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ABSTRACT

The recent experiments on the total inelastic cross sections for 190-Mev deuterons on various target nuclei demand a considerably larger nuclear radius than is usually accepted to explain the experimental results. It is shown that this result is not inconsistent with the total nuclear cross sections for 90-Mev neutrons if a nonsquare-well nuclear shape is taken. Taking for numerical simplicity a parabolic shape, the radius (measured to the edge of the distribution) that is obtained is $R = 1.6\ A^{1/3} \times 10^{-13}$ cm. The average radius is $R = 1.0\ A^{1/3} \times 10^{-13}$ cm. This result is compared with other determinations of the nuclear radius.

NUCLEAR CROSS SECTIONS AND THE SIZE OF THE NUCLEUS

Warren Heckrotte

The recent experiment of Millburn[1] et al. on the total inelastic cross sections for 190-Mev deuterons on various target nuclei yields the following expression for this cross section as a function of A, the mass number of the target nuclei:

$$\sigma_D = (1.68\, A^{1/3} + 0.64)^2 \times 10^{-26}\ \text{cm}^2 . \qquad (1)$$

The principal contribution to this inelastic cross section is from the breakup of the deuteron.

If it is assumed that the interaction of either of the nucleons of the deuteron with the target nucleus always leads to the breakup of the deuteron*, then the cross section is given by

$$\sigma_D = \pi R^2 + \frac{\pi}{2} R_D R , \qquad (2)$$

where R is the nuclear radius and R_D is the mean distance of separation of the particles of the deuteron. The term $\pi R_D R/2$ comes from Serber's calculation of the stripping process[2]. Equation (2) can be written as

$$\sigma_D = \pi(R + R_D/4)^2 - \frac{\pi R_D^2}{16} .$$

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* This would imply that the deuteron is also broken up without the excitation of the target nucleus.

It is a good approximation to neglect the latter term; thus for the deuteron inelastic cross section one may write

$$\sigma_D = \pi(R + R_D/4)^2 . \qquad (3)$$

Comparing this with Eq. (1), the nuclear radius and R_D are given by

$$R = 1.68\, A^{1/3} \times 10^{-13} \text{ cm.},$$
$$R_D = 2.56 \times 10^{-13} \text{ cm.} \qquad (4)$$

This is a reasonable value for R_D, but the value for the nuclear radius is considerably larger than values usually mentioned. Also, this is the minimum value to be deduced from the data. If there is an effective transparency for deuterons, the extent of nuclear matter is even greater.

Although this value for the radius is large, it is not incompatible with the idea of a fringe region about the main bulk of the nuclear matter, where the nuclear density falls off from its interior value to zero. That is, the nucleus is not of a square-well configuration, but is in a sense smoothed out. This outer fringe, though of smaller density than the interior region, is capable of stripping deuterons.

To understand further the above experimental results and interpretation, the total nuclear cross sections for 90-Mev neutrons[3] have been interpreted in terms of a nonsquare-well configuration for the nucleus. The optical model of Fernbach, Serber, and Taylor[4] has been used. As in FST it is assumed that the incident neutron wave passes through the nucleus with negligible refraction. The change in phase of the incident wave as it passes through the nucleus is then readily calculated. The change in phase of that part of the wave which strikes the nucleus at an impact parameter y and travels the distance $2S = 2(R^2 - y^2)^{1/2}$ through the nucleus, is given by

$$\phi = \int_{-S}^{S} (k + k_1 + i\frac{K}{2})dS' \quad , \qquad (5)$$

where k is the wave number of the incident wave, $(k + k_1)$ the real part of the wave number inside the nucleus, and K the absorption coefficient. Then k_1 is given by

$$k_1 = k\left[(1 + V/E)^{1/2} - 1\right] \simeq \frac{1}{2}\frac{kV}{E} \quad , \qquad (6)$$

where V is the nuclear potential and E is the energy of the incident neutrons. In order to simplify the calculations the approximate value of k_1 is used.

It is necessary now to choose a radial dependence for V(r) and K(r), the potential and the absorption coefficient. These will be taken to be proportional to the nuclear-density distribution $\rho(r)$, which in turn will be chosen to be of the form

$$\rho(r) = C(1 - \frac{r^2}{R^2}) \quad , \qquad 0 \leq r \leq R \quad , \qquad (7)$$

$$C = \frac{A}{(\frac{2}{5})(\frac{4\pi R^3}{3})} \quad .$$

Other than the square well, this is the simplest distribution to handle and was chosen for this reason. V(r) and K(r) becomes then,

$$V(r) = V_o(1 - \frac{r^2}{R^2}) ,$$

$$K(r) = \sigma \rho(r) = K_o(1 - \frac{r^2}{R^2}) , \qquad (8)$$

$$K_o = \sigma C ,$$

where σ is the mean neutron-nucleon cross section in nuclear matter.

