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Child Mortality and Fertility Decline: Does the Barro-Becker Model Fit the Facts?*

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Abstract

I compare the predictions of three variants of the altruistic-parent model by Barro and Becker for the relationship between child mortality and fertility. In the baseline model, fertility choice is continuous, and there is no uncertainty over the number of surviving children. The baseline model is contrasted to an extension with discrete fertility choice and stochastic mortality, and a setup with sequential fertility choice. The quantitative predictions of the models are remarkably similar. While in each model the total fertility rate falls as child mortality declines, the number of surviving children increases. The results suggest that factors other than declining infant and child mortality are responsible for the large decline in net reproduction rates observed in industrialized countries over the last century.

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1 Introduction

In 1861, the average woman in England had five children over her lifetime. However, only 70 percent of newborn children would live to see their tenth birthday. By 1951, average fertility had fallen to just over two children per woman, and only five percent of children would die in their first ten years of life. A similar pattern of declining fertility and mortality rates, collectively known as the demographic transition, has been observed in every industrializing country. Recently, a number of economists have developed macroeconomic theories that integrate an account of the demographic transition with theories of long-run economic growth. However, in most cases these studies have concentrated on the fertility aspect of the demographic transition, while abstracting from mortality decline (see, for example, Galor and Weil 2000 and Greenwood and Seshadri 2002). Demographers, in contrast, have pointed out that in many cases mortality decline precedes fertility decline, which suggests a causal link from falling mortality to falling fertility.

One reason why the macroeconomic literature has abstracted from mortality decline as a cause for fertility decline is that commonly used economic models of fertility are inconsistent with such a link. In particular, this is true for the model of Barro and Becker (1989), where parents are altruistic towards their surviving children. In the Barro-Becker model, infant and child mortality rates affect choices only to the degree that they influence the overall cost of a surviving child. Falling mortality rates tend to lower the cost of having a surviving child, hence fertility actually increases, not decreases, as mortality declines (this is discussed in Boldrin and Jones 2002 and Fernández-Villaverde 2001). Instead of emphasizing mortality decline, the Barro-Becker framework points to the quantity-quality tradeoff as an explanation for fertility decline: parents choose to have smaller families in order to invest more in the education of each child.

In this paper, I examine whether simple extensions of the Barro-Becker model can overturn its predictions for the link of mortality and fertility. In the baseline Barro-Becker model, fertility is treated as a continuous choice, all fertility decisions are made at one point in time, and there is no uncertainty over the number of surviving children. Richer models that allow for uncertainty and sequential fertility choice may lead to different implications. In particular, when mortality is stochastic and parents

want to avoid the possibility of ending up with very few (or zero) surviving children, a “precautionary” demand for children arises. Kalemli-Ozcan (2002) argues that when this effect is taken into account, declining child mortality can have a negative impact on fertility. If fertility is chosen sequentially, there is also a “replacement” effect: parents may condition their fertility decisions on the survival of children that were born previously.

To analyze whether these effects are quantitatively important, I examine three extensions of the basic Barro-Becker framework. The first model allows for different costs per birth and per surviving child, but is otherwise identical to the Barro-Becker setup. In the second model, fertility choice is restricted to be an integer, and there is mortality risk. The third extension adds sequential fertility choice.¹ The three models are compared with regards to their theoretical and quantitative implications regarding the link between infant and child mortality and fertility.

The main conclusion is twofold. All three models are consistent with a falling total fertility rate in response to declining child mortality. However, none of the models predicts that the net fertility rate (i.e., the number of surviving children) declines with child mortality. In other words, the analysis suggests that mortality decline may be one factor behind falling fertility during the demographic transition, but certainly not the only or even the main factor.

2 Three Variations on Altruistic Parents and Fertility

As the benchmark case, I consider the model by Barro and Becker (1989) with continuous fertility choice. In this model, parents care about their own consumption c and the number n and utility V of their surviving children.² The utility function is:

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} + \beta n^\epsilon V.$$

¹Fertility models with stochastic outcomes and sequential choices have been used in the empirical fertility literature, see Wolpin (1997). A model with sequential fertility choices and child mortality, but without stochastic outcomes has been considered by Eckstein, Mira, and Wolpin (1999).

²If parents can choose education, V becomes an endogenous variable. Since mortality is concentrated in the first few years of life, while education occurs later, child mortality and education decisions do not interact. Therefore, I abstract from education choice.

Throughout the paper, it is assumed that $\sigma, \beta, \epsilon \in (0, 1)$ and $V > 0$.³ Let b denote the number of births, and s is the probability of survival for each child, where $0 < s \leq 1$. Mortality is deterministic in the sense that s is the fraction of children surviving. Consequently, the number of surviving children is not constrained to be an integer. The full income of a parent is denoted by w . Since w is taken as given, the distinction between time and goods costs for children is irrelevant. It is assumed that each birth is associated with a cost of p , and each surviving child entails an additional cost of q . The budget constraint is then $c + pb + qn \leq w$ or, after plugging in the survival function $n = sb$:

$$c + (p + qs)b \leq w.$$

Income and cost parameters satisfy $w > 0$, $p \geq 0$, $q \geq 0$, and $p + q > 0$. At least one of the costs has to be strictly positive; otherwise, the optimal fertility choice is infinity. Both consumption and fertility are restricted to be nonnegative. The decision problem in the standard version of the Barro-Becker model is:

