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Quark spin content of vector mesons

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The European Muon Collaboration [1] results on polarized proton structure functions have sparked an interest in the fraction of the proton's spin born by quarks. A naive interpretation yielded the improbable result that none of the proton's spin was carried by the quarks and all the spin had to be in the gluon sector. This is modified when one takes the QCD chiral anomaly into account [2]. Veneziano [3] and Shore and Veneziano [3] obtained a Goldberger-Treiman-type relation for the proton's quark spin content. Efremov, Soffer, and Törnqvist [4,5] used such relations, together with an assumption on the large momentum behavior of a certain matrix element and reasonable estimate for the \( \eta' \)-nucleon coupling constant, to show that most of the proton's spin is carried by quarks. In this work we obtain a similar relation for the quark spin content of vector mesons. For the \( \rho \) and \( \omega \) mesons, vector dominance applied to radiative \( \eta' \) decays provides us with the relevant coupling constants, and a bound on the rate for \( \phi \to \eta' \gamma \) gives a limit on the \( \eta' \)-\( \phi \) coupling. With the above assumptions, we find that the quarks carry only about 30% of the spin of these particles.

First we briefly review the results of Refs. [3,4,5]. The usual flavor-singlet, axial-vector current is

\[
j_{\mu}^A = \sum_{f=1}^{N_f} \bar{q}_f \gamma_\mu \gamma_5 q_f ,
\]

while \( k_\mu \) is the gauge-dependent topological current whose divergence is \( Q \):

\[
Q = \frac{\alpha_s}{8\pi} N_f G_{a,\mu\nu} G^{\mu\nu}_a ;
\]

\( N_f \) is the number of flavors and \( G \) is the gauge field strength tensor. In the chiral limit we have

\[
\partial^\mu j_\mu = Q .
\]

The matrix elements of these currents between proton states are

\[
\langle p',s'| j_{\mu}^5 | p,s \rangle = \bar{u}(p',s') \gamma_\mu \gamma_5 G_1^{(P)}(q^2) + q_\mu \gamma_5 G_2^{(P)}(q^2) \rangle u(p,s) ,
\]

\[
\langle p',s'| k_{\mu}^5 | p,s \rangle = \bar{u}(p',s') \gamma_\mu \gamma_5 G_1^{(P)}(q^2) + q_\mu \gamma_5 G_2^{(P)}(q^2) \rangle u(p,s) .
\]

This form for the matrix element for \( k_{\mu}^5 \) is valid in any covariant gauge. A consequence of the gauge variance of the topological current is that the form factor \( G_2^{(P)}(q^2) \) has a ghost pole at \( q^2 = 0 \) [6]. It is the gauge-invariant residue of this pole that determines the quark content of the proton's spin \( \Delta \Sigma^P \) [2]:

\[
\Delta \Sigma^P = G_1^{(P)}(0) - G_1^{(P)}(q^2) = \frac{q^2 G_2^{(P)}(q^2)}{2\mathcal{M}_P} \Big|_{q^2=0} ;
\]

\( \mathcal{M}_P \) is the proton's mass and the second equality is due to Eq. (3). \( q^2 G_2^{(P)}(q^2) \) has a pole at \( m_\pi^2 \); it may also have a constant contribution from the direct coupling of the ghost pole to nucleons [3,5]. The presence or absence of this direct ghost coupling is equivalent to the question of whether \( q^2 G_1^{(P)}(q^2) \) satisfies a subtracted or unsubtracted dispersion relation. The last statement requires some care as even though \( q^2 G_2^{(P)}(q^2) \big|_{q^2=0} \) is gauge invariant it is not invariant away from \( q^2 = 0 \). Changing gauges, at least within the class of covariant gauges, adds polynomials in \( q^2 \) which vanish at \( q^2 = 0 \), to \( q^2 G_2^{(P)}(q^2) \); we may consider a dispersion relation for \( q^2 G_2^{(P)}(q^2) \) in the gauge in which it has at most a constant behavior for large \( q^2 \). The usual Goldberger-Treiman relation is based on the assumption that the divergence of the isospin axial-vector current is soft at high momenta and satisfies an unsubtracted dispersion relation; it is thus dominated by the pion pole. In Ref. [3] a similar assumption was made for \( G_2^{(P)}(q^2) \), i.e., that it is unsubtracted and dominated by the \( \eta' \) pole. The quark content of the proton's spin was found to be

\[
\Delta \Sigma^P = \frac{\sqrt{2N_f f_{\eta'}}}{2\mathcal{M}_P} g_{\eta'} NN ,
\]

where \( f_{\eta'} \) is the \( \eta' \) decay constant (normalization is such
that \( f_\pi = 93 \text{ MeV} \) and \( g_{\eta NN} \) is the \( \eta' \)-nucleon coupling constant. Estimates for \( f_\eta \) [7] and for \( g_{\eta NN} \) yield \( \Delta \Sigma^p = 1 \). Corrections due to isospin and SU(3) breaking in the quark masses [5] were found to be small.

