $y_{t,k,s}^{\min}$, and $c_{t,k,s}^{\max}$ characterize the step-wise linear real-time price quota curve $\phi_{t,k}(y_{t,k})$. For example, based on the curves in Figs. 1 and 2, we have $b_{t,k,1}^{\max} = 8$, $b_{t,k,2}^{\max} = 7$, $b_{t,k,3}^{\max} = 5$, etc.

B. Resulted Mixed-Integer Linear Program

After applying the changes in the four steps in Section III-A, we can reformulate optimization problem (1) as

$$\begin{aligned}
\mathbf{min} \quad & \frac{1}{K} \sum_{k=1}^{K} \sum_{t=\alpha}^{\beta} C_{t,k} + \sum_{s=1}^{o_{t,k}} \phi_{t,k,s} \left(c_{t,k,s} + w_{t,k,s} y_{t,k,s}^{\min} \right) \\
\mathbf{s.t.} \quad & \sum_{t=\alpha}^{\beta} q_{t,k} + y_{t,k} = e, \quad \forall k, \\
& x_t - \theta_{t,k} L \leq q_{t,k}, \quad \forall t, k, \\
& q_{t,k} \leq x_t, \quad \forall t, k, \\
& q_{t,k}^{\text{th}} - (1 - \theta_{t,k}) L \leq q_{t,k}, \quad \forall t, k, \\
& q_{t,k} \leq q_{t,k}^{\text{th}}, \quad \forall t, k, \\
& \sum_{s=1}^{m_{t,k}} \lambda_{t,k,s} \left(b_{t,k,s} + v_{t,k,s} x_{t,k,s}^{\min} \right) \\
& \quad - \theta_{t,k} L \leq C_{t,k}, \quad \forall t, k, \\
& C_{t,k} \leq \sum_{s=1}^{m_{t,k}} \lambda_{t,k,s} \left(b_{t,k,s} + v_{t,k,s} x_{t,k,s}^{\min} \right), \quad \forall t, k, \\
& \sum_{s=1}^{n_{t,k}} x_{t,k,s}^{\min} \left(c_{t,k,s} + v_{t,k,s} p_{t,k,s}^{\min} \right) \\
& \quad - (1 - \theta_{t,k}) L \leq C_{t,k}, \quad \forall t, k, \\
& C_{t,k} \leq \sum_{s=1}^{n_{t,k}} x_{t,k,s}^{\min} \left(c_{t,k,s} + v_{t,k,s} p_{t,k,s}^{\min} \right), \quad \forall t, k, \\
& q_{t,k}^{\text{th}} = \sum_{i=1}^{n_{t,k}} x_{t,k,s}^{\min} u_{t,k,s}, \quad \forall t, k,
\end{aligned}$$

$$
\begin{aligned}
& p_t = \sum_{i=1}^{n_{t,k}} \left(a_{t,k,s} + u_{t,k,s} p_{t,k,s}^{\min} \right), \quad \forall t, k, \\
& x_t = \sum_{s=1}^{m_{t,k}} \left(b_{t,k,s} + v_{t,k,s} x_{t,k,s}^{\min} \right), \quad \forall t, k, \\
& y_{t,k} = \sum_{s=1}^{o_{t,k}} \left(c_{t,k,s} + w_{t,k,s} y_{t,k,s}^{\min} \right), \quad \forall t, k, \\
& 0 \le a_{t,k,s} \le u_{t,k,s} a_{t,k,s}^{\max}, \quad \forall t, k, s, \\
& 0 \le b_{t,k,s} \le v_{t,k,s} b_{t,k,s}^{\max}, \quad \forall t, k, s, \\
& 0 \le c_{t,k,s} \le w_{t,k,s} c_{t,k,s}^{\max}, \quad \forall t, k, s, \\
& \sum_{s=1}^{n_{t,k}} u_{t,k,s} = 1, \quad \forall t, k, \\
& \sum_{s=1}^{m_{t,k}} v_{t,k,s} = 1, \quad \forall t, k, \\
& \sum_{s=1}^{o_{t,k}} w_{t,k,s} = 1, \quad \forall t, k, \\
& u_{t,k,s}, w_{t,k,s}, v_{t,k,s} \in \{0, 1\}, \quad \forall t, k, s
\end{aligned} \tag{42}
$$

where the optimization variables are x_t, p_t, $y_{t,k}$, $\theta_{t,k}$, $q_{t,k}$, $q_{t,k}^{\text{th}}$, $C_{t,k}$, $a_{t,k,s}$, $b_{t,k,s}$, $c_{t,k,s}$, $u_{t,k,s}$, $v_{t,k,s}$, and $w_{t,k,s}$ for any time slot t, any market scenario k, and any step number s. The problem in (42) is a mixed-integer linear program.

IV. More Complex Time-Shiftable Loads

The model that we used in our analysis so far describes a time-shiftable load in its most generic form. In this section, we explain how other characteristics of time-shiftable loads can also be incorporated into the analysis. More specifically, we show that the optimal bidding framework in this paper can include any other feature of time-shiftable loads, as long as the feature can be modeled as linear mixed-integer constraints.

