Three-body decays of sleptons with general flavor violation and left-right mixing

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(Received 13 April 2009; published 22 June 2009)

We determine the widths of three-body decays of sleptons, $\tilde{\ell}^- \to \tilde{\ell}^{\pm}\ell^-\ell^{\mp}$, $\tilde{\ell}^- \nu\bar{\nu}$, $\tilde{\ell}^- q\bar{q}$, in the presence of arbitrary slepton flavor violation and left-right mixing. These decays are important in scenarios in which the lightest supersymmetric particle is the gravitino, a generic possibility in models with gauge- and gravity-mediated supersymmetry breaking. Three-body decays have been discussed previously, assuming flavor conservation and left-right mixing in only the stau sector. Flavor violation and general left-right mixing open up many new decay channels, which provide new avenues for precision mass measurements and may play an essential role in solving the standard model flavor problem. We present results for toy models with two-generation mixing, and discuss the implementation of these results in SPICE, a program that simplifies collider event simulations of flavor-violating supersymmetric models.

DOI: 10.1103/PhysRevD.79.116009 PACS numbers: 11.30.Hv, 12.15.Ff, 12.60.Jv, 14.60.Pq

I. INTRODUCTION

Fermion masses are one of the least understood parts of the standard model (SM). Even the charged fermion masses span over 5 orders of magnitude from the top quark to the electron, begging for a theoretical explanation. The majority of these masses and mixing angles are precisely constrained by experiment. Still, this wealth of data does not conclusively single out any theory of flavor.

In the near future, this may change dramatically with the discovery of new particles at the weak scale. New particles may only deepen the mystery, as would be the case if a fourth generation were discovered. On the other hand, the masses and mixings of the new particles may be governed by the same principles that determine the SM fermion masses. In this case, rather than extending the fermion sector, the new particles will shed light on the existing fermion spectrum.

Weak-scale supersymmetry provides examples of both possibilities. In pure gauge-mediated models, for example, squark and slepton masses are set by flavor-blind contributions, with no connection to the SM fermion masses. However, in gravity-mediated models and hybrid models with both gauge- and gravity-mediated contributions, squark and slepton masses may receive contributions that are governed by flavor symmetries that also determine the SM fermion masses [1–3]. The latter possibility leads to nontrivial flavor effects in high-energy experiments [4–6], opening the possibility for real progress on the SM flavor problem at the Large Hadron Collider, as has been emphasized recently by many authors (see, for example, Refs. [7–16]).

Here we study the implications of flavor violation for the three-body decays of charged sleptons. Such processes are most relevant for colliders in models with a gravitino lightest supersymmetric particle (LSP) and a slepton next-to-lightest supersymmetric particle (NLSP), a generic possibility in models with both gauge-mediated supersymmetry breaking [17,18] and gravity-mediated supersymmetry breaking [19–23]. We consider a situation where the lightest particles are a gravitino LSP, several light sleptons, and the lightest neutralino, with the masses of each light slepton less than the mass of the lightest neutralino. This results in the typical two-body decays of these light sleptons being highly suppressed or kinematically inaccessible, leaving the three-body decays as the dominant decay modes. In these cases, the three-body decays are also often the last visible step in cascade decays of squarks and gluinos, and so they impact nearly all supersymmetry searches and studies.

Three-body slepton decays have been studied previously in an important, flavor-conserving case, where the authors considered $\tilde{e}_R \to e\tau\tilde{\tau}_1$ and $\tilde{\mu}_R \to \mu\tau\tilde{\tau}_1$, with the $\tilde{\tau}_1$ a mixture of left- and right-handed staus [24]. These decays are characterized by two distinct channels: a "charge-preserving" channel $\tilde{\ell}^- \to \tilde{\ell}^-\ell^-\ell^+$ with opposite-sign leptons and a "charge-flipping" channel $\tilde{\ell}^- \to \tilde{\ell}^+\ell^-\ell^-$ with same-sign leptons. Flavor-conserving three-body decays of squarks have also been considered [25], as have flavor-conserving three-body decays with sneutrinos as parent or daughter particles [26]. Our work generalizes the charged slepton analysis to the case of arbitrary lepton flavor violation (LFV) and arbitrary left-right mixing. In the presence of general LFV, any three-body decay of $\tilde{\ell}_i \to \tilde{\ell}_j$ has up to 9 possible charge-preserving modes $\tilde{\ell}_i^- \to \tilde{\ell}_j^-\ell_k^-\ell_m^+$ and 6 possible charge-flipping modes $\tilde{\ell}_i^- \to \tilde{\ell}_j^+\ell_k^-\ell_m^-$, where ℓ_k, $\ell_m = e$, μ, τ. In addition, LFV and left-right mixing bring additional complications that are absent in the flavor-conserving case, including new processes mediated by Higgs and Z bosons, new final states with neutrinos and quarks, and new interference effects in charge-flipping processes.

These complications are well worth confronting, however, as there is a wealth of information in these branching ratios, which may shed light on the SM flavor problem. These decays, if present, are also relevant more broadly, for example, for supersymmetric searches and precision mass measurements. For example, in the flavor-conserving examples studied previously [24], the final states necessarily contain τ leptons. Since these decay with missing energy, they degrade searches based on energetic leptons, and they greatly reduce the prospects for precision mass measurements. With LFV, however, even if the lightest slepton is a stau, there may also be decay modes with two electrons, two muons, or an electron and a muon. Even if these branching ratios are suppressed, they may be the more obvious signals at colliders, and they may also provide better opportunities for precision mass measurements. It is therefore of interest to know the size of these branching ratios, and what determines them.

In the following three sections, we begin with a general discussion of three-body decays and move gradually to more specific scenarios and concrete calculations. In Sec. II we present the new final states and new Feynman diagrams that are relevant to three-body decays once general flavor and left-right mixing are introduced. In Sec. III we discuss these results in more detail and explain the relative phenomenological importance of the various contributing diagrams in particular scenarios. In Sec. IV we then show concrete results in two toy models with two-generation slepton mixing to illustrate our results. Finally, in Sec. V, we present our conclusions and explain how our results have been incorporated into SPICE, a publicly available computation package for generating supersymmetric spectra and branching ratios in scenarios with arbitrary slepton mixing.

We stress that, although we strive to give readers an intuitive feel for our results by considering concrete cases in the body of the paper, our analysis is valid for fully general LFV and left-right mixing. The complete, model-independent calculation is lengthy, but the full expressions for all three-body decay modes are given in a series of appendixes. Our conventions and notations are defined in Appendix A. These are consistent with those of Ref. [27], where full details may be found.

II. FLAVOR VIOLATION IN THREE-BODY SLEPTON DECAYS

As mentioned in the Introduction, an important special case of three-body slepton decays has been discussed previously by Ambrosanio, Kribs, and Martin [24]. Motivated by pure gauge-mediated models, these authors considered the flavor-conserving decays $\tilde{e}_R \to e\tau\tilde{\tau}_1$ and $\tilde{\mu}_R \to \mu\tau\tilde{\tau}_1$. These decays are mediated solely by neutralinos. The charge-preserving modes $\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+$ are shown in Fig. 1, and the charge-flipping modes $\tilde{\ell}_i^- \to \tilde{\ell}_j^+ \ell_k^- \ell_m^-$ are shown in Fig. 2. The charge-flipping mode is made possible by the Majorana nature of the neutralino, and detection of the charge-flipping mode would provide strong evidence that neutralinos are Majorana fermions.

In the presence of more general slepton mixing, the gauge eigenstates $\tilde{e}_R$, $\tilde{\mu}_R$, $\tilde{\tau}_R$, $\tilde{e}_L$, $\tilde{\mu}_L$, and $\tilde{\tau}_L$ mix to form six mass eigenstates, $\tilde{\ell}_i$, $i = 1, \ldots, 6$, with increasing mass, and the lepton-slepton-neutralino interactions are no longer flavor diagonal. The neutralino diagrams of Figs. 1 and 2 are then modified by the inclusion of 6×6 mixing matrix factors at the interaction vertices.

In addition, new diagrams contribute. If the initial and final state charged sleptons contain left-handed components, there is the chargino-mediated decay to neutrinos shown in Fig. 1. This decay requires neither flavor violation nor left-right mixing. There are also charge-preserving decays mediated by neutral Higgs bosons and the Z boson, as shown in Fig. 3. The Higgs bosons mediate decays to same-flavor $\ell^+\ell^-$ and $q\bar{q}$ pairs, and the Z diagram mediates decays to same-flavor $\nu\bar{\nu}$, $\ell^+\ell^-$, and $q\bar{q}$ pairs.

Both the H_a and Z diagrams are present independent of LFV, but both require left-right mixing. In the case of the Higgs-mediated diagrams, the Higgs couplings to sleptons have the form $H_a\tilde{\ell}_L^*\tilde{\ell}_R$, $H_a\tilde{\ell}_L^*\tilde{\ell}_L$, and $H_a\tilde{\ell}_R^*\tilde{\ell}_R$. The first is generated by left-right mixing terms in the Lagrangian, and thus clearly requires left-right mixing to be nonzero. The other two come from D terms, and in the absence of left-right mixing, they are flavor diagonal in the separate LL and RR sectors, even with the presence of LFV in one or both. Thus, in the absence of left-right mixing, the Higgs coupling to sleptons is flavor diagonal in the six-

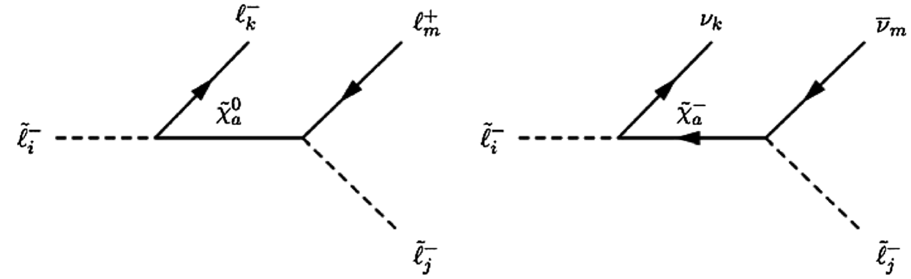

FIG. 1. Feynman diagrams for charge-preserving decays $\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+$ mediated by neutralinos and $\tilde{\ell}_i^- \to \tilde{\ell}_j^- \nu_k \bar{\nu}_m$ mediated by charginos.

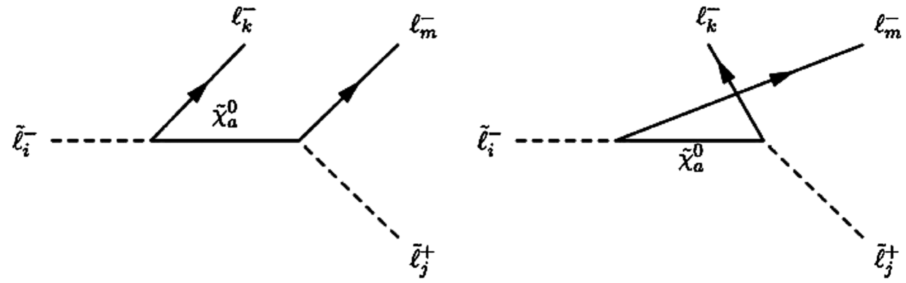

FIG. 2. Feynman diagrams for the charge-flipping decays $\tilde{\ell}_i^- \to \tilde{\ell}_j^+ \ell_k^- \ell_m^-$ mediated by neutralinos.

dimensional space of left and right sleptons and does not contribute to three-body decays. Similarly, the Z couples to the LL and RR sleptons separately, so in the absence of left-right mixing, its couplings are also flavor diagonal. Note, however, there are no photon-mediated diagrams at tree level, as the photon couples with equal strength to the left- and right-handed sleptons, and so its couplings are always flavor diagonal. Also note that in the case of nonzero left-right mixing but no flavor mixing, the H_a and Z diagrams exist, but their contributions are only appreciable for certain mass orderings. Namely, only when neither of the two mass eigenstates produced by left-right mixing possess kinematically allowed two-body decays will the three-body H_a and Z contributions be relevant.

