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Supersymmetry Breaking and the Supersymmetric Flavour Problem: An Analysis of Decoupling the First Two Generation Scalars [1]

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Abstract

The supersymmetric contributions to the Flavor Changing Neutral Current processes may be suppressed by decoupling the scalars of the first and second generations. It is known, however, that the heavy scalars drive the stop mass squareds negative through the two-loop Renormalization Group evolution. This tension is studied in detail. Two new items are included in this analysis: the effect of the top quark Yukawa coupling and the QCD corrections to the supersymmetric contributions to Δm_K. Even with Cabibbo-like degeneracy between the squarks of the first two generations, these squarks must be heavier than $\sim$ 40 TeV to suppress Δm_K. This implies, in the case of a high scale of supersymmetry breaking, that the boundary value of the stop mass has to be greater than $\sim$ 7 TeV to keep the stop mass squared positive at the weak scale. Low-energy supersymmetry breaking at a scale that is of the same order as the mass of the heavy scalars is also considered. In this case the finite parts of the two-loop diagrams are computed to estimate the contribution of the heavy scalar masses to the stop mass squared. It is found that for Cabibbo-like mixing between the squarks, the stop mass at the boundary needs to be larger than $\sim$ 2 TeV. Thus, for both cases, the large boundary value of the stop masses leads to an unnatural amount of fine tuning to obtain the correct Z mass.

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1 Introduction

The origin of electroweak symmetry breaking (EWSB) and the subsequent gauge hierarchy problem [1] are two large mysteries of the Standard Model (SM). Supersymmetry (SUSY)[2] provides a promising solution to these problems, by both stabilising the weak scale against radiative corrections[3], and by naturally breaking the electroweak symmetry through the quantum corrections of the superpartner of the top quark to the Higgs boson mass [4]. It is known, however, that generic weak scale values for the masses of the first two generation scalars give rates for many flavour violating processes that are in disagreement with the experimental observation. The measured value of Δm_K and detection limits for $\mu \to e\gamma$, and $\mu \to 3e$, for example, require that the first two generation scalars be degenerate to within a few tenths of a percent if their masses are at the weak scale [5, 6]. Constraints from CP violation are generally even more severe. Understanding the origin of this degeneracy is the supersymmetric flavour problem. Attempts to resolve this puzzle without introducing any fine tuning include: using approximate non-abelian or abelian symmetries[7]; communicating supersymmetry breaking to the visible sector by gauge interactions that do not distinguish between flavours [9]; squark-quark mass matrix alignment [8]; and raising the soft masses of the first two generation scalars to the tens of TeV range [10, 11, 12, 13, 14, 15, 16, 17].

The phenomenological viability and naturalness of this last scenario is the subject of this paper. To suppress flavour changing processes, the heavy scalars must have masses between a few TeV and a hundred TeV. The actual value depends on the degree of degeneracy and mixing between the masses of the first two generation scalars. As discussed in Reference [18], the masses of the heavy scalars cannot be made arbitrarily large without breaking colour and charge. This is because the heavy scalar masses contribute to the two-loop Renormalisation Group Equation (RGE) for the soft masses of the light scalars, such that the stop soft mass squared become more negative in RG scaling to smaller energy scales. This negative contribution is large if the scale at which supersymmetry breaking is communicated to the visible sector is close to the Grand Unification scale[18]. With the first two generation soft scalar masses ≈ 10 TeV, the initial value of the soft masses for the light stops must be $\approx$ (few TeV)^2 to cancel this negative contribution [18] to obtain the correct vaccum. This requires, however, an unnatural amount of fine tuning to correctly break the electroweak symmetry[19, 20].

In this paper we analyse these issues and include two new items not previously discussed within this context: the effect of the large top quark Yukawa coupling, λ_t, in the RG evolution, that drives the stop soft mass squared more negative; and QCD radiative corrections in the Δm_K constraint [21]. This modifies the bound on the heavy scalar masses which is consistent with the measured value of Δm_K. This, in turn, affects the minimum value of the initial scalar masses that is required to keep the scalar masses positive at the weak scale.

We note that the severe constraint obtained for the initial stop masses assumes that supersymmetry breaking occurs at a high scale. This leaves open the possibility that requiring positivity of the scalar masses is not a strong constraint if the scale of supersymmetry breaking is not much larger than the mass scale of the heavy scalars. In this paper we investigate this possibility by computing the finite parts of the same two-loop diagrams responsible for the negative contribution to the light scalar RG equation, and use these results as an *estimate* of the two-loop contribution in an actual model of low-energy supersymmetry breaking. We find that in certain classes of models, requiring positivity of the soft masses may place strong necessary conditions that such models must satisfy in order to be phenomenologically viable.

Our paper is organized as follows. In Section 2 an overview of the ingredients of our analysis is presented. Some philosophy and notation is discussed. Section 2.1 discusses the constraints on the masses and mixings of the first two generation scalars obtained from Δm_K after including QCD corrections. It is found, in particular, that Cabibbo-like mixing among both the first two generation left-handed squarks and right-handed squarks requires them to be heavier than 40 TeV. Section 2.2 discusses the logic of our RG analysis, and some formulas are presented. This analysis is independent of the Δm_K analysis. Sections 3 and 4 apply this machinery to the cases of low-energy and high-energy supersymmetry breaking, respectively. Section 3 deals with the case in which the scale at which SUSY breaking is communicated to the SM sparticles is close to the mass of the heavy scalars. We use the finite parts of the two-loop diagrams to estimate the negative contribution of the heavy scalars. We find that Cabibbo-like mixing among the left-handed and right-handed first two generation squarks implies that the boundary value of the stop masses has to greater than ~ 2 TeV to keep the stop mass squareds positive at the weak scale. This results in a fine tuning of naively 1% in electroweak symmetry breaking [19]. We also discuss the cases where there is $O(1)$ mixing among only the right or left squarks of the first two generations, and find that requiring positivity of the slepton mass squareds implies a constraint on the stop masses of ~ 1 TeV if gauge-mediated boundary conditions are used to relate the two masses. This is comparable to the direct constraint on the initial stop masses. In Section 4, we consider the case where the SUSY breaking masses for the SM sparticles are generated at a high scale ($\sim 10^{16}$ GeV). In this case, the negative contribution of the heavy scalars is enhanced by a large logarithm. We consider various boundary conditions for the stop and Higgs masses and find that with $O(0.22)$ degeneracy between the first two generation squarks, the boundary value of the stop mass needs to be larger than ~ 7 TeV. This gives a fine tuning of naively 0.02%[19]. For $O(1)$ mixing between the left (right) squarks only, the minimum initial value of the stop is $\sim 4(2)$ TeV. In Section 5 the scale of supersymmetry breaking is varied between 50 TeV and 2×10^{16} GeV. Uppers bounds on the amount of degneracy required between the first two generation scalars, that is consistent with positivity of the light scalar masses, naturalness in electroweak symmetry breaking, and the measured value of Δm_K, are obtained. These results are summarized in Figures 12 and 13. We conclude in Section 6. In the Appendix, we discuss the computation of the two-loop diagrams which give the negative contribution of the heavy scalars to the light scalar mass squareds.

2 Overview.

The chiral particle content of the Minimal Supersymmetric Standard Model (MSSM) contains 3 generations of $\bar{\mathbf{5}}+\mathbf{10}$ representations of $SU(5)$. The supersymmetry must be softly broken to not be excluded by experiment. Thus the theory must also be supplemented by some 'bare' soft supersymmetry breaking parameters, as well as a physical cutoff, M_{SUSY}. The 'bare' soft supersymmetry breaking parameters are then the coefficients appearing in the Lagrangian, defined with a cutoff M_{SUSY}. It will be assumed for simplicity that the bare soft masses, $\tilde{m}^2_{i,0}$, the bare gaugino masses $M_{A,0}$, and a bare trilinear term for the stops, $\lambda_t A_{t,0}$, are all generated close to this scale. The MSSM is then a good effective theory at energies below the scale M_{SUSY}, but above the mass of the heaviest superpartner.

The physical observables at low-energies will depend on these parameters. If an unnatural degree of cancellation is required between the bare parameters of the theory to produce a measured observable, the theory may be considered to be fine tuned. Of course, it is possible that a more fundamental theory may resolve in a natural manner the apparent fine tuning. The gauge-hierarchy problem is a well-known example of this. The Higgs boson mass of the SM is fine tuned if the SM is valid at energies above a few TeV. This fine tuning is removed if at energies close to the weak scale the SM is replaced by a more fundamental theory that is supersymmetric[3].

One quantification of the fine tuning of an observable $\mathcal{O}$ with respect to a bare parameter λ_0 is given by Barbieri-Giudice [19] to be

$$\Delta(\mathcal{O};\lambda_0) = (\delta\mathcal{O}/\mathcal{O})/(\delta\lambda_0/\lambda_0) = \frac{\lambda_0}{\mathcal{O}}\frac{\partial}{\partial\lambda_0}\mathcal{O}. \qquad (1)$$

It is argued that this only measures the sensitivity of $\mathcal{O}$ to λ_0, and care should be taken when interpreting whether a large value of Δ necessarily implies that $\mathcal{O}$ is fine tuned [20]. It is not the intent of this paper to quantify fine tuning; rather, an estimate of the fine tuning is sufficient and Equation 1 will be used. In this paper the value of $\mathcal{O}$ is considered extremely unnatural if $\Delta(\mathcal{O};\lambda_0) > 100$.

The theoretical prediction for Δm_K (within the MSSM) and its measured value are an example of such a fine tuning: Why should the masses of the first two generation scalars be degenerate to within 1 GeV, when their masses are $O(500\text{ GeV})$? Phrased differently, the first two generation scalars must be extremely degenerate for the MSSM to not be excluded by the measured value of Δm_K. An important direction in supersymmetry model building is aimed at attempting to explain the origin of this degeneracy.

One proposed solution to avoid this fine tuning is to decouple the first two generation scalars since their masses are the most stringently constrained by the flavour violating processes [10, 11, 12, 13, 15, 16, 17]. In this scenario, some of the first two generation scalars have masses $M_S \gg m_Z$. To introduce some notation, n_5 (n_{10}) will denote the number of $\bar{\mathbf{5}}$ ($\mathbf{10}$) scalars of the MSSM particle content that are very heavy [4]. Thus at energy scales $E \ll M_S$ the particle

[4] It is assumed that the heavy scalars form complete $SU(5)$ multiplets to avoid a large Fayet-Illiopoulus $D-$

content is that of the MSSM, minus the n_5 $\bar{\mathbf{5}}$ and n_{10} $\mathbf{10}$ scalars. In the literature this is often referred to as 'The More Minimal Supersymmetric Standard Model'[13].

There are, however, other possible and *equally valid* sources of fine tunings. The measured value of the Z mass is such an example [19]. The minimum of the renormalized Higgs potential determines the value of the Z mass which is already known from experiment. The vacuum expectation value (vev) of the Higgs field is, in turn, a function of the bare parameters of the theory. The relation used here, valid at the tree-level, is

$$\frac{1}{2}m_Z^2 = -\mu^2 + \frac{m_{H_d}^2(\mu_G) - m_{H_u}^2(\mu_G)\tan^2\beta}{\tan^2\beta - 1}. \tag{2}$$

It is clear from this Equation that requiring correct electroweak symmetry breaking relates the value of the soft Higgs masses at the weak scale, $m_{H_d}^2(\mu_G)$ and $m_{H_u}^2(\mu_G)$, to the supersymmetric Higgs mass μ. A numerical computation determines the dependence of $m_{H_u}^2(\mu_G)$ and $m_{H_d}^2(\mu_G)$ on the bare parameters $M_{A,0}$, $m_{\tilde{t}_i,0}^2$ and M_S. In the MSSM, the cancellation required between the bare parameters of the theory for it not to be excluded by the Z mass increases as the scale of supersymmetry breaking is increased. Typically, the bare mass of the gluino, stops, and the first two generation squarks must be less than a few TeV and ten TeV, respectively, for successful electroweak symmetry breaking not to be fine tuned at more than the one per cent level [19, 20, 17].

These two potential fine tuning problems- the supersymmetric flavour problem and that of electroweak symmetry breaking- are not completely independent, for they both relate to the size of supersymmetry breaking [17, 18]. Thus the consistency of any theoretical framework that attempts to resolve one fine tuning issue can be tested by requiring that it not reintroduce any comparable fine tunings in other sectors of the theory. This is the situation for the case under consideration here. Raising the masses of the first two generation scalars can resolve the supersymmetric flavour problem. As discussed in [17], this results in a fine tuning of m_Z^2 through the two-loop dependence of $m_{H_u}^2(\mu_G)$ on M_S. There is, however, another source of fine tuning of m_Z due to the heavy scalars: these large masses require that the bare masses of the stops, in particular, be typically larger than a few TeV to keep the soft masses squared positive at the weak scale [18]. This large value for the bare stop mass prefers a large value for the vev of the Higgs field, thus introducing a fine tuning in the electroweak sector. Further, this fine tuning is typically not less than the original fine tuning in the flavour sector. This is the central issue of this paper.

2.1 Δm_K Constraints

At the one-loop level the exchange of gluinos and squarks generates a $\Delta S = 2$ operator. In the limit $M_3 << M_S$ that we are interested in, the $\Delta S = 2$ effective Lagrangian at the scale M_S

term at the one-loop level[17, 13].

obtained by integrating out the squarks is

$$\mathcal{L}_{eff} = \frac{\alpha_S^2(M_S)}{216 M_S^2}\left(C_1\mathcal{O}_1 + \tilde{C}_1\tilde{\mathcal{O}}_1 + C_4\mathcal{O}_4 + C_5\mathcal{O}_5 + \text{h.c.}\right). \tag{3}$$

Terms that are $O(M_3^2/M_S^2)$ are subdominant and neglected. We expand the exact result in powers of $\delta_{LL,RR} = s_{L,R}c_{L,R}\eta_{L,R}(\tilde{m}_1^2 - \tilde{m}_2^2)_{LL,RR}/\tilde{m}^2_{AV,L,R}$, where $\tilde{m}^2_{AV}$ is the average mass of the scalars, and where $\eta_{L,R}$ is the phase and $s_{L,R}$ is the $1-2$ element of the $W_{L,R}$ matrix that appears at the gluino-squark-quark vertex[5]. This approximation underestimates the magnitude of the exact result, so our analysis is conservative[18]. The coefficients C_i to leading order in δ_{LL}, δ_{RR}, are

$$\begin{aligned} C_1 &= -22\delta^d_{LL} \\ C_4 &= 24\delta^d_{LL}\delta^d_{RR} \\ C_5 &= -40\delta^d_{LL}\delta^d_{RR}. \end{aligned} \tag{4}$$

The coefficient $\tilde{C}_1$ is obtained from C_1 with the replacement $\delta^d_{LL} \to \delta^d_{RR}$. The operators $\mathcal{O}_i$ are

$$\begin{aligned} \mathcal{O}_1 &= \bar{d}^a_L\gamma_\mu s_{L,a}\bar{d}^b_L\gamma^\mu s_{L,b} \\ \mathcal{O}_4 &= \bar{d}^a_R s_{L,a}\bar{d}^b_L s_{R,b} \\ \mathcal{O}_5 &= \bar{d}^a_R s_{L,b}\bar{d}^b_L s_{R,a} \end{aligned} \tag{5}$$

and $\tilde{\mathcal{O}}_1$ is obtained from $\mathcal{O}_1$ with the replacement $L \to R$. The Wilson coefficients, $C_1 - C_5$, are RG scaled from the scale of the squarks, M_S , to 900 MeV using the anomalous dimensions of the operators, $\mathcal{O}_1 - \mathcal{O}_5$. The anomalous dimension of $\mathcal{O}_1$ is well known [22] and is $\mu dC_1/d\mu = \alpha_s C_1/\pi$. We have computed the other anomalous dimensions and our result agrees with that of [21] (see this reference for a more general analysis of QCD corrections to the SUSY contributions to $K - \bar{K}$ mixing). These authors , however, choose to RG scale to μ_{had}, defined by $\alpha_s(\mu_{had})$=1. The validity of the pertubation expansion is questionable at this scale; we choose instead to RG scale to 900 MeV, where $\alpha_s(900\text{ MeV}) \sim .4$. The result is

$$\begin{aligned} C_1(\mu_{had}) &= \kappa_1 C_1(M_S) \\ \tilde{C}_1(\mu_{had}) &= \kappa_1\tilde{C}_1(M_S) \\ C_4(\mu_{had}) &= \kappa_4 C_4(M_S) + \frac{1}{3}(\kappa_4 - \kappa_5)C_5(M_S) \\ C_5(\mu_{had}) &= \kappa_5 C_5(M_S) \end{aligned} \tag{6}$$

where

$$\begin{aligned} \kappa_1 &= \left(\frac{\alpha_s(m_c)}{\alpha_s(900\text{ MeV})}\right)^{6/27}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{6/25}\left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{6/23}\left(\frac{\alpha_s(\mu_G)}{\alpha_s(m_t)}\right)^{6/21}\left(\frac{\alpha_s(M_S)}{\alpha_s(\mu_G)}\right)^{6/(9+(n_5+3n_{10})/2)} \\ \kappa_4 &= \kappa_1^{-4} \\ \kappa_5 &= \kappa_1^{1/2} \end{aligned} \tag{7}$$

[5]In this paper only 1-2 generation mixing is considered. Direct $L - R$ mass mixing is also neglected.

