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Essays on Asymmetric Information in Financial Markets

by

Bradyn Mitchel Breon-Drish

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

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Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Christine Parlour, Chair
Professor Johan Walden
Professor Robert M. Anderson

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Essays on Asymmetric Information in Financial Markets

Abstract

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Bradyn Mitchel Breon-Drish

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This dissertation studies the effects of asymmetric information and learning on asset prices and investor decision-making. Two main themes run through the work. The first is the linkage between investor decisions and the information used to make those decisions; that is, portfolio choices reflect the nature and quality of available information. The second theme is the interaction between investor learning and price informativeness. The information held by individual investors is reflected in market prices through their trading decisions, and prices thus transmit this information to other investors.

In the first chapter, *Asymmetric Information in Financial Markets: Anything Goes*, I study a standard Grossman and Stiglitz (1980) noisy rational expectations economy, but relax the usual assumption of the joint normality of asset payoff and supply. The primary contribution is to characterize how the equilibrium relation between price and fundamentals depends on the way in which investors react to the information contained in price. My solution approach dispenses with the typical "conjecture and verify" method, which allows me to analytically solve an entire class of previously intractable nonlinear models that nests the standard model. This simple generalization provides a purely information-based channel for many common phenomena. In particular, price jumps and crashes may arise endogenously, purely due to learning effects, and observation of the net trading volume may be valuable for investors in the economy as it can provide a refinement of the information conveyed by price. Furthermore, the value of acquiring information may be non-monotonic in the number of informed traders, leading to multiple equilibria in the information market. I show also that the relation between investor disagreement and returns is ambiguous and depends on higher moments of the return distribution. In short, many of the standard results from noisy rational expectations models are not robust. I introduce monotone likelihood ratio conditions that determine the signs of the various comparative statics, which represents the first demonstration of the implicit importance of the MLRP in the noisy rational expectations literature.

In the second chapter, *Do Fund Managers Make Informed Asset Allocation Decisions?*, a joint work with Jacob S. Sagi, we derive a dynamic model in which mutual fund managers

make asset allocation decisions based on private and public information. The model predicts that the portfolio market weights of better informed managers will mean revert faster and be more variable. Conversely, portfolio weights that mean revert faster and are more variable should have better forecasting power for expected returns. We test the model on a large dataset of US mutual fund domestic equity holdings and find evidence consistent with the hypothesis of timing ability, especially at three- to 12-month forecasting horizons. Nevertheless, whatever timing ability may be reflected in portfolio weights does not appear to translate into higher realized returns on funds' portfolios.

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Chapter 1

Asymmetric Information in Financial Markets: Anything Goes

1.1 Introduction

Since at least Hayek (1945) economists have recognized that an important role of financial markets is the aggregation and transmission of information held by individual traders. There is a vast literature, both theoretical and empirical, that seeks to understand how well prices reflect information and what frictions best explain apparent deviations from market efficiency.[1] Following this literature, I seek to explore several questions: How do traders react to the information in prices? Are asset prices a sufficient statistic for all public information? Are prices necessarily more informative when more traders are informed? How is disagreement among investors reflected in prices and future returns?

Addressing the interaction between price informativeness and investor behavior presupposes, of course, that traders have asymmetric information, for without information asymmetry there is no role for learning from price. The workhorse model for studying asymmetric information in (competitive) financial markets is the noisy rational expectations (RE) model of Grossman and Stiglitz (1980), and similar ones due to Hellwig (1980) and Diamond and Verrecchia (1981).[2] Unfortunately, the standard model is ill-suited for a full consideration of the questions posed above. It makes counterfactual assumptions about the distribution of asset payoffs and supply, and it leads to overly-simplistic descriptions of investor behavior.

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[1]Fama (1991) is a standard reference on empirical tests of market efficiency. Brunnermeier (2001) discusses the theory.

[2]There is a distinct but related literature, following Kyle (1985), that studies the consequences of asymmetric information in markets in which some traders behave strategically. Other models in this vein include Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), and Foster and Viswanathan (1993, 1996), in which traders submit market orders, and Kyle (1989) and Bhattacharya and Spiegel (1991), in which traders submit demand schedules. There is also a literature, initiated by Admati (1985), that considers noisy RE models with multiple assets.

In the standard model, all random variables are jointly normally distributed, and demand curves and asset prices are linear functions. Hence, all price observations are equally informative, and traders always react in the same way to changes in price. In practice, asset returns and supply are not jointly normally distributed. Fama (1965) and Mandelbrot (1963) were among the first to make note of this fact. Kon (1984) finds that discrete mixtures of normal distributions better describe stock returns than either normal, Student-t, or stable Paretian distributions. (Tucker (1992) reinforces this point with additional statistical tests, and Hall, Brorsen, and Irwin (1989) provide complementary evidence in the context of futures markets.) Recently, nonnormality has also received much coverage in the popular press in the context of heavy tails (see, for instance, Taleb (2007)).

In this paper, I investigate the effects of asymmetric information in a class of noisy rational expectations models in which I relax the standard Grossman and Stiglitz (1980) model to admit fundamentals and supply that do not follow a normal distribution. This seemingly minor change can have dramatic consequences for the standard results on the shape of demand curves (and consequently the possibility of information-based price crashes), price informativeness, the value of acquiring information, and the relation between disagreement and returns. The simplicity of the classic Grossman and Stiglitz (1980) economy makes it an ideal setting in which to illustrate the fragility of noisy RE models. My point is strengthened by the limited number of moving parts; more general models can be expected to provide even richer results as they afford more degrees of freedom for constructing examples.

My primary contribution is to characterize how the relation between price and fundamentals depends on the strength and direction in which investors react to information in price. I provide a purely information-based channel for many common phenomena. More specifically, I show how price-informativeness varies with the price level and how learning effects can cause uninformed investors to submit backward-bending demand curves, leading to price "jumps" and "crashes" in response to small changes in fundamentals.[3] Next, I show that observation of (signed) trading volume may be valuable for uninformed investors because it provides a refinement of the information contained in price alone. Third, I show that the value of information can be non-monotonic in the number of informed investors in the economy. That is, information acquisition can be a strategic complement. Finally, I demonstrate that the relation between investor disagreement and returns depends on the relation between conditional expected returns and conditional volatility and on the skewness of fundamentals.

In standard noisy RE models price crashes are impossible in the absence of other frictions. Demand curves of uninformed investors are downward-sloping at all prices, and because of linearity they always respond in the same way to perturbations in price.[4] As such, authors

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[3]Strictly speaking, my model is static, so there are no changes to which traders can react. I follow the convention of many papers in this literature, e.g., Gennotte and Leland (1990) and Barlevy and Veronesi (2003), and interpret comparative statics as approximating dynamic effects in a repeated version of the model.

[4]In the multi-asset noisy RE model of Admati (1985), demand curves can be globally upward sloping.

modeling crashes in settings with asymmetric information often introduce context-specific frictions. To rationalize the October 1987 crash, Gennotte and Leland (1990) introduce imperfectly anticipated hedging demand, which can cause large price reactions in response to small changes in fundamentals. Similarly, Romer (1993) introduces higher-order uncertainty over the information quality of other traders. In his model, small changes in price can reveal this information quality and lead to discontinuous drops in price. Yuan (2005) considers the effects of borrowing constraints, which make low prices less informative: uninformed investors have difficulty disentangling whether low price is due to a low fundamental or a binding borrowing constraint that has prevented informed from trading fully on the basis of information. Similarly, Barlevy and Veronesi (2003) study a model with risk-neutral traders who are both short-sale and borrowing constrained. My model differs from all of the above in that I focus solely on the effect of asymmetric information with fully-rational unconstrained investors. It turns out that crashes arise naturally in settings with asymmetric information because, in general, different price realizations are not equally informative.

It is also difficult to reconcile the empirical evidence on the information content of trading volume and other market-generated statistics with standard rational expectations models.[5] Typically, price is a sufficient statistic for all public information, and other market-generated statistics are redundant. Schneider (2009) gives a clear statement of this point, noting that "the fact that volume is helpful to an outside observer of the economy does not imply that investors within the economy can learn from observing volume. If investors are rational, then it is not clear how trading volume can contain information beyond the information that is already incorporated into prices." Schneider (2009) and Blume, Easley, and O'Hara (1994) introduce higher-order uncertainty and propose that volume can be informative about the quality of others' information. In my model, (signed) volume can be valuable, but the mechanism is different. Without normality, price may not be a sufficient statistic for public information and in such situations, signed volume refines the uninformed investors' information set.

Following the original Grossman and Stiglitz (1980) paper, the standard intuition is that as the number of informed investors in a market increases, it becomes easier to free-ride on their information by simply observing price. As such, the incentive for other traders to acquire information decreases–information acquisition is a strategic substitute. There is a small recent literature investigating the opposite situation, strategic complementarity in information acquisition. Barlevy and Veronesi (2000, 2008) introduce correlated fundamentals and supply, which makes it more difficult for uninformed investors to disentangle whether price changes are due to fundamentals of supply. Ganguli and Yang (2009) and Manzano and Vives (2010) allow investors to also observe signals about the supply. Veldkamp (2006a,b) considers a dynamic Grossman and Stiglitz (1980) model with economies of scale in the

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However, as they are still linear they never bend backwards and generate crashes of the sort discussed here.

[5]See, e.g., Karpoff (1987), Gallant, Rossi, and Tauchen (1992), and Gervais, Kaniel, and Mingelgrin (2001) for evidence on the relation between volume and returns.

(competitive) information market. In such a setting, information in higher demand is supplied at a lower price, generating complementarities. My model generates complementarity through a pure information channel: when the number of informed investors increases, price may become more difficult to read, causing uninformed investors to submit demand curves that are *less* closely aligned with those of informed investors.

While the model allows for essentially general distributions of uncertainty for both the asset payoff and supply, one need not depart too far from normality to obtain the results above. Normal mixture distributions have been proposed as an empirically plausible alternative to normal distributions (Kon, 1984), and as shown later, simple normal mixture specifications can produce a rich set of examples. In the standard model, the strong unimodality of the normal distribution implies that uninformed investors are able to learn from prices relatively "easily" (later, I make the unimodality condition precise and describe learning effects explicitly). High price is an unambiguous signal of high payoff (and vice versa), so that the informed and uninformed investors react in the same direction to a change in the fundamental and the equilibrium price function is monotone. When the number of informed investors increases, the information contained in price is "more correlated" with the fundamental, and uninformed investors are able to make portfolio decisions that are closer to those of the informed and therefore better aligned with the true state. Similarly, the fact that the conditional variance of a normal random variable, given another jointly normal random variable, is constant implies that there are no price levels at which an uninformed investor learns more or less than any other price level. With more general distributions this is not true without further restrictions on the distribution of uncertainty. I give examples of the above effects in Sections 1.4 below.

In order to derive the results above, I provide a tractable solution to a particular class of nonlinear noisy rational expectations models that nests the standard model. Instead of the usual solution method of conjecturing and verifying a (linear) price function, I approach the problem by solving a general version of the uninformed investors' optimization problem given an arbitrary price function and then utilize the market clearing condition to write down an equation that pins down the price as an implicit function of the primitive quantities in the model. In principle, the technique I use would also allow for uncertainty about quantities other than the conditional mean of asset payoffs, such as the variance, number of informed, or risk aversion.

Other notable exceptions to the normality assumption in the literature include Ausubel (1990a,b), Peress (2004), and Vanden (2008). However, these authors must make unappealing concessions and use a non-standard model setup or approximation methods. Bernardo and Judd (2000) develop a computational procedure for solving rational expectations models and demonstrate the non-robustness of some results from the standard Grossman and Stiglitz (1980) model. An advantage of their approach is the large class of models that it handles, but without an explicit characterization of price, it is difficult to pin down the conditions on distributions or preferences that drive standard results. The economy of Barlevy and Veronesi (2000, 2003) is similar to that in this paper, except that their traders

are risk-neutral and face a portfolio constraint, and they focus on a particular parametric distribution for random variables. Gibbons, Holden, and Powell (2010) consider a noisy RE model of an intermediate goods market in which all random variables are uniformly distributed and demonstrate non-robustness of some of the Grossman and Stiglitz (1980) results in their setting. DeMarzo and Skiadas (1998, 1999) study the properties a class of economies that nests the non-noisy economy of Grossman (1976); they demonstrate uniqueness of Grossman's (1976) fully-revealing linear equilibrium and give robust examples of partially revealing equilibria when payoffs are non-normal. Foster and Viswanathan (1993) study (linear) equilibria in the Kyle (1985) model when random variables are elliptically distributed, and Bagnoli, Viswanathan, and Holden (2001) derive necessary and sufficient conditions on probability distributions for existence of linear equilibria in various market making models. Finally, Rochet and Vila (1994) study existence and uniqueness properties in a model similar to Kyle (1985) with non-normal distributions.

The rest of the paper proceeds as follows. Section 1.2 lays out the model and characterizes the equilibrium. Section 1.3 discusses the monotone likelihood ratio property and previews its role in many of the results in the paper. Section 1.4 lays out the general results described above, and Section 1.5 concludes. Section 1.6.1 collects results on sign-regular and single crossing functions that are used to prove some of my propositions. Proofs are relegated to Section 1.6.2. Since my results speak to a number of different literatures, I postpone additional detailed discussion of related papers to the sections in which I present the relevant findings.

1.2   Model

The economy has three dates $t \in \{0, 1, 2\}$. At the first date, $t = 0$, agents choose whether to become informed. At the second, $t = 1$, agents trade financial assets. At the final date, $t = 2$, assets make liquidating payouts. Figure 1.2 shows a timeline. There are two assets, a risky asset that has a payoff D and a risk-free asset that pays 1 and has price normalized to 1. The risky-asset payoff D is the sum of two components μ and ε. The distribution of the fundamental μ has density f_μ, while ε is independently distributed $N(0, \sigma_\varepsilon^2)$. Note that because of the normal distribution for ε, the conditional distribution of D given μ is normal.

The assumption that ε has a normal distribution is not critical for my results, but it greatly simplifies the analysis. One way to motivate this assumption is to consider that even after the fundamental μ is known, there are a "large" number of "small" additive and independent idiosyncratic factors that can affect the final payoff. By the central limit theorem, the sum of these disturbances will be approximately normally distributed, and one can just as well aggregate them into a single term, namely ε. While this interpretation is plausible, I do not model it rigorously here.

To prevent fully-revealing prices, the risky asset is in random supply Z, which is independent of other random variables in the model and has density f_Z. To simplify the proofs

$$t = 0 \qquad\qquad t = 1 \qquad\qquad t = 2$$

Information acquisition decision   Financial market opens   Payoffs realized
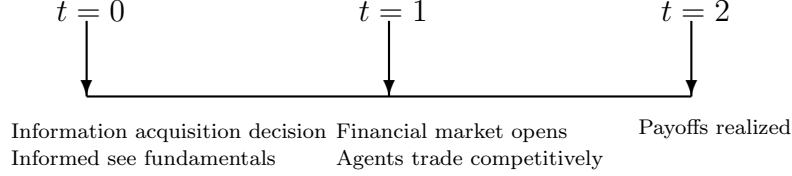Informed see fundamentals   Agents trade competitively

Figure 1.1: This timeline shows the sequence of moves in the model. At $t = 0$ investors decide whether to pay for information. After this decision, agents trade in the financial market at $t = 1$. Final payoffs occur at $t = 2$.

of various results, I assume that both μ and Z have absolutely continuous distributions supported on the entire real line and that their densities are continuously differentiable.[6]

A unit mass of ex-ante identical agents have CARA utility over wealth at $t = 2$ with common risk aversion α, so $u(w) = -e^{-\alpha w}$. Investors are endowed with x_0 shares of the risky asset and y_0 dollars in the risk-free asset that they can trade in the financial market. Without loss of generality, I let $x_0 = y_0 = 0$ because a CARA investor's demand for risky assets is independent of initial wealth. Information about the risky asset payoff is available at a fixed dollar cost $c > 0$; before trading, investors choose whether to pay c to observe μ. Those who choose to buy information ("informed investors") see μ immediately before the financial market opens. The remaining agents ("uninformed investors") do not see μ, but can use all public information – price and signed trading volume of the informed and noise traders – to make an inference about it. Note that all informed agents observe μ perfectly; they do not receive conditionally independent signals as in Hellwig (1980) or Diamond and Verrecchia (1981).

All agents are price takers. All probability distributions and other parameters of the economy are common knowledge, and therefore, agents are only asymmetrically informed about the fundamental μ.

The set of normal mixture distributions is useful for constructing counterexamples without straying too far from the standard model. I illustrate most results in the paper for an economy in which μ and Z follow independent normal mixture distributions

$$\mu \sim \beta N(\overline{\mu}_1, \sigma_\mu^2) + (1 - \beta)N(\overline{\mu}_2, \sigma_\mu^2), \quad 0 \leq \beta \leq 1$$
$$Z \sim \eta N(\mu_{Z1}, \sigma_Z^2) + (1 - \eta)N(\mu_{Z2}, \sigma_Z^2), \quad 0 \leq \eta \leq 1.$$

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[6]It can be verified that all results are also true as stated for absolutely continuous distributions with supports other than the real line.

1.2.1 Equilibrium

Let P_λ denote the equilibrium risky-asset price when the fraction of informed agents is λ. Let $X_I(\mu, P_\lambda)$ denote the number of shares demanded by an informed agent as a function of fundamental μ and price. Since the informed types know the realized value of μ, their demand takes the standard mean-variance form $X_I(\mu, P_\lambda) = \frac{\mu - P_\lambda}{\alpha \sigma_\varepsilon^2}$.

When the uninformed agents choose their demands, they have access to the price and the signed volume (order flow) of the informed and noise traders.[7] Noise traders supply Z shares, so the signed trading volume of the informed and noise traders is $\lambda X_I(\mu, P_\lambda) - Z$. However, it will turn out to be more convenient to work with the informationally-equivalent *adjusted volume,* defined as follows.

Definition 1.2.1 (Adjusted volume). The *adjusted volume* $\widehat{\mu}_\lambda$ of the informed and noise traders is

$$\widehat{\mu}_\lambda := \mu - \frac{\alpha \sigma_\varepsilon^2}{\lambda} Z.$$

The adjusted volume $\widehat{\mu}_\lambda$ is a transformation of the price and signed volume.[8] To compute adjusted volume, the uninformed need only to multiply the signed volume by the constant $\frac{\alpha \sigma_\varepsilon^2}{\lambda}$ and then add the price P_λ. Their information set provides sufficient information to perform these calculations.

Let $X_U(\widehat{\mu}_\lambda, P_\lambda)$ denote the demand of the uninformed as a function of the adjusted volume and price. The definition of equilibrium in the financial market is standard.

Definition 1.2.2 (Financial market equilibrium). A rational expectations equilibrium in the financial market is a (measurable) price function $P_\lambda(\mu, Z)$ mapping $\mathbb{R}^2 \mapsto \mathbb{R}$, and demand functions for the informed agents X_I and uninformed agents X_U such that all agents maximize expected utility, conditional on their information sets

$$X_I(\mu, P_\lambda) \in \arg\max_{x \in \mathbb{R}} \mathbb{E}\left[u(x(D - P_\lambda))|\mu, P_\lambda\right]$$

$$X_U(\widehat{\mu}_\lambda, P_\lambda) \in \arg\max_{x \in \mathbb{R}} \mathbb{E}\left[u(x(D - P_\lambda))|\widehat{\mu}_\lambda, P_\lambda\right],$$

and markets clear for each possible (μ, Z) pair

$$\lambda X_I(\mu, P_\lambda) + (1 - \lambda)X_U(\widehat{\mu}_\lambda, P_\lambda) = Z.$$

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[7]In the standard model signed volume provides redundant information; traders need only to observe price. In more general settings, signed volume may refine the information contained in price. In some results in this section I distinguish between the case in which investors condition only on price and the case in which they also observe signed volume.

[8]A similar variable, labeled w_λ, appears in the original Grossman and Stiglitz (1980) paper. I choose the $\widehat{\mu}_\lambda$ notation to emphasize that this variable will be interpreted as a signal about μ.

All statements in the paper about equality of random variables, such as the market clearing condition in Definition 1.2.2, should be understood to mean that the equality holds almost surely under the joint distribution of (μ, Z, ε), and all statements about equality of functions on $\mathbb{R}^n$ should be understood to mean that the equality holds almost everywhere with respect to Lebesgue measure on $\mathbb{R}^n$. To avoid unnecessary technical clutter, I refrain from making this explicit in the results and exposition below.

If the random variables in the model were jointly normally distributed, I would now solve for the equilibrium by conjecturing a price function that is linear in the fundamental μ and supply Z, solving the uninformed investors' inference and portfolio problem given the price function, and then substituting the resulting demand into the market clearing condition. This would produce a system of three equations with three unknowns (the coefficients in the price function). In this simple setting, the coefficient equations would have explicit closed-form solutions. With a non-normal joint distribution, this solution technique is not possible since the functional form of the price is not clear *a priori.* Indeed, the best outcome one can hope for is to characterize the price as an implicit function of μ and Z.

The following result characterizes the equilibrium asset price, assuming that it exists. (Proofs of all results are relegated to Section 1.6.2.) The key step is to first solve a general version of the uninformed investors' optimization problem, assuming that they conjecture an arbitrary price function. Since, in equilibrium, the quantity demanded by the uninformed must be equal to the residual supply of the noise traders after subtracting the informed demand, substituting in from the market clearing condition pins down a risky-asset price that both clears the market and is consistent with the uninformed investors' beliefs.

Proposition 1.2.1. *If there exists an equilibrium price function, $P_\lambda(\mu, Z)$, then it is implicitly defined as*

$$\int_{\mathbb{R}} \left((1-\lambda)y + \lambda\left(\mu - \tfrac{\alpha\sigma_\varepsilon^2}{\lambda}Z\right) - P_\lambda \right) e^{\frac{\lambda}{1-\lambda}\frac{\mu - \frac{\alpha\sigma_\varepsilon^2}{\lambda}Z - P_\lambda}{\sigma_\varepsilon^2}y} f_Z\left(\tfrac{\lambda}{\alpha\sigma_\varepsilon^2}\left(y - \left(\mu - \tfrac{\alpha\sigma_\varepsilon^2}{\lambda}Z\right)\right)\right) f_\mu(y)\, dy = 0.$$

$$(1.1)$$

Looking at the relation in eq. (1.1), it is apparent that price depends on μ and Z only through the adjusted volume $\widehat{\mu}_\lambda = \mu - \frac{\alpha\sigma_\varepsilon^2}{\lambda}Z$, which leads immediately to the following Corollary.

Corollary 1.2.2. *If it exists, the equilibrium price P_λ in Proposition 1.2.1 can be characterized as a function of the adjusted volume $\widehat{\mu}_\lambda$ only,*

$$\int_{\mathbb{R}} ((1-\lambda)y + \lambda\widehat{\mu}_\lambda - P_\lambda)\, e^{\frac{\lambda}{1-\lambda}\frac{\widehat{\mu}_\lambda - P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{\mu}_\lambda)\, dy = 0. \qquad (1.2)$$

From this point forward, I treat P_λ as a univariate function mapping realizations $\widehat{m}$ of adjusted volume $\widehat{\mu}_\lambda$ into an equilibrium price, as in Corollary 1.2.2. There is no loss, however,

in continuing to think of price as a function of the fundamental μ and supply Z that depends on them only through the linear combination $\widehat{\mu}_\lambda$.

To better understand the meaning of the integral in eq. (1.2), note that one can also write it as an integral over realizations of the payoff $D = \mu + \varepsilon$ rather than just the fundamental μ. Rearranging the resulting expression and writing out the utility function in general terms, $u(\cdot)$, gives

$$P_\lambda = \frac{\int_{\mathbb{R}} y u'\left(-\frac{\lambda}{1-\lambda}\frac{\widehat{\mu}_\lambda - P_\lambda}{\alpha\sigma_\varepsilon^2}(y - P_\lambda)\right) f_{D|\widehat{\mu}_\lambda}(y|\widehat{\mu}_\lambda)\,dy}{\int_{\mathbb{R}} u'\left(-\frac{\lambda}{1-\lambda}\frac{\widehat{\mu}_\lambda - P_\lambda}{\alpha\sigma_\varepsilon^2}(y - P_\lambda)\right) f_{D|\widehat{\mu}_\lambda}(y|\widehat{\mu}_\lambda)\,dy}.$$

This looks like the standard representative agent pricing formula except that the "endowment" of the agent, $-\frac{\lambda}{1-\lambda}\frac{\widehat{\mu}_\lambda - P_\lambda}{\alpha\sigma_\varepsilon^2}$, is endogenous. Accordingly, one can interpret eq. (1.2) as a representative agent pricing formula in which the representative uninformed agent's equilibrium risky asset holding is the residual supply of the noise traders, net of the demand of the informed investors.

It is clear from eq. (1.2) that the price is a nonlinear function of the fundamental μ and supply Z. However, because the residual uncertainty ε is normally distributed and utility functions are exponential, the information conveyed by price is still a linear combination $\widehat{\mu}_\lambda$ of the quantities μ and Z, as in the standard model. While this fact simplifies the analysis of the information content of prices, it is not vital for my results. Indeed, allowing for a more general distribution for ε would provide one more degree of freedom with which to construct counterexamples to standard results. Similarly, one could criticize the restriction to exponential utility, which precludes income effects. However, this also makes construction of counterexamples more difficult. Indeed, including nontrivial income effects would tend to *strengthen* most of the results in the paper.

Proposition 1.2.1 assumes existence of an equilibrium in the financial market. The following proposition gives a sufficient condition for existence.

Proposition 1.2.3. *If for each fixed $\widehat{m} \in \mathbb{R}$ the conditional moment generating function of μ given $\widehat{\mu}_\lambda = \widehat{m}$ exists in an open neighborhood around zero, then there exists an equilibrium price function P_λ.*

The restriction that μ has a moment generating function (mgf) is needed so that expected utility exists (expected utility for a CARA investor is essentially a moment-generating function) and the integral in eq. (1.2) converges. As long as the integral is finite, then the existence of an equilibrium price that satisfies eq. (1.2) follows from the intermediate value theorem. Unfortunately, the restriction on the distribution of fundamentals in the proposition rules out fat-tailed distributions, along with lognormal distributions.[9]

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[9]Difficulty incorporating fat-tailed distributions into an otherwise-standard economy with expected-utility-maximizing investors is not specific to my model. Geweke (2001) points out the same problem in a setting with a CRRA representative investor when log returns follow a t-distribution.

So far, I have said nothing of uniqueness. In principle, for some realizations of $\widehat{\mu}_\lambda$ there could be multiple values of P_λ that satisfy (1.2).[10] Fortunately, that is not the case, and the equilibrium price defined by eq. (1.2) is unique, at least within the class of continuously differentiable functions.

Proposition 1.2.4. *The function defined by* (1.2) *is the unique continuously differentiable price function.*

In some sense, the uniqueness result should not be surprising. In models in which agents do not learn from prices, multiplicity can arise if wealth effects are sufficiently strong to prevent aggregate demand from sloping downward at all prices. Exponential utility rules out wealth effects, and as indicated in Proposition 1.4.2, one can reduce the model to one in which the uninformed do not condition on price but instead observe only adjusted volume $\widehat{\mu}_\lambda$. Hence, the presence of only substitution effects means that for given $\widehat{\mu}_\lambda$, aggregate demand is downward sloping, and therefore the equilibrium price is unique.

A Corollary of Proposition 1.2.4 is that the linear equilibrium in the standard model is in fact (essentially) unique, not merely unique among *linear* equilibria. To my knowledge, this was still an open question.

Corollary 1.2.5. *When μ and Z are independently normally distributed, the linear price function given in Grossman and Stiglitz (1980) is the unique continuously differentiable price function.*

This completes the analysis of equilibrium in the financial market. Next, I address equilibrium in the information market.

Information market equilibrium

While it is not the focus of this paper, for completeness I now define equilibrium in the information market. Let $CE(\lambda)$ denote the ex-ante certainty-equivalent gain CE (gross of cost c) from becoming informed as a function of the fraction λ of informed agents

$$CE(\lambda) := \underbrace{-\frac{1}{\alpha}\log \mathbb{E}[e^{-\alpha X_I(\mu, P_\lambda)(D-P_\lambda)}]}_{\text{Informed certainty-equivalent wealth}} - \underbrace{-\frac{1}{\alpha}\log \mathbb{E}[e^{-\alpha X_U(\widehat{\mu}_\lambda, P_\lambda)(D-P_\lambda)}]}_{\text{Uninformed certainty-equivalent wealth}}. \tag{1.3}$$

I use a standard definition of equilibrium in the information market, identical to that used by Grossman and Stiglitz (1980). To add realism, one could more explicitly model an information production sector as in, for example, Admati and Pfleiderer (1986) or Veldkamp (2006b). However, for simplicity and comparability with earlier work I choose the most basic possible setup.

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[10]Technically speaking, eq. (1.2) defines a correspondence from which an equilibrium price must be selected. In principle, this correspondence might not be single-valued.

Definition 1.2.3 (Information market equilibrium)**.** An equilibrium in the information market is a fraction $\lambda^* \in [0, 1]$ such that,

- $\lambda^* = 0$ and $CE(0) \leq c$, or

- $\lambda^* \in (0, 1)$ and $CE(\lambda^*) = c$, or

- $\lambda^* = 1$ and $CE(1) \geq c$.

Definition 1.2.3 says that an equilibrium in the information market falls into one of three possible cases. Either (1) no one buys information because the cost of doing so exceeds the benefit, even if no one else buys any, or (2) there is an interior value for λ^* such that at the margin the gain from acquiring information is exactly equal to the cost of doing so, or (3) everyone buys information because it is sufficiently cheap that the benefit always outweighs the cost, regardless of how many others buy information.