A. Per-Time-Slot Consumption Limits

Some time-shiftable loads may have limitations on their consumption level at each time slot. Let $Z^{\min}$ and $Z^{\max}$ denote the minimum and maximum consumption levels that the time-shiftable load of interest can support. We must have

$$Z^{\min} r_{t,k} \le z_{t,k} \le Z^{\max} r_{t,k} \quad \forall t, \forall k \tag{43}$$

where for each time slot t and random scenario k, we have

$$z_{t,k} = q_{t,k} + y_{t,k} \tag{44}$$

and $r_{t,k}$ is a new binary variable to indicate whether the load is switched "on" or "off" at time slot t and under scenario k. This new variable will be useful also later in Section IV-C.

B. Ramp Constraints

The ramp up and ramp down constraints do not allow the time-shiftable load to change its consumption level faster than certain rates within two consecutive time slots:

$$z_{t,k} - z_{t-1,k} \le U^{\max} \quad \forall t \ge 2, \forall k \tag{45a}$$

$$z_{t-1,k} - z_{t,k} \le D^{\max} \quad \forall t \ge 2, \forall k \tag{45b}$$

where $U^{\max}$ and $D^{\max}$ denote the maximum ramp up and maximum ramp down rates, respectively. Note that the constraints in (45) address one type of *inter-temporal dependency* in time-shiftable loads. Another type is discussed next.

C. Uninterruptible Loads

If a time-shiftable load is *uninterruptible*, then as soon as it switches "on" to start operation, it must continue its operation until it finishes its intended task. Based on the notations that we defined in Section IV-A, the following constraints must hold for an uninterruptable time-shiftable load:

$$r_{t,k} \le r_{t+1,k} + \frac{1}{e} \sum_{\tau=1}^{t} z_{\tau,k} \quad \forall t \le T-1, \forall k. \tag{46}$$

From (46), at time slot t and under scenario k, we can choose $r_{t,k} = 1$ only if either $r_{t+1,k} = 1$, i.e., the operation of the load continues in the next time slot, or $\sum_{\tau=1}^{t} z_{\tau,k} = e$, i.e., the operation of the load finishes by the end of the current time slot [9]. Note that, the constraints in (46) address yet another type of inter-temporal dependency in time-shiftable loads.

D. Aggregated Small Sub-Loads

In some cases, a time-shiftable load may consist of several smaller time-shiftable subloads or subtasks [22]. In that case, besides selecting the day-ahead and real-time market bids, we must also optimally schedule the operation of all subloads. Let $S \ge 1$ denote the number of time-shiftable subloads. For each subload $s = 1, \ldots, S$, let α_s and β_s denote the beginning and the end of the time interval at which the subload can be scheduled. Also let e_s denote the total energy that must be consumed in order to finish the operation of subload s. We can incorporate the problem of scheduling subloads by adding the following constraints into the problem formulation:

$$\sum_{s=1}^{S} z_{t,k,s} = z_{t,k} \quad \forall t, \forall k, \tag{47}$$

$$\sum_{t=\alpha_l}^{\beta_l} z_{t,k,s} = e_l \quad \forall s, \forall k. \tag{48}$$

Note that, if there is only one subload, i.e., $S = 1$, then (47) reduces to (44); and (48) reduces to the first constraint in (42).

V. Case Studies

A. Case Study 1: A Detailed Illustrative Example

In this section, we present a detailed illustrative example. Suppose we would like to procure energy for a time-shiftable load with start time $\alpha = 1$, deadline $\beta = T = 3$, and total energy consumption $e = 75$ MWh. The uncertainty in the electricity market is modeled using $K = 2$ scenarios.

1) Basic Time-Shiftable Loads: First, consider the most generic time-shiftable load model in Section II-D. The price quota curves for the day-ahead and the real-time markets and the corresponding optimal bids are shown in Fig. 3. These optimal solutions are first obtained by solving the mixed-integer linear program in (42) and then the results are verified using exhaustive search. Using a computer with a 2.40-GHz CPU

