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Abstract

A detailed shell-model calculation for the lower excited levels and high-spin, alpha-emitting isomer of Po^{212} is reported. Our result shows that the main configuration of the isomeric state of Po^{212} is $[(h_{9/2})^2_8 (g_{9/2}\ i_{11/2})_{10}]$ and has spin J=18. The calculated spectrum also suggests the existence of a few low-lying levels which have not been observed experimentally, including a possible isomer with J=10. Alpha reduced widths and electromagnetic transition probabilities have been calculated and compared with those of the previous works, and with experiment. The results are generally satisfactory. The wave functions are strongly configuration admixed including non-negligible higher seniority components.

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†On leave of absence from Japan Atomic Energy Research Institute.

1. Introduction

The nucleus Po^{212} has two protons and two neutrons outside the doubly magic Pb^{208} core. Experimentally five excited levels are known, and three of them show long-range α-particle decays. Recently, Perlman et al.[1]) discovered a long-lived (45 sec) isomeric state at 2.93 MeV above the ground state.

Several calculations for Po^{212} have been reported earlier, but their results are more or less different from each other. A brief summary of each follows.

1.1 BAND, SLIV, AND KHARITONOV[2]) (hereafter referred to as BSK)

They tried to fit the lower levels and obtained fairly good results. They include a collective coordinate in their calculation, which describes the Pb^{208} core. Therefore their wave functions are inconvenient for the calculation of alpha-particle reduced widths γ_α^2.* They used the same forces for p-p, n-n, and p-n residual interactions, and their interaction could not reproduce the marked differences between Po^{210} and Pb^{210} spectra.

1.2 GLENDENNING[3])

This calculation was done primarily to show that an isomeric level of high spin could be accounted for by the shell model and to suggest what the spin of the observed isomer might be. No attempt was made to fit the lower levels. According to this calculation, the spin of the isomeric state is 18, and its main configuration† is $[(h_{9/2})_8^2\ (i_{11/2})_{10}^2]_{18}$. However, the single-particle levels which were used were taken from Mottelson and Nilsson's paper[4]). They are somewhat different from the experimental ones.

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*Their wave function gives a smaller alpha width for Po^{212} than for Po^{210}, contrary to the experiment.

†When referring to configurations of both protons and neutrons, we shall always refer first to the protons.

1.3 HARADA[5])

This is a simple calculation based on the pairing plus quadrupole-quadrupole Hamiltonian, and discussed mainly the effects of the p-n force on the α-decay rate.

1.4 AUERBACH AND TALMI[6])

They assumed pure configurations for Po^{210}, Pb^{210}, and Bi^{210} which allowed them to evaluate the matrix elements relevant to Po^{212} if one also assumes that no additional configurations are important there. However, the assumption of pure configurations for these heavy nuclei probably is violated strongly in nature, as it certainly is in calculation. According to their analysis, the spin of the isomeric state is 16 (the maximum spin for the assumed configuration), and the state appears at 2.68 MeV above the ground state. This is lower than the experimental value by 250 keV.

In both of the above 2nd and 4th calculations, the existence of the isomer is solely attributed to the effect of the p-n force because the binding energy of a pair of equivalent particles decreases with its total spin.

In order to obtain more definite information on Po^{212}, we have tried again a detailed shell-model calculation. We neglected the tensor force entirely in this calculation. The effects of the tensor force will be discussed briefly in sec. 3. Section 2 of this paper will discuss the method of calculation and the parameters which we used. In sec. 3 the results of this calculation will be given and compared with other theoretical calculations and with experimental results.

2. Calculation

2.1 PARAMETERS

Harmonic oscillator wave functions are used to represent the single-particle wave functions. The oscillator parameter ν is determined in the following way.

Blomqvist and Wahlborn[7]) have calculated the eigenfunctions in a Woods-Saxon potential for the single-particle levels around Pb^{208}. To determine ν, we postulated the following condition:

$$\int r^2 \psi^2_{H.O.} \, d\vec{r} = \int r^2 \psi^2_{B.W.} \, d\vec{r} \quad , \tag{1}$$

where $\psi_{H.O.}$ and $\psi_{B.W.}$ are respectively the harmonic and Blomqvist-Wahlborn wave functions. The calculated ν-values from this relation are given in table 1. We fixed our ν-value as 0.18 which is an average value over the seven ν values.

Table 1

Harmonic oscillator parameters $\nu = m\omega/\hbar$

orbits	ν	orbits	ν
$1h_{9/2}$	0.187	$2g_{9/2}$	0.170
$2f_{7/2}$	0.190	$1i_{11/2}$	0.188
$1i_{13/2}$	0.180	$1j_{15/2}$	0.180
		$3d_{5/2}$	0.135

The residual p-p, n-n, and p-n interactions are so determined as to reproduce the experimental Po^{210}, Pb^{210}, and Bi^{210} spectra. Gaussian-shaped forces,

$$V = -V_o \exp(-r/\beta)^2 \tag{2}$$

for each exchange type are used throughout this calculation. We changed the force parameters V_o and β entirely arbitrarily without making allowances for the properties of the free nucleon-nucleon force because of the expected strong modification of the latter. We even allow the p-p and n-n forces to be different since the residual shell-model force can depend, among other factors, on the particular shells involved.

The experimental single-particle levels relevant to Po^{212} are shown in fig. 1. We included the neutron levels below the dashed line in the figure. We will discuss our results for Po^{210}, Pb^{210}, and Bi^{210} nuclei successively.

2.2 Po^{210}

The experimental pairing energy of two protons in Po^{210} is about 1.24 MeV. If we plot the Po^{210} spectrum with the ground state at -1.24 MeV, the energy of the 6+ state is at about +240 keV. Even if we take into account the Coulomb energy, the energy of this state due to the attractive force is almost zero. This fact suggests a short-range for the p-p residual interaction since the interaction energy is in this case very small for large J. On the other hand, the spacing between the 2+ and 4+ levels is more characteristic of a long-range force. Therefore, it seems to us that the energy spectrum of Po^{210} might not be reproducible by a central interaction only. The parameters which best fit the relative spacing are given in table 2.

Table 2

p-p force

	V_o(MeV)	β(fermi)
singlet even	-64.54	1.40
triplet odd*	12.84	2.15

*We assumed triplet odd potential is effective only for J=0 state. This is a simulative form of the tensor force.

Figure 2 compares our result with the experimental spectrum and to the calculation of Kim and Rasmussen[8], referred to as K-R. Newby and Konopinski[9] have calculated the Coulomb energies for the various spin states of the $(h_{9/2})^2$ configuration. We adjusted their values by using the relation,

$$R \propto (\nu)^{-1/2}, \qquad (3)$$

and added them in both of the calculated results. The pairing energy, after the Coulomb correction is 157 keV greater than experiment. The calculated spectrum has been renormalized by this amount in fig. 2, and the spectrum of K-R by 83 keV.

Our eigenfunctions are very similar to Kim and Rasmussen's, even with greatly different interactions.

2.3 Pb^{210}

Recently, Redlich[10] has made a detailed analysis of this nucleus. We chose very similar force parameters to his. The best fit parameters and results are given in table 3 and fig. 2, respectively.

Table 3

n-n force

	V_o(MeV)	β(fermi)
singlet even	-36.07	1.90

We took into account only four single-neutron levels in our calculation, but Redlich includes seven levels. Nevertheless, we obtained very similar eigenfunctions except, of course, for the very high states. For our purpose we need only the lower ones.

2.4 Bi^{210}

Since we neglected the tensor force, we adjusted the p-n force parameters so as to reproduce the experimental interaction energy of the 9- state. The reason for this choice is that the 1- and 0- inversion cannot be reproduced by a purely central force[8,9]. The best fit parameters and results are given in table 4 and fig. 3, respectively. This force would lead to the Nordheim coupling rules.

Table 4

p-n force

	V_o(MeV)	β(fermi)
singlet even	-43.07	1.58
triplet even	-96.05	1.30

Except for the 1- and 0- inversion, our calculation could reproduce the correct level ordering. Kim and Rasmussen suggested spin and parity assignments for many other levels below 2.6 MeV in Bi^{210}. Figure 4 compares the theoretical results with experiment for those levels. Both calculations give very similar results except for the 10- state.

2.5 METHOD OF CALCULATION

From the above calculations on Po^{210}, Pb^{210}, and Bi^{210} we could fix the purely phenomenological p-p, n-n, and p-n interactions. We will use them for the calculation of Po^{212}. Since the complete solution to this four-body problem would involve the calculation and diagonalization of enormous matrices, some means of truncating the problem has to be found. One such method, particularly suitable for high-spin states, was described earlier[3]. For the lower-spin states we employ the following method: The products of the eigenfunctions of the two-

proton (Po^{210}) and two-neutron (Pb^{210}) systems, found in the preliminary calculations mentioned above, are used as the basic representation for Po^{212}. Since the like-nucleon interaction is already diagonal in this representation, only the lower levels are retained for the second step which consists in calculating the matrix elements of the p-n interactions between these product states. The product states are

$$\psi_\kappa^J = \phi_\mu^{J_p}(Po^{210}) \times \phi_\nu^{J_n}(Pb^{210}) \qquad (4)$$

where $\phi_\mu^{J_p}(Po^{210})$ denotes the μth level of spin J_p in Po^{210}, enumerated in order of increasing energy above the ground state. Here κ enumerates the product states and J, J_n and J_p must satisfy the vector addition rule.