The charge-flipping decay is unaffected by the Higgs and Z boson diagrams, but in the presence of flavor violation, the fermions in the final state may be interchanged. This implies a new interference effect in the charge-flipping mode which is absent in the flavor-conserving case. In principle, there should also be a similar interference term in the neutrino decay modes if neutrinos possess Majorana masses, but such a term is negligible in the limit of zero neutrino mass.

In summary, for charge-preserving decays to charged leptons of different generations, the analysis of Ref. [24] may be straightforwardly modified to the general case. The only modification needed is the insertion of rotation matrices from flavor eigenstates to gauge interaction eigenstates. For charge-preserving decays to charged leptons of the same generation, however, the Higgs and Z diagrams introduce new contributions to the decay width. There are also new charge-preserving decays to neutrinos, mediated by charginos and Z bosons, and to quarks, mediated by Higgs and Z bosons. Finally, in the case of the charge-flipping decays to like-sign leptons, the daughter leptons may be interchanged and thus the decay width must include an interference term between these two diagrams; however, there are no Higgs and Z contributions to this mode and so there are no further interference terms.

III. ANALYTIC RESULTS AND QUALITATIVE OBSERVATIONS

Given an understanding of the qualitatively new features introduced by flavor and left-right mixing described in Sec. II, we can now calculate the resulting decay widths. The full results are lengthy and are given in the appendixes. In this section, we summarize the results of our calculations and provide qualitative insight into the relative importance of each contributing mode.

Our slepton interaction Lagrangian is

$$\begin{aligned}\mathcal{L}_{\tilde{\ell}}^{\text{int}} = & [\tilde{\ell}_i^* \bar{\tilde{\chi}}_a^0 (\beta_{aik}^{(1)} P_L + \beta_{aik}^{(2)} P_R) \ell_k + \gamma_{aik} \tilde{\ell}_i^* \bar{\tilde{\chi}}_a P_L \nu_k + \text{H.c.}] \\ & + i\zeta_{ij}^{(2)} (\tilde{\ell}_i^* \partial_\mu \tilde{\ell}_j - \tilde{\ell}_j \partial_\mu \tilde{\ell}_i^*) Z^\mu + \sigma_{ij}^{(2)} \tilde{\ell}_i^* \tilde{\ell}_j h^0 \\ & + \sigma_{ij}^{(3)} \tilde{\ell}_i^* \tilde{\ell}_j H^0 + i\sigma_{ij}^{(4)} \tilde{\ell}_i^* \tilde{\ell}_j A^0 + i\sigma_{ij}^{(5)} \tilde{\ell}_i^* \tilde{\ell}_j G^0, \qquad (1)\end{aligned}$$

where the coefficients β, γ, ζ, and σ contain gauge and Yukawa couplings along with the mixing matrix elements for sleptons, Higgs bosons, neutralinos, and charginos. These coefficients are defined in Appendix A. Here, the indices are summed over the six slepton, three lepton and neutrino, four neutralino, and two chargino mass eigenstates. We have kept only the terms relevant to the three-body decays in question.

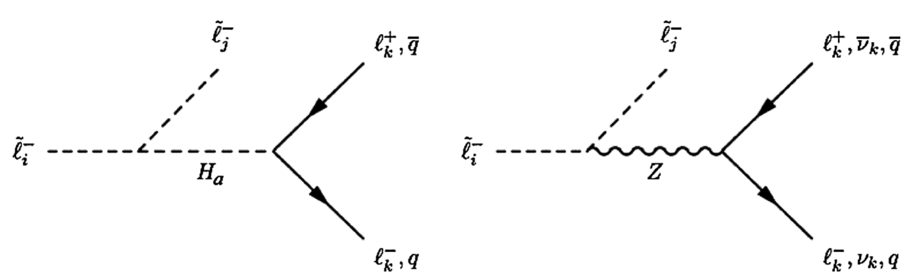

FIG. 3. Feynman diagrams for charge-preserving decays mediated by Higgs and Z bosons. The Higgs scalar H_a may be any of the neutral Higgs bosons: h^0, H^0, A^0, or (in Feynman gauge) G^0.

Our lepton interaction Lagrangian is

$$\mathcal{L}_{\ell}^{\rm int} = \frac{y_k^{(\ell)}}{\sqrt{2}} \bar{\ell}_k \bar{\ell}_k (h^0 \sin\theta_H - H^0 \cos\theta_H + iA^0 \gamma^5 \sin\beta - iG^0 \gamma^5 \cos\beta) \ell_k + \frac{ig}{2\cos\theta_W} \times Z_\mu (1 - 2\sin^2\theta_W) \bar{\ell}_k \gamma^\mu \ell_k. \quad (2)$$

The amplitudes for charge-preserving decays to charged leptons $\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+$ are

$$\mathcal{M}_{\tilde{\chi}_a^0} = -i\bar{u}(p_k)(\beta_{aik}^{1*} P_R + \beta_{aik}^{2*} P_L) \frac{(\not{p}_k - \not{p}_i) + m_{\tilde{\chi}_a^0}}{(p_k - p_i)^2 - m_{\tilde{\chi}_a^0}^2} \times (\beta_{ajm}^1 P_L + \beta_{ajm}^2 P_R) v(p_m), \quad (3)$$

$$\mathcal{M}_Z = \frac{ig\zeta_{ji}^{(2)}}{2\cos\theta_W} \frac{1}{(p_i - p_j)^2 - m_Z^2} \bar{u}(p_k)(\not{p}_i + \not{p}_j) \times (2\sin^2\theta_W - P_L) v(p_m) \delta_{km}, \quad (4)$$

$$\mathcal{M}_{h^0} = \frac{-i\sigma_{ji}^{(2)} y_k^{(\ell)} \sin\theta_H}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{h^0}^2} \bar{u}(p_k) v(p_m) \delta_{km}, \quad (5)$$

$$\mathcal{M}_{H^0} = \frac{i\sigma_{ji}^{(3)} y_k^{(\ell)} \cos\theta_H}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{H^0}^2} \bar{u}(p_k) v(p_m) \delta_{km}, \quad (6)$$

$$\mathcal{M}_{A^0} = \frac{-i\sigma_{ji}^{(4)} y_k^{(\ell)} \sin\beta}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{A^0}^2} \bar{u}(p_k) \gamma^5 v(p_m) \delta_{km}, \quad (7)$$

$$\mathcal{M}_{G^0} = \frac{i\sigma_{ji}^{(5)} y_k^{(\ell)} \cos\beta}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_Z^2} \bar{u}(p_k) \gamma^5 v(p_m) \delta_{km}, \quad (8)$$

where the indices i, j, k, and m correspond to the subscripts specifying the mass eigenstates of initial and final state particles in the decay mode $\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+$, and δ_{km} is the Kronecker delta function. Here and throughout the rest of this section we suppress these indices on matrix elements and decay widths.

The overall decay width is given by

$$\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+) = \frac{1}{64\pi^3 m_i} \int_{\rm PS_3} \sum_{\rm spins} |\mathcal{M}|^2 \quad (9)$$

$$= \frac{1}{64\pi^3 m_i} \int_{\rm PS_3} \sum_{\rm spins} \bigg(\sum_{a,b=1}^{4} \mathcal{M}_{\tilde{\chi}_a^0} \mathcal{M}_{\tilde{\chi}_b^0}^* + \mathcal{M}_Z \mathcal{M}_Z^* + \sum_{a,b} \mathcal{M}_{H_a} \mathcal{M}_{H_b}^* + 2\,{\rm Re}\bigg[\sum_{b=1}^{4} \mathcal{M}_Z \mathcal{M}_{\tilde{\chi}_b^0}^* + \sum_a \sum_{b=1}^{4} \mathcal{M}_{H_a} \mathcal{M}_{\tilde{\chi}_b^0}^* + \sum_b \mathcal{M}_Z \mathcal{M}_{H_b}^* \bigg] \bigg) \quad (10)$$

$$= \Gamma_{\tilde{\chi}^0 \tilde{\chi}^0} + \delta_{km} (\Gamma_{ZZ} + \Gamma_{HH} + 2\Gamma_{Z\tilde{\chi}^0} + 2\Gamma_{H\tilde{\chi}^0} + 2\Gamma_{ZH}), \quad (11)$$

where $\int_{\rm PS_3}$ is the integral over three-body phase space discussed in Appendix B which includes a sum over fermion spins, and we use an intuitively obvious notation for widths, so that, for example, $\Gamma_{\tilde{\chi}^0 \tilde{\chi}^0}$ is the partial width from $\sum_{a,b} \mathcal{M}_{\tilde{\chi}_a^0} \mathcal{M}_{\tilde{\chi}_b^0}^*$.

To develop a qualitative understanding of which matrix elements are typically dominant and subdominant, we first note that there is a suppression to the Higgs and Z modes from left-right slepton mixing. In fact, the explicit Higgs and Z couplings to sleptons depend on the left-right slepton mixing squared,

$$\sigma_{ij}^{(a)}, \zeta_{ij}^{(2)} \sim \frac{m_{LR}^2}{m_{\tilde{\ell}}^2}. \quad (12)$$

In particular, for μ not significantly larger than the slepton mass, this is roughly the same order as Yukawa suppression. We thus expect that, at the matrix element level, the Higgs or Z radiated modes are generically suppressed by two powers of left-right mixing relative to the neutralino and chargino modes.

Second, to simplify phase space factors, we assume a typical amount of mass squared splitting $\Delta m_{\tilde{\ell}}^2 / m_{\tilde{\ell}}^2 \lesssim 5\%$; we also neglect lepton masses when they are subdominant. Under these assumptions, the difference in propagator structure between the neutralino mode and that of the Higgs and Z modes becomes apparent. The neutralino propagator is inversely proportional to $(p_k - p_i)^2 - m_{\tilde{\chi}_a^0}^2 \approx m_{\tilde{\ell}_i}^2 - m_{\tilde{\chi}_a^0}^2$, while the Higgs ($Z$) propagator is inversely proportional to $(p_i - p_j)^2 - m_{H_a,Z}^2 \approx m_{\tilde{\ell}_i}^2 - m_{\tilde{\ell}_j}^2 - m_{H_a,Z}^2$, which simply reduces to $-m_{H_a,Z}^2$ if the slepton masses are not too far above m_Z. Thus, in models where the lightest neutralino and light slepton masses are close, the lightest neutralino pole contribution will be enhanced over the Higgs and Z mass-suppressed contributions. Conversely, as the slepton and neutralino mass scale grows, the Higgs and Z contributions will drop off more slowly than the neutralino contribution.