The effective Lagrangian at the hadronic scale is then

$$\mathcal{L}_{eff} = \frac{\alpha_s^2(M_S)}{216M_S^2}\left(-22(\delta^d_{LL})^2\kappa_1\mathcal{O}_1 - 22(\delta^d_{RR})^2\kappa_1\tilde{\mathcal{O}}_1 + \delta^d_{LL}\delta^d_{RR}(\frac{8}{3}(4\kappa_4+5\kappa_5)\mathcal{O}_4 - 40\kappa_5\mathcal{O}_5) + \text{h.c.}\right). \tag{8}$$

The SUSY contribution to the $K-\bar{K}$ mass difference is

$$(\Delta m_K)_{SUSY} = 2\text{Re} < K|\mathcal{L}_{eff}|\bar{K} > . \tag{9}$$

The relevant matrix elements (with bag factors set to 1) are

$$\begin{aligned} < K|\mathcal{O}_1|\bar{K} > &= \frac{1}{3}m_K f_K^2 \\ < K|\mathcal{O}_4|\bar{K} > &= \left(\frac{1}{24}+\frac{1}{4}\left(\frac{m_K}{m_s+m_d}\right)^2\right)m_K f_K^2 \\ < K|\mathcal{O}_5|\bar{K} > &= \left(\frac{1}{8}+\frac{1}{12}\left(\frac{m_K}{m_s+m_d}\right)^2\right)m_K f_K^2 \end{aligned} \tag{10}$$

in the vacuum insertion approximation. We use [23] $m_K = 497$ MeV, $f_K = 160$ MeV, $m_s = 150$ MeV , $(\Delta m_K)_{exp} = 3.5\times10^{-12}$ MeV, and $\alpha_s(M_Z) = 0.118$. This gives $\alpha_s(m_b) = .21$, $\alpha_s(m_c) = .29$ and $\alpha_s(900 \text{ MeV}) = .38$ using the one-loop RG evolution. Once values for $(n_5, n_{10}, \delta^d_{LL}, \delta^d_{RR})$ are specified, a minimum value for M_S is gotten by requiring that $(\Delta m_K)_{SUSY} = (\Delta m_K)_{exp}$. In the case that both $\delta_{RR} \neq 0$ and $\delta_{LL} \neq 0$, we assume that both the left-handed and right-handed squarks are heavy, so that $(n_5, n_{10}) = (2,2)$. In this case we require that only the dominant contribution to Δm_K, which is $\sim \delta^d_{LL}\delta^d_{RR}$, equals the measured value of Δm_K. If $\delta_{RR} \neq 0$ and $\delta_{LL} = 0$, we assume that only the right-handed squarks are heavy, and thus $(n_5, n_{10}) = (2,0)$. Similarly, if $\delta_{LL} \neq 0$ and $\delta_{RR} = 0$ then $(n_5, n_{10}) = (0,2)$. Limits are given in Tables 1 and 2 for some choices of these parameters. These results agree with Reference [21] for the same choice of input parameters. For comparison, the limits gotten by neglecting the QCD corrections are also presented in Tables 1 and 2. We consider δ^d_{LL} (δ^d_{RR}) = (i) 1, (ii) .22, (iii) .1, and (iv) 0.04. These correspond to :(i) no mixing and no degeneracy; (ii) Cabibbo-like mixing; (iii) Cabibbo-like mixing and $\sim .5$ degeneracy; and (iv) Cabibbo-like mixing and Cabibbo-like degeneracy. We expect only cases (i), (ii) and (iii) to be relevant if the supersymmetric flavour problem is resolved by decoupling the first two generation scalars. From Table 2 we note that for $(n_5, n_{10}) = (2,0)$, M_S must be larger than ~ 30 TeV if it is assumed there is no small mixing or degeneracy $(\delta^d_{RR} = 1)$ between the first two generation scalars.

The limits gotten from the mesaured rate of CP violation are now briefly discussed. Recall that the CP violating parameter ϵ is approximately

$$|\epsilon| \sim \frac{|\text{Im} < K|\mathcal{L}_{eff}|\bar{K} > |}{\sqrt{2}\Delta m_K}, \tag{11}$$

and its measured value is $|\epsilon| \sim |\eta_{00}| = 2.3\times10^{-3}$ [23]. In this case, the small value of ϵ implies either that the phases appearing in the soft scalar mass matrix are extremely tiny, or that the

$\sqrt{\mathrm{Re}(\delta^d_{LL}\delta^d_{RR})}$	$(n_5, n_{10}) = (2,2)$	$(n_5, n_{10}) = (2,2)$
	QCD incl.	no QCD
1	182 TeV	66 TeV
.22	40 TeV	15 TeV
.1	18 TeV	7.3 TeV
0.04	7.3 TeV	3.1 TeV

Table 1: Minimum values for heavy scalar masses M_S obtained from the measured value of Δm_K assuming $M_3^2/M_S^2 \ll 1$. The limits labeled 'QCD incl.' include QCD corrections as discussed in the text. Those labeled as 'no QCD' do not.

$\mathrm{Re}(\delta^d_{RR})$ $(\delta^d_{LL} = 0)$	$(n_5, n_{10}) = (2,0)$	$(n_5, n_{10}) = (2,0)$
	QCD incl.	no QCD
1	30 TeV	38 TeV
.22	7.2 TeV	8.9 TeV
0.1	3.4 TeV	4.1 TeV
0.04	1.4 TeV	1.7 TeV

Table 2: Minimum values for heavy scalar masses M_S obtained from the measured value of Δm_K assuming $M_3^2/M_S^2 \ll 1$. The limits labeled as 'QCD incl.' include QCD corrections as discussed in the text. Those labeled as 'no QCD' do not. The limits for $(n_5, n_{10}) = (0,2)$ obtained by $\delta^d_{LL} \leftrightarrow \delta^d_{RR}$ are similar and not shown.

masses of the heavy scalars are larger than the limits given in Tables 1 and 2. In the case where the phases are $O(1)$, Im $< K|\mathcal{L}_{eff}|\bar{K} > \sim$ Re $< K|\mathcal{L}_{eff}|\bar{K} >$ and thus the stronger constraint on M_S is obtained from ϵ and not Δm_K, for the same choice of input parameters. In particular, the constraint from CP violation increases the minimum allowed value of M_S by a factor of $1/\sqrt{2\sqrt{2}\epsilon} \sim 12.5$. This significantly increases the minimum value of the initial light scalar masses that is allowed by the positivity requirement.

2.2 RGE analysis

The values of the soft masses at the weak scale are determined by the RG evolution. In the $\overline{DR}'$ scheme [25, 26, 27], the RG equations [6] for the light scalar masses are, including the gaugino, A-term and λ_t contributions at the one-loop level and the heavy scalar contribution at the two-loop level [28],

$$\begin{aligned}\frac{d}{dt}m_i^2(t=\ln\mu) &= -\frac{2}{\pi}\sum_A \alpha_A(t)C_A^i M_A^2(t) + \frac{4}{16\pi^2}\sum_A C_A^i\alpha_A^2(t)(n_5 m_5^2 + 3n_{10}m_{10}^2) \\ &+\frac{8}{16\pi^2}\frac{3}{5}Y_i\alpha_1(t)\left(\frac{4}{3}\alpha_3(t) - \frac{3}{4}\alpha_2(t) - \frac{1}{12}\alpha_1(t)\right)(n_5 m_5^2 - n_{10}m_{10}^2) \\ &+\frac{\eta_i\lambda_t^2(t)}{8\pi^2}(m_{H_u}^2(t) + m_{\tilde{u}_3^c}^2(t) + m_{\tilde{Q}_3}^2(t) + A(t)^2) \\ &+\frac{6}{5}\frac{1}{4\pi}Y_i\alpha_1(t)\mathrm{Tr}Ym^2(t), \qquad (12)\end{aligned}$$

with $\eta = (3,2,1)$ for $\tilde{f}_i = H_u, \tilde{t}^c, \tilde{t}$, respectively, and zero otherwise. For simplicity it is assumed that $M_{A,0}/\alpha_{A,0}$ are all equal at M_{SUSY}. The initial value of the gluino mass, $M_{3,0}$, is then chosen to be the independent parameter. To avoid a large Fayet-Illiopoulus D-term at the one-loop level, we assume that the heavy scalars form complete $SU(5)$ representations[17, 13]. We use $SU(5)$ normalisation for the $U(1)$ coupling constant and $Q = T_3+Y$. Finally, C_A^i is the quadratic Caismir for the gauge group G_A that is 4/3 and 3/4 for the fundamental representations of $SU(3)$ and $SU(2)$, and $3/5Y_i^2$ for the $U(1)$ group. The cases $(n_5, n_{10})=$ (I) $(2,2)$, (II) $(2,0)$, (III) $(0,2)$ are considered. The results for the case $(3,0)$ is obtained, to a good approximation, from Case (II) by a simple scaling, and it is not discussed any further.

Inspection of Equation (12) reveals that in RG scaling from a high scale to a smaller scale the two-loop gauge contribution to the soft masses is negative, and that of the gauginos is positive. The presence of the large λ_t Yukawa coupling in the RGE drives the value of the stop soft mass squared even more negative. This effect increases the bound on the initial value for the stop soft masses and is included in our analysis. In our analysis the top quark mass in $\overline{MS}$ scheme is fixed at 167 GeV.

[6] An earlier version of the analysis presented in this manuscript did not include the one-loop hypercharge $D-$term in the RG equations. This has been corrected. This has not changed our conclusions though, since the numerical effect of this error is small. The authors thank B. Nelson for drawing our attention to this omission.

In the MSSM there is an extra parameter, $\tan\beta$, which is the ratio of the vacuum expecations values of the Higgs fields that couple to the up-type and down-type quarks respectively. Electroweak symmetry breaking then determines the top quark mass to be $m_t = \lambda_t/\sqrt{2}v\sin\beta$ with $v \sim 247$ GeV. In our analysis we consider the regime of small to moderate $\tan\beta$, so that all Yukawa couplings other than λ_t are neglected in the RG evolution. In this approximation the numerical results for $\tilde{f}_i \neq \tilde{t}$ or $\tilde{t}^c$ are independent of $\tan\beta$. In the numerical analysis of Sections 3 and 4 $\tan\beta$=2.2 is considered. In Section 5 $\tan\beta = 10$ is also considered.

In the case of low-energy supersymmetry breaking, the scale M_{SUSY} is not much larger than the mass scale of the heavy scalars. Then the logarithm $\sim\ln(M_{SUSY}/M_S)$ that appears in the solution to the previous RG equations is only $O(1)$. In this case the finite parts of the two-loop diagrams may not be negligible and should be included in our analysis. We use these finite parts to *estimate* the size of the two-loop heavy scalar contribution in an actual model.

The full-two loop expression for the soft scalar mass at a renormalisation scale μ_R is $m^2_{full}(\mu_R) = m^2_{\overline{DR}'}(\mu_R) + m^2_{finite}(\mu_R)$, where $m^2_{\overline{DR}'}(\mu_R)$ is the solution to the RG equation in $\overline{DR}'$ scheme, and $m^2_{finite}(\mu_R)$ is the finite part of the one-loop and two-loop diagrams, also computed in $\overline{DR}'$ scheme. The finite parts of the two-loop diagrams that contain internal heavy scalars are computed in the Appendix and the details are given therein. The answer for these two-loop finite parts is (assuming all heavy scalars are degenerate with common mass M_S^2)

$$\begin{aligned} m^2_{i,finite}(\mu_R) &= -\frac{1}{8}(\ln(4\pi) - \gamma + \frac{\pi^2}{3} - 2 - \ln\left(\frac{M_S^2}{\mu_R^2})\right) \times \sum_A \left(\frac{\alpha_A(\mu_R)}{\pi}\right)^2 (n_5 + 3n_{10}) C_A^i M_S^2 \\ &\quad -\frac{3}{5}\frac{1}{16\pi^2}\alpha_1(\mu_R)(n_5 - n_{10})Y_i \left(6 - \frac{2}{3}\pi^2 + 2(\ln(4\pi) - \gamma) - 4\ln(\frac{M_S^2}{\mu_R^2})\right) \\ &\quad \times \left(\frac{4}{3}\alpha_3(\mu_R) - \frac{3}{4}\alpha_2(\mu_R) - \frac{1}{12}\alpha_1(\mu_R)\right) M_S^2 \end{aligned} \tag{13}$$

where the gaugino and fermion masses are neglected. Since we use the $\overline{DR}'$ scheme to compute the finite parts of the soft scalar masses, the limits we obtain on the initial masses are only valid, strictly speaking, in this scheme. This is especially relevant for the case of low scale SUSY breaking. So while these finite parts should be viewed as semi-quantitative, they should suffice for a discussion of the fine tuning that results from the limit on the bare stop mass. For the case of high scale SUSY breaking, the RG logarithm is large and so the finite parts are not that important.

Our numerical analysis for either low-energy or high-energy supersymmetry breaking is described as follows.

The RG equations are evolved from the scale M_{SUSY} to the scale at which the heavy scalars are decoupled. This scale is denoted by μ_S and should be $O(M_S)$. The RG scaling of the heavy scalars is neglected. At this scale the finite parts of the two-loop diagrams are added to $m^2_{\tilde{f}_i}(\mu_S)$. We note that since the two-loop information included in our RG analysis is the leading $O(M_S^2)$ effect, it is sufficient to only use tree-level matching at the scale μ_S. Since the heavy scalars are not included in the effective theory below M_S and do not contribute to the gauge coupling beta functions, the numerical results contain an implicit dependence on the number of heavy scalars. This results in a smaller value for $\alpha_3(\mu_S)$ compared to its value if instead all the scalars have a $\sim$ 1TeV mass. This tends to weaken the constraint, and so it is included in our analysis [7]. The soft masses are then evolved using the one-loop RGE to the mass scale at which the gluinos are decoupled. This scale is fixed to be μ_G=1 TeV.

A constraint on the initial value of the soft masses is obtained by requiring that at the weak scale the physical scalar masses are positive. The experimental limit is $\sim$ 70 GeV for charged or coloured scalars[24]. The physical mass of a scalar is equal to the sum of the soft scalar mass, the electro-weak $D-$term, the supersymmetric contribution, and some finite one-loop and two-loop contributions. As mentioned in the previous paragraph, in the effective theory below M_S the finite two-loop part from the heavy scalars is included in value of the soft scalar mass of the light sparticles at the boundary, defined at $\mu_R = \mu_S \sim M_S$. The finite one-loop contributions are proportional to the gaugino and other light scalar masses, and are smaller than the corresponding logarithm that is summed in $\tilde{m}^2_i(\mu_R)$. So we neglect these finite one-loop parts. Further, the electroweak $D-$terms are less than 70 GeV. For the scalars other than the stops, the supersymmetric contribution is negligible. In what follows then, we will require that $\tilde{m}^2_i(\mu_G) > 0$ for scalars other than the stops. The discussion with the stops is complicated by both the large supersymmetric contribution, m^2_t, to the physical mass and by the $L - R$ mixing between the gauge eigenstates. This mixing results in a state with mass squared less than $\min(m^2_{\tilde{t}} + m^2_t, m^2_{\tilde{t}^c} + m^2_t)$, so it is a conservative assumption to require that for both gauge eigenstates the value of $m^2_{\tilde{t}_i} + m^2_t$ is larger than the experimental limit. This implies that $m^2_{\tilde{t}_i} \gtrsim$(70 GeV)$^2-$(175 GeV)^2 $= -$(160 GeV)^2. In what follows we require instead that $m^2_{\tilde{t}_i} \geq 0$. This results in an error that is $(160\text{GeV})^2/2m_{\tilde{t}_{i,0}} \approx 26$ GeV if the constraint obtained by neglecting m_t is $\sim$ 1 TeV. For the parameter range of interest it will be shown that the limit on the initial squark masses is $\sim$ 1 TeV, so this approximation is consistent.