The two-particle states for protons and neutrons have the respective forms

$$\phi_\mu^{J_p} = \sum_i \alpha_{J_p}^{i}(\mu)|i\rangle, \quad \phi_\nu^{J_n} = \sum_k \beta_{J_n}^{k}(\nu)|k\rangle \qquad (5)$$

where i denotes a two-particle proton configuration made up from the configurations shown in fig. 1, and k has a similar meaning for neutrons. In our case for positive parity, the first function has in general 4 terms and the second 7. The product function eq (4) therefore has in general 28 terms, and the matrix element of the p-n interactions has 28 × 28 terms:

$$\langle J_p(\mu),\ J_n(\nu);\ J|V_{np}|J_p'(\mu'),\ J_n'(\nu');\ J\rangle$$

$$= \sum_{i,k} \sum_{i',k'} \alpha_{J_p}^{i}(\mu)\ \beta_{J_n}^{k}(\nu)\ \alpha_{J_p'}^{i'}(\mu')\ \beta_{J_n'}^{k'}(\nu') \qquad (6)$$

$$\langle i\ J_p,\ k\ J_n;J|V_{np}|i'J_p',\ k'J_n';J\rangle\ .$$

In fig. 5 we show the zero-order energy spectra for several spins of Po^{212}. We include in our diagonalization all levels within at least 2 MeV of the lowest level, and checked the truncation error by including some additional configurations.

3. Results

3.1 ENERGY LEVELS AND WAVE FUNCTIONS OF Po^{212}

Figure 6 compares our calculated spectrum with the experimental one for the low excitation region. Dotted lines are used in order to indicate correspondences between the calculated and experimental levels. They are inferred from the alpha decay rate calculations in sec. 3.2. We see the general agreement is not as good as in the two-particle systems.

In the same energy region covered by fig. 6 we also find higher spin states, as noted in the figure captions. These have not yet been observed, but this is not surprising since the levels of Po^{212} are fed by the β-decay of Bi^{212} (1-).

Our calculated mutual binding energy of the last four nucleons in the ground state becomes -3.875 MeV after the same normalization as is done in Po^{210}. From the mass data, the experimental binding energy of four particles outside the Pb^{208} core can be calculated from

$$BE(Po^{212}) + 3\ BE(Pb^{208}) - 2\ BE(Bi^{209}) - 2\ BE(Pb^{209}), \quad (7)$$

and the result is -3.955 MeV. The agreement seems very good. If we look back to fig. 3, however, we can see that this agreement is accidental; our matrix element $\langle hg,0-|V_{np}|hg,0-\rangle$ is much larger than experiment indicates. Although

the apparent discrepancy is only 80 keV, it actually could be larger if the above matrix element were closer to the experimental value. It should be remarked, however, that the quoted experimental energy has a possible error of ±200 keV.

The calculated spectrum of the lowest level for each even J is given in fig. 7. For J=16 we also show the second state. Our calculation shows the spin of the isomeric state is 18. But it appears lower by 670 keV than the energy of the observed isomer. The second J=16 level has $[(h_{9/2})^2_8 (g_{9/2})^2_8]_{16}$ as the main configuration, and it appears 2.696 MeV above the ground state. This is the level that Auerbach and Talmi[6]) identified as the isomer. However, as fig. 7 shows, we do not confirm this assignment.

According to Auerbach and Talmi[6]), the second "energy gap" which comes from the vanishing of the pairing forces in both of the proton and neutron pairs appears between J=8 and J=10 states. This is a direct consequence of the fact that they considered only the $h_{9/2}$ and $g_{9/2}$ levels. In our case it appears between J=10 and J=12 states, and our result suggests that a new isomer, J=10, may exist around 1.2 MeV.

The calculated wave functions for typical states are listed in the appendix. All functions are characterized by a large amount of configuration mixing. Even the very small components are important for the transition rates, as we shall show in the next section. Especially we notice quite a large admixture of the higher seniority components in the ground state. We took into account more configurations and used a different residual interaction than BSK. Nevertheless, the amounts of the mixture of the higher seniority components are very similar to BSK, in regard to the 0_1 and 2_1 wave functions. However, as regards the higher roots, the differences are more pronounced. That is, their 0_2 and 2_3 levels correspond to our 0_3 and 2_5 levels, respectively. This identification

was established by comparing the wave functions and the numerical results of alpha reduced widths, which will be discussed in sec. 3.2 Our calculation suggests the existence of one more excited 0+ level and two more excited 2+ levels below 1.8 MeV.

From the tables in the appendix we can see that the main configurations of the high-spin states are $[(h_{9/2})^2\ (g_{9/2}\ i_{11/2})]$ and $[(h_{9/2}\ f_{7/2})\ (g_{9/2}\ i_{11/2}]$; neither $[(h_{9/2})^2\ (i_{11/2})^2]$ nor $[(h_{9/2})^2\ (g_{9/2})^2]$ are important. Figure 8 illustrates the changes of the p-n force diagonal matrix elements versus J_{np}. As is done frequently, we can classify two-particle systems into parallel pairs ($j_1=\ell_1\pm 1/2$, $j_2=\ell_2\pm 1/2$) and anti-parallel pairs ($j_1=\ell_1\pm 1/2$, $j_2=\ell_2\mp 1/2$). For both kinds of pairs the p-n force matrix elements of the maximum J are much larger than for intermediate J_{np}. The spin dependence of the matrix elements for like-nucleon pairs is shown in fig. 9. The binding energy of the parallel pair decreases with its spin, but the binding energy of the anti-parallel pair increases with spin. From the above graphical discussion, we can understand our wave function of the isomeric state, and the spectrum in fig. 7. The behavior shown in figs.8 and 9 in turn can be understood in detail in terms of the forces (tables 2, 3, and 4) and pictorial representations of the spins.

Conceivable reasons for the discrepancies in energy shown in fig. 6 for some of the low-spin states and for the ground-state isomer spacing mentioned above could include the following: 1) Truncation error. This is not expected to be serious. For the spin zero states, we at first included the lowest 8 zero-order configurations of fig. 5 and the ground state moved downward by 1,300 keV. We added 20 additional ones, and it moved down by an additional 70 keV. We cannot expect much further movement. 2) Residual interaction. Figure 4 shows our p-n level $(h_{9/2}\ i_{11/2})_{10}$, appearing at a much lower energy than the observed level that was tentatively assigned this spin by K-R[8]). As stated,

the main configuration of this isomer is $(h_{9/2}^2, g_{9/2}\, i_{11/2})$. If the 10- spin assignment is correct, the above fact ought to be responsible for our failure to obtain the correct energy of the isomer. In this situation some other p-n force would have to be used, perhaps one with repulsion in odd states which would tend to move states with $J \geq 12$ to higher energy.

We have not included a tensor component in our residual interactions, although according to a suggestion by Rasmussen, we introduced a repulsive triplet-odd potential for pairs having J=0 to simulate one of the effects of a tensor force.

According to the detailed analysis of Bi^{210} which was performed by Kim and Rasmussen[8]), the tensor-force affects the spin-1 state in an opposite sense to the spin-0 state, and it has a rather small effect on the states of spin-2 and higher. To see the effect, if any, of the tensor force on the high spin states let us examine the proton-neutron force matrix element between non-anti-symmetrized four-particle states, which is given by

$$\langle (j_1 j_2) J_p, (j_3 j_4) J_n; J | V_{13} | (j_1' j_2') J_p', (j_3' j_4') J_n'; J \rangle$$

$$= \delta_{j_2 j_2'} \delta_{j_4 j_4'} \sum_{I,K} [I][K] ([J_p][J_n][J_p'][J_n'])^{1/2} \qquad (8)$$

$$\begin{Bmatrix} j_1 & j_3 & I \\ j_2 & j_4 & K \\ J_p & J_n & J \end{Bmatrix} \begin{Bmatrix} j_1' & j_3' & I \\ j_2' & j_4' & K \\ J_p' & J_n' & J \end{Bmatrix} \langle (j_1 j_3) I | V_{13} | (j_1' j_3') I \rangle .$$

In our case the maximum value of K is 14, belonging to $(i_{13/3}\, j_{15/2})$ configuration. If $J \geq 16$, then $I \geq 2$. Therefore, we can infer that the tensor force will not affect the isomeric state.

In order to see the effects of the tensor force on the lower-spin states, we calculated a factor (see eq. (7)),

$$T(J) = [K]([J_p][J_n][J'_p][J'_n])^{1/2} \begin{Bmatrix} h & g & 0 \\ h & g & K \\ J_p & J_n & J \end{Bmatrix} \begin{Bmatrix} h & g & 0 \\ h & g & K \\ J'_p & J'_n & J \end{Bmatrix} \quad (9)$$

for several J. In table 5 we give T(J=0) for various J_p, J_n, J'_p, J'_n combinations.