Existence and uniqueness of equilibrium in the information market is a more delicate matter than existence and uniqueness in the financial market. Since information market equilibria depend on the value of acquiring information, I postpone further discussion until I take up the value of information in Section 1.4.3.

In noisy REE models, investors use available signals to make an inference about fundamentals. In my model, from the standpoint of the uninformed investors the signal is the adjusted volume $\widehat{\mu}_\lambda$, and the variable that they are trying to learn about is the fundamental μ. The main results presented later on the value of technical analysis, the shape of uninformed investor demand curves, and the value of acquiring information depend on how uninformed investors react to the information contained in price. It turns out that a monotone likelihood ratio property (MLRP) for signals is the appropriate concept for characterizing reaction to information.

1.3 Discussion of the monotone likelihood ratio property

While the MLRP is familiar to most readers, in this section I will briefly restate the definition, discuss the implications for investor behavior, and tie the MLRP to a restriction on the distribution of noise in the economy.[11] Briefly, in my model the MLRP guarantees that uninformed investors' demand reacts in the "correct" direction to changes in adjusted volume $\widehat{\mu}_\lambda$, and it requires that the distribution of noise satisfy a particularly stringent unimodality condition. In more general terms, requiring the MLRP for endogenous signals in a noisy REE model guarantees that "good news" for one agent is also "good news" for other agents who learn about her information by looking at the price (or other market data), and it requires that signals be affiliated with fundamentals.

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[11]Milgrom (1981) is a standard reference for the MLRP.

I begin by stating the definition of the monotone likelihood ratio property.

Definition 1.3.1 (Monotone likelihood ratio property)**.** Consider any two random variables X and Y. The family of conditional densities $\{f_{X|Y}(\cdot|y)\}_{y \in \text{Support}(Y)}$ satisfies the *monotone likelihood ratio property* (MLRP) if for all $x' > x$ and $y' > y$, the following inequality holds

$$f_{X|Y}(x'|y')f_{X|Y}(x|y) \geq f_{X|Y}(x'|y)f_{X|Y}(x|y'). \tag{1.4}$$

If the inequality in eq. (1.4) is strict at every set of points, the family is said to satisfy the *strict MLRP*.

It is helpful to think of the random variable X as a signal providing information about Y. If the conditional densities of the signal X have the MLRP, then increases in X shift "up" the posterior distribution of Y in the monotone likelihood ratio (MLR) stochastic order (i.e., given $x' > x$, the likelihood ratio of the *posteriors* $\frac{f_{Y|X}(y|x')}{f_{Y|X}(y|x)}$ is increasing in y). The MLR ordering is a strengthening of the well-known first-order stochastic dominance (FOSD) ordering.[12] In fact, Proposition 2 of Milgrom (1981) shows that a family of conditional densities $\{f_{X|Y}\}$ has the MLRP if and only if higher values of the signal X improve the posterior distribution of Y in the FOSD sense under *any* prior for Y. With only two random variables, the MLRP is equivalent to their being affiliated. See Milgrom and Weber (1982) for a detailed discussion of affiliation.

In my model, if the conditional densities of adjusted volume $f_{\widehat{\mu}_\lambda|\mu}$ have the MLRP then higher values of adjusted volume $\widehat{\mu}_\lambda$ imply MLR improvements in the risky asset payoff. Recalling that $\widehat{\mu}_\lambda$ can be written as a linear combination of the fundamental and supply, $\widehat{\mu}_\lambda = \mu - \frac{\alpha\sigma_\varepsilon^2}{\lambda}Z$, this implies that for a given realization of supply Z, both the informed and uninformed experience an MLR improvement in response to an increase in the fundamental μ; their beliefs react in the same direction to fundamentals. I provide more detail for this result below when discussing implications for investor demand.

Since adjusted volume $\widehat{\mu}_\lambda$ is not one of the primitives of the model, it is helpful to have a condition on the underlying random variables that determines whether $f_{\widehat{\mu}_\lambda|\mu}$ has the MLRP. The following Lemma provides an equivalent condition on the distribution f_Z of the asset supply.

Lemma 1.3.1. *The conditional densities $f_{\widehat{\mu}_\lambda|\mu}$ satisfy the (strict) MLRP if and only if* $\log f_Z$ *is (strictly) concave.*[13]

An alternative, but equivalent, characterization of Lemma 1.3.1 is that $f_{\widehat{\mu}_\lambda|\mu}$ satisfies the MLRP if and only if the distribution of asset supply is strongly unimodal.[14] Requiring

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[12]Eeckhoudt and Gollier (1995) provide a proof, as well as an example that satisfies FOSD but not MLR. See also Chapter 1.C of Shaked and Shanthikumar (2007) for further discussion of the MLR order.

[13]An (1998) and Bagnoli and Bergstrom (2005) discuss logconcavity and related properties.

[14]A density on the real line is unimodal if for all $K > 0$, the set $\{x \in \mathbb{R}|f(x) \geq K\}$ is convex. A density is strongly unimodal if its convolution with any other unimodal distribution is unimodal. If a distribution is strongly unimodal then it is unimodal, but the converse is not true.

a logconcave distribution rules out fat-tailed noise distributions (Karlin, 1968, Proposition 7.1.4) as well as both multimodal distributions and unimodal ones that are not strongly unimodal. Counterexamples to logconcavity (and thus the MLRP) need not be pathological. The normal mixture examples used in this paper to break the MLRP can be thought of as a world with higher-order uncertainty: noise trade is typically drawn from a normal distribution with, say, a mean of one, but with some small probability it is drawn from a distribution with a much higher mean, in which case noise traders flood with market with a large number of shares. Similarly, neither lognormal nor binomial distributions are logconcave. Even if one restricts attention to unimodal distributions with "nice" intuitive properties, logconcavity does not necessarily follow. Given a (bounded) distribution for the underlying state μ, Chambers and Healy (2009) exhibit a unimodal, symmetric, mean-zero error term (the analogue to supply in my model) and a range of signals over which posteriors deteriorate in the sense of FOSD.

In Figure 1.2(a), I plot the posterior beliefs about μ for increasing realizations of adjusted volume $\widehat{\mu}_\lambda$ when distributions are normal and therefore the MLRP is satisfied. The solid line corresponds to the lowest realization of $\widehat{\mu}_\lambda$, followed by the dashed line, and then the dotted line. Notice that higher signal realizations correspond unambiguously to "higher" posterior distributions. On the other hand, if the supply distribution Z is, say, a bimodal mixture of normals, then signals do not have the MLRP. Figure 1.2(c) plots the analogous posterior beliefs in such an economy for increasing realizations of $\widehat{\mu}_\lambda$. In this case, the posterior distribution corresponding to the *highest* signal (dotted line) is actually "lower" than the other two, which illustrates the fact that once signals fail to have the MLRP, the uninformed posterior beliefs do not necessarily move in the "correct" direction when μ changes.

Moving beyond probability assessments, it should be apparent that the way in which beliefs react to signals has implications for the way in which agents trade in response to signals. An upward shift in an asset's payoff distribution (for instance, an FOSD improvement) has both substitution and income effects in general (though income effects are absent with exponential utility). However, the signs and magnitudes of the effects vary, so the overall effect on demand is ambiguous without adding further restrictions on either utility functions or random variables.[15] To obtain clear comparative statics, one must restrict the class of upward shifts considered. As first noted by Landsberger and Meilijson (1990), MLR improvements are sufficient for all nonsatiated investors to demand a greater quantity of the risky asset in a single-risky-asset portfolio problem. Athey (2002, Lemma 5) proves the stronger result that MLR shifts are in fact necessary and sufficient for any nonsatiated investor to rebalance her portfolio in the expected direction regardless of the price of the risky

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[15]It is a common misconception that FOSD improvements in a risky asset are sufficient for increased demand. As first pointed out by Fishburn and Porter (1976), this is not true without further restrictions on the utility function. Note that despite the lack of wealth effects, FOSD is not sufficient even for exponential utility. Fishburn and Porter (1976), Kira and Ziemba (1980), Cheng, Magill, and Shafer (1987), and Hadar and Seo (1990) provide conditions on utility functions that guarantee the "expected" comparative statics results for stochastic dominance shifts in various formulations of the portfolio problem.

(a) Beliefs (with MLRP)

(b) Uninformed demand (with MLRP)

(c) Beliefs (without MLRP)

(d) Uninformed demand (without MLRP)
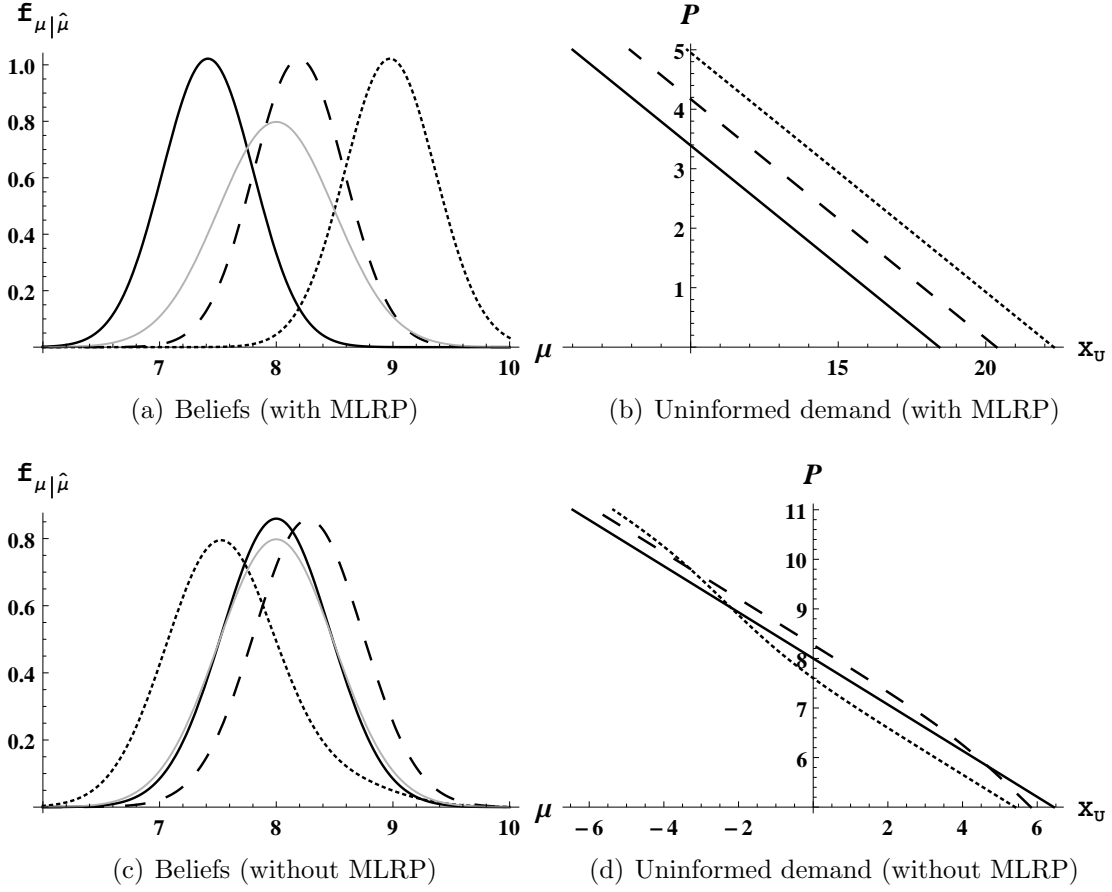
Figure 1.2: Plots of uninformed demand curves for increasing realizations of $\widehat{\mu}_\lambda$, along with the associated posterior beliefs. The lowest realization of $\widehat{\mu}_\lambda$ is represented by the solid line, next highest by the dashed line, and highest by the dotted line. For comparability, in the plots of beliefs the prior f_μ is included as the light gray line. In panels (a) and (b), $\eta = 1, \mu_Z = 1$, while in panels (c) and (d), $\eta = 8/10, \mu_{Z1} = 1, \mu_{Z2} = 4$. Other parameters are the same: $\beta = 1, \overline{\mu} = 8, \sigma_\mu = 1/2, \sigma_Z = 1/2, \sigma_\varepsilon = 1/2, \alpha = 1$.

asset. In other words, requiring the MLRP is equivalent not only to requiring that beliefs of both types respond in the same direction to changes in μ, but more importantly that their *demands* move in the same direction, regardless of the price that prevails in the market.

The intuition for why an MLR shift increases demand is straightforward. In short, it moves probability mass from bad (high marginal utility) states to good (low marginal utility) states and does so in such a way that the worst states lose the most mass and the best states gain the most most mass.[16] Regardless of risk preferences, any agent will desire to hold more of an asset that undergoes such an improvement. The following heuristic proof may help clarify. Suppose that the conditional densities of adjusted volume, $f_{\widehat{\mu}_\lambda|\mu}$ have the MLRP and consider a small increase in adjusted volume from $\widehat{m}$ to $\widehat{m} + \Delta\widehat{m}$. The conditional density of the fundamental μ changes from $f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})$ to $f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m} + \Delta\widehat{m})$. Using a first-order Taylor expansion, the new density can be written

$$f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m} + \Delta\widehat{m}) \approx f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m}) + \frac{\partial f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})}{\partial \widehat{m}}\Delta\widehat{m}$$

$$= f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})\left[1 + \frac{\frac{\partial f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})}{\partial \widehat{m}}}{f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})}\Delta\widehat{m}\right].$$

Therefore, the new conditional density is the original one corresponding to the lower signal multiplied by a factor that scales it up or down depending on whether the local likelihood ratio $\ell(m|\widehat{m}) := \frac{\frac{\partial f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})}{\partial \widehat{m}}}{f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})}$ is greater or less than zero. The MLRP implies that $\ell(m|\widehat{m})$ is increasing in m, and it is straightforward to show that $\int_{\mathbb{R}} \ell(m|\widehat{m}) f_{\mu|\widehat{\mu}_\lambda}(m|\widehat{m})\, dm = 0$. Hence there exists some state m_0 below which the new density subtracts mass ($\ell(m|\widehat{m}) < 0$) and above which it adds mass ($\ell(m|\widehat{m}) > 0$); moreover, the further to the left of m_0 one moves, the more mass is removed (as a proportion of the original mass) and the further to the right one moves, the more mass is added. The worst states (low μ) lose the most mass and the best states (high μ) gain the most mass. This makes the asset unambiguously more attractive for any investor who prefers more to less, regardless of their risk preferences.

Consider again an economy in which signals have the MLRP. Figure 1.2(b) illustrates the uninformed demand functions corresponding to the beliefs in Figure 1.2(a). Notice that demand shifts outward at every price when signals increase. Conversely, as illustrated in Figure 1.2(d), if signals do not have the MLRP then uninformed demand may not shift outward at every price. In that case an increase in signal can redistribute probability mass in an essentially arbitrary way and therefore can decrease demand at certain price levels. As seen in Figure 1.2(c), increasing the signal does not induce MLR (or even FOSD) shifts in the posterior, and therefore uninformed demand does not move in the same direction as informed demand with changes in the fundamental μ.

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[16]For instance, consider a risky asset with two-point support. Moving probability mass from the low state to the high state is an MLR improvement.

Up to now, I have said nothing of market clearing, but have merely described how agents react to signals, holding all else fixed. However, it should be unsurprising to learn that since the MLRP guarantees that the uninformed react in the "correct" direction to signals, it also guarantees that the market-clearing price reacts in the correct direction. To understand why this is, it may be helpful to first think about a model where there are no uninformed types. Consider an economy populated by only informed investors and noise traders. In this case, price always reacts in the expected direction. If the fundamental μ increases then, since changes in μ represent MLR shifts under the informed information set, the informed investors demand a greater quantity of risky asset at every price; therefore, in order to clear the market the equilibrium price must increase with μ, holding supply Z constant. Conversely, an increase in supply Z means that the informed must accommodate a greater number of shares at any level of the fundamental μ, so price must decrease in Z for fixed μ.

Now consider the same thought experiment of making a small change to the fundamental μ or supply Z but introduce uninformed investors who try to infer μ. If μ increases, then the informed demand curve still shifts outward at every price. The uninformed, on the other hand, do not observe μ, only a noisy signal in the form of the adjusted volume $\widehat{\mu}_\lambda$.[17] Since the uninformed demand does not always shift outward with higher signals, it follows that the *aggregate* demand will not necessarily shift outward either. As such, higher signals (corresponding, for instance, to higher realizations of μ) will not necessarily correspond to higher prices. The only way to guarantee a monotone price function for any prior f_μ is to impose the MLRP for the signal distribution $f_{\widehat{\mu}_\lambda|\mu}$ so that uninformed demand, and hence aggregate demand, shifts in the same direction as the fundamental.

1.4 General results

1.4.1 Uninformed demand and the information content of prices

In rational expectations models, price affects uninformed investor demand in multiple ways. First, a change in price will cause uninformed investors to modify their demand due to a standard substitution effect. Secondly, since price conveys a signal about the fundamental, there is an information effect: if higher prices signal higher fundamentals, uninformed demand will increase with price. In the standard model, all prices are equally informative, and equilibrium demand curves for the uninformed slope down. In other words, the substitution effect of an increase in price dominates the information effect. However, this depends on the joint distribution of fundamentals and price. In general, conditional moments of fundamentals will vary nontrivially with the price level. For certain regions of price, the information effect may dominate, leading to backward-bending demand. In such regions price responds

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[17]Note that here I am simply endowing the uninformed with the signal $\widehat{\mu}_\lambda$ and considering how their demand changes; there is no explicit learning from price. Proposition 1.4.2 below implies that doing so does not change the results.

abruptly to small changes in the fundamental, which can be interpreted as a price crash or jump.

Other papers in the literature generate backward bending demand in asymmetric information models, but must introduce other features, such as hedgers who follow strategies that are nonlinear in price (Gennotte and Leland, 1990), uncertainty over the number of informed traders (Romer, 1993), or borrowing constraints for informed types (Yuan, 2005, 2006). Here, the effect arises for purely informational reasons. In this sense it is complementary to the model proposed by Barlevy and Veronesi (2003). They consider a single-asset noisy RE model with risk-neutral traders who are subject to a constraint on position size (otherwise price would always be fully-revealing) and in which the fundamental follows a two-point distribution. While my model differs slightly from theirs, Proposition 1.4.1 below suggests the reason that they are able to generate crashes: any two-point distribution is not logconcave, and therefore uninformed demand can appear to "overreact" to information about the fundamental.

(a) Uninformed, informed, and aggregate demand

(b) Price function

Figure 1.3: Panel (a) plots uninformed demand (solid line), informed demand assuming μ equals its unconditional mean (dotted line), and overall aggregate demand (dashed line). Panel (b) shows price function for this economy as a function of $\widehat{\mu}_\lambda$. Parameters are: $\beta = 9/10, \eta = 1, \lambda = 1/10, \overline{\mu}_1 = 1, \overline{\mu}_2 = 6, \sigma_\mu = 1/5, \mu_Z = 1, \sigma_Z = 1/4, \sigma_\varepsilon = 1/2, \alpha = 1$.

To illustrate a price crash in my model, consider the normal mixture setting described above in which the mixing parameter for the supply distribution satisfies $\eta = 1$ so that only the fundamental μ is non-normal. Lemma 1.3.1 implies that the MLRP is satisfied and therefore by Proposition 1.4.2 below, I am justified in treating uninformed demand as a function purely of price. Figure 1.3(a) shows the uninformed demand curve (solid), along with the informed demand (dotted), and aggregate demand (dashed). Uninformed demand is clearly backward-bending. For "intermediate" prices, the information effect dominates

the substitution effect, and demand rises with price. However, once price is sufficiently high or low, the uninformed are again relatively certain about the state of the world, and the demand curve is downward-sloping. This fits with the intuition that an "extreme" price is more informative than an intermediate price: when price is very high or low, small *changes* have mostly substitution effects.

Backward-bending uninformed demand can lead to a price function that is particularly steep over narrow regions of fundamentals. In Figure 1.3(b), I plot the price function for the same economy as the demand functions in Figure 1.3(a). Notice that for values of adjusted volume $\widehat{\mu}_\lambda$ near 1.75 small shocks can cause large changes in price. Such extreme price reactions to small disturbances to fundamentals can be interpreted as crashes or jumps.

The example above is an illustration of the following result.

Proposition 1.4.1 (Backward-bending demand)**.** *Assume that the distribution of adjusted volume given the fundamental $f_{\widehat{\mu}_\lambda|\mu}$ has the strict MLRP and that the price function is differentiable.*[18]

- *If there exists $\widehat{m} < \widehat{m}'$ such that for $\widehat{\mu}_\lambda \in [\widehat{m}, \widehat{m}']$ the price function satisfies $\frac{\partial P_\lambda}{\partial \widehat{m}} > 1$, then uninformed demand slopes up for prices in the interval $[P_\lambda(\widehat{m}), P_\lambda(\widehat{m}')]$.*

- *If $\log f_\mu$ is concave, then $\frac{\partial P_\lambda}{\partial \widehat{m}} \leq 1$ and uninformed demand is everywhere downward-sloping.*

Recalling that the adjusted volume can be written $\widehat{\mu}_\lambda = \mu - \frac{\alpha \sigma_\varepsilon^2}{\lambda} Z$, one can interpret Proposition 1.4.1 as a condition on how strongly price reacts to a change in the fundamental μ. If there is a region of fundamentals in which prices "overreact" in the sense of moving more than dollar-for-dollar with μ, then that coincides with the region in which uninformed demand is backward-bending. This makes sense intuitively: A sufficiently strong information effect means that a change in price is self-reinforcing since the uninformed demand moves in the same direction as the price. A sufficient condition for downward-sloping demand is that the distribution of the fundamental is logconcave. This essentially guarantees that small changes in $\widehat{\mu}_\lambda$ are not "too informative," in the sense of moving the posterior expectation of μ by more than the change in $\widehat{\mu}_\lambda$ itself.

In this section, I have assumed that $f_{\widehat{\mu}_\lambda|\mu}$ has the MLRP. If it does not, then as discussed later in Proposition 1.4.2, in general, direct observation of $\widehat{\mu}_\lambda$ provides information not contained in price, and price does not uniquely determine uninformed demand. Thus, if one naively plots uninformed demand along with the associated equilibrium price, the result can appear multivalued, as in Figure 1.4(a). It is important to note, however, that for a given realization of adjusted volume, equilibrium demand is still uniquely determined since (from Proposition 1.4.2) observation of adjusted volume allows uninformed investors to determine the region of the price function in which the current equilibrium lies. In this case, as Figure

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[18]More generally, if the price function is not everywhere differentiable, the Proposition can be stated in terms of whether the function $\widehat{m} - P_\lambda(\widehat{m})$ is increasing or decreasing in $\widehat{m}$.

1.4(b) illustrates (and as discussed in detail in Section 1.4.2), changes in fundamentals can cause price to be decreasing in adjusted volume over certain regions.

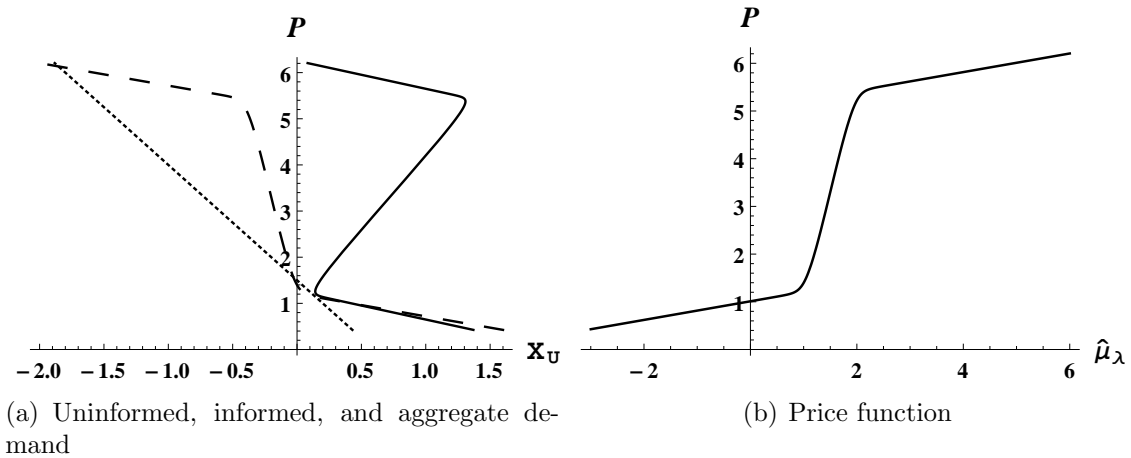(a) Uninformed, informed, and aggregate demand   (b) Price function
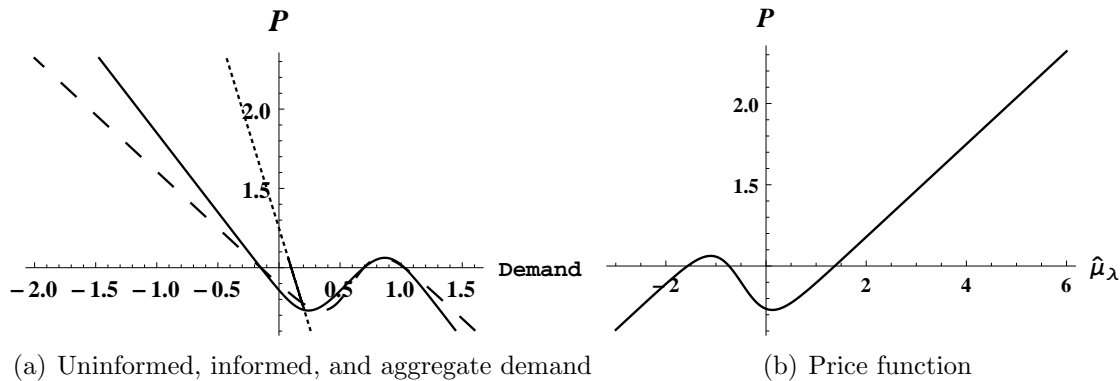
Figure 1.4: Panel (a) plots uninformed demand (solid line), informed demand assuming μ equals its unconditional mean (dotted line), and overall aggregate demand (dashed line). Panel (b) shows price function for this economy as a function of $\widehat{\mu}_\lambda$. Parameters are: $\beta = 1, \eta = 8/10, \lambda = 1/10, \overline{\mu} = 5/4, \sigma_\mu = 1/2, \mu_{Z1} = -1, \mu_{Z2} = 2, \sigma_Z = 1/2, \sigma_\varepsilon = 1/2, \alpha = 1$

Unlike other models in the literature, in my setting crashes and jumps arise without adding nonlinear hedgers, portfolio constraints, or additional uncertainty. All that is required is that in some regions, the uninformed learn at a particularly fast rate as price changes. This suggests that crashes may arise naturally for purely informational reasons, as long as there is asymmetric information in the economy.

1.4.2   The value of observing signed volume

In the standard model the asset price provides all possible information that the uninformed can glean from public sources, and observing (signed) volume, aggregate order flow, or any public quantity other than the current price provides no additional information.[19] Blume, Easley, and O'Hara (1994) and Schneider (2009) study more complex models in which observing trading volume is useful for investors. Blume, Easley, and O'Hara (1994) consider a dynamic model in which the precision of some traders' signals is random. However, they assume that investors are not able to condition on current price, only past prices. Combined with observations of the current price, observing volume provides a way to learn about the unknown signal precisions. Schneider (2009) studies an otherwise-standard static model but assumes that the correlation between investors' signals is random; trading volume allows

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[19]This is not necessarily true in models in which traders have diverse information such as Diamond and Verrecchia (1981). In those models, if traders can condition on volume and the sign of their own trade, then a fully-revealing equilibrium exists. See Blume, Easley, and O'Hara (1994) for a heuristic proof.

them to determine whether they are in the high correlation or low correlation state. Two other related papers in the technical analysis literature are Brown and Jennings (1989) and Grundy and McNichols (1989), both of which study three-date dynamic models in which past prices are informative when used in conjunction with the current price.

While these other studies require introduction of either uncertainty about some quantity other than fundamentals or additional rounds of trade, the following proposition shows that a similar result can hold even when investors need only to learn about the asset payoff, and there is no uncertainty over information quality or diversity.

Proposition 1.4.2 (Information content of adjusted volume)**.**

   (i) *If the conditional distribution of adjusted volume given the fundamental, $f_{\widehat{\mu}_\lambda|\mu}$, has the strict MLRP, then the price function is strictly increasing in adjusted volume $\widehat{\mu}_\lambda$ and observing P_λ provides the same information to the uninformed as observing $\widehat{\mu}_\lambda$.*

   (ii) *If $f_{\widehat{\mu}_\lambda|\mu}$ does not have the strict MLRP, then observing $\widehat{\mu}_\lambda$ provides more information than P_λ in the following sense: depending on the distribution of the fundamental, μ, there may exist realizations $\widehat{m}' \neq \widehat{m}$ of $\widehat{\mu}_\lambda$ for which $P_\lambda(\widehat{m}') = P_\lambda(\widehat{m})$.*

The first result in Proposition 1.4.2 is equivalent to the fact that with the MLRP, the price function is strictly increasing in adjusted volume $\widehat{\mu}_\lambda$. In that case, each realization of P_λ is associated with a unique value of $\widehat{\mu}_\lambda$, and price provides exactly the same information as direct observation of adjusted volume $\widehat{\mu}_\lambda$. An increase in fundamental μ MLR-improves the investment opportunity set for both types which increases the aggregate demand and therefore increases the price. Figure 1.5(a) illustrates a monotone price function for the normal mixture setting introduced above in which the mixing parameter for the fundamental satisfies $\beta = 1$ so that only supply is non-normal.