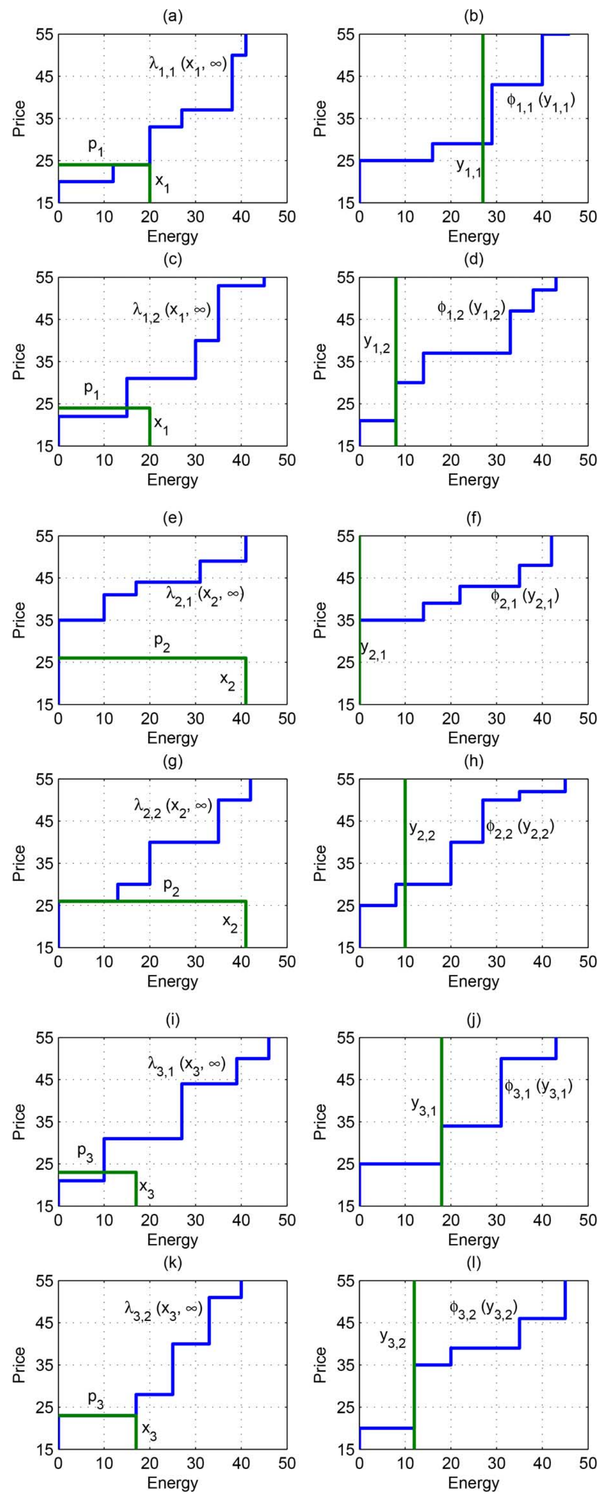

Fig. 3. Price quota curves and optimal bids for Case Study 1, where the time-shiftable load has its basic features as addressed in Section V-A1. Sub-figures (a), (c), (e), (g), (i), and (k) correspond to the day-ahead market and sub-figures (b), (d), (f), (h), (j), and (l) correspond to the real-time market.

and 80 GB of shared RAM, the mixed-integer linear program in (42) was solved in *less than 1 second*. However, it took *multiple days* for the exhaustive search with several *for loops* to finish the search and give the exact same solution.

From Fig. 3, we can see that the bidding outcome and the schedule of the time-shiftable load across time slots highly de-

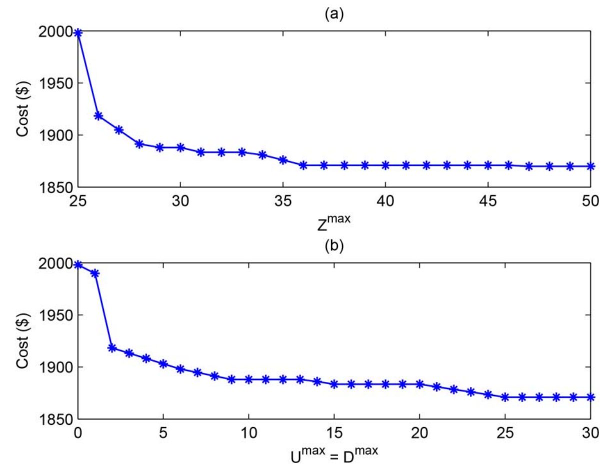

Fig. 4. Cost of energy procurement for Case Study 1 when the time-shiftable has (a) per-time-slot consumption limits and (b) ramp constraints.

pends on the realization of the market scenario. For example, if scenario $k = 1$ occurs, then the power consumption at time slot $t = 1$ becomes 47 MWh, out of which 20 MWh is procured from the day-ahead market at 24 \$/MW and 27 MWh is procured from the real-time market at 29 \$/MW. In this scenario, because the prices are high at time slot $t = 2$, no energy usage is scheduled at this time slot. Finally, the power consumption at time slot $t = 3$ and scenario $k = 1$ is $75 - 47 = 28$ MWh, out of which 10 MWh is procured from the day-ahead market at 23 \$/MW and 18 MWh is procured from the real-time market at 25 \$/MW. The total cost of power purchase from the day-ahead market in this scenario is $C_{1,1} + C_{2,1} + C_{3,1} = 20 \times 24 + 0 \times 0 + 10 \times 23 = \$710$. Also, the total cost of power procurement from the real-time market is obtained as $27 \times 29 + 0 \times 0 + 18 \times 25 = \$1\,233$.

We can similarly calculate the total cost of power procurement from the day-ahead market and the total cost of power procurement from the real-time market under scenario $k = 2$ as \$1089 and \$708, respectively. Therefore, the *expected overall cost of power procurement*, i.e., the objective value in optimization problem (1) becomes \$1870. Note that if we use the price-maker Self-Schedule bidding in [30], then the total expected cost of power procurement becomes \$1904, i.e., \$34 higher than our proposed price-maker economic bidding method. Also, if we do *even load distribution*, i.e., we distribute the total load $e = 75$ MWh equally across the $\beta - \alpha + 1 = 3$ time-slots and also equally across the day-ahead and real-time markets, then the total expected cost of power procurement becomes \$2169, i.e., \$299 higher than our proposed price-maker economic bidding method.