Table 5

T(J=0)

$(J'_pJ'_n)$ \ (J_pJ_n)	(0,0)	(2,2)	(4,4)	(6,6)	(8,8)
(0,0)	0.0100				
(2,2)	0.0224	0.0500			
(4,4)	0.0300	0.0671	0.0900		
(6,6)	0.0361	0.0806	0.1082	0.1300	
(8,8)	0.0412	0.0922	0.1237	0.1487	0.1700

A similar trend was seen in other T(J). We can infer from the above table that the tensor force will affect more strongly the higher (J_p,J_n) components than the lower components in the lower-spin state of Po^{212}.

3.2 ALPHA REDUCED WIDTHS

Rasmussen[11] has calculated alpha reduced widths γ_α^2 of Po^{212} using the BSK wave functions. He has used a delta function approximation for the size of the alpha particle, and used the radius R_o=8 fermi. For the purpose of comparison, we have calculated γ_α^2 based on the same approximation, but used R_o=9.5 fermi.*

*In calculating γ^2 we use the Blomqvist-Wahlborn radial functions which are needed at R_o, to avoid the incorrect asymptotic behavior of the harmonic oscillator functions.

The calculated γ_α^2 for our ground state is 29.8 times larger than for the pure configuration $[(h_{9/2})_0^2\ (g_{9/2})_0^2]_0$, showing a remarkable enhancement due to the correlations introduced by the residual interaction. Perhaps even more remarkable is the fact that fully one half of this enhancement is due to configurations each having less than one percent probability. According to Rasmussen's calculation, the BSK ground state wave function gives only 2.6 times larger γ_α^2 than the pure state. In table 6 we give the reduced hindrance factors of the various excited states, i.e., the ratios of the γ_α^2 of the ground state to the γ_α^2 of the relevant states, together with Rasmussen's. This relative quantity is somewhat similar in the two calculations.

Comparing the γ_α^2 for the $|Po^{212},0_1\rangle \times |Pb^{210},0_1\rangle$ configuration, to our ground state γ_α^2, we can estimate the effect of p-n force on the ground state γ_α^2. We found the p-n force increases the ground state γ_α^2 by about 15%, this factor in refs. 5 and 11 is 24% and 20%, respectively. Again the agreement is qualitatively good. The like-nucleon and p-n force if acting alone each induce desired correlations for alpha decay, but acting together each tends to undo somewhat the others work[5]).

Many of the components in the wave functions shown in the appendix have probability less than 1%. Nonetheless they are important as far as transition rates are concerned, because in each of the lowest energy states of every spin, all configurations contribute constructively to the alpha decay.* Table 7 summarizes the situation for the transition between ground states. There the reduced widths relative to that for the pure configuration $(h^2)_0(g^2)_0$, is shown (a) for the complete 0_1 wave function in the appendix; (b) for the wave function that obtains when the p-n interaction (which introduces the higher seniority

*The constructive interference means that our residual interaction would lead to an alpha particle structure with properties similar to those assumed for it (i.e., it has a strong singlet-even part). If the residual interaction in heavy nuclei were assumed to be quite different, in exchange character, than the free two-nucleon force, the interference for alpha decay could be destructive.

components) is absent; (c) when higher seniority components in the 0_1 wave function having amplitudes less than 0.1 are omitted; and (d) when all components having amplitudes less than 0.1 are omitted.

Table 6

Calculated alpha reduced hindrance factor

State	Present	(BSK) Rasmussen
0_1	1.	1.
0_2	17.4	
0_3	8.6	6.3
2_1	2.8	4.3
2_2	68.5	
2_3	33.7	610.
2_4	4950.	
2_5	2.6	6.6
4_1	6.8	
4_2	5360.	
4_3	164.	
10_1	20.2	
16_1	119.	
18_1	162.	

The experimental ratio of the ground state γ_α^2 of Po^{210} to Po^{212} is about 1/10. We have calculated the ground state γ_α^2 of Po^{210} by using True and Ford's[12]) wave function for Pb^{206}. The calculated ratio is 1/5.1, which is smaller than the experiment by factor 2. A similar disagreement exists in refs. 13 and 14. In view of the importance of small configurations revealed in table 7 we estimated

the effect of adding the three higher neutron configurations shown in fig. 1 by including the admixing induced by the like-nucleon interaction, but neglecting the p-n interaction. The result is that the ground state transition is enhanced by an additional factor of 1.65. This result is shown in row (e) of table 7. The p-n force would further enhance this by an estimateed 15% by comparison of columns (a) and (b). With the addition of these configurations the calculated ratio for the Po^{210}/Po^{212} ground state alpha decay is 1/8.5, in very satisfactory agreement with experiment.

Table 7

Reduced width for ground state alpha decay of Po^{212} in units of the reduced width of the dominant configuration $(h^2)_0(g^2)_0$. Results for the complete wave function are compared with various approximations to bring out the role of several features

Description	$\gamma_\alpha^2/\gamma_\alpha^2(h^2g^2)$
(a) complete wave function	30
(b) p-n force absent	26
(c) higher seniority amplitudes < 1% neglected	22
(d) all amplitudes < 1% neglected	15
(e) p-n force absent but higher neutron configurations s, g, d added	43

Zeh[14]) has discussed γ_α^2 of the isomeric state of Po^{212} assuming three different configurations for it. He has also pointed out that the WKB approximation for the calculation of penetrability is unsatisfactory near the inner turning point. We calculated the penetrability numerically making use of a program which was developed by Rawitscher[15]). The experimental ratio of the decay rates of the isomeric state to the ground state is 0.65×10^{-8}. The calculated ratio (including the estimated effect of the higher neutron configurations

as mentioned above) is 2.9×10^{-7} for R_o=9.5 fermi and 1.2×10^{-7} for R_o=9.0 fermi. These are larger than the experiment by a factor ~45 and ~18, respectively. This unfortunate dependence on R_o illustrates the shortcomings of current alpha decay theory. We conclude that our wave functions reproduce the experimentally observed ratio for these transitions within the uncertainty of alpha decay theory.

As we noted in sec. 3.1, there may be a new isomer in Po^{212} occurring at 1-2 MeV above the ground. Its reduced hindrance factor is shown in table 6. Since the penetration factor is very sensitive to the alpha-decay energy, we have considered two positions for the isomer, one at 1.2 MeV as calculated. It is also calculated to lie 1.07 MeV below the J=18 isomer. Since the latter position is in error, we therefore chose another alternate position, 1.07 MeV below the observed isomer. The resulting lifetimes are 4×10^{-5} and 3×10^{-6} sec respectively (normalized to the experimental ground state lifetime).* Such a short lifetime, if the state is indeed isomeric would account for its being undetected in the experiment of ref. 1 since there was a time lapse between the time of the bombardment that produced Po^{212} and the time at which the counting began.

3.3 γ-TRANSITION PROBABILITIES

For the purpose of further testing the accuracy of our wave functions, we have calculated the reduced transition probabilities $B(\lambda)$ for several transitions. Here we use the same definition for $B(\lambda)$ as in BSK; that is $B(\lambda)$ is connected with the total transition probability $T(\lambda)$ by the relation,

*Here again these answers depend upon the choice of nuclear radius, so that there is an additional uncertainty of perhaps an order of magnitude.

$$T(\lambda;\ I \rightarrow I') = B(\lambda; I \rightarrow I')(\Delta E)^{2\lambda+1}. \qquad (10)$$

As for the effective charge which is necessary for the calculation of B(E2), we took 2.15 e and 1.15 e for a proton and for a neutron, respectively. The latter is the same as used by True and Ford in their Pb^{206} calculation[12]). We used free space gyromagnetic ratios, g_ℓ and g_s for the calculation of B(M1).

Table 8 lists our calculated $B(\lambda)$, together with BSK's. The BSK wave functions have components with one or two phonons which describe the excitation of Pb^{208} core, and they do not use the effective charge. In spite of such an apparent difference between BSK and our wave functions, both results are almost the same except for several cases. As mentioned in paragraph 3.1, our 0_3 and 2_5 states correspond to BSK's 0_2 and 2_3 states, respectively. We put BSK's $B(\lambda)$ into suitable columns taking into account this correspondence. The two calculations differ most for the following transitions: $B(E2;\ 2_2 \rightarrow 2_1)$, $B(E2;\ 2_2 \rightarrow 0_1)$, $B(E2;\ 1_1 \rightarrow 2_1)$, $B(M1;\ 1_1 \rightarrow 0_1)$, and $B(M1;\ 2_5 \rightarrow 2_1)$. Both calculations show $B(M1;\ 2_2 \rightarrow 2_1) > B(E2;\ 2_2 \rightarrow 2_1)$; this result means Po^{212} cannot be described by a simple vibrational model.