The second result in Proposition 1.4.2 says that without the MLRP, there may exist prices that are consistent with two or more distinct realizations of adjusted volume $\widehat{\mu}_\lambda$.[20] If the distribution of $\widehat{\mu}_\lambda$ does not have the MLRP then one cannot guarantee that both informed and uninformed types react in the same direction (as in Figure 1.2(d) above) and thus that the price moves in the same direction. Nonmonotonicity of the price function may seem surprising at first; however, in light of the discussion in Section 1.3 it is in some sense obvious. Informed and uninformed demand may shift in opposite directions in response to increases in the fundamental μ, and over certain regions the uninformed reaction may be sufficiently strong to *decrease* the price. In those situations, the price is no longer a sufficient statistic for learning about μ, and the adjusted volume allows the uninformed to distinguish between the various regions of the price function. This is illustrated in Figure 1.5(b) in which the price function is non-monotone, and there are some realizations of price that correspond to three possible values of $\widehat{\mu}_\lambda$.

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[20]Note that if the uninformed were able to observe only the price, it follows from Lemma 1.6.3 that a $\widehat{\mu}_\lambda$-measurable equilibrium would fail to exist in these situations.

Nonmonotonic price functions also arise in asymmetric information models with feedback effects due to endogenous firm investment decisions (Dow and Rahi, 2003) or regulatory intervention (Bond, Goldstein, and Prescott, 2010). For instance, in Bond, Goldstein, and Prescott (2010), a given price may correspond to both a low-fundamental, positive-intervention state as well as a high-fundamental, no-intervention state. In my model, cash flows are exogenous, but there is a similar explanation with respect to the fundamental and noisy supply. Recall that in the normal mixture setting, one can think of the noisy supply realization as coming from a two-step procedure: first choose the 'high' or 'low' supply distribution, and then draw the supply from that distribution. In such a situation, the same price can arise in three distinct states. As the adjusted volume moves from 'high' values to 'low' values, the uninformed become become relatively confident that the low realization can be attributed to a draw from the 'high mean' supply distribution. Hence, they are willing to accommodate more of the asset than if they believed that the low realization was due to a low value for the fundamental μ. Hence, depending on the distribution of supply, a relatively low realization of $\widehat{\mu}_\lambda$ may actually be good news.

(a) Monotone price function   (b) Non-monotone price function
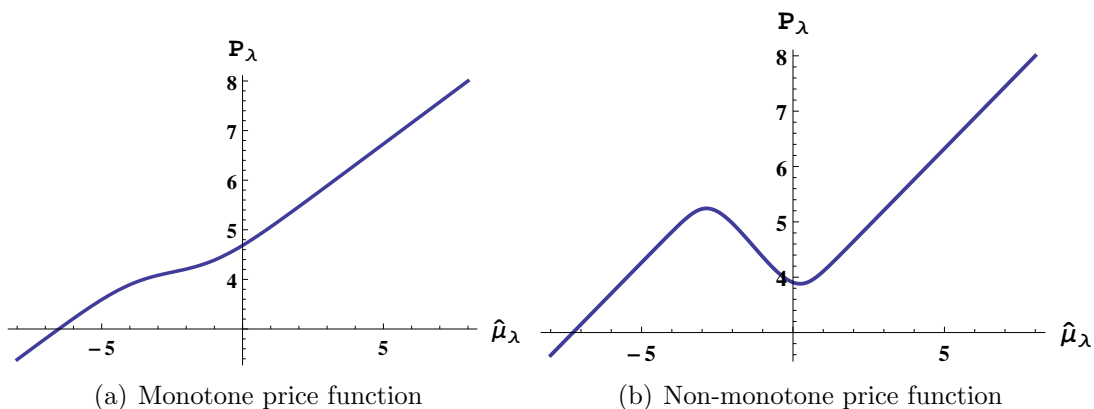
Figure 1.5: Plots of equilibrium price functions. In panel (a), $\mu_{Z2} = 3, \sigma_Z = 1$, while in panel (b), $\mu_{Z2} = 5, \sigma_Z = 1/2$. Other parameters are the same: $\beta = 1, \eta = 9/10, \overline{\mu} = 8, \sigma_\mu = 1, \mu_{Z1} = 0, \sigma_\varepsilon = 1/2, \alpha = 1, \lambda = 1/10$.

As the empirical analogue of adjusted volume is net order flow, this provides a potential explanation of the value of observing order flow: it contains information about aggregate demand that is not contained in price alone. This is reminiscent of the point made by Gallmeyer, Hollifield, and Seppi (2005) that the trading process itself can reveal information about the trading motive of one's counterparties. The results presented here are complementary in that their model focuses on learning effects with regard to unknown preferences and the consequences for future resale prices, while my model focuses on learning effects with regard to cash flows in a static model.

The result that adjusted volume conveys incremental information is similar in spirit to those of Blume, Easley, and O'Hara (1994) and Schneider (2009). In all cases, observation of a non-price statistic allows uninformed investors to more effectively learn from prices by making a previously non-invertible price function invertible.[21] However, my result makes clear that it is not necessary to add additional uncertainty to the model to achieve this. Rather, what is required is to change the distribution of uncertainty in the economy in such a way that the price function is no longer invertible. Whether one achieves this by, say, making signal quality random or simply changing the underlying probability distributions is immaterial. What is key is that the change makes the price function nonmonotonic in the fundamental.

1.4.3   The value of acquiring information

In the standard model as the number of informed investors increases, price becomes more informative, and the uninformed are better able to free-ride on the informed types' information. It follows that the value of observing the fundamental μ decreases with the number of informed. In other words, information acquisition is a strategic substitute: as more investors learn about μ, the incentive for others to do the same decreases. It has been an open question whether the opposite case (strategic complementarity in information acquisition) is also possible in an otherwise-standard noisy RE model.

In these models, an increase in the number of informed investors has two competing effects. It typically drives the asset price closer to the fundamental in each state (price effect), but it also changes the equilibrium allocations (share effect). The price effect tends to reduce the total surplus that both types enjoy at the expense of the noise traders, and it reduces the share of that surplus taken by the informed investors. On the other hand, the share effect requires that in equilibrium the remaining uninformed hold less advantageous positions in the asset in order to accommodate the increased number of informed investors. In the standard model with normal distributions, the price effect is sufficiently strong to offset the share effect. Price is responsive to information, which causes the price effect to dominate the share effect and make information acquisition a strategic substitute.

In my model, I can characterize the price and share effects of a change in the fraction of informed, λ, directly.

Lemma 1.4.3 (Utility gain). *Assume that the price function is differentiable with respect to $\widehat{m}$ and λ. The derivative of the utility gain function CE with respect to the number of*

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[21]I do not mean invertible in the sense of fully-revealing. Here, invertible means that the price is monotonic in some (possibly noisy) aggregate of the information of all traders.

informed λ can be written as the sum of a price effect and a share effect

$$CE'(\lambda) = -\underbrace{\iint_{\mathbb{R}^2} \left(X_I(m, P_\lambda(\widehat{m}))e^I - X_U(\widehat{m}, P_\lambda(\widehat{m}))e^U\right) \left[\frac{\partial P_\lambda}{\partial \widehat{m}} \frac{m-\widehat{m}}{\lambda} + \frac{\partial P_\lambda}{\partial \lambda}\right] f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm \, d\widehat{m}}_{Price\ effect}$$

$$-\underbrace{\iint_{\mathbb{R}^2} \left(m - P_\lambda(\widehat{m}) - \alpha\sigma_\varepsilon^2 X_U(\widehat{m}, P_\lambda(\widehat{m}))\right) \left[\frac{\partial X_U}{\partial \widehat{m}} + \frac{\partial X_U}{\partial p} \frac{\partial P_\lambda}{\partial \widehat{m}}\right] \frac{m-\widehat{m}}{\lambda} e^U f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm \, d\widehat{m}}_{Share\ effect},$$

$$(1.5)$$

where e^I and e^U are functions defined in the proof in Section 1.6.2.

As alluded to above, the interpretation of Proposition 1.4.3 is that a change in the number of informed agents affects the equilibrium in two ways. First, price changes in each state of the world both because the relative number of both types changes, and because the remaining uninformed hold different information relative to the previous world in which there were fewer informed investors. This is represented by the price effect term in eq. (1.5). Second, holding the price constant, the remaining uninformed investors must accommodate a larger number of informed types in each state of the world. This is represented by the share effect term in eq. (1.5) The first effect affects both the informed and uninformed. On the other hand, the second effect only affects the uninformed, since the informed investors' demand depends on $\widehat{\mu}_\lambda$ only indirectly through the price.

The next proposition gives sufficient conditions for signing the share effect.

Proposition 1.4.4 (Sign of the share effect)**.**

- *If $\frac{\partial P_\lambda}{\partial \widehat{m}} \leq 1$ then the share effect is positive.*

A price that moves less than dollar-for-dollar with fundamentals makes the share effect positive. Increasing λ, the fraction of informed investors, increases the correlation between μ and $\widehat{\mu}_\lambda$, making $\widehat{\mu}_\lambda$ "higher when μ is high" and "lower when μ is low" (compared to before the increase in λ). The equilibrium risky asset holding of the informed and noise traders is given by $\frac{\lambda}{\alpha\sigma_\varepsilon^2}\left(\widehat{\mu}_\lambda - P_\lambda(\widehat{\mu}_\lambda)\right)$. As long as $\frac{\partial P_\lambda}{\partial \widehat{m}} \leq 1$, this expression will be increasing in $\widehat{\mu}_\lambda$ and hence the holdings of the informed and noise traders will also tend to be higher when μ is high and lower when μ is low. As the uninformed investors must hold the opposite position, it follows that an increase in λ causes the uninformed investors' equilibrium allocation of the risky asset to be relatively smaller when μ is high and relatively higher when μ is low in order to accommodate the greater number of informed investors. As their positions move in the opposite direction of the fundamental, this increases the ex-ante benefit of becoming informed.

I have been unable to sign the price effect in general, but numerical experimentation suggests that the MLRP and downward-sloping demand is sufficient to guarantee that it is negative and that it dominates the share effect. To understand why, it is helpful to refer

back to the discussion in Section 1.3. As explained there, with the MLRP, the uninformed investors' demand reacts in the same direction as the informed to changes in the fundamental μ. An increase in the number of informed λ makes the signal $\widehat{\mu}_\lambda$ more highly correlated with the fundamental μ, and therefore makes uninformed demand more highly correlated with informed demand. This drives the price closer to the fundamental (on average), reducing the profit that the informed investors make from their information.

More precisely, it is straightforward to show that under the MLRP assumption, increases in the fraction informed, λ, improve the *accuracy* (sometimes also called *effectiveness*) of the signal $\widehat{\mu}_\lambda$. Accuracy was introduced by Lehmann (1988) in the context of statistical decision theory as a generalization of Blackwell (1951, 1953) sufficiency, which is another common criteria for comparing signals. Persico (1996, 2000) introduced the use of accuracy in economic contexts. Blackwell sufficiency was the definition of informativeness used by Grossman and Stiglitz (1980). Unfortunately, many signals cannot be compared using Blackwell sufficiency. Accuracy allows the comparison of many more signals, as discussed by Lehmann (1988). It also nests Blackwell sufficiency; if one signal is sufficient for another, then it is also more accurate.

The overall sign of the utility gain expression depends on the sign and strength of the two effects discussed above. As noted above, I have been unable to determine general conditions under which it is either increasing or decreasing. I present here some simple numerical examples to illustrate situations in which, contrary to the standard result, information acquisition is a strategic complement.

Example 1.4.1 (Failure of the MLRP is not necessary for complementarity)**.** Figure 1.6(a) plots the certainty equivalent gain of becoming informed as a function of λ for a normal mixture example in which $\eta = 1$, so that the fundamental is not symmetric but supply is symmetric and normally distributed. The parameter values are those used in the backward-bending demand example in Section 1.4.1 above. Notice that the plot is increasing until λ reaches about 0.05; over this region, information acquisition is a strategic complement. Due to the normality of supply, this example satisfies the MLRP.

Example 1.4.2 (Failure of downward-sloping demand is not necessary for complementarity)**.** Figure 1.6(b) plots the gain from becoming informed when the fundamental is distributed uniformly on $[0, 10]$, and supply is drawn from a binomial distribution on $\{-1, 1\}$ in which both realizations are equally likely. The binomial distribution for supply breaks the MLRP.[22]. In particular, for $\lambda \leq 1/5$, the equilibrium is fully revealing, while for $\lambda > 1/5$, it is only partially revealing. This provides a stark illustration that without the MLRP, changes in the fraction informed can have surprising effects on price informativeness.

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[22]A two-point distribution for supply is not consistent with the standing assumption that all random variables are continuously distributed, but a similar result holds if one approximates the binomial distribution with a continuous, 'U'-shaped, distribution centered at zero and having peaks at -1 and 1.

(a) Complementarity with MLRP for $\widehat{\mu}_\lambda$ (b) Complementarity without MLRP for $\widehat{\mu}_\lambda$
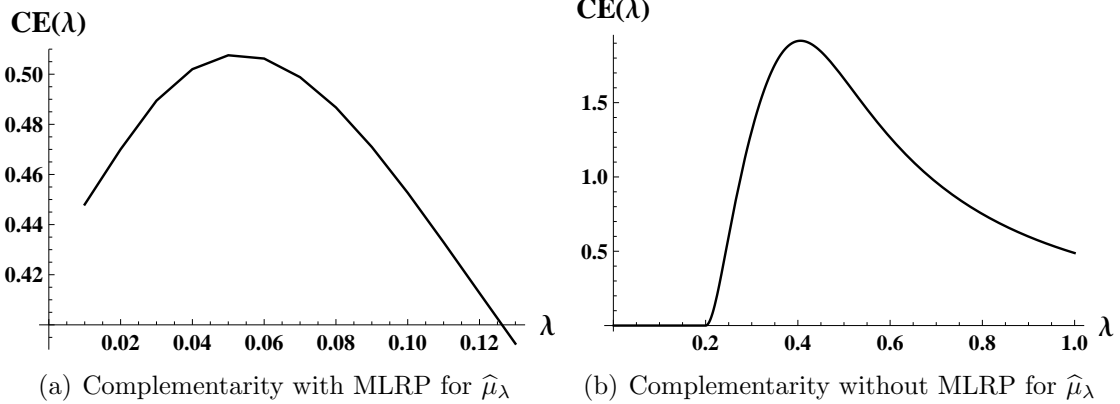
Figure 1.6: Panel (a) plots the certainty equivalent gain $CE(\lambda)$ from becoming informed as a function of λ for the normal mixture model with parameters $\beta = 8/10, \eta = 1, \overline{\mu}_1 = 1, \overline{\mu}_2 = 4, \sigma_\mu = 1/2, \mu_Z = 1, \sigma_Z = 1/2, \sigma_\varepsilon = 1/2, \alpha = 1$.. Panel (b) plots the certainty equivalent gain from becoming informed when the fundamental μ follows a $U[0, 10]$ distribution, and supply a binomial $\{-1, 1\}$. Other parameters are $\sigma_\varepsilon^2 = \alpha = 1$.

There is a small recent literature investigating situations in which information acquisition is a strategic complement. The model closest to my own is that of Barlevy and Veronesi (2000). They study a noisy RE model with risk-neutral investors in which the fundamental follows a two point distribution, and supply an exponential distribution. They claim that the change in distributions leads to complementarity in their model, but as pointed out by Chamley (2008) they compute the value of information in a non-standard way. After correcting this, information acquisition remains a strategic substitute. Barlevy and Veronesi (2008) resurrect their previous model by appealing to positively correlated fundamentals and supply. Effectively, it appears that introducing correlation generates complementarity by breaking the MLRP. Barlevy and Veronesi (2008) exhibit an economy in which increases in price induce *downward* MLR shifts in the asset payoff. They note that if one allows the fundamental and supply to be correlated in the standard Grossman and Stiglitz (1980) model, strategic complementarity arises if the signal from price (i.e., the adjusted volume $\widehat{\mu}_\lambda$) is negatively correlated with the fundamental.[23] In a jointly-normal world, this is exactly when the MLRP does not hold. From a practical standpoint, however, if noisy supply is meant to represent trade unrelated to fundamentals, it is not clear that a nonzero correlation is an appropriate modeling choice. On the other hand, if noise trade is motivated by liquidity needs, then a negative correlation (traders bail out of the asset when fundamentals are poor

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[23]Note that this is consistent with the positive correlation between the fundamental μ and supply Z. Since Z enters $\widehat{\mu}_\lambda$ with a negative sign, it induces a *negative* correlation between μ and $\widehat{\mu}_\lambda$ as long as the correlation with μ is sufficiently positive.

and vice versa) may better fit our intuition.

Ganguli and Yang (2009) demonstrate strategic complementarity in a CARA-normal model in which investors can become informed about both the fundamental and aggregate supply of the asset. When used in concert with the information in price, supply information is valuable since it allows investors to more precisely ascertain whether the prevailing price is due to the fundamental or merely a supply shock. With two dimensions of information, investors can coordinate on an equilibrium in which information acquisition is a strategic complement and price becomes less informative when the number of informed investors increases. If an additional trader becomes informed, the weight on the fundamental in the price increases. The increased weight on the fundamental makes all other investors trade more intensely on their supply information, and if this effect is sufficiently strong, it can swamp the increased informativeness from the increased weight on the fundamental. Manzano and Vives (2010) generalize the Ganguli and Yang (2009) setting to permit correlation between signal errors and demonstrate that strategic complementarity in the information market is not robust, in that such equilibria are unstable in the multi-signal CARA-normal setting.

Other authors generate complementarity in models that differ more substantially from the one presented here. Chamley (2007) solves a sequential-trade model in which investors have short horizons. In this setting, an increase in the number of informed investors drives today's price closer to the fundamental, but can also make tomorrow's price more uncertain. If the second effect is sufficiently strong, information acquisition can be a strategic complement. Veldkamp (2006b) introduces an information production sector into an overlapping generations version of the standard Grossman and Stiglitz (1980) model. The endogenous price of information generates complementarity in information acquisition and can lead to 'frenzies' in the information market in which many investors seek to buy the same information. The information market is such that it costs the provider nothing to distribute the information once it has been discovered, so to maximize profits, she reduces the price of information as demand increases to deter entry by competitors. Such price reduction can feed back and further increase demand for information. In recent work, García and Strobl (2010) show that relative wealth concerns can generate complementarities in information acquisition.

Despite these advances, to my knowledge, no one has yet demonstrated strategic complementarity in a standard static noisy REE model. Here, I have shown that even a simple model can generate the effect for purely informational reasons.

Implications for information market equilibria

Here I take up briefly the question of equilibrium in the information market, which was deferred from Section 1.2.1

In the examples presented in Figure 1.6 above, one can determine the equilibrium value of λ for various values of c by drawing a horizontal 'cost' line at level c. Interior equilibria are points at which the cost line intersects the certainty equivalent gain, while corner equilibria

$\lambda = 0$ or 1 occur when $CE(0) < c$ or $CE(1) > c$, respectively. In both examples in Figure 1.6, there are multiple equilibria in the information market. First, there is a zero-information equilibria in which no one finds it profitable to purchase information. Secondly, there are two interior equilibria[24] in which a strictly positive number of agents gather information. This example is an illustration of the following Corollary to Proposition 1.4.4.

Corollary 1.4.5.

- *If information acquisition is a strategic substitute for all $\lambda \in (0, 1)$, then the equilibrium in the information market is unique.*

- *If information acquisition is a strategic complement for $\lambda \in (0, k)$ for some constant $k \in (0, 1)$, then there may exist multiple equilibria in the information market: for certain values of c, there exist both an equilibrium in which no investors buy information and interior equilibria in which a strictly positive number of investors buy information.*

Since complementarity can generate multiple equilibria, my model, as those of Veldkamp (2006b) or Chamley (2007), provides a potential explanation for time-varying price informativeness. If markets can jump between zero-information and positive-information equilibria, then the information content of price can change abruptly over time.

1.4.4 The relation between disagreement and returns

A number of empirical papers document a negative relation between investor disagreement and future returns (see, e.g., Diether, Malloy, and Scherbina, 2002; Goetzmann and Massa, 2005).[25] The "difference of opinions" (abbreviated DO) model of Miller (1977) in which investors "agree to disagree" and do not condition on prices implies that if investors are short-sale constrained then stocks about which there is more disagreement will tend to have higher valuations (and hence lower returns). The reason is that the most pessimistic investors are prevented from taking negative positions. Chen, Hong, and Stein (2002) build on the Miller (1977) insight and provide further empirical support. Accordingly, based on these models, the documented negative relation between disagreement and returns is often taken as evidence that investors do not fully condition on prices

Banerjee (2010) points out the difficulty of distinguishing RE and DO models in a static setting with no short-sale constraints. Static RE and DO models can be made observationally equivalent by appropriate parameter choices. To distinguish the hypotheses, Banerjee (2010) considers how disagreement relates to the dynamic properties of returns and trading volume in a setting in which investors care about future resale prices. His empirical results support the hypothesis that investors do in fact condition on prices on average.

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[24]Only the interior equilibria at which CE is downward-sloping is stable in the *tântonnement* sense.

[25]There is some contrary evidence from Doukas, Kim, and Pantzalis (2006), but most studies find a negative relation.

In my model, with only two classes of investors, disagreement can be defined as the cross-sectional variance of beliefs about D at $t = 1$ and is given by

$$\lambda(1 - \lambda)(\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2.$$

As the next proposition shows, if return distributions are sufficiently negatively skewed or the relation between conditional volatility and excess returns is sufficiently weak, nonnormality enhances the difficulty of distinguishing RE and DO in a static setting.

Proposition 1.4.6. *The unconditional covariance between investor disagreement and future returns is given by*

$$\mathrm{Cov}(D - P_\lambda, \lambda(1 - \lambda)(\mu - \mathbb{E}[\mu|\widehat{\mu}])^2) = \lambda(1 - \lambda)\bigg(\mathrm{Cov}\left(\mathbb{E}[\mu - P_\lambda|\widehat{\mu}_\lambda], \mathrm{Var}[\mu - P_\lambda|\widehat{\mu}_\lambda]\right)$$
$$+ \mathbb{E}\left[\mathrm{Skew}[\mu - P_\lambda|\widehat{\mu}_\lambda]\right]\bigg).$$

Proposition 1.4.6 shows that the relation between disagreement and returns is given by the sum of two terms. The first is the covariance between conditional volatility and expected returns, which intuition typically suggests should be positive.[26] Holding the fraction of informed λ constant, greater disagreement is associated with greater uncertainty about fundamentals, which leads to a higher risk premium. The second term is the expected conditional skewness of the excess return $\mu - P_\lambda$.[27] This reflects the fact that for, say, a positively skewed distribution, the realizations of μ associated with the greatest degree of disagreement tend to be those that are above the mean.

The following Corollary emerges immediately from Proposition 1.4.6 and implies that the existing empirical evidence of a negative unconditional relation between disagreement and returns is not necessarily inconsistent with a fully-rational noisy RE model.

Corollary 1.4.7. *If the covariance between excess returns and conditional volatility,* $\mathrm{Cov}\left(\mathbb{E}[\mu - P_\lambda|\widehat{\mu}_\lambda], \mathrm{Var}[\mu - P_\lambda|\widehat{\mu}_\lambda]\right),$ *or the expected conditional skewness,* $\mathbb{E}\left[\mathrm{Skew}[\mu - P_\lambda|\widehat{\mu}_\lambda]\right],$ *is sufficiently negative, then the covariance between excess returns and disagreement is negative.*

Empirically, there is evidence that the conditional skewness of the market is negative. French, Schwert, and Stambaugh (1987) document "asymmetric volatility"–the fact that volatility tends to be higher when returns are negative. More recently, Chen, Hong, and Stein (2001) estimate conditional skewness directly and find that it is negative for the market

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[26]Empirically, however, Whitelaw (1994) shows that the relation is not as consistent as it is in most theoretical models.

[27]For notational convenience, I refer to the third central moment of a random variable as its skewness. The typical definition divides the third moment by the cube of the standard deviation. The nature of the Proposition is unchanged if one uses the standard definition.

as a whole (but is often positive for individual stocks). Furthermore, Chen, Hong, and Stein (2001) find that stocks for which measures of disagreement, such as lagged turnover, are highest tend to have more negatively skewed returns.

Even if one believes that conditional skewness is positive, given the conflicting evidence on the sign of the relation between conditional volatility and returns found by French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993), and Whitelaw (1994), it is not clear that a negative relation between disagreement and returns indicates a failure of investors to condition on price. Corollary 1.4.7 implies that such a result can be consistent with efficient use of the information in price, and demonstrates a heretofore unconsidered link between higher moments and the return-disagreement relation.

1.5   Conclusion

This paper studies a standard noisy rational expectations model in the spirit of Grossman and Stiglitz (1980) but relaxes the usual assumption of joint normality of fundamentals and supply. Results suggest that the normality assumption is not innocuous.

I show that, in general, price-informativeness varies with the price level and learning effects can cause uninformed investors to submit backward-bending demand curves, leading to price jumps and crashes in response to small changes in fundamentals. If signals do not satisfy a monotone likelihood ratio property, observation of signed trading volume may be valuable for uninformed investors because it provides a refinement of the information contained in price alone. Furthermore, the intuition that increasing the number of informed traders makes it easier for uninformed investors to free-ride on their information is not always true. The value of obtaining information can be non-monotonic in the number of informed investors in the economy. Finally, the relation between disagreement and returns depends on the relation between conditional expected returns and volatility and on the skewness of fundamentals.

While the model presented is not the most general possible, the intuition should carry over to more complex models in which investors observe diverse information, have utility functions that are not exponential, or face an opportunity set with multiple risky assets. The key insight is that many important comparative statics results depend on whether investors' beliefs and demands move in the same direction and with the comparable strength in response to changes in fundamentals. Restricting the conditional distribution of signals by requiring the MLRP guarantees that they do. Without such a restriction, the signs of many important comparative statics are ambiguous. As such, I provide the first demonstration of the importance of the MLRP for the noisy RE literature.

Overall, my results emphasize that most existing noisy RE models tend to be very special cases that are easy to analyze, but whose conclusions are not robust to perturbations of the economy. For the most part, the ubiquity of the standard model is due to the fact that it is technically demanding to analyze equilibrium when price must both clear the market and

convey information (but not be fully-revealing). The assumptions of exponential utility and joint normality ease the technical burden significantly and lead to a tractable, elegant model. However, a number of resulting properties that we take to be "properties of well-functioning financial markets" may be more appropriately considered to be "properties of the normal distribution."

1.6 Appendix

1.6.1 Some results on single-crossing and sign-regular functions

I collect in this Section some general results on single-crossing and sign-regular functions that are used to prove various results in the paper. I begin with the relevant definitions.

For my purposes, a single-crossing function is defined as follows.

Definition 1.6.1 (Single-crossing). A function $f : \mathbb{R} \to \mathbb{R}$ is single-crossing from below if there exists some point x_0 such that $x \geq x_0$ implies $f(x) \geq 0$. A function f is single-crossing from above if $-f$ is single-crossing from below.

The following two definitions are taken directly from Definitions 1.1 and 1.2 in Chapter 1 of Karlin (1968).

Definition 1.6.2 (Sign regularity). A function $K : \mathbb{R}^2 \to \mathbb{R}$ is *sign regular of order* 2 (SR_2) if there exist constants $\kappa_1, \kappa_2 \in \{-1, 1\}$ such that $\kappa_1 K(x, y) \geq 0$ and for for any choice of $x_1 < x_2$ and $y_1 < y_2$

$$\kappa_2 \begin{vmatrix} K(x_1, y_1) & K(x_1, y_2) \\ K(x_2, y_1) & K(x_2, y_2) \end{vmatrix} \geq 0, \tag{1.6}$$

where $|\cdot|$ represents the determinant. The function is *strictly sign regular of order* 2 (SSR_2) if the inequalities are strict.

The more commonly-encountered property of total positivity is a special case of sign-regularity

Definition 1.6.3 (Total positivity). A function $K : \mathbb{R}^2 \to \mathbb{R}$ is *totally positive of order* 2 (TP_2) if it is SR_2 and $\kappa_1 = \kappa_2 = 1$. The function is *strictly totally positive of order* 2 (STP_2) if it is SSR_2 and $\kappa_1 = \kappa_2 = 1$.

To clarify the importance of sign regularity, recall that a conditional density $f_{X|Y}(x|y)$ has the (strict) MLRP if and only if it is $(S)TP_2$. Similarly, a utility function u has (strictly) decreasing absolute risk aversion if and only if $u'(x - y)$ is $(S)TP_2$. Jewitt (1987, 1991) discusses these and other economic applications of sign regularity.

In the following Lemma, I generalize Theorem 11.2 from Chapter 6 Karlin (1968) to functions that are SR_2. I also provide a "strong" version of the result for STP_2 and SSR_2 functions.