2) Time-Shiftable Loads With Consumption Limits: Next, we consider the basic time-shiftable load model, but we also assume that there exist per-time-slot consumption limits as in Section IV-A. The results are shown in Fig. 4(a), where $Z^{\min} = 0$ MWh and $Z^{\max}$ varies from 25 to 50 MWh. We can see that the optimal energy procurement cost is high if the operation of the time-shiftable load is highly restricted due to the per-time-slot power consumption constraints. However, as we increase $Z^{\max}$, the cost reduces and finally reaches its original level as in Section V-A1, where $Z^{\max}$ is not binding.

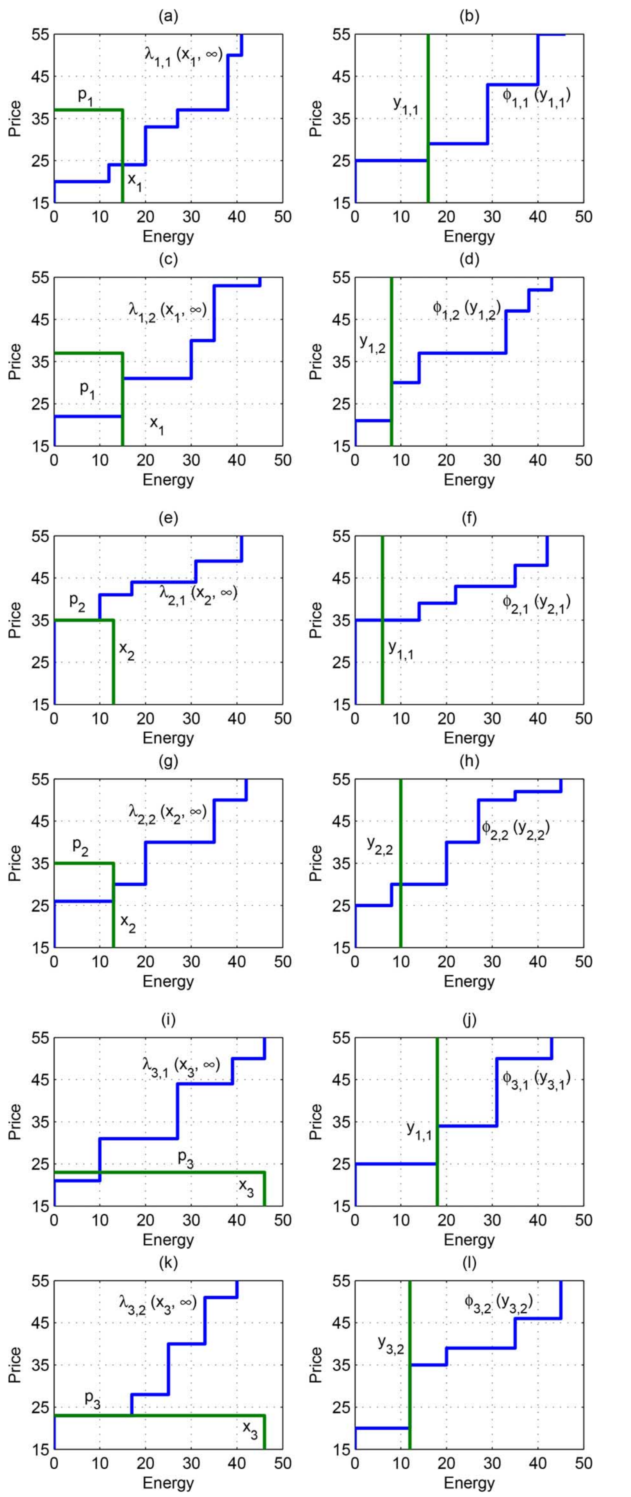

Fig. 5. Price quota curves and optimal bids for Case Study 1, where the time-shiftable load uninterrupitble as addressed in Section V-A4. Sub-figures (a), (c), (e), (g), (i), and (k) correspond to the day-ahead market and sub-figures (b), (d), (f), (h), (j), and (l) correspond to the real-time market.

3) Time-Shiftable Loads With Ramp Constraints: Again, consider the basic time-shiftable load model, but this time assume that there exist ramp constraints as in Section IV-B. The results are shown in Fig. 4(b), where $U^{\max} = D^{\max}$ vary from 0 to 30 MWh. Note that, if $U^{\max} = D^{\max} = 0$, then the load does not tolerate any inter-temporal variation. We can see that ramp constraints can significantly increase the energy procurement cost. However, as we increase $U^{\max}$ and $D^{\max}$, the cost reduces and finally reaches its original level as in Section V-A1, where the ramp constraints are not binding.

