Title
ENHANCED PLASMA. CONFINEMENT BY MHD OSCILLATIONS

Author
Cary, John R.

Publication Date
1978-09-01

Submitted to PHYSICAL REVIEW LETTERS

ENHANCED PLASMA CONFINEMENT BY MHD OSCILLATIONS

John R. Cary and James H. Hammer

September 1978

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

LBL-7993

ENHANCED PLASMA CONFINEMENT BY MHD OSCILLATIONS

John R. Cary and James H. Hammer

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

We propose using MHD oscillations to enhance plasma confinement in mirror machines. A low frequency oscillation with nodes at the mirror throats produces a ponderomotive-potential well in the machine center, trapping ions with small parallel energy. Small oscillations ($\delta B^2/4\pi n T_i \approx .1$) trap enough ions to stabilize the DCLC mode. Large oscillations ($\delta B^2/4\pi n T_i \gtrsim 1$) enhance confinement beyond classical. It is shown that plasma heating does not limit the use of MHD oscillations for enhancing confinement.

RF plugging of open-ended magnetoplasma systems has been discussed for some time. A survey of early work[1] covers the basics of the ponderomotive potential idea and its applicability to a fusion reactor. More recent work[2,3] concentrates on waves with frequency ω near the ion cyclotron frequency $\Omega_i \equiv eB/m_i c$ in order to take advantage of the amplifying denominator $(\omega^2 - \Omega_i^2)^{-1}$ in the ponderomotive potential. In these schemes the ponderomotive potential is positive (i.e. repulsive) and hence the oscillations must be localized near the ends of the magnetoplasma system.

We propose a quite different scheme. We work in the low frequency regime $(\omega \ll \Omega_i)$ in which the ponderomotive potential is negative. Thus, to confine plasma, the oscillations must be largest in the center of the machine. By performing a self-consistent calculation we show that small oscillations $(\delta B^2/4\pi n T_i \approx 0.1)$ trap enough ions to stabilize the drift-cyclotron-loss-cone (DCLC) mode, while large oscillations reduce the loss rate below the classical value. We include estimates of plasma heating, and show that (in contrast with early schemes[1]) it is not excessive. We comment briefly on other mechanisms which may limit the applicability of the scheme. Although these ideas may be applied to other open-ended devices such as Tormac,[4] here we consider the application to the axisymmetric mirror.

We first consider an oscillation in a uniform magnetized plasma: $\underset{\sim}{B}(\underset{\sim}{x},t) = \hat{z}B_0 + [\underset{\sim}{\mathcal{B}}(\underset{\sim}{x})\exp(-i\omega t) + c.c.]$ and $\underset{\sim}{E}(\underset{\sim}{x},t) = \underset{\sim}{\mathcal{E}}(\underset{\sim}{x},t)\exp(-i\omega t) + c.c.$

We take $\omega \sim c_A \partial \ln |\underset{\sim}{\mathcal{E}}| / \partial z$, typical of low frequency ($\omega \ll \Omega_i$) oscillations (c_A is the Alfven speed). If $v_{i\parallel} \equiv (T_{i\parallel}/m_i)^{1/2} \ll c_A$ is satisfied, then ions do not move appreciably in one oscillation period, validating the ponderomotive-potential concept.[5] The ion ponderomotive potential is [1,6,7]

$$\psi_i(\underset{\sim}{x}) = -\frac{m_i c^2 |\underset{\sim}{\mathcal{E}}_\perp|^2}{B_o^2} - \frac{iec(\mathcal{E}_x^*\mathcal{E}_y - \mathcal{E}_y^*\mathcal{E}_x)}{\omega B_o} + \frac{e^2|\mathcal{E}_z|^2}{m_i\omega^2} \tag{1}$$

in the low frequency ($\omega \ll \Omega_i$) limit. For $T_e/T_i \approx 0.1$ and $\beta \lesssim 0.5$ one can show from the work of references 9-12 that the last term on the right side of (1) is smaller than the first by the factor $(k_\perp\rho_i)^2(k_\parallel\rho_i)^2(\Omega_i/\omega)^2(T_e/T_i)^2 \approx 10^{-4}$ to 10^{-2} for a mode with $k_\perp\rho_i$ = 0.1 - 1.0 and $k_\parallel \sim \pi/L$ in a typical mirror reactor.[13] Hence we may neglect the last term of (1), and write $\psi_i = \Psi + e\phi_\parallel$, where

$$\Psi(\underset{\sim}{x}) = -m_i c^2 |\underset{\sim}{\mathcal{E}}_\perp|^2/B_o^2 \tag{2}$$

and

$$\phi_\parallel(\underset{\sim}{x}) = -ic(\mathcal{E}_x^*\mathcal{E}_y - \mathcal{E}_y^*\mathcal{E}_x)/(\omega B_o) \tag{3}$$