The change in phase ϕ is then readily evaluated from Eq. (5), which yields

$$\phi = 2kS + 2\frac{\alpha S^3}{R^2} + i\frac{\overline{K_o}}{R^2} S^3 , \qquad (9)$$

$$\alpha = \frac{1}{3}\frac{kV_o}{E}$$

$$\overline{K_o} = \frac{2}{3} K_o .$$

Here α and $\overline{K_o}$ represent, in a sense, average values of k_1 and K, respectively.

From this expression for ϕ, expressions for the diffraction and absorption cross sections can be written down in the same manner as in FST. These become

$$\sigma_d = 2\pi R^2 \int_0^1 \left| 1 - e^{-\overline{K_o}\, R\, \rho^3 + 2i\;\; R\, \rho^3} \right|^2 \rho \, d\rho \, ,$$

$$\sigma_a = 2\pi R^2 \int_0^1 \left(1 - e^{-\,2\overline{K_o}\, R\, \rho^3}\right) \rho \, d\rho \, . \qquad (10)$$

The total cross section is the sum of the two and is given by

$$\frac{\sigma_t}{\pi R^2} = 2 - 4\, I \, ,$$

$$I = \int_0^1 e^{-\overline{K_o}\, R\, \rho^3} \cos(2\alpha\, R\, \rho^3)\, \rho \, d\rho \, . \qquad (11)$$

Letting $\overline{K_o} R = x$ and $\frac{2\alpha}{K_o} = \beta$, the integral I can be expressed in the following power series form:

$$I = \sum_{n=0}^{\infty} \frac{(-)^n}{n!} \frac{x^n}{3n+2} (1 + \beta^2)^{n/2} \cos(n\theta) \, ,$$

$$\theta = \tan^{-1} \beta \, . \qquad (12)$$

For values of x and β larger than one, the evaluation of the series becomes rather tedious. However, the integral can be expressed as the solution of the following differential equation,

$$\frac{dI}{dx} = \frac{2}{3x}\left[\frac{1}{2}e^{-x}\cos x\beta - I\right],$$ (13)

$$I\ (x=0) = 1/2\ .$$

This equation may be easily solved on the differential analyzer at the Radiation Laboratory; solutions were found for values of $x > 1$. The results of the calculation are given in Fig. 1, where $\sigma_t/\pi R^2$ is plotted as a function of x for various values of β; $\sigma_a/\pi R^2$ is also shown as a function of x.

From these numerical results, and using the experimental cross sections[3], the variation of R with $A^{1/3}$ can be determined. This dependence is plotted in Fig. 2 for the values $\overline{K_o} = 3.2 \times 10^{12}\ cm^{-1}$ and $\beta = 2.5$. These values of the parameters yield an adequate straight-line fit to the experimental points. The slope of the line is 1.6, which yields

$$R = 1.6\ A^{1/3} \times 10^{-13}\ cm.$$ (14)

for the nuclear radius as a function of mass number.

If $\overline{K_o}$ and β are varied so as to keep the product $\overline{K_o}\,\beta$ constant, a considerable range of values for these parameters still yields satisfactory straight-line fits of essentially the same slope to the experimental points. As the product $\overline{K_o}\,\beta$ is proportional to V_o, the total cross sections determine only V_o and not the absorption coefficient[4]. The absorption coefficient is of course to be determined from the neutron-absorption cross sections. The values of the radius obtained from the nuclear-absorption cross sections for 84-Mev neutrons[5] and the above parameters are shown in Fig. 2 also. Because of the uncertainties in the absorption cross sections, the value of $\overline{K_o}$ can only be regarded as falling in the range between 3.6×10^{12} cm^{-1} and 2.7×10^{-12} cm^{-1}.

The angular distributions of the elastically scattered neutrons have also been calculated. It was found that this model yields essentially the same angular distributions as the square-well model[5]. For the lighter target nuclei the experimental angular distributions show greater peaking in the forward directions than is obtained from either model, although the agreement for the heavier target nuclei is reasonably good.

The values of V_o and σ obtained from β and $\overline{K_o}$ are

$$V_o = 52 \text{ Mev},$$

$$\sigma = 3.3 \times 10^{-26} \text{ cm}^2.$$

The value of V_o is considerably larger than the usual value of 30 Mev. The average value of the potential, however, as given by Eq. (9), is about 35 Mev. The mean value of σ that would be expected from the 125(90 + 35)-Mev nucleon-nucleon cross sections is about 4.5×10^{-26} cm^2. When this is multiplied by a factor of 2/3 to account for the exclusion principle[6], the agreement with the above result is satisfactory.