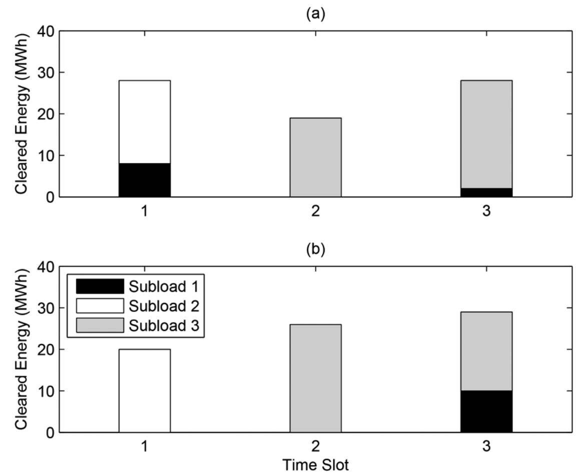

Fig. 6. Procured energy for different time-shiftable sub-loads at different time slots as in Section V-A5. (a) Scenario $k = 1$. (b) Scenario $k = 2$.

4) Uninterruptible Time-Shiftable Loads: Recall from Section IV-C that if a time-shiftable load is uninterruptible, then it is still flexible with respect to its operation start time; however, once it starts operation, it cannot be interrupted until it finishes its task. Here, interruption is defined as selecting $z_{t,k} < Z^{\min}$, which requires choosing $r_{t,k} = 0$, i.e., switching the load off. The optimal bids when the time-shiftable load is uninterruptible is shown in Fig. 5, where $Z^{\min} = 15$. We can see that, the time-shiftable load procures energy from all three time slots, including the second time slot which has high prices. This is because, unlike in Fig. 3, here, the operation cannot be interrupted during the second time slot and then resumed during the third time slot. Note that, since an uninterruptible time-shiftable load is less flexible than a basic time-shiftable load, it pays 14$ more for its energy procurement compared to the case in Section V-A1.

5) Aggregated Time-Shiftable Subloads: To study the impact of time-shiftable subloads on the choice of demand bids, the total load $e = 75$ MWh is now divided into three sub-loads as follow: 1) $e_1 = 10$, $\alpha_1 = 1$, $\beta_1 = 3$, 2) $e_2 = 20$, $\alpha_2 = 1$, $\beta_2 = 2$, and 3) $e_3 = 45$, $\alpha_3 = 2$, $\beta_3 = 3$. Fig. 6 shows the procured energy for different loads at scenarios $k = 1$ and $k = 2$. We can see that the different start and end-times for sub-loads affects the amount of total energy that needs to be procured under each scenario and at each time slot.

B. Case Study 2: California Energy Market

In this section, we present some additional case studies, this time based on the California energy market. To create the price quota curves, we used the hourly generator bids data from the public bids database in [35] at one dollar price bid resolution. The day-ahead and real-time prices are also obtained from the prices database in [35], where we averaged real-time market prices in each hour to make them comparable with the hourly price data from the day-ahead market. Finally, since the focus

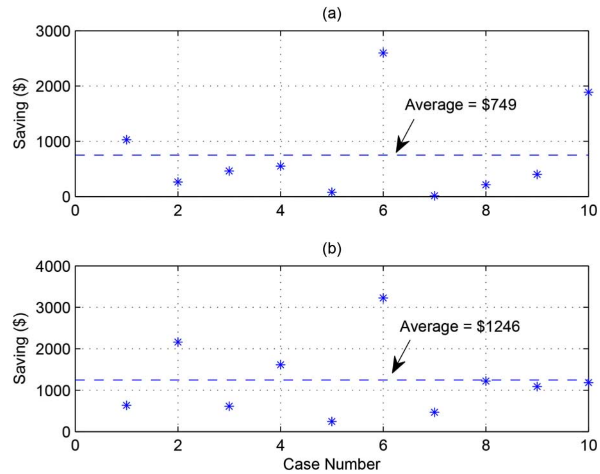

Fig. 7. Savings due to using optimal price-maker economic bidding over optimal price-maker self-scheduling. (a) Off-peak hours. (b) Peak hours.

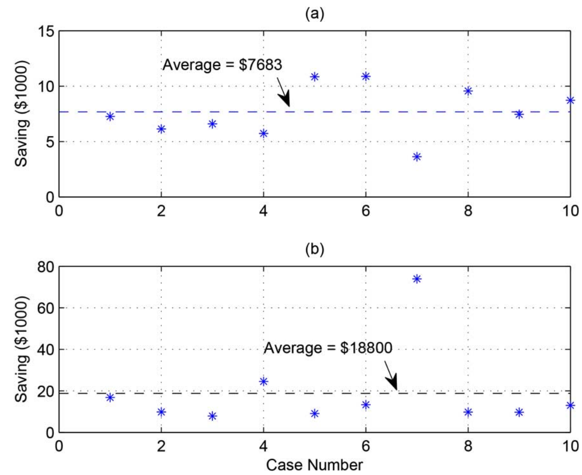

Fig. 8. Savings due to using optimal price-maker economic bidding over even load distribution. (a) Off-peak hours. (b) Peak hours.

in this paper is on pool-based markets, the grid topology and transmission constraints are not considered in our simulations.