These two terms have simple interpretations. To derive the first term Ψ, we think of a magnetized ion on a magnetic field line as a bead on a string.[14,15] The Lagrangian for the constrained motion of the bead is $\mathcal{L}(x,\dot{x},t) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m(\partial y/\partial t + \dot{x}\partial y/\partial x)^2$, where $y(x,t)$ gives the displacement of the string. For an oscillation with $\omega y \gg \dot{x}\partial y/\partial x$

(analogous to $c_A >> v_{i\parallel}$) we apply the method of averaging[16] to obtain $<\mathcal{L}> = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m<(\partial y/\partial t)^2>$, which describes the average motion along x. Since $\mathcal{L}$ = T-V, we have derived the ponderomotive potential $V = -\frac{1}{2} m <(\partial y/\partial t)^2>$. For a field line, $|\partial y/\partial t| = c|\underset{\sim}{E}_\perp|/B_o$, and therefore we find V = Ψ. To derive the second term $e\phi_\parallel(\underset{\sim}{x})$, one can use Faraday's law $\nabla \times \underset{\sim}{\mathcal{E}} = i\omega\underset{\sim}{\mathcal{B}}/c$ and the approximation $|\mathcal{E}_z| << |\varepsilon_\perp|$ to show that $\phi_\parallel$ is simply the parallel-electric-field potential produced directly by the wave,[17] i.e.

$$-\partial\phi_\parallel/\partial z = <\underset{\sim}{E}\cdot\underset{\sim}{B}/|\underset{\sim}{B}|> \tag{4}$$

In addition to these potentials, a self-consistent potential $\phi_{sc}(\underset{\sim}{x})$ may arise. If we define $\Phi = \phi_\parallel + \phi_{sc}$, then the total ion potential is Ψ + eΦ. We take the ion density response to the sum of these potentials to be[18]

$$n_i(\underset{\sim}{x}) = n_o \exp[-(\Psi(\underset{\sim}{x}) + e\Phi(\underset{\sim}{x}))/T_{i\parallel}] \quad . \tag{5}$$

For electrons the ponderomotive-potential concept is invalid since $v_e \equiv (T_e/m_e)^{1/2} >> c_A$. Instead the electrons respond via the Boltzmann factor to the sum Φ of the parallel-electric-field potential and the self-consistent potential:

$$n_e(\underset{\sim}{x}) = n_o \exp[e\Phi(\underset{\sim}{x})/T_{e\parallel}]. \tag{6}$$

We now determine Φ by imposing quasineutrality, $n_e = n_i$. By using Eqs. (5) and (6) we solve for the total ion potential energy

$$\Psi + e\Phi = T_{i\parallel}\Psi/(T_{i\parallel} + T_{e\parallel}) \approx \Psi, \tag{7}$$

and the electron potential energy

$$-e\Phi = T_{e\parallel}\Psi/(T_{i\parallel} + T_{e\parallel}) \approx T_{e\parallel}\Psi/T_{i\parallel}, \tag{8}$$

where the final equality in both cases follows from $T_{e\parallel} << T_{i\parallel}$.

Now we consider the application of these ideas to an axisymmetric mirror. Since Ψ is negative, we must drive an oscillation for which the maximum amplitude is in the center of the machine, and therefore the nodes of $\underset{\sim}{\mathcal{E}}_\perp$ are near the mirror throats (see Fig. 1a). In the central part of the plasma the uniform-$\underset{\sim}{B}_o$ analysis just given is valid and so the relations (7) and (8) hold. In the mirror region the oscillation is small, so we expect the ambipolar electrostatic potential drop of the usual mirror theory to dominate.[19-23] Hence the total ion potential $\Psi + e\Phi$ increases from zero in the mirror throat to $e\Delta\Phi$ (the ambipolar potential) just inside the mirror region and then decreases by the amount $\Psi_o + e\Phi_o$, where $\Psi_o + e\Phi_o$ is the value of $\Psi + e\Phi$ in the center of the machine (see Fig. 1b). On the other hand the electron potential $-e\Phi$ (easily seen by turning Fig. 1b upside down) decreases by the amount $-e\Delta\Phi$ in going through the mirror region and further decreases by the amount $-e\Phi_o$ in going to the plasma center.

