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Excitation of a Closed Cylindrical Cavity by a Charged Ring Moving Along the Axis at Constant Velocity

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Barbara S. Levine and Andrew M. Sessler

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EXCITATION OF A CLOSED CYLINDRICAL CAVITY BY A
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CONSTANT VELOCITY

Barbara S. Levine and Andrew M. Sessler

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EXCITATION OF A CLOSED CYLINDRICAL CAVITY BY A CHARGED RING
MOVING ALONG THE AXIS AT CONSTANT VELOCITY*

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Summary

An important aspect of the design of high-current devices is the beam loading of the accelerating structure. The phenomenon is sensitive to the charge distribution of the beam and to the geometry of the structure. Calculations for realistic geometry are difficult; to gain insight, a number of authors have studied the simplified problem of beam interaction with a closed, right cylindrical cavity.^{1,2,3} A formal solution, for arbitrary charge distribution, has been given as a (double) sum over the modes of the cavity.³ It has been emphasized by all previous workers that the simplified problem approximates reality only for those modes having wavelengths large compared to the holes in an actual structure. We have developed a computational program aimed at evaluating the energy transfer to the cavity when the geometry of the beam is characterized by inner and outer radii as well as an axial length. Sums over selected sets of (or all) modes are available. Parametric studies are presented of the total cavity excitation by a charged ring.

Formalism

The problem considered is that of a ring with total charge Q moving at constant velocity $v = \beta c$ in the z -direction, and encountering a single closed right cylindrical cavity of radius b and axial extension from $z = 0$ to $z = L$. The cavity is perfectly conducting, and we calculate the energy given up by the ring to the electromagnetic excitation of the cavity. The ring is assumed to move through the infinitely thin cavity walls. The problem was solved for a point charge by Kolpakov and Kotov¹ and by Morton and Neil²; and for a more general distribution by Winterbon.³ The analysis follows the classic paper by Condon.⁴

The result for a point charge is divergent, as is the energy loss for a ring of major radius, a , having zero minor radii. We consider, therefore, a ring which is uniformly charged between major radii a_1 and a_2 , and for z between $-h$ and h . The energy loss by the charged ring in going through the cavity ΔU is given by

$$\Delta U = \frac{32Q^2\beta^2}{Lb^2} \sum_{s=1}^{\infty} \sum_{p=0}^{\infty} \left(\frac{1+\delta_{p0}}{2} \right) \left[\frac{v \sin \frac{\omega_s h}{v}}{\omega_s h} \right]^2 \times \left[\frac{a_2 J_1(\mu_s a_2) - a_1 J_1(\mu_s a_1)}{\mu_s^2 (a_2^2 - a_1^2) J_1(\mu_s b)} \right]^2 \frac{[1 - (-1)^p \cos \frac{\omega_s L}{v}]^2}{[1 + \left(\frac{p\pi}{\gamma L \mu_s} \right)^2]^2}, \quad (1)$$

where s and p are integers, δ is a Kronecker delta, $\gamma = (1 - \beta^2)^{-1/2}$, the resonant frequencies of the cavity are

$$\omega_s = c \left[\mu_s^2 + \left(\frac{p\pi}{L} \right)^2 \right]^{1/2}, \quad (2)$$

and the quantities μ_s are determined from

$$J_0(\mu_s b) = 0; \quad s = 1, 2, \dots \quad (3)$$

Letting $h \rightarrow 0$ and $a_2 \rightarrow a_1 = a$, we obtain the (divergent) formula for a ring of zero minor dimensions:

$$\Delta U = \frac{8Q^2\beta^2}{b^2 L} \sum_{p,s} \left(\frac{1+\delta_{p0}}{2} \right) \left| \frac{J_0(\mu_s a)}{\mu_s J_1(\mu_s b)} \right|^2 \frac{[1 - (-1)^p \cos \frac{\omega_s L}{v}]^2}{[1 + \left(\frac{p\pi}{\gamma L \mu_s} \right)^2]^2}. \quad (4)$$

That the double sum (4) diverges can be inferred from the corresponding integral approximation for the terms with large values of p and s ,

$$\Delta U \rightarrow \frac{4Q^2\beta^2}{\pi L a} \int dp \int \frac{d\mu}{\mu^2} \frac{[1 - (-1)^p \cos \frac{\omega L}{v}]^2}{[1 + \left(\frac{p\pi}{\gamma L \mu} \right)^2]^2}, \quad (5)$$

which relies on asymptotic forms of the Bessel functions. The term in $\cos(\omega L/v)$ averages to zero, and making the change of variables

$$\frac{p\pi}{\gamma L} = \mu \tan \theta, \quad (6)$$

we obtain

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$$\Delta U \rightarrow \frac{4Q^2 \beta^2 \gamma}{\pi^2 a} \int_0^\infty \frac{d\mu}{\mu} \int_0^{\pi/2} \cos^2 \theta d\theta, \quad (7)$$

which clearly diverges--logarithmically--for large values of μ (or s). In the special case that $a = 0$ (truly point particle) the $J_0(\mu a)$ in (4) is replaced by unity rather than its asymptotic form, and the divergence is more severe (linear in μ) than in the case $a \neq 0$.

Computational Results

Numerical evaluation of (1) is difficult when γ is large and h is small, as the summation converges only very slowly. Winterbon quotes some numerical results;³ we have developed a general Fortran program which is available to anyone interested in obtaining values of ΔU for various sets of input parameters.