In total, we examined 10 cases. Each case has a time shiftable load with $E = 10$ GWh and $K = 3$ market scenarios. For Case 1, the three scenarios are based on the price and bid data during January 1, 2014 to January 3, 2014. For Case 2, the three scenarios are based on the price and bid data during January 4, 2014 to January 6, 2014. The rest of the cases are setup similarly, all together using data for 30 days. For each case, three design options are compared: 1) Optimal price-marker self-schedule bidding, which is an extension of the design in [30] to both day-ahead and real-time markets; 2) Optimal price-marker economic bidding, which is based on the design in this paper; and 3) Even load distribution, which distributes the load equally across time-slots and markets.

The amount of savings due to using optimal price-maker economic bidding over optimal price-maker self-schedule bidding across the 10 cases are shown in Fig. 7(a) and (b), during some off-peak hours from $\alpha = 10:00$ AM to $\beta = 12:00$ PM and also during some on-peak hours from $\alpha = 15:00$ PM to $\beta = 17:00$ PM, receptively. Similarly, the amount of savings due to using optimal price-maker economic bidding over even load distribution across the 10 cases are shown in Fig. 8.

Finally, the detailed simulation results for the example of Case 1 during off-peak hours are shown in Figs. 9 and 10. We can see major differences across the three designs, in terms of both the average cleared energy and the average purchase price. Note that, the averaging here is done across the $K = 3$ random market scenarios. For instance, on average, if optimal price-maker economic bidding is employed, then 29.8%, 33.7% and 36.5% of the total needed energy is purchased from the day-ahead and real-time markets during hours 10:00 AM, 11:00 AM, and 12:00 PM, respectively. These percentages change to 30.6%, 38.1%, and 21.3% if optimal price-maker self-scheduling is being employed. As for the price results in Fig. 10, an interesting observation is that optimal price-maker economic bidding is more successful in *smoothing down* the prices across the three operational hours and also to some extent across the day-ahead and real-time markets.

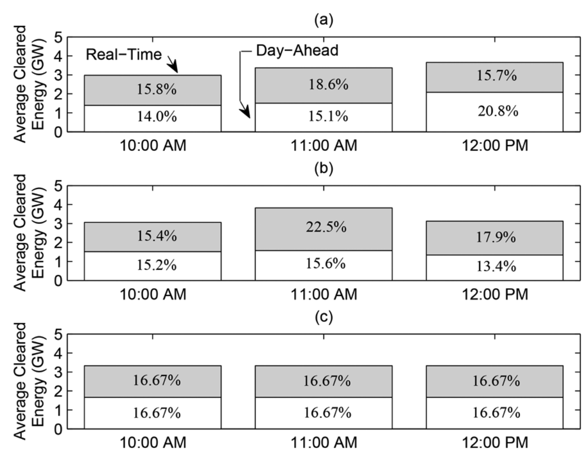

Fig. 9. Comparison between the three designs in terms of average cleared energy for Case 1 during off-peak hours. (a) Optimal price-maker economic bidding. (b) Optimal price-maker self-scheduling. (c) Even load distribution.

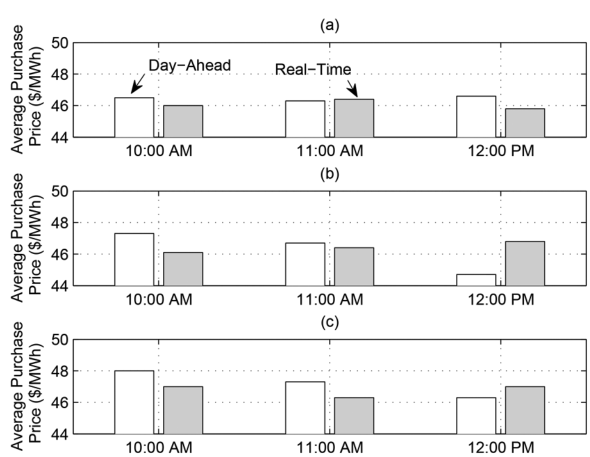

Fig. 10. Comparison between the three designs in terms of average purchase price for Case 1 during off-peak hours. (a) Optimal price-maker economic bidding. (b) Optimal price-maker self-scheduling. (c) Even load distribution.

Finally, the computation time of the proposed method versus the number of random scenarios is shown in Table I. As one would expect, increasing the number of random scenarios results in increasing the number of optimization variables, i.e., in-

TABLE I
COMPUTATION TIME VERSUS THE NUMBER OF SCENARIOS

# of Scenarios	# of Variables	# of Binary Variables	Computation Time (Sec)
1	228	105	0.7
2	406	188	0.8
3	570	264	2.7
4	760	353	6.2
5	958	446	23
6	1080	501	37
7	1214	562	91
8	1422	660	289
9	1616	751	897
10	1804	839	1478

creasing the size of the optimization problem. Accordingly, the computation time increases. When it comes to solving mixed-integer linear programs, the computation time particularly depends on the number of binary variables, because it indicates the maximum branching steps needed when we use a branch and bound algorithm [43]. From Table I, despite the increased computational complexity, one can still use the proposed method in this paper under larger sets of random scenarios. If needed, one can lower the price resolution at price quota curve, e.g., by setting resolution to \$2 instead of \$1, so as to decrease the number of steps in the price quota curve in order to further lower the computation time.