With such a potential configuration, those ions whose parallel energy $(\frac{1}{2}m_i v_{\parallel c}^2)$ in the machine center is less than the total ion potential well depth $(|\Psi_o + e\Phi_o|)$ are energetically confined and reflected before reaching the mirror throats. On the other hand, ions with $\frac{1}{2} m_i v_{\parallel c}^2 > |\Psi_o + e\phi_o|$ are confined by magnetic moment conservation if they are inside the usual hyperbolic boundary in velocity space: $\frac{1}{2} m_i v_{\parallel c}^2 < \frac{1}{2} (R-1) m_i v_{\perp c}^2 + |\Psi_o + e\phi_o| - |e\Delta\phi|$ (R is the mirror ratio). The confined region of velocity space is shown in Fig. 2.

To estimate the ambipolar potential $e\Delta\Phi$ in the presence of this oscillation, we follow the analysis of Kaufman.[19] The ion loss rate is taken to be $\dot{N}_i = -\nu_i N_o \exp(-|\Psi_o + e\Phi_o|/T_i)$, where ν_i is the ion collision frequency and N_o is the number of ions in the plasma. The electron loss rate is taken to be $\dot{N}_e = -\nu_e N_o \exp[-(|e\Phi_o| + |e\Delta\Phi|)/T_e]$. To maintain neutrality we must have $\dot{N}_e = \dot{N}_i$, which, together with Eq. (7), allows us to solve for the ambipolar potential: $|e\Delta\Phi| = T_e \ell n(\nu_i/\nu_e)$. This result is the same as that obtained[19] in the absence of oscillations.

We have shown that by driving an MHD oscillation in a mirror machine we can create a potential well $\Psi_o + e\Phi_o \approx \Psi_o$ which helps confine ions. In addition, quasineutrality implies that the electron potential well depth is increased by the amount $-e\Phi_o \approx T_e\Psi_o/T_i$ over what it would be in a mirror machine without MHD oscillations. We now discuss the particular modes of oscillation, the specific ways in which plasma confinement is affected, and some of the possible deleterious effects of this oscillation.

To minimize power supply requirements we propose the use of external coils to drive a normal mode of the mirror-machine plasma. Any mode with nodes at the mirror throats and sufficiently small heating rate will do. A possible mode is the m = 1, n = 0 (no radial nodes) oscillation. A detailed investigation[24] shows that the large magnetic field gradients in the mirror throats force the nodes of the fundamental (one half wavelength from end to end) to be within a few percent of the ends of the machine.

Since the ponderomotive potential traps low energy ions, stabilization of the drift-cyclotron-loss-cone (DCLC) mode occurs in a manner similar to stream stabilization.[25] To estimate the magnitude of ponderomotive potential needed for stabilization, we consider the DCLC stability of an ion distribution $f_i(v_\perp, v_\parallel)$ which is Maxwellian inside the confined region of velocity space (shown in Fig. 2) and which vanishes outside. The DCLC mode is an electrostatic flute mode for which the ions can be considered unmagnetized.[26] Hence DCLC instability is driven by positive slope in the projected ion distribution,

$$g(v_x) = \int dv_y dv_\parallel f_i[\,(v_y^2 + v_x^2)^{\frac{1}{2}}, v_\parallel\,]\,. \qquad (9)$$

We define the threshold value Ψ_t to be the minimum value of $|\Psi_o|$ for which g has no regions of positive slope. For $|\Psi_o| > \Psi_t$ the system is DCLC stable. Actually this is a conservative estimate, since the system can be stable even if g has some regions of positive slope.[27,28] Numerically integrating Eq. (9) with the aforementioned f_i, we find $\Psi_t/T_i \approx 0.1$ for $R = 2$ and $|e\Delta\Phi|/T_i \lesssim 0.3$. Therefore DCLC stability can be achieved with rather small oscillations, $m_i|c\mathcal{E}_\perp|^2/B_o{}^2 \approx T_i/10$. With $|\mathcal{E}_\perp| \approx c_A|\delta B|/c$, this condition is just $|\delta B|^2/4\pi n_o T_i \approx 0.1$. (In fact, considering the analyses of references 27 and 28, one may be able to achieve DCLC stability by injecting very cold plasma into a mirror machine with a very small ponderomotive well.) We note that this stabilization scheme differs from stream stabilization in that it causes much less plasma power loss, since the stabilizing plasma is confined for a collision time. Thus the electron temperature will be higher than in stream stabilization, and the ion power drain due to electron drag will be diminished.[29,30]