The three-body decays with two neutrinos or two quarks have much the same form as above, with minor changes (refer to the appendixes for details). These changes are as follows: For the neutrino mode, $\beta^1_{aik} \to \gamma_{aik}$ and $\beta^2_{aik} \to 0$, and the lepton masses are set to zero, hence simplifying the phase space calculation and removing the off-shell Higgs contribution. For the quark modes, there is no off-shell fermion intermediary, and the Higgs and Z modes are only modified with adjusted couplings and quark masses. The quark modes are, however, enhanced by the color factor and a sum over light flavors. Because of these changes, however, all of these modes are suppressed relative to the dilepton mode by left-right mixing. The quark modes only have contributions from the Higgs and Z diagrams, and thus are suppressed by left-right mixing as noted above. In the majority of models with a light slepton NLSP, the $\tilde{\ell}_R$ gauge eigenstates are generically lighter than the $\tilde{\ell}_L$ gauge eigenstates; for such a model the neutrino modes are suppressed since the neutrino only couples to $\tilde{\ell}_L$ gauge eigenstates while the light sleptons are primarily $\tilde{\ell}_R$ states. Indeed, in such a model

$$\gamma_{aik} \sim \left(\frac{m^2_{LR}}{m^2_{\tilde{\ell}}}\right)^{1/2}, \tag{13}$$

so the neutrino and quark decay widths are all suppressed by $(m^2_{LR}/m^2_{\tilde{\ell}})^2$ relative to the dilepton modes.

Finally, for the charge-flipping dilepton decay, the matrix element is given by

$$\mathcal{M}^{(1)}_{\tilde{\chi}^0_a} = -i\bar{u}(p_k)(\beta^{1*}_{aik}P_R + \beta^{2*}_{aik}P_L)\frac{(\not{p}_k - \not{p}_i) + m_{\tilde{\chi}^0_a}}{(p_i - p_k)^2 - m^2_{\tilde{\chi}^0_a}} \times (\beta^{2*}_{ajm}P_L + \beta^{1*}_{ajm}P_R)v(p_m), \tag{14}$$

$$\mathcal{M}^{(2)}_{\tilde{\chi}^0_a} = -i\bar{u}(p_k)(\beta^{1*}_{aim}P_R + \beta^{2*}_{aim}P_L)\frac{(\not{p}_k - \not{p}_i) + m_{\tilde{\chi}^0_a}}{(p_i - p_k)^2 - m^2_{\tilde{\chi}^0_a}} \times (\beta^{2*}_{ajk}P_L + \beta^{1*}_{ajk}P_R)v(p_m), \tag{15}$$

$$\mathcal{M}_{\tilde{\chi}^0_a} = \mathcal{M}^{(1)}_{\tilde{\chi}^0_a} - \mathcal{M}^{(2)}_{\tilde{\chi}^0_a}, \tag{16}$$

where the negative sign comes from Fermi statistics. The decay width is then

$$\Gamma(\tilde{\ell}^-_i \to \tilde{\ell}^+_j \ell^-_k \ell^-_m) = C_{km}(\Gamma_{11} + \Gamma_{22} - 2\Gamma_{21}), \tag{17}$$

with

$$\Gamma_{ij} = \frac{1}{64\pi^3 m_i}\int_{\mathrm{PS}_3}\sum_{\mathrm{spins}}\sum_{a,b=1}^{4}\mathrm{Re}[\mathcal{M}^{(i)}_a \mathcal{M}^{(j)*}_b], \tag{18}$$

and C_{km} is a phase space factor which has a value of 1 for two outgoing leptons of different flavor and 1/2 for two outgoing leptons of the same flavor due to indistinguishable particle statistics. Γ_{11} and Γ_{22} both have the same basic form as the result from [24], where the only change is the insertion of flavor-mixing coefficients and, for Γ_{22}, interchange of ℓ_k and ℓ_m between the two terms; the new Γ_{21} is presented in Appendix D 4. We find that this charge-flipping decay width is of the same order of magnitude as the charge-preserving dilepton mode, though the flavor structure is markedly different: this is discussed in the next section.

IV. TWO-SLEPTON MIXING: ILLUSTRATIVE EXAMPLES

To validate our results and investigate their phenomenological implications, we examine some simple cases of two-slepton mixing. In these examples we consider spectra with fairly degenerate sleptons and the lightest neutralino. The slepton-neutralino degeneracy is motivated by simple gauge-mediated supersymmetry breaking scenarios with not too many messengers. Larger splittings are, of course, possible if there are many messengers or in other frameworks, such as minimal supergravity. The sleptons are taken fairly degenerate so that large mixing angles are consistent with low-energy constraints and so may be considered. We note, however, that these examples are merely illustrative, and our results are valid in any chosen framework with arbitrary mass splittings.

First, we consider $\tilde{e}_R - \tilde{\mu}_R$ mixing parametrized by

$$\begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix}\begin{pmatrix} \tilde{e} \\ \tilde{\mu} \end{pmatrix}. \tag{19}$$

Figure 4 shows the flavor-violating decay widths $\tilde{\ell}^-_2 \to \tilde{\ell}^\pm_1 \ell^- \ell^\mp$ with electron and/or muon leptons as a function of the mixing angle θ_{12}. Here both leptons are taken to be explicitly massless and left-right slepton mixing is set to zero, thereby cutting off the Higgs and Z modes. $\tilde{\ell}_1$ and $\tilde{\ell}_2$ are assigned masses of 100 GeV and 105 GeV, respectively, and the lightest neutralino is given a mass of 110 GeV (heavier neutralino contributions are small in this case).

The left plot in Fig. 4, which shows the decay widths for the charge-preserving channel, demonstrates the typical structure of two-slepton mixing: all decay widths are at most π periodic, the sleptons interchange roles ($\mu^- \to e^-$ and $e^+ \to \mu^+$) at $\theta_{12} = \pi/2$, the e^-e^+ and $\mu^-\mu^+$ modes are equal at all mixing angles, and at $\theta_{12} = \pi/4$ all decay widths are equal. In contrast, the right plot in Fig. 4, which shows the charge-flipping channel, demonstrates a different flavor structure. Since the $e^-\mu^-$ and μ^-e^- modes, in general, contribute to the same decay width, all widths are $\pi/2$ periodic. Also, the e^-e^- and $\mu^-\mu^-$ modes are again equal at all mixing angles. In addition, note that the μ^-e^- mode drops to zero at $\theta_{12} = \pi/4$, because the charge-flipping mode has two diagrams at tree level which cancel at $\theta_{12} = \pi/4$ for the μ^-e^- mode. Then, as expected, the e^-e^- and $\mu^-\mu^-$ modes have decay widths that are half the total width at $\theta_{12} = \pi/4$.

Next we consider $\tilde{e}_R - \tilde{\tau}_R$ mixing, with massive leptons and a mixing angle θ_{13} given by

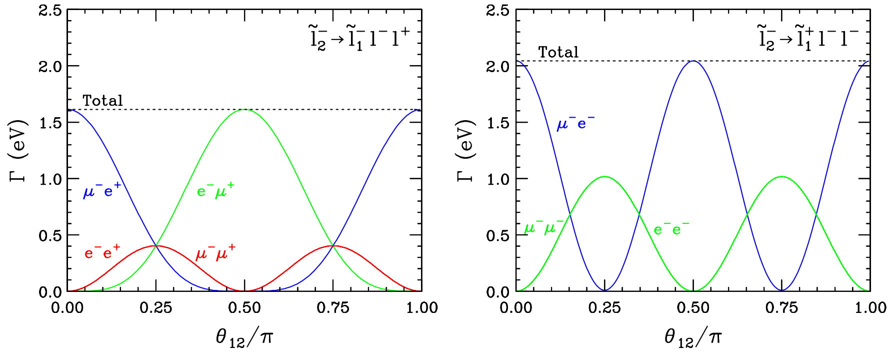

FIG. 4 (color online). Dilepton decay widths as a function of the mixing angle between a smuon and a selectron with left-right mixing neglected. The left plot shows the charge-preserving channel; the right plot shows the charge-flipping channel. In both plots the leptons are taken to be massless, $m_{\tilde{\ell}_1} = 100$ GeV, $m_{\tilde{\ell}_2} = 105$ GeV, and $m_{\tilde{\chi}_1^0} = 110$ GeV.

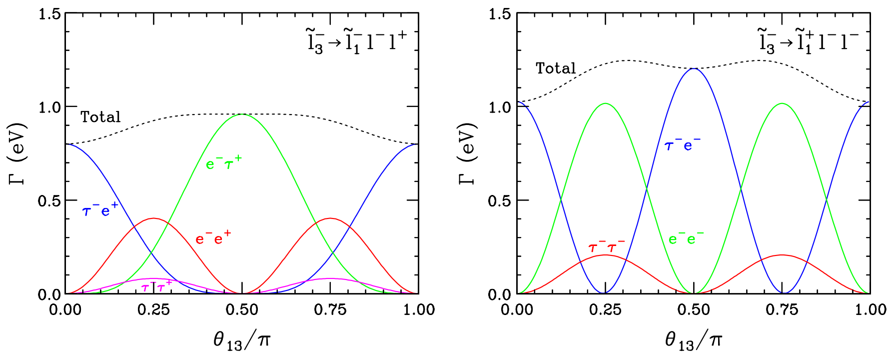

FIG. 5 (color online). Dilepton decay widths as a function of the mixing angle between a stau and a selectron with left-right mixing neglected. The left plot shows the charge-preserving channel; the right plot shows the charge-flipping channel. Here the leptons are both massive, $m_{\tilde{\ell}_1} = 100$ GeV, $m_{\tilde{\ell}_3} = 105$ GeV, and $m_{\tilde{\chi}_1^0} = 110$ GeV.

$$\begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_{13} & \sin\theta_{13} \\ -\sin\theta_{13} & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \tilde{\tau} \end{pmatrix}. \qquad (20)$$

Figure 5 shows the widths for $\tilde{\ell}_3^- \to \tilde{\ell}_1^\pm \ell^- \ell^\mp$, where the leptons are electrons and/or taus. $\tilde{\ell}_1$ and $\tilde{\ell}_3$ have masses of 100 GeV and 105 GeV, and the lightest neutralino mass is again 110 GeV. Left-right mixing is again neglected.

The first notable feature in Fig. 5 is the separation between the $e^- e^\pm$ and $\tau^- \tau^\pm$ modes: the $e^- e^\pm$ mode is almost unchanged numerically from the previous case, while the $\tau^- \tau^\pm$ mode is suppressed by phase space constriction, as expected. Likewise, the $\tau^- e^\pm$ and $e^- \tau^\pm$ modes are also suppressed relative to the massless case, but certainly less suppressed than the $\tau^- \tau^\pm$ mode. Note that the total decay width is no longer constant because of the nonzero tau mass.

V. CONCLUSIONS AND IMPLEMENTATION IN SPICE

We have determined the decay widths of three-body slepton decays, including both "charge-preserving" processes $\tilde{\ell}^- \to \tilde{\ell}^- \ell^- \ell^+$ and "charge-flipping" processes $\tilde{\ell}^- \to \tilde{\ell}^+ \ell^- \ell^-$, in the presence of arbitrary LFV and arbitrary left-right mixing. Such processes are particularly relevant in scenarios with a gravitino LSP and a slepton NLSP, where they are typically the dominant decay of some of the non-NLSP sleptons and are present as the last step in many supersymmetric cascade decays.

Arbitrary 6×6 slepton mixing leads to many new diagrams and new decade modes, as well as new interference effects. Our results are fully general, but we have illustrated them for two simple cases with two-generation LFV. The branching ratios to new modes may be significant, even accounting for low-energy constraints on LFV. The new modes may substitute final state electrons and muons for taus, potentially providing more obvious signals and better prospects for precision mass measurements. In addition, the measurement of these branching ratios may help pin down the supersymmetric flavor parameters and shed light on the SM flavor problem. We note that our illustrative examples have assumed fairly small slepton mass splittings so that large mixing angles are consistent with low-energy constraints. It is also possible, of course, that the splittings are larger and the mixing angles are smaller. The calculations presented here are valid in those cases as well, and it would also be very interesting to determine whether such small mixings may be established through collider studies.

