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Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

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Printed for the U.S. Atomic Energy Commission

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ABSTRACT

The polarization of deuterons produced at a center-of-mass angle of 115° in the $p + p \rightarrow \pi^+ + d$ reaction has been measured using 340-Mev unpolarized protons. The deuterons were detected in coincidence with the mesons and were analyzed by scattering from a carbon target at 24°. In conjunction with previous experiments on the reaction, this measurement completes the specification of the phenomenological parameters of Rosenfeld and Gell-Mann and Watson. Specifically it permits the determination of the relative phases of the several modes of meson production, which in turn are related to the p-p phase shifts at this energy.

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INTRODUCTION

As $p + p \rightarrow \pi^+ + d$ is a two-body reaction, the investigation and interpretation of meson production in nucleon-nucleon collisions through this reaction are particularly simple, and thus it has been thoroughly studied both experimentally and theoretically. Crawford and Stevenson[1] have measured the total and differential cross sections for the reaction in the region near threshold from 310 to 338 Mev, and have measured the asymmetric part of the cross section, using polarized protons of 315 Mev. Similar experiments have been performed at 437 Mev by Fields et al.[2] The results fit well with the phenomenological theory of Rosenfeld[3] and Gell-Mann and Watson,[4] which is developed for S- and P-wave meson production. The theory is thus expected to hold only over a limited energy region. Recently reported experiments by Meshcheryakov et al.[5] from 460 to 660 Mev show expected deviations in the energy dependence of the cross section but as yet give no indication of the presence of higher waves.

Marshak and Messiah[6] have analyzed the reaction, using polarized protons, and this analysis has been extended by Wolfenstein[7] and by Mandl and Regge[8] to include higher partial waves as well as a discussion of the polarization state of the deuteron. The investigation of deuteron polarization in this reaction was first suggested by Watson and Richman.[9]

Restricting consideration to S- and P-wave meson production, the following reactions conserve total angular momentum and parity:

Initial p-p state	Final π-d state	Relative transition amplitude
^1S_o	P	$\delta_o e^{i\tau_o}$
$^3P_1$	S	$\delta_1 e^{i\tau_1}$
$^1D_2$	P	1

where in the notation of Gell-Mann and Watson $\delta_o e^{i\tau_o}$ and $\delta_1 e^{i\tau_1}$ are the ratios of the complex amplitudes of production from S- and P-wave protons respectively to that from D-wave protons. Including the amplitude for production from D-wave protons but neglecting the over-all phase there are then five quantities to be determined in the phenomenological theory. The four experiments so far reported-- the total cross section, angular distribution, energy dependence of the cross section, and the asymmetric part of the cross section (using polarized protons)--plus the experiment described in this paper, in which the polarization of the outgoing deuteron is determined by use of unpolarized protons, are sufficient to completely specify the reaction near threshold.

DEUTERON POLARIZATION AND CALIBRATION OF ANALYZER

Before discussion of the experiment it will first be necessary to consider the description of the deuteron polarization and the detection of this polarization with an analyzer whose analyzing power has previously been established by means of a double-scattering experiment with deuterons.

The polarization state of a deuteron has been studied by Lakin,[10] who shows that its complete specification requires not only the expectation value of the spin T_{1M}, but also of the second-rank tensors T_{2M}. Explicit forms of the tensor operators in terms of the spin operator $\underline{S}$ are given by Lakin. He then shows that in two successive identical scatterings of a deuteron the angular distribution can be written as

$$I(\theta, \phi) = I_o(\theta)\left[1 + \langle T_{20}\rangle^2 + 2\left\{\left|\langle T_{11}\rangle\right|^2 - \langle T_{21}\rangle^2\right\}\cos\phi + 2\langle T_{22}\rangle^2 \cos 2\phi\right], \quad (1)$$

where θ and ϕ are the polar and azimuthal angles of scattering, I_o is the unpolarized differential cross section, and $\langle T_{20}\rangle$, $\langle T_{11}\rangle$, $\langle T_{21}\rangle$, and $\langle T_{22}\rangle$ are the expectation values of the tensor components of deuteron polarization present in the beam after the first scattering. Hence four parameters are required to describe the result of a double-scattering experiment, in contrast to the one parameter needed for a spin $\frac{1}{2}$ particle.

Angular-distribution experiments on the double scattering of deuterons from various elements have been performed by Chamberlain et al.[11] in the energy region from 165 Mev to 100 Mev and will be reported in more detail in a forthcoming paper. In these experiments large cos ϕ-dependent terms were observed in the cross sections for elastic scattering from carbon and other elements down to an energy of about 100 Mev. A search was made for a cos 2 ϕ-dependent term and a change in the azimuthally independent part of the cross section using polarized deuterons, but to within the 4% accuracy of the experiments none were detected. Thus $\langle T_{20}\rangle$ and $\langle T_{22}\rangle$ are consistent with zero at the double scattering angle. For identical double scatterings from carbon at 17^o the cos ϕ-dependent term is large and positive, indicating that $\langle T_{11}\rangle \gg \langle T_{21}\rangle$. No further experimental means are available for the separate determination of $\langle T_{11} \rangle$ and $\langle T_{21} \rangle$ without, as pointed out by Lakin, recourse to rather infeasible magnetic deflection between the first and second scatterer. However, theoretical calculations of deuteron polarization from elastic scattering by H. P. Stapp[12] give agreement with experiment and indicate that all second-rank tensor components of polarization are small at the scattering angles and energies of interest here. Specifically he finds that $\left|\frac{T_{21}}{T_{11}}\right|$ is less than 0.15. We shall use this as a basis for excluding $\langle T_{21} \rangle$ and establishing the analyzing power of the analyzer for $\langle T_{11} \rangle$.