Problem A: (Barro-Becker with continuous fertility choice)

$$\max_{0 \leq b \leq w/(p+qs)} \left\{ \frac{(w - (p + qs)b)^{1-\sigma}}{1 - \sigma} + \beta(sb)^\epsilon V. \right\}$$

I will now consider two variations of the Barro-Becker framework which add realism to the benchmark model. The first extension introduces stochastic survival and restricts fertility choice to be an integer. In this model, the realized number of children is uncertain. I assume that for each birth there is a constant probability of death, implying that the distribution of surviving children is Binomial. Apart from the integer restriction and stochastic survival, the model is identical to the benchmark. The decision problem is now given by:

Problem B: (Stochastic Barro-Becker with discrete fertility choice)

$$\max_{b \in \{\mathcal{N} \cup 0\}, b \leq w/(p+q)} \left\{ \sum_{n=0}^b \left(\frac{(w - pb - qn)^{1-\sigma}}{1 - \sigma} + \beta n^\epsilon V \right) \binom{b}{n} s^n (1-s)^{b-n} \right\}.$$

³The deterministic model can be extended to risk-aversion parameters equal to one (log utility) or bigger than one. However, in those cases the utility associated with having zero children is negative infinity, so that the choice problem under uncertainty (where zero surviving children occur with positive probability) is not well defined.

The second extension adds yet more realism by allowing sequential fertility choice, while preserving the integer constraint and stochastic survival of Problem B. In the sequential model, the period is divided into $T + 1$ subperiods, running from 0 to T . Parents have a fixed income of w in each subperiod. The parameter $\gamma \in (0, 1)$ is the discount factor between periods. In each period, parents can give birth to a single child. Since children live for multiple periods, the setup allows to distinguish infant and child mortality. Newborn infants survive with probability s_i until the next period. If the child survives, the probability of surviving the second period of life is s_y . Once a child has survived for two periods, it will survive until adulthood for sure.⁴ $b_t \in \{0, 1\}$ denotes the birth decision in period t , $y_t \in \{0, 1\}$ represents a young child (born in the preceding period), and n_t is the number of older children (born at least two periods prior) alive in period t . The cost per birth b_t is given by p , a young child y_t is associated with cost q , and older children n_t do not involve further expenses.⁵ The budget constraint in period t is $c_t + pb_t + qy_t \leq w$.

In the sequential model, parents are able to decide on fertility conditional on the survival of older children. Formally, the choice object of the parent is a sequence of decision rules $\{b_t : H_t \rightarrow \{0, 1\}\}_{t=0}^T$ which map the state h_t at time t into a birth decision. A parent is fecund only until period K , which imposes the additional constraint $b_t = 0$ for $K < t \leq T$. This constraint is imposed to provide a motive for “hoarding” of children. If a child dies after period K , it cannot be replaced. The state at time t is given by $h_t = \{n_t, y_t\}$, where $n_t \geq 0$ is the number of children that were born at least two periods ago and survived, and $y_t \in \{0, 1\}$ denotes whether there is a young child that was born in the preceding period. Since there is at most one birth per period, the maximum number of children is K . The state space is therefore $H_t = \{0, 1, \dots, K\} \times \{0, 1\}$. The evolution of the number of children depends on the number of older children n_t , whether there is a newborn b_t and a young child y_t , and on the survival probabilities. Specifically, for a parent that has n_t older children today, the probability of having $n_t + 1$ tomorrow is zero when there is no young child,

⁴The model could be extended to allow for a richer set of age-specific survival probabilities, but two survival probabilities are sufficient to contrast the sequential setup to the case of simultaneous fertility choice. In the data, mortality is highly concentrated in the first few years of a child’s life.

⁵This assumption can be justified through the economic benefits of older children in terms of child labor and help in the household. The model could be extended to a richer cost profile. It is important, however, that children do not cause expenses forever, because then late-born children would be cheaper overall than older children.

and s_y if a young child exists. Similarly, the probability of having a young child y_t in the next period is s_i if there is a newborn in this period, and zero otherwise. The probabilities over states are therefore defined recursively as:

$$\begin{aligned}
P_{t+1}(n, y) = & P_t(n, 0) (1 - y + (2y - 1)b_t(n, 0)s_i) \\
& + P_t(n, 1) (1 - y + (2y - 1)b_t(n, 1)s_i) (1 - s_y) \\
& + P_t(n - 1, 1) (1 - y + (2y - 1)b_t(n - 1, 1)s_i) s_y.
\end{aligned} \tag{1}$$

For example, consider the probability of having three old children and one young child in period six ($n = 3, y = 1$). This state can only be reached if in period five there are either three old children, or two old children and a young child. Therefore (1) sums over the respective probabilities in period five. Also, there has to be a birth in period five, and the infant has to survive, since otherwise there would be no young child in period six. Therefore, each probability is multiplied by $b_5(n, y)s_i$. If the state in period five is $\{3, 1\}$, there are three old children in period six only if the young child dies. Therefore, the respective probability is also multiplied by $1 - s_y$. Finally, if there are only two old children in period five, the young child has to survive if there are to be three old children in period six. Hence, the last term is multiplied by s_y . The probability of having n children survive into adulthood is:

$$P(n) = P_T(n, 1) (1 - s_y) + P_T(n, 0) + P_T(n - 1, 1) s_y. \tag{2}$$

Birth decisions do not enter here, since there are no births in the final period of adulthood T . The decision problem in the sequential model is:

Problem C: (Stochastic Barro-Becker with discrete and sequential fertility choice)

$$\max_{\{b_t\}_{t=0}^T} \left\{ \sum_{t=0}^T \sum_{h_t \in H_t} \gamma^t \frac{(w - pb_t(h_t) - qy_t)^{1-\sigma}}{1 - \sigma} P_t(h_t) + \beta \sum_{n=0}^N n^\epsilon VP(n) \right\},$$

where the probabilities over states $P_t(h_t)$ and surviving children $P(n)$ are functions of the birth decisions as defined in (1) and (2) above, and the initial probabilities are given by $P_0(0, 0) = 1$ and $P_0(h_0 \neq \{0, 0\}) = 0$ (adults start without children).