Comparison with experiment can be made in the several cases where the γ-branching ratio of a given state is known. The experimental and theoretical branching ratios are shown in table 9. Our worst disagreement occurs in case b. We calculated a very weak M1 transition for $1_1 \rightarrow 0_1$ which makes the indicated branching ratio large. This weak transition results from an almost complete cancellation of the contributions from the various configurations. The inclusion of the $g_{7/2}$ level would improve the situation. Redlich's calculation shows a 3% admixture which is significant for this result. A further remark concerns the type of radiation predicted for case b. The 1_1-2_1 transition, according to experiment is M1. Our results (table 8) agree with this. On the other hand

Table 8

Reduced γ-transition probabilities $B(\lambda)$

Transition	$B(E2)\times10^{-11}$ sec^{-1} MeV^{-5}		$B(M1)\times10^{-11}$ sec^{-1} MeV^{-3}	
	Present	BSK	Present	BSK
$2_1 \rightarrow 0_1$	8.63	4.30		
$0_2 \rightarrow 2_1$	1.41			
$4_1 \rightarrow 2_1$	9.53	7.20		
$2_2 \rightarrow 4_1$	0.23			
$2_2 \rightarrow 0_2$	2.99			
$2_2 \rightarrow 2_1$	0.86	0.04	52.4	61.0
$2_2 \rightarrow 0_1$	1.24	0.05		
$1_1 \rightarrow 2_2$	0.0047		0.10	
$1_1 \rightarrow 0_2$			0.069	
$1_1 \rightarrow 2_1$	0.026	1.60	0.45	0.38
$1_1 \rightarrow 0_1$			0.0064	0.69
$0_3 \rightarrow 2_1$	2.66	2.70		
$2_5 \rightarrow 2_2$	1.07	0.44	1.54	4.9
$2_5 \rightarrow 4_1$	0.13	0.38		
$2_5 \rightarrow 2_1$	6.68	2.20	0.33	0.01
$2_5 \rightarrow 0_1$	0.41	0.20		

BSK obtain an E2 multipolarity for this transition. Therefore we do not show an entry in table 9.

Table 9

γ-branching ratios for some transitions in Po^{212}

Case	Transition	Experiment	Theory	
			Present	BSK
a	$2_2\text{-}2_1/2_2\text{-}0_1$	3.43	2.63	75.2
b	$1_1\text{-}2_1/1_1\text{-}0_1$	0.255	12.3	
c	$2_5\text{-}2_1/2_5\text{-}0_1$	7.82	1.26	0.817

The agreement is generally satisfactory considering how sensitive electromagnetic transition rates are to details of the wave functions. We hope more checks will be made possible by the addition of new and accurate experimental information.

4. Summary

Determining the residual interaction from the level scheme of adjacent (two-particle) nuclei, we have been able to explain the general properties of the low-lying levels of Po^{212}. Prominent features include the high-spin isomer, its alpha-decay rate compared to the ground state, and the ratio of the ground state alpha decays of Po^{212} to Po^{210}. The position of the isomer was reproduced with a 20% error. The alpha decay rates are in excellent agreement with observation. The γ-ray branching ratios for the several levels for which they have been satisfactorily measured are fairly good, and the correct type of radiation is predicted.

We predict levels in the low-energy spectrum additional to those already observed, including another 0+ and two 2+ levels and several of higher spin, one of which, a 10+ may be isomeric. The latter would not be populated sufficiently to be observed in decay-scheme studies but it can be produced in nuclear reactions. Since alpha radiation from it does not appear in the spectrum of ref. 1, either it is not populated as highly as the J=18 isomer under the conditions of the preparation (Bi^{209} + 40 MeV α), has a short half-life not greater than a few seconds, or is not isomeric.

To further check our understanding of the nucleus, supplementary decay-scheme studies aimed at finding the additional low-spin states predicted, and determining accurately the γ-ray branching ratios of these and the observed levels would be very useful.

More of theoretical interest, our detailed calculations for this nucleus show a not insignificant admixture of higher seniority components and multipoles higher than 2 (omitted in many random-phase calculations). These higher components play a role in transition rates, especially in alpha decay because of their constructive interference in the lowest level of each spin, and their large number.

Acknowledgments

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Appendix

In the tables of this appendix we show the wave functions of some of the levels of Po^{212}. They are identified by their spin and sequence in the spin by the notation J_μ. Thus 0_2 refers to the second J=0 state.

In all shell-model calculations certain (conscious or unconscious) choices of phase have to be made. We have used the following conventions:

1) Condon-Shortley spherical harmonics

2) Spin-orbit functions coupled in the order $(s\ell)j$.

3) Radial functions have positive slope at the origin.

The phases of our wave functions therefore correspond to these conventions.

Some relevant formulae for four particle matrix elements are given in ref. 3 and others needed are very similar.

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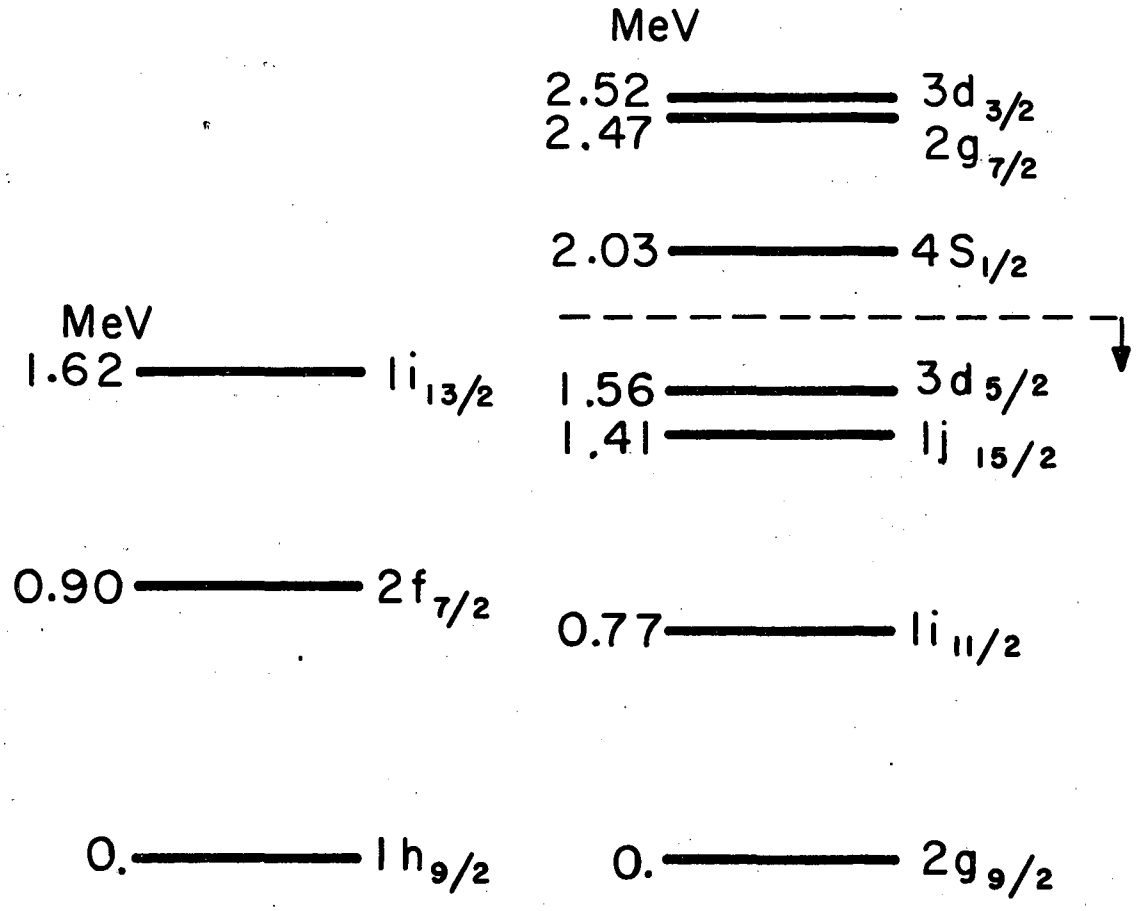

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Fig. 1. The experimental single-particle levels of Bi^{209} and Pb^{209} taken from R. W. Hoff and J. M. Hollander, Phys. Rev. 109 (1958) 447, and P. Mukherjee and B. L. Cohen, Phys. Rev. 127 (1962) 1284.