We then combine the above two analyses as follows. The Δm_K constraints of Section 2.1 determine a minimum value for M_S once some theoretical preference for the δ's is given. Either a natural value for the δ's is predicted by some model, or the δ's are arbitrary and chosen solely by naturalness considerations. Namely, in the latter case the fine tuning to suppress Δm_K is roughly $2/\delta$. Further, a model may also predict the ratio M_3/M_S. Otherwise, Equations 1 and 2 may be used as a rough guide to determine an upper value for M_3, based upon naturalness considerations of the Z mass. Without such a limitation, the positivity requirements are completely irrelevant if the bare gluino mass is suffuciently large; but then the Z mass is fine tuned. Using these values of M_3 and M_S, the RGE analysis gives a minimum value for the initial stop masses which is consistent with Δm_K and positivity of the soft masses. This translates into some fine tuning

[7] This is the origin of a small numerical discrepancy of $\sim$ 10% between our results and the analysis of [18] in the approximation $\lambda_t = 0$.

of the Z mass, which is then roughly quantified by Equations 1 and 2.

Finally, we remark that our analysis may also be extended to include models that contain a Fayet-Illiopoulos hypercharge $D-$term, ζ_D, at the tree-level. The effect of the $D-$term is to shift the soft scalar masses, $m^2_{i,0} \to \tilde{m}^2_{i,0} = m^2_{i,0} + Y_i\zeta_D$. In this case, the positivity analysis applies to $\tilde{m}^2_{i,0}$, rather than $m^2_{i,0}$.

3 Low Energy Supersymmetry Breaking

In this Section we investigate the positivity requirement within a framework that satisifes both of the following: (i) supersymmetry breaking is communicated to the visible sector at low energies; and (ii) multi-TeV scale soft masses, M_S, are generated for some of the first two generation scalars. This differs from the usual low-energy supersymmetry breaking scenario in that we assume $M_S^2 \gg m^2_{\tilde{t}_i,0}$. In the absence of a specific model, however, it is difficult to obtain from the positivity criterion robust constraints on the scalar spectra for the following reasons. At the scale M_{SUSY} it is expected that, in addition to the heavy scalars of the MSSM, there are particles that may have SM quantum numbers and supersymmetry breaking mass parameters. All these extra states contribute to the soft scalar masses of the light particles. The sign of this contribution depends on, among other things, whether the soft mass squared for these additional particles is positive or negative-clearly very model-dependent. The total two-loop contribution to the light scalar masses is thus a sum of a model-dependent part and a model independent part. By considering only the model-independent contribution we have only isolated one particular contribution to the total value of the soft scalar masses near the supersymmetry breaking scale. We will, however, use these results to *estimate* the typical size of the finite parts in an actual model. That is, if in an actual model the sign of the finite parts is negative and its size is of the same magnitude as in Equation (13), the constraint in that model is identical to the constraint that we obtain. The constraint for other values for the finite parts is then obtained from our results by a simple scaling.

Before discussing the numerical results, the size of the finite contributions are estimated in order to illustrate the problem. Substituting $M_S \sim 25$ TeV, $\alpha_3(25 \text{ TeV}) \sim 0.07$ and $\alpha_1(25 \text{ TeV}) \sim 0.018$ into Equation 13 gives

$$\delta m^2_{\tilde{q}} \approx -(410 \text{ GeV})^2(n_5 + 3n_{10}) \left(\frac{M_S}{25 \text{ TeV}}\right)^2 \tag{14}$$

for squarks, and

$$\delta m^2_{\tilde{e}^c} \approx -\left((n_5 + 3n_{10})(70 \text{ GeV})^2 + (n_5 - n_{10})(100 \text{ GeV})^2\right) \left(\frac{M_S}{25 \text{ TeV}}\right)^2 \tag{15}$$

for the right-handed selectron. The negative contribution is large if $M_S \sim 25$ TeV. For example, if $n_5 = n_{10} = 2$ then $\delta m^2_{\tilde{e}^c} \approx -(200 \text{ GeV})^2$ and $\delta m^2_{\tilde{q}} \approx -(1.2 \text{ TeV})^2$. If $n_5 = 2$, $n_{10} = 0$, then $\delta m^2_{\tilde{e}^c} \approx -(170 \text{ GeV})^2$ and $\delta m^2_{\tilde{q}} \approx -(580 \text{ GeV})^2$.

In this low-energy supersymmetry breaking scenario, it is expected that $M_{SUSY} \sim M_S$. In our numerical analysis we will set $M_{SUSY} = \mu_S$ since the actual messenger scale is not known. The scale μ_S is chosen to be 50 TeV. At the scale μ_S =50 TeV the μ_S-independent parts of Equation (13) are added to the initial value of the soft scalar masses. The soft masses are then evolved using the RG equations (not including the two-loop contribution) to the scale μ_G= 1TeV.

First we discuss the constraints the positivity requirement imply for $\tilde{f}_i \neq \tilde{t}_L$ or $\tilde{t}_R$. In this case $m^2_{\tilde{f}_i}$ is renormalised by $M^2_{3,0}$, M^2_S, $m^2_{\tilde{f}_i,0}$ and the initial value of $\mathrm{Tr}Ym^2 \equiv D_{Y,0}$. We find

$$\begin{aligned} m^2_{\tilde{f}_i}(\mu_G) &= m^2_{\tilde{f}_i,0} + (0.243C^i_3 + 0.0168C^i_2 + 0.00156Y_i^2)M^2_{3,0} + c_D \times 10^{-3} Y_i D_{Y,0} \\ &\quad -(0.468C^i_3 + .095C^i_2 + .0173Y_i^2)\frac{1}{2}(n_5 + 3n_{10}) \times 10^{-3} M^2_S \\ &\quad -0.0174(n_5 - n_{10})Y_i \times 10^{-3} M^2_S \\ &\quad -(n_5 - n_{10})\left((-0.00058 + 0.0016(n_5 + 3n_{10}))M^2_S - .925M^2_{3,0}\right) Y_i \times 10^{-3}, \quad (16) \end{aligned}$$

where the strongest dependence on (n_5, n_{10}) has been isolated. The coefficient appearing in front of $D_{Y,0}$ is $c_D = -6$. The numerical coefficients in Equation (16) also depend on (n_5, n_{10}) and the numbers presented in Equation(16) are for (n_5, n_{10}) = (2,0). This sensitivity is, however, only a few percent between the four cases under consideration here [8]. Requiring positivity of the soft scalar masses directly constrains $m^2_{\tilde{f}_i,0}/M^2_S$ and $M^2_{3,0}/M^2_S$.

The value of $D_{Y,0}$ depends on the spectrum at the supersymmetry breaking scale, and is therefore model-dependent. To obtain model-independent constraints from the positivity requirement, we therefore only constrain the combination $\tilde{m}^2_{\tilde{f}_i,0} \equiv m^2_{\tilde{f}_i,0} + c_D Y_i D_{Y,0}$. Only this combination appears in the weak-scale value for the scalar mass of $\tilde{f}_i$. The numerical effect is small, since with $D_{Y,0} \sim O(m^2_{\tilde{f}_i,0})$, the coefficient of $m^2_{\tilde{f}_i,0}$ is shifted from 1 to $\sim 1 - 6. \times 10^{-3} Y_i$.

The positivity requirement $\tilde{m}^2_{\tilde{f}_i}$ for $\tilde{f}_i \neq \tilde{t}$ or $\tilde{t}^c$ is given in Figure 1 for different values of n_5 and n_{10}. That is, in Figure 1 the minimum value of $\tilde{m}_{\tilde{f}_i,0}/M_S$ required to keep the soft masses positive at the scale μ_G is plotted versus $M_{3,0}/M_S$. We conclude from these figures that the positivity criterion is weakest for n_5=2 and n_{10}=0. This is expected since in this case the heavy particle content is the smallest. We note that even in this 'most minimal' scenario the negative contribution to the masses are rather large. In particular, we infer from Figure 1 that for $(n_5 = 2, n_{10} = 0)$ and $M_S \sim$ 25 TeV, $\delta m^2_{\tilde{e}^c} \approx -(190 \text{ GeV})^2$ for $M_{3,0}$ as large as 1 TeV. In this case it is the two-loop contribution from the hypercharge D-term that is responsible for the large negative mass squared. In the case (n_5, n_{10})=(2, 2), we obtain from Figure 1 that for $M_S \sim$ 25 TeV, $\delta m^2_{\tilde{e}^c} \approx -(210 \text{ GeV})^2$ and $\delta m^2_{\tilde{b}^c} \approx -(1.1 \text{ TeV})^2$ for $M_{3,0}$ as large as 1 TeV.

We now apply the positivity requirement to the stop sector. In this case it is not possible to directly constrain the boundary values of the stops for the following simple reason. There are only two positivity constraints, whereas the values of $m^2_{\tilde{t}}(\mu_G)$ and $m^2_{\tilde{t}^c}(\mu_G)$ are functions of the three soft scalar masses $m^2_{\tilde{t},0}$, $m^2_{\tilde{t}^c,0}$ and $m^2_{H_u,0}$. To obtain a limit some theoretical assumptions

[8] This dependence is included in Figure 1.

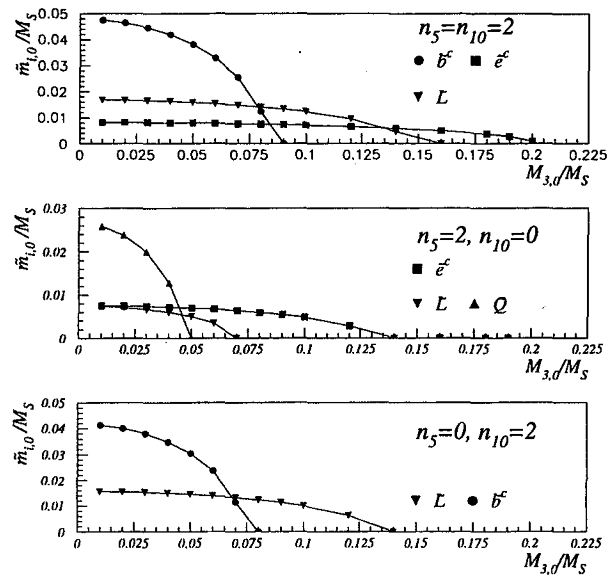

Figure 1: Limits for $m_{\tilde{f}_i,0}/M_S$ from the requirement that the mass squareds are positive at the weak scale, for low-energy supersymmetry breaking. The regions below the curves are excluded. For the case (2,0), the limits for the other squarks are very similar to that for $\tilde{Q}$ and are therefore not shown.

must be made to relate the three initial soft scalar masses.

The numerical solutions to the RG equations for $\tan\beta$=2.2 and $(n_5, n_{10}) = (2,0)$ are

$$\begin{aligned}
m_{\tilde{t}}^2(\mu_G) &= -0.0303A_t^2 + 0.00997A_tM_{3,0} + 0.322M_{3,0}^2 + c_D \times \frac{1}{6} \times 10^{-3}D_{Y,0} \\
&\quad -0.0399(m_{H_u,0}^2 + m_{\tilde{t}^c,0}^2) + 0.960m_{\tilde{t},0}^2 - 0.000645c_LM_S^2 \\
m_{\tilde{t}^c}^2(\mu_G) &= -0.0606A_t^2 + 0.0199A_tM_{3,0} + 0.296M_{3,0}^2 + c_D \times \frac{-2}{3} \times 10^{-3}D_{Y,0} \\
&\quad 0.920m_{\tilde{t}^c,0}^2 - 0.0797(m_{H_u,0}^2 + m_{\tilde{t},0}^2) - 0.000492c_RM_S^2 \\
m_{H_u}^2(\mu_G) &= -0.0909A_t^2 + 0.0299A_tM_{3,0} - 0.0289M_{3,0}^2 + c_D \times \frac{1}{2} \times 10^{-3}D_{Y,0} \\
&\quad +0.880m_{H_u,0}^2 - 0.119(m_{\tilde{t},0}^2 + m_{\tilde{t}^c,0}^2) + 0.0000719c_HM_S^2. \qquad (17)
\end{aligned}$$

The numerical coefficients other than that of M_S do not vary more than a few percent between the different values for (n_5, n_{10}), and thus this dependence is not shown. For M_S, we find that (c_L, c_R, c_H) is $(1,1,1)$, $(3.62, 3.84, 4.59)$, $(2.78, 3.04, 3.92)$, for (n_5, n_{10}) = (2,0), $(2,2)$ and $(0,2)$, respectively. Also, $c_D = -6$. We find from Equations 1 and 2 that to keep m_Z^2 fine tuned at less than 1% ($\Delta \leq 100$) in each of the bare parameters, we must have: $\mu \lesssim$ 460 GeV; $M_{3,0} \lesssim 2.3$ TeV; $m_{\tilde{t},0} \lesssim 1.7$ TeV; $m_5 \lesssim 80$ TeV and $m_{10} \lesssim 50$ TeV for $(n_5, n_{10}) = (2,2)$. Finally, for other values of these parameters the fine tuning increases as $\Delta = 100 \times \tilde{m}^2/\tilde{m}_0^2$, where $\tilde{m}_0$ is the value of $\tilde{m}$ that gives $\Delta = 100$.

It is possible to show, using the fact that $Y_{H_u} + Y_Q + Y_{u^c} = 0$, that the solutions in Equations 17 are unchanged if we replace $m^2_{i,0}$ with $\tilde{m}^2_{i,0} = m^2_{i,0} + c_D \times 10^{-3} Y_i D_{Y,0}$ *and* set $D_{Y,0} = 0$. In what follows then, we will use the posivitity analysis to constrain $\tilde{m}^2_{i,0}$ for the stops. We note though, that the difference between $\tilde{m}^2_{i,0}$ and $m^2_{i,0}$ is small, owing to the small coefficient appearing in front of $D_{Y,0}$. In the remainder of this Section the tilde on $\tilde{m}^2_{i,0}$ will be removed to simplify the notation.

To constrain the initial values of the stop masses we will only consider gauge-mediated supersymmetry breaking mass relations. From Equation 17 we see that to naturally break electroweak symmetry a small hierarchy $m^2_{\tilde{t}_i,0} > m^2_{H_u,0}$ is required. This is naturally provided by gauge-mediated boundary conditions [9]. The relations between the soft scalar masses when supersymmetry breaking is communicated to the visible sector by gauge messengers are [9]

$$m^2_{i,0} = \frac{3}{4} \sum_A C^i_A \frac{\alpha^2_A(M_{SUSY})}{\alpha^2_3(M_{SUSY}) + \alpha^2_1(M_{SUSY})/5} m^2_{\tilde{t}^c,0}. \tag{18}$$

Substituting these relations into Equations (17) and assuming $A_{t,0}$ =0 determines $m^2_{\tilde{t}}(\mu_G)$ and $m^2_{\tilde{t}^c}(\mu_G)$ as a function of $M_{3,0}$, M^2_S and $m^2_{\tilde{t}^c,0}$. In Figure 2 we have plotted the minimum value of $m_{\tilde{t}^c,0}/M_{3,0}$ required to maintain both $m^2_{\tilde{t}}(\mu_G) \geq 0$ and $m^2_{\tilde{t}^c}(\mu_G) \geq 0$.

Another interesting constraint on these class of models is found if it is assumed that the initial masses of all the light fields are related at the supersymmetry breaking scale by some gauge-mediated supersymmetry breaking (GMSB) mass relations, as in Equation (18). This ensures the degeneracy, as required by the flavour changing constraints, of any light scalars of the first two generations. This is required if, for example, one of n_5 or n_{10} are zero. Then in our previous limits of $m_{\tilde{f}_i,0}$ for $\tilde{f}_i \neq \tilde{t}$ or $\tilde{t}^c$, constraints on the initial value of $m_{\tilde{t}^c}$ are obtained by relating $m_{\tilde{f}_i,0}$ to $m_{\tilde{t}^c,0}$ using Equation (18). In this case the slepton masses provide the strongest constraint and they are also shown in Figure 2. This result may be understood from the following considerations. The two-loop hypercharge D-term contribution to the soft mass is $\sim Y_i(n_5 - n_{10})\alpha_1\alpha_3 M^2_S$ and this has two interesting consequences. The first is that for $n_5 \neq n_{10}$, the resulting $\delta\tilde{m}^2$ is always negative for one of $\tilde{e}^c$ or $\tilde{L}$. Thus in this case there is always a constraint on $m^2_{\tilde{t}^c}$ once gauge-mediated boundary conditions are assumed. That this negative contribution is large is seen as follows. The combined tree-level mass and two-loop contribution to the selectron mass is approximately $m^2_{\tilde{e}^c,0} - k\alpha_1\alpha_3 M^2_S$ where k is a numerical factor. Substituting the gauge-mediated relation $m^2_{\tilde{e}^c,0} \sim \alpha^2_1/\alpha^2_3 m^2_{\tilde{t}^c,0}$, the combined selectron mass is $\alpha^2_1/\alpha^2_3(m^2_{\tilde{t}^c,0} - k(\alpha_3/\alpha_1)\alpha^2_3 M^2_S)$. Since the combined mass of the stop is $\sim m^2_{\tilde{t}^c,0} - k'\alpha^2_3 M^2_S$, the limit for $m^2_{\tilde{t}^c,0}$ obtained from the positivity requirement for $m^2_{\tilde{e}^c}$ is comparable or larger than the constraint obtained from requiring that $m^2_{\tilde{t}^c}$ remains positive. For example, with $n_5 = 2$, $n_{10} = 0$ and $M_S \sim 25$ TeV, the right-handed slepton constraint requires that $m_{\tilde{t}^c,0} \sim 1.1$ TeV. For n_{10}=2, n_5=0 and $M_S \sim 25$ TeV, $\tilde{L}_3$ is

[9] In fact, low-energy gauge-mediated supersymmetry breaking provides "too much" electroweak symmetry breaking [29].

