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Modified Firefly Algorithm for Solving Multireservoir Operation in Continuous and Discrete Domains

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Abstract: Reservoir systems are essential for water resources management. The application and development of optimization techniques for optimal reservoir operation is therefore a valuable undertaking. This paper presents a modified firefly algorithm (MFA) and applies it to optimally solve reservoir operation problems. Three well-known benchmark multireservoir operation problems are optimized for energy production. The results of the MFA are compared with results obtained with other mathematical programming approaches, such as linear programming (LP), differential dynamic programming (DDP), and discrete DDP (DDDP), the genetic algorithm (GA), the multicolony ant algorithm (MCAA), the honey-bee mating optimization (HBMO) algorithm, the water cycle algorithm (WCA), the bat algorithm (BA), and the biogeography-based optimization (BBO) algorithm. The MFA was found to be more effective than alternative optimization methods in solving the test problems demonstrating its strong potential to tackle multireservoir operation problems. This paper's results indicate that the MFA differed by 0.01 and 0.79% with the LP global optimal solutions of a continuous four-reservoir problem (CFP) and a continuous 10-reservoir problem (CTP), respectively. The objective function of a discrete four-reservoir problem (DFP) obtained with the MFA is equal to the LP's objective function. This paper demonstrates that the MFA is a competitive optimization method with which to solve a variety of reservoir operation problems. **DOI: 10.1061/(ASCE)WR.1943-5452.0000644.** *© 2016 American Society of Civil Engineers.*

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Introduction

Water resources management is a very important and pressing topic, particularly when dealing with reservoir operation issues. A step forward in solving reservoir operation problems is to use optimization methods, including mathematical programming approaches and evolutionary or metaheuristic algorithms. The fact that mathematical programming approaches are time consuming and cannot solve complex problems, in some cases (Bozorg-Haddad et al. 2016), reinforces the need to resort to more reliable algorithms. To achieve this goal, researchers have increasingly resorted to evolutionary or metaheuristic algorithms because of their effectiveness and versatility, instead of mathematical programming approaches such as linear programming (LP) and dynamic programming (DP).

Novel optimization techniques are commonly tested with well-known benchmark problems whose optimal solutions are known. A brief outline of several approaches that have been used to solve well-known benchmark reservoir problems, namely, discrete four-reservoir problem (DFP), continuous four-reservoir problem (CFP), and continuous ten-reservoir problem (CTP), is presented below.

The application of DFP was first reported by Larson (1968). Subsequently, other researchers applied other optimization techniques such as DP, differential DP (DDP), and discrete DDP (DDDP) to solve DFP as reported by Heidari et al. (1971) and Murray and Yakowitz (1979). The limitations of mathematical programming techniques in solving complex real multireservoir operation problems contributed to the rise of evolutionary or metaheuristic algorithms owing to their solution capacity and versatility. DFP subsequently served as an example for the genetic algorithm (GA) in research by Wardlaw and Sharif (1999) and HInçal et al. (2011). Bozorg-Haddad et al. (2011) implemented the honey-bee mating optimization (HBMO) algorithm to demonstrate its advantageous performance over other optimization techniques in solving multireservoir operation problem.

The second benchmark problem, CFP, was introduced by Chow and Cortes-Rivera (1974) who applied LP and DDDP to solve this problem. This problem was solved by several researchers using mathematical programming optimization techniques such as Murray and Yakowitz (1979). Mathematical programming optimization techniques exhibit several limitations. For this reason, CFP has relied primarily on evolutionary and metaheuristic algorithms as the optimization technique of choice, which replaced mathematical programming optimization techniques. As an example, Bozorg-Haddad et al. (2011) applied the HBMO algorithm in solving the CFP. Recently, Bozorg-Haddad et al. (2015a, b) evaluated the performance of two new metaheuristic algorithms, including the water cycle algorithm (WCA) and the bat algorithm (BA), in solving the CFP. The latter two studies concluded that the WCA and the BA perform better than the GA in solving the CFP. More recently, Bozorg-Haddad et al. (2016) used biogeography-based optimization (BBO) algorithm to reservoir operation problems, particularly

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CFP, and indicated that the application of BBO seems appropriate and timely. The latter authors also compared BBO's and GA's performances in the solution of CFP. Parameter-tuning for BBO is easier and faster than GA, making it a user-friendly optimization method.

The third benchmark problem, CTP, was introduced by Murray and Yakowitz (1979), who showed that their modified DDP method could solve large problems. However, owing to the realization that evolutionary or metaheuristic algorithms are more reliable and simpler for solving complex optimization problems than mathematical programming ones, several researchers examined the performance of the former optimization techniques in solving the CTP. Walrdlaw and Sharif (1999) solved CTP using the GA and showed that the results of the GA were satisfactory. Thereafter, Jalali et al. (2007) and Bozorg-Haddad et al. (2011) used other modern optimization algorithms in solving efficiently the CTP. The former authors proposed multicolony ant algorithm (MCAA) to solve the CTP and reported that the performance of the MCAA was better in reaching the near-optimal solution than those of other approaches previously reported in the literature. The latter authors solved the CTP employing HMBO and showed the superior performance of HBMO by comparing its results with those from different analytical and evolutionary algorithms.

The firefly algorithm (FA) was introduced by Yang (2008). The FA is a stochastic, nature-inspired, metaheuristic algorithm that has gained acceptance and notoriety in several areas of optimization problems over the last decade. Yang (2009) implemented the FA to solve 10 multimodal optimization test problems and compared FA with GA and particle swarm optimization (PSO) algorithm. Yang (2009) demonstrated that the FA performs better than the GA and the PSO algorithm in terms of the success rate in obtaining the global solution of a variety of optimization problems. Yang (2011) used chaos with the FA and compared the results with the results of the standard FA obtained by solving the well-known problem of the welded beam, and showed the relative effectiveness of the proposed stochastic variant of the FA. Farahani et al. (2011) stabilized the fireflies' movements and increased the convergence speed of the FA by reformulating by means of a Gaussian distribution that moves the fireflies according to a random walk. The proposed algorithm was tested with five functions, and the results proved the better performance and greater accuracy of the proposed algorithm than those of the standard FA. Yan et al. (2012) developed an adaptive FA (AFA) to overcome the inability of the standard FA in obtaining the global optima of large dimensional problems. They concluded the greater accuracy of the AFA compared with those of FA, differential evolution (DE) algorithm, and PSO algorithm. Afnizanfaizal et al. (2012) attempted to improve the searching accuracy of the FA by means of a technique named hybrid evolutionary FA (HEFA) that combined the FA and DE algorithms. The HEFA was applied to predict the parameters of a complex, nonlinear, biological model of large dimensionality and showed a better searching accuracy than those of the GA, the PSO algorithm, and evolutionary programming (EP). Yang (2014) applied different stochastic distributions such as uniform, Gaussian, levy flight, chaotic maps, and random sampling with a turbulent fractal cloud to examine the performance of the FA. The latter author concluded that the performance of stochastic distributions depends strongly on the type of problem solved. Thus, the proper stochastic distribution should be selected based on the nature of the problem being solved.

Previous research on the FA indicates that: (1) it can deal with highly multimodal, nonlinear, optimization problems; (2) it avoids premature convergence observed with the PSO algorithm; and (3) it is capable of controlling its modality and adapt to a problem's landscape by means of its scaling parameters. The FA is, in fact, a generalization of simulated annealing (SA) algorithm, the PSO algorithm, and the DE algorithm (Fister et al. 2013).

The complexity of water resources management problems, particularly in the field of optimal reservoir operation, makes mathematical programing approaches less efficient and enhances the popularity of evolutionary or metaheuristic optimization algorithms given their proven computational speed and convergence properties. One of the recently introduced metaheuristic optimization algorithm is the FA, which has some limitations such as setting its parameters, damping too quickly, and its low convergence speed in solving reservoir operation problems. To overcome these limitations, this study defines, implements, and tests a novel metaheuristic algorithm, namely, modified firefly algorithm (MFA), to derive optimal operation policies for three well-known benchmark reservoir operation problems, namely, the DFP, CFP, and CTP. The MFA developed in this study resulted in a more efficient reservoir-operation solving algorithm than the FA. It is shown that the MFA has a high capacity to efficiently achieve a better solution with a streamlined process for setting the model parameters, a higher convergence speed, and without being damped quickly. This paper shows that the results calculated with the MFA for three benchmark reservoir problems are better than those reported by previous studies measured in terms of the value of the objective function values calculated with similar number of algorithmic evaluations. Also, the first obtained results from the MFA are presented to encourage water-resources scientists and engineers to apply it in various other fields.

Benchmark Multireservoir Operation Model

The objective function, constraints, and simulation equations of the DFP, CFP, and CTP are presented first. It is worth noting that these problems offer an opportunity to verify the performance of the MFA against LP (global optimum) and other previous optimization algorithms' solutions published in the literature.

The objective function of the three benchmark reservoir problems is to maximize the total benefit from the hydropower production over 12 operating periods expressed as

$$\text{Maximize}\, B = \sum_{r=1}^{R}\sum_{t=1}^{T} b_r(t)\text{Re}_r(t) \tag{1}$$

in which B = total benefit of the benchmark multireservoir system; r = index for the reservoir number; R = total number of reservoirs; t = index for the periods of reservoir operation; T = total number of operation periods; $b_r(t)$ = unit benefit of release from reservoir r during period t; and $\text{Re}_r(t)$ = reservoir release (the sum of release and spill) of reservoir r during period t. The reservoir releases are the decision variables.