The full calculation presented here is lengthy, but has been incorporated into SPICE: Simulation Package for Including Flavor in Collider Events, a publicly available code. SPICE takes as input a flavor-conserving model, such as minimal gauge-mediated supersymmetry breaking, and arbitrary lepton flavor-violating parameters, and generates the full supersymmetric spectrum, complete with flavor-violating branching ratios. The three-body decays discussed here are included in SPICE. Details on obtaining and running SPICE are described in the SPICE user manual [27].

ACKNOWLEDGMENTS

We are grateful to Y. Nir for many helpful discussions and a careful reading of the manuscript. The work of J. L. F., D. S., and F. Y. was supported in part by NSF Grant No. PHY-0239817 and No. PHY-0653656 and the Alfred P. Sloan Foundation. I. G. thanks the UC Irvine particle theory group for their hospitality while this work was in progress. Research was supported in part by the United States–Israel Binational Science Foundation (BSF) under Grant No. 2006071. The research of I. G. and Y. S. is also supported by the Israel Science Foundation (ISF) under Grant No. 1155/07.

APPENDIX A: LAGRANGIAN

As given in Eq. (1), the relevant flavor-general interactions for this study are given by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\tilde{\ell}}^{\rm int} = {}& [\tilde{\ell}_i^*(\bar{\tilde{\chi}}_a^0 \beta_{aik}^{(1)} P_L + \beta_{aik}^{(2)} P_R)\ell_k + \gamma_{aik}\tilde{\ell}_i^* \bar{\tilde{\chi}}_a P_L \nu_k \\ & + {\rm H.c.}] + i\zeta_{ij}^{(2)}(\tilde{\ell}_i^* \partial_\mu \tilde{\ell}_j - \tilde{\ell}_j \partial_\mu \tilde{\ell}_i^*) Z^\mu \\ & + \sigma_{ij}^{(2)} \tilde{\ell}_i^* \tilde{\ell}_j h^0 + \sigma_{ij}^{(3)} \tilde{\ell}_i^* \tilde{\ell}_j H^0 + i\sigma_{ij}^{(4)} \tilde{\ell}_i^* \tilde{\ell}_j A^0 \\ & + i\sigma_{ij}^{(5)} \tilde{\ell}_i^* \tilde{\ell}_j G^0. \end{aligned} \tag{A1}$$

The coefficients are

$$\beta_{aik}^{(1)} = \frac{1}{\sqrt{2}}(g O_{2,a}^* + g' O_{1,a}^*) U_{k,i}^{(\tilde{\ell})*} - y_k^{(\ell)} O_{3,a}^* U_{k+3,i}^{(\tilde{\ell})*}, \tag{A2}$$

$$\beta_{aik}^{(2)} = -\sqrt{2} O_{1,a} g' U_{k+3,i}^{(\tilde{\ell})*} - y_k^{(\ell)} O_{3,a} U_{k,i}^{(\tilde{\ell})*}, \tag{A3}$$

$$\gamma_{aik} = \begin{cases} -g\cos\theta_L U_{k,i}^{(\tilde{\ell})*} + y_k^{(\ell)} \sin\theta_L U_{k+3,i}^{(\tilde{\ell})*} & a=1 \\ g\sin\theta_L U_{k,i}^{(\tilde{\ell})*} + y_k^{(\ell)} \cos\theta_L U_{k+3,i}^{(\tilde{\ell})*} & a=2, \end{cases} \tag{A4}$$

$$\zeta_{ij}^{(2)} = \frac{g}{2\cos\theta_W}[U_{c,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})} - 2\sin^2\theta_W \delta_{ij}], \tag{A5}$$

$$\begin{aligned}\sigma_{ij}^{(2)} = {}& -\bigg[\bigg(\frac{g m_W}{2}(1-\tan^2\theta_W)\sin(\theta_H+\beta) - \frac{g m_{\ell_c}^2 \sin\theta_H}{m_W\cos\beta}\bigg) \\ & \times U_{c,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})} + \bigg(g m_W \tan^2\theta_W \sin(\theta_H+\beta) \\ & - \frac{g m_{\ell_c}^2 \sin\theta_H}{m_W\cos\beta}\bigg) U_{c+3,i}^{(\tilde{\ell})*} U_{c+3,j}^{(\tilde{\ell})} - \frac{g m_{\ell_c}}{2 m_W \cos\beta} \\ & \times (\mu\cos\theta_H + A_c^{\tilde{\ell}}\sin\theta_H)(U_{c,i}^{(\tilde{\ell})*} U_{c+3,j}^{(\tilde{\ell})} + U_{c+3,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})})\bigg], \end{aligned} \tag{A6}$$

$$\begin{aligned}\sigma_{ij}^{(3)} = {}& \bigg[\bigg(\frac{g m_W}{2}(1-\tan^2\theta_W)\cos(\theta_H+\beta) - \frac{g m_{\ell_c}^2 \cos\theta_H}{m_W\cos\beta}\bigg) \\ & \times U_{c,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})} + \bigg(g m_W \tan^2\theta_W \cos(\theta_H+\beta) \\ & - \frac{g m_{\ell_c}^2 \cos\theta_H}{m_W\cos\beta}\bigg) U_{c+3,i}^{(\tilde{\ell})*} U_{c+3,j}^{(\tilde{\ell})} + \frac{g m_{\ell_c}}{2 m_W \cos\beta} \\ & \times (\mu\sin\theta_H - A_c^{\tilde{\ell}}\cos\theta_H)(U_{c,i}^{(\tilde{\ell})*} U_{c+3,j}^{(\tilde{\ell})} + U_{c+3,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})})\bigg], \end{aligned} \tag{A7}$$

$$\sigma_{ij}^{(4)} = \frac{g m_{\ell_c}}{2 m_W}(\mu + A_c^{\tilde{\ell}}\tan\beta)(U_{c,i}^{(\tilde{\ell})*} U_{c+3,j}^{(\tilde{\ell})} - U_{c+3,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})}), \tag{A8}$$

$$\sigma_{ij}^{(5)} = \frac{g m_{\ell_c}}{2 m_W}(\mu\tan\beta + A_c^{\tilde{\ell}})(U_{c,i}^{(\tilde{\ell})*} U_{c+3,j}^{(\tilde{\ell})} - U_{c+3,i}^{(\tilde{\ell})*} U_{c,j}^{(\tilde{\ell})}). \tag{A9}$$

$U_{c,a}^{(\tilde{\ell})}$ and $U_{c,a}^{(\tilde{\nu})}$ are the slepton and sneutrino mixing matrices, respectively. These relate the slepton and sneutrino gauge eigenstates,

$$\tilde{\ell}_\alpha = (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R)^T, \tag{A10}$$

$$\tilde{\nu} = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)^T, \tag{A11}$$

to the mass eigenstates $\tilde{\ell}_i$, $\tilde{\nu}_i$ through the relations $\tilde{\ell}_i = U_{i\alpha}^{(\tilde{\ell})\dagger}\tilde{\ell}_\alpha$ and $\tilde{\nu}_i = U_{i\alpha}^{(\tilde{\nu})\dagger}\tilde{\nu}_\alpha$. The mass eigenstates are defined in order of increasing mass. The neutralino gauge eigenstates $\tilde{\psi}^0 = (-i\tilde{B}, -i\tilde{W}, \tilde{\psi}_d^0, \tilde{\psi}_u^0)^T$ are related to the mass eigenstates χ_i^0 by $\chi_i^0 = O_{i\alpha}^\dagger \tilde{\psi}_\alpha^0$. Similarly, the mixing of the negative charginos is

$$\begin{pmatrix}\chi_1^- \\ \chi_2^-\end{pmatrix} = \begin{pmatrix}\cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L\end{pmatrix}\begin{pmatrix}-i\tilde{W}^- \\ \tilde{\psi}_d^-\end{pmatrix}. \tag{A12}$$

Finally, the neutral Higgs doublet is given by

$$\begin{pmatrix}H_u^0 \\ H_d^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}v_u \\ v_d\end{pmatrix} + \frac{1}{\sqrt{2}} R_{\theta_H}\begin{pmatrix}h^0 \\ H^0\end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix}G^0 \\ A^0\end{pmatrix}, \tag{A13}$$

where

$$R_{\theta_H} = \begin{pmatrix} \cos\theta_H & \sin\theta_H \\ -\sin\theta_H & \cos\theta_H \end{pmatrix} \quad \text{and} \quad R_\beta = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix}. \tag{A14}$$

These coefficients are consistent with those of Ref. [27], where a more complete discussion may be found.

APPENDIX B: PHASE SPACE INTEGRATION

In the three-body decays $\tilde{\ell}_i \to \tilde{\ell}_j f_k f_m$, we label the initial and final state particle masses m_i, m_j, m_k, and m_m. We will maintain these indices throughout the appendix, with i corresponding to the parent slepton, j to the daughter slepton, k to the daughter fermion with the same sign as the parent slepton, and m to the other daughter fermion. The decay widths are the squared matrix elements integrated over three-body phase space:

$$\Gamma = \frac{1}{64\pi^3 m_i}\int_{\text{PS}_3}|\mathcal{M}|^2 \equiv \frac{1}{64\pi^3 m_i}\int_{E_k^-}^{E_k^+} dE_k \int_{E_m^-}^{E_m^+} dE_m |\mathcal{M}|^2 = \frac{m_i}{256\pi^3}\int_{x_-}^{x_+} dx \int_{y_-}^{y_+} dy |\mathcal{M}|^2, \tag{B1}$$

where the dimensionless quantities

$$x = \frac{2E_k}{m_i} \quad \text{and} \quad y = \frac{2E_m}{m_i} \tag{B2}$$

have limits

$$x_+ = 1 + r_k^2 - (r_j + r_m)^2, \tag{B3}$$

$$x_- = 2r_k, \tag{B4}$$

$$y_\pm = \frac{1}{2(1 - x + r_k^2)}\left[(2 - x)(R^2 - x) \pm \sqrt{x^2 - 4r_k^2}\,\lambda^{1/2}(1 - x + r_k^2, r_m^2, r_j^2)\right], \tag{B5}$$

and we have defined

$$r_j = \frac{m_j}{m_i} \qquad r_k = \frac{m_k}{m_i} \qquad r_m = \frac{m_m}{m_i}, \tag{B6}$$

$$R^2 = 1 - r_j^2 + r_k^2 + r_m^2, \tag{B7}$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \tag{B8}$$

Throughout this work, we implicitly assume that the matrix element squared has been averaged and summed over initial and final state spins, respectively.

The squared matrix element has the form

$$|\mathcal{M}|^2 = \sum_{a,b}\mathcal{M}_a\mathcal{M}_b^*, \tag{B9}$$

where $\mathcal{M}_a$ and $\mathcal{M}_b$ are matrix elements corresponding to Feynman diagrams with intermediate particles a and b. To evaluate the decay widths, we choose to integrate over y analytically. The decay widths are then written in terms of dimensionless integrals over x, which are performed numerically.