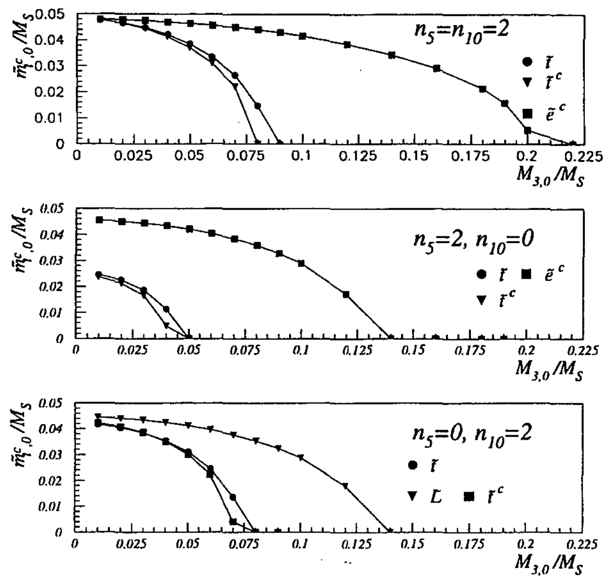

Figure 2: Limits for $m_{\tilde{t}^c,0}/M_S$ from the requirement that the stop and slepton mass squared are positive at the weak scale. The regions below the curves are excluded. Low-energy gauge-mediated supersymmetry breaking mass relations between the light sparticles and $\tan\beta$ =2.2 are assumed.

driven negative and implies that $m_{\tilde{t}^c,0} \sim 1$ TeV. From Figure 2 we find that these results are comparable to the direct constraint on $m_{\tilde{t}^c,0}$ obtained by requiring that colour is not broken.

The positivity analysis only constrains $m_{\tilde{t}_i,0}/M_S$ for a fixed value of $M_{3,0}/M_S$. To directly limit the initial scalar masses some additional information is needed. This is provided by the measured value of Δm_K. If some mixing and degeneracy between the first two generation scalars is assumed, parameterized by $(\delta_{LL}, \delta_{RR})$, a minimum value for M_S is obtained by requiring that the supersymmetric contribution to Δm_K does not exceed the measured value. We use the results given in Section 2 to calculate this minimum value. This result together with the positivity analysis then determines a minimum value for $m_{\tilde{t}^c,0}$ for a given initial gluino mass $M_{3,0}$. The RG analysis is repeated with $\mu_S = M_S$, rather than μ_S=50 TeV. We only present the results found by assuming GMSB mass relations between the scalars. These results are shown in Figure 3. The mass limits for other $\tilde{f}_i$ are easily obtained from the information provided in Figure 1 and Table 2 and are not shown. From Figure 3 we find that for $(n_5, n_{10}) = (2,2)$ and $M_{3,0}$ less than 2 TeV, $m_{\tilde{t}^c,0}$ must be larger than 8 TeV for $\sqrt{\delta_{LL}\delta_{RR}} = 1$, and larger than 1.8 TeV for $\sqrt{\delta_{LL}\delta_{RR}} = .22$. This results in $\Delta(m_Z^2, m_{\tilde{t},0}^2)$ of 2000 and 120, respectively. In this case both the squark and selectron limits for $m_{\tilde{t}^c,0}$ are comparable. The limits for other choices for $\sqrt{\delta_{LL}\delta_{RR}}$ are obtained from Figure 3 by a simple scaling, since to a good approximation $\Delta m_K \sim \delta_{LL}\delta_{RR}/M_S^2$. For the cases $(n_5, n_{10}) = (2,0)$ and $(0,2)$, the corresponding limits are much weaker. In the case $(n_5, n_{10}) = (2, 0)$, for example, only for $\delta_{RR} \sim 1$ does the selectron mass limit require that $m_{\tilde{t}^c,0} \sim$1 TeV. The limits for a smaller value of δ are not shown.

We conclude with some comments about how these results change if CP violation is present in these theories with $O(1)$ phases. Recall from Section 2 that for the same choice of input parameters, the limits on the initial stop masses increases by about a factor of 12. This may be interpreted in one of two ways. Firstly, this constrains those models that were relatively unconstrained by the Δm_K limit. We concentrate on those models with $n_5 = 2$ and $n_{10} = 0$, since this case is the most weakly constrained by the combined Δm_K and positivity analysis. The conclusions for other models will be qualitatively the same. We find from Figure 3 the limit $m_{\tilde{t}^c,0} >$1 TeV [10] is only true if $\delta_{RR} \sim O(1)$. Smaller values of δ_{RR} do not require large initial stop masses. From the CP violation constraint, however, smaller values for δ_{RR} are now constrained. For example, if $\delta_{RR} \sim$0.1 and $O(1)$ phases are present, then $m_{\tilde{t}^c,0} >$1 TeV is required. Secondly, the strong constraint from ϵ could partially or completely compensate a weakened constraint from the positivity analysis. This could occur, for example, if in an actual model the negative two-loop contribution to the stop mass squared for the same initial input parameters is smaller than the estimate used here. For example, if the estimate of the two-loop contribution in an actual model decreases by a factor of $\sim (12.5)^2$ and $O(1)$ phases are present, the limit in this case from ϵ for the same δ is identical to the values presented in Figure 3.

4 High Scale Supersymmetry Breaking

In this section, we consider the case in which SUSY breaking is communicated to the MSSM fields at a high energy scale, that is taken to be [11] $M_{GUT} = 2 \times 10^{16}$ GeV. In this case, the negative contribution of the heavy scalar soft masses to the soft mass squareds of the light scalars is enhanced by $\sim \ln(M_{GUT}/50 \text{ TeV})$, since the heavy scalar soft masses contribute to the RGE from M_{GUT} to mass of the heavy scalars. It is clear that as the scale of SUSY breaking is lowered the negative contribution of the heavy scalar soft masses reduces.

This scenario was investigated in Reference [18], and we briefly discuss the difference between that analysis and the results presented here. In the analysis of Reference [18], the authors made the conservative choice of neglecting λ_t in the RG evolution. The large value of λ_t can change the analysis, and it is included here. We find that for some pattern of initial stop and up-type Higgs scalar masses, e.g. universal scalar masses, this effect increases the constraint on the stop masses by almost a factor of two. This results in an increase of a factor of 3-4 in the amount of fine tuning required to obtain the correct Z mass. Further, in combining the positivity analysis

[10]For GMSB relations only. The direct constraint on the stop masses is slightly weaker.

[11]This choice for the high scale is done to remain agnostic about any physics appearing between the Grand Unification scale and the Planck scale. This also results in a conservative assumption, since the negative two-loop contribution is smaller with $M_{SUSY} = M_{GUT}$.

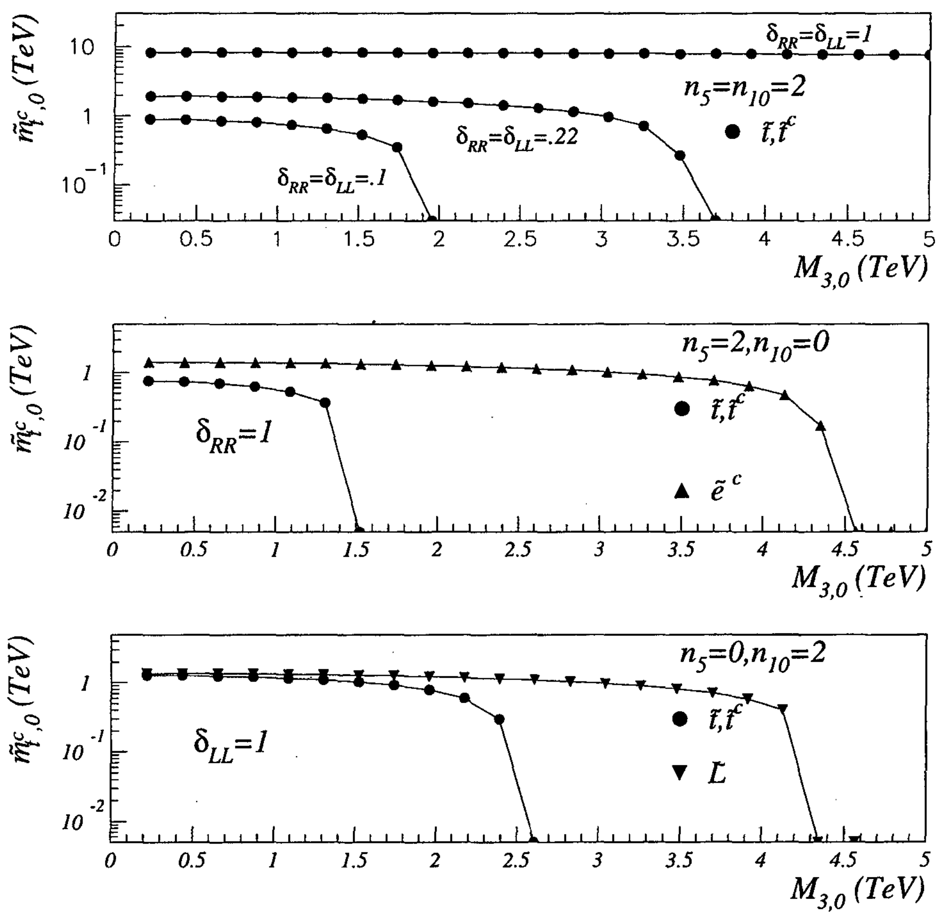

Figure 3: Limits for $m_{\tilde{t}^c,0}$ from the requirement that the stop and slepton mass squared are positive at the weak scale while suppressing Δm_K, for different values of (n_5, n_{10}), and $(\delta_{LL}, \delta_{RR})$. The regions below the curves are excluded. Low-energy gauge-mediated supersymmetry breaking mass relations between the light scalars and $\tan\beta$ =2.2 are assumed.

with the constraints from the Δm_K analysis, the QCD corrections to the Flavour Changing Neutral Current (FCNC) operators has been included, as discussed in Section 2. In the case $(n_5, n_{10}) = (2,2)$, this effect alone increases the positivity limit by a factor of $\sim 2-3$. The combination of these two elements imply that the positivity constraints can be quite severe.

We proceed as follows. First, we solve the RGEs from M_{GUT} to μ_S where the heavy scalars are decoupled. At this scale, we add the finite parts of the two-loop diagrams. Next, we RG scale (without the heavy scalar terms in the RGEs) from μ_S to μ_G using these new boundary conditions. Except where stated otherwise, the scales μ_S and μ_G are fixed to be 50 TeV and 1 TeV, respectively.

For $\tilde{f}_i \neq \tilde{t}, \tilde{t}^c$ we find,

$$\begin{aligned} m^2_{\tilde{f}_i}(\mu_G) &= m^2_{\tilde{f}_{i,0}} + (2.84C^i_3 + 0.639C^i_2 + 0.159Y_i^2)M^2_{3,0} + c_D Y_i D_{Y,0} \\ &\quad -(4.38C^i_3 + 1.92C^i_2 + 0.622Y_i^2)\frac{1}{2}(n_5 + 3n_{10}) \times 10^{-3} M^2_S \\ &\quad -0.829(n_5 - n_{10})Y_i \times 10^{-3} M^2_S \\ &\quad +(n_5 - n_{10})\left(17.2M^2_{3,0} + (.226 - 0.011(n_5 + 3n_{10}))M^2_S\right) Y_i \times 10^{-3}. \end{aligned} \qquad (19)$$

These results agree with Reference [18] for the same choice of input parameters. The term proportional to $D_{Y,0}$, and the terms in the last line result from integrating the one-loop hypercharge $D-$term. In this case $c_D = -0.051$. As in the previous Section, the numerical coefficients in Equation(19) depend on (n_5, n_{10}) through the gauge coupling evolution, and the numbers in Equation(19) are for $(n_5, n_{10}) = (2,0)$ [12]. Requiring the soft masses squared to be positive constrains $\tilde{m}^2_{i,0} = m^2_{i,0} + c_D Y_i D_{Y,0}$. In Figure 4 we plot the values of $\tilde{m}_{\tilde{f}_{i,0}}/M_S$ that determine $\tilde{m}^2_{\tilde{f}_i}(\mu_G) = 0$ as a function of M_3/M_S, for $\tilde{f}_i = \tilde{L}_i$, $\tilde{Q}_i$, $\tilde{u}^c_i$, $\tilde{d}^c_i$ and $\tilde{e}^c_i$. We emphasize that the results presented in Figure 4 are independent of any further limits that FCNC or fine tuning considerations may imply, and are thus useful constraints on any model building attempts.

For the stops, the numerical solutions to the RGEs for $\tan\beta = 2.2$ are

$$\begin{aligned} m^2_{\tilde{t}}(\mu_G) &= -0.021A^2_t + 0.068A_t M_{3,0} + 3.52M^2_{3,0} + c_D\frac{1}{6}D_{Y,0} \\ &\quad -0.142(m^2_{H_u,0} + m^2_{\tilde{t}^c,0}) + 0.858m^2_{\tilde{t},0} - c_L 0.00613M^2_S \\ m^2_{\tilde{t}^c}(\mu_G) &= -0.042A^2_t + 0.137A_t M_{3,0} + 2.33M^2_{3,0} + c_D\frac{-2}{3}D_{Y,0} \\ &\quad -0.283(m^2_{H_u,0} + m^2_{\tilde{t},0}) + 0.716m^2_{\tilde{t}^c,0} - c_R 0.00252M^2_S \\ m^2_{H_u}(\mu_G) &= -0.063A^2_t + 0.206A_t M_{3,0} - 1.72M^2_{3,0} + c_D\frac{1}{2}D_{Y,0} \\ &\quad -0.425(m^2_{\tilde{t},0} + m^2_{\tilde{t}^c,0}) + 0.574m^2_{H_u,0} + c_H 0.00193M^2_S \end{aligned} \qquad (20)$$

where $(c_L, c_R, c_H) = (1,1,1)$, $(3.57, 4.92, 5.15)$, $(2.7, 4.16, 4.27)$ for $(n_5, n_{10}) = (2,0)$, $(2,2)$ and $(0,2)$, respectively. Also, $c_D = -0.051$. The mixed two-loop contribution to the RG evolution is

[12] The numerical results presented in Figure 4 include this dependence.

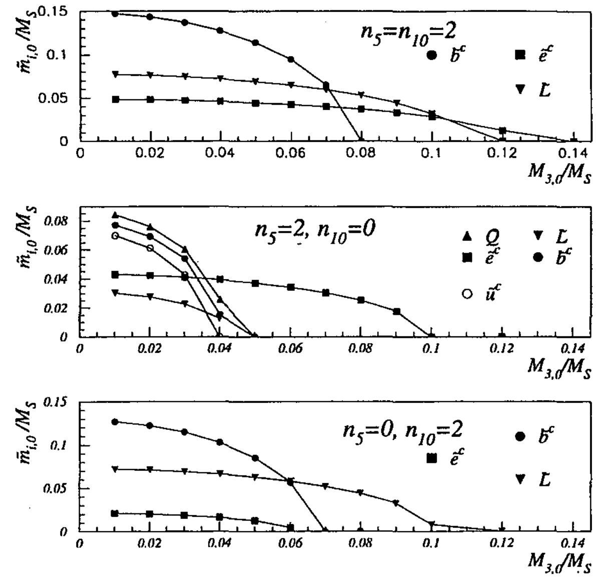

Figure 4: Limits for $m_{\tilde{f}_i,0}$ for different values of (n_5, n_{10}) from the requirement that the mass squareds are positive at the weak scale, assuming a supersymmetry breaking scale of M_{GUT}. The regions below the lines are excluded.