The reservoir's equation of water balance, or continuity equation, is written as follows:

$$S_r(t+1) = S_r(t) + Q_r(t) - \text{RCM}_{R\times R}\text{Re}_r(t) \qquad r = 1, \ldots, R; t = 1, \ldots, T \tag{2}$$

in which $S_r(t+1)$ = (water) storage of reservoir r at the beginning of period $t+1$; $S_r(t)$ = storage of reservoir r at the beginning of period t; $Q_r(t)$ = net inflow (the net of river inflow, precipitation on the reservoir's surface, evaporation from the reservoir lake, and net seepage on the reservoir bottom) to reservoir r during period t; $\text{RCM}_{R\times R}$ = R-order matrix of indexes of reservoir connections with -1's along the diagonal, $+1$'s off-diagonal entries that describe reservoir releases to a downstream reservoir, and zeros elsewhere.

The reservoir releases and reservoir storages have lower and upper bounds expressed as

$$\text{Remin}_r(t) \leq \text{Re}_r(t) \leq \text{Remax}_r(t) \quad r = 1, \ldots, R; t = 1, \ldots, T \tag{3}$$

$$S\text{min}_r(t) \leq S_r(t) \leq S\text{max}_r(t) \quad r = 1, \ldots, R; t = 1, \ldots, T \tag{4}$$

in which $\text{Remin}_r(t)$ and $\text{Remax}_r(t)$ = minimum and the maximum allowable releases from reservoir r during period t, respectively. Likewise, $S\text{min}_r(t)$ and $S\text{max}_r(t)$ = minimum and the maximum allowable storages of reservoir r during period t, respectively.

The last constraints of the benchmark problems specify the initial and final storages of each reservoir

$$S_r(1) = S\text{initial}_r \quad r = 1, \ldots, R \tag{5}$$

$$S_r(T+1) = S\text{target}_r \quad r = 1, \ldots, R \tag{6}$$

in which $S\text{initial}_r$ and $S\text{target}_r$ = initial and the final storages of reservoir r, respectively.

It is worth mentioning that the feasible results of reservoir operation are those in which all the stated constraints are satisfied. Penalty functions are required to specify constraints in the evolutionary or metaheuristics algorithms. The penalty functions produce solutions with negligible violations of the physical constraints [Eqs. (2)–(6)]. The penalty functions of the cited benchmark problems are expressed as follows:

$$P_{1,t} = \begin{cases} 0 & \text{if } S_r(t) > S\text{min}_r(t) \\ g \times [S\text{min}_r(t) - S_r(t)]^2 & \text{Otherwise} \end{cases} \tag{7}$$

$$P_{2,t} = \begin{cases} 0 & \text{if } S_r(t) < S\text{max}_r(t) \\ g \times [S_r(t) - S\text{max}_r(t)]^2 & \text{Otherwise} \end{cases} \tag{8}$$

$$P_{3,t} = \begin{cases} 0 & \text{if } S_r(t) > S\text{target}_r(t) \\ g \times [S\text{target}_r(t) - S_r(T+1)]^2 & \text{Otherwise} \end{cases} \tag{9}$$

with $r = 1, \ldots, R$; $t = 1, \ldots, T$, $P_{1,t}$, $P_{2,t}$, and $P_{3,t}$ = the penalties imposed on the violation of constraints stated in Eqs. (4)–(6), respectively, during period t. Additionally, g = the penalty constant which is considered equal to 40 for the DFP and CFP (Heidari et al. 1971) and 60 for the CTP (Wardlaw and Sharif 1999).

The above penalty functions are added to Eq. (1). Consequently, the modified objective function of the problem that is solved is expressed as follows:

$$\text{Maximize } B' = \sum_{r=1}^{R} \sum_{t=1}^{T} b_r(t) \text{Re}_r(t) - P_{1,t} - P_{2,t} - P_{3,t} \tag{10}$$

in which B' = modified objective function.

Methodology

This section is divided into two subsections. Subsection (1) describes the FA, and subsection (2) describes the MFA.

Fireflyalgorithm (FA)

The FA is inspired by the behavior of fireflies in nature. Fireflies emit their stored energy as a light to mate, hunt, or evade pesky insects, and protect themselves. Thus, that fireflies produce attractiveness through the emission of light. This unique biologic phenomenon served as the basis for the FA. Firefly behavior suggests the following rules: (1) All fireflies are unisex and their attractiveness depends only on the amount of flashed light; and (2) the attractiveness of fireflies is proportional to their brightness. Accordingly, for any two flashing fireflies, the firefly which flashes less intensely will move towards the firefly which flashes more intensely. The longer the distance between fireflies, the lower their attractiveness and brightness. The movement of fireflies continues guided by rules (1) and (2) until there is not a brighter firefly in a group. In this case, fireflies move randomly. Rule (3) states that the brightness of a firefly is determined by an objective function.

According to Yang's (2009) assumptions, the attractiveness of a firefly is calculated by Eq. (11)

$$\beta(\wp) = \beta_0 e^{-\gamma \wp^{im}} \quad im \geq 1 \tag{11}$$

in which $\beta(\wp)$ = and β_0 denote the firefly's attractiveness and the attractiveness at a distance $\wp = 0$, respectively. Also, γ = light absorption coefficient, and $\wp$ = the distance between any two fireflies. Yang (2009) proposed the value of im equal to two.

As noted by Yang (2009), the distance between any pair of i and j fireflies at the positions x_i and x_j, respectively, equals the Cartesian distance and is computed with Eq. (12)

$$\wp_{ij} = \|x_i - x_j\| = \sqrt{\sum_{d=1}^{TD} (x_{i,d} - x_{j,d})^2} \tag{12}$$

in which $\wp_{ij}$ = Cartesian distance between a pair of i and j fireflies; $\| \ \|$ = distance norm between a pair of i and j fireflies in space; d = index for spatial dimensions (decision variables); TD = total number of dimensions (decision variables); $x_{i,d}$ = dth dimension of the spatial coordinate of the ith firefly's position; and $x_{j,d}$ = dth dimension of the spatial coordinate of the jth firefly's position.

Yang (2009) pointed out that the distance $\wp_{ij}$ defined above is not limited to Cartesian (Euclidean) distance. Other formulations of $\wp_{ij}$ can be defined in the d-dimensional hyperspace, depending on the type of problem.

The movement of firefly i towards a more attractive (brighter) firefly j is determined by Eq. (13)

$$x\text{new}_i = x_i + \beta(\wp) \cdot (x_j - x_i) + \alpha \varepsilon_i \tag{13}$$

in which $x\text{new}_i$ = and x_i = new and the current positions of firefly i, respectively, where firefly i is less bright than firefly j; x_j = position of firefly j, which is brighter than firefly i; α = randomized parameter; and ε_i = vector of random numbers drawn from a Gaussian distribution or uniform distribution. Eq. (13) shows that the new position of any firefly with low brightness is associated with two factors. The first factor is the position of the firefly with more brightness, and the second one is a random walk biased towards the brighter fireflies. Given these factors, Eq. (14) is a rewritten form of Eq. (13) in which Eq. (11) is used instead of $\beta(\wp)$ and (rand − 0.5) is used instead of ε_i

$$x\text{new}_i = x_i + \beta_0 e^{-\gamma \wp_{ij}^2} (x_j - x_i) + \alpha(\text{rand} - 0.5) \tag{14}$$

in which (rand − 0.5) = the simplest form of ε_i, where rand is a random number generator uniformly distributed in [0, 1].

Because the performance of the FA is sensitive to the parameters' values, there are recommended ranges for them. In accordance with the Yang (2009) recommendation, it should be noted that in theory, $\gamma \in [0, \infty)$, but in most practical applications it typically varies from 0.1 to 10. However, the true range of this parameter depends on the system that is optimized. Moreover, Yang (2009) chose $\beta_0 = 1$ and $\alpha \in [0, 1]$.

The FA can be employed to solve complex problems with stationary or nonstationary objective functions, and continuous or discrete structure. Yet, it may have limitations in some case studies. For instance, Yang (2009) indicated that the distance $\wp_{ij}$ requires specification based on the solved problem. Moreover, selecting the proper value or determining the proper ranges for the parameters of the FA is another issue that may hinder its performance. To address these limitations, the authors modified the FA to produce a modified FA (MFA) that circumvents the cited limitations in solving multireservoir operation problems.

Modified Firefly Algorithm (MFA)

Multireservoir operation is a complex and multidimensional problem. The implementation of FA to solve such problems requires modifications with which the best solution can be calculated. Four steps are used to achieve the MFA. Before describing those four main steps, a predevelopment step is introduced.

Predevelopment Step: Determine the Range of $\beta(\wp)$

Eq. (13) shows that the new position of a firefly with low brightness is affected by two factors. The first factor [second term on the right-hand side of Eq. (13)] plays a role of modification so that the firefly with low brightness moves towards a brighter one. The second factor [third term on the right-hand side of Eq. (13)] is a random-movement operator. Thus, it is expected that the second term focuses only upon modification. The value of $\beta(\wp)$ is a percentage of the vector describing the distance between two fireflies and ranges from zero to one. If $\beta(\wp) = 0$, the position of the firefly with low brightness is not modified, and it does not move towards the firefly with more brightness. In such situations, the new position of a firefly with low brightness is only affected by the random operator in Eq. (13). If $\beta(\wp) = 1$, the new position of the firefly with low brightness is equal to the position of the firefly with more brightness. However, this first movement of the firefly with low brightness is not complete because a random walk is added to it and new positions are searched. If $\beta(\wp) > 1$, the firefly with low brightness can move towards the firefly with more brightness according to equations that may include a random part in the modification term. To recall a previous statement, the second term of Eq. (13) simply modifies a firefly's position without imposing any random walk. Thus, the values of the predefined parameters used in the second term of Eq. (13) cause $\beta(\wp)$ to range between zero and one.