In integrating over y, it is convenient to note that all terms $\mathcal{M}_a\mathcal{M}_b^*$ have a numerator that is either constant or linear in y and a denominator that is proportional to the product of two propagators, each of which is either of the form $(1 - x + r_k^2 - r_{a,b}^2)^{-1}$ or $(x + y - 1 + r_j^2 - r_{a,b}^2)^{-1}$, where $r_{a,b} = m_{a,b}/m_i$. We therefore define the following dimensionless functions of x:

$$f(x, r_a, r_b) = \int_{y_-}^{y_+}\frac{y dy}{(1 - x + r_k^2)(1 - x + r_k^2 - r_a^2)(1 - x + r_k^2 - r_b^2)} = \frac{k(x, r_b)}{(1 - x + r_k^2 - r_a^2)(1 - x + r_k^2)}, \tag{B10}$$

$$f^{(1)}(x, r_a, r_b) = \int_{y_-}^{y_+}\frac{dy}{(1 - y + r_m^2 - r_a^2)(1 - x + r_k^2 - r_b^2)} = -\frac{l(x, r_a^2 - 1 - r_m^2)}{(1 - x + r_k^2 - r_b^2)}, \tag{B11}$$

$$f^{(2)}(x, r_a, r_b) = \int_{y_-}^{y_+}\frac{y dy}{(1 - y + r_m^2 - r_a^2)(1 - x + r_k^2 - r_b^2)} = (1 + r_m^2 - r_a^2)f^{(1)}(x, r_a, r_b) - k(x, r_b), \tag{B12}$$

$$g^{(1)}(x, r_a, r_b) = \int_{y_-}^{y_+}\frac{dy}{(x + y - 1 + r_j^2 - r_a^2)(1 - x + r_k^2 - r_b^2)} = \frac{l(x, x - 1 + r_j^2 - r_a^2)}{(1 - x + r_k^2 - r_b^2)}, \tag{B13}$$

$$g^{(2)}(x, r_a, r_b) = \int_{y_-}^{y_+}\frac{y dy}{(x + y - 1 + r_j^2 - r_a^2)(1 - x + r_k^2 - r_b^2)} = k(x, r_b) + (1 - x - r_j^2 + r_a^2)g^{(1)}(x, r_a, r_b), \tag{B14}$$

$$h^{(1)}(x, r_a, r_b) = \int_{y_-}^{y_+} \frac{dy}{(x+y-1+r_j^2-r_a^2)(x+y-1+r_j^2-r_b^2)} = \frac{l(x, x-1+r_j^2-r_a^2) - l(x, x-1+r_j^2-r_b^2)}{(r_a^2 - r_b^2)}, \tag{B15}$$

$$h^{(2)}(x, r_a, r_b) = \int_{y_-}^{y_+} \frac{ydy}{(x+y-1+r_j^2-r_a^2)(x+y-1+r_j^2-r_b^2)} = l(x, x-1+r_j^2-r_b^2) + (1-x-r_j^2+r_a^2)h^{(1)}(x, r_a, r_b), \tag{B16}$$

$$i^{(1)}(x, r_a) = \int_{y_-}^{y_+} \frac{dy}{(x+y-1+r_j^2-r_a^2)^2} = \frac{y_+ - y_-}{(y_+ + x - 1 + r_j^2 - r_a^2)(y_- + x - 1 + r_j^2 - r_a^2)}, \tag{B17}$$

$$i^{(2)}(x, r_a) = \int_{y_-}^{y_+} \frac{ydy}{(x+y-1+r_j^2-r_a^2)^2} = l(x, x-1+r_j^2-r_a^2) + (1-x-r_j^2+r_a^2)i^{(1)}(x, r_a), \tag{B18}$$

where

$$k(x, r_b) = \frac{y_+ - y_-}{1 - x + r_k^2 - r_b^2}, \tag{B19}$$

$$l(x, z) = \ln\left|\frac{y_+ + z}{y_- + z}\right|. \tag{B20}$$

These functions will appear frequently in the decay widths to be discussed below. The first function f contains an extra factor in the denominator to conform to the notation used in Ref. [24]. The other functions are simply integrals over y with all relevant combinations of propagators in the denominator and a numerator either constant or linear in y.

APPENDIX C: $\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+)$

1. Total width

For the charge-preserving case, the matrix elements for all contributing modes are presented in Eqs. (3)–(8). The total decay width is

$$\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- \ell_k^- \ell_m^+) = \frac{1}{64\pi^3 m_i} \int_{\mathrm{PS}_3} |\mathcal{M}|^2 \tag{C1}$$

$$= \frac{1}{64\pi^3 m_i} \int_{\mathrm{PS}_3} \Bigg(\sum_{a,b=1}^{4} \mathcal{M}_{\tilde{\chi}_a^0} \mathcal{M}^*_{\tilde{\chi}_b^0} + \mathcal{M}_Z \mathcal{M}_Z^* + \sum_{a,b} \mathcal{M}_{H_a} \mathcal{M}^*_{H_b} + 2\,\mathrm{Re}\Bigg[\sum_{b=1}^{4} \mathcal{M}_Z \mathcal{M}^*_{\tilde{\chi}_b^0} + \sum_a \sum_{b=1}^{4} \mathcal{M}_{H_a} \mathcal{M}^*_{\tilde{\chi}_b^0} + \sum_b \mathcal{M}_Z \mathcal{M}^*_{H_b} \Bigg] \Bigg) \tag{C2}$$

$$= \Gamma_{\tilde{\chi}^0\tilde{\chi}^0} + \delta_{km}(\Gamma_{ZZ} + \Gamma_{HH} + 2\Gamma_{Z\tilde{\chi}^0} + 2\Gamma_{H\tilde{\chi}^0} + 2\Gamma_{ZH}), \tag{C3}$$

where δ_{km} is the Kronecker delta function. For $\Gamma_{\tilde{\chi}^0\tilde{\chi}^0}$ the general case of independent k and m is taken; for the remaining widths, $k = m$ is assumed to simplify the expressions.

2. $\tilde{\chi}^0\tilde{\chi}^0$ contribution

The neutralino width corresponds to the width given in Ref. [24] with the addition of flavor-violating vertices. The decay width is given by

$$\Gamma_{\tilde{\chi}^0\tilde{\chi}^0} = \frac{m_{\tilde{\ell}_i}}{512\pi^3} \sum_{t=1}^{6} \sum_{a,b=1}^{4} C^{(t)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} I^{(t)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0}, \tag{C4}$$

where t labels the coefficients and integrals, and a and b label the neutralinos. The dimensionless integrals are defined by

$$I^{(1)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = \int_{x_-}^{x_+} dx(x - 2r_k^2)(1 - x + r_k^2)(R^2 - x)f(x, r_a, r_b), \tag{C5}$$

$$I^{(2)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_a r_b \int_{x_-}^{x_+} dx(x - 2r_k^2)(R^2 - x)f(x, r_a, r_b), \tag{C6}$$

$$I^{(3)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = 2r_m r_b \int_{x_-}^{x_+} dx(x - 2r_k^2)(1 - x + r_k^2)f(x, r_a, r_b), \tag{C7}$$

$$I^{(4)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = 2r_k r_b \int_{x_-}^{x_+} dx(1 - x + r_k^2)(R^2 - x)f(x, r_a, r_b), \tag{C8}$$

$$I^{(5)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = 2r_k r_m r_a r_b \int_{x_-}^{x_+} dx(1 - x + r_k^2)f(x, r_a, r_b), \tag{C9}$$

$$I^{(6)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = 2r_k r_m \int_{x_-}^{x_+} dx(1 - x + r_k^2)^2 f(x, r_a, r_b), \tag{C10}$$

with coefficients

$$C^{(1)}_{\tilde{\chi}^0_a\tilde{\chi}^0_b} = \beta^{1*}_{aik}\beta^{1}_{ajm}\beta^{1}_{bik}\beta^{1*}_{bjm} + \beta^{2*}_{aik}\beta^{2}_{ajm}\beta^{2}_{bik}\beta^{2*}_{bjm}, \quad \text{(C11)}$$

$$C^{(2)}_{\tilde{\chi}^0_a\tilde{\chi}^0_b} = \beta^{1*}_{aik}\beta^{2}_{ajm}\beta^{1}_{bik}\beta^{2*}_{bjm} + \beta^{2*}_{aik}\beta^{1}_{ajm}\beta^{2}_{bik}\beta^{1*}_{bjm}, \quad \text{(C12)}$$

$$C^{(3)}_{\tilde{\chi}^0_a\tilde{\chi}^0_b} = 2\,\mathrm{Re}[\beta^{1*}_{aik}\beta^{1}_{ajm}\beta^{1}_{bik}\beta^{2*}_{bjm} + \beta^{2*}_{aik}\beta^{2}_{ajm}\beta^{2}_{bik}\beta^{1*}_{bjm}], \quad \text{(C13)}$$

$$C^{(4)}_{\tilde{\chi}^0_a\tilde{\chi}^0_b} = -2\,\mathrm{Re}[\beta^{1*}_{aik}\beta^{1}_{ajm}\beta^{2}_{bik}\beta^{1*}_{bjm} + \beta^{2*}_{aik}\beta^{2}_{ajm}\beta^{1}_{bik}\beta^{2*}_{bjm}], \quad \text{(C14)}$$

$$C^{(5)}_{\tilde{\chi}^0_a\tilde{\chi}^0_b} = -4\,\mathrm{Re}[\beta^{1*}_{aik}\beta^{2}_{ajm}\beta^{2}_{bik}\beta^{1*}_{bjm}], \quad \text{(C15)}$$

$$C^{(6)}_{\tilde{\chi}^0_a\tilde{\chi}^0_b} = -4\,\mathrm{Re}[\beta^{1*}_{aik}\beta^{1}_{ajm}\beta^{2}_{bik}\beta^{2*}_{bjm}]. \quad \text{(C16)}$$

3. *ZZ* contribution

The Z boson contribution to the decay width is

$$\Gamma_{ZZ} = \frac{m_i}{512\pi^3}\left|\frac{g\zeta^{(2)}_{ji}}{\cos\theta_W}\right|^2 \sum_{t=1}^{2} C^{(t)}_{ZZ} I^{(t)}_{ZZ}, \quad \text{(C17)}$$

where the dimensionless integrals are

$$I^{(1)}_{ZZ} = \int_{x_-}^{x_+} dx(2x - 2 - r_k^2)i^{(2)}(x, r_Z) + [2 - (2 + r_k^2)x - 2r_j^2 + 3r_k^2 + r_k^2 r_j^2]i^{(1)}(x, r_Z), \quad \text{(C18)}$$

$$I^{(2)}_{ZZ} = r_k^2 \int_{x_-}^{x_+} dx i^{(2)}(x, r_Z) + (x - 3 - r_j^2)i^{(1)}(x, r_Z), \quad \text{(C19)}$$

and the coefficients are

$$C^{(1)}_{ZZ} = 8\sin^4\theta_W - 4\sin^2\theta_W + 1, \quad \text{(C20)}$$

$$C^{(2)}_{ZZ} = 8\sin^4\theta_W - 4\sin^2\theta_W. \quad \text{(C21)}$$

4. *HH* contribution

The purely Higgs-mediated contribution actually consists of several pieces with similar phase space structure. Furthermore, it simplifies into the sum of contributions from the real and pseudoscalar Higgs bosons, since the interference term between a real scalar and a pseudoscalar vanishes.