We shall now extend these ideas to the light vector mesons \( \rho, \omega, \phi \). The matrix elements between any of these meson states with momenta \( p, p' \) and helicities \( \lambda, \lambda' \) of the singlet axial-vector and topological currents are defined as

\[
\langle p', \lambda' | j_\mu^A(p, \lambda) \rangle = -ie^{\alpha\mu\nu} \epsilon^*_\phi(\lambda') \epsilon_\alpha(\lambda)(p + p')_\nu \times [g_{\alpha\mu} \bar{G}^{(V)}_1(q^2) + (p - p')_\alpha(p - p')_\mu \bar{G}^{(V)}_2(q^2)] + \cdots ,
\]

(7)

\[
\langle p', \lambda' | j_\mu^B(p, \lambda) \rangle = -ie^{\alpha\mu\nu} \epsilon^*_\phi(\lambda') \epsilon_\alpha(\lambda)(p + p')_\nu \times [g_{\alpha\mu} \bar{G}^{(V)}_1(q^2) + (p - p')_\alpha(p - p')_\mu \bar{G}^{(V)}_2(q^2)] + \cdots ,
\]

where the ellipsis represents other invariant structures which are automatically conserved and play no role in the determination of the spin content of these mesons. The quark spin content of these mesons \( \Delta \Sigma^V \) is given by an expression similar to that of Eq. (5):

\[
\mathcal{A}_{\eta' \gamma \gamma}^{(\text{vector dominance})} = \sum_\nu \left[ \frac{e}{2\gamma} \right]^2 \frac{g_{\eta' \gamma \gamma}}{2M_V} e^{\mu\nu\rho} \epsilon^*_\phi(\lambda') \epsilon_\alpha(\lambda)(p + p')_\mu(p - p')_\nu .
\]

Comparing with the experimental results for [10]

\[ \Gamma(\eta' \rightarrow \rho \gamma), \quad \Gamma(\eta' \rightarrow \omega \gamma) \] and the limit on \( \Gamma(\phi \rightarrow \eta' \gamma) \) we find \( g_{\eta' \rho \gamma} = 1.27 \pm 0.04, \quad g_{\eta' \omega \gamma} = 1.55 \pm 0.07, \quad \text{and} \quad g_{\eta' \phi \gamma} \leq 2.15 \). The results are

\[
\Delta \Sigma^{(\rho)} \approx 0.25 ,
\]

\[
\Delta \Sigma^{(\omega)} \approx 0.30 ,
\]

\[
\Delta \Sigma^{(\phi)} < 0.33 .
\]

How should we view this result? (i) We could accept the conclusion that only a small portion of the spin of the vector mesons is due to quarks. This is somewhat unnatural as at least a major part of the nucleon's spin is due to quarks [4]. (ii) Vector dominance could be inapplicable to this problem and the \( \eta' \)-vector-meson coupling is a factor of 3 larger than the estimates presented here. This is unlikely as all the radiative decays of the \( \eta' \) are consistent with vector dominance in that they give essentially the same coupling. (iii) The assumption that \( \eta' \) dominates the various form factors could be wrong.
There is then the question of why the results on the nucleon are so close to the expectation that the spin of that particle is due to quarks. We cannot exclude the possibility that the dispersion relation for \( q^2 G_V(z) \) requires a subtraction, while the one for \( q^2 G_P(z) \) does not, or that the subtraction in the nucleon case is much smaller than in the vector-meson one. In terms of direct ghost couplings, we would have the situation where the ghost coupling to vector mesons is weaker than its coupling to nucleons.

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[7] As in Ref. [3], we shall use \( f_\nu = 1.25 f_\mu \).
[9] We use the measured rates for \( V \rightarrow e^- e^+ \) (Ref. [10], pp. vii.22) to obtain the \( \gamma \)-vector-meson couplings. The results are
\[ 4\pi/\gamma_V^2 = 1.98 \pm 0.09, \]
\[ 4\pi/\gamma_P^2 = 0.17 \pm 0.005, \]
\[ 4\pi/\gamma_V^2 = 0.30 \pm 0.01. \]