$\propto (n_5 - n_{10})$ and is not negligible. Thus there is no simple relation between the c's for different values of n_5 and n_{10}. From Equations 2 and 1 we find that to keep m_Z^2 fine tuned at less than 1% ($\Delta \leq 100$) in each of the bare parameters, we must have: $\mu \lesssim 460$ GeV; $M_{3,0} \lesssim 300$ GeV; $m_{\tilde{t}_i,0} \lesssim .87$ TeV; $m_{5,i} \lesssim 16$ TeV; and $m_{10,i} \lesssim 10$ TeV, for $(n_5, n_{10}) = (2,2)$. The fine tuning of the Z mass with respect to the heavy scalars is discussed in [17]. Finally, for other values of these parameters the fine tuning increases as $\Delta = 100 \times \tilde{m}^2/\tilde{m}_0^2$, where $\tilde{m}_0$ is the value of $\tilde{m}$ that gives $\Delta = 100$.

As in Section 3, we rewrite Equations 20 in terms of $\tilde{m}_{i,0}^2 = m_{i,0}^2 + c_D Y_i D_{Y,0}$. This is equivalent to setting $D_{Y,0} = 0$ in Equations 20, and relabeling $m_{i,0}^2 \rightarrow \tilde{m}_{i,0}^2$. In what follows, we use the positivity analysis to constrain $\tilde{m}_{i,0}^2$. Since c_D is small and $D_{Y,0} \sim O(m^2)$, the difference between $\tilde{m}_{i,0}^2$ and $m_{i,0}^2$ is small. To simplify the notation, in the remainder of this Section we will also remove the tilde from $\tilde{m}_{i,0}^2$.

As was also discussed in Section 3, some relations between $m_{\tilde{t},0}^2$, $m_{\tilde{t}^c,0}^2$ and $m_{H_u,0}^2$ are needed to obtain a constraint from Equation(20), using $m_{\tilde{t}}^2(\mu_G) > 0$ and $m_{\tilde{t}^c}^2(\mu_G) > 0$. We discuss both model-dependent and model-independent constraints on the initial values of the stop masses. The outline of the rest of this Section is as follows. First, we assume universal boundary conditions. These results are presented in Figure 5. Model-independent constraints are obtained by the following. We assume that $m_{H_u,0}^2 = 0$ and choose $A_{t,0}$ to maximize the value of the stop masses at the weak scale. These results are presented in Figure 6. We further argue that these constraints represent minimum constraints as long as $m^2_{H_u,0} \geq 0$. To obtain another set of model independent constraints, we use the electroweak symmetry breaking relation to eliminate $m^2_{H_u,0}$ in favour of μ. Then we present the positivity limits for different values of $\tilde{\mu}/M_S$, where $\tilde{\mu}^2 = \mu^2 + \frac{1}{2}m_Z^2$, and assume that $m^2_{H_d,0} = 0$ to minimize the value of μ [13]. These limits are model-independent and are presented in Figure 7, for the case $n_5 = n_{10} = 2$. We then combine these analyses with the limits on M_S obtained from Δm_K. We conclude with some discussion about the anomalous $D-$term solutions to the flavour problem.

We first consider universal boundary conditions for the stop and Higgs masses. That is, we assume that $m^2_{\tilde{t},0} = m^2_{\tilde{t}^c,0} = m^2_{H_u,0} = \tilde{m}_0^2$. In Figure 5 we plot for $\tan\beta = 2.2$ the minimum value of $\tilde{m}_0/M_S$ required to maintain $m^2_{\tilde{t}}(\mu_G) > 0$ and $m^2_{\tilde{t}^c}(\mu_G) > 0$. This value of $\tan\beta$ corresponds to $\lambda_t(M_{GUT}) = .88$, in the case that $(n_5, n_{10}) = (2, 0)$. For comparison, the results gotten assuming $\lambda_t = 0$ may be found in Reference [18]. For $n_5 = n_{10} = 2$ we note from Figure 5 that if $M_S = 20$ TeV and the gaugino masses are small, the limit on the stop mass is $m_{\tilde{t}^c,0} \geq 6.2$ TeV. This limit is weakened to 6 TeV if $M_{3,0} \lesssim 300$ GeV is allowed. Even in this case, this large initial stop mass requires a fine tuning that in this case is $\Delta \sim (6\ \text{TeV})^2/m_Z^2 \sim 4200$, i.e. a fine tuning of $\lesssim 10^{-3}$ is needed to obtain the correct Z mass.

We now assume $m^2_{H_u,0} = 0$ and choose the initial value of $A_{t,0}$ to *maximize* the value of $m^2_{\tilde{t}_i}(\mu_G)$. The values of $m^2_{\tilde{t},0}$ and $m^2_{\tilde{t}^c,0}$ are chosen such that $m^2_{\tilde{t}}(\mu_G) > 0$ and $m^2_{\tilde{t}^c}(\mu_G) > 0$. We note that in this case the constraint is weaker because the λ_t contribution to the RG evolution of the stop masses is less negative. These results are plotted in Figure 6.

We discuss this case in some more detail and argue that the minimum value of $m_{\tilde{t}_i,0}$ obtained in this way will be valid for all $m^2_{H_u} \geq 0$ and all $A_{t,0}$. Eliminate the $A_{t,0}$ term by choosing $A_{t,0} = kM_{3,0}$ such that the A_t contributions to $m^2_{\tilde{t}_i}(\mu_G)$ is maximized. Other choices for $A_{t,0}$ require larger values for $m^2_{\tilde{t}_i,0}$ to maintain $m^2_{\tilde{t}_i}(\mu_G) = 0$. The value of k is determined by the following. A general expression for the value of the soft masses of the stops at the weak scale is

$$m^2_{\tilde{t}}(\mu_G) = -aA^2_{t,0} + bA_{t,0}M_{3,0} + cM^2_{3,0} + \cdots, \tag{21}$$

$$m^2_{\tilde{t}^c}(\mu_G) = -2aA^2_{t,0} + 2bA_{t,0}M_{3,0} + dM^2_{3,0} + \cdots, \tag{22}$$

with a, c and d positive. The maximum value of $m^2_{\tilde{t}_i}(\mu_G)$ is obtained by choosing $A_{t,0} = bM_{3,0}/2a$. The value of the stops masses at this choice of $A_{t,0}$ are

$$m^2_{\tilde{t}}(\mu_G) = (c + \frac{b^2}{4a})M^2_{3,0} + \cdots, \tag{23}$$

$$m^2_{\tilde{t}^c}(\mu_G) = (d + 2\frac{b^2}{4a})M^2_{3,0} + \cdots. \tag{24}$$

An inspection of Equation 20 gives $b = 0.068$ and $a = 0.021$ for $\tan\beta = 2.2$. In this case the 'best' value for $A_{t,0}$ is $A^B_{t,0} \sim 1.6M_{3,0}$. It then follows that the quantity $b^2/4a = 0.055$ is a small

[13] Strictly speaking, this last assumption is unnecessary. Only the combination $\tilde{\mu}^2_H \equiv \tilde{\mu}^2 - m^2_{H_d,0}/\tan^2\beta$ appears in our analysis. Thus for $m^2_{H_d,0} \neq 0$ our results are unchanged if the replacement $\tilde{\mu} \to \tilde{\mu}_H$ is made.

correction to the coefficient of the gaugino contribution in Equation 20. Thus the difference between the minimum initial stop masses for $A_{t,0} = 0$ and $A_{t,0}= A^B_{t,0}$ is small. Next assume that $m^2_{H_u,0} = 0$. Requiring that both $m^2_{\tilde{t}}(\mu_G) = 0$ and $m^2_{\tilde{t}^c}(\mu_G) = 0$ determines a minimum value for $m^2_{\tilde{t},0}$ and $m^2_{\tilde{t}^c,0}$. Now since the $m^2_{H_u,0}$ contribution to both the stop soft masses is negative (see Equation 20), the minimum values for $m^2_{\tilde{t}_i,0}$ found by the preceeding procedure are also minimum values if we now allow any $m^2_{H_u,0} > 0$.

We conclude that for all $A_{t,0}$ and all $m^2_{H_u,0} \geq 0$, the limits presented in Figure 6 represent lower limits on the initial stop masses if we require that the soft masses remain positive at the weak scale. Further, the limits in this case are quite strong. For example, from Figure 6 we find that if $M_S \sim 20$ TeV and $M_{3,0} \sim 200$ GeV (so that $M_{3,0}/M_S \sim 10^{-2}$), then the initial stop masses must be greater than 3.5 TeV in the case that $(n_5, n_{10}) = (2,2)$ The results are stronger in a more realistic scenario, *i.e.* $m^2_{H_u,0} > 0$. If, for example, $m^2_{H_u,0} \sim m^2_{\tilde{t}^c,0}/9$ the constraints are larger by only a few percent. In the case that $m^2_{H_u,0} = m^2_{\tilde{t}^c,0} = m^2_{\tilde{t},0}$, presented in Figure 5, however, the constraint on the initial $\tilde{t}^c$ mass increases by almost a factor of two.

To obtain constraints on the initial stop masses we have thus far had to assume some relation between $m^2_{H_u,0}$ and $m^2_{\tilde{t}^c,0}$; e.g., $m^2_{H_u,0} = 0$ or $m^2_{H_u,0} = m^2_{\tilde{t}^c,0}$. Perhaps a better approach is to use the EWSB relation, Equation (2), to eliminate $m^2_{H_u,0}$ in favour of μ^2. This has the advantage of being model-independent. It is also a useful reorganization of independent parameters since the amount of fine tuning required to obtain the correct Z mass increases as μ is increased. To obtain some limits we choose $m^2_{H_d,0} = 0$ [14] to minimize the value of μ^2, and require that $m^2_{H_u,0}$ is positive. The minimum value of $m_{\tilde{t}^c,0}/M_S$ and $m_{\tilde{t},0}/M_S$ for different choices of $\tilde{\mu}/M_S$ are gotten by solving $m^2_{\tilde{t}^c}(\mu_G) = 0$ and $m^2_{\tilde{t}}(\mu_G) = 0$. These results are presented in Figure 7. In this Figure the positivity constraints terminate at that value of $M_{3,0}$ which gives $m^2_{H_u,0} = 0$.

As discussed in the above, reducing the value of $m^2_{H_u,0}$ decreases the positivity limit on $m_{\tilde{t}_i,0}$. Consequently the fine tuning of m^2_Z with respect to $m^2_{\tilde{t}_i,0}$ is also reduced. But using Equations 20 and 2, it can be seen that decreasing $m^2_{H_u,0}$ while keeping $m^2_{\tilde{t}^c}(\mu_G) = 0$ and $m^2_{\tilde{t}}(\mu_G) = 0$ results in a larger μ, thus increasing the fine tuning with respect to μ. This can also be seen from Figure 7. We find, for example, that if $M_{3,0}/M_S \sim 0.01$, the small value $\tilde{\mu}/M_S = 0.01$ requires $m_{\tilde{t}_i,0}/M_S \sim .25$. For $M_S = 10$ TeV, this corresponds to $\mu \sim 100$ GeV and $m_{\tilde{t}_i,0} \geq$ 2.5 TeV. A further inspection of Figure 7 shows that for the same value of $M_{3,0}/M_S$, a value of $m_{\tilde{t},0}/M_S = 0.17$ is allowed (gotten by decreasing $m^2_{H_u,0}$) only if $\tilde{\mu}/M_S$ is increased to .14. This corresponds to $\mu = 1.4$ TeV for $M_S = 10$ TeV; this implies that $\Delta(m^2_Z;\mu) \sim 930$. We find that the limit on the initial stop masses can only be decreased at the expense of increasing μ.

Finally, the limits become weaker if $m^2_{H_u,0} < 0$. This possibility is theoretically unattractive on two accounts. Firstly, a nice feature of supersymmetric extensions to the SM is that the dynamics of the model, through the presence of the large top quark Yukawa coupling, naturally leads to the breaking of the electroweak symmetry[4]. This is lost if electroweak symmetry

[14]This assumption is unnecessary. See the previous footnote.

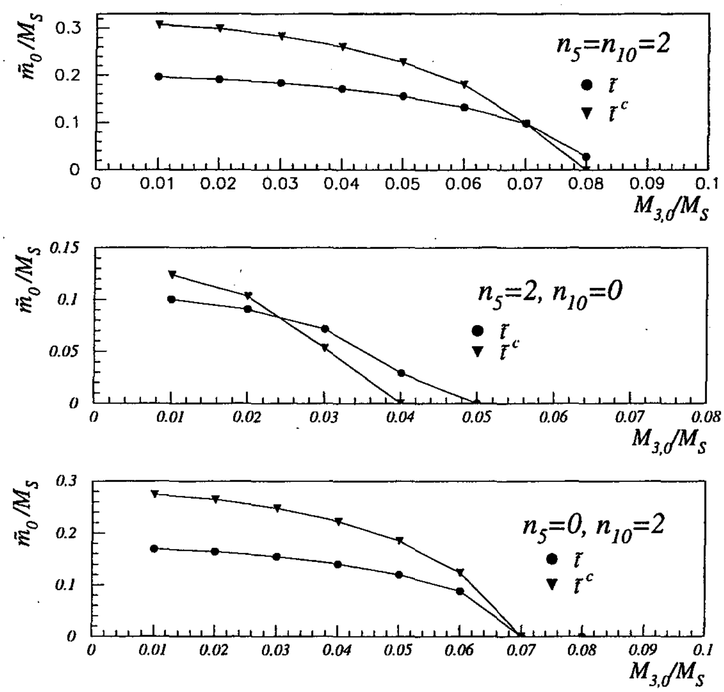

Figure 5: Limits for $\tilde{m}_0/M_S$ from the requirement that the stop mass squareds are positive at the weak scale, for $\tan\beta = 2.2$, $A_{t,0} = 0$ and assuming universal scalar masses at M_{GUT} for the stop and Higgs scalars. The regions below the curves are excluded.

breaking is already present at the tree-level. Secondly, the fine tuning required to obtain the correct Z mass is increased. From Figure 7 we infer that while reducing $m^2_{H_u,0}$ below zero does reduce the limit on the initial stop masses, the value of μ increases beyond the values quoted in the previous paragraph, thus further increasing the fine tuning of the Z mass. This scenario is not discussed any further.

We now combine the positivity analysis of this Section with the results of Section 2 to place lower limits on the soft scalar masses. For given values of δ_{LL}, δ_{RR}, a minimum value of M_S, $M_{S,min}$, is found using the results of Section 2. This is combined with the positivity analysis in Figure 6, to produce the results shown in Figure 8. We also show other limits gotten by assuming $m^2_{H_u,0} = m^2_{\tilde{t}^c,0}$. These results are presented in Figure 9. In Figure 10 we also present the stop mass limits for different values of μ, and restrict to $m^2_{H_u,0} \geq 0$ and $\sqrt{\delta_{LL}\delta_{RR}} = 0.04$. In all cases the heavy scalars were decoupled at $M_{S,min}$, rather than 50 TeV, and so the positivity analysis was repeated. The value of $A_{t,0}$ was chosen to maximize the value of the stop masses at the weak scale. For completeness, the results for the cases $(n_5, n_{10}) = (2,0)$ and $(0,2)$ and $m^2_{H_u,0} = 0$ are presented in Figure 11. We repeat that the minimum allowable values for the stop masses consistent with $m^2_{H_u,0} > 0$, gotten by setting $m^2_{H_u,0} = 0$, are given in Figures 8 and 11.