The width is given by

$$\Gamma_{HH} = \frac{1}{256\pi^3 m_i}(C_{h^0h^0}I^r_{h^0h^0} + C_{H^0H^0}I^r_{H^0} + 2C_{h^0H^0}I^r_{h^0H^0} + C_{A^0A^0}I^p_{A^0A^0} + C_{G^0G^0}I^p_{G^0G^0} + 2C_{A^0G^0}I^p_{A^0G^0}), \quad \text{(C22)}$$

where the dimensionless integrals are

$$I^r_{H_aH_a} = \int_{x_-}^{x_+} dx i^{(2)}(x, r_{H_a}) + (x - R^2 - 2r_k^2)i^{(1)}(x, r_{H_a}), \quad \text{(C23)}$$

$$I^r_{H_aH_b} = \int_{x_-}^{x_+} dx h^{(2)}(x, r_{H_a}, r_{H_b}) + (x - R^2 - 2r_k^2)h^{(1)}(x, r_{H_a}, r_{H_b}), \quad \text{(C24)}$$

$$I^p_{H_aH_a} = \int_{x_-}^{x_+} dx i^{(2)}(x, r_{H_a}) + (x - 1 + r_j^2)i^{(1)}(x, r_{H_a}), \quad \text{(C25)}$$

$$I^p_{H_aH_b} = \int_{x_-}^{x_+} dx h^{(2)}(x, r_{H_a}, r_{H_b}) + (x - 1 + r_j^2)h^{(1)}(x, r_{H_a}, r_{H_b}), \quad \text{(C26)}$$

where $a \neq b$, and the coefficients are

$$C_{h^0h^0} = |\sigma^{(2)}_{ji} y^{(\ell)}_k \sin\theta_H|^2, \quad \text{(C27)}$$

$$C_{H^0H^0} = |\sigma^{(3)}_{ji} y^{(\ell)}_k \cos\theta_H|^2, \quad \text{(C28)}$$

$$C_{h^0H^0} = -\mathrm{Re}[\sigma^{(2)}_{ji}\sigma^{(3)*}_{ji} y^{(\ell)2}_k \sin\theta_H \cos\theta_H], \quad \text{(C29)}$$

$$C_{A^0A^0} = |\sigma^{(4)}_{ji} y^{(\ell)}_k \sin\beta|^2, \quad \text{(C30)}$$

$$C_{G^0G^0} = |\sigma^{(5)}_{ji} y^{(\ell)}_k \cos\beta|^2, \quad \text{(C31)}$$

$$C_{A^0G^0} = -\mathrm{Re}[\sigma^{(4)}_{ji}\sigma^{(5)*}_{ji} y^{(\ell)2}_k \sin\beta\cos\beta]. \quad \text{(C32)}$$

5. $Z\tilde{\chi}^0$ contribution

The width from the $Z\tilde{\chi}^0$ interference term is

$$\Gamma_{Z\tilde{\chi}^0_b} = -\frac{m_i}{512\pi^3}\frac{g\zeta^{(2)}_{ji}}{\cos\theta_W}\sum_{b=1}^{4}\sum_{t=1}^{2} C^{(t)}_{Z\tilde{\chi}^0_b} I^{(t)}_{Z\tilde{\chi}^0_b}, \quad \text{(C33)}$$

where the integrals are

$$I^{(1)}_{Z\tilde{\chi}^0_b} = \int_{x_-}^{x_+} dx[2(1 - x + r_k^2)g^{(2)}(x, r_Z, r_b) + (2x - 2R^2 + r_k^2 - r_j^2 r_k^2)g^{(1)}(x, r_Z, r_b)], \quad \text{(C34)}$$

$$I^{(2)}_{Z\tilde{\chi}^0_b} = r_k r_b \int_{x_-}^{x_+} dx[g^{(2)}(x, r_Z, r_b) - (1 + x - r_j^2)g^{(1)}(x, r_Z, r_b)], \quad \text{(C35)}$$

$$I^{(3)}_{Z\tilde{\chi}^0_b} = r_k r_b \int_{x_-}^{x_+} dx[g^{(2)}(x, r_Z, r_b) + (1 - x - r_j^2)g^{(1)}(x, r_Z, r_b)], \quad \text{(C36)}$$

$$I^{(4)}_{Z\tilde{\chi}^0_b} = r_k^2 \int_{x_-}^{x_+} dx[(3 - 2x + r_j^2)g^{(1)}(x, r_Z, r_b)], \quad \text{(C37)}$$

and the coefficients are

$$C^{(1)}_{Z\tilde{\chi}^0_b} = \mathrm{Re}[(2\sin^2\theta_W - 1)\beta^1_{bik}\beta^{1*}_{bjm} + 2\sin^2\theta_W\beta^2_{bik}\beta^{2*}_{bjm}], \quad \text{(C38)}$$

$$C^{(2)}_{Z\tilde{\chi}^0_b} = \mathrm{Re}[(2\sin^2\theta_W - 1)\beta^1_{bik}\beta^{2*}_{bjm} + 2\sin^2\theta_W\beta^2_{bik}\beta^{1*}_{bjm}], \quad \text{(C39)}$$

$$C^{(3)}_{Z\tilde{\chi}^0_b} = \mathrm{Re}[2\sin^2\theta_W\beta^1_{bik}\beta^{2*}_{bjm} + (2\sin^2\theta_W - 1)\beta^2_{bik}\beta^{1*}_{bjm}], \quad \text{(C40)}$$

$$C^{(4)}_{Z\tilde{\chi}^0_b} = \mathrm{Re}[2\sin^2\theta_W\beta^1_{bik}\beta^{1*}_{bjm} + (2\sin^2\theta_W - 1)\beta^2_{bik}\beta^{2*}_{bjm}]. \quad \text{(C41)}$$

6. $H\tilde{\chi}^0$ contribution

The width from the $H\tilde{\chi}^0$ interference is

$$\Gamma_{H\tilde{\chi}^0} = \frac{1}{256\sqrt{2}\pi^3}\sum_{b=1}^{4}\sum_{t=1}^{2}(C^{(t)}_{h^0\tilde{\chi}^0_b}I^{H(t)}_{h^0\tilde{\chi}^0_b} + C^{(t)}_{H^0\tilde{\chi}^0_b}I^{H(t)}_{H^0\tilde{\chi}^0_b} + C^{(t)}_{A^0\tilde{\chi}^0_b}I^{P(t)}_{A^0\tilde{\chi}^0_b} + C^{(t)}_{G^0\tilde{\chi}^0_b}I^{P(t)}_{G^0\tilde{\chi}^0_b}), \quad \text{(C42)}$$

where the dimensionless integrals are

$$I^{H(1)}_{H_a\tilde{\chi}^0_b} = r_b \int_{x_-}^{x_+} dx[g^{(2)}(x, r_{H_a}, r_b) + (x - R^2 - 2r_k^2)g^{(1)}(x, r_{H_a}, r_b)], \quad \text{(C43)}$$

$$I^{H(2)}_{H_a\tilde{\chi}^0_b} = r_k \int_{x_-}^{x_+} dx(2x - R^2 - 2r_k^2)g^{(1)}(x, r_{H_a}, r_b), \quad \text{(C44)}$$

$$I^{P(1)}_{H_a\tilde{\chi}^0_b} = r_b \int_{x_-}^{x_+} dx[g^{(2)}(x, r_{H_a}, r_b) + (x - 1 + r_j^2)g^{(1)}(x, r_{H_a}, r_b)], \quad \text{(C45)}$$

$$I^{P(2)}_{H_a\tilde{\chi}^0_b} = r_k \int_{x_-}^{x_+} dx(1 - r_j^2)g^{(1)}(x, r_{H_a}, r_b), \quad \text{(C46)}$$

and the coefficients are

$$C^{(1)}_{h^0\tilde{\chi}^0_b} = \mathrm{Re}[\sigma^{(2)}_{ji}y^{(\ell)}_k \sin\theta_H(\beta^1_{bik}\beta^{2*}_{bjm} + \beta^2_{bik}\beta^{1*}_{bjm})], \quad \text{(C47)}$$

$$C^{(2)}_{h^0\tilde{\chi}^0_b} = \mathrm{Re}[\sigma^{(2)}_{ji}y^{(\ell)}_k \sin\theta_H(\beta^1_{bik}\beta^{1*}_{bjm} + \beta^2_{bik}\beta^{2*}_{bjm})], \quad \text{(C48)}$$

$$C^{(1)}_{H^0\tilde{\chi}^0_b} = -\mathrm{Re}[\sigma^{(3)}_{ji}y^{(\ell)}_k \cos\theta_H(\beta^1_{bik}\beta^{2*}_{bjm} + \beta^2_{bik}\beta^{1*}_{bjm})], \quad \text{(C49)}$$

$$C^{(2)}_{H^0\tilde{\chi}^0_b} = -\mathrm{Re}[\sigma^{(3)}_{ji}y^{(\ell)}_k \cos\theta_H(\beta^1_{bik}\beta^{1*}_{bjm} + \beta^2_{bik}\beta^{2*}_{bjm})], \quad \text{(C50)}$$

$$C^{(1)}_{A^0\tilde{\chi}^0_b} = \mathrm{Re}[\sigma^{(4)}_{ji}y^{(\ell)}_k \sin\beta(\beta^1_{bik}\beta^{2*}_{bjm} - \beta^2_{bik}\beta^{1*}_{bjm})], \quad \text{(C51)}$$

$$C^{(2)}_{A^0\tilde{\chi}^0_b} = \mathrm{Re}[\sigma^{(4)}_{ji}y^{(\ell)}_k \sin\beta(\beta^1_{bik}\beta^{1*}_{bjm} - \beta^2_{bik}\beta^{2*}_{bjm})], \quad \text{(C52)}$$

$$C^{(1)}_{G^0\tilde{\chi}^0_b} = -\mathrm{Re}[\sigma^{(5)}_{ji}y^{(\ell)}_k \cos\beta(\beta^1_{bik}\beta^{2*}_{bjm} - \beta^2_{bik}\beta^{1*}_{bjm})], \quad \text{(C53)}$$

$$C^{(2)}_{G^0\tilde{\chi}^0_b} = -\mathrm{Re}[\sigma^{(5)}_{ji}y^{(\ell)}_k \cos\beta(\beta^1_{bik}\beta^{1*}_{bjm} - \beta^2_{bik}\beta^{2*}_{bjm})]. \quad \text{(C54)}$$

7. ZH contribution

The ZH interference decay width is

$$\Gamma_{ZH} = \frac{g\zeta^{(2)}_{ji}}{256\sqrt{2}\pi^3\cos\theta_W}(C_{Zh^0}I^r_{Zh^0} + C_{ZH^0}I^r_{ZH^0} + C_{ZA^0}I^p_{ZA^0} + C_{ZG^0}I^p_{ZG^0}), \quad \text{(C55)}$$

where

$$I^r_{ZH_a} = r_k \int_{x_-}^{x_+} dx[h^{(2)}(x, r_Z, r_{H_a}) - xh^{(1)}(x, r_Z, r_{H_a})], \quad \text{(C56)}$$

$$I^p_{ZH_a} = r_k \int_{x_-}^{x_+} dx(1 - r_j^2)h^{(1)}(x, r_Z, r_{H_a}), \quad \text{(C57)}$$

and

$$C_{Zh^0} = -\mathrm{Re}[\sigma^{(2)}_{ji}y^{(\ell)}_k \sin\theta_H(4\sin^2\theta_W - 1)], \quad \text{(C58)}$$

$$C_{ZH^0} = \mathrm{Re}[\sigma^{(3)}_{ji}y^{(\ell)}_k \cos\theta_H(4\sin^2\theta_W - 1)], \quad \text{(C59)}$$

$$C_{ZA^0} = -\mathrm{Re}[\sigma^{(4)}_{ji}y^{(\ell)}_k \sin\beta], \quad \text{(C60)}$$

$$C_{ZG^0} = \mathrm{Re}[\sigma^{(5)}_{ji}y^{(\ell)}_k \cos\beta]. \quad \text{(C61)}$$

APPENDIX D: $\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^+ \ell_k^- \ell_m^-)$

1. Total width

For the charge-flipping decay, the calculation is complicated by the fact that the same-sign daughter leptons create an interference term. It is convenient to break the matrix element into two parts, as shown in Eqs. (14)–(16). It is correspondingly convenient to separate the decay width into three terms:

$$\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^+ \ell_k^- \ell_m^-) = C_{km}(\Gamma_{11} + \Gamma_{22} - 2\Gamma_{21}), \quad \text{(D1)}$$

where

$$\Gamma_{ij} = \frac{1}{64\pi^3 m_i} \int_{\mathrm{PS}_3} \sum_{a,b=1}^{4} \mathrm{Re}[\mathcal{M}_a^{(i)} \mathcal{M}_b^{(j)*}], \quad \text{(D2)}$$

and C_{km} is 1 when the two outgoing leptons are of different generations and $1/2$ when they are of the same generation.