The procedure used to calibrate the analyzer is then as follows. The internal circulating deuteron beam is scattered at 16° from a carbon target and brought into the experimental area ("cave") in a manner similar to that for polarized protons. There they are scattered again at 16° by a carbon target into a counter telescope with the energy threshold set to accept only elastically scattered deuterons. With $\langle T_{20} \rangle^2$ and $\langle T_{22} \rangle^2$ measured to be zero and $\langle T_{21} \rangle$ assumed to be negligible, the asymmetry e_B obtained gives, by Eq. (1).

$$\left|\langle T_{11} \rangle_B\right| = \sqrt{\frac{e_B}{2}} = 0.32 \pm .02$$

for the deuteron beam entering the cave. The sign of $\langle T_{11} \rangle_B$ is not determined by this experiment, but can be deduced to be positive if the shell-model sign for the spin-orbit potential is used. This choice of sign for the spin-orbit potential is justified by the fact that it gives the experimentally determined[13] sign for proton polarization in high-energy elastic scattering from complex nuclei.

The beam is then degraded to 138 Mev to correspond to the energy of deuterons formed in the $p + p \rightarrow \pi^{+} + d$ reaction, and is scattered by a target consisting of two defining counters and a carbon target between them. The scattered deuterons are detected by a two-counter telescope with an energy threshold set to accept only elastically scattered deuterons. The geometrical arrangement of the deuteron-defining counters, carbon target, and analyzing counters is identical with that used for the measurement of the asymmetry of deuterons formed in $p + p \rightarrow \pi^+ + d$, and is shown as part of the meson-production experiment illustrated in Fig. 1.

The most suitable conditions of the analyzer for measuring deuteron polarization were established at this time by use of the polarized deuteron beam. The criterion used to determine the optimum analyzing-target thickness and analyzing angle for the minimum fractional uncertainty $\Delta e_c / e_c$ in the asymmetry e_c measured with the analyzer was that αe_c^2 should be a maximum, where α is the fraction of incident deuterons scattered into the counter telescope. The thickness of the carbon target chosen in this way was 0.75 inch (a greater thickness would have resulted in some scatterings at energies less than 100 Mev where the polarization is small), and a mean analyzing angle of 24°. Under these conditions $\alpha = 1/1150$ and the analyzing power

$$\left|\langle T_{11}\rangle_C\right| = \frac{e_c}{2\left|\langle T_{11}\rangle\right|_B} = 0.33 \pm .03$$

In the notation of Gell-Mann and Watson the polarization of the deuterons in the $p + p \rightarrow \pi^+ + d$ reaction is given by

$$P = \frac{\sin \omega_o}{\left[\left(\frac{1}{2} + X\right) / \sqrt{X^2 + X}\right] + \cos \omega_o} \cdot \frac{\sin \theta \cos \theta}{A + \cos^2 \theta} \qquad (2)$$

where P is the expectation value of the deuteron spin and is related to $\langle T_{11}\rangle$ by $P = \frac{2}{\sqrt{3}} \langle T_{11}\rangle$. θ is the center-of-mass angle of meson production and A and X are found by Crawford and Stevenson to be $A = 0.25 \pm .03$ (at 340 Mev) and $X = 0.082 \pm .034$. With this value of A the maximum deuteron polarization occurs at $\theta = 66^\circ$ and 114° or at meson laboratory angles of $\theta = 35.5^\circ$ and 66.5°. A measurement of the deuteron polarization then determines ω_o, which is related to the phase angles τ_o and τ_1.

EXPERIMENTAL ARRANGEMENT

The arrangement of the apparatus for the $p + p \rightarrow \pi^+ + d$ reaction is shown in Fig. 1. An unpolarized proton beam of 3×10^8 protons/sec. entered the cave through a 2-inch-diameter collimator, having passed through a set of three strong-focusing coils which caused almost the entire beam to be contained in a rectangle 1 by 0.75 inch, thereby avoiding scattering from the collimator. The beam energy, measured by a Bragg curve, was 342.5 Mev.

The liquid hydrogen target consisted of 1 g/cm^2 hydrogen contained in a 4-mil stainless steel cylinder. After traversing the hydrogen, the beam passed through a 7-foot helium bag to reduce scattering, a monitoring ion chamber, and then into a rear cave, eliminating back-scattering into the analyzer.