The value of the nuclear radius obtained from the neutron data is still smaller than the value obtained from the inelastic-deuteron cross section experiments. However, the model here adopted still suffers from the same defect as the square-well model; i.e., a cutoff at some distance R. The main point is that the inelastic-deuteron cross sections demand an extended nuclear-density distribution, and it appears that the 90-Mev neutron total and absorption cross sections can be fitted by an extended distribution. A more extensive analysis is needed using various well shapes that tail off. This program is, however, more suited to high-speed machine computation. This is also true from the point of view of accuracy, since the errors introduced by the neglect of refraction are probably not altogether negligible*. Such a program of computation is being undertaken by S. Fernbach at UCRL, Livermore.

Finally, a comparison between these results and other determinations of nuclear radii should be noted. The radius of the square well distribution used by FST to fit the 90-Mev neutron scattering data is $R = 1.37 A^{1/3} 10^{-13}$ cm. Though the extent of the parabolic well is greater than this, a comparison of the average and/or root-mean-square radius is more significant. This comparison is given in Table 1. These values of the first two moments for the two models agree rather closely, though it appears that a smoothing out of the square well causes a reduction of the average and r-m-s radii.

These values of the r-m-s radius can be compared with the results of the electron scattering and mesic-atom experiments, which effectively measure the r-m-s radius of the charge distribution[7]. The r-m-s radius obtained in these experiments is about $R = 0.9 A^{1/3} 10^{-13}$ cm, or about 15 percent less than the r-m-s radius obtained from the optical model. Now it may be that with the introduction of a well shape which extends to infinity, the optical model will yield an r-m-s radius of about this value and resolve the difference between these two types of measurements. On the other hand, Johnson and Teller[8] have recently suggested that such a difference in the radius of the proton

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* For example, a Gaussian shaped well may be considered in the same manner as was the parabolic well. However, the real potential obtained is such as to render the approximations invalid.

distribution and neutron distribution might exist. However, the existence of such an effect would be better resolved, not by the considerations of the optical model which admits of many variations, but by resort to experimental measurements which in some way would depend on the N/P ratio at the surface of the nucleus.

I wish to express my appreciation to Drs. Birnbaum, Crandall, Millburn, and Schecter for the many discussions pertaining to the experimental data on the scattering of particles from nuclei. I am also indebted to the various members of the computing group who assisted in the numerical computation. This work was performed under the auspices of the Atomic Energy Commission.

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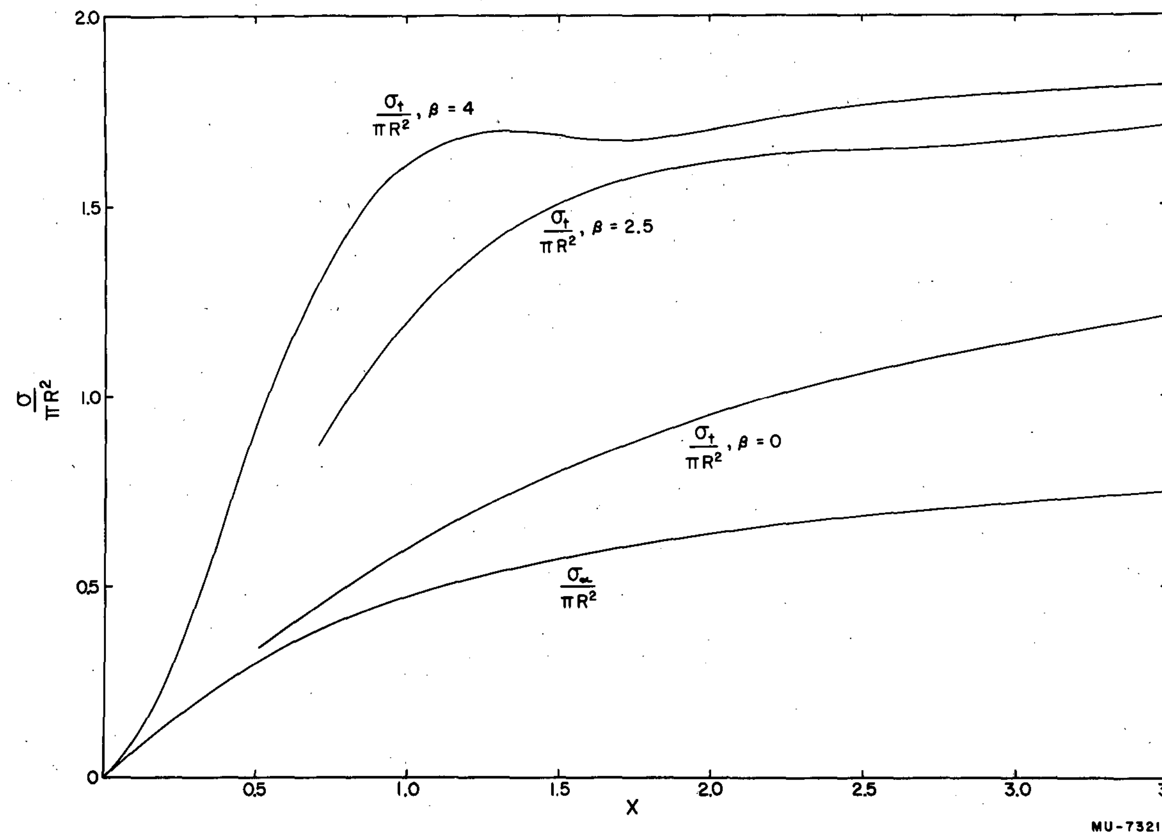
TABLE I