For illustrative purposes, we present numerical results for a particular cavity, a limited range of ring parameters, and with the sum in (1) taken over all modes. The results are expressed in terms of B , where

$$\Delta U = \frac{1}{2} Q^2 B(b, L, a_1, a_2, h, \gamma). \quad (8)$$

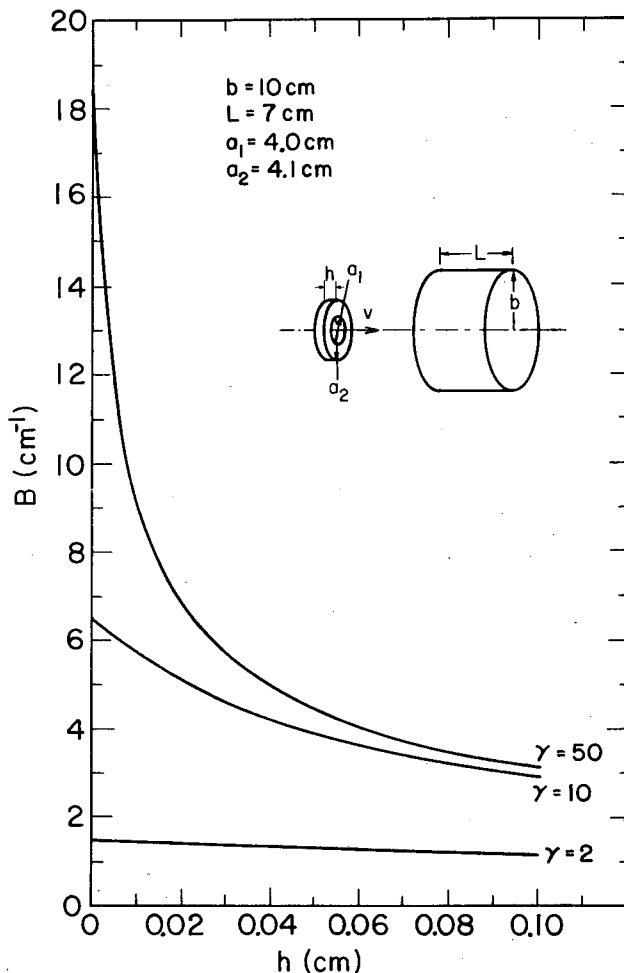
In the case presented, the cavity radius b is 10.0 cm, the cavity axial extent L is 7.0 cm, the inner major ring radius a_1 is 4.0 cm, and the outer major ring radius a_2 is 4.1 cm. In Fig. 1 we show the dependence of B on h for three values of γ . It can be seen that the value of B is finite as h approaches 0 (since $a_2 - a_1 > 0$). In Fig. 2 we present values of B as a function of γ for two values of h . It can be seen that B approaches a constant as $\gamma \rightarrow \infty$, the asymptotic value being a function of h . In Fig. 3 we show B as a function of γ for a case in which the ring axial dimension remains constant (0.1 cm) in the ring frame, so that in the laboratory, due to the Lorentz contraction, h decreases with increasing γ . The reader is cautioned that the example presented here is not immediately applicable to typical accelerating columns; the energy loss by the ring to excitation of a cavity having entrance and exit ports is certainly not given by the complete sum in (1), but rather, perhaps, by the sum over a restricted set of modes.^{1,3}

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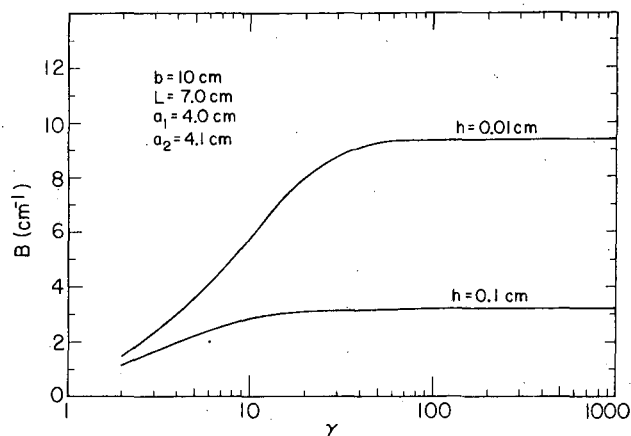
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4. E. V. Condon. J. Appl. Phys. **12**, 129 (1941).



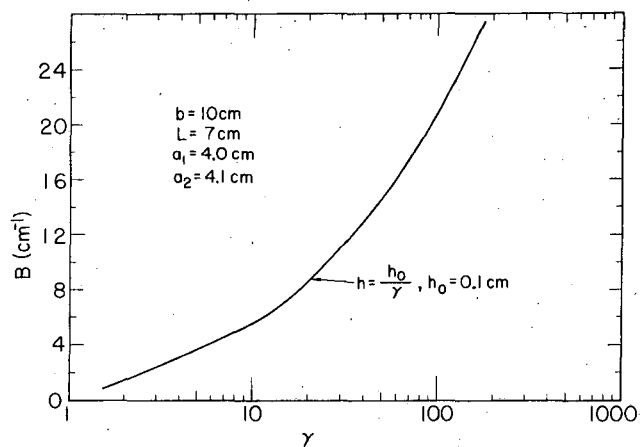
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Fig. 1. Cavity excitation energy [see Eqs. (1) and (8)] for a uniformly charged ring of major radii 4.0 cm and 4.1 cm, passing through a cavity of length 7 cm and radius 10 cm. The ring's axial extent is h , and its speed is characterized by $\gamma = (1 - \beta^2)^{-1/2}$.



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Fig. 2. Cavity excitation energy as a function of ring speed for two different ring axial extents. Other parameters are the same as in Fig. 1.



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Fig. 3. Cavity excitation energy as a function of ring speed for a Lorentz-contracted ring which has an axial extent of 0.1 cm in the ring frame. Other parameters are the same as in Fig. 1.

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