All deuterons are produced in a cone of $\pm 6^\circ$ to the beam direction, with the deuteron laboratory angle a double-valued function of the center-of-mass angle, approximately symmetrical about 90° c.m. Since Eq. (2) is antisymmetrical about 90° c.m., it follows that it is necessary to detect the meson in coincidence with the deuteron in order to measure the deuteron polarization.

Elastically scattered protons at 35° were 300 times as numerous as mesons and would have made accidental coincidences between meson counters and deuteron counters prohibitively large if means were not employed to eliminate them. Consideration was given to Cerenkov counter detection of the meson, but at this energy the difference in velocity between the mesons and protons was not sufficient to make a clean separation. Momentum analysis, however, was feasible, and mesons produced at $35^\circ \pm 8^\circ$ were bent by a magnetic field through 60° and detected by a pair of 4-by-10-inch plastic scintillation counters. One-eighth-inch copper between the meson counters was found to be effective in reducing low-energy background without decreasing the π-d coincidence rate.

The deuteron-detecting counters, consisting of a pair of 2-by-2-inch and a pair of 2.25-by-2.25-inch plastic scintillators, were placed at a distance of 10 feet from the hydrogen target. No effort was made to make either the deuteron or meson counters defining, but instead the counter sizes were chosen in such a way as to minimize the accidental rate. The deuteron counters and a 0.75-inch carbon target between them served as a target for the elastic scattering of deuterons into the analyzing telescope composed of a pair of 2-by-6-inch counters.

The signals from each of the six counters were amplified by two Hewlett-Packard A Amplifiers and clipped with 4-foot shorted stubs before being fed into four- and sixfold coincidence circuits. The π-d coincidences were recorded in the fourfold channel, while the sixfold coincidence required in addition that the deuteron be scattered into the analyzing counters. The deuteron counters were first plateaued on deuterons accelerated by the cyclotron and were operated near the knee of the plateau curve so that high-energy protons, which constituted the major source of background and whose pulse heights were one-third as high as deuterons, were counted with reduced efficiency. Frequent checks were made to verify that the counters were on plateau for deuterons.

In order to further supress the proton background valuable use was made of the 1.3×10^{-8}-sec difference in time of flight between the deuterons and protons. A curve of π-d coincidences vs delay in the meson counter is shown in Fig. 2. The time resolution was adequate to strongly discriminate against fast protons.

Range curves were taken on both the mesons and deuterons, and are shown in Fig. 3. Both curves agree well with the ranges and range spreads to be expected.

Accidental rates were obtained by delaying the meson counters or deuteron-analyzing counters by one cyclotron rf pulse. The beam level was such that the accidental π-d coincidences did not exceed 10% of the effect. Accidentals between the analyzing counters and the π-d counters were also about 10%, and were nearly symmetrical for left and for right scattering of deuterons. As shown in Fig. 1, lead shielding was abundantly used to shield the deuteron counters from the main proton beam, which passed near the counters.

The analyzing-counter apparatus was approximately positioned optically. Then the deuteron-beam centerline was precisely located by sweeping the analyzing counters through the deuteron beam. By this technique the centerline was located to $\pm 0.10^{\circ}$, with a resultant uncertainty in the asymmetry given by

$$\Delta e \approx \frac{d\ln\sigma}{d\theta} \Delta\theta = 0.17 \times 0.10^{\circ} = \pm .017.$$

The deuteron-defining counters were accurately located over the center of rotation. By shielding various regions of the counters and observing the effect upon the π-d coincidence rate, it was determined that the illumination was approximately uniform over the entire surface, thus assuring that the beam passed over the center of rotation. More detailed discussion of these alignment methods, which have been applied in other polarization experiments, are contained in Reference 14.

Asymmetries were measured by setting the analyzing counters alternately to 24° left and 24° right and using the relation $e = R-L/R+L$. The π-d coincidence rate was used as a primary monitor, and agreement between it and the ion chamber was within 1%. Accidental coincidences of both varieties discussed previously were subtracted. The counting rates were about 30 π-d coincidences per second, and 1.5 counts per minute in the sixfold channel recording analyzed deuterons.

An attempt was made to also measure the polarization of the deuteron in the forward center of mass angle by observing the meson at $\textcircled{H} = 75$ degrees in coincidence. By equation (2) this should reverse the polarization and would have been a useful check on any inherent asymmetries in the experiment. However range discrimination against protons scattered into the meson counters proved insufficiently positive to make the experiment successful.

RESULTS

The asymmetry measured for deuterons produced at a center-of-mass angle of 115^{o} and scattered from carbon at 24^{o} is $e = 0.043 \pm .033$. Dividing by the analyzing power of the analyzer, we find that the polarization of the deuterons is $P = 0.076 \pm .059$. For an analyzer with $\left| \frac{\langle T_{21} \rangle}{\langle T_{11} \rangle} \right| < 0.15$, any sensitivity of the analyzer to $\langle T_{21} \rangle$ will not alter the value of polarization by an amount greater than the statistical uncertainty.