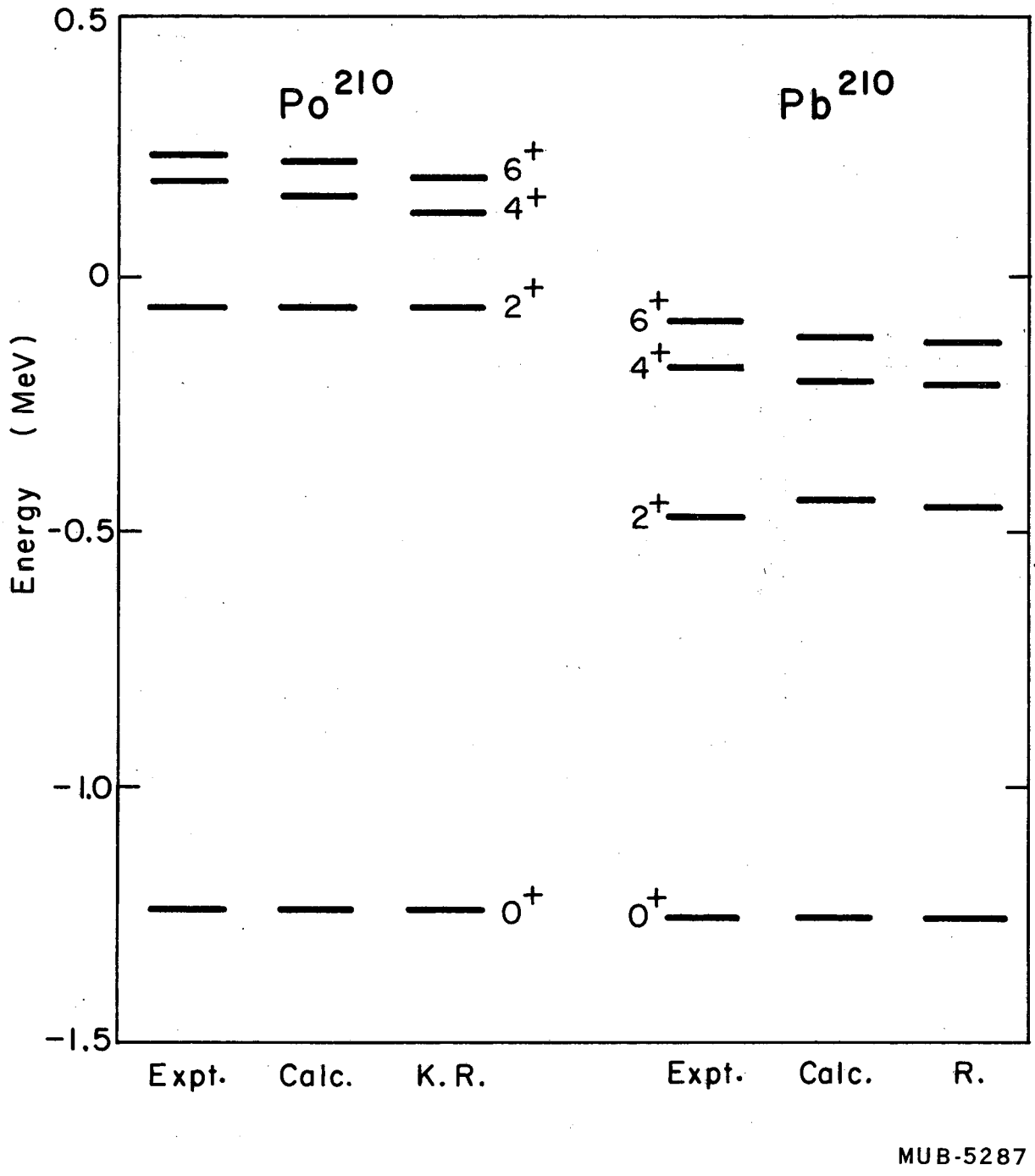

Fig. 2. Spectra of Po^{210} and Pb^{210} calculated in the present work and by Kim and Rasmussen (Po^{210}) and by Redlich (Pb^{210}) are compared with experiment.

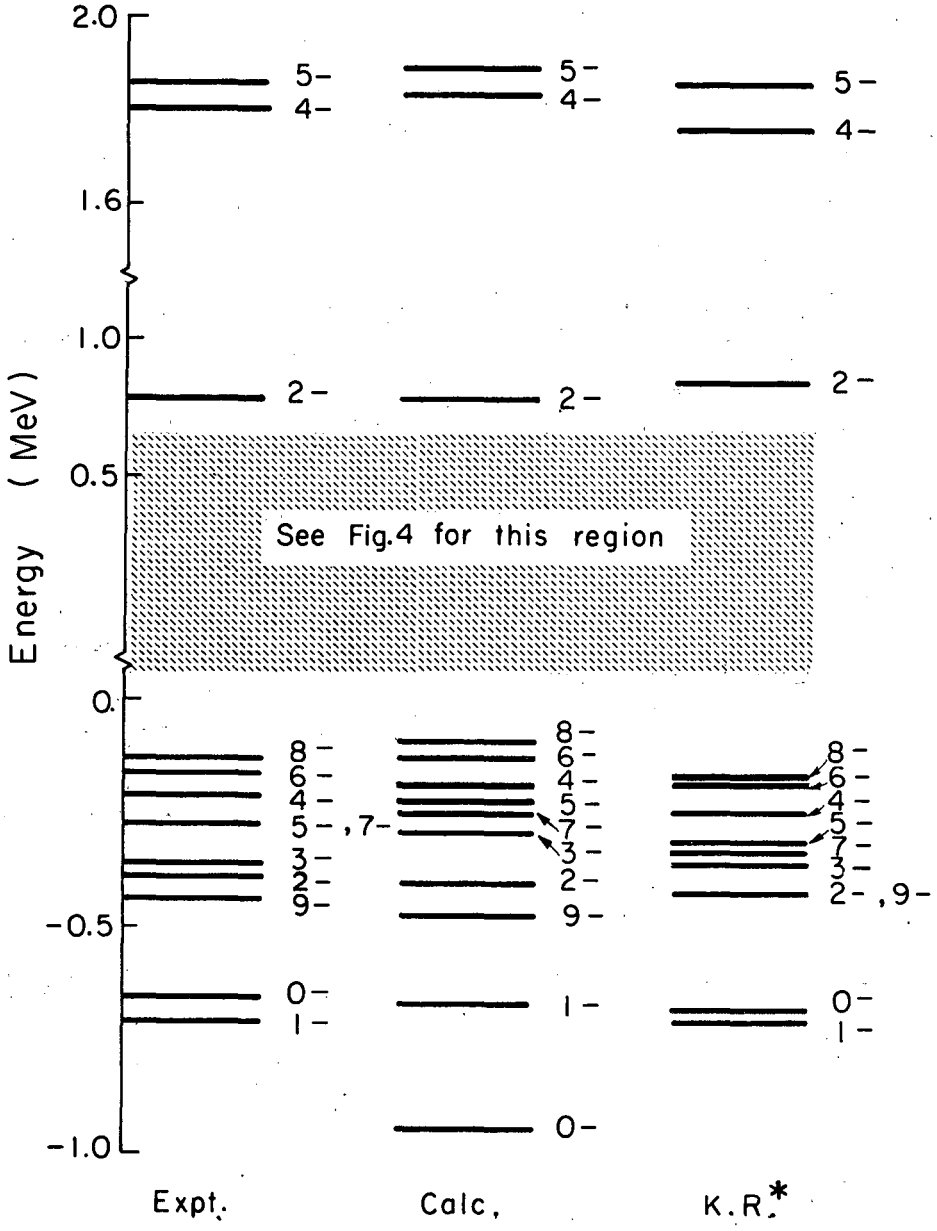

Fig. 3. The spectra for Bi^{210} calculated in the present work and by Kim and Rasmussen are compared with experiment. The results of K-R have been normalized downward.

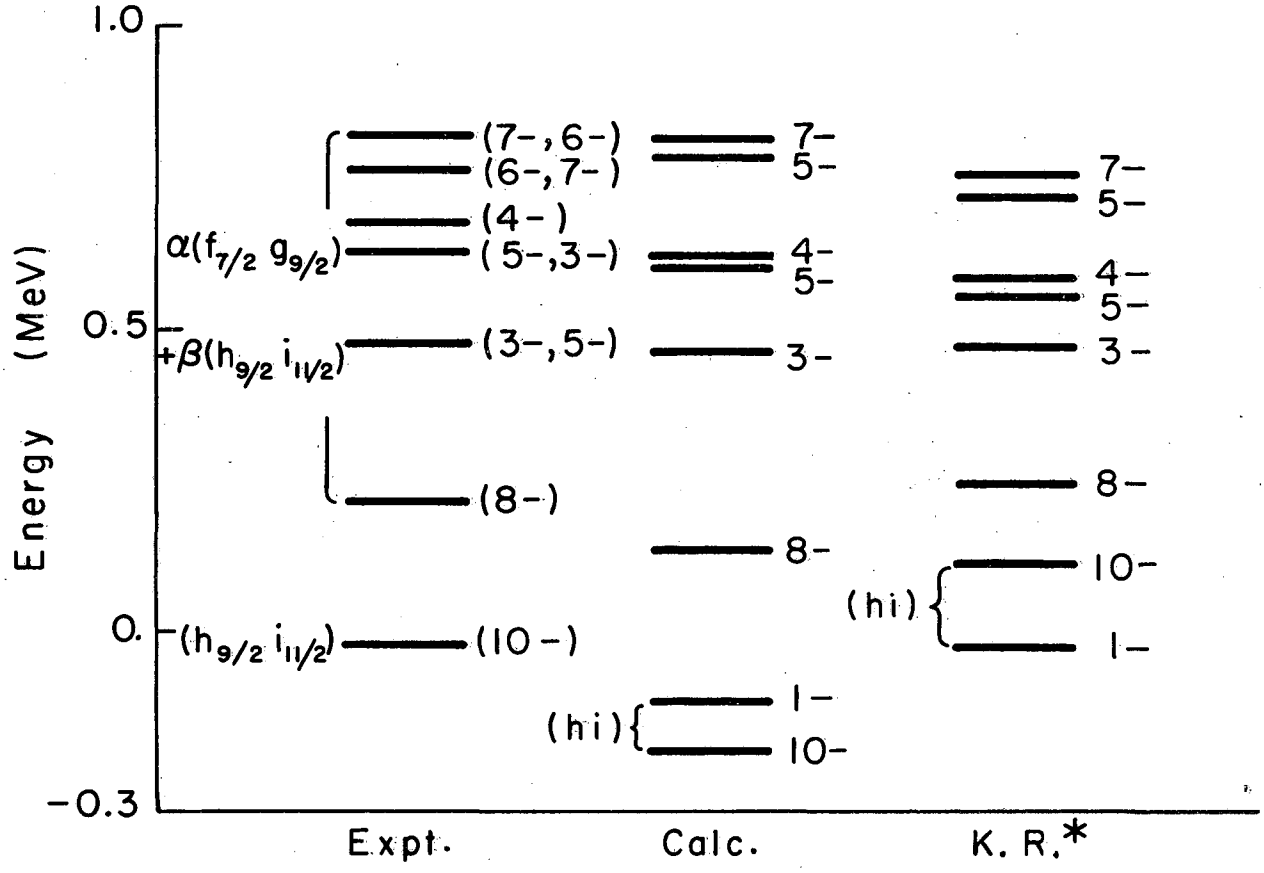

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Fig. 4. The spectrum in Bi^{210} from the shaded region of Fig. 3. Spins and parities for levels shown in parentheses were suggested by Kim and Rasmussen on the basis of their calculation.