If $|\Psi_o|/T_i$ is increased beyond unity, the end loss of ions may be significantly reduced. To estimate the effect we use the following approximate formula for the ion energy loss rate P_{iL}:

$$P_{iL} = \nu_i N_o (T_i + |\Psi_o|) e^{-|\Psi_o|/T_i} . \qquad (10)$$

This formula assumes that electron drag is negligible, as one would expect to be true in a mirror reactor without stream stabilization. In calculating Q (the ratio of fusion power to the circulating power) we can simply use the ion energy loss rate P_{iL} for the circulating power. The fusion power should be the same with or without oscillations. Thus Q in the oscillating mirror is enhanced by the factor q over the classical mirror, where

$$q \equiv (1+|\Psi_o|/T_i)^{-1} \exp\,(|\Psi_o|/T_i). \qquad (11)$$

For example, if $|\Psi_o|/T_i = 2$ then $q = 3.7$. Of course, at this large value of q the ponderomotive potential, not the mirror effect, is producing confinement; the mirrors only serve to define the normal mode of oscillation.

Finally, we ask what might limit this scheme. Previous ponderomotive confinement schemes have been plagued by excessive plasma heating,[31] i.e. the plasma absorbs wave energy much faster than fusion power is generated. To analyze this aspect we consider the ion-temperature evolution equation (consideration of the electron-temperature evolution equation gives similar results[24]):

$$N_o \frac{dT_i}{dt} = 2\gamma_i W - P_{iL} + P. \tag{12}$$

The first term on the right is the rate at which the wave heats ions, $W \equiv 2\int d^3x < |\delta \underset{\sim}{B}|^2 >/8\pi$ is the wave-energy, and γ_i is the linear damping rate of the wave due to ions. The second term is the ion energy loss rate [c.f. Eq. (10)]. The last term contains any additional power sources such as neutral beams.

In steady state operation $dT_i/dt = 0$. At the maximum allowable amplitude the oscillation provides all the plasma heating, i.e. $P = 0$. Of course, as long as the mirror machine is a power amplifier ($Q>1$), the wave heating power is less than the fusion power. Using the approximation $|\underset{\sim}{\mathcal{E}}_\perp| \approx c_A \underset{\sim}{\mathcal{B}}/c$ we find $W \approx 2N_o|\Psi_o|$, which we substitute into (12) together with P_{iL} from Eq. (10) and $P = dT_i/dt = 0$. This determines Ψ_{max}, the maximum allowable value of Ψ_o:

$$(\Psi_{max} + T_i)^{-1}\, \Psi_{max} \exp(\Psi_{max}/T_i) = \nu_i/2\gamma_i. \tag{13}$$

We proceed to estimate the damping rate by reference to the work of Hasegawa and Chen,[10,11] and Ott, Wehrsinger, and Bonoli.[12] We infer that the plasma response to an antenna oscillating in the Alfven wave frequency range is a sum of a bulk oscillation and a short perpendicular wavelength mode, i.e. $E = E_b + E_k$, where $\partial E_b/\partial r = O(E_b/a)$ and $\partial E_k/\partial r = O(E_k/\rho_i)$. In a uniform plasma E_b and E_k correspond to two different waves, a long perpendicular wavelength ($k_\perp \rho_i << 1$) compressional wave and a short perpendicular wavelength ($k_\perp \rho_i \sim 1$) kinetic Alfven wave.[10,11] The first wave damps due to parallel viscosity[24,32] at a rate $\gamma_i \approx \nu_i \beta/6$. The second wave damps because of strong perpendicular shear viscosity[10,11] at a rate $\gamma_i \sim \nu_i k_\perp^2 \rho_i^2 \sim \nu_i$. In nonuniform plasma theory E_b and E_k are coupled by the density gradient. The relative amount of short wavelength energy to long wavelength energy will vary depending on the system under consideration. However, knowing the uniform medium damping rates we can at least give estimates for the upper and lower bounds: $\nu_i \beta/6 \lesssim \gamma_i \lesssim \nu_i$. Taking $\beta = 1/3$ and using Eq. (12) these bounds give