3 Mortality Decline and Fertility: Analytical Findings

In this section, I examine the effect of mortality decline on fertility in the three variants of the altruistic-parents model. All proofs are contained in the appendix.

Proposition 1 *Let $b(s)$ denote the solution to Problem A as a function of s . $b(s)$ has the following properties:*

- *The number of surviving children $sb(s)$ is non-decreasing in s .*
- *If $p > 0$ and $q = 0$, fertility $b(s)$ is increasing in s .*
- *If $p = 0$ and $q > 0$, fertility $b(s)$ is decreasing in s and $sb(s)$ is constant.*

The intuition for these results is simple. Since parents care only about surviving children and there is no uncertainty, the survival probability s affects choices only through the full cost of a surviving child $p/s + q$. Raising s lowers this cost, and through the substitution effect therefore increases the number of surviving children. In the special case where the cost p for each birth is zero, the total cost of a surviving child is independent of s , and consequently parents choose the preferred number of surviving children irrespective of s . Total fertility can fall as mortality declines if the cost of births is relatively low, but net fertility (the number of surviving children) never declines as mortality falls. I turn to the stochastic models next.

Proposition 2 *Let $b(s)$ denote the solution to Problem B as a function of s . If $p = 0$, the optimal choice $b(s)$ is non-increasing in s .*

Proposition 3 *Let $b_t(h_t)(s_i)$ denote the solution to Problem C as function of the infant survival probability s_i at a given state h_t . If $p = 0$ and $s_y = 1$, $b_t(h_t)(s_i)$ is non-increasing in s_i .*

Thus in both stochastic models, we find that if births are costless, the optimal number of births declines as survival rates increase. In the sequential model, we get the additional implication that age at first birth increases. However, there are no clear-cut results regarding net fertility. If utility is highly concave in n , parents want to

avoid a low number of surviving children. If mortality is high, this can give rise to a “precautionary” demand for children, which declines as mortality (and therefore uncertainty) decreases. However, the opposite effect is also possible, since utility is concave in consumption as well. If parents are very risk averse in terms of consumption, they might want to avoid the risk of having too many surviving children (and thereby high expenditures on children), which would lower the number of births when mortality is high. While these effects apply in principle to both Problem B and Problem C, the model with sequential fertility choice is in some sense in between the deterministic and the stochastic model. Since choices are spread out over time, parents have the possibility of replacing children that die early in the life cycle, leading to less uncertainty over the realized number of children than in Problem B, where all children are born simultaneously.

4 Mortality Decline and Fertility: Quantitative Findings

The analytical results show that all models are consistent with declining fertility rates in response to falling mortality. However, while the deterministic model predicts that the number of surviving children rises as mortality falls, the more elaborate models do not make clear-cut predictions. Therefore, I assess the quantitative predictions of the models with a calibration exercise. Each model is parameterized to reproduce mortality and fertility rates in England in 1861, when infant and child mortality was still high. I then increase the survival parameters to correspond to mortality rates in 1951 (by which time most of the fall in infant and child mortality had been completed) and compare the predictions of each model for the impact on fertility rates.

The models are parameterized as follows. In the sequential model, we set $T = 14$ and $K = 12$, so that the maximum number of births is 13. Income w is a scale parameter and is set to 1 per period in the sequential model and 14 in the other models. The parameter p corresponds to the cost of a child until its first birthday, while the parameter q accounts for the remaining cost. In terms of goods, it is natural to assume that the yearly cost increases until the child is able to work and partly pay for itself. The time cost, on the other hand, decreases over time. In addition, the cost per birth should account for the cost of pregnancy and the risk of the mother’s death during childbirth. Since time and goods cost move in opposite directions, I assume as the

baseline case that overall cost is proportional to age, and that children are no longer a net burden once they are six years old. I therefore set $q/p = 5$. The overall level of the cost parameters is set such that in the sequential model, a household with both an infant and a young child spends half of its income on the children. This gives $p = 1/12$ and $q = 5/12$. The curvature parameters in the utility function are set to $\sigma = \epsilon = 1/2$, and the discount factor in the sequential model is $\gamma = 0.95$. The children's utility level V is equated to the parent's utility in each case (i.e., the steady-state utility that would obtain with constant income and mortality rates). The survival parameters are chosen to correspond to the situation in England in 1861. According to Preston, Keyfitz, and Schoen (1972) the infant mortality rate (death rate until first birth rate) was 16 percent, while the child mortality rate (death rate of between first and fifth birthday) was 13 percent. Accordingly, I set $s_i = 0.84$ and $s_y = 0.87$ in the sequential model, and $s = s_i s_y = 0.73$ in the other models. Finally, the altruism factor β is set in each model to match the total fertility rate, which was 4.9 in 1861 (Chesnais 1992). Since fertility choice is discrete in Models B and C, I chose a total fertility rate of 5.0 as the target.