The average value of $\sin\theta \cos\theta / (A + \cos^2\theta)$ for this experiment is calculated to be $0.83 \pm .03$, yielding from Eq. (2) a value of $\omega_o = 15.6^{o}$ or 175.0^{o}. The value ω_o is related to the phase angle τ_o between the two P-wave meson states by

$$|\delta_o| e^{i\tau_o} = \frac{1}{\sqrt{2}} (1 + 3X) + \frac{3}{\sqrt{2}} X(X+1) e^{i\omega_o} .$$

Figure 4 shows δ_o plotted in the complex plane. The dashed lines indicate the region of the plane in which δ_o is located by the experiments of Crawford and Stevenson, and the shaded areas show the additional limits set by this experiment. The area near the origin corresponds to meson creation, predominantly from the $^1D_2$ p-p state, and is to be preferred to creation from the $^1S_0$ p-p state for a strong meson-nucleon interaction in the isotopic spin 3/2, total angular momentum 3/2 state.

From the asymmetry in meson-deuteron production using polarized protons, Crawford and Stevenson obtain $\sin(\psi - \tau_1) = 0.70 \pm .14$, where ψ is given by the relation $\delta_o + \frac{1}{\sqrt{2}} = \left|\delta_o + \frac{1}{\sqrt{2}}\right| e^{i\psi}$. The determination of ψ thereby establishes τ_1. Table I lists the four possible values of τ_1 associated with the two values of ω_o, δ_o, and τ_o; the ambiguity arises from the fact that the experiments determine not the angles themselves but trigonometric functions of the angles.

Table I

The possible values of ω_o, δ_o, τ_o and τ_1. The first set corresponds to a dominant interaction in the initial $^1D_2$ p-p state

ω_o	δ_o	τ_o	τ_1
15.6	0.32	148.0	65.7
			156.9
175.0	1.51	177.9	40.5
			131.7

Gell-Mann and Watson state that τ_o and τ_1, are related to the p-p phase shifts by

$$\tau_o = \alpha\,(^1S_o) - \alpha\,(^1D_2) + n\pi\,,$$

$$\tau_1 = \alpha\,(^3P_1) - \alpha\,(^1D_2) + (n' + 1/2)\pi\,.$$

A phase-shift analysis[12] of the various p-p scattering and polarization experiments[15] at 315 Mev provide an independent means of determining τ_o and τ_1. Figure 5 is a plot of τ_o vs τ_1 containing the four $p + p \rightarrow \pi^+ + d$ solutions with their statistical errors. The dots show the values of τ_o and τ_1 for 28 solutions of the p-p phase shifts obtained from single-, double-, and triple-scattering experiments. Additional experimental information can, however, give a unique set. About half of these 28 solutions can be discarded, since they do not give the observed destructive interference between Coulomb and nuclear scattering at small angles. Of the remainder, the six best fits have been compared with the recent triple-scattering experiment of James Simmons involving a rotation of the spin into the direction of motion of the proton between the first and second scattering. Only one solution is compatible with that experiment, and this solution is indicated by a cross in Fig. 5. Its value of $\tau_o = 143.1^o$ and $\tau_1 = 79.3^o$ is in good agreement with the first of the above four solutions given by the $p + p \rightarrow \pi^+ + d$ experiments.

Valuable advice and assistance in π-d coincidence techniques were provided by Drs. Frank S. Crawford and M. Lynn Stevenson, and much assistance during the course of the experiment was rendered by Messrs. John Baldwin and Richard Weingart. Drs. Owen Chamberlain, Emilio Segrè, Clyde Wiegand, and Tom Ypsilantis generously supplied much of the experimental apparatus. I am indebted to Professor Lincoln Wolfenstein for discussions on theoretical aspects of deuteron polarization and to Dr. Henry P. Stapp for making available his calculations on the polarization of elastically scattered deuterons.

This work was done under the auspices of the U.S. Atomic Energy Commission.

* Fermi's Varenna Lecture Notes (Supplemento Nuovo Cimento 2, 54 (1955) show the general approach to these relationships through the unitarity and symmetry of the S matrix.

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LEGENDS

Fig. 1. Geometry of the $p + p \rightarrow \pi^+ + d$ experiment

Fig. 2. π-p Coincidences vs time delay in meson counter

Fig. 3. Meson and deuteron range curves

Fig. 4. Complex δ_o plane. Dashed lines indicate region of the plane in which δ_o was located by the experiments of Crawford and Stevenson, who determined $X = 0.082 \pm .034$. Shaded areas show the limits set by this experiment.

Fig. 5. The four possible $p + p \rightarrow \pi^+ + d$ solutions are shown with their statistical errors (standard deviations). The dots indicate 28 solutions of the p-p scattering experiments, and the point indicated by a superposed cross is the one to be preferred according to more recent triple-scattering experiments.