Modification Step 1—Change the Fireflies' Positions under All Conditions of Brightness

This modification increases the conditions under which solutions are modified. This means that if firefly i is less bright than firefly j, the first modification of the solution is applied and a random walk is conducted. Otherwise, if firefly i is as bright as or brighter than firefly j, a random walk is applied to the solution. This modifies the FA, which does not impose movement under this condition. This modification produces more new positions of fireflies in each iteration of the MFA than in the FA.

Modification Step 2—Change the Application of the Random Walk Operator

This is the key modification in the MFA. Recall that the random walk is conducted by the third term of Eq. (13) in the FA. Therefore, a value with a range between [0, 1] is applied to all the decision variables of a firefly in each iteration as shown in Fig. 1(a). Applying random walk in this manner is not appropriate when the FA is used to solve a multireservoir operation problem. The proof of this statement is that the values of the decision variables in a multireservoir operation problem are mostly large values, which indicates that in some cases one must apply random-walk values larger than those in the range [0, 1] to the decision variables. The application of random walk operator in the MFA is as follows:

1. The decision variables of a firefly are randomly placed in different classes. The number of classes is defined by the user after a preliminary trial and error process. However, it is recommended that further study be conducted about the number of classes.
2. For each class, a different range from which a random value is selected is defined. It should be noted that these ranges include small and large ranges.
3. The ranges of classes with large ranges are decreased with a decreasing (damping) coefficient. However, the ranges of classes with small ranges remain unchanged. In other words, the MFA has the opportunity of reaching more solutions in the decision space in its first iterations. This is in contrast with the FA, which uses an identical range in all iterations, and a smaller solution space is searched. Thus, in some cases, the FA might be quickly damped or stopped at local optima. Fig. 1(b) depicts this modification.

The number of the classes used in this step depends on the type of problem solved and can be determined after a quick trial and error process.

Modification Step 3—Define the Distance between Fireflies in the Case of Reservoir Operation Problems

According to Yang's (2009) recommendation, the formulation of the distance between a pair of fireflies should change based on the problem. Any mathematical formulation of the firefly distance that causes the better solutions can be used. In reservoir operation problems, the values of decision variables cause the value of $\wp_{ij}$ to become large so that the term $e^{-\gamma\wp^2}$, regardless of what the value of γ is, becomes nearly zero. For this reason, the second term in Eq. (14) vanishes, and the next movement of a firefly is conducted randomly. Hence, the values of the objective that are primarily in the range [0, 1] are used as expressed in Eq. (15) instead of

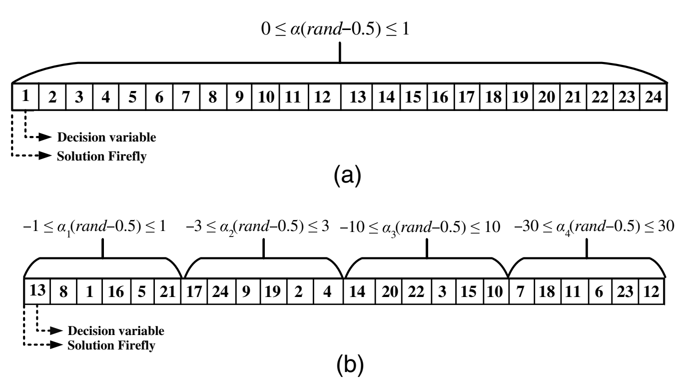

Fig. 1. Manner of applying the random walk operator in (a) the FA; (b) the MFA

the values of decision variables to formulate the distance between fireflies in MFA

$$\wp_{ij} = \mathrm{OF}_j - \mathrm{OF}_i \tag{15}$$

in which OF_i and OF_j are the values of objective functions of the ith and jth fireflies, respectively, where firefly i is less bright than firefly j. Thus, the value of $\wp_{ij}$ in Eq. (15) is always positive in maximization problems. It is noted that in minimization problems, Eq. (15) changes to $\wp_{ij} = \mathrm{OF}_i - \mathrm{OF}_j$.

Modification Step 4—Select the Unique Final Solutions for the Next Iteration

In all evolutionary or metaheuristic algorithms, the solutions at the end of each iteration are sorted based on the objective function values so as to be used in the next iteration. If some solutions are similar so that the decision variables and the objective functions are the same, each of them is considered as a separate solution. Thus, repetitive solutions may appear in the process of selecting the best sorted solutions. In such a situation, the number of repetitive solutions is increased during the sequential iterations so that in the last iterations the solutions that are transferred to the next iteration are equal to each other. This issue causes the evolutionary or metaheuristic algorithms to be quickly damped, and the solutions do not change significantly.

The fourth modification is concerned with selecting unique final solutions for the next iteration. With this modification, the MFA is not damped quickly.

Recommended MFA Parameters

The performance of the MFA is sensitive to the parameters' values. The following are ranges recommended by the authors for use with the MFA:

1. β_0: The value of this parameter is equal to one as Yang (2009) suggested.
2. α_n: The decision variables of a firefly need to be classified into different classes randomly in the MFA. Thus, the number of randomization parameters is equal to the number of classes, and for each class n, a different range of the randomization parameter (α_n) is defined. α_n may vary within a narrow range in some of these classes (for instance from -1 to $+1$) and in other classes it may change in a wide range (-30 to 30). It is noted that damping the values of the randomization parameter causes the wide ranges to change into narrow ranges during the implementation of the iterative MFA.
3. γ: The proper values for this parameter depends on the type of problem being solved. In other words, the values of γ depends on $\wp$. Table 1 lists several values of $\wp$ and γ and the calculated $\beta(\wp)$. It is seen in Table 1 that the values of $\beta(\wp)$ and $\wp$ vary from 0 to 1. In addition, the values of γ range from 0.0001 to 10,000, which covers the practical range recommended by Yang (2009). Table 1 shows that $\beta(\wp)$ changes most when $\gamma \in [0.1, 10]$. Therefore, the value of γ recommended for the MFA is [1, 5] according to Table 1.

The complete flowchart of the MFA is depicted in Fig. 2 in which the development steps are shown with light gray rectangular shapes. It is noted that the MFA is not a simple application of the FA reported by Yang (2009). The four modifications detailed above and Fig. 2 show that the MFA is a substantially improved FA well suited for solving multireservoir optimization problems. On the whole, the strength and novelty of the MFA in comparison with FA consist of (1) the distance $\wp_{ij}$ has been specified based on the characteristics of the reservoir problem in the MFA. This modification contributes to enhance the convergence speed and searching process; (2) the parameters of the MFA can be set more readily and properly than in the FA, whose parameter specification is a time consuming process; (3) whereas in the FA the fireflies' positions only change under specific conditions, the fireflies' positions in the MFA change under all conditions of brightness. This modification

Table 1. Values of $\beta(\wp)$ for Different Values of $\wp$ and γ

	$\wp$																				
γ	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1
0.0001	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.0005	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.001	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.005	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.01	1	1	1	1	1	1	1	1	1	1	1	0.99	1	0.99	1	0.99	1	0.99	1	0.99	1
0.05	1	1	1	0.99	1	0.99	1	0.98	1	0.98	1	0.97	1	0.97	1	0.96	1	0.96	1	0.95	1
0.10	1	1	1	0.99	1	0.98	1	0.97	1	0.96	1	0.95	0.9	0.94	0.9	0.93	0.9	0.92	0.9	0.91	1
0.15	1	0.99	1	0.98	1	0.96	1	0.95	0.9	0.93	0.9	0.92	0.9	0.91	0.9	0.89	0.9	0.88	0.9	0.87	1
0.20	1	0.99	1	0.97	1	0.95	0.9	0.93	0.9	0.91	0.9	0.9	0.9	0.88	0.9	0.86	0.9	0.84	0.8	0.83	1
0.50	1	0.98	1	0.93	0.9	0.88	0.9	0.84	0.8	0.8	0.8	0.76	0.7	0.72	0.7	0.69	0.7	0.65	0.6	0.62	1
0.70	1	0.97	0.9	0.90	0.9	0.84	0.8	0.78	0.8	0.73	0.7	0.68	0.7	0.63	0.6	0.59	0.6	0.55	0.5	0.51	0
0.90	1	0.96	0.9	0.87	0.8	0.8	0.8	0.73	0.7	0.67	0.6	0.61	0.6	0.56	0.5	0.51	0.5	0.47	0.4	0.43	0
1	1	0.95	0.9	0.86	0.8	0.78	0.7	0.7	0.7	0.64	0.6	0.58	0.5	0.52	0.5	0.47	0.4	0.43	0.4	0.39	0
3	1	0.86	0.7	0.64	0.5	0.47	0.4	0.35	0.3	0.26	0.2	0.19	0.2	0.14	0.1	0.11	0.1	0.08	0.1	0.06	0
4	1	0.78	0.6	0.47	0.4	0.29	0.2	0.17	0.1	0.11	0.1	0.06	0	0.04	0	0.02	0	0.01	0	0.01	0
7	1	0.70	0.5	0.35	0.2	0.17	0.1	0.09	0.1	0.04	0	0.02	0	0.01	0	0.01	0	0	0	0	0
10	1	0.61	0.4	0.22	0.1	0.08	0	0.03	0	0.01	0	0	0	0	0	0	0	0	0	0	0
13	1	0.52	0.3	0.14	0.1	0.04	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	0.47	0.2	0.11	0	0.02	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	0.43	0.2	0.08	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0.37	0.1	0.05	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	1	0.08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	1	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1,000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10,000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