Each model is thus calibrated to reproduce the relationship of fertility and infant and child mortality in 1861. I now examine how fertility adjusts when mortality rates fall to the level observed in 1951, which is 3 percent for infant mortality and 0.5 percent for child mortality. The results for fertility can be compared to the observed total fertility rate of 2.1 in 1951. In Model A (Barro-Becker with continuous fertility choice), the total fertility rate falls from 5.0 (the calibrated target) to 4.2 when mortality rates are lowered to the 1951 level. The expected number of surviving children increases from 3.7 to 4.0. Thus while there is a small decline in total fertility, the net fertility rate increases. While given Proposition 1 this was to be expected, it is surprising that Model B (stochastic Barro-Becker with discrete fertility choice) generates very similar results. In the stochastic model, total fertility falls from 5.0 to 4.0, and net fertility increases from 3.7 to 3.9. While fertility falls more than in the continuous model, the difference is small. In Model C (sequential fertility choice), the total fertility rate is not an integer since it depends on the random individual mortality outcomes. Therefore, β was chosen to move the total fertility rate to 5.2, which is the closest possible match. When mortality is lowered to 1951 levels, fertility falls only to 5.0, while net fertility increases substantially from 3.8 to 4.8. These results are partly due to the fact that the sequential model distinguishes infant and child mortality, while the other models

do not. The models line up more closely if we set $s_y = 1$ and assign the entire fall in mortality to infant mortality s_i (as we do implicitly in the other two models). In this case, total fertility falls from 5.1 to 4.0, while net fertility increases from 3.7 to 3.9. This is identical to the results with Model B.⁶ Figures 1 to 3 show that the predictions of the models are similar for the entire range of possible infant mortality rates (solid line is total fertility, dotted line net fertility rate; for Figure 3, child mortality was set to $s_y = 1$). The sequential model yields additional predictions for the age at first birth, which increases with the survival probability once s_i is at least 10 percent. This increase not only reflects the corresponding decline in total fertility, but also narrower spacing of births. When mortality is high, parents start to have children early so that there is time to make up for children who die. This replacement motive is less important when survival rates are high.

In summary, each model predicts that total fertility falls with infant mortality, but none of the models predicts a fall in net fertility rates. Relative to the data, the models suggest that only a small proportion of observed fertility decline, and none of the net fertility decline, is accounted for by declining infant mortality. The question arises whether the results are specific to the calibrated parameter values. In other words, are there reasonable parameters for which any of the models predicts a substantial decline in net fertility as infant mortality declines? We know from Proposition 1 that this can never be the case in Model A. In the other models, however, a “precautionary” demand for children can arise if parents’ utility is highly concave in the number of children, but close to linear in consumption. Indeed, if we choose the (somewhat extreme) utility parameters $\sigma = \epsilon = .01$ and adjust β to keep fertility at 5.0 given 1861 mortality rates, in Model B total fertility falls from 5.0 to 2.0, and net fertility from 3.7 to 1.9 when mortality rates drop to their values in 1951. This effect disappears entirely, however, when we move (with the same parameters) to the more realistic sequential model, where parents can replace children who die early. Here, despite the extreme risk-aversion with regards to the number of children, total fertility drops only to 4.0, and net fertility rises to 3.9, just as with the benchmark parameters. Figures 5 and 6 show fertility rates over the entire range of mortality rates in the two

⁶In the computations, the children’s utility V was held constant. However, results are virtually unchanged if V is adjusted to reflect the new (higher) steady-state utility. We also disregard the increase in income per capita over the period, since with the chosen functional forms fertility is independent of the level of income (assuming that the cost of children is proportional to income).

models. Thus, once we allow for sequential fertility choice, the conclusion that only a minor fraction of observed fertility decline is explained by mortality decline is robust with regards to the choice of parameters.

5 Conclusions

All three models discussed in this paper lead to the same conclusion: declines in child mortality lower total fertility rates, but do not cause substantial decreases in net fertility. It is in line with this finding that van de Walle (1986) finds only a loose association between the exact timing of infant mortality and fertility decline in a number of European countries and regions. This is particularly true for England, where rapid fertility decline started in 1880, but infant mortality stayed relatively high until early in the twentieth century. Thus, while mortality decline may contribute to an overall explanation of fertility decline, our findings provide no reason to discard alternative theories based on the quantity-quality tradeoff, old-age security, or the role of child labor.

In terms of modeling choices, we find that the implications of the baseline model are virtually the same as those of the more sophisticated setup with sequential choice and stochastic outcomes. While the sequential model yields additional predictions regarding the timing and spacing of births which are important in certain applications (see Caucutt, Guner, and Knowles 2002), as long as only total and net fertility rates are of interest, the standard Barro-Becker model appears to be a useful approximation.