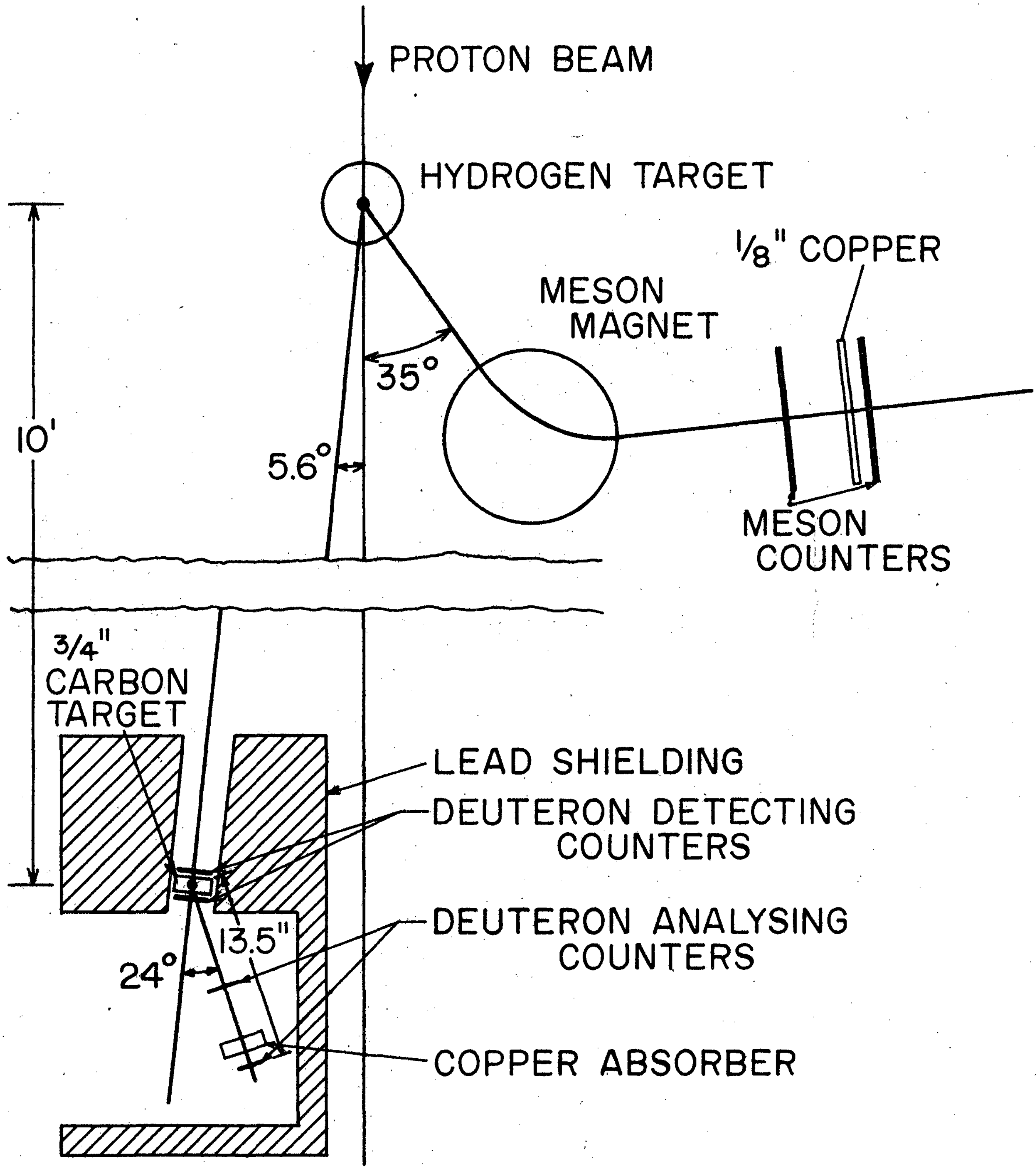

PROTON BEAM
HYDROGEN TARGET
1/8" COPPER
MESON
MAGNET
35°
10'
5.6°
MESON
COUNTERS
3/4"
CARBON
TARGET
LEAD SHIELDING
DEUTERON DETECTING
COUNTERS
DEUTERON ANALYSING
COUNTERS
13.5"
24°
COPPER ABSORBER