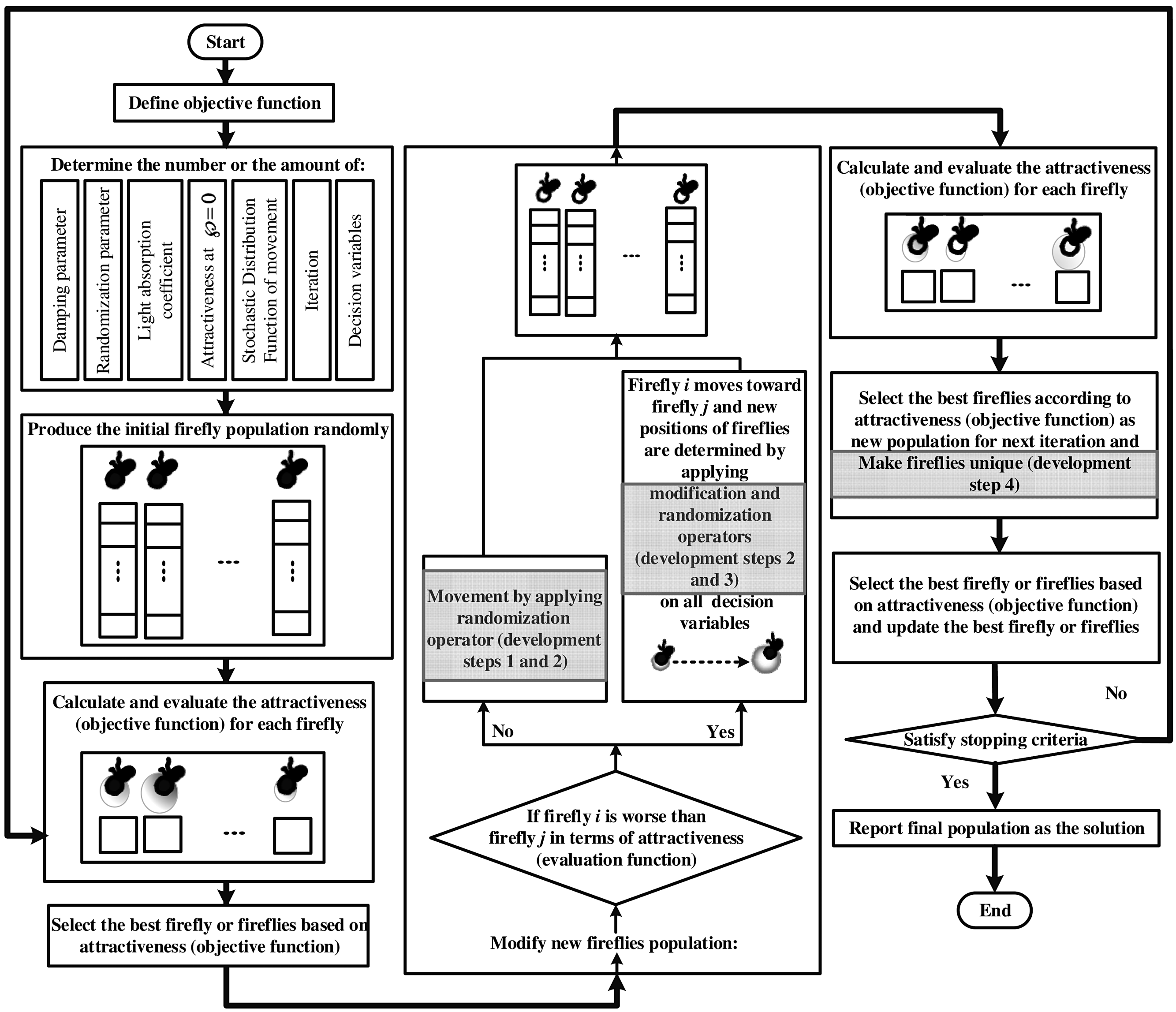

Fig. 2. Complete flowchart of the MFA (the modification steps are shown with light gray rectangular shapes)

produces more new positions of fireflies in each iteration of the MFA and enhances the convergence speed; (4) the application of the random walk operator in MFA is modified which leads to search a wider decision space and a better movement of fireflies until finding the best solution; and (5) selecting unique possible solutions for the next iteration prevents the appearance of repetitive solutions in the process of selecting the best sorted solutions.

Results and Discussions (Algorithm Application: Benchmark Multireservoir Operation Systems)

Three reservoir benchmark problems, DFP, CFP, and CTP, were chosen to test the capability of the MFA in solving multireservoir operation problems (Appendixes I, II, and III). These problems offer the opportunity to test the performance of MFA against LP (known global) solutions and previous solutions published in the literature.

The data used with the DFP, CFP, and CTP can be found in the studies of Larson (1968), Chow and Cortes-Rivera (1974), and Murray and Yakowitz (1979), respectively. Nevertheless, this paper compiled the complete datasets for the three reservoir benchmark problems for clarity and reproducibility purposes (see Appendix).

Three benchmark problems of reservoir operation were evaluated to test the performance of the MFA. The results of DFP, CFP, and CTP obtained from LP, the FA, and the MFA are presented in the next three subsections.

Results for the Discrete Four-Reservoir Problem (DFP)

The DFP was solved in this study with LP, the FA, and MFA. LP was implemented with the *Lingo 14.0* optimization software and produced a value of the objective function equal to 401.3, the same value as that reported by Larson (1968). The *MATLAB 12* software was used to implement the FA and the MFA. The parameters for the FA are the number of fireflies, the number of iterations, β_0, γ, and α, which were set equal to 50, 300, 2, 0.01, and 1, respectively, according to the FA's recommendations and preliminary trial and error evaluation. Likewise, the parameters of the MFA, which are the number of fireflies, the number of iterations, the number of classes, β_0, and γ were 50, 300, 10, 1, and 5, respectively, according to the MFA's recommendations and preliminary trial and error testing. Setting the number of functional evaluations (reporting the number of functional evaluations is more informative than

Table 2. Objective Function Outputs from Five Runs of the FA and MFA for the Benchmark Problems

Run number	Discrete four-reservoir (DFP)			Continuous four-reservoir (CFP)			Continuous 10-reservoir (CTP)		
	FA	MFA	LP	FA	MFA	LP	FA	MFA	LP
1	388.80	401.3	401.3	304.72	308.26	308.2915	1,101.06	1,184.07	1,194.44
2	395.90	401.3	—	305.86	308.23	—	1,100.32	1,184.99	—
3	394.70	401.3	—	306.35	308.13	—	1,088.23	1,181.32	—
4	391.00	401.3	—	305.13	308.23	—	1,107.85	1,182.59	—
5	389.00	401.3	—	305.50	308.20	—	1,089.58	1,185.00	—
Best	395.90	401.3	—	306.35	308.25	—	1,107.85	1,185.00	—
Average	391.88	401.3	—	305.51	308.21	—	1,097.41	1,183.59	—
Worst	388.80	401.3	—	304.72	308.13	—	1,088.23	1,181.32	—
Standard deviation	2.9070	0.0000	—	0.6647	0.0503	—	8.0142	1.5177	—
Percentage difference of the best run against LP	1.35	0	—	0.63	0.01	—	7.25	0.79	—

reporting the computational time because the number of functional evaluations is independent of the computing equipment, thus providing a fairer comparison) equal to 15,000, the FA and the MFA converged to the best objective function values equal to 395.90 and 401.3, respectively, in five independent solution runs. The objective function outputs from five runs of the FA and the MFA for the benchmark problems are listed in Table 2, from which it is concluded that the MFA reached the same result as LP in all five solution runs of the DFP. The best solution run of the FA differs approximately by 1.4% from that of LP. Fig. 3(a) shows the comparisons between FA and MFA in terms of their rates of convergence over five solution runs for the DFP. For better interpretation of the difference between FA and MFA, a narrower boundary of the objective function value is illustrated in Fig. 3(b) for DFP which highlights the difference between the FA and the MFA convergence rates using narrow bounds. According to Fig. 3(b), it is concluded that the MFA converges more rapidly than the FA in 15,000 functional evaluations, and yielded a better objective function value than that of the FA. Another important issue is that even though the objective function values obtained from the MFA are the same over five solution runs, four alternative solutions (solutions with different decision variables values but the same objective function values) were obtained. In other words, one may notice that the DFP has at least four other optimal alternative solutions that have been determined in this study. Bozorg-Haddad et al. (2011) stated that the HBMO detected at least two other optimal alternative solutions. Thus, the MFA determined more alternative solutions than any other previous study. Figs. 4 and 5 illustrate the monthly reservoir releases and storages, respectively, obtained from the FA and the MFA compared with those from LP. According to Fig. 4(a), regardless of the differences in releases schedule of MFA, the same value of 401.3 is obtained as the optimal value of the objective function.