$$0.5 \lesssim \Psi_{max}/T_i \lesssim 2.6. \tag{14}$$

We must also set an upper limit $|\underset{\sim}{B}|^2/B_o^2 \lesssim 1/2$. At this amplitude the sideways displacement of the plasma is comparable to its length. Practical considerations (e.g. vacuum chamber size) will probably dictate smaller oscillations. Again we take $|\underset{\sim}{\mathcal{E}}_\perp| \approx c_A|\underset{\sim}{B}|/c$, and we find this upper limit to be $\Psi_{max}/T_i \approx \beta^{-1}$. This upper limit is therefore more stringent than the upper bound in Eq.(13) for high β ($\gtrsim .4$) machines.

Other mechanisms, such as nonlinear heating via wave decay or induced scattering,[33] may also set upper limits on the allowable oscillation amplitude. However, we leave further study of this nonlinear nonuniform oscillating plasma for later research.

In summary, we have shown how one can energetically confine low energy ions using the ponderomotive potential of a driven low frequency bulk mode. The benefits of this scheme are shown graphically in figure 3, a plot of the Q enhancement factor q [from Eq. (11)] versus $|\Psi_o|/T_i$. At low oscillation amplitudes one can confine enough ions to effect DCLC stabilization. At larger amplitudes one can enhance q. The maximum amplitude Ψ_{max} obtainable has upper bounds due to plasma heating and excessively large plasma motion. Present estimates indicate that Ψ_{max} is at least large enough to permit DCLC stabilization. Whether Ψ_{max} is large enough for Q enhancement depends on the detailed normal mode properties.

The authors would like to acknowledge helpful discussions with Drs. A.N. Kaufman, W.B. Kunkel, A.M. Sessler, and H.L. Berk.

Work done under the auspices of the U.S. Department of Energy.

REFERENCES

1. H. Motz and C. Watson, Adv. Electronics & Electron Phys. 23, 153 (1967).

2. T. Watari, S. Hiroe, T. Sato, and S. Ichimaru, Phys. Fluids 17, 2107 (1974).

3. S. Miyake et al, J. Phys. Soc. Jap. 31, 265 (1971).

4. C. Gallagher, L. Combes, and M. Levine, Phys. Fluids 13, 1617 (1970).

5. Although $v_i^2/c_A^2 = \beta/2$, the condition $v_{i\parallel}/c_A \ll 1$ does not preclude $\beta \sim 1$, since the energy of mirror confined ions is mainly perpendicular.

6. J. Cary and A. Kaufman, Phys. Rev. Lett. 39, 402 (1977).

7. V. Krapchev and A. Bers, paper G5-1 in Proc. 3rd Top. Conf. RF Plasma Heating (1978).

8. We note that $\omega \sim c_A/L \ll \Omega_i$ holds for a typical mirror reactor[13] of length L.

9. L. Chen and A. Hasegawa, Phys. Fluids 17, 1399 (1974).

10. A. Hasegawa and L. Chen, Phys. Rev. Lett. 32, 454 (1974).

11. A. Hasegawa and L. Chen, Phys. Fluids 19, 1924 (1976).

12. E. Ott, J. Wersinger, and P. Bonoli, Cornell Univ. Lab. Plasma Studies Rep. 236 (1978).

13. F. Coensgen, Lawrence Livermore Lab. rep. LLL-Prop.-142.

14. T. Northrop, The Adiabatic Motion of Charged Particles (John Wiley and Sons, New York, 1963) pp. 20-23.

15. H. Grad, Phys. Fluids 9, 225 (1966).

16. A. Nayfeh, Perturbation Methods (John Wiley and Sons, New York, 1973) ch. 5.

17. J. Cary and J. Hammer, Lawrence Berkeley Lab. rep. LBL-7992 (1978), submitted to Phys. Fluids.

18. Strictly speaking, the concept of (parallel) temperature does not apply to ions in a mirror machine. Nevertheless, the response (5) is at least qualitatively correct; the ion density is small where the potential is more positive, and large where the potential is more negative.