Lemma 1.6.1. *Let f and K be functions mapping $X \times Y \mapsto \mathbb{R}$ for some intervals $X, Y \subset \mathbb{R}$, and let μ be any σ-finite measure on Y. Assume that the integral $g(x) := \int_Y f(x, y) K(x, y) d\mu(y)$ exists and is continuous in x.*

If for each fixed $y \in Y$, $f(x, y)$ is increasing in x, and either

(a) *For each fixed $x \in X$, $f(x, y)$ crosses zero at most once, and from below, as y increases, and K is TP_2; or*

(b) *For each fixed $x \in X$, $f(x, y)$ crosses zero at most once, and from above, as y increases, and K is SR_2 with $\kappa_1 = 1, \kappa_2 = -1$;*

then g crosses zero at most once, and from below, as x increases. (Note that the single-crossing "point" could in fact be an interval over which $g = 0$.) If the hypotheses are strengthened to require that f is strictly *increasing in x for fixed y and either (a) K is STP_2, or (b) K is SSR_2, then if g crosses zero it does so at a single point.*

Proof of Lemma 1.6.1. I prove Case (b). First, consider the "weak" version of Case (b) in which K is SR_2 and f is weakly increasing in x. The proof is similar to that of the weak version of Case (a) in Karlin (1968). Take x_0 such that $g(x_0) = 0$. It suffices to show that for any $x > x_0$, one has $g(x) \geq 0$. By the hypothesis that f is single-crossing in y for fixed x, it follows that there exists y_0 (which will, in general, depend on x_0) such that $f(x_0, y) \lesseqgtr 0$ as $y \gtreqless y_0$. Now, write

$$
\begin{aligned}
\frac{g(x)}{K(x, y_0)} &= \frac{g(x)}{K(x, y_0)} - \frac{g(x_0)}{K(x_0, y_0)} \\
&= \int_Y f(x, y) \frac{K(x, y)}{K(x, y_0)} \, d\mu(y) - \int_Y f(x_0, y) \frac{K(x_0, y)}{K(x_0, y_0)} \, d\mu(y) \\
&= \int_Y (f(x, y) - f(x_0, y)) \frac{K(x, y)}{K(x, y_0)} \, d\mu(y) + \int_Y f(x_0, y) \frac{K(x, y)}{K(x, y_0)} \, d\mu(y) \\
&\quad - \int_Y f(x_0, y) \frac{K(x_0, y)}{K(x_0, y_0)} \, d\mu(y) \\
&= \int_Y (f(x, y) - f(x_0, y)) \frac{K(x, y)}{K(x, y_0)} \, d\mu(y) + \int_Y f(x_0, y) \left[\frac{K(x, y)}{K(x, y_0)} - \frac{K(x_0, y)}{K(x_0, y_0)} \right] d\mu(y).
\end{aligned}
$$
(1.7)

The first equality holds since $g(x_0) = 0$, the second simply uses the definition of g to write out the integrals, the next-to-last equality adds and subtracts $f(x_0, y)$ in first integral, and the final line condenses the second and third integrals in the expression.

One can now use the expression in eq. (1.7) to sign $g(x)$. Since f is increasing in x, the term $f(x, y) - f(x_0, y)$ in the first integral is ≥ 0, and since $\kappa_1 = 1$, one has $\frac{K(x,y)}{K(x,y_0)} \geq 0$. Thus, the first integral is ≥ 0. In the second integral, $f(x_0, y) \lesseqgtr 0$ as $y \gtreqless y_0$ by the choice of y_0, and $\frac{K(x,y)}{K(x,y_0)} - \frac{K(x_0,y)}{K(x_0,y_0)} \lesseqgtr 0$ as $y \gtreqless y_0$ by the assumption that $\kappa_2 = -1$. It follows that the second integral is also ≥ 0. Hence, $\frac{g(x)}{K(x,y_0)}$, and therefore $g(x)$, is ≥ 0. This completes the "weak" version of Case (b). The "strong" version follows immediately since in that case, the weak inequalities involving f and K that were used to sign the two integrals on the last displayed line are strict. $\qquad \square$

For convenience, I restate, without proof, Lemma 1 from Persico (2000).

Lemma 1.6.2 (Lemma 1 from Persico (2000))**.** *Let $Y \subset \mathbb{R}$ be any interval. Let $g : \mathbb{R} \to \mathbb{R}$ be an increasing function, and let $h : \mathbb{R} \to \mathbb{R}$ cross zero at most once and from below. Assume that for some measure μ on Y, we have*

$$\int_Y h(y) \, d\mu(y) = 0.$$

Then,

$$\int_Y g(y)h(y) \, d\mu(y) \geq 0.$$

1.6.2 Proofs

I begin with two preliminary Lemmas that will be used in the proofs of Proposition 1.2.1 and 1.2.4.

Lemma 1.6.3. *If the uninformed investors can condition only in price, then any equilibrium price function must be one-to-one in Z for each fixed realization $\mu = m$ and one-to-one in μ for each fixed realization $Z = z$.*[28]

Proof of Lemma 1.6.3. I prove that price must be one-to-one in Z for fixed $\mu = m$. The proof of the second result is essentially identical. Assume to the contrary that P_λ is an equilibrium price function but that there exists some realization m such that $P_\lambda(m, \cdot)$ is not one-to-one in its second argument. Then there exist realizations $z \neq z'$ with $P_\lambda(m, z) = P_\lambda(m, z')$. This implies

$$
\begin{aligned}
0 &= (1 - \lambda)X_U(P(m, z)) + \lambda \frac{m - P_\lambda(m, z)}{\alpha\sigma_\varepsilon^2} - z \quad &&\text{market clearing in state } (m, z) \\
&= (1 - \lambda)X_U(P(m, z')) + \lambda \frac{m - P_\lambda(m, z')}{\alpha\sigma_\varepsilon^2} - z \quad &&\text{since } P_\lambda(m, z) = P_\lambda(m, z') \\
&\neq (1 - \lambda)X_U(P(m, z')) + \lambda \frac{m - P_\lambda(m, z')}{\alpha\sigma_\varepsilon^2} - z' \quad &&\text{since } z \neq z' \\
&= 0 \quad &&\text{market clearing in state } (m, z'),
\end{aligned}
$$

which is a contradiction. $\qquad\square$

---

[28]Of course, one can only make such statements up to a sets of Lebesgue measure zero. As in the body of the paper, I do not insist on such qualifications in this Section. It is straightforward but tedious to extend all proofs to be fully rigorous with respect to such measure-theoretic details.

Proof of Proposition 1.2.1. For clarity, I break the proof into steps. I begin by solving the agents' optimization problems assuming that the uninformed can condition only on the price, and that they have conjectured some (continuously differentiable) price function satisfying the conditions in Lemma 1.6.3 that they use for updating their beliefs. Next, I impose market clearing and simplify the resulting expression to produce the an implicit function that characterizes a candidate price function. There are then two cases to consider, one in which the candidate price function satisfies the condition in Lemma 1.6.3 and one in which it does not. I verify that when the candidate function does satisfy the condition then observing adjusted volume provides no additional information beyond that conveyed by price alone, which completes the proof in this case. To conclude, I show that when the candidate function does not satisfy the conditions Lemma 1.6.3, it nevertheless remains an equilibrium price function as long as the uninformed can condition on adjusted volume.

Fix a realization of $(\mu, Z) = (m, z)$. Consider the informed agent's maximization problem.

$$\max_{x \in \mathbb{R}} \mathbb{E}[-\exp\{-\alpha x(D - P_\lambda)\}|\mu = m, P_\lambda = p].$$

Since this problem is that of a CARA investor facing a conditionally normal risky asset, the demand function is standard

$$X_I(m, p) = \frac{m - p}{\alpha \sigma_\varepsilon^2}.$$

Next, consider an uninformed agent's maximization problem. Assume that the uninformed conjecture some price function $P_\lambda : \mathbb{R}^2 \to \mathbb{R}$ when computing their posterior beliefs given the observed price p, and this function satisfies the condition in Lemma 1.6.3

$$\max_{x \in \mathbb{R}} \mathbb{E}[-\exp\{-\alpha x(D - P_\lambda)\}|P_\lambda = p].$$

Since ε is independent of the other random variables, I can integrate it out, rewriting the maximization as

$$\max_{x \in \mathbb{R}} \mathbb{E}[-\exp\{-\alpha x(\mu - P_\lambda) + \frac{1}{2}\alpha^2 \sigma_\varepsilon^2 x^2\}|P_\lambda = p].$$

Differentiation yields the first-order condition for uninformed demand $X_U(p)$.[29] Since the utility function is strictly concave, the first-order condition defines the global maximum.

$$\mathbb{E}[(\mu - P_\lambda - X_U(p)\alpha\sigma_\varepsilon^2)\exp\{-\alpha X_U(p)(\mu - P_\lambda) + \frac{1}{2}\alpha^2\sigma_\varepsilon^2 (X_U(p))^2\}|P_\lambda = p] = 0. \quad (1.8)$$

---

[29]Differentiation through the expectation is justified since the expected utility is a conditional moment generating function of D evaluated at the point $-\alpha x$, and moment generating functions are infinitely continuously differentiable in their domain of existence.

After dividing out terms that are constant with respect to μ, I can rewrite (1.8) explicitly as an integral

$$\int_{\mathbb{R}} (y - p - \alpha\sigma_\varepsilon^2 X_U(p)) e^{-\alpha X_U(p)y} f_{\mu|P_\lambda}(y|p)\, dy = 0. \tag{1.9}$$

My next goal is to derive an expression for the joint distribution of μ and P_λ in terms of the distributions of the primitive random variables in order to characterize the conditional distribution in eq. (1.9). The condition that P_λ be one-to-one for each fixed $\mu = w$ guarantees that there exists a inverse function $P_\lambda^{-1}(p; w)$ that maps price to a unique value of supply for each fixed w. The inverse is defined implicitly by $p = P_\lambda(w, P_\lambda^{-1}(p; w))$. Thus, using Bayes rule and the standard transformation of random variables formula, the conditional distribution has a density which can be written as

$$f_{\mu|P_\lambda}(y|p) = \frac{f_Z(P_\lambda^{-1}(p; y)) \left| \frac{\partial}{\partial p} P_\lambda^{-1}(p; y) \right| f_\mu(y)}{\int_{\mathbb{R}} f_Z(P_\lambda^{-1}(p; w)) \left| \frac{\partial}{\partial p} P_\lambda^{-1}(p; w) \right| f_\mu(w)\, dw}. \tag{1.10}$$

The above steps characterize the uninformed investors' information set and optimal demand for a given function P_λ. Now I will impose market clearing and use eq. (1.9) to pin down the equilibrium price. The market clearing condition requires that at an equilibrium price $P_\lambda(m, z)$, the quantity demanded by the uninformed investors equals the negative of the quantity demanded by the informed investors and noise traders: $(1 - \lambda)X_U(P_\lambda(m, z)) = z - \lambda X_I(m, P_\lambda(m, z)) = z - \lambda\frac{m - P_\lambda(m,z)}{\alpha\sigma_\varepsilon^2}$. Hence, I can replace p with the candidate equilibrium price function $P_\lambda(m, z)$ in eq. (1.9) and use the market clearing condition to substitute for $X_U(P_\lambda)$ to obtain a functional equation in P_λ

$$\int_{\mathbb{R}} \left(y - \frac{\alpha\sigma_\varepsilon^2}{1-\lambda}z + \frac{\lambda}{1-\lambda}m - \frac{1}{1-\lambda}P_\lambda(m, z) \right) e^{-\frac{\alpha}{1-\lambda}\left(z - \lambda\frac{m - P_\lambda(m,z)}{\alpha\sigma_\varepsilon^2}\right)y}\, dF_{\mu|P_\lambda}(y|P_\lambda(m, z)) = 0. \tag{1.11}$$

It remains to derive a more explicit expression for the conditional distribution evaluated at the equilibrium price. Consider once again the negative of the quantity demanded by the informed investors and noise traders: $z - \lambda X_I(m, P_\lambda)$. If some other state (y, v) leads to the same price p as the state (m, z), then the uninformed demand must be the same in both states since they condition only on price. By market clearing, this implies that the effective supply must also be equal in both states. Hence, when evaluated at an equilibrium price $P_\lambda(m, z)$, the inverse function in eq. (1.10) must produce

$$P_\lambda^{-1}(P_\lambda(m, z); y) = \{v : v - \lambda X_I(y, P_\lambda(m, z)) = z - \lambda X_I(m, P_\lambda(m, z))\}$$

$$= z - \frac{\lambda}{\alpha\sigma_\varepsilon^2}(m - y). \tag{1.12}$$

It follows that the conditional density in eq. (1.10) can be written as

$$f_{\mu|P_\lambda}(y|P_\lambda(m,z)) = \frac{f_Z\left(z - \frac{\lambda}{\alpha\sigma_\varepsilon^2}(m-y)\right)f_\mu(y)\left|\frac{\partial}{\partial p}P_\lambda^{-1}(p;y)\right|\Big|_{p=P_\lambda(m,z)}}{\displaystyle\int_{\mathbb{R}} f_Z\left(z - \frac{\lambda}{\alpha\sigma_\varepsilon^2}(m-w)\right)f_\mu(w)\left|\frac{\partial}{\partial p}P_\lambda^{-1}(p;w)\right|\Big|_{p=P_\lambda(m,z)} dw}. \qquad (1.13)$$

To eliminate the remaining Jacobian terms, differentiate eq. (1.12) totally with respect to z to obtain

$$\frac{\partial}{\partial p}P_\lambda^{-1}(p;y)\Big|_{p=P_\lambda(m,z)}\frac{\partial}{\partial z}P_\lambda(m,z) = 1,$$

which can be rearranged to yield $\frac{\partial}{\partial p}P_\lambda^{-1}(p;y)\Big|_{p=P_\lambda(m,z)} = \frac{1}{\frac{\partial}{\partial z}P_\lambda(m,z)}$. This expression is constant with respect to y, so the Jacobian terms cancel from the numerator and denominator of eq. (1.13). Substituting this conditional density into eq. (1.11), canceling terms that do not depend on y, and rearranging produces eq. (1.1) in the text.

If the function P_λ defined implicitly by (1.1) is in fact one-to-one in each variable when the other is held fixed, then the condition from Lemma 1.6.3 is met and P_λ is an equilibrium price function for the case in which the uninformed can condition only on the asset price. Since the uninformed can also condition on the signed volume of the informed and noise traders, then in principle P_λ may not remain an equilibrium. This would be the case if the signed volume, $\lambda X_I(m,P_\lambda) - z$, reveals strictly more information that the linear combination, $\widehat{m}$, that is revealed by the price However, it does not do so. To see this, note that the signed volume also depends (directly) on m and z only through $\widehat{m}$:

$$\lambda X_I(m,P_\lambda) - z = \lambda\frac{m - P_\lambda}{\alpha\sigma_\varepsilon^2} - z$$

$$= \frac{\lambda}{\alpha\sigma_\varepsilon^2}(\widehat{m} - P_\lambda) \qquad (1.14)$$

This concludes the proof in the case that P_λ is monotone.

If the function P_λ defined by (1.1) is not one-to-one in m for fixed z, then the inversion in eq. (1.13) in the proof is not possible for all realizations p. Hence, if the uninformed can condition only on the asset price, then equilibrium does not exist in this case. However, as long as the uninformed can condition on signed volume, then the (non-one-to-one) P_λ from eq. (1.1) is still an equilibrium price function. To prove this, it suffices to show that observing the combination of P_λ from eq. (1.1) and signed volume is equivalent to observing $\widehat{m}$. Hence, the uninformed investors' updating step in eq. (1.13) will still reduce to updating from observations of the random variable $\widehat{\mu}_\lambda$. Given the signed volume can be written

as in eq. (1.14), the required observational equivalence is immediately apparent since the uninformed need only to multiply the signed volume by $\frac{\alpha \sigma_\varepsilon^2}{\lambda}$ and add P_λ to obtain $\widehat{m}$. This completes the proof. □

Proof of Proposition 1.2.3. Fix any realization $\widehat{\mu}_\lambda = \widehat{m}$. Suppose that the conditional moment generating function of μ given $\widehat{\mu}_\lambda = \widehat{m}$ exists in some open interval $(-\delta_0, \zeta_0)$, where $\delta_0, \zeta_0 > 0$, allowing the possibility of either endpoint being ∞. Consider eq. (1.2) as a function of p, after multiplying by $e^{-\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2} \widehat{m}}$, which is constant with respect to y and strictly positive, and thus does not affect the value of p at which eq. (1.2) equals zero

$$F(p) := \int_{\mathbb{R}} ((1-\lambda)y + \lambda\widehat{m} - p) \, e^{\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2}(y-\widehat{m})} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m}) \, dy.$$

If this integral exists and is continuous in p in some nonempty open set, then by the intermediate value theorem, it suffices to show that there exist p and p' in that open set such that $F(p) < 0$ and $F(p') > 0$.

Given the assumption on existence of the conditional moment-generating function, the integral is finite as long as p satisfies $\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2} \in (-\delta_0, \zeta_0)$. Furthermore, since moment generating functions are infinitely continuously differentiable in their domain of existence, it follows that F is continuously differentiable in p.[30] It remains to find the p and p' with $F(p) < 0$ and $F(p') > 0$.

Evaluating F at $\widehat{m}$ gives

$$F(\widehat{m}) = (1-\lambda) \int_{\mathbb{R}} (y - \widehat{m}) f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy.$$

If $\mathbb{E}[\mu|\widehat{\mu}_\lambda = \widehat{m}] = \widehat{m}$, existence is thus immediate, with $P_\lambda(\widehat{m}) = \widehat{m}$. If such is not the case, then the sign of $F(\widehat{m})$ depends on whether the conditional expectation of μ given $\widehat{\mu}_\lambda = \widehat{m}$ is greater or less than $\widehat{m}$ itself. Without loss of generality, consider the case in which $\mathbb{E}[\mu|\widehat{\mu}_\lambda = \widehat{m}] > \widehat{m}$. Then, $F(\widehat{m}) > 0$. It remains to find p such that $F(p) < 0$.

Parameterize $p(\delta) = \widehat{m} + \sigma_\varepsilon^2 \frac{1-\lambda}{\lambda} \delta$, where $0 < \delta < \delta_0$. I will show that one can choose δ such that $F(p(\delta)) < 0$. We have

$$F(p(\delta)) = (1-\lambda) \int_{\mathbb{R}} (y - \widehat{m}) e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy - \sigma_\varepsilon^2 \frac{1-\lambda}{\lambda} \delta \int_{\mathbb{R}} e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy$$

$$< (1-\lambda) \int_{\mathbb{R}} (y - \widehat{m}) e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy.$$

---

[30]To see this, let $M_{\mu|\widehat{\mu}_\lambda}$ denote the conditional moment generating function and write F as

$$F(p) = \left[(1-\lambda) M'_{\mu|\widehat{\mu}_\lambda} \left(\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2} \Big| \widehat{m} \right) + (\lambda\widehat{m} - p) \, M_{\mu|\widehat{\mu}_\lambda} \left(\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2} \Big| \widehat{m} \right) \right] e^{-\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2} \widehat{m}}.$$

Now, split the integral on the last line into positive and negative parts

$$\int_{\mathbb{R}} (y - \widehat{m}) e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy$$

$$= \int_{\mathbb{R}} \mathbf{1}_{\{y<\widehat{m}\}} (y - \widehat{m}) e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy + \int_{\mathbb{R}} \mathbf{1}_{\{y\geq\widehat{m}\}} (y - \widehat{m}) e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy$$

$$\leq \int_{\mathbb{R}} \mathbf{1}_{\{y<\widehat{m}\}} (y - \widehat{m}) e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy + \int_{\mathbb{R}} \mathbf{1}_{\{y\geq\widehat{m}\}} (y - \widehat{m}) f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy. \qquad (1.15)$$

If the first integral in line (1.15) can be made arbitrarily negative by appropriate choice of δ, it will dominate the second (positive) integral, and the proof will be complete. It is convenient, and equivalent, to show that the negative of the first integral, $\int_{\mathbb{R}} -(y - \widehat{m}) \mathbf{1}_{\{y<\widehat{\mu}\}} e^{-\delta(y-\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy$, can be made arbitrarily large. Consider first the case in which $\delta_0 = \infty$. As $\delta \to \infty$, the integrand converges almost everywhere to ∞. Hence, by Fatou's Lemma, one has

$$\liminf_{\delta\to\infty} \int_{\mathbb{R}} -(y - \widehat{m}) \mathbf{1}_{\{y<\widehat{m}\}} e^{-\delta(y-\widehat{m})} \, dy \geq \infty.$$

Thus, there exists some $\delta > 0$ such that $F(p(\delta)) < 0$, which completes the proof. Next, consider the case in which the moment generating function diverges at $-\delta_0$ for $\delta_0 < \infty$. Then, for $\delta \in (0, \delta_0)$, one has

$$\int_{\mathbb{R}} -(y - \widehat{m}) \mathbf{1}_{\{y<\widehat{m}\}} e^{-\delta(y-\widehat{m})} \, dy \geq \int_{\mathbb{R}} -(y - \widehat{m}) \mathbf{1}_{\{y<\widehat{m}-1\}} e^{-\delta(y-\widehat{m})} \, dy$$

$$\geq \int_{\mathbb{R}} \mathbf{1}_{\{y<\widehat{m}-1\}} e^{-\delta(y-\widehat{m})} \, dy.$$

Let $\delta \uparrow \delta_0$. Because the moment generating function diverges at $-\delta_0$, it follows that

$$\lim_{\delta\uparrow\delta_0} \int_{\mathbb{R}} \mathbf{1}_{\{y<\widehat{m}-1\}} e^{-\delta(y-\widehat{m})} \, dy = \infty.$$

Hence,

$$\liminf_{\delta\uparrow\delta_0} \int_{\mathbb{R}} -(y - \widehat{m}) \mathbf{1}_{\{y<\widehat{m}\}} e^{-\delta(y-\widehat{m})} \, dy \geq \infty,$$

which again guarantees the existence of $\delta > 0$ such that $F(p(\delta)) < 0$. $\qquad \square$

Proof of Proposition 1.2.4. To prove uniqueness in the case that the uninformed can condition only on price, it suffices to show that the price correspondence defined by eq. (1.2) is single-valued since the proof of Proposition 1.2.1 implied that in this case, any equilibrium

must be characterized by eq. (1.2). In the case that the uninformed can also condition on signed volume, to show uniqueness among $\widehat{\mu}_\lambda$-measurable price functions one must also show that any such price function is characterized by eq. (1.2).

I begin by showing that the price correspondence defined by eq. (1.2) is single-valued. Fix $\widehat{\mu}_\lambda = \widehat{m}$ and apply Lemma 1.6.1 to

$$\int_{\mathbb{R}} -\left((1-\lambda)y + \lambda\widehat{m} - p\right) e^{\frac{\lambda}{1-\lambda} \frac{\widehat{m}-p}{\sigma_\varepsilon^2} y} f_Z\left(\frac{\lambda}{\alpha\sigma_\varepsilon^2}(y - \widehat{m})\right) f_\mu(y)\, dy, \qquad (1.16)$$

which is simply eq. (1.2) multiplied by -1 (this multiplication, of course, does not affect the point p at which the expression equals zero). Since $e^{\frac{\lambda}{1-\lambda} \frac{\widehat{\mu}_\lambda - p}{\sigma_\varepsilon^2} y}$ is SSR_2 in (p, y) and $-((1-\lambda)y + \lambda\widehat{\mu}_\lambda - p)$ is strictly increasing in p and single-crossing from above in y it follows that (1.16) crosses zero at most once, at a single point, and from below, as p increases.

To prove that eq. (1.2) characterizes all $\widehat{\mu}_\lambda$-measurable price functions, note that in the proof of Proposition 1.2.1, I showed that observation of a $\widehat{\mu}_\lambda$-measurable price function along with signed volume provides the uninformed with the same information as direct observation of $\widehat{\mu}_\lambda$. Hence, any such equilibrium is equivalent to one in which the uninformed condition *only* on $\widehat{\mu}_\lambda$. Eq. (1.2) characterizes exactly this equilibrium. □

Proof of Lemma 1.3.1. This result is well-known, and follows immediately after writing out $f_{\widehat{\mu}_\lambda|\mu}$ in terms of the density of supply f_Z.

$$f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|m) = \frac{\lambda}{\alpha\sigma_\varepsilon^2} f_Z\left(\frac{\lambda}{\alpha\sigma_\varepsilon^2}(m - \widehat{m})\right). \qquad \square \quad (1.17)$$

The next lemma collects some useful expressions for the derivatives of uninformed demand and equilibrium price with respect to various quantities, assuming that the derivatives exist.

Lemma 1.6.4. *Considered as a function of the realizations $\widehat{\mu}_\lambda = \widehat{m}$ and $P_\lambda = p$, the derivatives of uninformed demand X_U are*

$$\frac{\partial X_U}{\partial \widehat{m}} = \frac{\int_{\mathbb{R}} \left(y - p - \alpha\sigma_\varepsilon^2 X_U\right) e^{-\alpha X_U y} \frac{\frac{\partial f_{\mu|\widehat{\mu}}(y|\widehat{m})}{\partial \widehat{m}}}{f_{\mu|\widehat{\mu}}(y|\widehat{m})} f_{\mu|\widehat{\mu}}(y|\widehat{m})\, dy}{\alpha \int_{\mathbb{R}} \left[\left(y - p - \alpha\sigma_\varepsilon^2 X_U\right)y + \sigma_\varepsilon^2\right] e^{-\alpha X_U y} f_{\mu|\widehat{\mu}}(y|\widehat{m})\, dy} \qquad (1.18)$$

$$\frac{\partial X_U}{\partial p} = \frac{-\int_{\mathbb{R}} e^{-\alpha X_U y} f_{\mu|\widehat{\mu}}(y|\widehat{m})\, dy}{\alpha \int_{\mathbb{R}} \left[\left(y - p - \alpha\sigma_\varepsilon^2 X_U\right)y + \sigma_\varepsilon^2\right] e^{-\alpha X_U y} f_{\mu|\widehat{\mu}}(y|\widehat{m})\, dy}. \qquad (1.19)$$

The derivatives of the price function P_λ with respect to the realization of $\widehat{\mu}_\lambda = \widehat{m}$ and the

parameter λ are

$$\frac{\partial P_\lambda}{\partial \widehat{m}} = \frac{\int_{\mathbb{R}} \left[(1-\lambda)y + \lambda\widehat{m} - P_\lambda\right] e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} \frac{\frac{\partial f_{\mu|\widehat{\mu}_\lambda}}{\partial \widehat{m}}(y|\widehat{m})}{f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}$$

$$+ \frac{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + \lambda\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-\widehat{P}_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-\widehat{P}_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy} \tag{1.20}$$

and

$$\frac{\partial P_\lambda}{\partial \lambda} = \frac{\int_{\mathbb{R}} \left((1-\lambda)y + \lambda\widehat{m} - P_\lambda\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} \frac{\frac{\partial f_{\mu|\widehat{\mu}_\lambda}}{\partial \lambda}(y|\widehat{m})}{f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}$$

$$+ \frac{\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2(1-\lambda)^2} \int_{\mathbb{R}} \left((1-\lambda)y + \lambda\widehat{m} - P_\lambda\right) y e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}$$

$$+ \frac{\frac{\widehat{m}-P_\lambda}{1-\lambda} \int_{\mathbb{R}} e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy}. \tag{1.21}$$

Proof of Lemma 1.6.4. The first-order condition defining $X_U(\widehat{m}, p)$ is

$$\int_{\mathbb{R}} \left(y - \alpha\sigma_\varepsilon^2 X_U - p\right) e^{-\alpha X_U y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})\, dy = 0.$$

The denominators in the expressions in the Lemma are non-zero due to Lemma 1.6.2. Hence, we can apply the implicit function theorem to obtain the results for $\frac{\partial X_U}{\partial \widehat{m}}$ and $\frac{\partial X_U}{\partial p}$ immediately.

Using the implicit function theorem on eq. (1.2), gives the results for $\frac{\partial P_\lambda}{\partial \widehat{m}}$. Applying the

implicit function theorem again for $\frac{\partial P_\lambda}{\partial \lambda}$ gives

$$
\frac{\partial P_\lambda}{\partial \lambda} = \frac{\int_{\mathbb{R}} \left((1-\lambda)y + \lambda\widehat{m} - P_\lambda\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} \frac{\frac{\partial f_{\mu|\widehat{\mu}}}{\partial \lambda}(y|\widehat{m})}{f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m})} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m}) \, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy}
$$

$$
+ \frac{\frac{1}{(1-\lambda)^2}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2} \int_{\mathbb{R}} \left((1-\lambda)y + \lambda\widehat{m} - P_\lambda\right) y e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m}) \, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy}
$$

$$
- \frac{\int_{\mathbb{R}} (y - \widehat{m}) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m}) \, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda]\frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1\right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}}(y|\widehat{m}) \, dy}.
$$

However, by rearranging eq. (1.2) one can see that

$$
- \int_{\mathbb{R}} (y - \widehat{m}) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m}) \, dy = \frac{\widehat{m} - P_\lambda}{1 - \lambda} \int_{\mathbb{R}} e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\mu|\widehat{\mu}_\lambda}(y|\widehat{m}) \, dy.
$$

Substituting this for the numerator in the last term above gives the desired result. □

Proof of Proposition 1.4.1. Under the assumption that $f_{\widehat{\mu}_\lambda|\mu}$ has the MLRP, Proposition 1.4.2 below implies that the price function is invertible. This implies that uninformed demand is in fact a function of P_λ. Given that P_λ is increasing in $\widehat{\mu}_\lambda$, is follows also that uninformed demand will be increasing in price if and only if it is increasing in $\widehat{m}$. Assuming that the price and uninformed demand functions are differentiable, differentiating the equilibrium demand $X_U(\widehat{m}, P_\lambda(\widehat{m}))$ totally with respect to $\widehat{m}$ gives

$$
\frac{d}{d\widehat{m}} X_U(\widehat{m}, P_\lambda(\widehat{m})) = \frac{\partial X_U}{\partial \widehat{m}} + \frac{\partial X_U}{\partial p}\frac{\partial P_\lambda}{\partial \widehat{m}}.
$$

Substitute in from Lemma 1.6.4 for all three partial derivatives on the right-hand side of the equation. It is then tedious but straightforward to manipulate the resulting expression and show that it is ≥ 0 if and only if $\frac{\partial P_\lambda}{\partial \widehat{m}} \geq 1$.