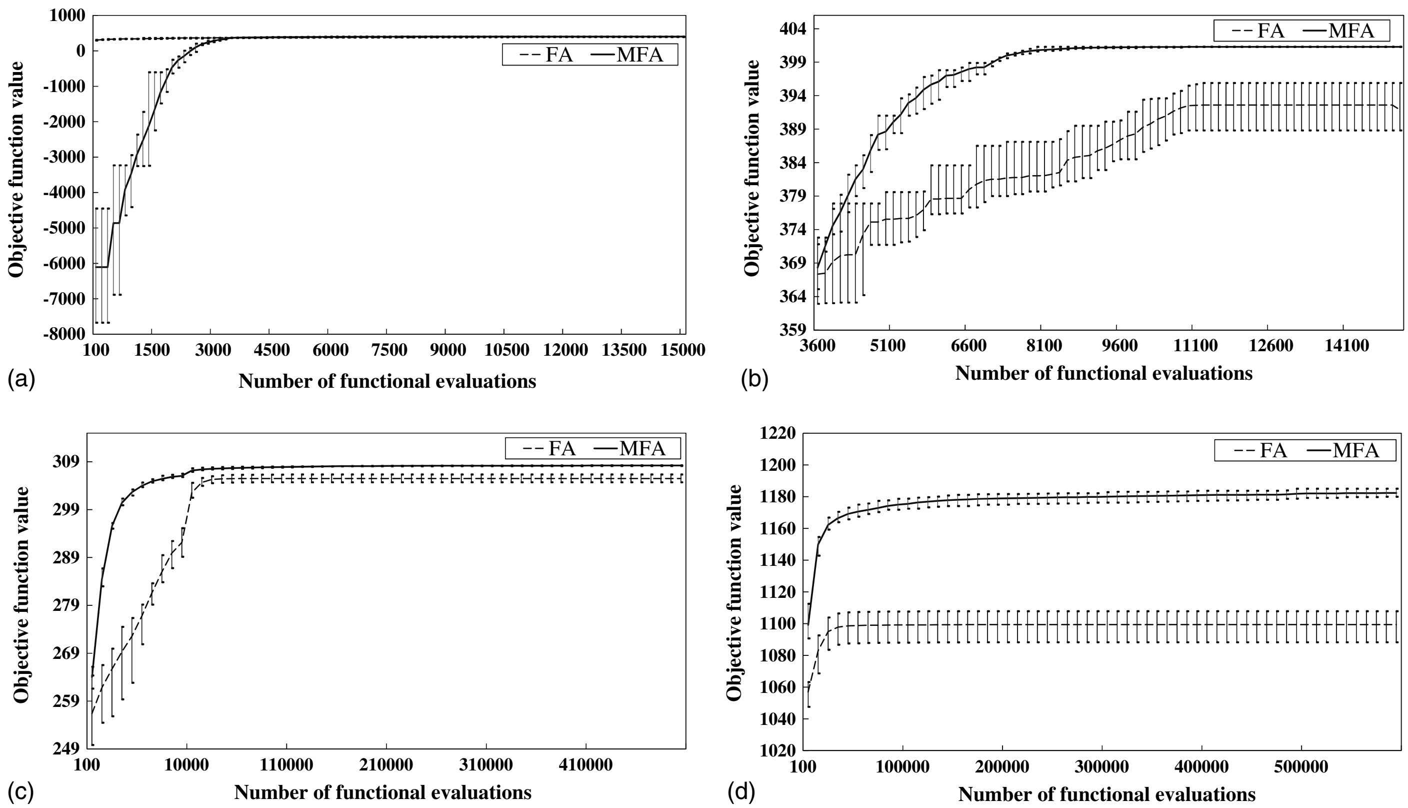

Fig. 3. Comparisons of FA and MFA rates of convergence over five runs for (a) the DFP; (b) the DFP with narrower bounds for better interpretation; (c) the CFP; (d) the CTP; highest curve is best for maximization

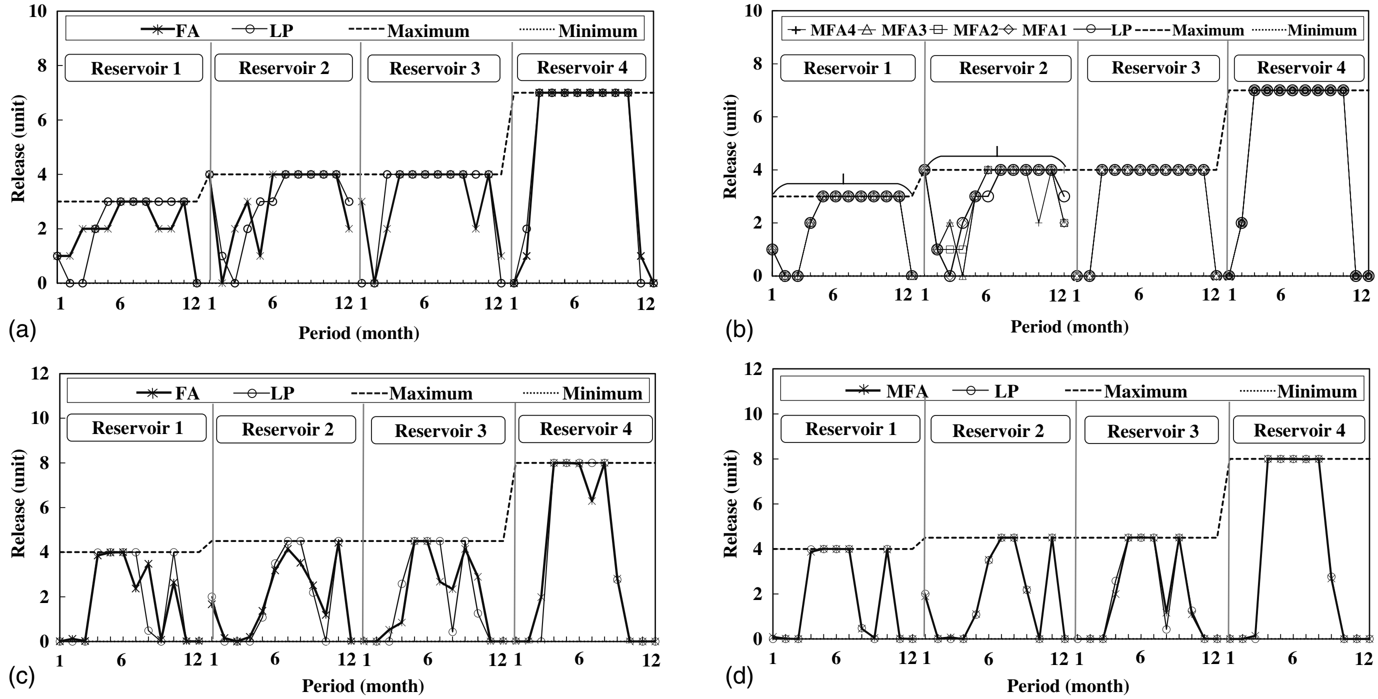

Fig. 4. Monthly reservoir releases of the best run of five runs for DFP obtained from (a) the FA; (b) the MFA; for the CFP obtained from (c) the FA; (d) the MFA

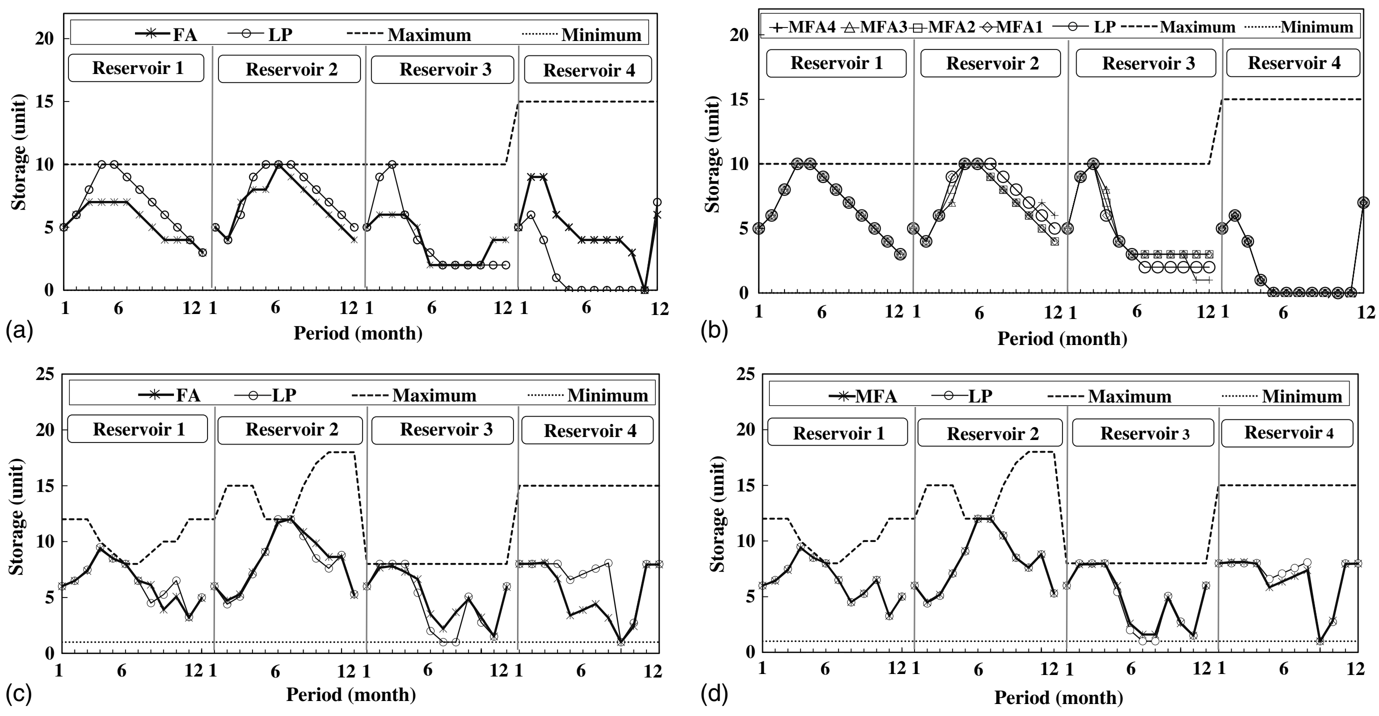

Fig. 5. Monthly reservoir storages of the best run of five runs for DFP obtained from (a) the FA; (b) the MFA; for the CFP obtained from (c) the FA; (d) the MFA