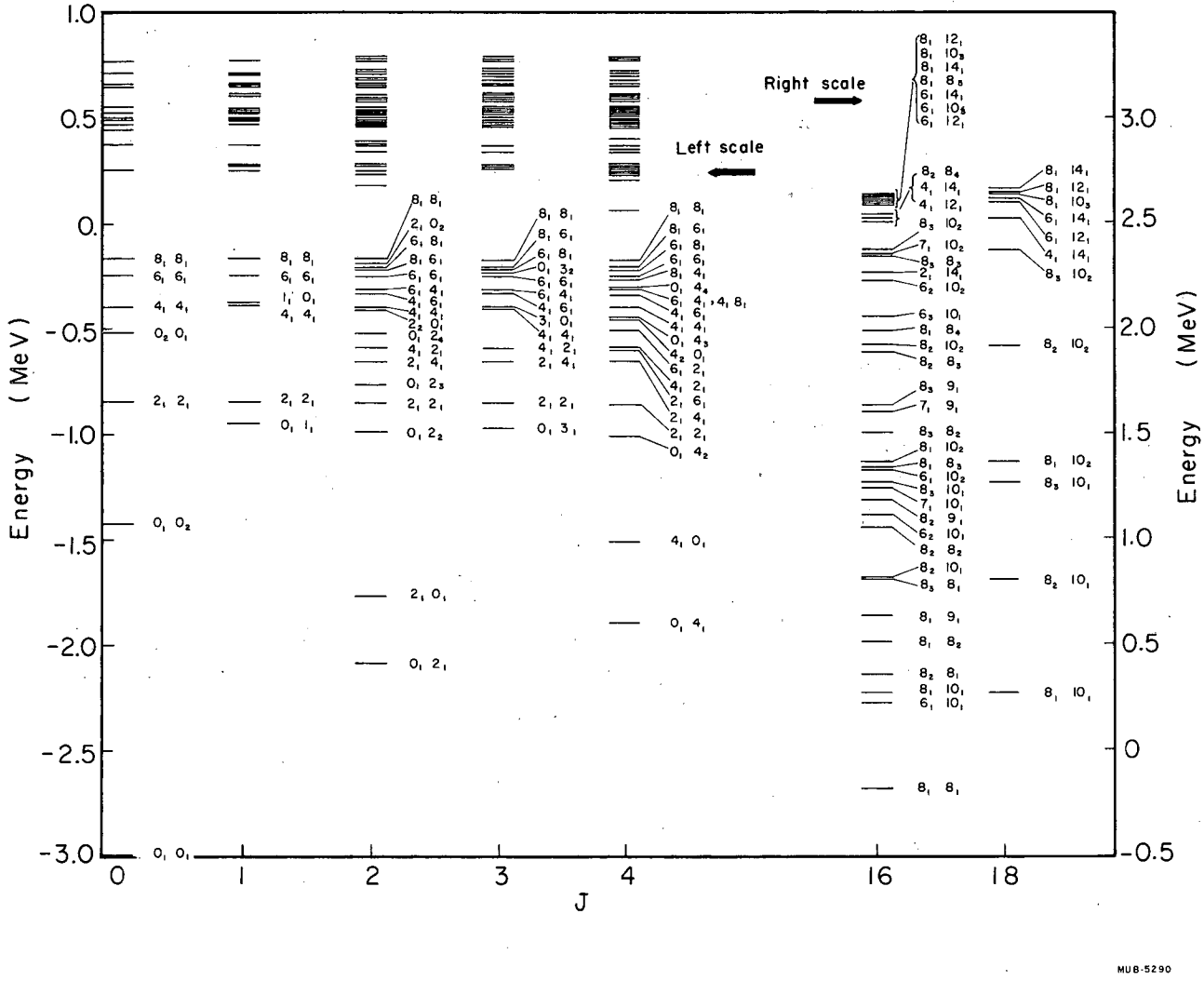

Fig. 5. Zero-order energy spectrum for Po^{212} compounded from Po^{210} and Pb^{210} eigenvalues. Only a few spins are shown and very high energy levels (above the scale) are omitted. The notation, for example $(0_1\ 2_2)$ means the following: the 0_1 refers to the first 0+ state of the protons (Po^{210}) and the 2_2 to the second 2+ state of the neutrons (Pb^{210}).

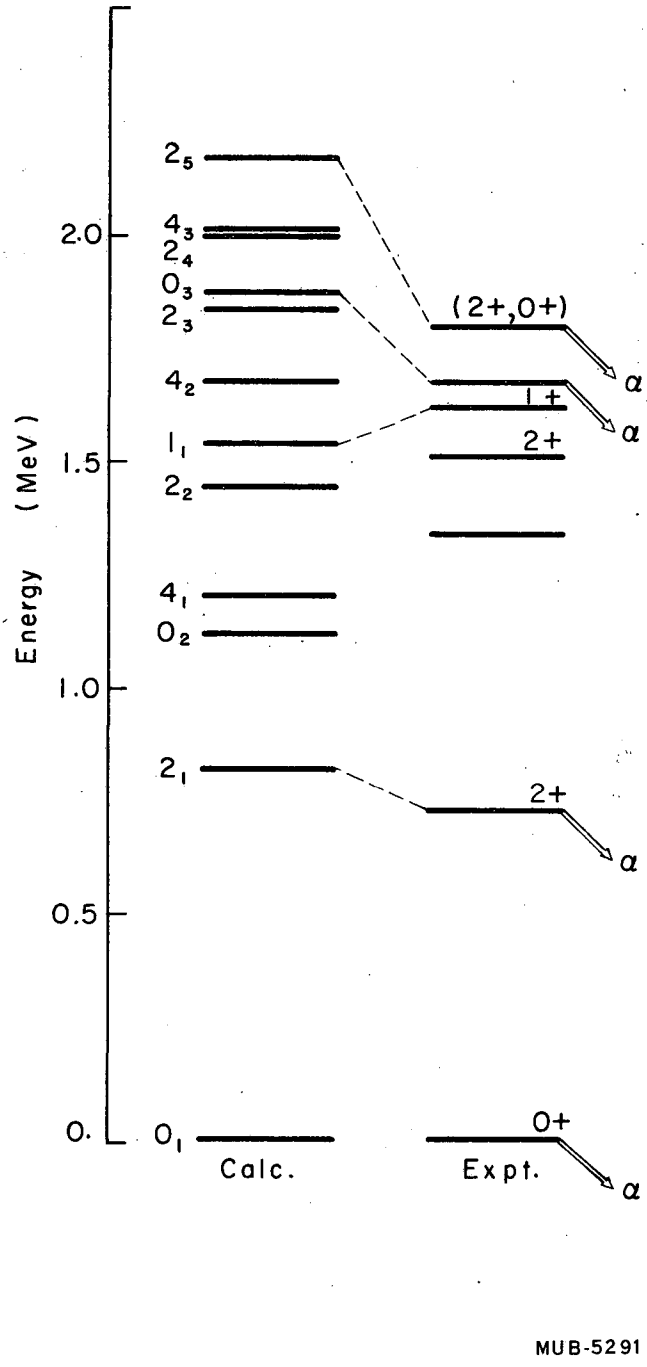

Fig. 6. Calculated spectrum for Po^{212} is compared with experiment. Only the lower spins are shown. The dashed lines are correspondences inferred from the alpha-decay properties. In the same region covered by this figure the following levels are also calculated: 6+ at 1.36 MeV, 8+ at 1.41 MeV, and 10+ at 1.19 MeV. In addition around 2 MeV there may be a 3+, 5+, and 7+ level.