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A Mathematical Appendix

Proof of Proposition 1: Problem A is given by:

$$\max_{0 \leq b \leq w/(p+qs)} \left\{ \frac{(w - (p + qs)b)^{1-\sigma}}{1 - \sigma} + \beta(sb)^\epsilon V. \right\} \quad (3)$$

The assumptions on parameter values ($\sigma, \epsilon, \beta \in (0, 1), s \in (0, 1], p, q \geq 0, w, V, p + q > 0$) guarantee that (3) is strictly concave in b and that an interior optimum exists. The optimal number of births $b(s)$ as a function of the survival probability s is characterized by the first-order condition:

$$\frac{(p + qs) s^{-\epsilon} b(s)^{1-\epsilon}}{(w - (p + qs)b(s))^\sigma} = \beta\epsilon V, \quad (4)$$

which can be written as:

$$\frac{(p + qs) (sb(s))^{1-\epsilon}}{s^{1-\sigma} (ws - (p + qs)sb(s))^\sigma} = \beta\epsilon V$$

or:

$$\frac{(p/s + q)^{1-\sigma} (sb(s))^{1-\epsilon}}{(w/(p/s + q) - sb(s))^\sigma} = \beta\epsilon V. \quad (5)$$

Clearly, there is a unique $b(s)$ which satisfies (5) for any s . Notice that the term $(p/s + q)$ is non-increasing in s (strictly decreasing if $p > 0$), while the term $w/(p/s + q)$ is non-decreasing in s (strictly increasing if $p > 0$). Since (5) has to be satisfied for all s , $sb(s)$ is therefore non-decreasing in s (strictly increasing if $p > 0$), which proves the first part of the claim.

If $q = 0$, (4) simplifies to:

$$\frac{p b(s)^{1-\epsilon}}{s^\epsilon (w - pb(s))^\sigma} = \beta\epsilon V. \quad (6)$$

Since the left-hand side is strictly decreasing in s^ϵ and (6) has to be satisfied for all s , $b(s)$ is strictly increasing in s , which proves the second part of the claim.

Finally, if $p = 0$ (4) simplifies to:

$$\frac{q (sb(s))^{1-\epsilon}}{(w - qsb(s))^\sigma} = \beta\epsilon V. \quad (7)$$

Since s only enters through $sb(s)$, net fertility $sb(s)$ has to be constant for all s to satisfy (7), which proves the last part of the claim. \square

Proof of Proposition 2: We are considering Problem B under the assumption that the

per-birth cost is zero, $p = 0$. In this case, Problem B is a special case of the model analyzed by Sah (1991), and the results derived there apply. Specifically, define:

$$u(n) = \frac{(w - qn)^{1-\sigma}}{1-\sigma} + \beta n^\epsilon V,$$

and:

$$U(b, s) = \sum_{n=0}^b u(n) \binom{b}{n} s^n (1-s)^{b-n}.$$

Problem B is to maximize $U(b, s)$ by choice of b , and $U(n)$ is strictly concave in n and does not depend on b or s . The model is now in the form of Sah (1991), and since the concavity assumption is satisfied, the proof for Proposition 2 in Sah (1991) applies here as well.

□

To prove Proposition 3, it is useful to first develop some additional notation. The assumptions $p = 0$ and $s_y = 1$ are maintained throughout. Let $V_t(h_t)$ be the utility at time $t \leq T$ given that state h_t has been realized. These utilities are given by:

$$V_T(n, y) = \frac{(w - qy)^{1-\sigma}}{1-\sigma} + \beta [(1-y)n^\epsilon + y(n+1)^\epsilon] V \quad (8)$$

for $t = T$ and:

$$V_t(n, y) = \frac{(w - qy)^{1-\sigma}}{1-\sigma} + \gamma b_t(n, y) s_i \left[(1-y) V_{t+1}(n, 1) + y V_{t+1}(n+1, 1) \right] \quad (9)$$

$$+ \gamma (1 - b_t(n, y) s_i) \left[(1-y) V_{t+1}(n, 0) + y V_{t+1}(n+1, 0) \right]$$

for $0 \leq t \leq T$. Optimal birth decisions are determined by:

$$b_t(n, y) = \operatorname{argmax}_{b \in \{0,1\}} \left\{ b \left[(1-y) V_{t+1}(n, 1) + y V_{t+1}(n+1, 1) \right] \right. \quad (10)$$

$$\left. + (1-b) \left[(1-y) V_{t+1}(n, 0) + y V_{t+1}(n+1, 0) \right] \right\},$$

with the additional restriction that $b_t(n, y) = 0$ for $t > K$. I assume that when a parent is just indifferent, a birth takes place and $b_t(n, y) = 1$. This assumption is for ease of exposition only and does not affect results. We will also need to consider derivatives with respect to s_i . Since the usual derivative may not be well defined for all s_i (b_t is a

step function), we will use left-hand derivatives. We have $\frac{\partial V_T(n,y)}{\partial s_i} = 0$ and for $t < T$:

$$\begin{aligned} \frac{\partial V_t(n,y)}{\partial s_i} &= \gamma b_t(n,y) \left[V_{t+1}(n+y,1) - V_{t+1}(n+y,0) \right] \\ &\quad + \gamma \left[(1 - b_t(n,y)s_i) \frac{\partial V_{t+1}(n+y,0)}{\partial s_i} + b_t(n,y)s_i \frac{\partial V_{t+1}(n+y,1)}{\partial s_i} \right]. \end{aligned} \quad (11)$$

Notice that (10) and (11) imply:

$$b_t(n,1) = b_t(n+1,0) \quad (12)$$

and:

$$\frac{\partial V_t(n,1)}{\partial s_i} = \frac{\partial V_t(n+1,0)}{\partial s_i}. \quad (13)$$

These relations will be used below. The following lemma can now be established:

Lemma 1 *For all t and y , $V_t(n,y)$ is strictly monotone increasing and weakly concave in n . $b_t(n,y)$ is non-increasing in n . $V_t(n,1) - V_t(n,0)$ is non-increasing in n .*

Proof: First, notice that $V_t(n,1)$ is equal to $V_t(n+1,0)$ apart from the first term, which does not depend on n . Concavity of $V_t(n,0)$ (i.e., $V_t(n+1,0) - V_t(n,0)$ is non-increasing in n) is therefore equivalent to $V_t(n,1) - V_t(n,0)$ being non-increasing in n . The last part of the claim is therefore implied once we prove the first part. We also have:

$$V_t(n,1) - V_t(n-1,1) = V_t(n+1,0) - V_t(n,0). \quad (14)$$

Monotonicity and concavity of $V_t(n,0)$ therefore imply the same properties for $V_t(n,1)$. In the induction step below, it therefore suffices to establish these properties for $V_t(n,0)$.