Results of the Continuous Four-Reservoir Problem (CFP)

The CFP was solved in this work with LP, the FA, and the MFA. The LP was implemented with the *Lingo 14.0* optimization software which produced an optimal value of the objective function equal to 308.2915, the same value reported by Bozorg-Haddad et al. (2011). The *MATLAB 12* software was used to implement the FA and the MFA. The parameters for the FA are the number of fireflies, the number of iterations, β_0, γ, and α were set equal to 50, 10,000, 2, 0.1, and 1, respectively, according to the FA's recommendations and preliminary trial and error evaluation. Likewise, the parameters for the MFA are the number of fireflies, the number of iterations, the number of classes, β_0, and γ were set equal to 50, 10,000, 10, 1, and 5, respectively, according to the MFA's recommendations and preliminary trial and error testing. The number of functional evaluations (which is independent of the type of computer used) was 500,050 (almost 0.5 million). The FA and the MFA converged to the best objective function values equal to 306.35 and 308.25, respectively, over five independent runs. It is worth mentioning that this optimal value obtained with the FA (308.25) is larger than those reported in previous studies by other authors. In fact, with the same number of functional evaluations (0.5 million) Bozorg-Haddad et al. (2015a) found the best value to be 308.20. Moreover, Bozorg-Haddad et al. (2011) reported a value equal to 308.24 with 14 million functional evaluations, which is 28 times the 0.5 million evaluations of the MFA and yielded a result inferior to that of the MFA. The objective function outputs from five runs of the FA and the MFA are listed in Table 2. From the values in Table 2, it is concluded that the best objective function value of the MFA differs approximately 0.01% from LP's solution, whereas the best objective function value of the FA differs approximately 0.63% from LP's solution. Fig. 3(c) shows the comparisons between FA and MFA in terms of their rates of convergence over five solution runs for the CFP. Based on Fig. 3(c), it is concluded that the MFA converged quickly to an objective function better than that of the FA with 0.5 million functional evaluations. Figs. 4 and 5 illustrate the monthly reservoir releases and storages, respectively, obtained with the FA and the MFA and compared with those calculated with LP. According to Figs. 4(c and d) and 5(c and d), it is evident that the releases and the storages obtained with the MFA are very close or identical to those from LP in most periods. However, the results of the FA differ significantly from those of LP.

Results of the Continuous Ten-Reservoir Problem (CTP)

The CTP was solved in this work with LP, the FA, and the MFA. LP was implemented with the *Lingo 14.0* optimization software and calculated the best value of the objective function to be equal to 1,194.44, the same value reported by Bozorg-Haddad et al. (2011). The *MATLAB 12* software was used to implement the FA and the MFA. The parameters for the FA are the number of fireflies, the number of iterations, β_0, γ, and α were set to 50, 20,000, 2, 0.01, and 0.5, respectively, according to the FA's recommendations and preliminary trial and error testing. Likewise, the parameters for MFA are the number of fireflies, the number of iterations, the number of classes, β_0, and γ are considered as 50, 20,000, 10, 1, and 5, respectively, according to the MFA's recommendations and preliminary trial and error evaluations. Setting the number of functional evaluations (which is independent of the type of computer used) equal to approximately one million, the FA and the MFA converged to best objective function values equal to 1,107.85 and 1,185.00, respectively, over five independent runs. It is worth mentioning that this optimal obtained with the MFA (1,185.00) is larger than that obtained by Bozorg-Haddad et al. (2011) with 1.32 million functional evaluations. The objective function outputs from five runs of the FA and the MFA are listed in Table 2, which shows that the best objective function value of the MFA differs less than 1% from the LP's best value of the objective function, while the best objective function value of the FA differs approximately 7% from the LP's best solution. Fig. 3(d) shows the comparisons between FA and MFA in terms of their rates of convergence over five solution runs for the CTP. Based on the displayed convergence rates, it is concluded that the MFA converged more rapidly than the FA and yielded a better objective function value. Figs. 6 and 7 illustrate the monthly reservoir releases and storages, respectively, obtained with the FA and the MFA and compared with the LP solutions. Figs. 6 and 7 demonstrate that the releases and the storages obtained with the MFA are closer to those of LP than those calculated with the FA.

Apart from comparing the results of MFA with those of FA and LP for three benchmark problems, this work compared its performance with previous published results using different methods.

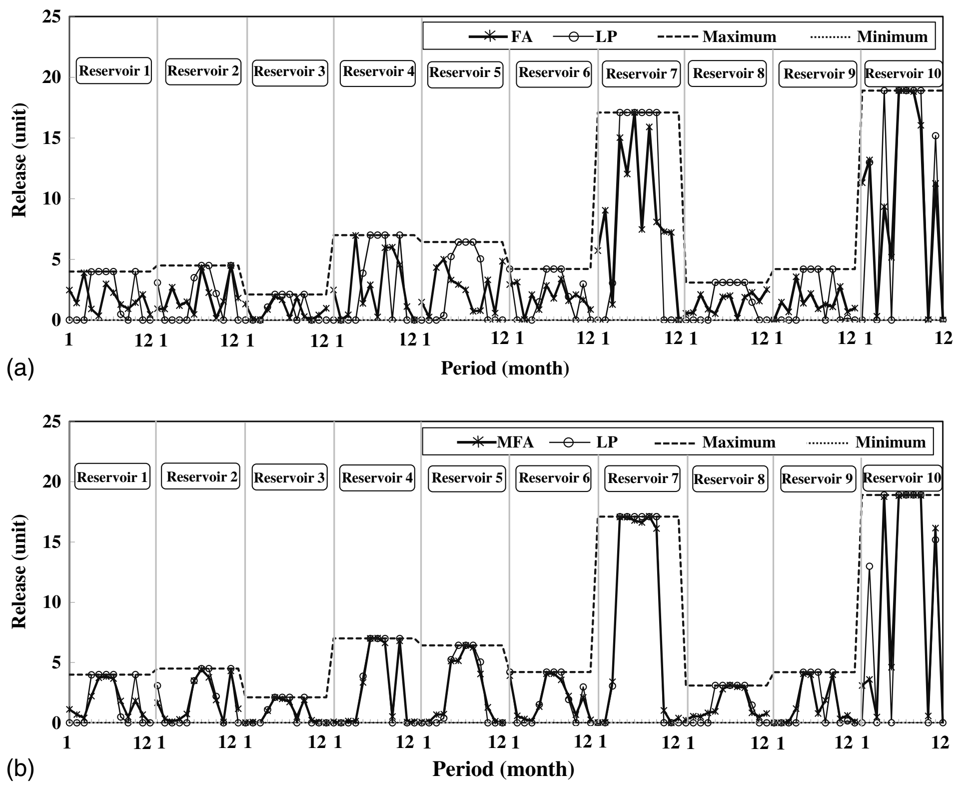

Fig. 6. Monthly reservoir releases of the best run of five runs for the CTP obtained from (a) the FA; (b) the MFA

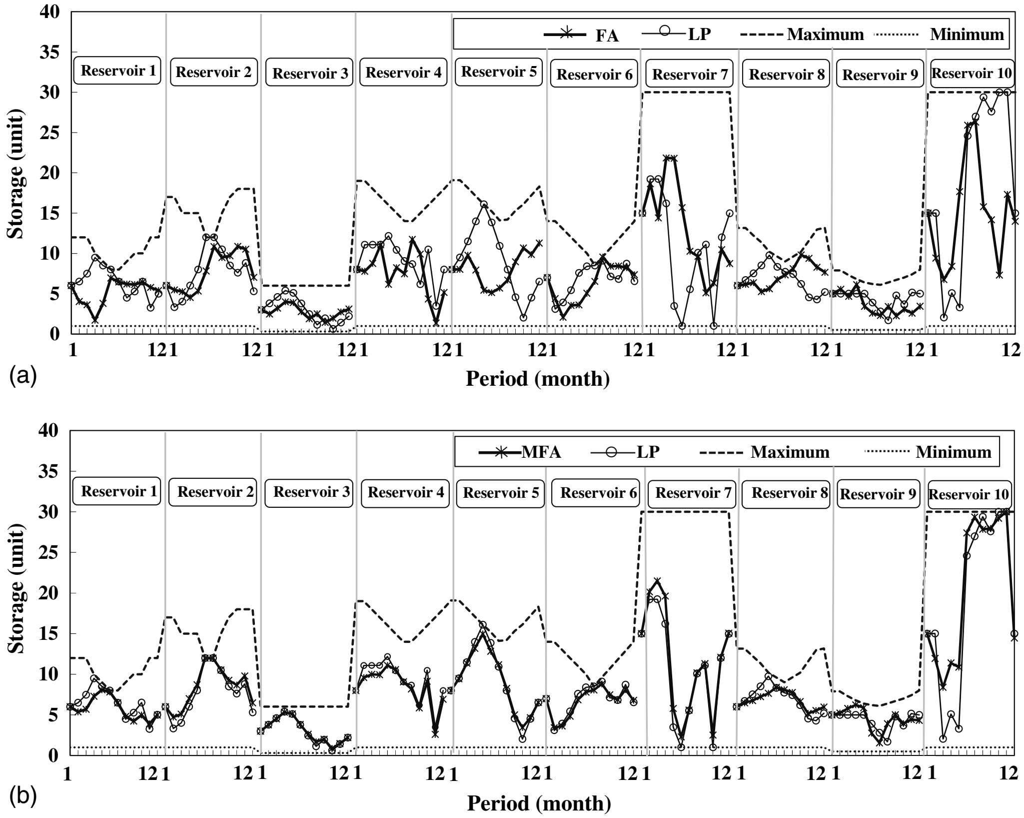

Fig. 7. Monthly reservoir storages of the best run of five runs for the CTP obtained from (a) the FA; (b) the MFA