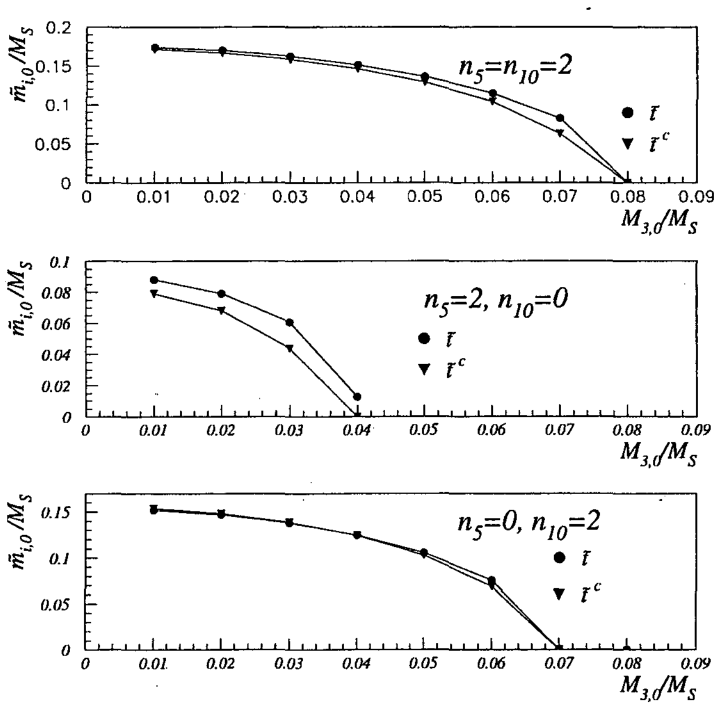

Figure 6: Limits for $m_{\tilde{t},0}/M_S$, $m_{\tilde{t}^c,0}/M_S$, from the requirement that the stop mass squareds are positive at the weak scale, for $M_{SUSY} = M_{GUT}$, $\tan\beta = 2.2$ and assuming that $m^2_{H_u,0} = 0$. The value of $A_{t,0}$ is chosen to maximize the value of the stop soft masses at the weak scale. The regions below the curves are excluded.

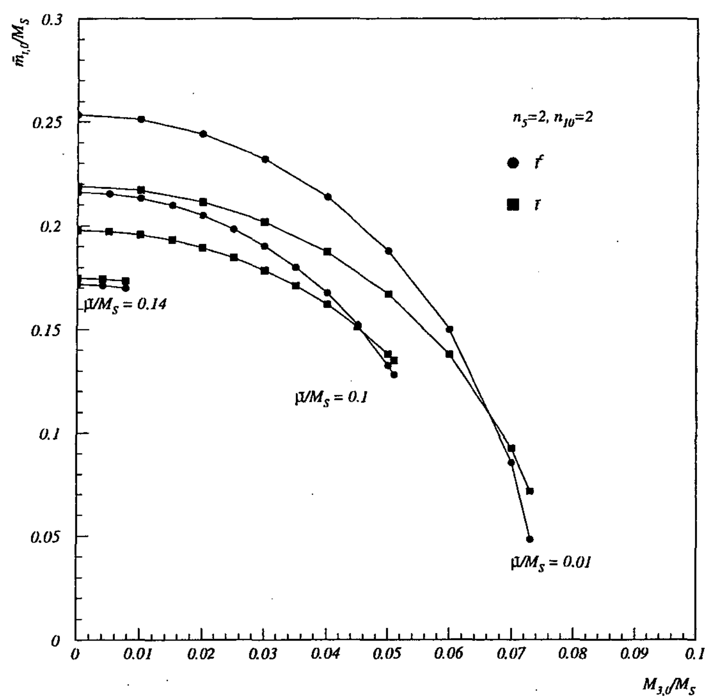

Figure 7: Limits for $m_{\tilde{t},0}/M_S$, $m_{\tilde{t}^c,0}/M_S$, from the requirement that the stop mass squareds are positive at the weak scale, for $(n_5, n_{10}) = (2,2)$, $M_{SUSY} = M_{GUT}$, $\tan\beta = 2.2$, and different values of $\tilde{\mu}/M_S$. The contours end at that value of $M_{3,0}/M_S$ that gives $m_{H_u,0}/M_S = 0$. The value of $A_{t,0}$ is chosen to maximize the value of the stop soft masses at the weak scale. The regions below the lines are excluded.

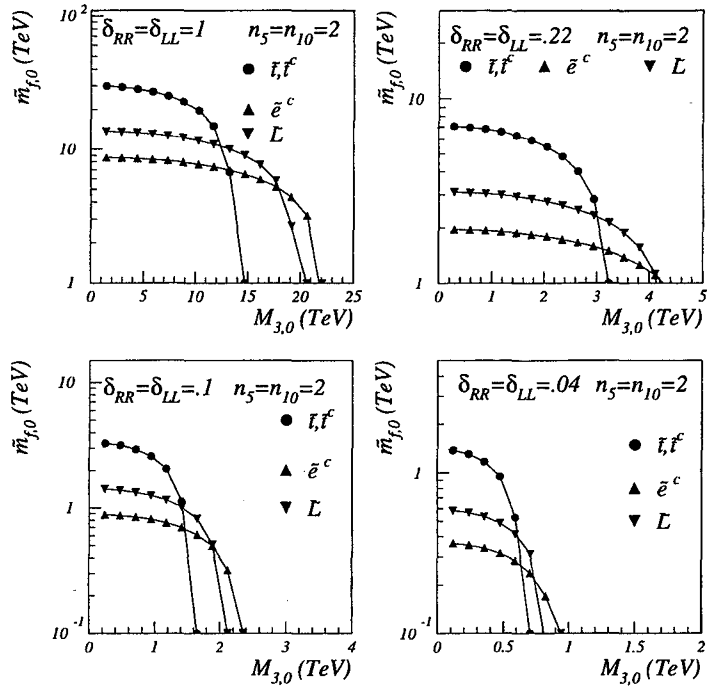

Figure 8: Limits for $m_{\tilde{t},0}$ and $m_{\tilde{t}^c,0}$, $m_{\tilde{e}^c}$, and $m_{\tilde{L}}$ from the requirement that the mass squareds are positive at the weak scale while suppressing Δm_K. It was assumed that $M_{SUSY} = M_{GUT}$, $\tan\beta = 2.2$ and that $m^2_{H_u,0} = 0$. The value of $A_{t,0}$ was chosen to maximize the value of the stop soft masses at the weak scale. The heavy scalars were decoupled at the minimum value allowed by Δm_K. The regions below the lines are excluded.

We next briefly discuss some consequences of this numerical analysis. We concentrate on the case $n_5 = n_{10} = 2$, since this is the relevant case to consider if the supersymmetric flavour problem is explained by decoupling the heavy scalars. Other choices for n_5 and n_{10} requires additional physics to explain the required degeneracy or alignment of any light non-third generation scalars. From Figures 8 and 9 we find that for $\sqrt{\delta_{LL}\delta_{RR}} = 0.22$ and $M_{3,0} \leq 1$ TeV, $m_{\tilde{t}_i,0} \lesssim 7$ TeV is required. If instead we restrict both $\Delta(m^2_Z; M^2_S)$ and $\Delta(m^2_Z; M_{3,0})$ to be less than 100, then we must have $M_S \lesssim 10$ TeV and $M_{3,0} \lesssim 300$ GeV. To not be excluded by Δm_K, we further require that $\sqrt{\delta_{LL}\delta_{RR}} \lesssim .06$. For this value of $\sqrt{\delta_{LL}\delta_{RR}} = 0.06$, a minimum value for $m_{\tilde{t},0}$ of $\sim$1.5$-$2.5 TeV is gotten by rescaling the results in Figures 8 and 9 for $\sqrt{\delta_{LL}\delta_{RR}} = 0.04$ by an amount 0.06/0.04. The range depends on the value of $m^2_{H_u,0}$, with the lower (upper) limit corresponding to $m^2_{H_u,0} = 0$ ($m^2_{\tilde{t}^c,0}$). Thus $\Delta(m^2_Z; m^2_{\tilde{t}_i,0}) \sim 400 - 800$. This fine tuning can be reduced only by either increasing $M_{3,0}$—which increases $\Delta(m^2_Z, M_{3,0})$ beyond 100—or by reducing M_S— which requires a smaller value for $\sqrt{\delta_{LL}\delta_{RR}}$. We conclude that unless $\sqrt{\delta_{RR}\delta_{LL}}$ is naturally small, decoupling the heavy scalars does not provide a natural solution to the flavour problem.

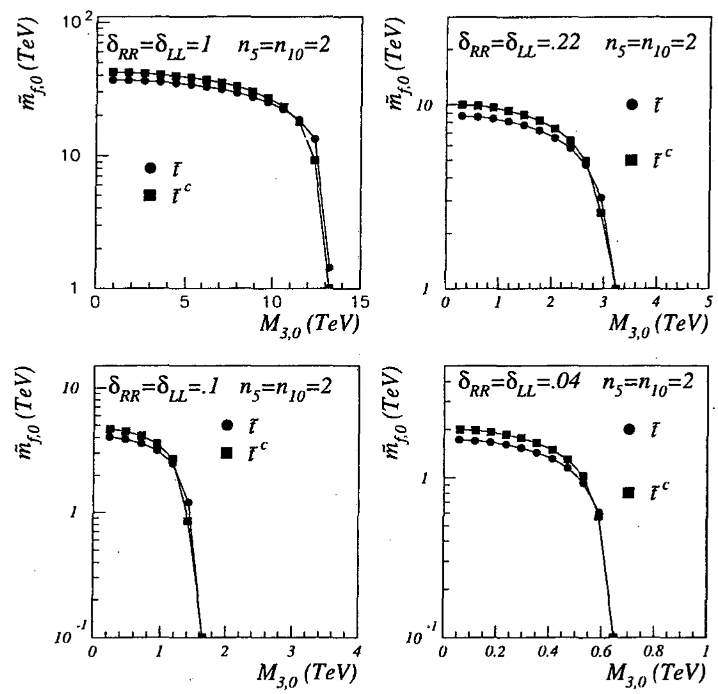

Figure 9: Limits for $m_{\tilde{t},0}$ and $m_{\tilde{t}^c,0}$ from the requirement that the stop mass squareds are positive at the weak scale while suppressing Δm_K. It was assumed that $M_{SUSY} = M_{GUT}$, $\tan\beta = 2.2$ and that $m^2_{H_u,0} = m^2_{\tilde{t}^c,0}$. The value of $A_{t,0}$ was chosen to maximize the value of the stop soft masses at the weak scale. The heavy scalars were decoupled at the minimum value allowed by Δm_K. The regions below the lines are excluded.

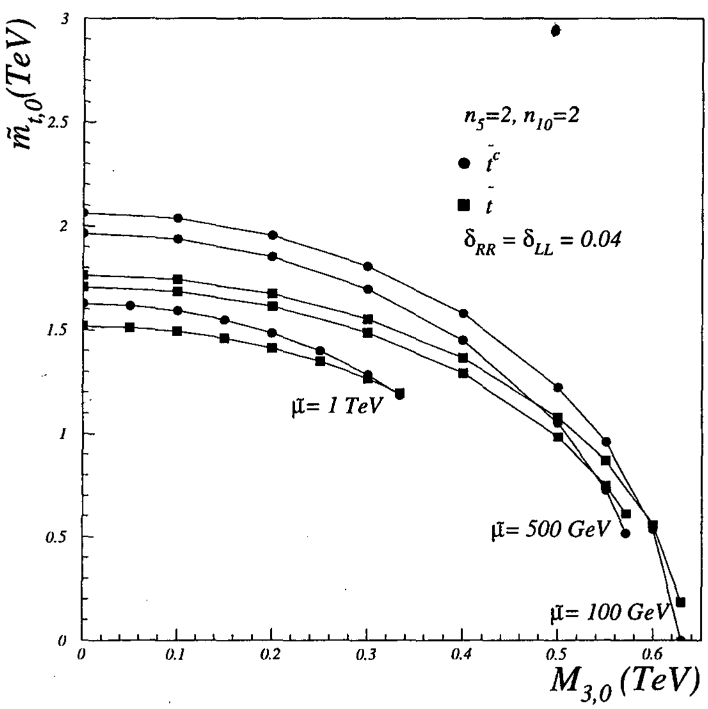

Figure 10: Limits for $m_{\tilde{t},0}$ and $m_{\tilde{t}^c,0}$ from the requirement that the stop mass squareds are positive at the weak scale while suppressing Δm_K, for $(n_5, n_{10}) = (2, 2)$, $\sqrt{\delta_{LL}\delta_{RR}} = 0.04$, and different values of μ. The contours terminate at $m^2_{H_u,0} = 0$. It was assumed that $M_{SUSY} = M_{GUT}$ and $\tan\beta = 2.2$. The value of $A_{t,0}$ was chosen to maximize the value of the stop soft masses at the weak scale. The heavy scalars were decoupled at the minimum value allowed by Δm_K. The regions below the lines are excluded.

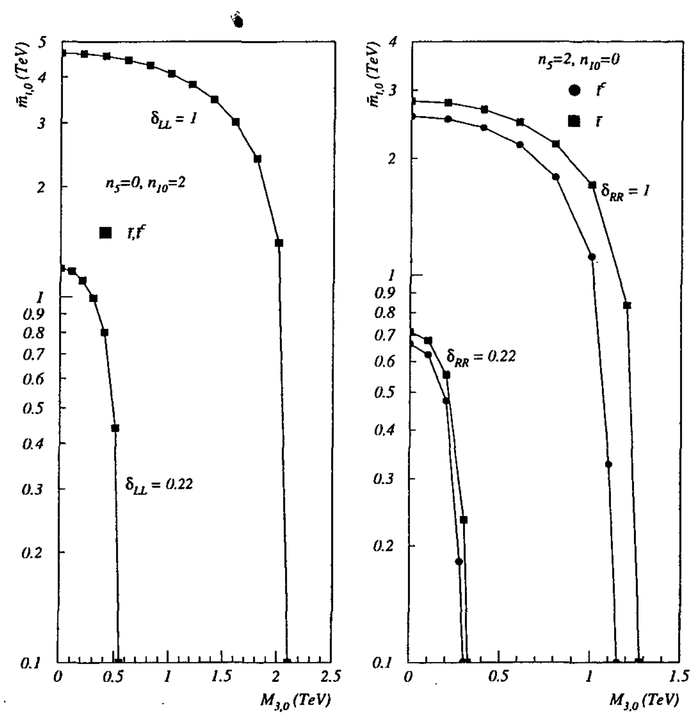

Figure 11: Limits for $m_{\tilde{t},0}$, $m_{\tilde{t}^c,0}$ from the requirement that the stop mass squareds are positive at the weak scale while suppressing Δm_K, for the cases $(n_5, n_{10}) = (2,0)$ and $(0,2)$. It was assumed that $M_{SUSY} = M_{GUT}$, $\tan\beta = 2.2$ and that $m^2_{H_u,0} = 0$. The value of $A_{t,0}$ was chosen to maximize the value of the stop soft masses at the weak scale. The heavy scalars were decoupled at the minimum value allowed by Δm_K. The regions below the lines are excluded.

To conclude this Section we discuss the constraint this analysis implies for those models which generate a split mass spectrum between different generations through the D-term contributions of the anomalous $U(1)$ gauge symmetry[12, 16, 15]. In the model of set D of [15], there are two $\bar{\mathbf{5}}$s at 7 TeV and 6.1 TeV and two **10**s at 6.1 and 4.9 TeV, respectively, so that Δm_K is suppressed. These values must be increased by a factor of 2.5 to correct for the QCD enhancement of the SUSY contribution to Δm_K, as discussed in Section 2. To obtain a conservative bound on the initial stop masses from the positivity requirement, we first assume that all the heavy scalars have a common mass $M_S = 2.5 \times 5$TeV$=$ 12.5 TeV. (It would have been 5 TeV without the QCD correction.) Then assuming a weak scale value of the gluino mass that is less than 710 GeV and setting $m^2_{H_u,0} = 0$ $(m^2_{\tilde{t}^c,0})$, we find from Figure 6 (5) that $m_{\tilde{t},0} \geq 2.1$ (3.6) TeV is required. This leads to $\Delta(m^2_Z; m^2_{\tilde{t},0}) \geq 580$ (1700). To obtain a better bound, we repeat our analysis using $n_5 m^2_5 + 3 n_{10} m^2_{10} = ((7 \text{ TeV})^2 + (6.1 \text{ TeV})^2 + 3 \times (6.1 \text{ TeV})^2 + 3 \times (4.9 \text{ TeV})^2) \times (2.5)^2$. It is possible to do this since only this combination appears in the RG analysis for $(n_5, n_{10}) = (2, 2)$. We find (assuming $m^2_{H_u,0} = 0$ and the gluino mass at the weak scale is less than 710 GeV) that $m_{\tilde{t},0} \gtrsim 2.4$ TeV. In the model of [16], $\delta_{RR} \approx \delta_{LL} \approx 0.01$. To obtain a limit on the initial stop masses, we use the bound obtained from either Figures 8 or 9 for $\delta_{RR} = \delta_{LL} \approx 0.04$, and divide the limit by a factor of 4. By inspecting these Figures we find that this model is only weakly constrained, even if $m^2_{H_u,0} \sim m^2_{\tilde{t},0}$. We now discuss the limits in this model when $O(1)$ CP violating phases are present. To obtain the minimum value of M_S in this case, we should multiply the minimum value of M_S obtained from the Δm_K constraint for $\delta_{LL} = \delta_{RR} = 0.04$ by 12.5/4; dividing by 4 gives the result for $\delta_{LL} = \delta_{RR} = 0.01$, and multiplying by 12.5 gives the constraint on M_S from ϵ. The result is $M_S \gtrsim$ 23 TeV. Next, we assume that $M_{3,0}$ is less than 300 GeV, so that the value of the gluino mass at the weak scale is less than 710 GeV. This gives $M_{3,0}/M_S \leq 0.013$. Using these values of $M_{3,0}$ and M_S, an inspection of Figures 5 and 6 implies that $m_{\tilde{t},0}$ must be larger than 3.9 TeV to 6.9 TeV, depending on the value of $m^2_{H_u,0}$. This gives $\Delta(m^2_Z; m^2_{\tilde{t},0}) \geq 2000$. In the model of [12], $M_{3,0}/M_S \approx 0.01$ and $m_{\tilde{f},0}/M_S \approx 0.1$. Inspecting Figures 5 and 6 we find that these values are excluded for $(n_5, n_{10}) = (2, 2)$ and $(0, 2)$. The case $(2, 0)$ is marginally allowed. The model of [12] with $(n_5, n_{10}) = (2, 2)$ and $\lambda_t = 0$ was also excluded by the analysis of Reference [18].