The proof proceeds by induction. The first step is to show that $V_T(n,y)$ is strictly increasing and concave in n . These properties follow directly from the definition (8). Since $T > K$, we also have that $b_T(n,y) = 0$, thus $b_T(n,y)$ is non-increasing in n .

Now assume that $V_{t+1}(n,y)$ is strictly increasing and weakly concave in n for $y \in \{0,1\}$. To complete the induction, we need to show that $V_t(n,0)$ has the same properties and that $b_t(n,y)$ is non-increasing in n . For the last part, it follows from (10) that $b_t(n,0) = 1$ if and only if:

$$V_{t+1}(n,1) - V_{t+1}(n,0) \geq 0.$$

Since we assume that V_{t+1} is increasing and concave, the difference on the left-hand side is non-increasing in n , and therefore $b_t(n,0)$ is non-increasing in n . The same argument applies to $b_t(n,1)$. Next, notice that in (9) $V_t(n,0)$ is a strictly increasing

function of the V_{t+1} on the right-hand side. Since the V_{t+1} are assumed to be strictly increasing in n , raising n therefore strictly increases $V_t(n, 0)$ even if the birth decision is held constant. $V_t(n, 0)$ is therefore strictly increasing.

Concavity requires more work. We want to show that $V_t(n + 1, 0) - V_t(n, 0)$ does not increase with n :

$$[V_t(n + 1, 0) - V_t(n, 0)] - [V_t(n, 0) - V_t(n - 1, 0)] \leq 0 \quad (15)$$

for all n . Three cases can be distinguished. Assume first that for a given n , $b_t(n + 1, 0) = b_t(n, 0) = b_t(n - 1, 0) = b$. Writing out (15) for this case gives:

$$\begin{aligned} bs_i [V_{t+1}(n + 1, 1) - 2V_{t+1}(n, 1) + V_{t+1}(n - 1, 1)] \\ + (1 - bs_i) [V_{t+1}(n + 1, 0) - 2V_{t+1}(n, 0) + V_{t+1}(n - 1, 0)] \leq 0, \end{aligned}$$

which holds because of the assumed concavity of V_{t+1} , regardless of b . Next, assume $b_t(n + 1, 0) = b_t(n, 0) = 0$ and $b_t(n - 1, 0) = 1$ (notice that we already established that b_t is non-increasing in n given the induction hypothesis). In this case, writing out (15) gives:

$$V_{t+1}(n + 1, 0) - 2V_{t+1}(n, 0) + [s_i V_{t+1}(n - 1, 1) + (1 - s_i) V_{t+1}(n - 1, 0)] \leq 0 \quad (16)$$

Notice that since $b_t(n, 0) = 0$, we must have $V_{t+1}(n, 0) > V_{t+1}(n, 1)$. Since the left-hand side is increased relative to (16), it is therefore sufficient to show:

$$\begin{aligned} V_{t+1}(n + 1, 0) - V_{t+1}(n, 0) \\ - [s_i [V_{t+1}(n, 1) - V_{t+1}(n - 1, 1)] + (1 - s_i) [V_{t+1}(n, 0) - V_{t+1}(n - 1, 0)]] \leq 0. \end{aligned} \quad (17)$$

Because of (14), this is equivalent to:

$$\begin{aligned} V_{t+1}(n + 1, 0) - V_{t+1}(n, 0) \\ - [s_i [V_{t+1}(n + 1, 0) - V_{t+1}(n, 0)] + (1 - s_i) [V_{t+1}(n, 0) - V_{t+1}(n - 1, 0)]] \leq 0, \end{aligned}$$

which is satisfied because of the assumed concavity of $V_{t+1}(n, 0)$. The last case is $b_t(n + 1, 0) = 0$ and $b_t(n, 0) = b_t(n - 1, 0) = 1$. Writing out (15) gives:

$$\begin{aligned} V_{t+1}(n + 1, 0) - 2[s_i V_{t+1}(n, 1) + (1 - s_i) V_{t+1}(n, 0)] \\ + [s_i V_{t+1}(n - 1, 1) + (1 - s_i) V_{t+1}(n - 1, 0)] \leq 0. \end{aligned} \quad (18)$$

This time, since $b_t(n, 0) = 1$, we must have $V_{t+1}(n, 0) \leq V_{t+1}(n, 1)$. By the same argument as before, it is sufficient to establish the following condition where the left-

hand side has is increased relative to (18):

$$V_{t+1}(n+1,0) - V_{t+1}(n,0) - [s_i [V_{t+1}(n,1) - V_{t+1}(n-1,1)] + (1-s_i) [V_{t+1}(n,0) - V_{t+1}(n-1,0)]] \leq 0.$$

This is (17) and therefore satisfied. $V_t(n,0)$ is therefore concave, which completes the proof. \square