To prove the second statement, it suffices to show that logconcavity of f_μ implies that $\frac{\partial P_\lambda}{\partial \widehat{m}} \leq 1$. The nondifferentiable case can be handled by rearranging the uninformed first-order condition and applying Corollary 2.3 of Ma (1999).

Consider the expression for the derivative of the price function in eq. (1.20) By a simple application of Bayes rule on the conditional density in the integrand, one can express the

numerator in the first term in eq. (1.20) as

$$\int_{\mathbb{R}} [(1-\lambda)y + \lambda\widehat{m} - P_\lambda] \, e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} \frac{\partial f_{\widehat{\mu}_\lambda|\mu}}{\partial\widehat{m}}(\widehat{m}|y) f_\mu(y)\, dy \qquad (1.22)$$

From eq. (1.17), it follows that $\frac{\partial f_{\widehat{\mu}_\lambda|\mu}}{\partial\widehat{m}}(\widehat{m}|y) = -\frac{\partial f_{\widehat{\mu}_\lambda|\mu}}{\partial y}(\widehat{m}|y)$. Use this fact to perform integration by parts on eq. (1.22) to obtain the following expression (the boundary terms are zero since the densities have mgfs, and therefore tails that converge to zero at least exponentially)

$$= (1-\lambda)\int_{\mathbb{R}} e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|y) f_\mu(y)\, dy + \frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}\int_{\mathbb{R}} [(1-\lambda)y + \lambda\widehat{m} - P_\lambda] e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|y) f_\mu(y)\, dy$$

$$+ \int_{\mathbb{R}} [(1-\lambda)y + \lambda\widehat{m} - P_\lambda] e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|y) \frac{f_\mu'(y)}{f_\mu(y)} f_\mu(y)\, dy.$$

The second term in this sum vanishes, due to the price equation eq. (1.2). Plugging the remaining two terms back into eq. (1.20) and rearranging gives

$$\frac{\partial P_\lambda}{\partial\widehat{m}} = 1 + \frac{\int_{\mathbb{R}} [(1-\lambda)y + \lambda\widehat{m} - P_\lambda] e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} \frac{f_\mu'(y)}{f_\mu(y)} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|y) f_\mu(y)\, dy}{\int_{\mathbb{R}} \left([(1-\lambda)y + \lambda\widehat{m} - P_\lambda] \frac{\lambda}{1-\lambda}\frac{y}{\sigma_\varepsilon^2} + 1 \right) e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|y) f_\mu(y)\, dy}. \qquad (1.23)$$

If $\log f_\mu$ is concave, then $\frac{f_\mu'(y)}{f_\mu(y)}$ is decreasing in y. The numerator of the second term in eq. (1.23) is therefore ≤ 0 by Lemma 1.6.2, and as noted in the first section of this proof, the denominator is ≥ 0. Hence, the entire term is ≤ 0, and therefore $\frac{\partial P_\lambda}{\partial\widehat{m}} \le 1$. $\qquad\square$

Proof of Proposition 1.4.2. It suffices to show that under the MLRP the price function is strictly increasing in the realization of $\widehat{\mu}_\lambda$ and hence invertible. I do so by showing that the derivative in eq. (1.20) is greater than zero.

From eq. (1.2) we know that $[(1-\lambda)y + \lambda\widehat{m} - P_\lambda] e^{\frac{\lambda}{1-\lambda}\frac{\widehat{m}-P_\lambda}{\sigma_\varepsilon^2}y}$ integrates to 0 against $f_{\mu|\widehat{\mu}}$. Furthermore, this function crosses zero once, and from below, as y increases. Since the MLRP guarantees that $\frac{\frac{\partial f_{\mu|\widehat{\mu}}}{\partial\widehat{m}}(y|\widehat{m})}{f_{\mu|\widehat{\mu}}(y|\widehat{m})}$ is increasing in y, Lemma 1.6.2 implies that the numerator of the first term in eq. (1.20) is greater than zero. Similarly, since y is increasing, the numerator of the second term and the denominator of each term are also greater than zero. It follows that eq. (1.20) is greater than zero. $\qquad\square$

Proof of Lemma 1.4.3. Fix $\lambda \in (0,1)$, and choose any $\tau \in (\lambda,1)$. Following Persico (1996, 2000), define the function $T_{\lambda,\tau,m}(\widehat{m}) = m + \frac{\lambda}{\tau}(\widehat{m} - m)$. Persico (1996) shows that conditional on $\mu = m$ the random variable $\widehat{\mu}_\tau$ is equal in distribution to $T_{\lambda,\tau,m}(\widehat{\mu}_\lambda)$. Hence, when fraction

τ are informed, the difference in certainty equivalent wealth is

$$CE(\tau) = -\frac{1}{\alpha} \log \iint_{\mathbb{R}^2} e^{-\alpha X_I(m-P_\tau)+\frac{1}{2}\alpha^2 X_I^2 \sigma_\varepsilon^2} f_{\widehat{\mu}_\tau|\mu}(\widehat{m}|m) f_\mu(m)\, dm\, d\widehat{m}$$

$$+ \frac{1}{\alpha} \log \iint_{\mathbb{R}^2} e^{-\alpha X_U(m-P_\tau)+\frac{1}{2}\alpha^2 X_U^2 \sigma_\varepsilon^2} f_{\widehat{\mu}_\tau|\mu}(\widehat{m}|m) f_\mu(m)\, dm\, d\widehat{m}$$

$$= -\frac{1}{\alpha} \log \iint_{\mathbb{R}^2} e^{\alpha c - \alpha X_I(m,P_\tau(T(\widehat{m})))(m-P_\tau(T(\widehat{m})))+\frac{1}{2}\alpha^2 X_I^2(m,P_\tau(T(\widehat{m})))\sigma_\varepsilon^2} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|m) f_\mu(m)\, dm\, d\widehat{m}$$

$$+ \frac{1}{\alpha} \log \iint_{\mathbb{R}^2} e^{-\alpha X_U(T(\widehat{m}),P_\tau(T(\widehat{m})))(m-\widehat{P}_\tau(T(\widehat{m})))+\frac{1}{2}\alpha^2 X_U^2(T(\widehat{m}),P_\tau(T(\widehat{m})))\sigma_\varepsilon^2} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|m) f_\mu(m)\, dm\, d\widehat{m}.$$

Differentiate this expression with respect to τ and evaluate the derivative at $\tau = \lambda$. Since the integrand is continuously differentiable in τ, it is permissible to differentiate through the integral sign.

$$\frac{d}{d\tau} CE(\tau)\Big|_{\tau=\lambda}$$

$$= -\frac{1}{\alpha\, EI} \iint_{\mathbb{R}^2} \frac{d}{d\tau} e^{\alpha c - \alpha X_I(m,P_\tau(T(\widehat{m})))(m-P_\tau(T(\widehat{m})))+\frac{1}{2}\alpha^2 X_I^2(m,P_\tau(T(\widehat{m})))\sigma_\varepsilon^2}\Big|_{\tau=\lambda} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|m) f_\mu(m)\, dm\, d\widehat{m}$$

$$+ \frac{1}{\alpha\, EU} \iint_{\mathbb{R}^2} \frac{d}{d\tau} e^{-\alpha X_U(T(\widehat{m}),P_\tau(T(\widehat{m})))(m-P_\tau(T(\widehat{m})))+\frac{1}{2}\alpha^2 X_U^2(T(\widehat{m}),P_\tau(T(\widehat{m})))\sigma_\varepsilon^2}\Big|_{\tau=\lambda} f_{\widehat{\mu}_\lambda|\mu}(\widehat{m}|m) f_\mu(m)\, dm\, d\widehat{m},$$

where

$$EI = \mathbb{E}\left[e^{-\alpha X_I(m-P_\lambda)+\frac{1}{2}\alpha^2 X_I^2 \sigma_\varepsilon^2}\right],$$

$$EU = \mathbb{E}\left[e^{-\alpha X_U(m-P_\lambda)+\frac{1}{2}\alpha^2 X_U^2 \sigma_\varepsilon^2}\right]$$

are the negative of the unconditional expected utilities of each type of investor.

I now proceed to differentiate the integrand. To conserve space, I use the notation e^I and e^U for the exponentials in the informed and uninformed expected utility, divided by EI and EU, respectively.

$$= -\iint_{\mathbb{R}^2} \left\{ X_I(m, P_\tau(T(\widehat{m}))) \left[\frac{\partial P_\tau}{\partial \widehat{m}} \frac{\partial T}{\partial \tau} + \frac{\partial P_\tau}{\partial \tau} \right] \right.$$

$$\left. + (m - P_\tau(T(\widehat{m})) - \alpha X_I(m, P_\tau(T(\widehat{m})))\sigma_\varepsilon^2) \left[\frac{\partial X_I}{\partial p} \left[\frac{\partial P_\tau}{\partial \widehat{m}} \frac{\partial T}{\partial \tau} + \frac{\partial P_\tau}{\partial \tau} \right] + \frac{\partial X_I}{\partial \tau} \right] \right\} e^I \Big|_{\tau=\lambda} f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm\, d\widehat{m}$$

$$+ \iint_{\mathbb{R}^2} \left\{ X_U(T(\widehat{m}), P_\tau(T(\widehat{m}))) \left[\frac{\partial P_\tau}{\partial \widehat{m}} \frac{\partial T}{\partial \tau} + \frac{\partial P_\tau}{\partial \tau} \right] \right.$$

$$\left. - (m - P_\tau(T(\widehat{m})) - \alpha X_U(T(\widehat{m}), P_\tau(T(\widehat{m})))\sigma_\varepsilon^2) \left[\frac{\partial X_U}{\partial \widehat{m}} \frac{\partial T}{\partial \tau} + \frac{\partial X_U}{\partial p} \left[\frac{\partial P_\tau}{\partial \widehat{m}} \frac{\partial T}{\partial \tau} + \frac{\partial P_\tau}{\partial \tau} \right] + \frac{\partial X_U}{\partial \tau} \right] \right\} e^U \Big|_{\tau=\lambda} f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm\, d\widehat{m}.$$

The informed investors' first-order condition implies that

$$m - P_\tau(T(\widehat{m})) - \alpha X_I(m, P_\tau(T(\widehat{m})))\sigma_\varepsilon^2 = 0,$$

which eliminates the second term in the first integral. Similarly, the uninformeds' first-order condition implies that the terms in the second integral involving $\frac{\partial X_U}{\partial p} \times \frac{\partial P_\tau}{\partial \tau}$ and $\frac{\partial X_U}{\partial \tau}$ vanish

when integrating over m since these expressions depend only on $\widehat{m}$. Hence, the expression becomes

$$
= -\iint_{\mathbb{R}^2} X_I(m, P_\tau(T(\widehat{m}))) \left[\tfrac{\partial P_\tau}{\partial \widehat{m}}\tfrac{\partial T}{\partial \tau} + \tfrac{\partial P_\tau}{\partial \tau}\right] e^I \Big|_{\tau=\lambda} f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm \, d\widehat{m}
$$

$$
+ \iint_{\mathbb{R}^2} \left\{ X_U(T(\widehat{m}), P_\tau(T(\widehat{m}))) \left[\tfrac{\partial P_\tau}{\partial \widehat{m}}\tfrac{\partial T}{\partial \tau} + \tfrac{\partial P_\tau}{\partial \tau}\right] \right.
$$

$$
\left. - \left(m - P_\tau(T(\widehat{m})) - \alpha X_U(T(\widehat{m}), P_\tau(T(\widehat{m})))\sigma_\varepsilon^2\right) \left[\tfrac{\partial X_U}{\partial \widehat{m}} + \tfrac{\partial X_U}{\partial p}\tfrac{\partial P_\tau}{\partial \widehat{m}}\right] \tfrac{\partial T}{\partial \tau} \right\} e^U \Big|_{\tau=\lambda} f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm \, d\widehat{m}.
$$

Finally, substitute for $\frac{\partial T}{\partial \tau}$ and use the fact that $T_{\lambda,\lambda,m}(\widehat{m}) = \widehat{m}$ to evaluate the integrand explicitly at $\tau = \lambda$ to derive the expression in the text,

$$
CE'(\lambda) = -\iint_{\mathbb{R}^2} \left(X_I(m, P_\lambda(\widehat{m}))e^I - X_U(\widehat{m}, P_\lambda(\widehat{m}))e^U\right) \left[\tfrac{\partial P_\lambda}{\partial \widehat{m}}\tfrac{m-\widehat{m}}{\lambda} + \tfrac{\partial P_\lambda}{\partial \lambda}\right] f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm \, d\widehat{m}
$$

$$
- \iint_{\mathbb{R}^2} \left(m - P_\lambda(\widehat{m}) - \alpha X_U(\widehat{m}, P_\lambda(\widehat{m}))\sigma_\varepsilon^2\right) \left[\tfrac{\partial X_U}{\partial \widehat{m}} + \tfrac{\partial X_U}{\partial p}\tfrac{\partial P_\lambda}{\partial \widehat{m}}\right] \tfrac{m-\widehat{m}}{\lambda} \right\} e^U f_{\widehat{\mu}_\lambda|\mu} f_\mu \, dm \, d\widehat{m}. \quad \square
$$

Proof of Proposition 1.4.4. The expression I desire to sign is

$$
- \iint_{\mathbb{R}^2} \left(m - P_\lambda(\widehat{m}) - \alpha X_U(\widehat{m}, P_\lambda(\widehat{m}))\sigma_\varepsilon^2\right) \left[\tfrac{\partial X_U}{\partial \widehat{m}} + \tfrac{\partial X_U}{\partial p}\tfrac{\partial P_\lambda}{\partial \widehat{m}}\right] \tfrac{m-\widehat{m}}{\lambda} \right\} e^U f_{\mu|\widehat{\mu}_\lambda} f_{\widehat{\mu}_\lambda} \, dm \, d\widehat{m}. \tag{1.24}
$$

Note that

$$
\int_{\mathbb{R}} \left(m - P_\lambda(\widehat{m}) - \alpha X_U(\widehat{m}, P_\lambda(\widehat{m}))\sigma_\varepsilon^2\right) e^U f_{\mu|\widehat{\mu}_\lambda} \, dm = 0
$$

from the uniformed investors' first order condition. Given that $m - \widehat{m}$ is increasing in m, Lemma 1.6.2 will once again imply that the inner integral in eq. (1.24) (and hence the entire expression) is ≥ 0 as long as $\left[\frac{\partial X_U}{\partial \widehat{m}} + \frac{\partial X_U}{\partial p}\frac{\partial P_\lambda}{\partial \widehat{m}}\right] \leq 0$ for all $\widehat{m}$. However, the proof of Proposition 1.4.1 shows that this is the case if and only if uninformed demand is everywhere downward-sloping. Hence, under this assumption eq. (1.24) is ≥ 0. $\square$

Proof of Corollary 1.4.5. Interior equilibria in the information market are defined as λ^* such that $CE(\lambda^*) = c$, and zero-information-acquisition equilibria, $\lambda^* = 0$, occur when $CE(0) \leq c$. Therefore, if CE is nonmonotonic, then for appropriate c there can be multiple values of λ^*, including zero, consistent with equilibrium. $\square$

Proof of Proposition 1.4.6. Using the law of total covariance and the fact that ε is indepen-

dent of all other random variables, write

$$
\begin{aligned}
\mathrm{Cov}\left(D - P_\lambda, (\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2\right) &= \mathrm{Cov}\left(\mathbb{E}\left[\mu - P_\lambda|\widehat{\mu}_\lambda\right], \mathbb{E}[(\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2|\widehat{\mu}_\lambda]\right) \\
&\quad + \mathbb{E}\left[\mathrm{Cov}\left(\mu - P_\lambda, (\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2|\widehat{\mu}_\lambda\right)\right] \\
&= \mathrm{Cov}\left(\mathbb{E}\left[\mu - P_\lambda|\widehat{\mu}_\lambda\right], \mathrm{Var}[\mu||\widehat{\mu}_\lambda]\right) + \mathbb{E}\left[\mathrm{Cov}\left(\mu - P_\lambda, (\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2|\widehat{\mu}_\lambda\right)\right] \\
&= \mathrm{Cov}\left(\mathbb{E}\left[\mu - P_\lambda|\widehat{\mu}_\lambda\right], \mathrm{Var}[\mu|\widehat{\mu}_\lambda]\right) + \mathbb{E}\left[\mathrm{Cov}\left(\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda], (\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2|\widehat{\mu}_\lambda\right)\right] \\
&= \mathrm{Cov}\left(\mathbb{E}[\mu - P_\lambda|\widehat{\mu}_\lambda], \mathrm{Var}[\mu|\widehat{\mu}_\lambda]\right) + \mathbb{E}\left[\mathrm{Skew}[\mu|\widehat{\mu}_\lambda]\right].
\end{aligned}
$$

The second equality uses the definition of conditional variance to simplify the first term. The third inequality uses the fact that P_λ and $\mathbb{E}[\mu|\widehat{\mu}_\lambda]$ are conditionally constant to manipulate the conditional covariance in the second term. The last equality uses the fact that $\mathbb{E}[\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda]] = 0$ to rewrite $\mathrm{Cov}\left(\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda], (\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^2|\widehat{\mu}_\lambda\right) = \mathbb{E}[(\mu - \mathbb{E}[\mu|\widehat{\mu}_\lambda])^3|\widehat{\mu}_\lambda]$. Multiplying the above expression by $\lambda(1 - \lambda)$ and using the fact that the covariance is a bilinear operator to pull this constant inside the expression gives the desired result. $\qquad\square$

Proof of Corollary 1.4.7. Follows immediately from Proposition 1.4.6. $\qquad\square$

Chapter 2

Do Fund Managers Make Informed Asset Allocation Decisions

Note: This chapter represents joint work with Jacob S. Sagi.

2.1 Introduction

Every active fund manager has to make asset allocation decisions, and changes in the portfolio weights of major asset classes should be viewed as a function of the manager's information set and her or her ability to optimally use that information set.[1] This paper develops a fully dynamic model of the asset allocation decision, where each portfolio manager accounts for a historical time series of public *and* private information available to him or her. Such information, if used optimally, will reflect itself in the dynamics of portfolio weights. The model leads to testable and, to our knowledge, novel predictions. The main prediction, after suitably controlling for the conditional volatility of market returns, is that aggregate equity weights that are more volatile reflect better information and should better forecast future market returns.

   We apply the model to the portfolio weights of US mutual funds holding primarily US common equity from 1979Q3 until 2006Q4. We find evidence for the model predictions at forecasting horizons from three to twelve months, but weak contrary evidence at one-month forecasting horizons. The timing ability we find at horizons beyond one month is consistent with a small number of recent studies, and we interpret it as indicative of better information about medium-term future market returns. If one believes our model assumptions and empirical methodology are sound, one can attribute the general absence of timing ability at a shorter horizons as evidence against the existence of much private information about short-run expected returns. However, even for forecasting horizons over which some managers

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[1]The decision by an active manager to *not* attempt to time the market is also a decision that is presumably based on the manager's information.

appear to have ability, timing behavior does not reflect itself in higher returns on funds' portfolios. This could be because of measurement error or because the portfolios are not held long enough to benefit from the predictability.

2.1.1 Literature review and motivation

The literature on portfolio management generally views 'market timing' as the shifting of funds between broad asset categories (such as 'US equities', or 'US Government bonds') in an attempt to capture higher risk-adjusted returns. Skillful market timers are said to divine those times during which returns on one major asset class will exceed those of another.

Early tests of market timing ability were largely based on the regression methodologies of Treynor and Mazuy (1966) and Henriksson and Merton (1981), and looked for a convex relationship between fund-level returns and contemporaneous market returns. Results from these early experiments have largely shaped researchers' perception that professional portfolio managers do not exhibit timing ability.[2] This is despite the fact that such tests are susceptible to numerous criticisms (see Ferson and Schadt, 1996; Goetzmann, Ingersoll, and Ivković, 2000; Jagannathan and Korajczyk, 1986; Kothari and Warner, 2001). For instance, bias can result if a portfolio is rebalanced more frequently than the observation intervals used in the test. In addition, the residuals may be highly non-normal and correlated across funds, leading one to potentially question the use of pooled statistics employed by all the early studies (this is similar to the criticism offered by Kosowski, Timmermann, Wermers, and White, 2006). Finally, even if managers possessed timing ability, one might not detect it using fund-level returns because the latter incorporate fees which, in a competitive market, could offset the value added by market timing (e.g., Berk and Green, 2004). Ferson and Schadt (1996) also point out that market timing tests should condition on public information relevant to predicting the market. When making the appropriate modifications, the negative results of earlier tests disappear and they even find weak support for timing ability, although their inference from pooled statistics does not account for cross-sectional correlations and non-normalilty when computing standard errors. Edelen (1999) confirms these findings, pointing out that fund flows account for the negative results obtained in earlier studies.[3]

A number of subsequent studies have focused on holdings, often testing market timing in a multiple-asset allocation context. Those finding no evidence for timing include Daniel, Grinblatt, Titman, and Wermers (1997); Kacperczyk and Seru (2007); Kacperczyk, Sialm, and Zheng (2005); Kosowski, Timmermann, Wermers, and White (2006); Wermers (2000).[4]

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[2] Among the early tests are Chang and Lewellen (1984); Henriksson (1984); Henriksson and Merton (1981); Kon (1983); Treynor and Mazuy (1966).

[3] Eckbo and Smith (1998) apply the conditional performance test approach of Ferson and Schadt (1996) to assess the performance of insiders on the Oslo Stock Exchange. They find no evidence of timing ability among these insiders.

[4] Recent related studies of mutual funds that employ holdings to construct performance measures include

Save for Kosowski, Timmermann, Wermers, and White (2006), these holding-based studies employ pooled statistics and, to our knowledge, do not fully adjust for the cross-sectional correlation in funds' weights when reporting significance. Such correlation may be particularly important when testing for timing because funds presumably attempt to time the same macroeconomic variables.[5] Standing in contrast with the other holdings-based mutual fund studies of timing is Jiang, Yao, and Yu (2007), who estimate aggregate portfolio betas from mutual fund holdings and find that funds tend to hold higher beta securities prior to when market returns are high. Taliaferro (2009), however, suggests that their results depend strongly on the period 2000-2002 and that managers do not necessarily show timing ability continuously over their sample. Recent work by Simutin (2009) finds a relation between forecasting ability and *excess* cash holdings of funds.

All of the studies mentioned thus far examine monthly and/or quarterly holdings. Several recent studies that examine timing ability at a higher frequency have tended to yield positive evidence. Busse (1999) and Fleming, Kirby, and Ostdiek (2001) study volatility timing, while Chance and Hemler (2001) examine timing strategies based on the recommendations of 30 Registered Financial Advisers that, when pooled, yield significant evidence for market timing ability. The latter authors document that the use of daily data is key to their findings, confirming the negative results of Graham and Harvey (1996) who look at newsletter recommendations on a monthly basis. Bollen and Busse (2001) also find significant evidence for ability in their Treynor and Mazuy (1966) and Henriksson and Merton (1981) regression tests using daily data and bootstrapped standard errors, although they do not control for cross-sectional correlations in their tests.

Overall, the picture that emerges from reviewing the literature is that early studies, though flawed, found no or negative evidence for timing ability, while a survey of recent literature provides a substantially more mixed view of the topic. Of 14 mentioned papers written since the 1990's on the subject of timing, seven find supportive evidence. By and large, there does not appear to be a definitive answer to whether the average portfolio manager can create value through asset allocation. When one further considers that many of the papers can potentially be criticized on econometric grounds, the picture becomes hazier still.

We attempt to shed additional light on this question by offering a new set of tests for timing ability, and design our empirical methodology keeping in mind the various econometric pitfalls we've identified above. In addition, our tests address at least one additional issue ignored in this literature: If market returns are forecasted to be higher than average but the Sharpe Ratio is forecasted to be lower than average, it is not clear that a rational market timer would elect to increase her exposure to equities. As far as we know, all of the studies

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Huang, Sialm, and Zhang (2008) and Shumway, Szefler, and Yuan (2009).

[5]While Kosowski, Timmermann, Wermers, and White (2006) use a bootstrap methodology, they only mention their results for timing in a footnote without reporting their test methodology. It is therefore not clear how they controlled for cross-sectional correlations in market weights when conducting their bootstrap tests for timing ability.

looking for market timing ability ignore the potential importance of conditional volatility on market timing, whereas this consideration plays a key role in our methodology.

2.1.2 Our contribution

We examine 'market timing' from several new perspectives. Consistent with the literature on the predictability of aggregate returns, we assume that fund managers receive noisy *private* signals about future market returns.[6] This information comes in two forms: information about the slow-moving expected equity premium, some of which is known to be present in macroeconomic variables such as Lettau and Ludvigson's (2001) cay; and information about the shock to market returns.[7] The first type of information provides the manager with a refined sense of the premium of the market over bonds, while the second type of information can provide the manager with more dramatic insight such as whether bonds may fare better than stocks in the immediate future. We also consider three types of managers. 'Type 1', or non-highly informed, have access to private and public information about the slow moving equity premium, while 'type 2', or highly informed managers, in addition have access to information about the shock to market returns. Together, we refer to 'type 1' and 'type 2' managers as informed managers. On the other hand, 'type 3', or noise traders, believe that they have access to information about the shock to market returns but in actuality observe only noise. Correspondingly, we characterize the Bayesian-optimal changes in weights of the three types of managers assuming they can invest in the market and/or in short-term bonds; the managers are assumed to optimize a mean-variance myopic objective function and can condition on their current private information as well as *all* past private information, and *all* past market returns.

The primary prediction of the model is that, controlling for conditional volatility, informed managers whose portfolio weight in equity exhibits a greater time-series variance must possess better information. The reason is intuitive. A manager who has more precise information will be able to take more extreme positions in response to her signals without increasing the portfolio risk. Conversely, for noise traders, the greater the variance of portfolio equity weights, the less information about future market returns should be reflected

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[6]The predictability of market returns has been documented in Breen, Glosten, and Jagannathan (1989); Campbell (2002); Campbell and Shiller (1988a,b); Keim and Stambaugh (1986); Lettau and Ludvigson (2001). The potential benefits from making use of this information in asset allocation decisions is documented in, among other papers, Andrade, Babenko, and Tserlukevich (2006); Kandel and Stambaugh (1996); Whitelaw (1997). Because investors can rebalance their exposure to market risk in reaction to changes in publicly available information, managers may not feel compelled to adjust their portfolio weights in reaction to such information. Thus it is better to view our managers' information as private in this paper. In a separate addendum, available upon request, we discuss and empirically compare publicly available predictors of the equity premium to aggregate changes in funds' equity weights. There, we also calculate the potential utility loss to investors who ignore this predictability.

[7]We do not assume that the information about the slow-moving equity premium is public. In principal, such information can be correlated with public information.

in equity weights. This too is intuitive, a noise trader who believes that she has better information will take more extreme positions, but in so doing she merely adds idiosyncratic noise to her portfolio returns. In short, if managers have timing ability, we should observe a positive relation between the variability of portfolio weights and their forecasting power for future market returns, while if managers are trading mostly on noise, we should observe a negative relation.

The model also implies that the portfolio equity weights of *every* non-highly informed manager ought to have an autocorrelation coefficient equal to that of the time-varying expected equity premium. Roughly, the intuition for this result is that all non-highly informed managers are attempting to forecast the time-varying expected equity premium. Their Bayesian-optimal forecasts should, therefore, exhibit this very same persistence. A further implication is that the portfolio equity weights of highly informed managers and noise traders will have a lower autocorrelation than a non-highly informed manager, and that this autocorrelation will decrease with the signal-to-noise ratio of their high-frequency signals. The intuition for this result is straightforward: information about the shock to the market returns is, by definition, high frequency and iid. The more information that a manager has about the shock, the more her portfolio weights will respond to that information. Hence, for informed managers, the smaller the autocorrelation of a managers equity exposure, the more information she ought to have about future market returns, and this should be reflected in the equity exposure of the fund.

The fact that our model is set in a partial equilibrium setting might invite criticism. In particular, in a general equilibrium setting the fact that all portfolio positions must sum to the 'market' precludes *every* investor from shifting weights with the equity premium. To address this, we first emphasize that, as in practice, managers in the model base their trades on a heterogeneous mix of public and private information and do not necessarily tilt their portfolios in the same direction. Second, we do not presuppose that informed fund managers span the entire market. Third, it is possible to embed a model of *profitable* informed trading in a general equilibrium setting (see, for example, Grossman and Stiglitz, 1980); to our knowledge, there is nothing in the model or the industrial organization of the mutual fund industry that suggests that general equilibrium considerations will nullify the model's predictions.

The model predictions are made under the assumption that one can control for non-informational changes in portfolio weights (e.g., fund flows, or portfolio insurance strategies), and we attempt to do this in our empirical tests. While our predictions ought to be fairly robust because the economic rationale behind them is more general than the particular model we investigate, in our battery of empirical tests we focus on the relation between the variability of portfolio equity weights and their market forecasting power. There are a number of reasons to believe that the our predictions on weight autocorrelation will be less robust. If the market autocorrelation is low, even minor misspecification in our control variables may lead to spurious results. Additionally, to the extent that managers disagree on the data generating process for market returns, it is difficult to interpret deviations from

the predictions. Conversely, the prediction relating weight variability and forecasting power relies only on the assumption of manager rationality, as long as a forecast that incorporates a manager's signal is more precise that one that does not. Nevertheless, the model used to derive our results is rich in allowing a great deal of heterogeneity in fund manager characteristics and their private information. We test the model predictions on a large panel of US mutual fund holdings, and are able to, at least partially, assess the degree to which asset allocation decisions reflect information.