	Square-well distribution $R = 1.37\ A^{1/3}\ 10^{-13}$ cm	Parabolic distribution $R = 1.6\ A^{1/3}\ 10^{-13}$ cm
$\overline{R}$	$1.03\ A^{1/3}\ 10^{-13}$ cm	$1.0\ A^{1/3}\ 10^{-13}$ cm
$(\overline{R^2})^{\frac{1}{2}}$	$1.06\ A^{1/3}\ 10^{-13}$ cm	$1.05\ A^{1/3}\ 10^{-13}$ cm

FIGURE CAPTION

Figure 1: Plot of the total and absorption cross sections as a function of $x = \overline{K_o}R$. The total cross sections are plotted for several values of $\beta = 2kV_o/3E\,\overline{K_o}$.

Figure 2: Nuclear radii deduced from the total nuclear cross sections for 90-Mev neutrons (circular points) and from the absorption cross sections for 84-Mev neutrons (triangular points) as a function of the cube root of the mass number.



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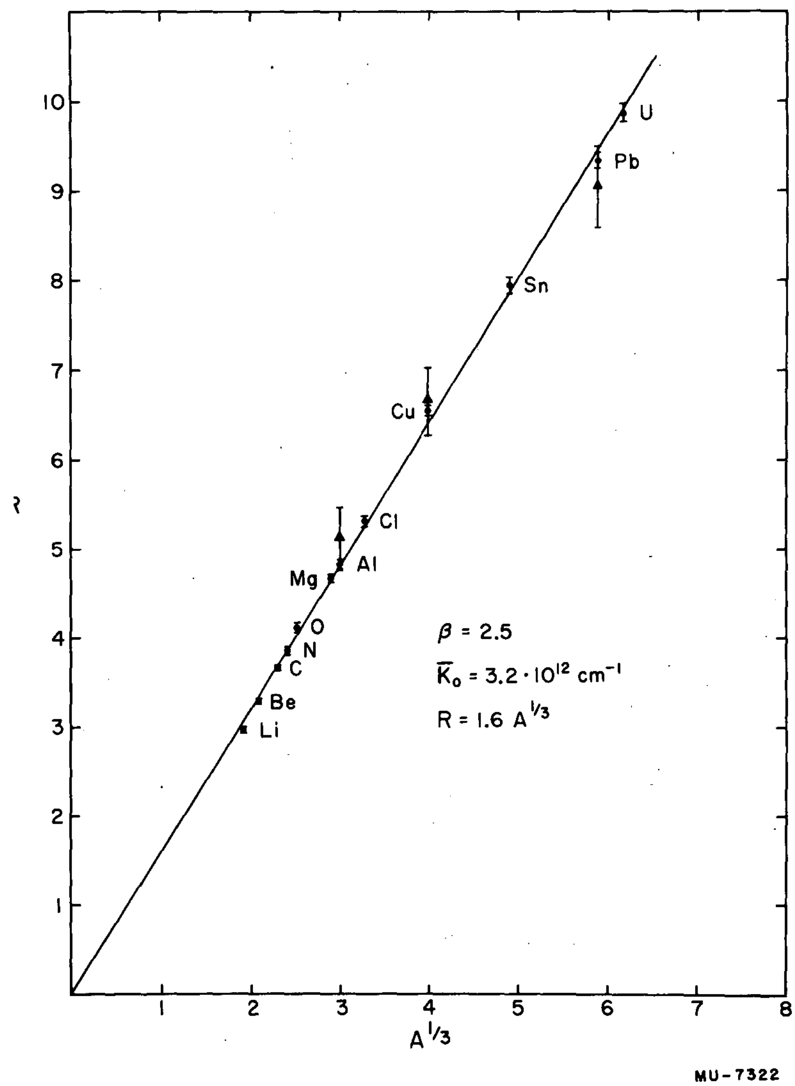

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