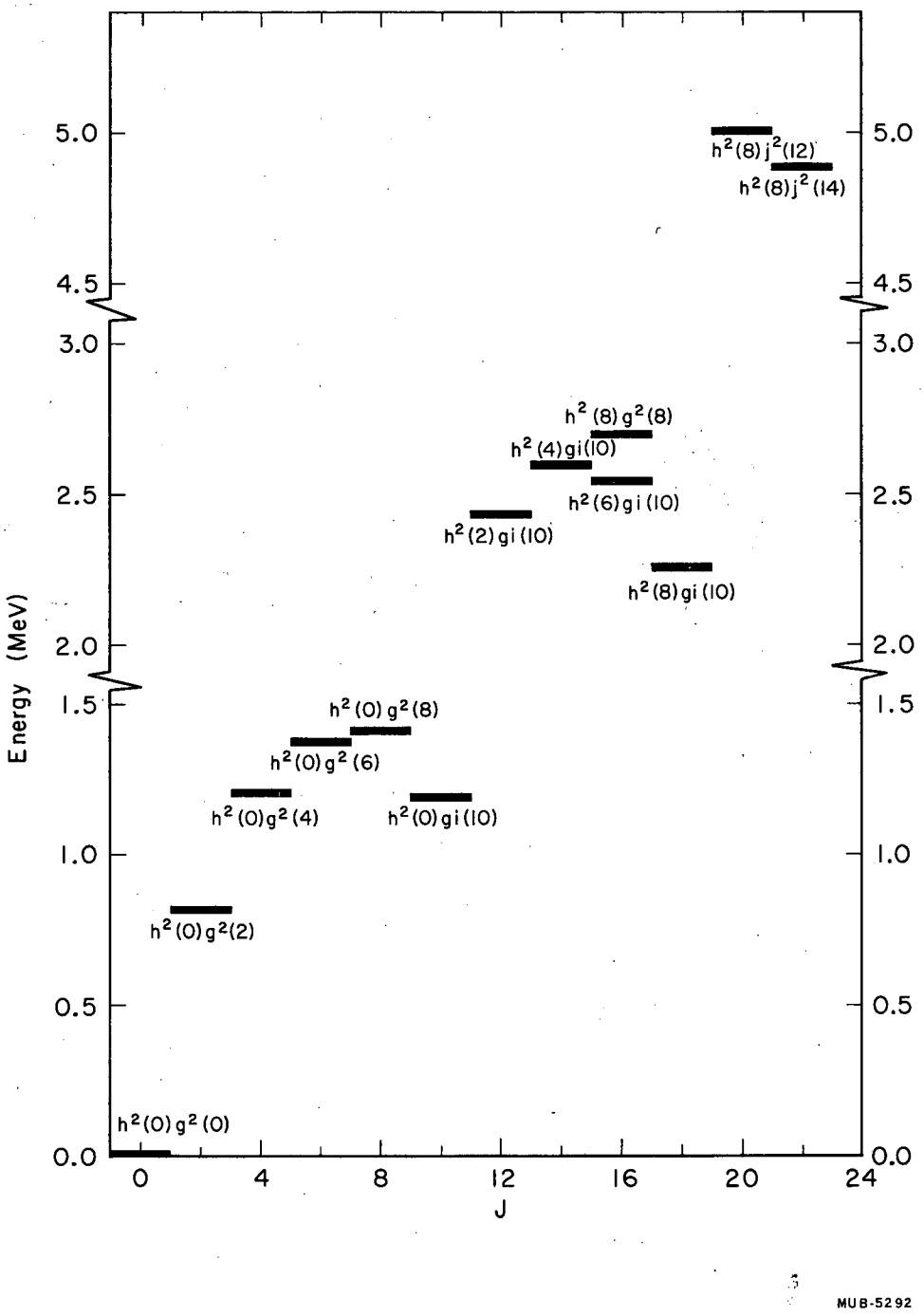

Fig. 7. The lowest level of each even spin, calculated for Po^{212} are shown. This figure illustrates the isomerism. The dominant configuration of each level is also shown.

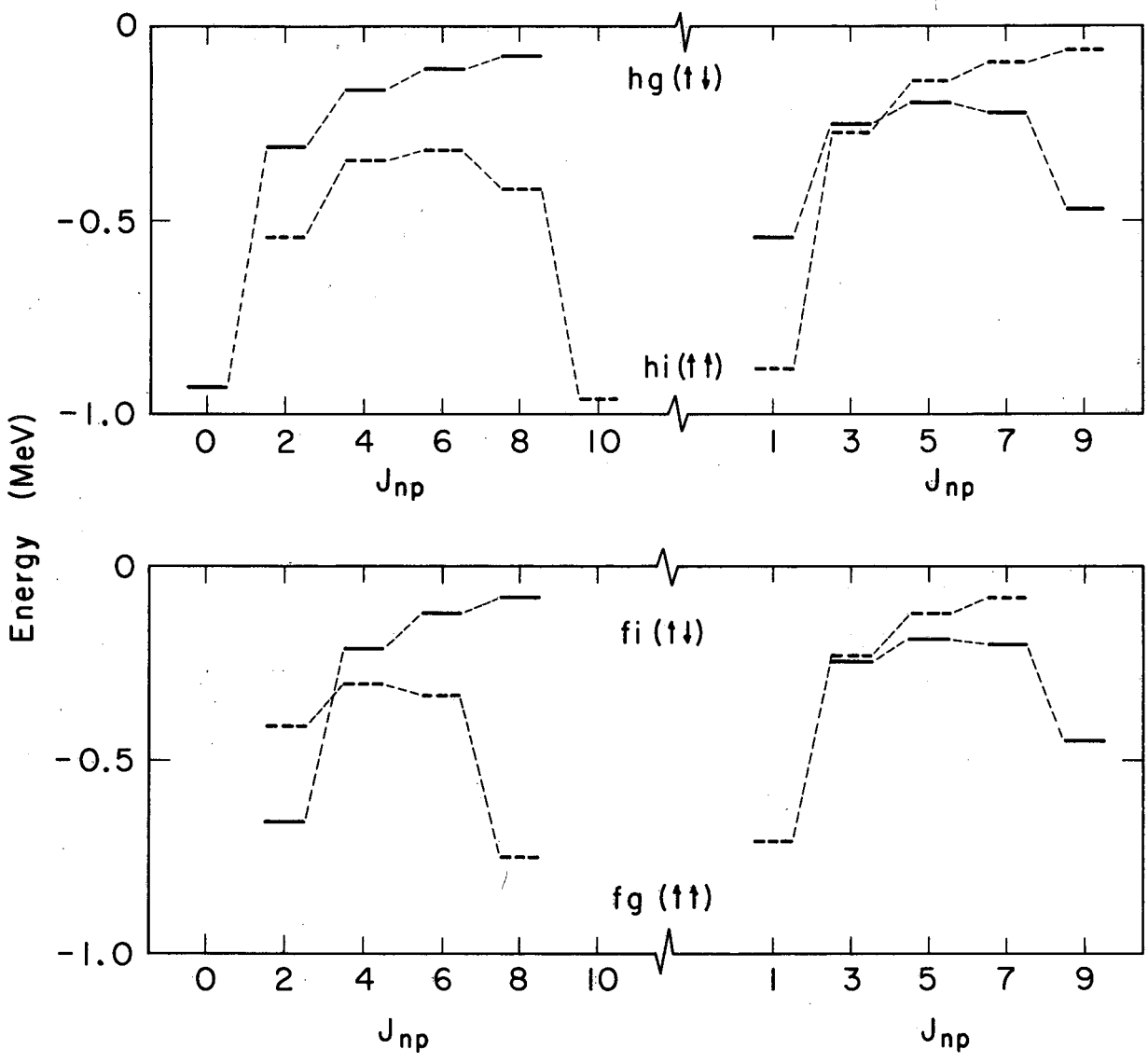

Fig. 8. The J-dependence of the p-n force diagonal matrix elements for several configurations.

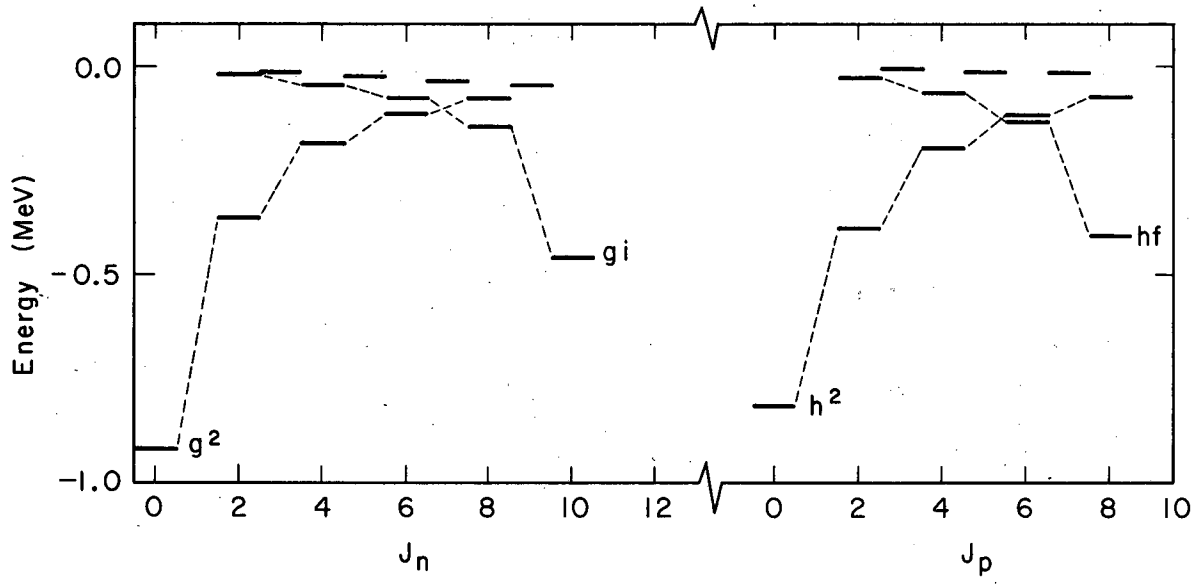

Fig. 9. The J-dependence of the p-p and n-n force diagonal matrix elements for several configurations.