Fig 1

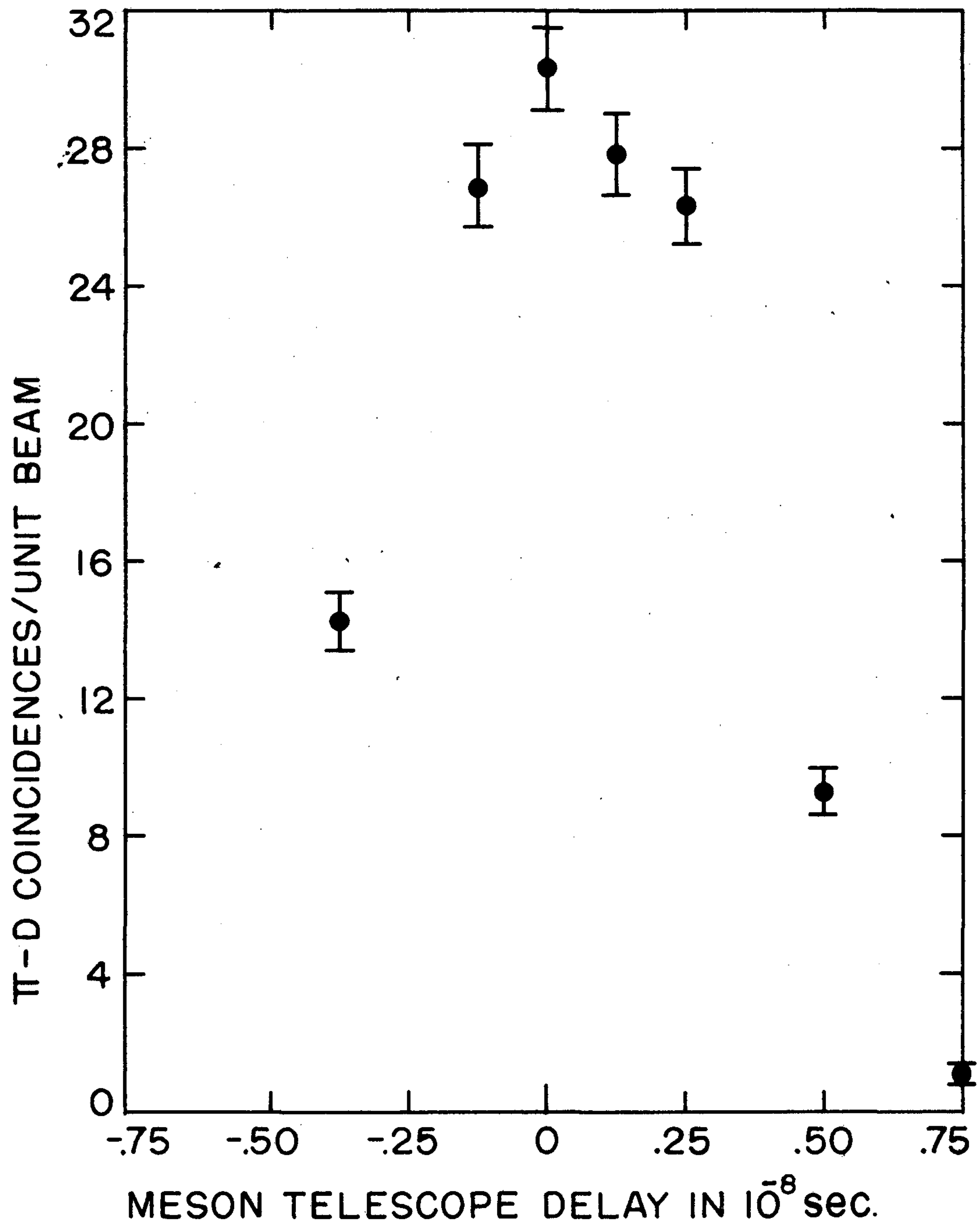
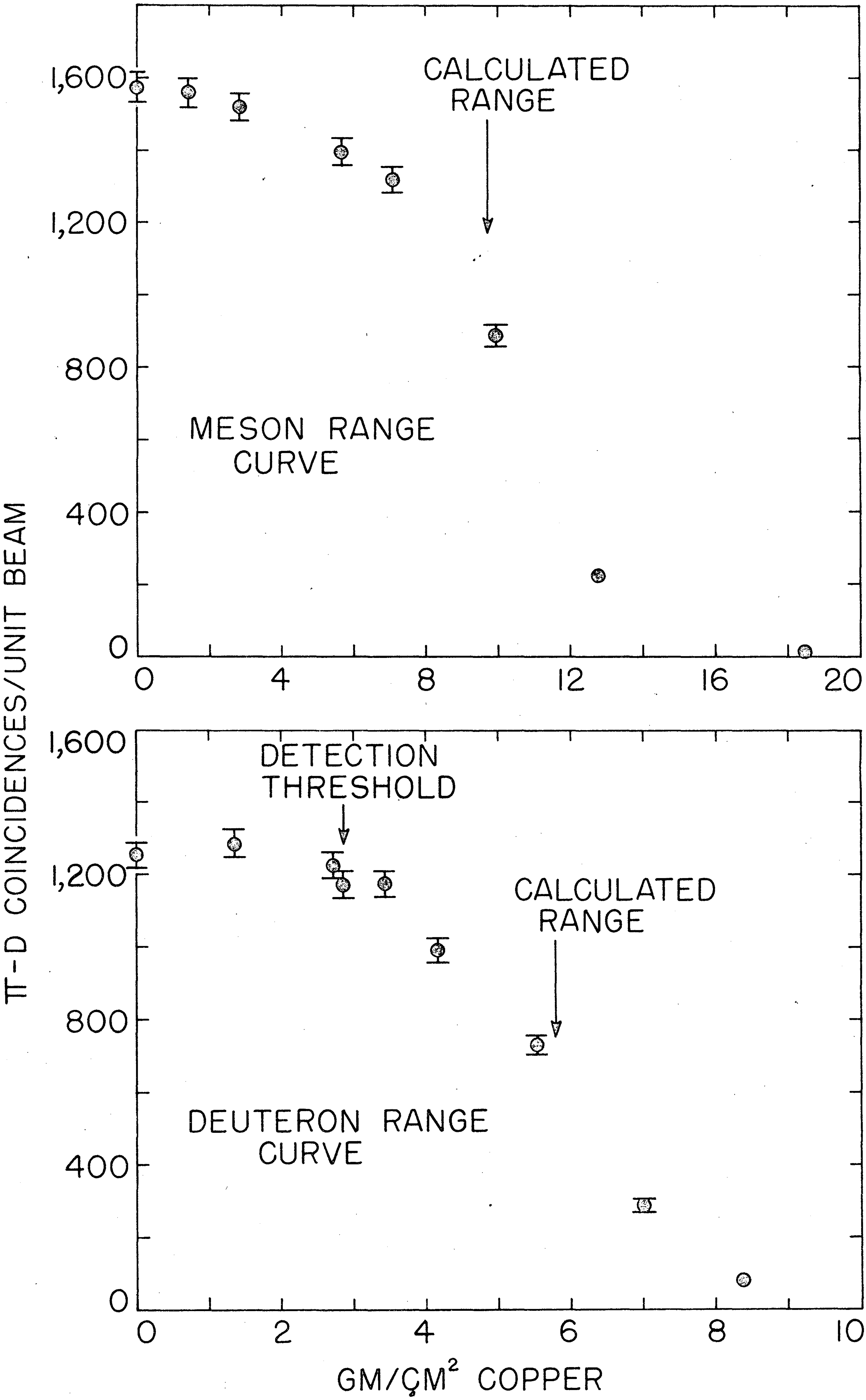