VI. CONCLUSION

We formulated and efficiently solved a new scenario-based stochastic mixed-integer linear programming framework for price-maker economic bidding of time-shiftable loads with deadlines in day-ahead and real-time markets. On the application side, the results in this paper extended some recent results in price-taker operation of time-shiftable loads in wholesale electricity markets. Both basic and complex time-shiftable load types are addressed, where the latter includes time-shiftable loads that are uninterruptible, have per-time-slot consumption limits or ramp constraints, or comprise several smaller time-shiftable subloads. On the methodology side, the results in this paper also extended the existing results on price-maker self-scheduling of both loads and generators, because price-maker self-scheduling is a restricted special case of price-maker economic bidding. To investigate the performance of our design, a highly detailed illustrative case study along with multiple case studies based on the California energy market data are presented. We showed that the proposed optimal price-maker economic bidding approach outperforms both optimal price-maker self-scheduling and even-load-distribution.

REFERENCES

[1] S. Shao, T. Zhang, M. Pipattanasomporn, and S. Rahman, "Impact of TOU rates on distribution load shapes in a smart grid with PHEV penetration," in *Proc. IEEE PES Transmission and Distribution Conf. Expo.*, New Orleans, LA, USA, Apr. 2010.

[2] H. Sherif, Z. Zhu, and S. Lambotharan, "An optimization framework for home demand side management incorporating electric vehicles," in *Proc. IEEE PES ISGT'14*, Kuala Lumpur, Malaysia, May 2014.

[3] G. Marks, E. Wilcox, D. Olsen, and S. Goli, Opportunities for Demand Response in California Agricultural Irrigation: A Scoping Study, Lawrence Berkeley Nat. Lab., Berkeley, CA, USA, Jan. 2013, Tech. Rep.

[4] B. Aksanli, J. Venkatesh, L. Zhang, and T. Rosing, "Utilizing green energy prediction to schedule mixed batch and service jobs in data centers," in *Proc. ACM Workshop Power-Aware Computing and Systems*, Cascais, Portugal, Oct. 2011.

[5] Z. Liu, I. Liu, S. Low, and A. Wierman, "Pricing data center demand response," in *Proc. ACM Sigmetrics*, Austin, TX, USA, Jun. 2014.

[6] M. Ghamkhari and H. Mohsenian-Rad, "Energy and performance management of green data centers: A profit maximization approach," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 1017–1025, Jun. 2013.

[7] S. Meyn, P. Barooah, A. Busic, and J. Ehren, "Ancillary service to the grid from deferrable loads: The case for intelligent pool pumps in Florida," in *Proc. IEEE Conf. Decision and Control*, Florence, Italy, Dec. 2013.

[8] K. Vanthournout, R. D'hulst, D. Geysen, and G. Jacobs, "A smart domestic hot water buffer," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 2121–2127, Dec. 2012.

[9] A. Gholian, H. Mohsenian-Rad, Y. Hua, and J. Qin, "Optimal industrial load control in smart grid: A case study for oil refineries," in *Proc. IEEE PES General Meeting*, Vancouver, BC, Canada, Jul. 2013.

[10] P. Yang, G. Tang, and A. Nehorai, "A game-theoretic approach for optimal time-of-use electricity pricing," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 884–892, May 2013.

[11] Z. Yu, L. Jia, M. C. Murphy-Hoye, A. Pratt, and L. Tong, "Modeling and stochastic control for home energy management," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2244–2255, Dec. 2013.

[12] A. S. Al-Sumaiti, M. H. Ahmed, and M. M. A. Salama, "Electric power components and systems," *Smart Home Activ.: Lit. Rev.*, vol. 42, no. 3–4, pp. 294–305, Feb. 2014.

[13] H. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 120–133, Sep. 2010.

[14] H. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Trans. Smart Grid*, vol. 1, no. 3, pp. 320–331, Dec. 2010.

[15] Z. Zhu, J. Tang, S. Lambotharan, W. Chin, and Z. Fan, "An integer linear programming based optimization for home demand-side management in smart grid," in *Proc. IEEE PES ISGT*, Washington, DC, USA, Jun. 2012.

[16] X. Chen, T. Wei, and S. Hu, "Uncertainty-aware household appliance scheduling considering dynamic electricity pricing in smart home," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 932–941, Jun. 2013.

[17] M. Alizadeh, T. H. Chang, and A. Scaglione, "On modeling and marketing the demand flexibility of deferrable loads at the wholesale level," in *Proc. IEEE HICSS*, Grand Wailea, HI, USA, Jan. 2013.

[18] M. C. Caramanis, E. Goldis, P. A. Ruiz, and A. Rudkevich, "Power market reform in the presence of flexible schedulable distributed loads. new bid rules, equilibrium and tractability issues," in *Proc. Annu. Allerton Conf. Communication, Control, and Computing*, Urbana, IL, USA, Oct. 2012.

[19] D. Materassi, M. Roozbehani, and M. A. Dahleh, "Equilibrium price distributions in energy markets with shiftable demand," in *Proc. IEEE Conf. Decision and Control*, Maui, HI, USA, Dec. 2012.