Proof of Proposition 3: We would like to show that $b_t(n,y)(s_i)$ is non-increasing in s_i . From (10), we have that $b_t(n,y)(s_i) = 1$ if and only if:

$$V_{t+1}(n+y,1) \geq V_{t+1}(n+y,0).$$

It is therefore sufficient to show that for all t and n :

$$\frac{\partial V_t(n,1)}{\partial s_i} \leq \frac{\partial V_t(n,0)}{\partial s_i}. \quad (19)$$

The proof is once again by induction. At time T , condition (19) is trivially satisfied since $\frac{\partial V_T(n,y)}{\partial s_i} = 0$ for all n and y . Now assume that:

$$\frac{\partial V_{t+1}(n,1)}{\partial s_i} \leq \frac{\partial V_{t+1}(n,0)}{\partial s_i} \quad (20)$$

is satisfied for all n . To complete the proof, we need to show that (19) follows at time t for all n . Using (11), condition (19) can be written as:

$$\begin{aligned} \frac{\partial V_t(n,1)}{\partial s_i} - \frac{\partial V_t(n,0)}{\partial s_i} = & \gamma b_t(n,1) [V_{t+1}(n+1,1) - V_{t+1}(n+1,0)] - \gamma b_t(n,0) [V_{t+1}(n,1) - V_{t+1}(n,0)] \\ & + \gamma \left[(1-b_t(n,1)s_i) \frac{\partial V_{t+1}(n+1,0)}{\partial s_i} + b_t(n,1)s_i \frac{\partial V_{t+1}(n+1,1)}{\partial s_i} \right] \\ & - \gamma \left[(1-b_t(n,0)s_i) \frac{\partial V_{t+1}(n,0)}{\partial s_i} + b_t(n,0)s_i \frac{\partial V_{t+1}(n,1)}{\partial s_i} \right] \leq 0. \quad (21) \end{aligned}$$

The first term is less than or equal to zero since Lemma 1 shows:

$$V_{t+1}(n+1,1) - V_{t+1}(n+1,0) \leq V_{t+1}(n,1) - V_{t+1}(n,0)$$

and Lemma 1 together with (10) implies that $b_t(n,0) \leq b_t(n,1)$. It therefore suffices

to show that:

$$\begin{aligned} & \gamma \left[(1 - b_t(n, 1)s_i) \frac{\partial V_{t+1}(n+1, 0)}{\partial s_i} + b_t(n, 1)s_i \frac{\partial V_{t+1}(n+1, 1)}{\partial s_i} \right] \\ & - \gamma \left[(1 - b_t(n, 0)s_i) \frac{\partial V_{t+1}(n, 0)}{\partial s_i} + b_t(n, 0)s_i \frac{\partial V_{t+1}(n, 1)}{\partial s_i} \right] < 0. \quad (22) \end{aligned}$$

This condition is satisfied since (13) and the induction hypothesis (20) imply:

$$\frac{\partial V_{t+1}(n+1, 1)}{\partial s_i} \leq \frac{\partial V_{t+1}(n+1, 0)}{\partial s_i} = \frac{\partial V_{t+1}(n, 1)}{\partial s_i} \leq \frac{\partial V_{t+1}(n, 0)}{\partial s_i},$$