π-D COINCIDENCES/UNIT BEAM
32
28
24
20
16
12
8
4
0
-.75
-.50
-.25
0
.25
.50
.75
MESON TELESCOPE DELAY IN 10⁻⁸ sec.

Fig 2



CALCULATED RANGE
MESON RANGE CURVE
π-D COINCIDENCES/UNIT BEAM
1,600
1,200
800
400
0
0
4
8
12
16
20
DETECTION THRESHOLD
CALCULATED RANGE
DEUTERON RANGE CURVE
0
2
4
6
8
10
GM/CM² COPPER

Fig 3

24123-2

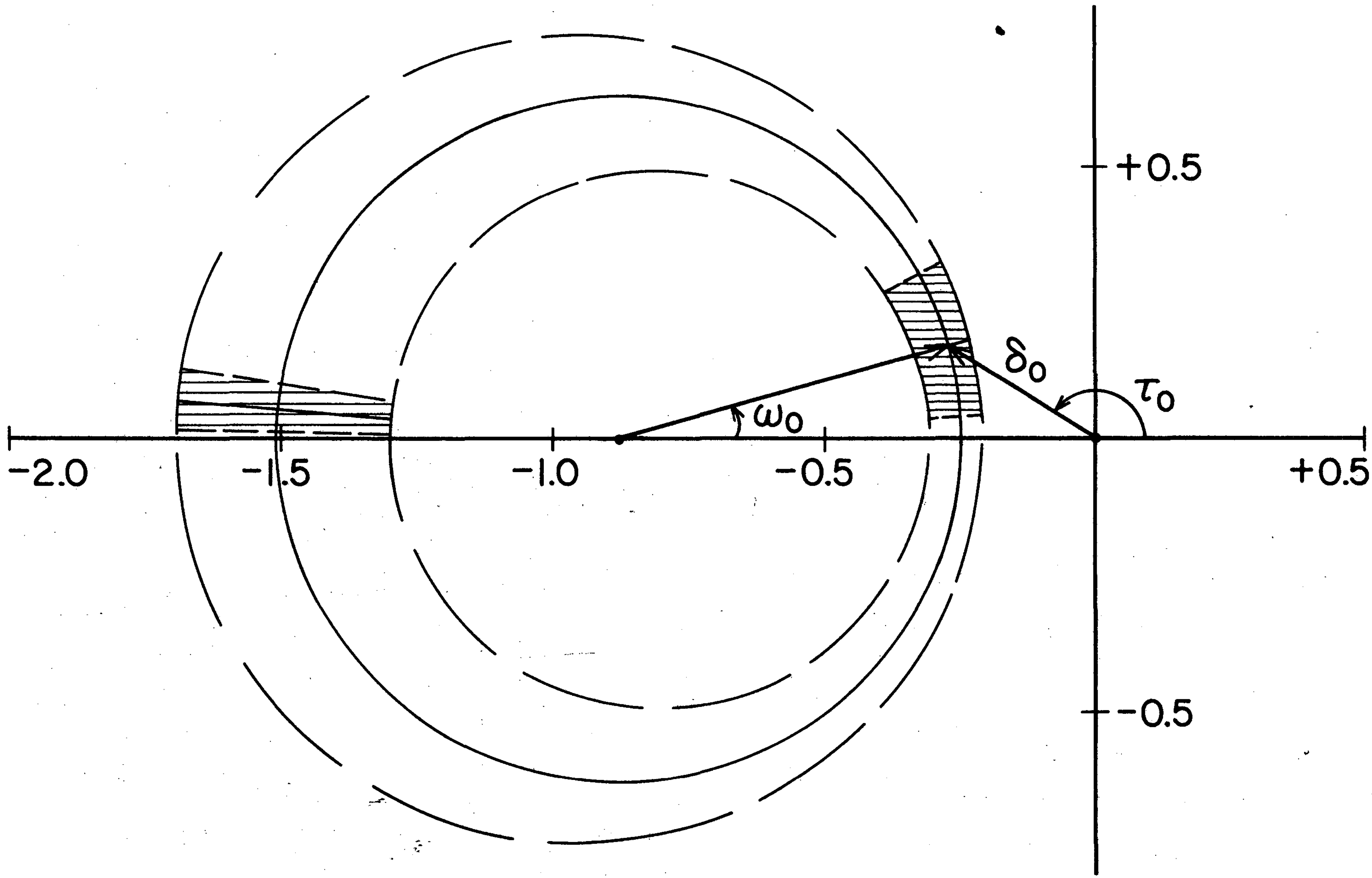
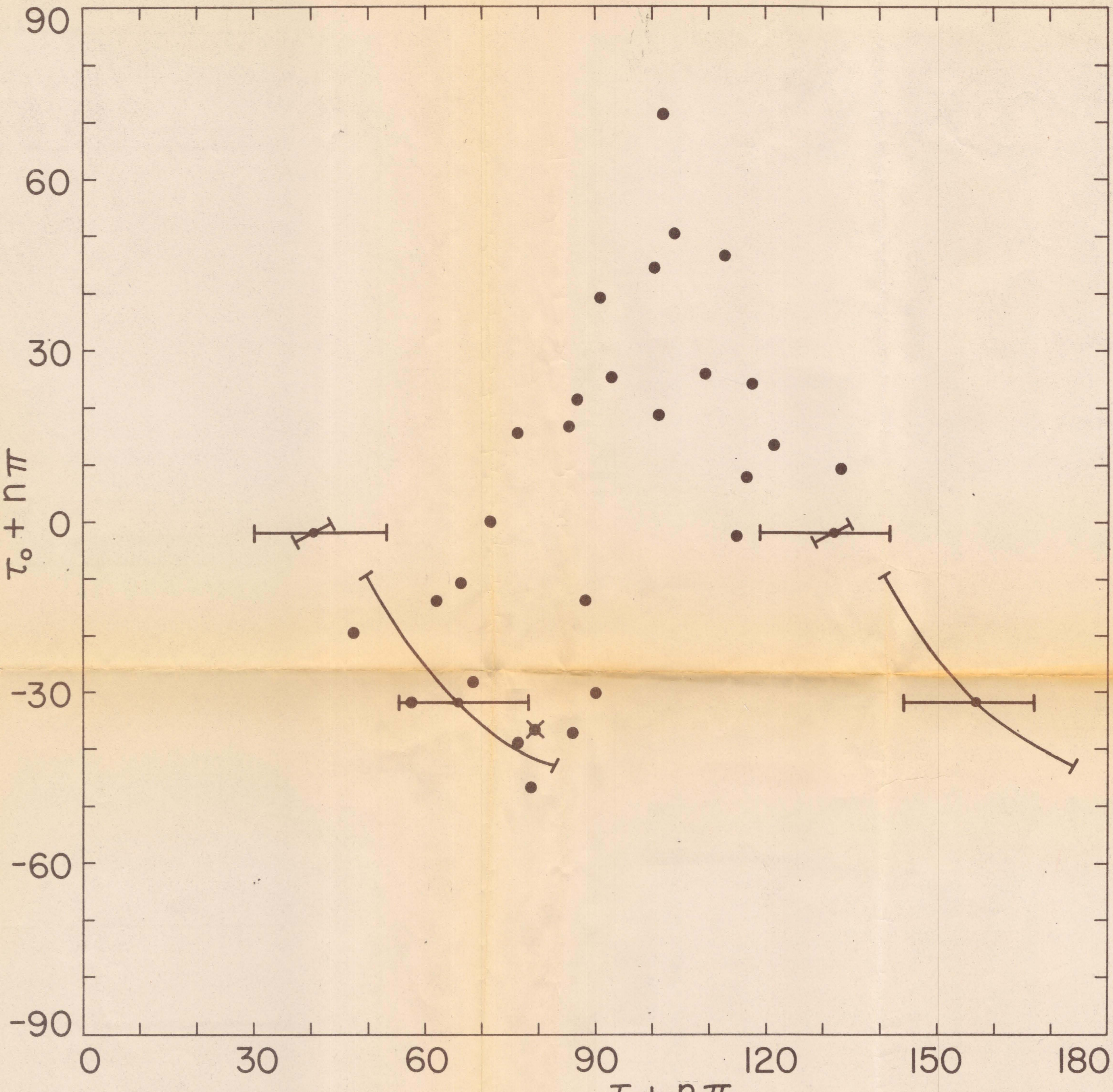


Fig 4

[redacted]

24199-1



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