5 Using Finetuning to Constrain δ

In this section, we vary the messenger scale, M_{SUSY}, between the GUT scale and a low scale $\sim$ 50 TeV, and restrict the boundary values of the stop and gluino masses so that EWSB is not fine tuned. This gives us an upper limit to δ if we require both positivity of the stop mass squareds at the weak scale and suppression of Δm_K. In other words, we determine the values for (δ, M_{SUSY}) which are allowed by the following requirements: 1. Suppression of the SUSY contribution to Δm_K by making the mass of the first two generation scalars, M_S, large. 2. Positivity of the stop mass squareds and 3. Fine tuning in electroweak symmetry breaking does not exceed 1% or 10% (*i.e.*, both $\Delta(m_Z^2, m_{\tilde{t},0}^2)$ and $\Delta(m_Z^2, M_{3,0})$ are smaller than either 100 or 10).

An upper limit to δ satisfying the above requirements is obtained as follows. For a given M_{SUSY} we compute, using Equations 1 and 2, the boundary values of the stop mass, $m_{\tilde{t},max}$, and the gluino mass, $M_{3,max}$, such that both $\Delta(m_Z^2, m_{\tilde{t},0}^2)$ and $\Delta(m_Z^2, M_{3,0})$ are equal to some maximum value Δ_{max} which is chosen to be 100 or 10. [15] Substituting these values of the bare stop[16] and gluino masses into the expression for the weak-scale value of the stop mass squared, we determine the maximum value of M_S, $M_{S,max}$, such that the stop mass squareds at the weak scale are positive. Using this value for M_S and the analysis described in Section 2.1, an upper value to δ is gotten from the Δm_K constraint. This value of δ and M_{SUSY} then satisfies the above-mentioned three requirements. This can be seen as follows. For the given M_{SUSY}, if δ is larger than this limit, then to suppress Δm_K, M_S has to be larger than $M_{S,max}$. But, then to keep the stop mass squareds positive at the weak scale, the boundary value of either the stop or the gluino mass has to increase beyond $m_{\tilde{t},max}$ or $M_{3,max}$ respectively, leading to $\Delta(m_Z^2, m_{\tilde{t},0}^2)$ or $\Delta(m_Z^2, M_{3,0})$ larger than Δ_{max}, *i.e.*, increasing the fine tuning in EWSB.

We show the limits on $\sqrt{\delta_{LL}\delta_{RR}}$ as a function of M_{SUSY} for the case ($n_5 = 2, n_{10} = 2$) in Figures 12. In the top of Figure 12, $m_{H_u,0}^2 = 0$ is assumed. GMSB relations between the stop and Higgs masses are assumed in the bottom of Figure 12. For both cases, $\Delta_{max} = 100$, $\tan\beta = 2.2$ and 10 are considered. For other choices for Δ_{max}, the upper limit to δ roughly scales as $\sqrt{\Delta_{max}/100}$, since both $m_{\tilde{t},max}$, $M_{3,max}$ and therefore $M_{S,max}$ scale as $\sqrt{\Delta_{max}}$.

In the case of GMSB mass relations, the boundary value of the Higgs mass and the stop masses are comparable for high M_{SUSY}. Since $m_{H_u,0}^2$ results in a negative contributon to the stop mass squared, this tends to reduce the stop mass squared at the weak scale as compared to the case $m_{H_u,0}^2 = 0$. Then, from the above analysis, we can see that $M_{S,max}$ and, in turn, the limit on δ is smaller for the GMSB case as compared to the case $m_{H_u,0}^2 = 0$. This can be seen

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[15] In computing the Δ's, $\tan\beta$, in addition to $m_{H_u}^2(m_Z)$, should be regarded as a function of the bare parameters. However, this additional contribution to the Δ's is small for $\tan\beta \gtrsim 2$ and also makes the magnitude of Δ larger. We neglect this dependence which is a conservative choice.

[16] Strictly speaking, we should translate the upper bound on $m_{\tilde{t}_i,0}^2$ into an upper bound on $\tilde{m}_{\tilde{t}_i,0}^2$ using $\tilde{m}_{\tilde{t}_i,0}^2 = m_{\tilde{t}_i,0}^2 + c_D Y_{\tilde{t}_i} D_{Y,0} + Y_{\tilde{t}_i}\zeta_D$, *i.e.*, to that combination appearing in the positivity constraint. Instead, we use the same bound for both $m_{\tilde{t}_i,0}^2$ and $\tilde{m}_{\tilde{t}_i,0}^2$. This is reasonable, since c_D is generally small ($\lesssim 0.05$), and $D_{Y,0} \sim O(m^2)$. In any case, this effect is in the opposite direction for $\tilde{t}$ and $\tilde{t}^c$. In the case that $\zeta_D \neq 0$, a slightly larger ($O(30\%)$) value for δ may be allowed as compared to $\zeta_D = 0$. This is because if $\zeta_D < 0$, the maximum value for $\tilde{m}_{\tilde{t}^c,0}^2$ is larger than $m_{\tilde{t},max}^2$. This, in turn, allows for a larger value of M_S, and hence δ. Naturalness considerations limit $|\zeta_D|$, though. The EWSB relation for m_Z^2, Equation 2, contains a term linear in ζ_D. Requiring that $\Delta(m_Z^2, \zeta_D) < 100$ implies that $|\zeta_D| \lesssim \zeta_{D,max} \equiv (900\ \text{GeV})^2$. Thus for a high scale of supersymmetry breaking, the upper bound on $\tilde{m}_{\tilde{t}^c,0}^2$ may be increased to $\tilde{m}_{\tilde{t}^c,0}^2 \sim m_{\tilde{t},max}^2 + \frac{2}{3}\zeta_{D,max} \sim \frac{5}{3} m_{\tilde{t},max}^2$, while maintaining $\Delta(m_Z^2, m_{\tilde{t}^c,0}^2) = \Delta(m_Z^2, \zeta_D) = 100$. This roughly translates into an increase of $\sim \sqrt{5/3} = 1.3$ in the limit to δ. The actual limit will be smaller, since with this choice of sign for ζ_D, the positivity constraint for the left-handed stop is now stronger. It is thus reasonable to require that the maximum value of $\tilde{m}_{\tilde{t}_i,0}^2$ be comparable to $m_{\tilde{t}_i,max}^2$.

by comparing the top and bottom of Figure 12.

In Figure 13 the limits on δ_{RR} and δ_{LL} for $(n_5 = 2, n_{10} = 0)$ and $(n_5 = 0, n_{10} = 2)$ are shown, respectively. We assume $m^2_{H_u,0} = 0$ and consider $\tan\beta = 2.2$ and 10. If we choose Δ_{max} to be 100, then we get a constraint on δ ($\delta \lesssim 0.5$) only for high values of M_{SUSY}. So, we choose instead Δ_{max} to be 10.

We have checked that, for $\tan\beta = 10$, the limits on the boundary value of the stop mass from requring positivity of the mass squared at the weak scale do not differ by more than a few percent from the case $\tan\beta = 2.2$ (for the same values of the gluino and heavy scalar masses). However, the fine tuning of EWSB for the same gluino and stop mass is smaller for $\tan\beta = 10$ as compared to $\tan\beta = 2.2$. This is because, for $\tan\beta = 10$, λ_t is smaller than in the case $\tan\beta = 2.2$. Hence the sensitivity of the weak scale value of $m^2_{H_u}$ to $m^2_{\tilde{t},0}$ and $M_{3,0}$ is smaller. Also, the $\tan^2\beta/(\tan^2\beta - 1)$ factor in Equation 2 is smaller, further reducing the sensitivity of m^2_Z to $m^2_{\tilde{t},0}$ and $M_{3,0}$. In other words, for $\tan\beta = 10$, $m_{\tilde{t},max}$ and $M_{3,max}$ are larger so that $M_{S,max}$ and, in turn, the limit on δ is larger. This can be seen in Figures 12 and 13.

6 Conclusions

In this paper we have studied whether the SUSY flavor problem can be solved by making the scalars of the first and second generations heavy, with masses M_S ($\gtrsim$few TeV), without destabilising the weak scale. If the scale, M_{SUSY}, at which SUSY breaking is mediated to the SM scalars is close to the GUT scale, then the heavy scalars drive the light scalar (in particular the stop) mass squareds negative through two-loop RG evolution. In order to keep the mass squareds at the weak scale positive, the initial value of the stop (and other light scalar) soft masses, $m_{\tilde{f}_i,0}$, must typically be $\gtrsim 1$ TeV, leading to fine tuning in EWSB. We included two new effects in this analysis: the effect of λ_t in the RGEs which makes the stop mass squareds at the weak scale more negative and hence makes the constraint on the initial value stronger, and the QCD corrections to the SUSY box diagrams which contribute to $K - \bar{K}$ mixing.

Some results of our analysis for $M_{SUSY} = M_{GUT}$ can be summarized as follows. We restrict the gluino mass (at the weak scale) to be less than about 710 GeV, so that the fine tuning of m^2_Z with respect to the bare gluino mass, $M_{3,0}$, is not worse than 1%. This requires that $M_{3,0} \lesssim 300$ GeV. We also assume that $m^2_{H_u,0} = 0$ to maximize the value of the stop masses at the weak scale. We find that if $\sqrt{\delta_{LL}\delta_{RR}} = 0.22$ then $M_S \geq 40$ TeV is required to be consistent with Δm_K. With these assumptions, this implies that for $M_{3,0}$ less than 1 TeV, $m_{\tilde{t}_i,0} > 6.5$ TeV is needed to not break colour and charge at the weak scale. Even for $\sqrt{\delta_{LL}\delta_{RR}} = 0.04$, we find that we need $M_S \gtrsim 7$ TeV. This implies that $m_{\tilde{t},0} > 1$ TeV is required if $M_{3,0} \leq 300$ GeV. This results in a fine tuning of $\sim 1\%$. For $\delta_{LL} = 1$ and $\delta_{RR} = 0$, we find that $M_S \gtrsim 30$ TeV and $m_{\tilde{t},0} > 4.5$ TeV. For $\delta_{LL} = 0.22$ and $\delta_{RR} = 0$, we find that $M_S \gtrsim 7$ TeV and $m_{\tilde{t},0} > 1$ TeV. For $\delta_{LL} = 0$ and $\delta_{RR} = 1$, we find that $M_S \gtrsim 30$ TeV and $m_{\tilde{t}^c,0} > 2.5$ TeV. The constraints are

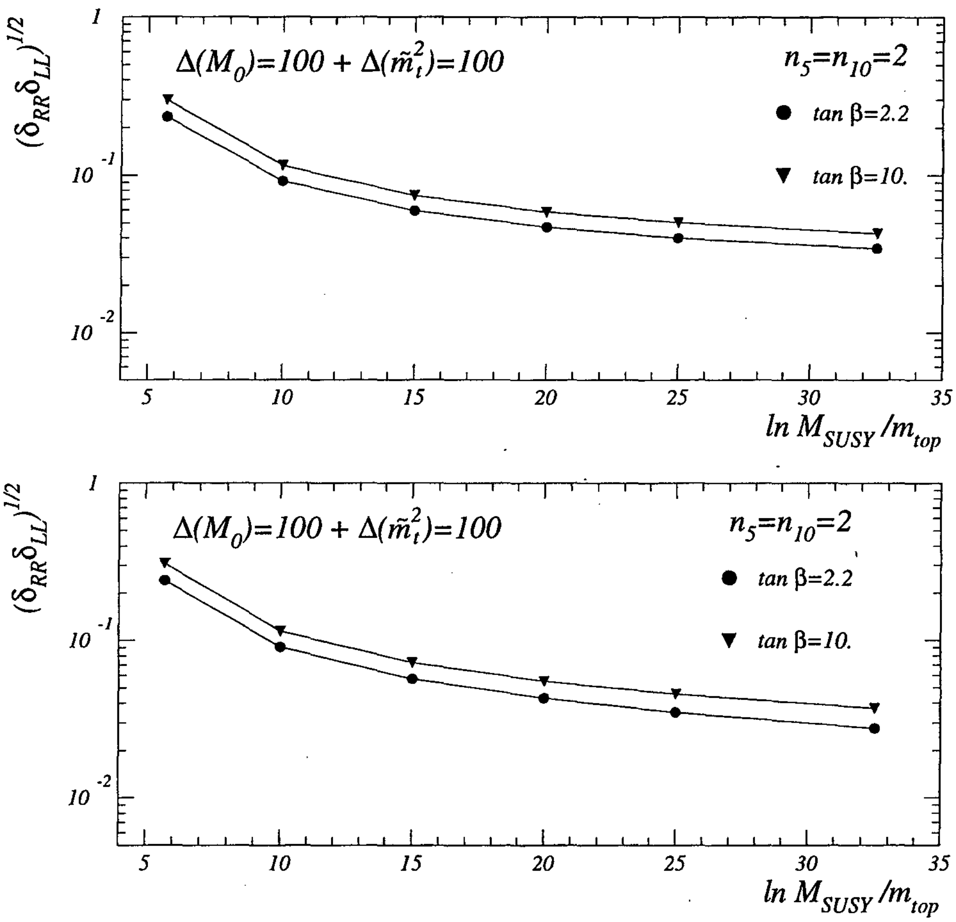

Figure 12: Maximum value for $(\delta_{LL}\delta_{RR})^{1/2}$ that is consistent with $\Delta(m_Z^2, M_{3,0}) < 100$, $\Delta(m_Z^2, m_{\tilde{t},0}^2) < 100$ and $(\Delta m_K)_{SUSY} < (\Delta m_K)_{exp}$. Two boundary conditions are considered: $m_{H_u,0}^2 = 0$ (top) and gauge-mediated relations (bottom). Two values for $\tan\beta$ are considered. The value of $A_{t,0}$ was chosen to maximize the value of the stop masses at the weak scale.