which completes the proof. □

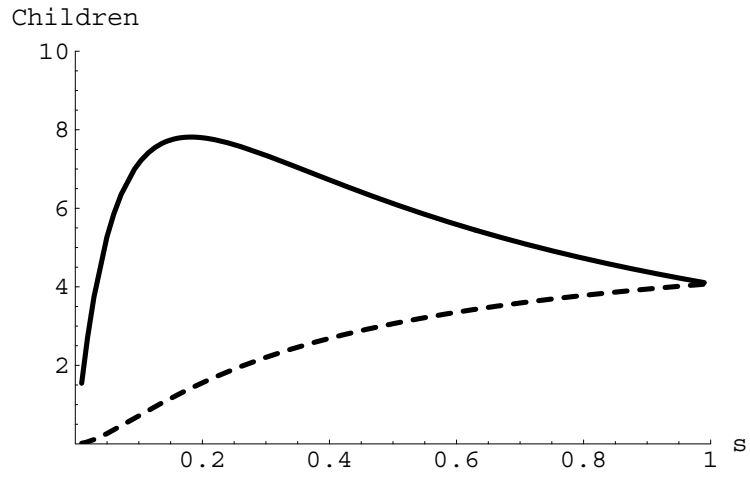


Figure 1: Births and Survivors in the Benchmark Model

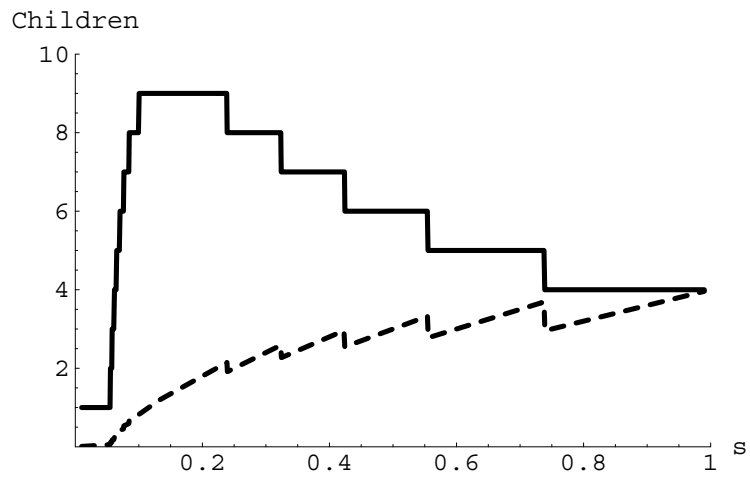


Figure 2: Births and Survivors in the Binomial Model

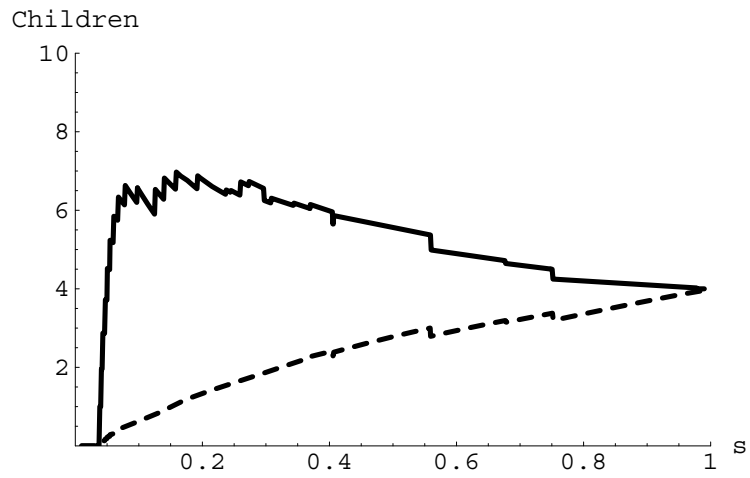


Figure 3: Births and Survivors in the Sequential Model

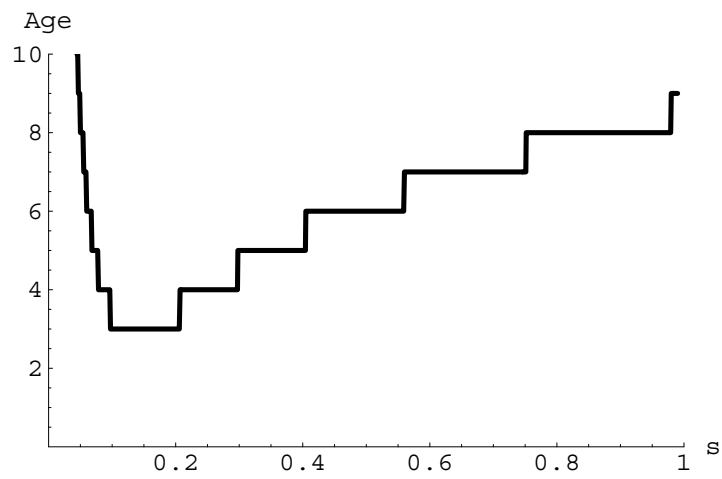


Figure 4: Age at First Birth in the Sequential Model

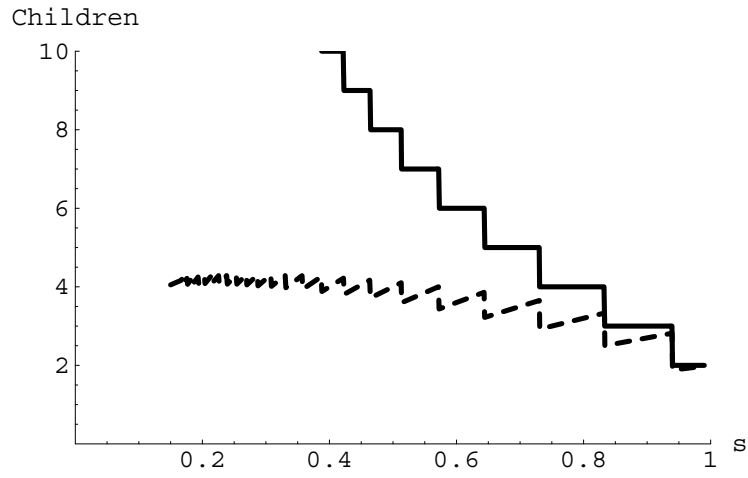


Figure 5: Births and Survivors in the Binomial Model, $\sigma = .01, \epsilon = .01$

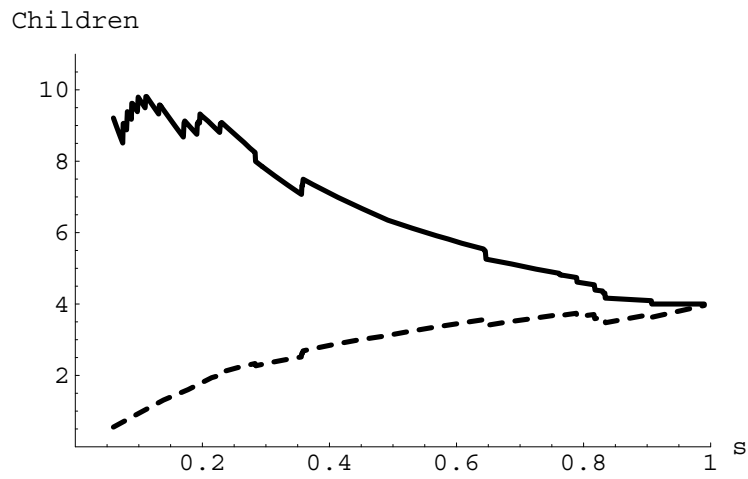


Figure 6: Births and Survivors in the Sequential Model, $\sigma = .01, \epsilon = .01$