2. Γ_{11} width

From inspection of the matrix element, $\mathcal{M}_a^{(i)}$ is identical to $\mathcal{M}_{\tilde{\chi}_a^0}$ except for the coefficients in the ajm vertex. Then Γ_{11} is identical to $\Gamma_{\tilde{\chi}^0\tilde{\chi}^0}$ in the charge-preserving case with the substitutions

$$\beta_{xjm}^1 \to \beta_{xjm}^{2*}, \qquad \beta_{xjm}^2 \to \beta_{xjm}^{1*}, \quad \text{(D3)}$$

and with the identical substitutions for the complex conjugates, where $x = a, b$.

3. Γ_{22} width

From further inspection of the matrix elements, $\mathcal{M}_a^{(1)}$ and $\mathcal{M}_a^{(2)}$ differ only in the interchange of the two outgoing leptons. Thus Γ_{22} may be obtained from Γ_{11} with the interchange $k \leftrightarrow m$.

4.Γ_{21} width

The Γ_{21} width from the interference term is given by

$$\Gamma_{21} = \frac{1}{64\pi^3 m_i} \int_{\mathrm{PS}_3} \sum_{a,b} \mathrm{Re}[\mathcal{M}_a^{(2)} \mathcal{M}_b^{(1)*}]$$
$$= \frac{m_i}{256\pi^3} \sum_{t=1}^{8} \sum_{a,b=1}^{4} \mathrm{Re}[D^{(t)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} J^{(t)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0}], \quad \text{(D4)}$$

where the integrals are

$$J^{(1)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = \int_{x_-}^{x_+} dx\{[2r_k^2 r_m^2 - x(1 + r_m^2) + R^2] f^{(1)}(x, r_a, r_b)\} - (1 - x + r_k^2) f^{(2)}(x, r_a, r_b), \quad \text{(D5)}$$

$$J^{(2)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_k r_m \int_{x_-}^{x_+} dx(1 + r_j^2 - r_k^2 - r_m^2) f^{(1)}(x, r_a, r_b), \quad \text{(D6)}$$

$$J^{(3)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_m r_b \int_{x_-}^{x_+} dx[R^2 f^{(1)}(x, r_a, r_b) - f^{(2)}(x, r_a, r_b)], \quad \text{(D7)}$$

$$J^{(4)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_k r_b \int_{x_-}^{x_+} dx[f^{(2)}(x, r_a, r_b) - 2r_m^2 f^{(1)}(x, r_a, r_b)], \quad \text{(D8)}$$

$$J^{(5)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_m r_a \int_{x_-}^{x_+} dx(x - 2r_k^2) f^{(1)}(x, r_a, r_b), \quad \text{(D9)}$$

$$J^{(6)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_k r_a \int_{x_-}^{x_+} dx(R^2 - x) f^{(1)}(x, r_a, r_b), \quad \text{(D10)}$$

$$J^{(7)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = r_a r_b \int_{x_-}^{x_+} dx[(x - R^2) f^{(1)}(x, r_a, r_b) + f^{(2)}(x, r_a, r_b)], \quad \text{(D11)}$$

$$J^{(8)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = 2r_k r_m r_a r_b \int_{x_-}^{x_+} dx f^{(1)}(x, r_a, r_b), \quad \text{(D12)}$$

and the coefficients are

$$D^{(1)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = \beta^{1*}_{aim}\beta^{2*}_{ajk}\beta^{2}_{bik}\beta^{1}_{bjm} + \beta^{2*}_{aim}\beta^{1*}_{ajk}\beta^{1}_{bik}\beta^{2}_{bjm}, \quad \text{(D13)}$$

$$D^{(2)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = -(\beta^{1*}_{aim}\beta^{2*}_{ajk}\beta^{1}_{bik}\beta^{2}_{bjm} + \beta^{2*}_{aim}\beta^{1*}_{ajk}\beta^{2}_{bik}\beta^{1}_{bjm}), \quad \text{(D14)}$$

$$D^{(3)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = \beta^{1*}_{aim}\beta^{2*}_{ajk}\beta^{2}_{bik}\beta^{2}_{bjm} + \beta^{2*}_{aim}\beta^{1*}_{ajk}\beta^{1}_{bik}\beta^{1}_{bjm}, \quad \text{(D15)}$$

$$D^{(4)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = -(\beta^{1*}_{aim}\beta^{2*}_{ajk}\beta^{1}_{bik}\beta^{1}_{bjm} + \beta^{2*}_{aim}\beta^{1*}_{ajk}\beta^{2}_{bik}\beta^{2}_{bjm}), \quad \text{(D16)}$$

$$D^{(5)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = -(\beta^{1*}_{aim}\beta^{1*}_{ajk}\beta^{1}_{bik}\beta^{2}_{bjm} + \beta^{2*}_{aim}\beta^{2*}_{ajk}\beta^{2}_{bik}\beta^{1}_{bjm}), \quad \text{(D17)}$$

$$D^{(6)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = \beta^{1*}_{aim}\beta^{1*}_{ajk}\beta^{2}_{bik}\beta^{1}_{bjm} + \beta^{2*}_{aim}\beta^{2*}_{ajk}\beta^{1}_{bik}\beta^{2}_{bjm}, \quad \text{(D18)}$$

$$D^{(7)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = -(\beta^{1*}_{aim}\beta^{1*}_{ajk}\beta^{1}_{bik}\beta^{1}_{bjm} + \beta^{2*}_{aim}\beta^{2*}_{ajk}\beta^{2}_{bik}\beta^{2}_{bjm}), \quad \text{(D19)}$$

$$D^{(8)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0} = \beta^{1*}_{aim}\beta^{1*}_{ajk}\beta^{2}_{bik}\beta^{2}_{bjm} + \beta^{2*}_{aim}\beta^{2*}_{ajk}\beta^{1}_{bik}\beta^{1}_{bjm}. \quad \text{(D20)}$$

APPENDIX E: $\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- \nu_k \bar{\nu}_m)$

1. Matrix elements

The decay to neutrinos is mediated by charginos $\tilde{\chi}_a^-$ and the Z boson, and so the matrix element is

$$\mathcal{M} = \sum_{a=1}^{2} \mathcal{M}_{\tilde{\chi}_a^-} + \mathcal{M}_Z, \tag{E1}$$

where

$$\mathcal{M}_{\tilde{\chi}_a^-} = -i\bar{u}(p_k)(i\gamma^*_{aik}P_R)\frac{(\not{p}_k - \not{p}_i) + m_a}{(p_i - p_k)^2 - m_a^2} \times (i\gamma_{ajm}P_L)v(p_m) = i(\gamma^*_{aik}\gamma_{ajm})\bar{u}(p_k)\frac{\not{p}_k - \not{p}_i}{(p_i - p_k)^2 - m_a^2}P_L v(p_m), \tag{E2}$$

$$\mathcal{M}_Z = \frac{ig\zeta_{ji}^{(2)}}{2\cos\theta_W}\frac{1}{(p_i - p_j)^2 - m_Z^2} \times \bar{u}(p_k)(\not{p}_i + \not{p}_j)P_L v(p_m). \tag{E3}$$

2. Total width

The width is therefore

$$\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- \nu_k \bar{\nu}_m) = \frac{1}{64\pi^3 m_i}\int_{\mathrm{PS}_3} |\mathcal{M}|^2 = \frac{1}{64\pi^3 m_i}\int_{\mathrm{PS}_3}\Big(\sum_{a,b=1}^{4} \mathcal{M}_{\tilde{\chi}_a^-}\mathcal{M}^*_{\tilde{\chi}_b^-} + \mathcal{M}_Z\mathcal{M}_Z^* + 2\sum_{b=1}^{4}\mathrm{Re}[\mathcal{M}_Z\mathcal{M}^*_{\tilde{\chi}_b^-}]\Big) = \Gamma_{\tilde{\chi}^-\tilde{\chi}^-} + \delta_{km}(\Gamma_{ZZ} + 2\Gamma_{Z\tilde{\chi}^-}), \tag{E4}$$

where the partial widths are defined below.

3. $\tilde{\chi}^-\tilde{\chi}^-$ contribution

The chargino-mediated width is

$$\Gamma_{\tilde{\chi}^-\tilde{\chi}^-} = \frac{m_i}{512\pi^3}\sum_{a,b=1}^{2}\gamma^*_{aik}\gamma_{ajm}\gamma_{bik}\gamma^*_{bjm} \times \int_0^{1-r_j^2} dx \frac{x^2(1-x-r_j^2)^2}{(1-x)(1-x-r_a^2)(1-x-r_b^2)} = \frac{m_i}{512\pi^3}\sum_{a,b=1}^{2}\gamma^*_{aik}\gamma_{ajm}\gamma_{bik}\gamma^*_{bjm} \times \int_{x_-}^{x_+} dx x(1-x)(1-x-r_j^2)f(x, r_a, r_b) = \frac{m_i}{512\pi^3}\sum_{a,b=1}^{2}\gamma^*_{aik}\gamma_{ajm}\gamma_{bik}\gamma^*_{bjm}I_{\tilde{\chi}_a^-\tilde{\chi}_b^-}. \tag{E5}$$

Here the integral $I_{\tilde{\chi}_a^-\tilde{\chi}_b^-}$ is the same as $I^{(1)}_{\tilde{\chi}_a^0\tilde{\chi}_b^0}$ for the charge-preserving case, except the neutralino mass is replaced by the chargino mass and lepton masses m_k and m_m are set to zero.

4. ZZ contribution

The Z-mediated width is

$$\Gamma_{ZZ} = \frac{m_i}{256\pi^3}\int_{x_-}^{x_+} dx \int_{y_-}^{y_+} dy \mathcal{M}_Z\mathcal{M}_Z^* = \frac{m_i}{512\pi^3}\left|\frac{g\zeta_{ji}^{(2)}}{\cos\theta_W}\right|^2\int_{x_-}^{x_+} dx[2(x-1)i^{(2)}(x, r_Z) + 2(1-x-r_j^2)i^{(1)}(x, r_Z)] = \frac{m_i}{512\pi^3}\left|\frac{g\zeta_{ji}^{(2)}}{\cos\theta_W}\right|^2 I_{ZZ}^{(1)}. \tag{E6}$$

Here again the integral is the same as the ZZ integral for the charge-preserving case except the lepton masses m_k and m_m are set to zero.