[20] M. C. Caramanis and J. M. Foster, "Coupling of day ahead and real-time power markets for energy and reserves incorporating local distribution network costs and congestion," in *Proc. Annu. Allerton Conf. Communication, Control, and Computing*, Urbana, IL, USA, Sep. 2010.

[21] C. O. Adika and L. Wang, "Demand-side bidding strategy for residential energy management in a smart grid environment," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1724–1733, Jul. 2014.

[22] H. Mohsenian-Rad, "Optimal demand bidding for time-shiftable loads," *IEEE Trans. Power Syst.*, to be published.

[23] M. C. Caramanis and J. M. Foster, "Uniform and complex bids for demand response and wind generation scheduling in multi-period linked transmission and distribution markets," in *Proc. IEEE Conf. Decision and Control*, Orlando, FL, USA, Dec. 2011.

[24] J. M. Foster and M. C. Caramanis, "Optimal power market participation of plug-in electric vehicles pooled by distribution feeder," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2065–2076, Aug. 2013.

[25] M. Zarif, M. H. Javidi, and M. S. Ghazizadeh, "Self-scheduling of large consumers with second-order stochastic dominance constraints," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 289–299, Feb. 2013.

[26] H. Yan and H. Yan, "Optimal energy purchase in deregulated California energy markets," in *Proc. IEEE PES Winter Meeting*, Las Vegas, NV, USA, Feb. 2000.

[27] California ISO [Online]. Available: http://www.caiso.com/1c78/1c788230719c0.pdf

[28] ERCOT [Online]. Available: http://www.ercot.com/content/services/programs/load/Load_Participation_in_the_ERCOT_Nodal_Market_v1_0_061107.pdf
[29] PJM [Online]. Available: http://www.pjm.com/~/media/committees-groups/task-forces/ttf/20120523/20120523-day-ahead-market-modeling-presentation.ashx
[30] S. de la Torre, J. M. Arroyo, A. J. Conejo, and J. Contreras, "Price maker self-scheduling in a pool-based electricity market: A mixed-integer LP approach," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 1037–1042, Nov. 2002.
[31] G. Steeger, L. A. Barroso, and S. Rebennack, "Optimal bidding strategies for hydro-electric producers: A literature survey," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1758–1766, Jul. 2014.
[32] A. G. Bakirtzis, N. P. Ziogos, A. C. Tellidou, and G. A. Bakirtzis, "Electricity producer offering strategies in day-ahead energy market with step-wise offers," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1804–1818, Nov. 2007.
[33] A. Baillo, M. Ventosa, M. Rivier, and A. Ramos, "Optimal offering strategies for generation companies operating in electricity spot markets," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 745–753, May 2004.
[34] G. Baslis and G. Bakirtzis, "Mid-term stochastic scheduling of a price-maker hydro producer with pumped storage," *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 1856–1865, Nov. 2011.
[35] [Online]. Available: http://oasis.caiso.com
[36] G. B. Sheblé, *Computational Auction Mechanisms for Restructured Power Industry Operation*. Norwell, MA, USA: Kluwer, 1999.
[37] G. Aneiros, J. M. Vilar, R. Cao, and A. M. S. Roque, "Functional prediction for the residual demand in electricity spot markets," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4201–4208, Nov. 2013.
[38] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems*. New York, NY, USA: Wiley, 2002.
[39] P. Kall and S. W. Wallace, *Stochastic Programing*, 2nd ed. New York, NY, USA: Wiley, 1994.
[40] K. Marti, *Stochastic Optimization Methods*. New York, NY, USA: Springer, 2005.
[41] [Online]. Available: http://www.ibm.com/developerworks/downloads/ws/ilogcplex
[42] [Online]. Available: http://www.mosek.com
[43] S. Rao, *Engineering Optimization*, 4th ed. Hoboken, NJ, USA: Wiley, 2009.

Mahdi Kohansal (S'14) received the M.Sc. degree in electrical engineering from Amirkabir University of Technology, Tehran, Iran, in 2013. He is now pursuing the Ph.D. degree at the University of California at Riverside, Riverside, CA, USA.

His research interests include the application of mathematical optimization in power systems, electricity markets, and smart power grids.

Hamed Mohsenian-Rad (S'04–M'09–SM'14) received the Ph.D. degree in electrical and computer engineering from the University of British Columbia, Vancouver, BC, Canada, in 2008.

Currently, he is an Assistant Professor of Electrical Engineering at the University of California at Riverside, Riverside, CA, USA. His research interests are the design, optimization, and game-theoretic analysis of power systems and electricity markets.

Dr. Mohsenian-Rad is the recipient of the National Science Foundation (NSF) CAREER Award 2012, the Best Paper Award from the IEEE Power and Energy Society General Meeting 2013, and the Best Paper Award from the IEEE International Conference on Smart Grid Communications 2012. He serves as Editor for the IEEE TRANSACTIONS ON SMART GRID, the IEEE COMMUNICATIONS SURVEYS AND TUTORIALS, and the IEEE COMMUNICATIONS LETTERS.