19. A. Kaufman, Conf. on Cont. Thermonuclear Reactions (1956), AEC rep. TID-7520 (pt. 2) p. 387.

20. D. BenDaniel, Plasma Phys. 3, 235 (1961).

21. J. Guillory and W. Kunkel, Plasma Phys. 12, 529 (1970).

22. V. Pastukhov, Nucl. Fusion 14, 3 (1974).

23. R. Cohen, M. Rensink, T. Cutler, and A. Mirin, Nucl. Fusion, to be published (1978).

24. J. Cary and J. Hammer, in preparation.

25. F. Coensgen et al, Plasma Phys. and Cont. Nucl. Fus. Res. (Proc. 4th Int. Conf., Madison, 1971) 2, IAEA, Vienna, 735 (1971).

26. R. Post and M. Rosenbluth, Phys. Fluids 9, 730 (1966).

27. M. Gerver, Phys. Fluids 19, 1646 (1976).

28. H. Berk and M. Gerver, Phys. Fluids 19, 1646 (1976).

29. D. Baldwin, Rev. Mod. Phys. 49, 317 (1977).

30. D. Baldwin et al, 7th Int. Conf. on Plasma Phys. and Cont. Nucl. Fus., Austria (1978) to be published.

31. See ref. 1, p. 238.

32. A study of normal modes in a low-β plasma slab[24] shows that the fundamental has even lower damping.

33. A. Hasegawa and L. Chen, Phys. Rev. Lett. 36, 1362 (1976).

FIGURE CAPTIONS

Fig. 1. (a) Sketch showing how the oscillating field line (dashed lines) deviate from their equilibrium position (solid lines) in the presence of an MHD oscillation. (b) Plot of the electrostatic potential in the absence of oscillations (solid line), the electrostatic potential in the presence of oscillations (--), and the effective ion potential $\Psi_o + e\Phi_o$ (-•-).

Fig. 2. Plot of ion velocity space in the center of the plasma showing the regions which are: unconfined (clear), confined by magnetic moment conservation (shaded), and confined by the ponderomotive potential (stippled).

Fig. 3. Plot of Q-enhancement factor q versus ponderomotive-potential-well depth $|\Psi_o|/T_i$ showing where DCLC stabilization occurs and where various effects may limit this scheme.