Table 3. Comparison of Computed Objective Function from Various Mathematical Programming Approaches or Algorithms for the Benchmark Problems

Benchmark problem	Study	Method	Number of functional evaluations	Objective function value
Discrete four-reservoir (DFP)	Larson (1968)	LP	—	401.300
	Heidari et al. (1971)	DDDP	—	399.060
	Murray and Yakowitz (1979)	Modified DDP	—	401.274
	Wardlaw and Sharif (1999)	GA	750,000	401.300
	HInçal et al. (2011)	New version of GA	25,000,000	401.300
	Bozorg-Haddad et al. (2011)	HBMO	1,100,000	401.300
	Present study	FA	15,000	395.900
	Present study	MFA	15,000	401.300
Continuous four-reservoir (CFP)	Chow and Cortes-Rivera (1974)	LP	—	308.2665
	Chow and Cortes-Rivera (1974)	DDDP	—	307.9800
	Murray and Yakowitz (1979)	Modified DDP	—	308.2340
	Bozorg-Haddad et al. (2011)	LP	—	308.2915
	Bozorg-Haddad et al. (2011)	HBMO	14,000,000	308.2400
	Bozorg-Haddad et al. (2011)	HBMO	1,100,000	308.0700
	Bozorg-Haddad et al. (2015b)	WCA	1,100,000	307.5000
	Bozorg-Haddad et al. (2015b)	BA	500,000	308.2000
	Bozorg-Haddad et al. (2016)	BBO	500,000	308.1200
	Present study	FA	500,000	306.35
	Present study	MFA	500,000	308.25
Continuous 10-reservoir (CTP)	Murray and Yakowitz (1979)	DDDP	—	1,190.25
	Wardlaw and Sharif (1999)	LP	—	1,194.00
	Wardlaw and Sharif (1999)	GA	500,000	1,190.25
	Jalali et al. (2007)	MCAA	3,000,000	1,192.39
	Bozorg-Haddad et al. (2011)	HBMO	14,000,000	1,192.54
	Bozorg-Haddad et al. (2011)	HBMO	1,320,000	1,156.79
	Present study	FA	1,000,000	1,107.85
	Present study	MFA	1,000,000	1,185.00

J. Water Resour. Plann. Manage., 2016, 142(9): 04016029

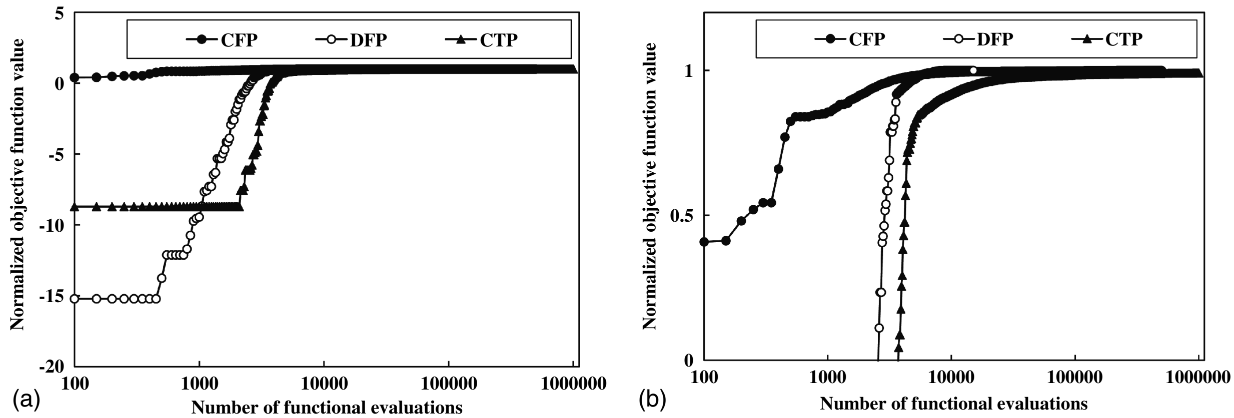

Fig. 8. Normalized average rates of convergence over five runs for the DFP, CFP, and CTP obtained from the MAF: (a) total range of objective function value; (b) smaller range of objective function value

Table 3 lists results of the comparison. It is evident in Table 3 that the MFA solved the CFP with the lowest number of functional evaluations (15,000) compared with other solution algorithms. Not only does it achieve the best solution equal to LP (global optimum), but it also produces four alternatives. Thus, it is asserted that the MFA can effectively solve discrete multireservoir operation problems with a reliability of 100% of obtaining the global solution. Similarly, in CFP, it is concluded that MFA achieved the nearest solution to the LP (global optimum) compared with all of the methods listed in Table 3. The results of Table 3 establish that the MFA dominated the BA, BBO, and the FA with the same number of functional evaluations (0.5 million). Therefore, it is concluded that the MFA achieved the best solution for the CFP with the fastest convergence among all the tested solution algorithms.

For the last benchmark problem, CTP, according to Table 3, the MFA dominated the HBMO and the FA when using about the same number of functional evaluations (1 million). The results of Table 3 indicate that the MFA was not able to dominate the GA with 1 million functional evaluations (see also results by Wardlaw and Sharif 1999). Yet, the procedure of setting the parameters of the MFA is easier and faster owing to modifications introduced in this paper's methodology.

Although the MFA performance was illustrated for each problem independently, a comparison of the algorithm'sperformance across these problems is pertinent. The average convergence rate of convergence of each problem was normalized so that all the objective values were divided by the LP result (global optimal solution). The three normalized average convergence rates are displayed in Fig. 8, where is shown in part a) the total range of objective function variation, and in Fig. 8(b) the normalized range ([0,1] without the negative values) for better comparison. It is concluded from Fig. 8 that as the number of reservoirs increases, the number of functional evaluations required to reach near-optimal solutions also increases. In addition, the algorithm converges more slowly compared with the convergence rate attained when the number of reservoirs (and of decision variables) is smaller. In other words, convergence to the solution of the CFP is faster and with a lower number of functional evaluations than that observed for the CTP. It is also seen in Fig. 8 that convergence to the CFP's solution is faster than that obtained for the DFP's solution.

Concluding Remarks

This paper has demonstrated that the MFA is a new metaheuristic algorithm capable of optimally solving multireservoir operation problems. Because of the limitations of FA to accurately solve complex optimization problems, FA was modified and entitled MFA. The MFA was employed to solve three benchmark reservoir operation problems. Based on the results of the first problem, the DFP, it is concluded that the MFA could calculate the best value of the objective function, equal to the global optimum achieved with LP (401.3). Another substantial achievement of applying the MFA to solve the DFP is that it identified four different alternative optimal solutions that had not been reported before by other investigators using other techniques. This finding confirms the wide range of the solution space searched by the MFA.

Concerning the CFP, the optimal solution obtained with the MFA (308.25) is 99.99% of the LP solution (308.29), and is larger than those reported by previous studies published in the literature considering approximately the same number of functional evaluations. This negligible difference reveals that the MFA is reliable when applied to continuous multireservoir operation problems. Concerning the results of the MFA for the third problem, the CTP, the advantages and superior performance of MFA were highlighted by solving the complex CTP reaching the best value of the objective function (1,185.00) being 99.21% of the LP solution (1,194.44). It is noted that this calculated value is larger than those obtained in previous studies published in the literature using approximately the same number of functional evaluations.

This study's results demonstrate that the MFA can efficiently handle problems in continuous and discrete domains. Our evaluation of the MFA's solving capacity with well-known benchmark problems indicates that it is reliable for optimizing multireservoir operation problems.

Appendix I. Discrete Four-Reservoir System Operation Problem (DFP) (Maximization)

The DFP system consists of four reservoirs with associated fluxes illustrated in Fig. 9(a). In accordance with the layout of the DFP shown in Fig. 9(a), the matrix of indexes of reservoir release connections is expressed as

$$\mathrm{RCM}_{4\times 4} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 \\ +1 & 0 & +1 & -1 \end{bmatrix} \quad (16)$$