5. $Z\tilde{\chi}^-$ contribution

The Z-chargino interference term is

$$\Gamma_{Z\tilde{\chi}_b^-} = \frac{m_i}{256\pi^3}\int_{x_-}^{x_+} dx \int_{y_-}^{y_+} dy \mathcal{M}_Z\mathcal{M}^*_{\tilde{\chi}_b^-} = -\frac{m_i}{512\pi^3}\frac{g\zeta_{ji}^{(2)}}{\cos\theta_W}\mathrm{Re}[\gamma_{bik}\gamma^*_{bjm}]\int_{x_-}^{x_+} dx[2(1-x) \times g^{(2)}(x, r_Z, r_b) + 2(x-1+r_j^2)g^{(1)}(x, r_Z, r_b)] = -\frac{m_i}{512\pi^3}\mathrm{Re}\Big[\frac{g\zeta_{ji}^{(2)}}{\cos\theta_W}\gamma_{bik}\gamma^*_{bjm}\Big]I_{Z\tilde{\chi}_b^-}. \tag{E7}$$

Here the integral $I_{Z\tilde{\chi}_b^-}$ is the same as $I^{(1)}_{Z\tilde{\chi}_b^0}$ for the charge-preserving case, except the neutralino mass is replaced by the chargino mass and lepton masses m_k and m_m are set to zero.

APPENDIX F: $\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- q_k \bar{q}_m)$

1. Matrix elements

The decay modes with daughter quarks are much the same as those for daughter leptons except the neutralino intermediary contribution is removed and the different couplings are substituted. The matrix elements for the up-type quarks are

$$\mathcal{M}_Z = \frac{3ig\zeta_{ji}^{(2)}}{2\cos\theta_W}\frac{1}{(p_i - p_j)^2 - m_Z^2}\bar{u}(p_k)(\not{p}_i + \not{p}_j) \times \Big(P_L - \frac{4}{3}\sin^2\theta_W\Big)v(p_m)\delta_{km}, \tag{F1}$$

$$\mathcal{M}_{h^0} = \frac{-3i\sigma_{ji}^{(2)}y_k^{(u)}\cos\theta_H}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{h^0}^2}\bar{u}(p_k)v(p_m)\delta_{km}, \tag{F2}$$

$$\mathcal{M}_{H^0} = \frac{-3i\sigma_{ji}^{(3)} y_k^{(u)} \sin\theta_H}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{H^0}^2} \bar{u}(p_k) v(p_m) \delta_{km}, \quad \text{(F3)}$$

$$\mathcal{M}_{A^0} = \frac{-3i\sigma_{ji}^{(4)} y_k^{(u)} \cos\beta}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{A^0}^2} \bar{u}(p_k) \gamma^5 v(p_m) \delta_{km}, \quad \text{(F4)}$$

$$\mathcal{M}_{G^0} = \frac{-3i\sigma_{ji}^{(5)} y_k^{(u)} \sin\beta}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_Z^2} \bar{u}(p_k) \gamma^5 v(p_m) \delta_{km}, \quad \text{(F5)}$$

where the factor of 3 is the color factor, and those for the down-type quarks are

$$\mathcal{M}_Z = \frac{3ig\zeta_{ji}^{(2)}}{2\cos\theta_W} \frac{1}{(p_i - p_j)^2 - m_Z^2} \bar{u}(p_k)(\not{p}_i + \not{p}_j) \times \left(\frac{2}{3}\sin^2\theta_W - P_L\right) v(p_m) \delta_{km}, \quad \text{(F6)}$$

$$\mathcal{M}_{h^0} = \frac{3i\sigma_{ji}^{(2)} y_k^{(d)} \sin\theta_H}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{h^0}^2} \bar{u}(p_k) v(p_m) \delta_{km}, \quad \text{(F7)}$$

$$\mathcal{M}_{H^0} = \frac{-3i\sigma_{ji}^{(3)} y_k^{(d)} \cos\theta_H}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{H^0}^2} \bar{u}(p_k) v(p_m) \delta_{km}, \quad \text{(F8)}$$

$$\mathcal{M}_{A^0} = \frac{3i\sigma_{ji}^{(4)} y_k^{(d)} \sin\beta}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_{A^0}^2} \bar{u}(p_k) \gamma^5 v(p_m) \delta_{km}, \quad \text{(F9)}$$

$$\mathcal{M}_{G^0} = \frac{-3i\sigma_{ji}^{(5)} y_k^{(d)} \cos\beta}{\sqrt{2}} \times \frac{1}{(p_i - p_j)^2 - m_Z^2} \bar{u}(p_k) \gamma^5 v(p_m) \delta_{km}. \quad \text{(F10)}$$

The total decay width is then

$$\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- q_k \bar{q}_k) = \Gamma_{ZZ} + \Gamma_{HH} + 2\Gamma_{ZH}. \quad \text{(F11)}$$

2. Up-type quarks

The decay width to up-type quarks is

$$\begin{aligned}\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- u_k \bar{u}_k) = {}& \frac{9m_i}{512\pi^3} \left| \frac{g\zeta_{ji}^{(2)}}{\cos\theta_W} \right|^2 \sum_{t=1}^{2} A_{ZZ}^{(t)} I_{ZZ}^{(t)} \\ &+ \frac{9}{256\pi^3 m_i} (A_{h^0h^0} I^r_{h^0h^0} + A_{H^0H^0} I^r_{H^0H^0} \\ &+ 2A_{h^0H^0} I^r_{h^0H^0} + A_{A^0A^0} I^p_{A^0A^0} \\ &+ A_{G^0G^0} I^p_{G^0G^0} + 2A_{A^0G^0} I^p_{A^0G^0}) \\ &+ 2\left[\frac{9g\zeta_{ji}^{(2)}}{256\sqrt{2}\pi^3 \cos\theta_W} (A_{Zh^0} I^r_{Zh^0} \right. \\ &\left. + A_{ZH^0} I^r_{ZH^0} + A_{ZA^0} I^p_{ZA^0} + A_{ZG^0} I^p_{ZG^0})\right], \end{aligned} \quad \text{(F12)}$$

with integrals from Appendix C, with quark masses substituted. The coefficients are

$$A_{ZZ}^{(1)} = \frac{32}{9}\sin^4\theta_W - \frac{8}{3}\sin^2\theta_W + 1, \quad \text{(F13)}$$

$$A_{ZZ}^{(2)} = \frac{32}{9}\sin^4\theta_W - \frac{8}{3}\sin^2\theta_W, \quad \text{(F14)}$$

$$A_{h^0h^0} = |\sigma_{ji}^{(2)} y_k^{(u)} \cos\theta_H|^2, \quad \text{(F15)}$$

$$A_{H^0H^0} = |\sigma_{ji}^{(3)} y_k^{(u)} \sin\theta_H|^2, \quad \text{(F16)}$$

$$A_{h^0H^0} = \mathrm{Re}[\sigma_{ji}^{(2)} \sigma_{ji}^{(3)*} y_k^{(u)2} \sin\theta_H \cos\theta_H], \quad \text{(F17)}$$

$$A_{A^0A^0} = |\sigma_{ji}^{(4)} y_k^{(u)} \cos\beta|^2, \quad \text{(F18)}$$

$$A_{G^0G^0} = |\sigma_{ji}^{(5)} y_k^{(u)} \sin\beta|^2, \quad \text{(F19)}$$

$$A_{A^0G^0} = \mathrm{Re}[\sigma_{ji}^{(4)} \sigma_{ji}^{(5)*} y_k^{(u)2} \sin\beta \cos\beta], \quad \text{(F20)}$$

$$A_{Zh^0} = -\mathrm{Re}[\sigma_{ji}^{(2)} y_k^{(u)} \cos\theta_H (\tfrac{8}{3}\sin^2\theta_W - 1)], \quad \text{(F21)}$$

$$A_{ZH^0} = -\mathrm{Re}[\sigma_{ji}^{(3)} y_k^{(u)} \sin\theta_H (\tfrac{8}{3}\sin^2\theta_W - 1)], \quad \text{(F22)}$$

$$A_{ZA^0} = -\mathrm{Re}[\sigma_{ji}^{(4)} y_k^{(u)} \cos\beta], \quad \text{(F23)}$$

$$A_{ZG^0} = -\mathrm{Re}[\sigma_{ji}^{(5)} y_k^{(u)} \sin\beta]. \quad \text{(F24)}$$

3. Down-type quarks

The decay width to down-type quarks is

$$\Gamma(\tilde{\ell}_i^- \to \tilde{\ell}_j^- d_k \bar{d}_k) = \frac{9m_i}{512\pi^3}\left|\frac{g\zeta_{ji}^{(2)}}{\cos\theta_W}\right|^2 \sum_{t=1}^{2} B_{ZZ}^{(t)} I_{ZZ}^{(t)} + \frac{9}{256\pi^3 m_i}(B_{h^0h^0}I^r_{h^0h^0} + B_{H^0H^0}I^r_{H^0H^0} + 2B_{h^0H^0}I^r_{h^0H^0} + B_{A^0A^0}I^p_{A^0A^0} + B_{G^0G^0}I^p_{G^0G^0} + 2B_{A^0G^0}I^p_{A^0G^0}) + 2\left[\frac{9g\zeta_{ji}^{(2)}}{256\sqrt{2}\pi^3\cos\theta_W}(B_{Zh^0}I^r_{Zh^0} + B_{ZH^0}I^r_{ZH^0} + B_{ZA^0}I^p_{ZA^0} + B_{ZG^0}I^p_{ZG^0})\right], \tag{F25}$$

where, again, the integrals are equivalent to those defined in Appendix C with quark masses substituted. The coefficients are

$$B_{ZZ}^{(1)} = \tfrac{8}{9}\sin^4\theta_W - \tfrac{4}{3}\sin^2\theta_W + 1, \tag{F26}$$

$$B_{ZZ}^{(2)} = \tfrac{8}{9}\sin^4\theta_W - \tfrac{4}{3}\sin^2\theta_W, \tag{F27}$$

$$B_{h^0h^0} = |\sigma_{ji}^{(2)} y_k^{(d)} \sin\theta_H|^2, \tag{F28}$$

$$B_{H^0H^0} = |\sigma_{ji}^{(3)} y_k^{(d)} \cos\theta_H|^2, \tag{F29}$$

$$B_{h^0H^0} = -\mathrm{Re}[\sigma_{ji}^{(2)}\sigma_{ji}^{(3)*} y_k^{(d)2} \sin\theta_H \cos\theta_H], \tag{F30}$$

$$B_{A^0A^0} = |\sigma_{ji}^{(4)} y_k^{(d)} \sin\beta|^2, \tag{F31}$$

$$B_{G^0G^0} = |\sigma_{ji}^{(5)} y_k^{(d)} \cos\beta|^2, \tag{F32}$$

$$B_{A^0G^0} = -\mathrm{Re}[\sigma_{ji}^{(4)}\sigma_{ji}^{(5)*} y_k^{(d)2} \sin\beta\cos\beta], \tag{F33}$$

$$B_{Zh^0} = -\mathrm{Re}[\sigma_{ji}^{(2)} y_k^{(d)} \sin\theta_H(\tfrac{4}{3}\sin^2\theta_W - 1)], \tag{F34}$$

$$B_{ZH^0} = \mathrm{Re}[\sigma_{ji}^{(3)} y_k^{(d)} \cos\theta_H(\tfrac{4}{3}\sin^2\theta_W - 1)], \tag{F35}$$

$$B_{ZA^0} = -\mathrm{Re}[\sigma_{ji}^{(4)} y_k^{(d)} \sin\beta], \tag{F36}$$

$$B_{ZG^0} = \mathrm{Re}[\sigma_{ji}^{(5)} y_k^{(d)} \cos\beta]. \tag{F37}$$

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