Our tests support the hypothesis that at three-, six-, and 12-month forecasting horizons, the market forecasting power of funds' weights does increase with the weights' variance (as predicted by the model). Thus, it appears that variation in timing ability does exist in the cross-section of mutual fund managers, at least at quarterly forecasting horizons and beyond. On the other hand, we find essentially no evidence of timing ability at a one-month horizon. Our findings appear broadly consistent with those of Jiang, Yao, and Yu (2007), who find evidence of timing ability at horizons of three and six months, but not at the one-month horizon. We confirm their findings with respect to portfolio betas in our sample. Nevertheless, further tests lead us to conclude that the forecasting power documented here and by Jiang, Yao, and Yu (2007) does *not* translate into higher portfolio returns.

It is important, however, to temper our results by noting that the holdings data used in our empirical tests are generally limited to US equity holdings only. We have no information on how funds invest outside of this asset class. Although our treatment of fund holdings is consistent with that of other studies, these funds could in principal make use of instruments such as index futures or high-yield bonds to change their effective equity exposure and our study would not pick this up.[8] Moreover, it is also possible that the reported portfolio holdings suffer from window dressing and do not truly reflect funds' portfolio strategies.

Section 2.2 develops the model. Section 2.3 describes our data set, the empirical methodology, and reports our tests of the model. Section 2.4 analyzes the extent to which forecasting ability is associated with improved portfolio returns. Section 2.5 concludes.

2.2   A model of optimal market timing

We begin by considering a typical market-timing fund manager, identified by the index i, who receives a noisy private signal each period about the market risk premium and adjusts her portfolio accordingly. Our assumptions represent a rich information environment, both across managers and across time. Doing so enables us to achieve a level of realism and generality beyond the typical static modeling of the asset allocation decision under asymmetric information.

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[8]Koski and Pontiff (1999) and Almazan, Brown, Carlson, and Chapman (2004) document that few funds use derivatives.

The market's excess return at date $t + 1$ is assumed to be:

$$\tilde{r}^e_{t+1} = \bar{\mu} + m_t + \varepsilon_{t+1}, \tag{2.1}$$

where $\bar{\mu}$ is the unconditional premium and m_t is its publicly unobserved time-varying component. The empirical literature notes that market return volatility is predictable. Consistent with this, we assume that ε_{t+1} has an observable date-t conditional variance of $\sigma^2_{\varepsilon t}$. [9]

There are three broad classes of portfolio managers. *All* types receive a noisy signal about m_t of the form:

$$s_{it} = n_{it} + m_t, \tag{2.2}$$

where i indexes the identity of the manager and n_{it} is the noise component of the manager's signal. The signal s_{it}, incorporates public information (available to all fund managers) as well as private information about m_t. Whereas managers in all classes receive a private signal of the form s_{it}, only managers belonging to the second class receive an additional signal of the form,

$$q^H_{it} = e_{it} + \varepsilon_{t+1}, \tag{2.3}$$

that provides information about the shock variable ε_{t+1}. We will refer to this second type of manager as being *highly informed* (the first class of managers will be referred to as *non-highly informed*).[10] For every highly informed manager the conditional variances of e_{it} and ε_{t+1} at date t have a ratio, denoted R_{qi}, that does not vary with time. That is to say, for the highly-informed manager, the signal-to-noise ratio of q_{it} is constant. Finally, there is a third class of managers, called *noise traders*, who believe that they receive a signal like that of the highly informed managers, but in actuality receive only noise. Specifically they update their beliefs as though they receive a signal of the form

$$q^{N'}_{it} = e'_{it} + \varepsilon_{t+1},$$

where the ratio of conditional variances of e'_{it} and ε_{t+1} is still denoted R_{qi}. However, the true 'signal' is purely noise

$$q^N_{it} = e_{it},$$

where the ratio of conditional variances of e_{it} and ε_{t+1} is $1 + R_{qi}$. This assumption on

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[9]It appears realistic to assume that investors observe the conditional volatility of market returns (using, for example, the S&P500 volatility index). In other words, $\mathrm{Var}[r^e_{t+1}|\mathcal{P}_t]$ is observable, with $\mathcal{P}_t$ representing a common knowledge (public) information set. Under the assumption that m_t is independent of $\tilde{\varepsilon}_{t+1}$, assuming the observability of $\sigma^2_{\varepsilon t}$ presupposes that $\mathrm{Var}[m_{t+1}|\mathcal{P}_t]$ is separately observable.

[10]There is no loss of generality in thinking of the non-highly informed managers as receiving a signal of the form q^H_{it} where $\mathrm{Var}[e_{it}]$ is arbitrarily large.

the variance ratio guarantees that the perceived variance coincides with the true variance, $\text{Var}(q_{it}^{N\prime}) = \text{Var}(q_{it}^N)$.

We further assume that

$$m_t = (1 - \phi_m)m_{t-1} + u_t,$$
$$n_{it} = (1 - \phi_{in})n_{it-1} + v_{it},$$

and that the conditional variances of u_t and v_{it} are constants, denoted as σ_u^2 and σ_{iv}^2, respectively. Finally, each shock in the collection, $\{\frac{\varepsilon_s}{\sigma_{\varepsilon s-1}}, \frac{e_{is}}{\text{Var}[e_{is}]}, \frac{u_s}{\sigma_u}, \frac{v_{is}}{\sigma_{iv}}\}_{s \leq t}$ is a standard normal iid random variable, independent of the process that generates $\sigma_{\varepsilon t}^2$. Under our assumptions, m_t and $\sigma_{\varepsilon t}^2$ are independent and universal to all managers while n_{it} (and e_{it} for highly informed and noise traders) may or may not be correlated across managers.[11] Moreover, the variance of noise in managers' signals is heterogeneous in precision as well as persistence.

Given our assumptions, $\text{Var}[m_t] = \frac{\sigma_u^2}{1-(1-\phi_m)^2}$ and $\text{Var}[n_{it}] = \frac{\sigma_{iv}^2}{1-(1-\phi_{in})^2}$; we'll refer to these unconditional variances as $\text{Var}[m]$ and $\text{Var}[n_i]$, respectively. Let I_{it} correspond to manager i's information set, consisting of observations of $s_{ix}, \sigma_{\varepsilon x}^2$ and r_{ix}^e (as well as q_{ix} for highly informed and noise traders) for all dates $x \leq t$. Finally, we assume that the manager seeks to myopically maximize a mean-variance function of her portfolio returns, implying that the optimal allocation at date t is

$$w_{it} = A_i \frac{E[\tilde{r}_{t+1}^e | I_{it}]}{\text{Var}[\tilde{r}_{t+1}^e | I_{it}]}. \tag{2.4}$$

The proportionality factor, A_i, can be viewed as a measure of relative risk tolerance and is assumed constant through time. Thus, if $\sigma_m = m_0 = 0$ and $\sigma_{\varepsilon t}^2$ is constant (i.e., there is no predictability in the market's Sharpe Ratio), then an informed manager follows a strategy of rebalancing to constant weights. When we test the model, we revisit this assumption and control for alternative specifications that are consistent with dynamic portfolio management for an optimizing agent (e.g., a buy and hold strategy, or a portfolio insurance strategy). Assuming a mean-variance objective function is consistent with the preferences of a log-investor who can rebalance continuously, but the assumption ignores the additional hedging demands of other types of investors. In neglecting a hedging demand component, we note that its sign and magnitude vary with investor preferences and horizon, while *all* investors place some (often considerable) weight on the myopic allocation given by Eq. (2.4).[12] Finally,

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[11]Without loss of generality and without changing our main results, one can replace $\bar{\mu}$ with $\xi\sigma_{\varepsilon t}^2$ plus a constant, consistent with various asset pricing models. Various studies explore the relationship between the market's conditional variance and expected returns. Whitelaw (1994) demonstrates that the theoretical relationship may not be monotonic, French, Schwert, and Stambaugh (1987) find a positive relationship while Breen, Glosten, and Jagannathan (1989) and Breen, Glosten, and Jagannathan (1989) do not. In light of this, we elected not to explicitly model such a relationship, although we account for its potential presence in the empirical section

[12]Kim and Omberg (1996) and Wachter (2002) demonstrate that when the Sharpe Ratio is an AR(1)

we note that, to the extent that the equity premium affects the expected returns of all stocks, Eq. (2.4) ought to apply to all managers of equity portfolios (i.e., both 'stock pickers' and 'market timers').

The following proposition establishes properties of the manager's optimal forecast of the time-varying component of the equity premium.

Proposition 2.2.1.

$$\hat{m}_{it} \equiv E[m_t|I_{it}] = \left(\sum_{j=0}^{\infty} a_{itj}s_{it-j} + \sum_{j=0}^{\infty} b_{itj}(r^e_{t-j} - \bar{\mu}) \right), \tag{2.5}$$

where the coefficients $\{a_{itj}, b_{itj}\}_{j=0}^{\infty}$ provide a solution to the following infinite set of linear equations:

$$(1 - \phi_m)^k \, Var[m] = \sum_{j=0}^{\infty} a_{ijt} \Big(Var[m](1 - \phi_m)^{|k-j|} + Var[n_i](1 - \phi_{in})^{|k-j|} \Big)$$

$$+ \sum_{j=0}^{\infty} b_{ijt} \, Var[m](1 - \phi_m)^{|k-j-1|}, \;\; k \geq 0$$

$$(1 - \phi_m)^{k+1} \, Var[m] = b_{ikt}\sigma^2_{\varepsilon t-k-1} + \sum_{j=0}^{\infty} a_{ijt} \, Var[m](1 - \phi_m)^{|k+1-j|} + \sum_{j=0}^{\infty} b_{ijt} \, Var[m](1 - \phi_m)^{|k-j|}, \;\; k \geq 0.$$

Moreover,

$$Var[m_t|I_{it}] = \sigma^2_{\varepsilon t-1}(1 - \phi_{in})b_{i0t} + Var[n_i]a_{i0t}\phi_{in}(2 - \phi_{in}). \tag{2.6}$$

Finally, for both highly informed managers and noise traders,

$$E[\varepsilon_{t+1}|I_{it}] = \frac{1}{1 + R_{qi}}q^j_{it}, \quad j \in \{H, N\} \tag{2.7}$$

$$Var[\varepsilon_{t+1}|I_{it}] = \frac{R_{qi}}{1 + R_{qi}}\sigma^2_{\varepsilon t}. \tag{2.8}$$

Proofs to all results are found in Section 2.6.1. Although being able to actually solve the infinite set of equations in Proposition 2.2.1 is not germane to our analysis in this paper, it is worth noting that the infinite set of coefficients in Proposition 2.2.1 can be approximated extremely well by truncating the higher order equations. In numerically experimenting with

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process in continuous time, any investor who can rebalance continuously and has utility over terminal wealth with constant relative risk aversion will allocate her wealth to equities by modifying Eq. (2.4) to include calendar-time dependence in A and an additional calendar-time dependent constant. Detemple, Garcia, and Rindisbacher (2003) find that variations in the hedging demand are significantly less pronounced than those of the myopic solution. Overall, this suggests that Eq. (2.4) captures much of the information content in changes to equity allocations even in the presence of a hedging demand.

the equations, we've found that for realistic parameter settings it suffices to keep only those coefficients for which $k \leq 5$.

2.2.1 Testable predictions

The next set of results pertains to the relation between the autocorrelation of funds' equity portfolio weights and their associated forecasting power.

Proposition 2.2.2. *The unconditional autocorrelation of $Var[r^e_{t+1}|I_{it}] \times w_{it} \propto E[r^e_{t+1}|I_{it}]$ for a non-highly informed manager is $1 - \phi_m$, coinciding with the unconditional autocorrelation of m_t.*

Thus, controlling for the denominator in (2.4) (say, by multiplying w_{it} by $\sigma^2_{\varepsilon t}$ and assuming $\sigma^2_{\varepsilon t} \gg Var[m_t|I_{it}]$), the autocorrelation of the optimal weight assigned to the market is the same across *non-highly informed* managers despite the rich heterogeneity in their information structure. One can understand the intuition for the result as follows: Every non-highly informed manager knows that the conditional equity premium has persistence of $(1 - \phi_m)$. If her estimate of the equity premium, $\hat{m}_{it}$, exhibits a different level of persistence, then the manager is either over-reacting or under-reacting to new information.

Corollary to Proposition 2.2.2. *The unconditional autocorrelation of $Var[r^e_{t+1}|I_{it}] \times w_{it} \propto E[r^e_{t+1}|I_{it}]$ for a* highly informed *manager or a* noise trader *is strictly smaller than $1 - \phi_m$, and monotonically decreases with $\frac{1}{1+R_{qi}}$.*

Recall that R_{qi} is the ratio of the conditional variances of e_{it} to the conditional variance of ε_{t+1}. A high value of $\frac{1}{1+R_{qi}}$ is therefore a measure of the signal-to-noise ratio of the corresponding highly informed manager. The corollary states that a higher signal-to-noise ratio is associated with less persistence in $E[r^e_{t+1}|I_{it}]$. Controlling for the denominator in (2.4), this implies that highly-informed managers and noise traders ought to exhibit *less* persistent equity portfolio weights. Putting together Propositions 2.2.2 and its corollary, one concludes that if managers are highly informed, the R^2 in a regression of $\tilde{r}^e_{t+1}$ against $Var[r^e_{t+1}|I_{it}] \times w_{it}$ (a measure of forecasting power of fund weights) should monotonically decrease with the persistence of the portfolio equity weights. Conversely, if managers act as noise traders, the R^2 should monotonically increase with the persistence of the portfolio equity weights.

As noted previously, Proposition 2.2.2 and its Corollary rely strongly on the assumption that the return generating process for r^e_{t+1} is common knowledge. Therefore, any tests of the Proposition or Corollary suffer from a joint hypothesis problem that managers both (1) agree on the return generating process and (2) use information optimally. As such, we refrain from attempting any direct tests of the Proposition or implications of its predictions.

The key result for our empirical tests relies on the observation that, controlling for the denominator in (2.4), the time-series variation in weights is related to the quality of the

manager's signal. If the quality of the signal is poor (e.g., consider a non-highly informed manager with $\frac{\text{Var}[m]}{\text{Var}[m]+\text{Var}[n_i]}$ small), then the manager will optimally react by being careful not to make dramatic changes in the weights, which are proportional to changes in $E[\tilde{r}^e_{t+1}|I_{it}]$. Likewise, a high quality signal will be associated with larger shifts in weights in response to the signal. Thus, a higher variance of portfolio weights ought to reflect better forecasting power for the market returns. This is the subject of the next result.

Proposition 2.2.3. *For non-highly informed and highly informed managers, the regression of $\tilde{r}^e_{t+1}$ on $E[\tilde{r}^e_{t+1}|I_{it}]$ yields a slope coefficient of $\beta = 1$, and for noise traders the regression yields a slope coefficient of $\beta < 1$. Moreover, the unconditional correlation of $\tilde{r}^e_{t+1}$ with $E[\tilde{r}^e_{t+1}|I_{it}]$ is*

$$\rho_{r\hat{r}_i} = \beta\sqrt{\frac{Var\left[E[\tilde{r}^e_{t+1}|I_{it}]\right]}{\sigma^2_r}} = \begin{cases} \frac{\sigma_{\hat{m}_i}}{\sigma_r} & \text{if } i \text{ is non-highly informed} \\ \frac{\sigma_{\hat{m}_i}}{\sigma_r}\sqrt{1 + \frac{\sigma^2_{\varepsilon t}}{(1+R_{qi})\sigma^2_{\hat{m}_i}}} & \text{if } i \text{ is highly informed} \\ \frac{\sigma_{\hat{m}_i}}{\sigma_r}\frac{1}{\sqrt{1 + \frac{\sigma^2_{\varepsilon t}}{(1+R_{qi})\sigma^2_{\hat{m}_i}}}} & \text{if } i \text{ is a noise trader,} \end{cases} \tag{2.9}$$

where σ_r is the unconditional standard deviation of equity returns and $\sigma_{\hat{m}_i}$ is the unconditional standard deviation of $\hat{m}_{it}$.

An implication of Eq. (2.9) is that, controlling for the conditional volatility in Eq. (2.4), if managers have timing ability the R^2 in a regression of the market excess return against lagged realizations of $\text{Var}[r^e_{t+1}|I_{it}] \times w_{it}$ (i.e., $\rho^2_{r\hat{m}_i}$) should be increasing in the time-series variance
$\text{Var}\left[\text{Var}[r^e_{t+1}|I_{it}] \times w_{it}\right] = \text{Var}\left[E[\tilde{r}^e_{t+1}|I_{it}]\right]$. Conversely, if managers act as noise traders, the R^2 should be decreasing in the above time-series variance.

Unlike the predictions on weight autocorrelations, the prediction from Proposition 2.2.3 does not rely as strongly on fund managers agreeing on the data generating process. Rather, it essentially depends only on the assumption of individual rationality (as long as a forecast that incorporates a manager's signal is more precise than one that does not). Accordingly, it lends itself better to empirical testing, to which we turn in the next Section.

2.3 Empirical investigation

Our empirical work is guided by the model results of Section 2.2.1 where we assumed that the portfolio weight assigned to the market is given by Eq. (2.4). Before we can apply the results of the model to open-end mutual funds holding domestic equity, we have to address three issues.

First, fund managers may not time the market by shifting between equities and cash. Rather, if a manager forecasts higher (lower) excess market returns in the near future, she

may move into (out of) high-beta stocks and leave her cash position unchanged. Even if managers do time the market by adjusting cash holdings, the cash balance of the fund is likely to be affected by other factors not related to market timing, such as liquidity needs for future redemptions.[13] To account for these possibilities, we compute a holdings-based portfolio beta for each fund at each report date and use the resulting beta, β_{it}, as the 'effective' portfolio weight in equities.[14] In a setting in which a manager holds only cash and a market proxy, then the portfolio beta, of course, coincides with the raw weight in the market proxy. To compute the holdings-based betas, we follow the procedure of Jiang, Yao, and Yu (2007), the details of which we outline in subsection 2.3.1 below.

A second issue is that managers may not follow a strategy that rebalances to constant weights in the absence of information. One example of this is a buy-and-hold strategy, while another is portfolio insurance. In the case of iid returns, it is well known (e.g., Cox and Leland, 2000; Leland, 1980) that these three dynamic strategic do not dominate each other in the sense that certain investors (e.g., those exhibiting a particular version of decreasing relative risk aversion) might prefer portfolio insurance while others might prefer a rebalancing strategy. Moreover, in the presence of trading costs, it may be optimal to allow weights to wander within an 'inaction' region (see Davis and Norman, 1990) and the optimal allocation of new funds might therefore exhibit a lag. Because all of these alternative reasons for weight changes are contingent on past returns or fund flows, which are orthogonal to the error term in forecasting $\tilde{r}_t^e$, one can still test the model by controlling for past returns and fund flows.

Finally, the Propositions pertain to the numerator of Eq. (2.4), but in practice portfolio weights incorporate the denominator as well:

$$
w_{it} = \begin{cases} A_i \dfrac{\bar{\mu}+\hat{m}_{it}}{\sigma_{\varepsilon t}^2+\text{Var}[m_t|I_{it}]} & \text{if manager } i \text{ is non-highly informed} \\[4mm] A_i \dfrac{\bar{\mu}+\hat{m}_{it}+\frac{q_{it}}{1+R_{qi}}}{\frac{R_{qi}}{1+R_{qi}}\sigma_{\varepsilon t}^2+\text{Var}[m_t|I_{it}]} & \text{if manager } i \text{ is highly informed or a noise trader.} \end{cases} \tag{2.10}
$$

Thus, in testing whether asset allocation is informed, one must control for the conditional volatility in the denominator of Eq. (2.10). To do so, we begin by assuming that $\frac{R_{qi}}{1+R_{qi}}\sigma_{\varepsilon t}^2 \gg \text{Var}[m_t|I_{it}]$, which appears reasonable given that the 'shock' component of market returns is harder to forecast than the equity premium and ought to vary substantially more.[15] Based

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[13]Chordia (1996) documents that fund cash holdings are affected by uncertainty over flows and the load structure of the fund, while Yan (2006) relates cash to the liquidity of funds' holdings and to the level and volatility of future flows. Simutin (2009) also documents a relation between excess cash holdings and the liquidity and riskiness of funds' stock holdings.

[14]The betas we calculate have a correlation with the raw equity weights of only about 0.33, which suggests that the factors that affect cash holdings differ from those that affect portfolio allocation decisions.

[15]For example, even if the equity premium m_t has a standard deviation of 4% per year, assuming that $R_{qi} = 1$, and that ε_t has an unconditional standard deviation of 16% per year, yields a ratio of $\frac{R_{qi}}{1+R_{qi}}\sigma_{\varepsilon t}^2$ to $\text{Var}[m_t|I_{it}]$ greater than 8.

on that assumption, one can approximate

$$\sigma_{\varepsilon t}^2 w_{it} \approx \begin{cases} A_i\left(\bar{\mu} + \hat{m}_{it}\right) & \text{if manager } i \text{ is non-highly informed} \\[2em] A_i\frac{1+R_{qi}}{R_{qi}}\left(\bar{\mu} + \hat{m}_{it} + \frac{q_{it}}{1+R_{qi}}\right) & \text{if manager } i \text{ is highly informed or a noise trader.} \end{cases}$$

$$(2.11)$$

One can proxy for $\sigma_{\varepsilon t}^2$ using a volatility index such as the vix, acknowledging that a market forecast of volatility will differ, though likely not by much, from $\sigma_{\varepsilon t}^2$.[16] Eq. (2.11) provides a proxy for $\text{Var}[r_{t+1}^e|I_{it}] \times w_{it}$, and it is to this proxy that we will apply the insights from Section 2.2.1.

2.3.1 Data

We obtain quarterly holdings information for all mutual funds, from 1979Q3 until 2006Q4, in the Thompson Financial CDA/Spectrum s12 database accessed through the Wharton Research Data Services (WRDS). The data is then linked to CRSP through WRDS' MFLinks service and the CRSP survivorship bias-free Mutual Fund Database (MFDB). For each quarter and each fund we obtain, whenever available, the portfolio weight corresponding to the total domestic equity holdings of the fund, the value-weighted return on those holdings in the three months immediately following the report date, the S&P objective, style and specialty fund codes (from CRSP MFDB), and the CDA/Spectrum s12 investment objective fund code.[17] We also obtain the return to fund investors, net of distributions, for each calendar quarter and document the dollar value of total assets managed by the fund. We augment this with quarterly data constructed from the monthly series of CRSP value-weighted returns and the risk-free rate (from WRDS), and quarterly data for the aggregate dividend yield and earnings-to-price ratio on the S&P500 index (Global Financial Data). Finally, our tests require a measure of conditional volatility, i.e., a proxy for $\sigma_{\varepsilon t}^2$. We compute four different such proxies of market volatilities: the first predictor is a naive monthly volatility calculated using the past month's daily CRSP value-weighted return data, the second corresponds to a fit of monthly CRSP value-weighted return data over the period 1954-2006 to a GARCH(1,3) model, and the third consists of the S&P100 volatility index (vox from

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[16]$\text{Var}[r_{t+1}^e|\mathcal{P}_t]$, where $\mathcal{P}_t$ represents a common knowledge (public) information set, should be approximately a constant proportion of $\text{Var}[r_{t+1}^e|I_{it}]$. The deviation from a constant proportionality should amount to the difference between $\text{Var}[m_{t+1}|\mathcal{P}_t]$ and $\text{Var}[m_{t+1}|I_{it}]$, which ought to be small relative to $\sigma_{\varepsilon t}^2$.

[17]Because funds report their holdings at different times, the holding returns are not contemporaneous across all funds. When a fund reports holdings more than once in a quarter, we consider only the earliest report for that quarter. Many funds only report twice a year, the minimum SEC requirement, resulting in substantial 'seasonality' in the number of funds that report each quarter.

WRDS).[18] The fourth proxy for $\sigma_{\varepsilon t}^2$ is a constant, corresponding to a situation in which fund managers ignore changes in volatility when selecting the optimal equity position (the actual magnitude of this constant is immaterial as it can be absorbed into A_i in the definition of w_{it}). In merging the volatility data with our quarterly observations, we choose the volatility predictor for the last month of each quarter.[19]

To compute holdings-based fund betas, we follow the procedure from Jiang, Yao, and Yu (2007). Closing prices from CRSP are used to first calculate the portfolio weights of the individual holdings of each fund on each reporting date.[20] Denote by ω_{ijt} the weight of stock j in fund i's portfolio at time t. Next, for each stock on each reporting date t, daily returns for the preceding one-year period are pulled from CRSP. The beta of stock j at date t is computed using the Dimson (1979) method. We run the regression

$$r_{j\tau}^e = a_{jt} + \sum_{q=-5}^{5} b_{jqt} r_{m,\tau-q}^e + \epsilon_{j\tau}, \qquad \tau \in \{t - 364, t\},$$

and the stock beta is estimated as the sum of the coefficients

$$b_{jt} = \sum_{q=-5}^{5} b_{jqt}.$$

At least 60 daily return observations are required for this regression. Stocks not meeting this criteria are assigned betas of one. All non-stock securities are assigned betas of zero. Combining the portfolio weights and stock beta estimates, the beta of fund i at time t is given by

$$\beta_{it} = \sum_j \omega_{ijt} b_{jt}.$$

We initially start with 5278 funds. We filter out funds that at any point reported an equity portfolio weight of more than 200% (76 funds), funds that report holdings in fewer than eight quarters (983 more funds), and funds with an average equity portfolio weight of

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[18]We found the GARCH(1,3) model to be the most parsimonious best fit nested within a GARCH(4,4) framework. Of the three, the GARCH measure is the only one that is incorporates information unavailable contemporaneously because the coefficient estimates use the full time series. This turns out to be inconsequential for our tests.

[19]For the GARCH measure of volatility, while we could have used an average of the forecast for all three months of the quarter based on the last quarter's information, this doesn't significantly impact our test results. Moreover, weights are typically reported towards the end of the quarter and would therefore reflect a volatility prediction for that month (given that turnover ratios for the average mutual fund tend to be larger than what might be suggested from reported quarterly weight changes).

[20]Individual stocks from the CDA/Spectrum s12 holdings file are matched with CRSP using CUSIPs and, when CUSIPs are not present in the holdings file, ticker symbols.

less than 50% (774 additional funds). We generally wish to investigate funds that invest in a broad enough range of domestic equity so that information about the US equity premium ought to particularly matter to them. Table 2.1 tabulates how the remaining 3445 funds are then categorized as 'broad domestic equity funds' using their CDA/Spectrum s12 investment objective codes, ICDI (MFDB) objective codes, and S&P objective codes.[21] Funds not highlighted in the table are considered 'broad domestic equity funds'. We exclude the other funds from the sample and are left with 2766 funds. Even for these remaining funds, some fields are missing for some (or all) quarters.

For each fund, we calculate the time-series average holdings-based beta, the average weight of domestic equity in the fund's portfolio, the average total net assets under management, the number of observations, the contemporaneous correlation of returns on the fund's domestic equity portfolio with the CRSP value-weighted index returns, the contemporaneous correlation between a fund's holdings-based beta and the various predictors of market return variance. We also calculate the correlation between the holdings-based betas of every pair of funds in our sample that have at least eight overlapping weight data (2,675,922 pairs). Finally, we estimate the CAPM $\hat{\beta}$ and $\hat{\alpha}$ from a regression of the fund's domestic equity returns, the t-statistic for the α, and the Sharpe Ratio of the non-CAPM returns (i.e., the $\hat{\alpha}$ divided by the standard deviation of the CAPM regression residual, or the 'information ratio'). Table 2.2 provides a summary of these statistics across funds.

The summary statistics indicate that the funds in our sample maintain a typical holdings-based beta of around 1, and that it is not unusual for this to fluctuate by 0.2 or more each quarter. The funds also have a high average portfolio weight in equities. Moreover, the equity portfolio held by the typical fund is highly correlated with the market portfolio (this is also confirmed by CAPM β of the equity portfolio held by the typical fund). A striking statistic is the low typical correlation between the holdings-based betas of any two funds. This suggests that much of the asset allocation taking place is largely due to noise. Overall, the typical fund in our sample is not particularly good at picking stocks either, although according to Kosowski, Timmermann, Wermers, and White (2006) it is likely that the highest ranked funds do exhibit ability.

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[21]Some funds change objective codes throughout the sample period. We assign a fund its modal investment code, and when there is more than one mode, we assign the 'largest' one (numerically or alphabetically). In classifying a fund, we rely firstly on its CDA/Spectrum s12 objective code and consider it not to be a broad domestic equity fund if the investment code is 1 ('International'), 5 ('Municipal Bonds'), 6 ('Bond and Preferred'), or 8 ('Metals'). Only 2289 of the 3445 funds surviving the initial filter have a CDA/Spectrum s12 objective code. Surprisingly, 170 funds have an investment code of 1, 5, 6, or 8, meaning that, despite their objective, they mostly hold U.S. domestic equity (150 of these are classified as 'International'). None of the unclassified funds have an entry for their MFDB policy code or Wiesenberger objective.

2.3.2 The market forecasting power inherent in weight dynamics

An implication of Proposition 2.2.3 is that for managers with timing ability, a larger estimated weight variance implies better forecasting power. On the other hand, for noise traders the above results should reverse. By testing these predictions, we test whether the cross-sectional differences in portfolio weights (with respect to their variance) are related to ability or noise.