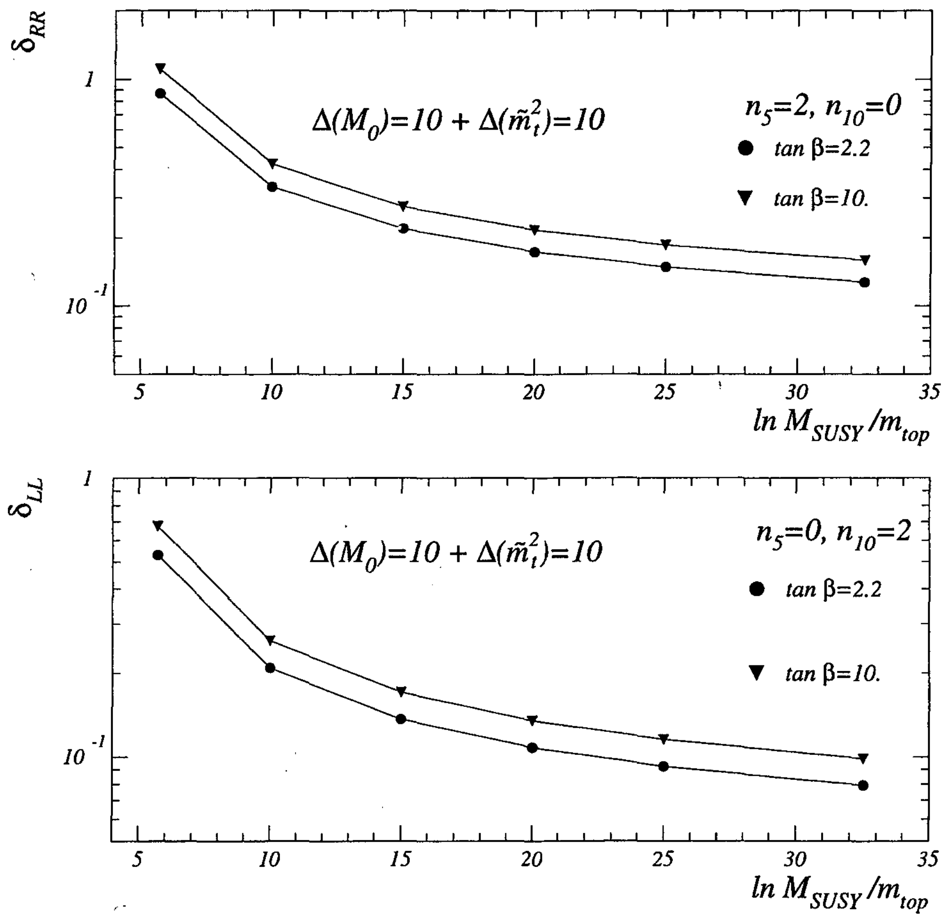

Figure 13: Maximum value for δ_{LL}, δ_{RR} that is consistent with $\Delta(m_Z^2, M_{3,0}) < 10$, $\Delta(m_Z^2, m_{\tilde{t},0}^2) < 10$ and $(\Delta m_K)_{SUSY} < (\Delta m_K)_{exp}$. It was assumed that $m_{H_u,0}^2 = 0$. Two values for $\tan\beta$ are considered. The value of $A_{t,0}$ was chosen to maximize the value of the stop masses at the weak scale.

weaker for smaller values of δ. In a realistic model, $m^2_{H_u,0}$ might be comparable to $m^2_{\tilde{t},0}$ and the constraints on $m_{\tilde{t},0}$ in this case are stronger. This is also discussed. We note that independent of the constraint from $K-\bar{K}$ mixing, our analysis can be used to check the phenomenological viability of any model that has heavy scalars. We also discuss the phenomenological viability of the anomalous $D-$term solution, and find it to be problematic.

We then considered the possibility that $M_{SUSY} = M_S$. In this case, there is no RG log enhancement of the negative contribution of the heavy scalar masses to the light scalar masses. For this case, we computed the finite parts of the two-loop diagrams and used these results as estimates of the two-loop contribution of the heavy scalars to the light scalar soft mass squareds. We then combined these results with the constraints from $K-\bar{K}$ mixing to obtain lower limits on the boundary values of the stops. As an example, we assumed gauge mediated SUSY breaking boundary conditions for the light scalars. If $n_5 \neq n_{10}$ then one of the selectron masses, rather than the stop masses, provides the stronger constraint on $m_{\tilde{t}_i,0}$ once gauge-mediated boundary conditions are used to relate $m_{\tilde{e}^c,0}$ and $m_{\tilde{L},0}$ to $m_{\tilde{t}_i,0}$. Some of our results can be summarized as follows. We restrict the gluino mass at the weak scale to be less than about 2.3 TeV, again to avoid more than 1% fine tuning of m^2_Z with respect to the gluino mass. For $\sqrt{\delta_{LL}\delta_{RR}} = .22$ we find that $m_{\tilde{t}_i,0} \geq 1.4$ TeV is required. The fine tuning of m^2_Z with respect to the stop mass is $\sim 1.5\%$ in this case. For the cases $\delta_{LL} = 0$ and $\delta_{RR} = 1$, and $\delta_{LL} = 1$ and $\delta_{RR} = 0$ we find that $m_{\tilde{t},0} \gtrsim 1$ TeV. As before, the constraints on $m_{\tilde{t},0}$ for smaller values of δ are weaker than ~ 1 TeV. Again, we emphasize that the constraints in an actual model of this low-energy supersymmetry breaking scenario could be different, and our results should be treated as estimates only. We also briefly discuss the CP violating constraints from ϵ, and find that these limits increase by a factor of ~ 12 if $O(1)$ phases are present.

Finally, in Section 5 the scale of supersymmetry breaking is varied between 50 TeV and 2×10^{16} GeV. Uppers bounds to δ, that are consistent with positivity of the light scalar masses, naturalness in electroweak symmetry breaking, and $(\Delta m_K)_{exp}$, are obtained. These results are summarized in Figures 12 and 13.

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8 Appendix: Two-loop calculation

In this Appendix we discuss the two-loop contribution of the heavy scalar soft masses to the light scalar soft masses. These contributions can be divided into two classes. In the first class, a vev for the hypercharge D-term is generated at two-loops. The Feynman diagrams for these contributions are given in Figure 14 and are clearly $\sim \alpha_1 \alpha_i$. These diagrams are computed in a later portion of this Appendix. In the other class, the two-loop diagrams are $\sim \alpha_i^2$. These have been computed by Poppitz and Trivedi[30]. So, we will not give details of this computation which can be found in their paper. However, our result for the finite parts of these diagrams differs slightly from theirs and we discuss the reason for the discrepancy. When one regulates the theory using dimensional reduction [25, 26] (compactifying to $D < 4$ dimensions), the vector field decomposes into a D-dimensional vector and $4-D$ scalars, called ϵ-scalars, in the adjoint representation of the gauge group. Thus the number of Bose and Fermi degrees of freedom in the vector multiplet remain equal. The ϵ-scalars receive, at one-loop, a divergent contribution to their mass, proportional to the supertrace of the mass matrix of the matter fields. Neglecting the fermion masses, this contribution is

$$\delta m_{\epsilon}^2 = -\frac{\alpha}{4\pi}(\frac{2}{\epsilon} + \ln 4\pi - \gamma)(n_5 + 3n_{10})M_S^2. \tag{25}$$

In our notation $D = 4-\epsilon$. Poppitz and Trivedi choose the counterterm to cancel this divergence in the MS scheme, *i.e.*, the counterterm consists only of the divergent part, proportional to $1/\epsilon$. When this counterterm is inserted in a one-loop ϵ-scalar graph with SM fields (scalars) as the external lines , one obtains a divergent contribution to the SM scalar soft masses (the $1/\epsilon$ of the counterterm is cancelled after summing over the ϵ adjoint scalars running in the loop). Poppitz and Trivedi use a cut-off, Λ_{UV}, to regulate this graph, giving a contribution from this graph that is:

$$m_i^2 = -\sum_A (n_5 + 3n_{10})C_A^i \frac{1}{16}(\frac{\alpha_A}{\pi})^2 M_S^2 \ln\Lambda_{UV}^2 \tag{26}$$

with no finite part. We, on the other hand, choose the ϵ-scalar mass counterterm in the $\overline{MS}$ scheme, *i.e.*, proportional to $2/\epsilon - \gamma + \ln 4\pi$ (where $\gamma \approx 0.58$ is the Euler constant) and use dimensional reduction to regulate the graph with the insertion of the counterterm. This gives a contribution

$$\begin{aligned} m_i^2 &= -\sum_A (n_5 + 3n_{10})C_A^i \frac{1}{16}\left(\frac{\alpha_A}{\pi}\right)^2 M_S^2 (\frac{2}{\epsilon} - \gamma + \ln 4\pi)^2 \epsilon \\ &= -\sum_A (n_5 + 3n_{10})C_A^i \frac{1}{8}\left(\frac{\alpha_A}{\pi}\right)^2 M_S^2 (2/\epsilon - 2\gamma + 2\ln 4\pi) \end{aligned} \tag{27}$$

In the first line the first factor of $(2/\epsilon - \gamma + \ln 4\pi)$ is from the counter-term insertion, the second factor is the result of the loop integral, and the over-all factor of ϵ counts the number of ϵ-scalars running in the loop. In the $\overline{MS}$ scheme, *i.e.*, after subtracting $2/\epsilon - \gamma + \ln 4\pi$, we are left with a finite part[17] proportional to $-\gamma + \ln 4\pi$. The remaining diagrams together give a finite result and we agree with Poppitz and Trivedi on this computation. Our result for the finite part of the two-loop diagrams (neglecting the fermion masses) is

$$\begin{aligned} m_{i,finite}^2(\mu) &= -\frac{1}{8}\left(\ln(4\pi) - \gamma + \frac{\pi^2}{3} - 2 - \ln\left(\frac{M_S^2}{\mu^2}\right)\right) \\ &\quad \times \sum_A \left(\frac{\alpha_A(\mu)}{\pi}\right)^2 (n_5 + 3n_{10})C_A^i M_S^2 \end{aligned} \tag{28}$$

whereas the Poppitz-Trivedi result does not have the $\ln(4\pi) - \gamma$ in the above result. The computation of the two-loop hypercharge D-term, which gives contribution to the soft scalar mass

[17] The same finite part is obtained in the MS scheme,regulated with DR' .

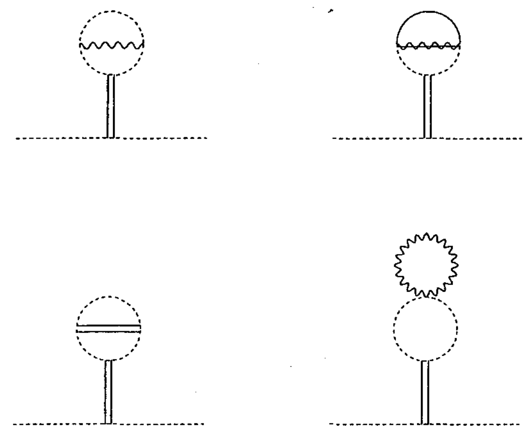

Figure 14: Mixed two-loop corrections to the scalar mass. Wavy lines, wavy lines with a straight line through them, solid lines, and dashed lines denote gauge boson, gaugino, fermion and scalar propagators,respectively. The double-line denotes the hypercharge D-term propagator.

squareds proportional to $\alpha_1\alpha_s$ and $\alpha_1\alpha_2$ (*i.e.*, the "mixed" two-loop contributon) is discussed below in detail.

Two-loop hypercharge D-term

We compute the two-loop diagrams of Figure 14 in the Feynman gauge and set all fermion and gaugino masses to zero. It is convienent to calculate in this gauge because both the scalar self-energy and the D_Y-term vertex corrections are finite at one-loop and thus require no counter-terms. We have also computed the two-loop diagrams in the Landau gauge and have found that it agrees with the calculation in the Feynman gauge. The calculation in the Landau gauge requires counter-terms, is more involved, and hence the discussion is not included. Finally, in the calculation a global $SU(5)$ symmetry is assumed so that a hypercharge D-term is not generated at one-loop [17, 13].

The sum of the four Feynman diagrams in Figure 14 is given in the Feynman gauge by

$$-i\tilde{\Pi}_{D,f} = i\frac{3}{5}g_1^2 Y_f \mathrm{Tr} Y_i \sum_{A,i} g_A^2 C_A^i (4I_1(m_i^2) - 4I_2(m_i^2) + I_3(m_i^2)) \tag{29}$$

where the trace is over the gauge and flavour states of the particles in the loops. If the particles in the loop form complete $\bar{5}$ and 10 representations with a common mass M_S, the sum simplifies to

$$-i\tilde{\Pi}_{D,f} = i\frac{3}{5}\alpha_1 Y_f (n_5 - n_{10})(\frac{4}{3}\alpha_3 - \frac{3}{4}\alpha_2 - \frac{1}{12}\alpha_1)(4I_1(M_S^2) - 4I_2(M_S^2) + I_3(M_S^2)). \tag{30}$$

The functions I_1, I_2 and I_3 are

$$I_1(m^2) = \int \frac{d^Dp}{(2\pi)^D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(p^2-m^2)^2} \frac{(2p-k)^2}{k^2} \frac{1}{(p-k)^2-m^2}, \tag{31}$$

$$I_2(m^2) = \int \frac{d^Dp}{(2\pi)^D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(p^2-m^2)^2} \frac{k^2-k\cdot p}{k^2} \frac{1}{(p-k)^2}, \tag{32}$$

$$I_3(m^2) = \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2-m^2)^2} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2-m^2}. \tag{33}$$

We now compute these functions.

Evaluating I_1

After a Feynman parameterization and performing a change of variables, $I_1 = J_1 + J_2$, where

$$J_1(m^2) = \Gamma(3) \int_0^1 dx(1-x) \int \frac{d^Dp}{(2\pi)^D} \int \frac{d^Dk}{(2\pi)^D} 4\frac{p^2}{k^2} \frac{1}{(p^2-(m^2-x(1-x)k^2))^3} \tag{34}$$

and

$$J_2(m^2) = \Gamma(3) \int_0^1 dx(1-x)(2x-1)^2 \int \frac{d^Dp}{(2\pi)^D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(p^2-(m^2-x(1-x)k^2))^3}. \tag{35}$$

After some algebra we find that

$$J_1(m^2) = \frac{\Gamma(3-D)}{(4\pi)^D}(m^2)^{D-3}\frac{2D}{D/2-1}B(2-D/2, 3-D/2), \tag{36}$$

$$J_2(m^2) = \frac{\Gamma(3-D)}{(4\pi)^D}(m^2)^{D-3} \times (4B(3-D/2, 2-D/2) - 4B(2-D/2, 2-D/2) + B(1-D/2, 2-D/2)), \tag{37}$$

where $B(p,q) = \Gamma[p]\Gamma[q]/\Gamma[p+q]$ is the usual Beta function.

Combining these two results gives

$$I_1(m^2) = \frac{\Gamma(3-D)}{(4\pi)^D}(m^2)^{D-3}\frac{1-D}{D-2}B(3-D/2, 2-D/2). \tag{38}$$

Evaluating I_2

$$\begin{aligned} I_2(m^2) &= \int \frac{d^Dp}{(2\pi)^D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(p^2-m^2)^2} \frac{k^2-k\cdot p}{k^2} \frac{1}{(p-k)^2} \\ &= \frac{1}{(4\pi)^D}\Gamma(3-D)(m^2)^{D-3}B(D/2, 1-D/2). \end{aligned}$$

Evaluating I_3

$$\begin{aligned} I_3(m^2) &= \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2-m^2)^2} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2-m^2} \\ &= \left(\frac{i}{(4\pi)^{D/2}}\Gamma(2-D/2)(m^2)^{D/2-2}\right)\left(\frac{i}{(4\pi)^{D/2}}\frac{\Gamma(2-D/2)}{D/2-1}(m^2)^{D/2-1}\right) \\ &= -\frac{1}{(4\pi)^D}(\Gamma(2-D/2))^2\frac{1}{D/2-1}(m^2)^{D-3}. \end{aligned}$$

We may now combine I_1, I_2 and I_3 to obtain

$$\begin{aligned} T(m^2) &\equiv 4I_1(m^2) - 4I_2(m^2) + I_3(m^2) \\ &= \frac{(m^2)^{D-3}}{(4\pi)^D} \times (4\left(\frac{1-D}{D-2}B(3-D/2, 2-D/2) - B(D/2, 1-D/2)\right)\Gamma(3-D) \\ &\quad -\frac{1}{D/2-1}\Gamma(2-D/2)^2). \end{aligned}$$

Writing $D = 4 - \epsilon$ and expanding in ϵ gives

$$T(m^2) = \frac{1}{(16\pi^2)^2}\left(\frac{4}{\epsilon} + \left(6 - \frac{2}{3}\pi^2 + 4(\ln(4\pi) - \gamma) - 4\ln m^2\right) m^2 + O(\epsilon)\right). \tag{39}$$

In the $\overline{MS}$ scheme the combination $2/\epsilon + \ln(4\pi) - \gamma$ is subtracted out. The finite piece that remains is

$$\frac{1}{(16\pi^2)^2}\left(6 - \frac{2}{3}\pi^2 + 2(\ln(4\pi) - \gamma) - 4\ln m^2\right) m^2. \tag{40}$$

Thus in the $\overline{MS}$ scheme

$$\begin{aligned} -i\tilde{\Pi}_{D,f} &= i\frac{3}{5}\frac{1}{(16\pi^2)^2} g_1^2 Y_f \mathrm{Tr}_i Y_i \sum_A g_A^2 C_A^i \left(6 - \frac{2}{3}\pi^2 + 2(\ln(4\pi) - \gamma) - 4\ln m_i^2\right) m_i^2 \\ &= i\frac{3}{5}\frac{1}{16\pi^2}\alpha_1(\mu_R)(n_5 - n_{10})Y_f\left(6 - \frac{2}{3}\pi^2 + 2(\ln(4\pi) - \gamma) - 4\ln(\frac{M_S^2}{\mu_R^2})\right) \\ &\quad \times \left(\frac{4}{3}\alpha_3(\mu_R) - \frac{3}{4}\alpha_2(\mu_R) - \frac{1}{12}\alpha_1(\mu_R)\right) M_S^2 \end{aligned} \tag{41}$$

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