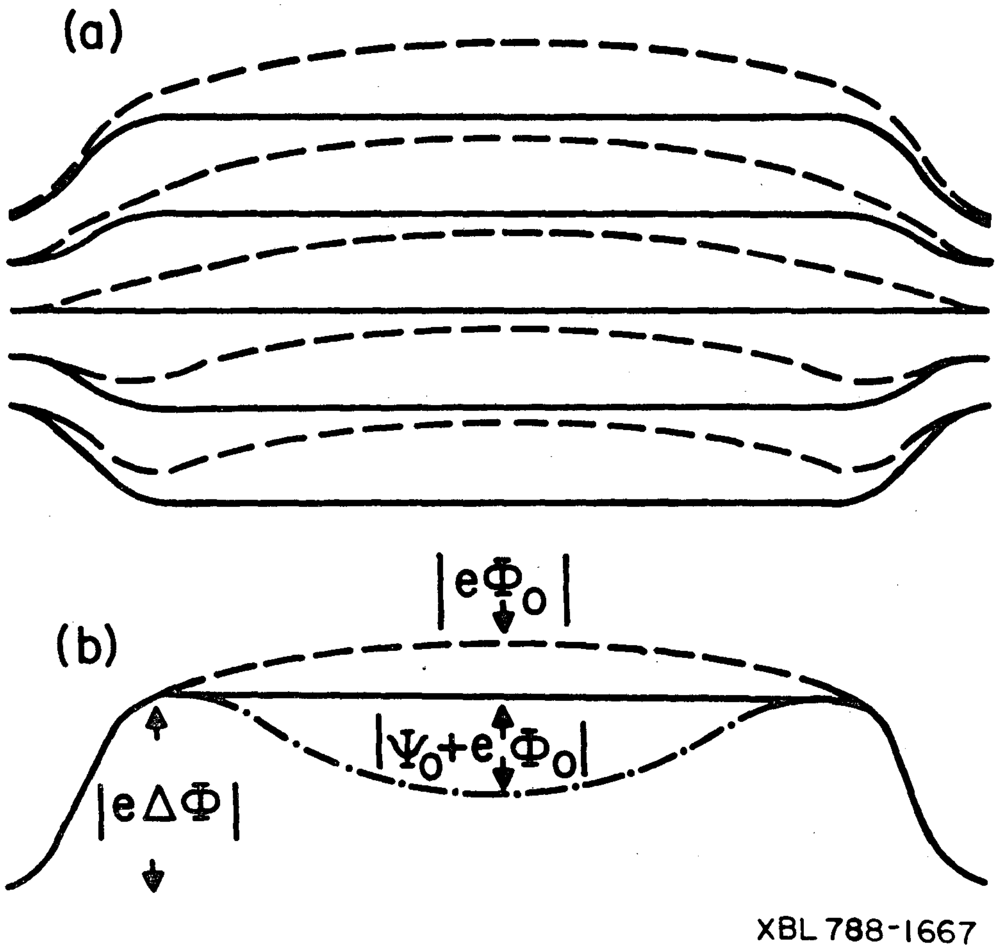


Fig. 1

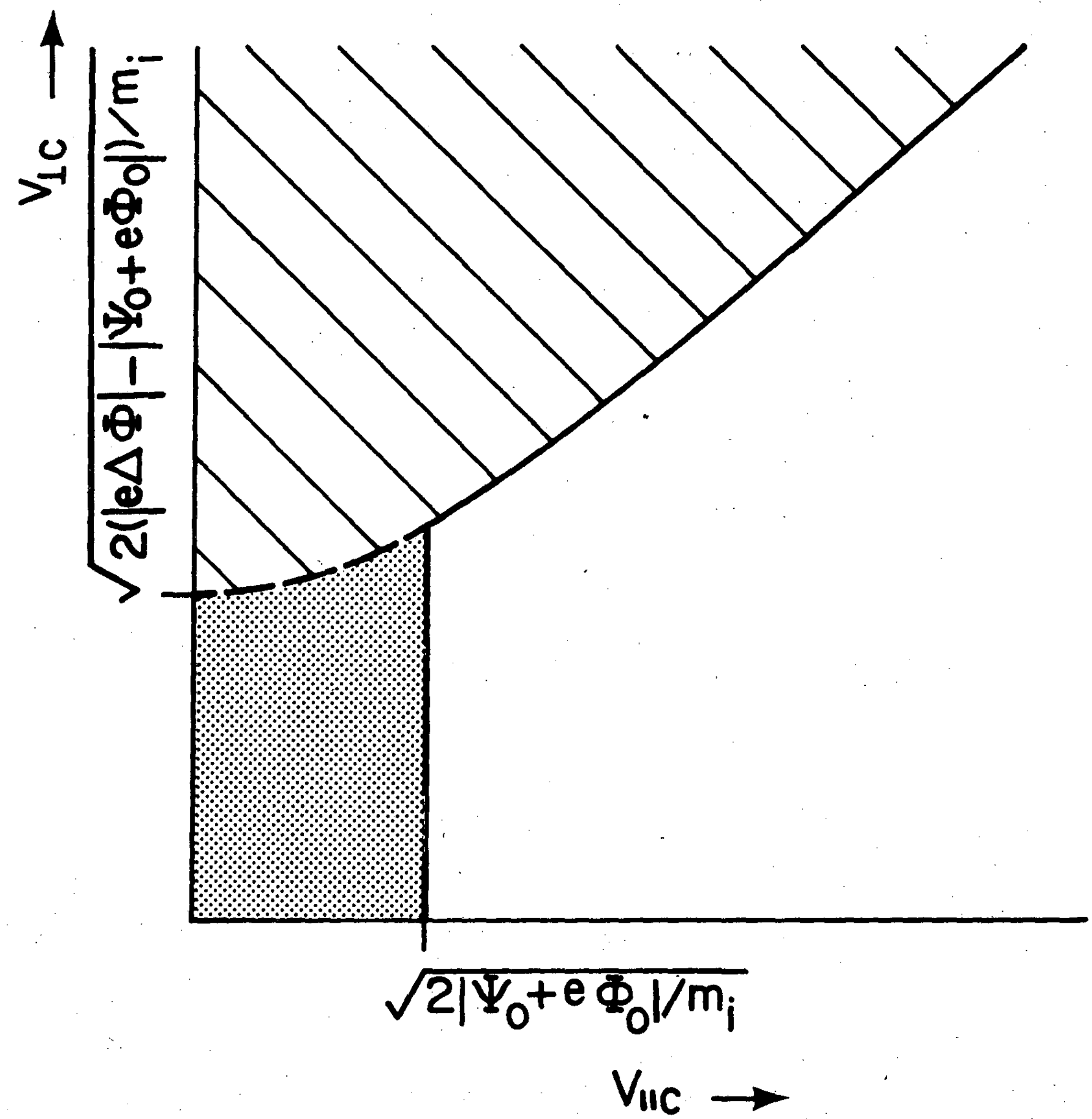