As mentioned earlier, the observed portfolio weight may deviate from the expression in Eq. (2.4). In order to control for strategy and for time-varying volatility, we posit that the fund's *observed* 'effective' weight in equity, $w_{it}^O (= \beta_{it})$ can be written as

$$w_{it}^O = w_{it} + B_i + \gamma_{i1} r_{it-1} + \gamma_{i2} r_{it-1}^2 + \delta_{i1} f_{it} + \delta_{i2} f_{it}^2, \tag{2.12}$$

where w_{it} is the expression from Eq. (2.4), r_{it-1} is the excess return on the fund's equity subportfolio, f_{it} is fund's growth in net assets due to net inflows, and B_i is a constant.[22] By including r_{it-1} and r_{it-1}^2 we are controlling for persistent changes in weights due to strategies such as buy-and-hold or portfolio insurance. These terms, along with f_{it} and f_{it}^2, also control for the presence of 'no trade' regions that arise in the presence of transaction costs or other forms of illiquidity.[23] The inclusion of squared flows helps account for any nonlinearity in the relation between weights and flows. Using the approximation in Eq. (2.11), we proxy for $\sigma_{\varepsilon t}^2$ using one of the four predictors of market variance described earlier, and denoted as $\sigma_{\text{p} t}^2$ (e.g., $\sigma_{\text{vox} t}^2$).

Consider the following forecasting regression

$$r_{t+1}^e = \text{constant} + \hat{\zeta}_i \sigma_{pt}^2 w_{it}^O + \hat{\gamma}_{i1} \sigma_{pt}^2 r_{it-1} + \hat{\gamma}_{i2} \sigma_{pt}^2 r_{it-1}^2 + \hat{\delta}_{i1} \sigma_{pt}^2 f_{it} + \hat{\delta}_{i2} \sigma_{pt}^2 f_{it}^2 + \hat{\tau}_i \sigma_{pt}^2 + \text{noise}. \tag{2.13}$$

Under our assumption, Eq. (2.12), $\sigma_{\text{p} t}^2 w_{it}^O$ corresponds to σ_{pt}^2 times the sum of the control variables, plus a constant multiple of $E[\tilde{r}_{t+1}^e | I_{it}]$ from Proposition 2.2.3 (where the constant of proportionality depends on A_i and R_{qi}, as in Eq. (2.11)). Moreover, under the model assumptions, the forecast error in $E[\tilde{r}_{t+1}^e | I_{it}]$ is orthogonal to the control variables in the equation so that ζ_i and the residual variance can be estimated via OLS.

Testing Proposition 2.2.3

To test the implication of Proposition 2.2.3, one needs to estimate the variance of $\text{Var}[r_{t+1}^e | I_{it}] \times w_{it}$. Because we do not observe this directly, we instead estimate the residual variance from

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[22]If, as discussed in footnote 11, the predictable part of the equity premium includes a term such as $\xi \sigma_{\text{p} t}^2$, then by including the constant B_i we ensure that w_{it} accounts for the remaining predictive variables.

[23]Fund flows are calculated as the difference between the growth in total net assets (TNA) under management less the rate of return on the fund including all distributions: $f_{it} = \frac{TNA_t - (1+r_t)TNA_{t-1}}{TNA_{t-1}}$.

the following equation, based on Eq. (2.12):

$$\sigma_{pt}^2 w_{it}^O = \text{constant} + B_i \sigma_{pt}^2 + \sigma_{pt}^2 \gamma_{i1} r_{it-1} + \gamma_{i2} \sigma_{pt}^2 r_{it-1}^2 + \delta_{i1} \sigma_{pt}^2 f_{it} + \delta_{i2} \sigma_{pt}^2 f_{it}^2 + \text{noise}. \quad (2.14)$$

Under our assumptions, the residual variance from Eq. (2.14) is equal to the variance of the expression in Eq. (2.11) (i.e., is proportional to the variance of $E[\tilde{r}_{t+1}^e|I_{it}]$ where the constant of proportionality depends on A_i and R_{qi}). If there is no relation between the long-run target of the fund's equity exposure, corresponding to A_i, and the quality of information available to the manager, then Proposition 2.2.3 implies an overall positive relationship between the residual variance in Eq. (2.14) and the incremental R^2 from Eq. (2.13) in the presence of informed and highly-informed managers and an overall negative relation in the presence of informed managers and noise traders.

Table 2.3 reports on this relationship at various forecasting horizons, using each of our four proxies for market volatility, by presenting the Spearman rank correlation between the incremental R^2 from Eq. (2.13) and the residual variance from Eq. (2.14). At a one-month forecasting horizon three of the 16 correlations between residual variance and incremental R^2 are significantly negative, all using the vox conditional volatility measure. This is consistent with the prediction of Proposition 2.2.3 for noise traders; however, the limited number of significant coefficients provides only weak evidence against timing ability. At three, six-, and 12-month forecasting horizons, the results appear broadly supportive of the hypothesis that some managers possess superior information on future market returns. Results under both the naive and vox volatility estimates indicate a significantly *positive* relation between weight variation and forecasting ability, which is consistent with the prediction of Proposition 2.2.3 when cross-sectional differences in weight variation are due to ability. This result is consistent with evidence presented by Simutin (2009) that funds with the most volatile excess cash holdings exhibit timing ability at three- to six-month forecasting horizons. It is also broadly consistent with the findings of Jiang, Yao, and Yu (2007), who find that that holdings-based betas predict three- and six-month ahead market returns.

It is notable that when we use the simple constant volatility proxy, the relation between weight variance and forecasting ability is consistently significantly negative. As there is ample evidence (see, e.g., French, Schwert, and Stambaugh, 1987; Schwert, 1989) that conditional volatility varies substantially over time, we place comparatively little weight on these results. However, it is interesting to note that were we to fail to control for time-varying volatility and consider only the constant-volatility proxy, we would conclude that managers exhibit no timing ability. Given that (to our knowledge) all of the previous studies that evaluate market timing ability ignore conditional volatility, this may partially explain some of the negative results in the market timing literature.

To be sure that our results are not the result of an unanticipated cross-sectional relationship between the A_i's and the variance of $E[\tilde{r}_{t+1}^e|I_{it}]$, we re-did the test by normalizing the residual variance of Eq. (2.14) by the squared sample mean of $\sigma_{pt}^2 w_{it}^O$, taking care to

adjust the denominator for bias.[24] There was no substantial difference in the conclusions, indicating that no systematic bias is introduced in Table 2.3 by ignoring the variation in A_i's. Thus, we conclude that at a three-, six-, and 12-month forecasting horizons there is a positive relationship between forecasting ability (as inferred from portfolio weights) and variation in weights.[25]

We report various robustness exercises in the next Section that seek to examine the sensitivity of these conclusions to our statistical methods.

2.3.3 Robustness checks

The results reported in Table 2.3, lead us to conclude that, at least at three- to 12-month forecasting horizons, there is information in managers' equity weight exposures. To confirm this, we test, using a bootstrapping methodology, whether the cross-sectional distribution of ζ_i's in the regression equation (2.13) is different than what would arise under the null of $\hat{\zeta}_i = 0$ for all i. The methodology proceeds as follows:

1. Using the data, the regression equation (2.13) is estimated for each fund, and the t-statistics for $\hat{\zeta}_i$, denoted as t_i, is saved along with the corresponding regression residual. Because the residuals in our model are heteroskedastic, White (1980) standard errors are used when computing t-statistics.

2. Next, the regression equation (2.13) with $\hat{\zeta}_i$ set to zero is estimated:

$$r_{t+1}^e = \check{\gamma}_{i1}\sigma_{pt}^2 r_{it-1} + \check{\gamma}_{i2}\sigma_{pt}^2 r_{it-1}^2 + \check{\delta}_i\sigma_{pt}^2 f_{it} + \check{\tau}_i\sigma_{pt}^2 + \check{\epsilon}_{it+1} + \text{const}_i,$$

   and the predicted returns, $\check{r}_{it+1}^e \equiv r_{t+1}^e - \check{\epsilon}_{it+1}$ are saved.

3. The set of dates $\{1979Q3, \ldots 2006Q4\}$ is randomly sampled, with replacement, to create 2000 sets of data, each of which has the same time-series length as the original sample. Denote by $\mathcal{T}(k, t)$ the random element from $\{1979Q3, \ldots 2006Q4\}$ corresponding to the tth item in the kth sample.

4. For each fund, we construct 2000 sets of bootstrapped sample returns under the null that $\hat{\zeta}_i = 0$ by combining randomly drawn residuals from the unrestricted model in step 1 with the predicted returns from step 2. Specifically, the return at date t of the

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[24]The normalization will, theoretically, remove any dependence on the A_i's. The bias mentioned in the text arises from dividing through by something with an estimation error. We correct for this using the 'delta'-method.

[25]We also ran all of our tests on a subsample of funds that we classified as 'timers' based on the specification of flexibility or dynamic asset allocation in their MFDB policy code, S&P objective code, S&P style code, or S&P specialist code. The limited number of funds in this sample yielded too little statistical power to draw any conclusions.

k^{th} bootstrapped sample is:

$$r_{ikt}^{e}{}^{*} = \check{r}_{it}^{e} + \hat{\epsilon}_{i\mathcal{T}(k,t)}.$$

This approach preserves the cross-sectional properties of the residuals in each of the 2000 bootstrapped panels.

5. For each fund, denoted by i, and bootstrapped sample, denoted by k, the following time-series regression is estimated:

$$r_{ikt+1}^{e}{}^{*} = \zeta_{ik}^{*}\sigma_{pt}^{2}w_{it} + \gamma_{ik1}^{*}\sigma_{pt}^{2}r_{it-1} + \gamma_{ik2}^{*}\sigma_{pt}^{2}r_{it-1}^{2} + \delta_{ik}^{*}\sigma_{pt}^{2}f_{it} + \tau_{ik}^{*}\sigma_{pt}^{2} + \epsilon_{ikt+1}^{*} + \mathrm{const}_{ik}^{*}.$$

The estimate for the t-statistic associated with each ζ_{ik}^{*} is saved. This exercise essentially samples the joint distribution of the t_i^{*}'s, the t-statistics associated with the ζ_i's, *under the null of no timing ability*. For the kth bootstrapped panel, let $\Gamma^{k}(\ell)$ denote the cross-sectional ℓ^{th} percentile of among the t_i^{*}'s.

6. The one-sided p-values for the cross-sectional percentiles, $\Gamma(\ell)$, of t_i's from step 1 are computed according to

$$p(\ell) = \frac{1}{2000}\sum_{k=1}^{2000}\mathbf{1}\{\Gamma^{k}(\ell) > \Gamma(\ell)\},$$

For instance, $p(50)$ corresponds to the likelihood, under the null, that we would observe by chance alone a sample median t_i as high or higher than the median t_i in step 1. In particular, if $p(50)$ is small, then this could be interpreted as evidence that the asset allocation decisions made by the median manager contain more information than would be expected under the null.

Table 2.4 reports various cross-section percentiles of t_i's when the regression is performed using our four different measures of market volatility, when various restrictions are imposed on funds' age in the panel, and for a one-month versus three-months forecast. The bootstrapped p-values are reported below each estimated percentile. At the one-month forecasting horizon, for virtually all values of ℓ, the percentile $\Gamma(\ell)$ is not significantly greater than what is obtained under the null that $\hat{\zeta}_i = 0$. There is no evidence that the cross sectional distribution of ζ_i's is shifted to the right of what would expected under a null of $\zeta_i = 0$. Table 2.5 reports the same results at a 3-month forecasting horizon. In this case, under all volatility proxies, the upper right-tail of the cross-sectional distribution of t-statistics is shifted significantly to the right. This provides confirmatory evidence of timing ability at a three-month horizon. Similar conclusions are reached when the bootstrapping exercise is repeated using the Kosowski, Timmermann, Wermers, and White (2006) approach. In summary, consistent with our previous tests, at horizons longer than one month, fund managers appear to move

portfolio weights between equity and non-equity in a manner that predicts future market returns.

Comparison with Jiang, Yao, and Yu (2007)

Jiang, Yao, and Yu (2007) report that lagged equity portfolio betas of open-end mutual funds predict market returns. Specifically, fund managers appear to be holding equity portfolios with higher betas prior to positive market outcomes, and tend to be in possession of equity portfolios with lower betas prior to negative market outcomes. This is viewed as supportive of timing ability on the part of active fund managers. Section 2.6.2 qualitatively confirms that, in our sample, equity portfolio betas also predict market returns. Our results differ somewhat from those of Jiang, Yao, and Yu (2007) (see their Table 3) in that we find mixed evidence that betas predict returns at the three-month horizon, whereas Jiang, Yao, and Yu (2007) do find such evidence. Both Jiang, Yao, and Yu (2007) and we fail to find evidence that portfolio betas forecast market returns at the one-month horizon. This is in line with the earlier results of this section, in that evidence of timing ability, as reflected in portfolio weights, appears to not be present at the one-month horizon.

In summary, the results reported in this subsection reinforce our conclusion that, at least at forecasting horizons of three months and beyond, cross-sectional differences in the time-series properties of portfolio weights appear to be related to ability.

2.4 Does successful 'timing' translate into higher returns?

There are several ways in which the seemingly positive results from Section 2.3.2 could arise, even in the absence of timing ability. For instance, it might be that our finding is not actually reflective of market timing ability, perhaps because of mis-measurement in the portfolio betas, because the shift into higher (lower) beta portfolios is accompanied by a shift into lower (higher) non-market systematic risk, or because changes in funds' equity portfolios might be taking place at a frequency that is too high to benefit from the predictability at 3- to 12-month horizons. It is also possible that our results are a mere statistical artifact.[26]

To help shed light on whether our findings are truly indicative of market timing ability, we perform Treynor-Matzuy (TM) and Henriksson-Merton (HM) regressions on the cross section of funds' *equity portfolio* returns and compare the standardized timing coefficients from these regressions with those from bootstrapped samples for which, by construction, there is no timing ability. Jiang, Yao, and Yu (2007) examine TM and HM return regressions for *fund* returns, which suffer from the various criticisms mentioned in the Introduction (fund

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[26]In this vein, Taliaferro (2009) reports evidence that the Jiang, Yao, and Yu (2007) results are strongly influenced by a few managers timing correctly during the downturn of 2000-2002. If one eliminates those years from their sample, managers appear to forecast no better than would be expected by chance.

returns do not reflect the equity portfolio returns because the former results from trading at a frequency greater than quarterly). By contrast, we look at the results of TM and HM bootstrapped tests for the same portfolios whose market betas are shown to forecast future market returns. Because the rebalancing period for these portfolios coincides with the observation frequency by construction, the criticism of Goetzmann, Ingersoll, and Ivković (2000) and Jagannathan and Korajczyk (1986) do not apply to our tests.

Our first bootstrapping procedure, similar to the procedure used to test the timing regression in Section 2.3.3, is aimed at addressing the possibility that t-statistics from the timing regressions might not be t-distributed, and that residuals are likely to be cross-sectionally correlated. We proceed similarly to the method outlined in Section 2.3.3 except that $\check{r}^e_{it}$ is now defined as $\check{r}^e_{it} \equiv \text{const}_i + \sum_{j \in \{\text{m,smb,hml,umd}\}} \beta_j r^e_{jt}$, and the pseudo returns are generated by combining $\hat{r}_{it}$ with reordered values of $\hat{\varepsilon}_{i\mathcal{T}(k,t)}$, where $\mathcal{T}(k,t)$ is defined as in Section 2.3.3. The timing regression is then re-run for each replication of each fund, and right-tail p-values are calculated for various percentiles.

The results, reported in Panels A and B of Table 2.6, suggest that there is no evidence of timing ability in funds' equity portfolios. We also re-ran the procedure using a monthly, rather than quarterly, forecasting horizon.[27] At the monthly horizon, we did find weak evidence of timing in the HM regression test, though not for the TM test.

The second procedure we performed proceeds similarly to Bollen and Busse (2001). The results are consistent with those for the first procedure, so we relegate discussion of the procedure and results to Section 2.6.3.

To recap, although the equity portfolios of actively managed funds, as reconstructed from quarterly holdings, exhibit portfolio betas that predict market returns at a horizon of three months and longer, we find no evidence that this translates into successful timing as measured in terms of quarterly portfolio returns. This could be because the portfolios are not held long enough to benefit from the predictability. Alternatively, this could be because of mis-measurement in the portfolio betas or because the shift into higher (lower) beta portfolios is accompanied by a shift into lower (higher) non-market systematic risk.

2.5 Conclusions

We derive a model of asset allocation based on a dynamic noisy information model. The model predicts that a more volatile equity exposure should be linked to higher quality of information about the equity premium. The model also predicts that the equity exposure of every portfolio manager who uses information optimally, whether they are market timers or not, ought to have an autocorrelation coefficient that decreases with the quality of information that the manager has.

We find evidence supportive of the first prediction, at forecasting horizons of three, six,

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[27]The table is available upon request.

and 12 months. Fund managers with more volatile effective equity portfolio weights appear to better forecast future market returns (as measured through forecasting regressions that include the weights). Various bootstrap exercises corroborate this result. While our tests support the hypothesis that some managers have information about future market returns, we find no evidence that this ability manifests itself in the form of higher portfolio returns.

2.6   Appendix

2.6.1   Proofs

Proof of Proposition 2.2.1. Since m_t, s_{it-j}, and $\tilde{r}_{t-j}$ are jointly normal, the conditional expectation takes the form of a linear projection of m_t onto s_{it-j} and $\tilde{r}_{t-j}$ (recall that q_{it} is orthogonal to m_t).

The equations defining the coefficients in such a linear projection are of the form

$$E[s_{it-k}m_t] = \sum_{j=0}^{\infty} a_{itj}E[s_{t-k}s_{t-j}] + \sum_{j=0}^{\infty} b_{itj}E[s_{t-k}(\tilde{r}_{t-j}^e - \bar{\mu})], \quad k \geq 0$$

$$E[(\tilde{r}_{t-k}^e - \bar{\mu})m_t] = \sum_{j=0}^{\infty} a_{itj}E[(\tilde{r}_{t-k}^e - \bar{\mu})s_{t-j}] + \sum_{j=0}^{\infty} b_{itj}E[(\tilde{r}_{t-k}^e - \bar{\mu})(\tilde{r}_{t-j}^e - \bar{\mu})], \quad k \geq 0.$$

Calculating the expectations in these expressions allows one to rewrite the first equation as

$$(1 - \phi_m)^k \mathrm{Var}[m] = \sum_{j=0}^{\infty} \Big(a_{itj}\left(\mathrm{Var}[m](1 - \phi_m)^{|k-j|} + \mathrm{Var}[n_i](1 - \phi_{in})^{|k-j|}\right)$$
$$+ b_{itj}\mathrm{Var}[m](1 - \phi_m)^{|k-j-1|}\Big). \tag{2.15}$$

and the second equation as

$$(1 - \phi_m)^{k+1}\mathrm{Var}[m] = b_{itk}\sigma_{\varepsilon t-k-1}^2 + \sum_{j=0}^{\infty} \Big(a_{itj}\mathrm{Var}[m](1 - \phi_m)^{|k+1-j|} +$$
$$b_{itj}\mathrm{Var}[m](1 - \phi_m)^{|k-j|}\Big) \tag{2.16}$$

As long as $1 - \phi_m, 1 - \phi_{in} \in (0, 1)$, Austin (1987) guarantees that a solution to this infinite set of equations exists. This establishes the first claim in the proposition.

Next, consider $\mathrm{Var}[m_t|I_{it}]$. The coefficients $\{a_{itj}, b_{itj}\}_{j=0}^{\infty}$ are chosen such that $m_t -$

$E[m_t|I_{it}]$ is orthogonal to $E[m_t|I_{it}]$, so $\text{Var}[m_t|I_{it}] = \text{Var}[m_t - E[m_t|I_{it}]]$. Hence,

$$\text{Var}[m_t|I_{it}] = \text{Var}[m] - 2\sum_{j=0}^{\infty}\left(a_{itj}\text{Var}[m](1-\phi_m)^j + b_{itj}\text{Var}[m](1-\phi_m)^{j+1}\right) +$$

$$\text{Var}\left[\sum_{j=0}^{\infty}\left(a_{itj}s_{it-j} + b_{itj}(\tilde{r}_{t-j}^e - \bar{\mu})\right)\right] \tag{2.17}$$

To simplify this, obtain an expression for the middle summation term by multiplying (2.15) by a_{itk} and summing over k and by multiplying (2.16) by b_{itk} and summing over k. Obtain an expression for the third term by expanding and simplifying it to write it as

$$\sum_{j,k} a_{itj}a_{itk}\left(\text{Var}[m](1-\phi_m)^{|k-j|} + \text{Var}[n_i](1-\phi_{in})^{|k-j|}\right) + \sum_{j,k} a_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k+1-j|}$$

$$+ \sum_{j,k} a_{itk}b_{itj}\text{Var}[m](1-\phi_m)^{|k-j-1|} + \sum_{j,k} b_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k-j|} + \sum_{k} b_{itk}^2\sigma_{\varepsilon t-k-1}^2$$

Substituting the results from these manipulations back into (2.17) gives

$$\text{Var}[m_t|I_{it}] = \text{Var}[m] - \sum_{j,k} a_{itj}a_{itk}\text{Var}[m](1-\phi_m)^{|k-j|} - \sum_{j,k} a_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k+1-j|}$$

$$- \sum_{j,k} a_{itk}b_{itj}\text{Var}[m](1-\phi_m)^{|k-j-1|} - \sum_{j,k} b_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k-j|}$$

$$- \sum_{j,k} a_{itj}a_{itk}\text{Var}[n_i](1-\phi_{in})^{|k-j|} - \sum_{k} b_{itk}^2\sigma_{\varepsilon t-k-1}^2. \tag{2.18}$$

To simplify this expression to one that does not depend on $\text{Var}(m)$, first multiply (2.15) by a_{itk} and sum over k, then multiply (2.16) by b_{itk} and sum over k, and finally sum the results of these manipulations to obtain

$$-\sum_{j,k} a_{itj}a_{itk}\text{Var}[m](1-\phi_m)^{|k-j|} - \sum_{j,k} a_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k+1-j|}$$

$$-\sum_{j,k} a_{itk}b_{itj}\text{Var}[m](1-\phi_m)^{|k-j-1|} - \sum_{j,k} b_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k-j|}$$

$$= \sum_{j,k} a_{itj}a_{itk}\text{Var}[n_i](1-\phi_{in})^{|k-j|} + \sum_{k} b_{itk}^2\sigma_{\varepsilon t-k-1}^2 - \sum_{k}\left(a_{itk}\text{Var}[m](1-\phi_m)^k + b_{itk}\text{Var}[m](1-\phi_m)^{k+1}\right)$$

$$\tag{2.19}$$

Take $k=0$ in eq. (2.15) to get an expression to substitute for

$$\sum_{k}\left(a_{itk}\text{Var}[m](1-\phi_m)^k + b_{itk}\text{Var}[m](1-\phi_m)^{k+1}\right),$$

then plug the result back to (2.18) to obtain

$$\text{Var}[m_t|I_{it}] = \sum_j a_{itj}\text{Var}[n_i](1 - \phi_{in})^j \tag{2.20}$$

Finally, we obtain the result in the proposition by stepping (2.15) forward from k to $k+1$, subtracting (2.16), and taking $k = 0$ to produce

$$b_{it0}\sigma^2_{\varepsilon t-1} = \sum_{j=0}^{\infty} a_{itj}\text{Var}[n_i](1 - \phi_{in})^{|1-j|}$$

After minor manipulation, substituting this expression back into (2.20) gives the desired formula.

The result for the highly-informed manager is standard and relies on the fact that the shocks are not serially correlated and the assumption that the conditional variances of e_{it} and ε_{t+1} at date t have a ratio, denoted R_{qi}, that is time-invariant. $\qquad\square$

Proof of Proposition 2.2.2. Begin by using the definition of autocorrelation to write

$$\frac{1}{\text{Var}[\hat{m}_{it}]}\text{Cov}\left(\sum_{j=0}^{\infty}\left(a_{itj}s_{it-j} + b_{itj}\left(\tilde{r}^e_{t-j} - \bar{\mu}\right)\right), \sum_{k=0}^{\infty}\left(a_{itk}s_{it-1-k} + b_{itk}\left(\tilde{r}^e_{t-1-k} - \bar{\mu}\right)\right)\right)$$

$$= \frac{1}{\text{Var}[\hat{m}_{it}]}\left[\sum_{j,k}\left(a_{itj}a_{itk}\left(\text{Var}[m](1-\phi_m)^{|k+1-j|} + \text{Var}[n_i](1-\phi_{in})^{|k+1-j|}\right) + b_{itj}a_{itk}\text{Var}[m](1-\phi_m)^{|k-j|}\right.\right.$$

$$\left.\left. + a_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k+2-j|} + b_{itj}b_{itk}\text{Var}[m](1-\phi_m)^{|k+1-j|}\right) + \sum_k b_{itk}b_{it(k+1)}\sigma^2_{\varepsilon t-2-k}\right] \tag{2.21}$$

To simplify this expression, take (2.15), advance k to $k+1$, multiply by a_{itk}, and sum over k. Similarly, take (2.16), advance k to $k+1$, multiply by b_{itk} and sum over k. Add the two results to conclude that the term in brackets in (2.21) equals

$$\text{Var}[m]\sum_k\left(a_{itk}(1-\phi_m)^{k+1} + b_{itk}(1-\phi_m)^{k+2}\right). \tag{2.22}$$

To simplify (2.22) further take $k = 0$ in (2.15), multiply the result by $1 - \phi_m$, and rearrange the result to obtain

$$\text{Var}[m]\sum_k\left(a_k(1-\phi_m)^{k+1} + b_k(1-\phi_m)^{k+2}\right) = (1-\phi_m)\left(\text{Var}[m] - \text{Var}[n_i]\sum_j a_{itj}(1-\phi_{in})^j\right)$$

$$= (1-\phi_m)\left(\text{Var}[m] - \text{Var}[m_t|I_{it}]\right), \tag{2.23}$$

where the second equality follows from (2.20).

Finally, substitute from (2.23) back into (2.21) and use the fact that $\text{Var}[\hat{m}_{it}] = \text{Var}[m] - \text{Var}[m_t - E[m_t|I_{it}]]$ to obtain the stated result. $\qquad\square$

Proof of Corollary to Proposition 2.2.2. One can write $E[r_{t+1}^e|I_{it}] = \hat{m}_{it} + E[\varepsilon_{t+1}|I_{it}] = \hat{m}_{it} + \frac{q_{it}}{1+R_{qi}}$. Because the q_{it}^j are serially independent, Proposition 2.2.2 can be used to write the autocorrelation of $E[r_{t+1}^e|I_{it}]$ for the highly informed managers and noise traders as

$$\frac{1 - \phi_m}{\sqrt{\left(1 + \frac{\sigma_{\varepsilon t}^2}{(1+R_{qi})\text{Var}(\hat{m})}\right)\left(1 + \frac{\sigma_{\varepsilon t-1}^2}{(1+R_{qi})\text{Var}(\hat{m})}\right)}}.$$

This establishes the corollary. $\qquad\square$

Proof of Proposition 2.2.3. Let $\hat{r}_{it+1}^e \equiv E[\tilde{r}_{t+1}^e|I_{it}]$. For the informed and highly informed managers, we have

$$\begin{aligned}
\beta &= \frac{\text{Cov}\left[\tilde{r}_{t+1}^e, \hat{r}_{it+1}^e\right]}{\text{Var}[\hat{r}_{it+1}^e]} \\
&= \frac{\text{Cov}\left[(\tilde{r}_{t+1}^e - \hat{r}_{it+1}^e) + \hat{r}_{it+1}^e, \hat{r}_{it+1}^e\right]}{\text{Var}[\hat{r}_{it+1}^e]} \\
&= 1,
\end{aligned}$$

where the last equality follows since $\hat{r}_{it+1}^e$ is by definition orthogonal to $\tilde{r}_{t+1}^e - \hat{r}_{it+1}^e$.

For the noise traders,

$$\begin{aligned}
\beta &= \frac{\text{Cov}\left[\tilde{r}_{t+1}^e, \hat{r}_{it+1}^e\right]}{\text{Var}[\hat{r}_{it+1}^e]} \\
&= \frac{\text{Cov}\left[m_t + \varepsilon_{t+1}, \hat{m}_{it} + \frac{q_{it}^N}{1+R_{qi}}\right]}{\text{Var}[\hat{m}_{it} + \frac{q_{it}^N}{1+R_{qi}}]} \\
&= \frac{\text{Cov}\left[m_t, \hat{m}_{it}\right]}{\text{Var}[\hat{m}_{it}] + \frac{\text{Var}[q_{it}^N]}{(1+R_{qi})^2}} \\
&= \frac{1}{1 + \frac{\sigma_{\varepsilon t}^2}{(1+R_{qi})\text{Var}[\hat{m}_{it}]}},
\end{aligned}$$

where the third equality follows from the fact that $\text{Cov}(\varepsilon_{t+1}, q_{it}^N) = 0$, and the last equality follows (after rearranging) since $\hat{m}_{it}$ is orthogonal to $m_t - \hat{m}_{it}$.

We can also express β as

$$\begin{aligned} \beta &= \frac{\mathrm{Cov}\left[\tilde{r}^e_{t+1}, \hat{r}^e_{it+1}\right]}{\mathrm{Var}\left[\hat{r}^e_{it+1}\right]} \\ &= \frac{\rho_{r\hat{r}_i}\sigma_{\hat{r}_i}\sigma_r}{\sigma^2_{\hat{r}_i}} \end{aligned}$$

Using the values for β derived above and rearranging, we obtain the results. $\qquad\square$

2.6.2 Reproducing the results from Jiang, Yao, and Yu (2007)

With the holdings-based beta estimates from Section 2.3.1, we follow Jiang, Yao, and Yu (2007) and estimate the Treynor-Mazuy and Henriksson-Merton timing measures directly from the regressions[28]

$$\begin{aligned} \beta_{it} &= \alpha_i + \gamma_i r^e_{m,t+1} + \eta_{i,t+1} \\ \beta_{it} &= \alpha_i + \gamma_i \mathbf{1}_{\{r^e_{m,t+1}>0\}} + \eta_{i,t+1}. \end{aligned}$$

To test the null hypothesis that funds have no timing ability, we first estimate the TM and HM regressions for each fund and save the regression coefficients and t-statistics. Following Jiang, Yao, and Yu (2007), only funds with at least 8 valid report dates are included in the analysis. Furthermore, the t-statistics are computed using the Newey-West procedure with a two-quarter lag window to correct for serial correlation in the residuals brought about by overlapping market returns. In the results below, four different horizons (one-, three-, six-, and 12-month) for the market excess return $r^e_{m,t+1}$ are reported.