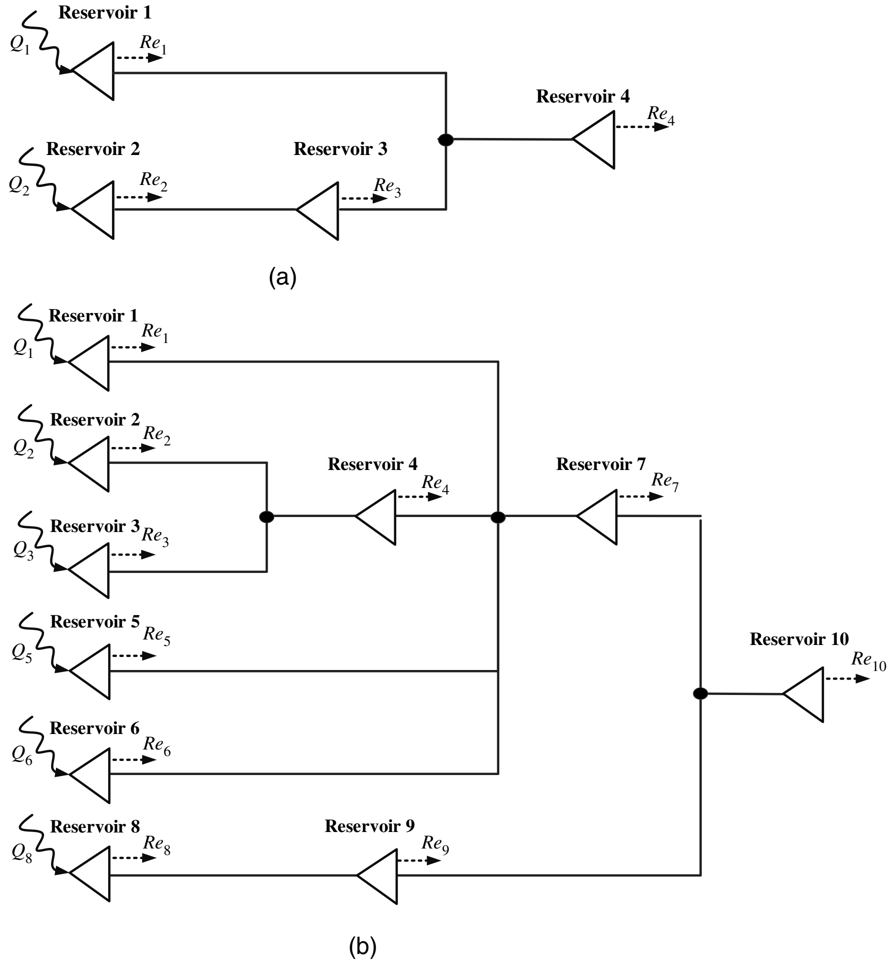

Fig. 9. Layout of the benchmark multi-reservoir operation problem with its associated fluxes: (a) the four reservoirs system; (b) the 10 reservoirs system

Table 4. Net Inflows to Reservoirs, the Maximum Allowable Reservoir Storages, and the Benefits Data of the DFP

		Period											
Data	Reservoir	1	2	3	4	5	6	7	8	9	10	11	12
The net inflows to reservoirs (unit)	1	2	2	2	2	2	2	2	2	2	2	2	2
	2	3	3	3	3	3	3	3	3	3	3	3	3
	3	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0
The maximum allowable reservoirs storages (unit)	1	10	10	10	10	10	10	10	10	10	10	10	10
	2	10	10	10	10	10	10	10	10	10	10	10	10
	3	10	10	10	10	10	10	10	10	10	10	10	10
	4	15	15	15	15	15	15	15	15	15	15	15	15
The benefits (unit)	1	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4
	2	1.4	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8
	3	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4	1.1
	4	2.6	2.9	3.6	4.4	4.2	4	3.8	1.4	3.6	3.1	2.7	2.5

The water released from reservoirs is utilized for hydropower generation and for irrigation supply. Hydropower generation is possible in each reservoir, and part of the release from reservoir number 4 is diverted for irrigation supply. In addition, the benefits of hydropower generation and irrigation supply are quantified by linear functions of discharge. Table 4 represents the net inflows to reservoirs, the maximum allowable reservoirs storages, and the benefits data for the DFP. Moreover, the minimum allowable reservoirs storages, the initial reservoirs storages, the target reservoirs storages, and the constraints imposed on reservoirs releases are listed in Table 5.

Table 5. Minimum Allowable Reservoir Storages, the Initial Reservoir Storages, the Target Reservoir Storages, and the Constraints Imposed on Reservoir Releases of the DFP

Reservoir	Minimum storage reservoir (unit) Smin	Target storage reservoir (unit) Sinitial	Initial storage reservoir (unit) Starget	Minimum release reservoir (unit) Remin	Maximum release reservoir (unit) Remax
1	0	5	5	0	3
2	0	5	5	0	4
3	0	5	5	0	4
4	0	5	7	0	7

Table 6. Net Inflows to Reservoirs, the Maximum Allowable Reservoir Storages, and the Benefits Data of CFP

Data	Reservoir	Period 1	2	3	4	5	6	7	8	9	10	11	12
The net inflows to reservoirs (unit)	1	0.5	1	2	3	3.5	2.5	2	1.25	1.25	0.75	1.75	1
	2	0.4	0.7	2	2	4	3.5	3	2.5	1.3	1.2	1	0.7
	3	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0
The maximum allowable reservoirs storages (unit)	1	—	12	12	10	9	8	8	9	10	10	12	12
	2	—	15	15	15	12	12	12	15	17	18	18	18
	3	—	8	8	8	8	8	8	8	8	8	8	8
	4	—	15	15	15	15	15	15	15	15	15	15	15
The benefits (unit)	1	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4
	2	1.4	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8
	3	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4	1.1
	4	2.6	2.9	3.6	4.4	4.2	4	3.8	4.1	3.6	3.1	2.7	2.5

Table 7. Minimum Allowable Reservoir Storages, the Initial Reservoir Storages, the Target Reservoir Storages, and the Constraints Posed on Reservoir Releases of CFP

Reservoir	Minimum storage reservoir (unit) Smin	Target storage reservoir (unit) Sinitial	Initial storage reservoir (unit) Starget	Minimum release reservoir (unit) Remin	Maximum release reservoir (unit) Remax
1	1	6	6	0.005	4
2	1	6	6	0.005	4.5
3	1	6	6	0.005	4.5
4	1	8	8	0.005	8

Appendix II. Continuous Four-Reservoir System Operation Problem (CFP) (Maximization)

The CFP system is similar to the DFP with the exception that the data used are continuous, rather than discrete (that is, integer-valued). Tables 6 and 7 list the required data and the constraints imposed in the CFP.

Appendix III. Continuous Ten-Reservoir System Operation Problem (CTP) (Maximization)

The CTP system consists of 10 reservoirs with its associated fluxes shown in Fig. 9(b). Unlike the four-reservoir system, CTP is complex in size and in the number of time-dependent constraints on reservoir storages. This system comprises reservoirs in parallel and in series whose matrix of indexes of reservoir release connections is as follows:

$$RCM_{10\times 10} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & +1 & +1 & +1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 & -1 \end{bmatrix} \quad (17)$$

In the CTP system the net benefit of the hydropower production is maximized over 12 operating periods. Net inflows are

Table 8. Net Inflows to Reservoirs, the Maximum Allowable Reservoir Storages, and the Benefits Data of CTP

Data	Reservoir	Period											
		1	2	3	4	5	6	7	8	9	10	11	12
The net inflows to reservoirs (unit)	1	0.5	1	2	3	3.5	2.5	2	1.25	1.25	0.75	1.75	1
	2	0.4	0.7	2	2	4	3.5	3	2.5	1.3	1.2	1	0.7
	3	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
	4	0	0	0	0	0	0	0	0	0	0	0	0
	5	1.5	2	2.5	2.5	3	3.5	3.5	3	2.5	2.5	2.5	1.5
	6	0.32	0.81	1.53	2.16	2.31	4.32	4.81	2.24	1.63	1.91	1.63	0.46
	7	0	0	0	0	0	0	0	0	0	0	0	0
	8	0.71	0.83	1	1.25	1.67	2.5	2.8	1.87	1.45	1.2	0.93	0.81
	9	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0
The maximum allowable reservoirs storages (unit)	1	—	12	12	10	9	8	8	9	10	10	12	12
	2	—	17	15	15	15	12	12	15	17	18	18	18
	3	—	6	6	6	6	6	6	6	6	6	6	6
	4	—	19	18	17	16	15	14	14	15	16	17	18
	5	—	19.1	18.1	17.1	16.1	15.2	14.1	14.2	15.3	16.1	17.2	18.3
	6	—	14	13	12	11	10	8.5	9.6	10.7	11.8	12.9	14
	7	—	30	30	30	30	30	30	30	30	30	30	30
	8	—	13.16	12.23	11.37	10.2	9.6	9	9.6	10.2	11.58	12.96	13.18
	9	—	7.9	7.3	6.8	6.4	6.2	6.1	6.4	6.7	7	7.4	8
	10	—	30	30	30	30	30	30	30	30	30	30	30
The benefits (unit)	1	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4
	2	1.4	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8
	3	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4	1.1
	4	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4
	5	1	1.1	1.2	1.3	1.4	1.5	1.67	1.56	1.45	1.34	1.25	1.14
	6	1.4	1.1	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8
	7	2.6	2.9	3.6	4.4	4.2	4	3.8	4.1	3.6	3.1	2.7	2.5
	8	1	1.1	1.2	1.3	1.4	1.5	1.67	1.56	1.45	1.34	1.25	1.14
	9	1	1	1.2	1.8	2.5	2.2	2	1.8	2.2	1.8	1.4	1.1
	10	2.7	3	2.8	3.2	2.9	3.9	4	3.6	3.7	2.8	3.5	2.1

Table 9. Minimum Allowable Reservoir Storages, the Initial Reservoir Storages, the Target Reservoir Storages, and the Constraints Posed on Reservoir Releases of CTP

	Minimum storage reservoir (unit)	Target storage reservoir (unit)	Initial storage reservoir (unit)	Minimum release reservoir (unit)	Maximum release reservoir (unit)
Reservoir	*S*min	*S*initial	*S*target	Remin	Remax
1	1	6	6	0.005	4
2	1	6	6	0.005	4.5
3	0.3	3	3	0.005	2.12
4	1	8	8	0.005	7
5	1	8	8	0.006	6.43
6	1	7	7	0.006	4.21
7	1	15	15	0.01	17.1
8	1	6	6	0.008	3.1
9	0.5	5	5	0.008	4.2
10	1	15	15	0.01	18.9

defined for each of the most upstream reservoirs as shown in Fig. 9(b). Table 8 lists the net inflows to reservoirs, the maximum allowable reservoir storages, and the benefits data of the CTP. Moreover, the minimum allowable reservoir storages, the initial reservoir storages, the target reservoir storages, and the constraints imposed on reservoir releases are listed in Table 9.

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