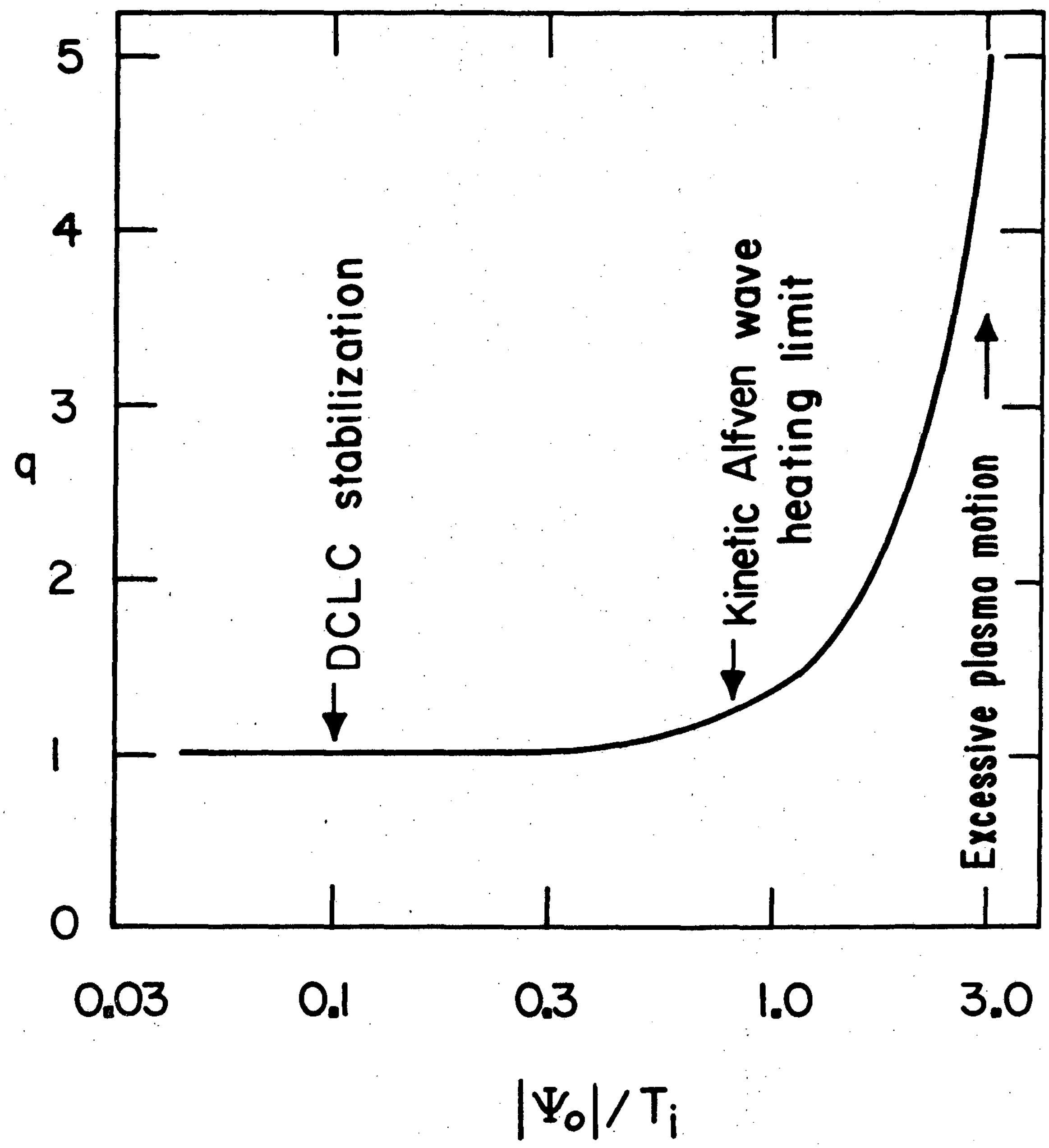


Fig. 2



XBL 788-1669

Fig. 3

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720