The cross-section of t-statistics, t_i, is analyzed with a bootstrap procedure. The procedure proceeds by randomly sampling with replacement the set of market excess returns to produce 2000 time series, each with the length of the original time series. The TM and HM regressions above are re-run on the bootstrapped excess market return series to produce 2000 distinct cross-sectional panels of regression slope coefficients and associated t-statistics, t^k_i.

Consider the ℓth percentile $\Gamma(\ell)$ of the cross-sectional distribution of "actual" γ_i t-statistics. To test whether the ℓth percentile is significantly greater than expected under the null, we compare it to the percentiles $\Gamma^k(\ell)$ of the bootstrapped t-statistics. The p-value for a one-sided test of $\Gamma(\ell)$ is computed as

$$p(\ell) = \frac{1}{2000} \sum_{k=1}^{2000} \mathbf{1}\{\Gamma^k(\ell) > \Gamma(\ell)\}.$$

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[28] Jiang, Yao, and Yu (2007) use characteristic-adjusted betas (see Daniel, Grinblatt, Titman, and Wermers, 1997) in their analysis. Because they mention that their results are not sensitive to this adjustment, we do not make it.

In short, a cross-sectional percentile is considered significantly larger than expected under the null if only a small number of bootstrap samples produce cross-sectional percentiles that are larger.

Table 2.7 reports the results of this test and confirms the findings in Jiang, Yao, and Yu (2007) that the equity portfolio betas forecast future market excess returns.

2.6.3   Robustness of equity portfolio timing regressions

Using quarterly data, we run the regression

$$r_{it}^e = \text{const}_i + \sum_{j \in \{\text{m,smb,hml,umd}\}} \beta_j r_{jt}^e + \gamma_i f(r_{mt}^e) + \varepsilon_{it}, \tag{2.24}$$

for each fund, where r_{it}^e is the excess return on the equity portion of the fund's portfolio, r_{kt}^e is the return for Fama-French-Carhart factor k, and $f(r_{mt}^e) = (r_{mt}^e)^2$ for the TM model and $f(r_{mt}^e) = \mathbf{1}\{r_{mt}^e > 0\}r_{mt}^e$ for the HM model. The fitted values, $\hat{r}_{it}^e \equiv \text{const}_i + \sum_{j \in \{\text{m,smb,hml,umd}\}} \beta_j r_{jt}^e + \gamma_i f(r_{mt}^e)$ and the residuals are saved. We next create 2000 bootstrapped panels as follows. To create a single bootstrapped panel the set of dates is randomly resampled, with replacement, and the residuals for each fund reordered accordingly. Then, the resampled residuals are merged back with the $\hat{r}_{it}$'s, producing a time-series panel of pseudo-return data for the funds' equity portfolios. The equity portfolio pseudo-returns for a given fund is considered missing if no residual is available for the resampled date. The regression in (2.24) is re-run for each replication of each fund. The bootstrapped standard error of each estimated γ_i parameter is computed using

$$\text{Std. Err.}(\gamma_i) = \frac{1}{2000 - 1} \sum_{k=1}^{2000} \left(\gamma_{ik} - \overline{\gamma_{ik}}\right)^2.$$

t-statistics are computed using the formula $t = \frac{\gamma_i}{\text{Std. Err.}(\gamma_i)}$, and are compared to ± 1.96 to assess significance. For consistency with the Jiang, Yao, and Yu (2007) replication results, we require a fund have a minimum of 8 quarters of data to qualify for inclusion in the sample. Panel A of Table 2.8 shows the results for this procedure and is analogous to Table III in Bollen and Busse (2001). The fact that the number and magnitude of negative and positive timing coefficients is roughly the same suggests that there is no serious negative bias of the sort suggested in Jagannathan and Korajczyk (1986) and Goetzmann, Ingersoll, and Ivković (2000), and found in the analysis of fund-level returns by Jiang, Yao, and Yu (2007). The fact that significant coefficients are no more frequent than might be expected is evidence against timing ability, as reflected in equity portfolio returns.

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CDA/Spectrum s12 objective

modal ioc	objective	Number of funds	
	1 International	131	
	5 Municiple Bonds	1	
	6 Bond & Preferred	1	
	8 Metals	17	
	2 Aggressive Growth	209	
	3 Growth	1218	1964
	4 Growth & Income	390	
	7 Balanced	147	
9 or NA	Unclassified or Unlisted	990	
	Total dropped (shaded)	150	

ICDI objective code	ICDI objective	Number of funds	
GB	Global Bond	1	
GE	Global Equity	8	
IE	International Equity	4	
MF	Monery Market Muni	2	
SF	Sector Fund	116	
UT	Utilities	3	
AG	Aggressive Growth	189	
BL	Balanced	8	
GI	Growth and Income	98	551
IN	Income Fund	7	
LG	Long-term Growth	244	
TR	Total Return Fund	5	
NA	Unclassified or unlisted	305	
	Total dropped (shaded)	134	

S&P objective code	S&P objective	Number of funds	
NA	NA	9	
EGG	Global Equity	5	
EGX	Global Equity	6	
EIG	International Equity	0	
FIN	Financial Sector	1	
HLT	Health Sector	9	
NTR	Natural Resources	4	
RLE	Real Estate	10	
SEC	Sector Equity	3	
TEC	Technology	4	
UTI	Utilities	3	
AGG	Aggressive growth	18	
BAL	Balanced	5	
FLX	Asset Allocation	7	
GMC	Midcap equity	31	251
GRI	Growth and Income	38	
GRO	Growth	82	
ING	Growth and Income	7	
SCG	Small-cap Equity	63	
	Total dropped (shaded)	54	

Table 2.1: This table depicts the objectives criteria we used for selecting funds for our sample. Funds were filtered sequentially depending on the availability of objectives from CDA/Spectrum s12, then ICDI, and then S&P. Funds with shaded objectives were dropped.

variable	mean	max	min	p5	p25	p50	p75	p95
$\overline{\beta}$	.98	2.06	.17	.55	.84	.96	1.13	1.42
σ_{β}	.21	1.23	.023	.081	.14	.19	.27	.40
Avg. tnam	597	28000	10	17	54	156	470	2313
# quarters	33	105	8	9	17	27	42	82
ρ_{mkt}	.79	1.0	-.26	.44	.74	.84	.92	.97
$\rho_{\sigma^2 \text{ naive}}$	-.16	.93	-.91	-.67	-.38	-.17	.049	.39
$\rho_{\sigma^2 \text{ GARCH}}$	-.084	.93	-.95	-.68	-.33	-.087	.16	.52
$\rho_{\sigma^2 \text{ vox}}$	-.16	1.0	-.97	-.70	-.40	-.16	.071	.43
$\rho_{\text{bet }\beta}$	.13	1.0	-.99	-.56	-.15	.15	.43	.75
$\overline{w}$	.88	1.1	.5	.59	.85	.92	.95	.99
σ_w	.085	.46	.0015	.017	.04	.067	.12	.21
$\hat{\beta}$	1.2	3.3	-.44	.81	1.0	1.1	1.4	1.8
$\hat{\alpha}$	-.0014	.12	-.084	-.025	-.0082	-.0012	.0056	.021
t_{α}	-.15	5.4	-7.4	-2.4	-1.1	-.17	.74	2.1
SR_{α}	-.088	2.8	-4.9	-1.1	-.4	-.063	.27	.85

Table 2.2: Summary statistics for the 2766 funds in our sample. $\overline{\beta}$ and σ_{β} correspond, respectively, to the time-series average and standard deviation of a fund's holdings-based beta. Avg. tnam refers to a fund's time series average of total net assets under management. ρ_{mkt} denotes the contemporaneous correlation between the return on a fund's domestic equity portfolio and the CRSP value weighted index returns. Each of $\rho_{\sigma^2 \text{ naive}}$, $\rho_{\sigma^2 \text{ GARCH}}$, and $\rho_{\sigma^2 \text{ vox}}$ is a contemporaneous correlation between the fund's domestic equity weight and a predictor of market variance (naive, GARCH(1,3), and the vox, respectively). $\rho_{\text{bet wts}}$ is the contemporaneous correlation between the domestic equity weights of two distinct funds. $\overline{w}$ and σ_w correspond, respectively, to a fund's time-series average and standard deviation of weight allocated to domestic equity. $\hat{\alpha}$ and $\hat{\beta}$ are the CAPM regression statistics for each fund's domestic equity returns (t_{α} is the t-statistic for the fund's alpha, while SR_{α} is the fund's α divided by the residual standard deviation).

one-month horizon				
$\sigma^2_{\mathrm{p}\,t}$	min obs	$\mathrm{Corr}(R^2_I, \mathrm{Var}[\hat{m}_i])$	p-val	# funds
	16	-0.02	0.493	1746
$\sigma^2_{\mathrm{naive}}$	24	0.04	0.198	1176
	32	0.05	0.161	759
	40	0.00	0.915	480
	16	-0.03	0.246	1746
$\sigma^2_{\mathrm{GARCH}}$	24	-0.01	0.677	1176
	32	-0.00	0.944	759
	40	-0.06	0.204	480
	16	-0.08	0.001	1698
σ^2_{vox}	24	-0.07	0.028	1127
	32	-0.09	0.015	711
	40	-0.11	0.018	426
	16	0.02	0.413	1746
$\sigma^2_{\mathrm{const}}$	24	0.03	0.369	1176
	32	-0.04	0.270	759
	40	-0.08	0.086	480
three-month horizon				
	16	0.15	0.000	1746
$\sigma^2_{\mathrm{naive}}$	24	0.20	0.000	1176
	32	0.18	0.000	759
	40	0.17	0.000	480
	16	-0.03	0.149	1746
$\sigma^2_{\mathrm{GARCH}}$	24	-0.01	0.770	1176
	32	-0.00	0.964	759
	40	-0.05	0.244	480
	16	0.07	0.003	1698
σ^2_{vox}	24	0.08	0.004	1127
	32	0.10	0.007	711
	40	0.05	0.295	426
	16	-0.08	0.000	1746
$\sigma^2_{\mathrm{const}}$	24	-0.09	0.002	1176
	32	-0.08	0.038	759
	40	-0.08	0.081	480
Continued on next page				

Table 2.3: This table reports Spearman rank correlations between the residual variance of Eq. (2.14) and the incremental R^2 from the forecasting Eq. (2.13) for a one-month and three-month forecasting horizons. The associated p-values are reported.

Table 2.3, continued.

six-month horizon				
$\sigma^2_{\mathrm{p}\,t}$	min obs	$\mathrm{Corr}(R_I^2, \mathrm{Var}[\hat{m}_i])$	p-val	# funds
	16	0.10	0.000	1746
σ^2_{naive}	24	0.15	0.000	1176
	32	0.11	0.002	759
	40	0.07	0.121	480
	16	-0.04	0.097	1746
σ^2_{GARCH}	24	-0.05	0.098	1176
	32	-0.07	0.067	759
	40	-0.11	0.019	480
	16	0.09	0.000	1698
σ^2_{vox}	24	0.07	0.017	1127
	32	0.07	0.075	711
	40	0.03	0.583	426
	16	-0.11	0.000	1746
σ^2_{const}	24	-0.14	0.000	1176
	32	-0.13	0.000	759
	40	-0.15	0.001	480
twelve-month horizon				
	16	0.16	0.000	1746
σ^2_{naive}	24	0.24	0.000	1176
	32	0.20	0.000	759
	40	0.15	0.001	480
	16	0.00	0.898	1746
σ^2_{GARCH}	24	0.02	0.514	1176
	32	0.00	0.913	759
	40	-0.05	0.305	480
	16	0.12	0.000	1698
σ^2_{vox}	24	0.14	0.000	1127
	32	0.14	0.000	711
	40	0.06	0.204	426
	16	-0.07	0.003	1746
σ^2_{const}	24	-0.06	0.039	1176
	32	-0.07	0.059	759
	40	-0.09	0.045	480

	min obs	1%	5%	10%	25%	50%	75%	90%	95%	99%
		t-stats with associated p-values for forecasting regression								

	min obs	1%	5%	10%	25%	50%	75%	90%	95%	99%
σ^2_{naive}	8	-7.502	-3.581	-2.450	-1.240	-.1518	0.9089	2.1190	3.3006	6.3357
		0.0275	0.2289	0.3993	0.6382	0.7396	0.8201	0.8816	0.9085	0.9990
	16	-4.771	-2.896	-2.101	-1.131	-.1810	0.7442	1.7067	2.4344	4.2153
		0.8721	0.8816	0.8366	0.8101	0.7786	0.7431	0.6062	0.4703	0.3253
	24	-4.118	-2.492	-1.841	-.9728	-.1347	0.7092	1.5781	2.2327	3.7641
		0.8741	0.8031	0.7431	0.6747	0.7146	0.7271	0.5977	0.4328	0.2469
	32	-3.464	-2.320	-1.749	-.9319	-.1449	0.6691	1.4425	1.8507	3.2522
		0.7506	0.7506	0.7021	0.6507	0.7221	0.7316	0.6792	0.7411	0.3443
	40	-3.260	-2.189	-1.653	-.9265	-.1472	0.5867	1.2373	1.6411	2.8819
		0.6562	0.6792	0.6387	0.6512	0.7031	0.7816	0.8341	0.8541	0.5672
σ^2_{GARCH}	8	-7.112	-3.420	-2.364	-1.073	-.0406	0.9562	2.0546	3.0701	5.7944
		0.0160	0.1894	0.3788	0.4583	0.5767	0.7236	0.8831	0.9460	0.9995
	16	-4.438	-2.757	-1.950	-.9275	-.0435	0.7977	1.6409	2.1718	3.5307
		0.8846	0.8926	0.7951	0.6082	0.5837	0.6157	0.5707	0.5957	0.5647
	24	-3.558	-2.149	-1.511	-.7388	0.0227	0.8091	1.5644	2.0473	3.0697
		0.8176	0.6477	0.4693	0.4213	0.4893	0.5137	0.4928	0.4748	0.4788
	32	-3.224	-1.871	-1.400	-.7235	0.0417	0.7922	1.4681	1.9386	2.8673
		0.8151	0.4598	0.4198	0.4438	0.4668	0.4903	0.5392	0.4973	0.4338
	40	-2.291	-1.549	-1.282	-.6782	0.0310	0.7077	1.4256	1.8508	2.7772
		0.2099	0.1964	0.3288	0.4093	0.4803	0.5757	0.5397	0.5172	0.4563
σ^2_{vox}	8	-6.200	-3.033	-2.142	-1.049	-.1115	0.7638	1.5885	2.2572	4.4383
		0.3988	0.5297	0.5912	0.6072	0.6777	0.7996	0.9260	0.9630	0.9705
	16	-4.076	-2.454	-1.881	-.9401	-.1267	0.6640	1.3248	1.8115	2.6567
		0.9290	0.8751	0.8751	0.7126	0.7166	0.7431	0.8306	0.8121	0.8981
	24	-3.089	-2.083	-1.517	-.7641	-.0635	0.6646	1.3041	1.7190	2.3788
		0.7486	0.7411	0.5992	0.5172	0.6252	0.6812	0.7316	0.7396	0.8611
	32	-3.055	-1.863	-1.396	-.6427	-.0126	0.6587	1.2451	1.6456	2.3575
		0.8621	0.5742	0.4928	0.3558	0.5292	0.6507	0.7471	0.7366	0.7101
	40	-2.417	-1.578	-1.222	-.5436	0.0795	0.6748	1.2339	1.6196	2.4245
		0.3938	0.2839	0.3023	0.2549	0.4078	0.6082	0.7111	0.7071	0.6217
σ^2_{const}	8	-6.200	-3.033	-2.142	-1.049	-.1115	0.7638	1.5885	2.2572	4.4383
		0.3988	0.5297	0.5912	0.6072	0.6777	0.7996	0.9260	0.9630	0.9705
	16	-4.076	-2.454	-1.881	-.9401	-.1267	0.6640	1.3248	1.8115	2.6567
		0.9290	0.8751	0.8751	0.7126	0.7166	0.7431	0.8306	0.8121	0.8981
	24	-3.089	-2.083	-1.517	-.7641	-.0635	0.6646	1.3041	1.7190	2.3788
		0.7486	0.7411	0.5992	0.5172	0.6252	0.6812	0.7316	0.7396	0.8611
	32	-3.055	-1.863	-1.396	-.6427	-.0126	0.6587	1.2451	1.6456	2.3575
		0.8621	0.5742	0.4928	0.3558	0.5292	0.6507	0.7471	0.7366	0.7101
	40	-2.417	-1.578	-1.222	-.5436	0.0795	0.6748	1.2339	1.6196	2.4245
		0.3938	0.2839	0.3023	0.2549	0.4078	0.6082	0.7111	0.7071	0.6217

Table 2.4: This table reports ℓ^{th} percentiles, $\Gamma(\ell)$ for $\ell = 1\%$, 5%, 10%, 25%, 50%, 75%, 90%, 95%, and 99%, of the cross-sectional distribution of t-statistics of $\hat{\zeta}_i$ in the regressions $r^e_{t+1} = \hat{\zeta}_i \sigma^2_{pt} w_{it} + \hat{\gamma}_{i1} \sigma^2_{pt} r_{it-1} + \hat{\gamma}_{i2} \sigma^2_{pt} r^2_{it-1} + \hat{\delta}_{i1} \sigma^2_{pt} f_{it} + \hat{\delta}_{i2} \sigma^2_{pt} f^2_{it} + \hat{\tau}_i \sigma^2_{pt} + \hat{\epsilon}_{it+1} + \text{const}_i$, with a 1-month forecasting horizon. Bootstrapped right-tailed p-values for a test of the null hypothesis $\hat{\zeta}_i = 0$ for all i are below each estimate. Each bootstrap sample is composed of 2000 replications.

	min obs	1%	5%	10%	25%	50%	75%	90%	95%	99%
						t-stats with associated *p*-values for forecasting regression					
σ^2_{naive}	8	-9.522	-3.952	-2.651	-1.102	0.1648	1.4646	2.8538	3.8623	7.5264
		0.1424	0.4913	0.6387	0.4623	0.2664	0.1514	0.2374	0.5867	0.9780
	16	-5.387	-2.976	-2.069	-.8801	0.2487	1.4785	2.7454	3.5284	5.8299
		0.9645	0.9265	0.8251	0.4793	0.1754	0.0220	0.0060	0.0080	0.0130
	24	-4.712	-2.676	-1.855	-.6912	0.3539	1.4547	2.5859	3.3526	5.1098
		0.9675	0.9080	0.7541	0.2884	0.0950	0.0245	0.0075	0.0065	0.0120
	32	-4.081	-2.392	-1.713	-.5588	0.3500	1.3870	2.5364	3.2066	4.7164
		0.9365	0.7861	0.6537	0.1849	0.1154	0.0390	0.0095	0.0120	0.0165
	40	-3.717	-2.161	-1.696	-.6604	0.2069	1.2738	2.2795	2.9694	3.9256
		0.8471	0.6142	0.6512	0.3378	0.2664	0.0860	0.0405	0.0340	0.1099
σ^2_{GARCH}	8	-8.491	-3.518	-2.372	-.9687	0.2044	1.3691	2.6730	3.9046	8.8164
		0.0725	0.2614	0.4313	0.3113	0.2284	0.2109	0.3503	0.5362	0.9075
	16	-4.250	-2.529	-1.775	-.7644	0.3365	1.3765	2.4873	3.4232	4.8873
		0.8446	0.7851	0.6227	0.3713	0.1114	0.0355	0.0140	0.0070	0.0385
	24	-3.633	-2.104	-1.506	-.5943	0.4096	1.3043	2.3673	2.9614	4.5827
		0.8646	0.6172	0.4628	0.2429	0.0705	0.0410	0.0105	0.0095	0.0070
	32	-3.279	-2.037	-1.398	-.4783	0.3922	1.2311	2.2332	2.7387	4.0483
		0.8421	0.6212	0.3973	0.1624	0.0895	0.0675	0.0210	0.0235	0.0165
	40	-3.004	-1.865	-1.380	-.5777	0.2969	1.0957	2.0184	2.4178	3.4382
		0.7016	0.5107	0.4343	0.3028	0.1829	0.1569	0.0580	0.0975	0.1269
σ^2_{vox}	8	-9.728	-3.484	-2.327	-.9131	0.3038	1.5233	2.9672	4.0483	7.1716
		0.2079	0.2359	0.3428	0.2114	0.1209	0.0965	0.1389	0.3918	0.9815
	16	-4.543	-2.602	-1.834	-.6864	0.4282	1.5323	2.8197	3.5813	6.1618
		0.8366	0.7251	0.5787	0.2149	0.0605	0.0220	0.0085	0.0115	0.0070
	24	-3.434	-2.268	-1.515	-.4785	0.5768	1.5202	2.5829	3.2713	5.5226
		0.6637	0.6462	0.3708	0.1054	0.0270	0.0200	0.0110	0.0100	0.0050
	32	-3.451	-2.109	-1.370	-.4109	0.5725	1.4203	2.3303	2.9711	4.0581
		0.8261	0.5992	0.2879	0.0975	0.0310	0.0410	0.0280	0.0185	0.0510
	40	-3.222	-1.821	-1.305	-.4582	0.5529	1.3778	2.2338	2.9519	3.8779
		0.7046	0.3578	0.2714	0.1434	0.0445	0.0540	0.0405	0.0175	0.0825
σ^2_{const}	8	-5.703	-2.727	-1.926	-.7804	0.2167	1.1798	2.1394	3.1220	5.9919
		0.2709	0.3178	0.3668	0.2399	0.2089	0.2584	0.4128	0.3863	0.6622
	16	-3.156	-2.109	-1.529	-.6211	0.3467	1.2142	2.0112	2.7946	4.4181
		0.5262	0.5862	0.4508	0.2339	0.0760	0.0575	0.0590	0.0160	0.0175
	24	-2.685	-1.829	-1.276	-.4908	0.4034	1.1964	1.8897	2.4796	3.8888
		0.4113	0.4248	0.2594	0.1464	0.0515	0.0485	0.0660	0.0350	0.0160
	32	-2.611	-1.784	-1.172	-.4406	0.3652	1.1525	1.7771	2.2501	3.5784
		0.5087	0.4483	0.1959	0.1244	0.0755	0.0690	0.1199	0.0975	0.0210
	40	-2.551	-1.595	-1.130	-.4682	0.2494	1.0788	1.6882	2.1098	2.9691
		0.4743	0.2714	0.1999	0.1779	0.1799	0.1139	0.1754	0.1854	0.2114

Table 2.5: This table reports ℓ^{th} percentiles, $\Gamma(\ell)$ for $\ell = 1\%$, 5%, 10%, 25%, 50%, 75%, 90%, 95%, and 99%, of the cross-sectional distribution of t-statistics of $\hat{\zeta}_i$ in the regressions $r^e_{t+1} = \hat{\zeta}_i \sigma^2_{pt} w_{it} + \hat{\gamma}_{i1} \sigma^2_{pt} r_{it-1} + \hat{\gamma}_{i2} \sigma^2_{pt} r^2_{it-1} + \hat{\delta}_{i1} \sigma^2_{pt} f_{it} + \hat{\delta}_{i2} \sigma^2_{pt} f^2_{it} + \hat{\tau}_i \sigma^2_{pt} + \hat{\epsilon}_{it+1} + \text{const}_i$, with a 3-month forecasting horizon. Bootstrapped right-tailed p-values for a test of the null hypothesis $\hat{\zeta}_i = 0$ for all i are below each estimate. Each bootstrap sample is composed of 2000 replications.

Panel A	t-stats with associated p-values for TM regression
	1%	5%	10%	25%	50%	75%	90%	95%	99%
8	-4.662	-2.661	-1.995	-.8863	0.1243	1.1031	2.2092	2.9289	5.6924
	0.0324	0.1118	0.2420	0.2141	0.2759	0.4162	0.4711	0.6846	0.7360
16	-3.669	-2.355	-1.781	-.8286	0.1109	0.9875	1.9261	2.5471	3.7780
	0.6003	0.5519	0.5544	0.3797	0.2959	0.3169	0.2580	0.2410	0.3698
24	-3.239	-2.108	-1.576	-.6920	0.1905	0.9920	1.8440	2.3498	3.3549
	0.5978	0.5005	0.4536	0.2849	0.2086	0.2335	0.1861	0.1966	0.3288
32	-2.912	-1.952	-1.439	-.5842	0.2693	0.9982	1.7817	2.2331	3.0041
	0.4651	0.4167	0.3358	0.1946	0.1432	0.2101	0.1976	0.2330	0.4491
40	-2.921	-1.770	-1.375	-.5950	0.2321	0.9015	1.6524	2.0989	2.9876
	0.5434	0.2735	0.3273	0.2430	0.1886	0.3069	0.2954	0.3164	0.3987

Panel B	t-stats with associated p-values for HM regression
8	-4.323	-2.525	-1.763	-.8962	0.1376	1.1014	2.1294	2.7054	5.1685
	0.0165	0.1043	0.1183	0.3154	0.2690	0.3733	0.5070	0.7994	0.8907
16	-3.279	-2.145	-1.644	-.8219	0.1413	1.0261	1.8897	2.4428	3.5407
	0.4566	0.4511	0.4905	0.4541	0.2520	0.2166	0.1971	0.2096	0.3548
24	-2.936	-1.895	-1.397	-.6770	0.2793	1.0513	1.8455	2.3107	3.0889
	0.5259	0.3762	0.3194	0.3278	0.1158	0.1352	0.1103	0.1362	0.3463
32	-2.824	-1.721	-1.297	-.5194	0.3305	1.0778	1.7706	2.2185	2.9021
	0.5993	0.2759	0.2720	0.1577	0.0913	0.1048	0.1317	0.1412	0.3378
40	-2.858	-1.657	-1.247	-.5110	0.3055	0.9981	1.5799	1.9427	2.8669
	0.6811	0.2839	0.2590	0.1826	0.1173	0.1732	0.2899	0.3558	0.3244

Table 2.6: Panels A and B reports percentiles, $\Gamma(\ell)$, of the cross-sectional distribution of t for the timing coefficient in the TM and HM regressions. Bootstrapped one-sided p-values for a test of the null hypothesis of $\gamma = 0$ are included below each percentile. Each bootstrap consists of 2000 replications.

	Mean	1%	5%	10%	25%	50%	75%	90%	95%	99%
1-month horizon										
γ	-.21	-4.3	-2.4	-1.7	-.76	-.11	0.44	1.12	1.71	3.21
p	0.83	0.91	0.92	0.90	0.83	0.72	0.63	0.56	0.51	0.54
t	-.19	-3.8	-2.3	-1.7	-.94	-.14	0.59	1.33	1.83	3.10
p	0.73	0.83	0.74	0.64	0.67	0.67	0.76	0.74	0.72	0.58
3-month horizon										
γ	0.08	-2.3	-1.1	-.70	-.24	0.12	0.45	0.79	1.09	2.00
p	0.24	0.87	0.73	0.62	0.33	0.12	0.10	0.20	0.25	0.27
t	0.27	-3.7	-2.1	-1.5	-.57	0.33	1.21	1.93	2.37	3.66
p	0.16	0.80	0.63	0.47	0.21	0.12	0.07	0.09	0.14	0.20
6-month horizon										
γ	0.10	-2.3	-1.1	-.69	-.20	0.15	0.45	0.81	1.07	2.04
p	0.13	1.00	0.99	0.96	0.51	0.02	0.00	0.01	0.01	0.00
t	0.40	-4.6	-2.5	-1.7	-.57	0.47	1.51	2.38	3.04	4.69
p	0.08	0.99	0.94	0.76	0.24	0.05	0.00	0.00	0.00	0.00
12-month horizon										
γ	0.06	-1.8	-.89	-.57	-.18	0.10	0.34	0.59	0.82	1.53
p	0.18	1.00	1.00	0.99	0.76	0.02	0.00	0.00	0.00	0.00
t	0.39	-5.3	-3.1	-2.0	-.72	0.46	1.58	2.77	3.71	5.68
p	0.08	1.00	1.00	0.96	0.44	0.06	0.00	0.00	0.00	0.00

Table 2.7: To generate this table we replicated the procedure used in Jiang, Yao, and Yu (2007) (see their Table 3). The table reports percentiles (and the mean) of the cross-sectional distribution of γ and t for the holdings-based Trenor-Mazuy regression along with the associated one-sided p-values given a null hypothesis that managers have no ability. The bootstrap consists of 2000 replications.

	+	-	++	- -
Fraction				
TM	0.5242	0.4758	0.0061	0.0042
HM	0.5326	0.4674	0.0076	0.0061
Mean timing coefficient				
TM	1.2298	-1.0387	1.2766	-2.1912
HM	0.5819	-0.3903	0.4035	-0.6677

Table 2.8: The first two rows of report the fraction of estimated γ_i's (timing coefficients as in Bollen and Busse, 2001) that are positive or negative (+,-) and significantly positive or negative (++/- -). The second two rows report the conditional means of the γ_i's, given that they are positive, negative, significantly positive, or significantly negative. Each bootstrap consists of 2000 replications.

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