state	0_1				
(J_p,J_n)	(0,0)	(2,2)	(4,4)	(6,6)	(8,8)
$h^2\ g^2$	0.6682	0.2385	0.1385	0.0844	0.0543
h^2 gi		-0.0211	-0.0328	-0.0270	-0.0225
$h^2\ i^2$	0.3900	0.1702	0 0102	0.0044	0.0024
h^2 gd		0.0646	0.0120	0.0048	
h^2 id			0.0026	0.0026	0.0038
$h^2\ j^2$	-0.2386	-0.0705	-0.0063	-0.0027	-0.0016
$h^2\ d^2$	0.1067	0.0176	0.0030		
hf g^2		0.0165	0.0187	0.0178	0.0134
hf gi		-0.0085	-0.0166	-0.0267	-0.0580
hf i^2		0.0141	0.0024	0.0024	0.0040
hf gd		0.0088	0.0024	0.0020	
hf id			0.0006	0.0014	0.0064
hf j^2		-0.0060	-0.0014	-0.0014	-0.0025
hf d^2		0.0017	0.0005		
$f^2\ g^2$	0.2305	0.0215	0.0066	0.0025	
f^2 gi		-0.0023	-0.0020	-0.0014	
$f^2\ i^2$	0.1064	0.0154	0.0005	0.0002	
f^2 gd		0.0060	0.0006	0.0002	
f^2 id			0.0001	0.0001	
$f^2\ j^2$	-0.0692	-0.0064	-0.0003	-0.0001	
$f^2\ d^2$	0.0361	0.0016	0.0001		
$i^2\ g^2$	-0.2030	-0.0311	-0.0096	-0.0041	-0.0021
i^2 gi		0.0030	0.0027	0.0022	0.0036
$i^2\ i^2$	-0.1665	-0.0222	-0.0007	-0.0003	-0.0003
i^2 gd		-0.0086	-0.0009	-0.0003	
i^2 id			-0.0002	-0.0002	-0.0004
$i^2\ j^2$	0.1026	0.0092	0.0005	0.0002	0.0002
$i^2\ d^2$	-0.0467	-0.0023	-0.0002		

state	0_2				
(J_p,J_n)	(0,0)	(2,2)	(4,4)	(6,6)	(8,8)
$h^2\ g^2$	0.3907	0.2458	0.1500	0.1113	0.0854
h^2 gi		0.0068	0.0257	0.0120	0.0092
$h^2\ i^2$	-0.6152	-0.3659	0.0057	0.0025	0.0009
h^2 gd		0.0290	0.0090	0.0042	
h^2 id			0.0016	0.0016	0.0014
$h^2\ j^2$	0.2392	0.1152	-0.0041	-0.0018	-0.0007
$h^2\ d^2$	-0.0009	-0.0018	0.0026		
hf g^2		0.0147	0.0155	0.0178	0.0173
hf gi		-0.0064	0.0007	0.0019	0.0151
hf i^2		-0.0274	0.0008	0.0004	0.0007
hf gd		0.0044	0.0011	0.0007	
hf id			0.0002	0.0002	-0.0011
hf j^2		0.0085	-0.0005	-0.0003	0.0004
hf d^2		-0.0000	0.0003		
$f^2\ g^2$	0.1481	0.0220	0.0070	0.0032	
f^2 gi		0.0003	0.0011	0.0003	
$f^2\ i^2$	-0.1424	-0.0330	0.0003	0.0001	
f^2 gd		0.0027	0.0004	0.0001	
f^2 id			0.0001	0.0000	
$f^2\ j^2$	0.0558	0.0104	-0.0002	-0.0001	
$f^2\ d^2$	0.0039	-0.0002	0.0001		
$i^2\ g^2$	-0.1735	-0.0319	-0.0102	-0.0052	-0.0031
i^2 gi		-0.0007	-0.0017	-0.0006	-0.0010
$i^2\ i^2$	0.2586	0.0477	-0.0004	-0.0001	0.0000
i^2 gd		-0.0039	-0.0006	-0.0002	
i^2 id			-0.0001	-0.0001	0.0000
$i^2\ j^2$	-0.1006	-0.0150	0.0003	0.0001	-0.0000
$i^2\ d^2$	-0.0003	0.0002	-0.0002		

state	0_3				
(J_p,J_n)	(0,0)	(2,2)	(4,4)	(6,6)	(8,8)
$h^2\ g^2$	0.2752	-0.6178	-0.3980	-0.3073	-0.2979
h^2 gi		0.0387	0.0440	0.0373	0.0416
$h^2\ i^2$	-0.0451	-0.2228	-0.0249	-0.0118	-0.0079
h^2 gd		-0.0226	-0.0311	-0.0148	
h^2 id			-0.0065	-0.0071	-0.0125
$h^2\ j^2$	-0.0456	0.0998	0.0159	0.0076	0.0053
$h^2\ d^2$	0.0568	0.0265	-0.0080		
hf g^2		-0.0253	-0.0368	-0.0399	-0.0336
hf gi		-0.0036	-0.0151	-0.0109	-0.0223
hf i^2		-0.0085	-0.0006	-0.0004	0.0009
hf gd		0.0036	-0.0016	-0.0012	
hf id			-0.0002	-0.0003	0.0014
hf j^2		0.0036	0.0006	0.0004	-0.0005
hf d^2		-0.0007	-0.0006		
$f^2\ g^2$	0.2740	-0.0547	-0.0184	-0.0086	
f^2 gi		0.0032	0.0014	0.0006	
$f^2\ i^2$	0.0459	-0.0197	-0.0011	-0.0003	
f^2 gd		-0.0018	-0.0014	-0.0004	
f^2 id			-0.0003	-0.0002	
$f^2\ j^2$	-0.0525	0.0088	0.0007	0.0002	
$f^2\ d^2$	0.0434	-0.0023	-0.0004		
$i^2\ g^2$	-0.1494	0.0798	0.0269	0.0140	0.0094
i^2 gi		-0.0048	-0.0023	-0.0011	0.0001
$i^2\ i^2$	0.0100	0.0288	0.0016	0.0005	0.0002
i^2 gd		0.0028	0.0021	0.0006	
i^2 id			0.0004	0.0003	0.0003
$i^2\ j^2$	0.0259	0.0129	0.0010	-0.0003	-0.0001
$i^2\ d^2$	0.0287	0.0034	0.0005		

state						$^1_1$										
(J_p,J_n)	(0,1)	(1,0)	(1,1)	(1,2)	(2,1)	(2,2)	(2,3)	(3,2)	(3,3)	(3,4)	(4,3)	(4,4)	(4,5)	(6,5)	(6,6)	(8,8)
$h^2\ g^2$						0.0124						0.0176			0.0166	0.0106
h^2 gi	-0.6814				-0.1780	0.3687	-0.3114				-0.1474	0.2057	-0.1597	-0.1064	0.1148	0.0506
$h^2\ i^2$						0.0154						-0.0171			-0.0074	-0.0031
h^2 gd						-0.0028	-0.0035				-0.0016	-0.0120	-0.0014	-0.0009	-0.0046	
h^2 id												-0.0038	-0.0011	-0.0007	-0.0042	-0.0049
$h^2\ j^2$						-0.0009						0.0084			0.0041	0.0019
$h^2\ d^2$						-0.0011						-0.0016				
hf g^2		0.0025		0.0042		0.0032		0.0031		0.0008		0.0010			0.0012	0.0422
hf gi			0.0054	-0.0048	-0.0192	0.0386	-0.0298	0.0008	-0.0017	0.0023	-0.0087	0.0121	-0.0094	-0.0079	0.0085	0.0426
hf i^2		0.0010		-0.0057		0.0095		-0.0017		-0.0002		-0.0010			-0.0006	-0.0021
hf gd				-0.0043		0.0063	-0.0003	-0.0012	-0.0000	-0.0001	-0.0001	-0.0007	-0.0001	-0.0001	-0.0003	
hf id										-0.0000		-0.0002	-0.0001	-0.0001	-0.0003	-0.0032
hf j^2		-0.0007		0.0020		-0.0032		0.0005		0.0001		0.0005			0.0003	0.0012
hf d^2		0.0004		-0.0004		0.0007		-0.0001		-0.0000		-0.0001				
$f^2\ g^2$						0.0012						0.0008			0.0004	
f^2 gi	-0.2232				-0.0164	0.0338	-0.0284				-0.0066	0.0093	-0.0072	-0.0028	0.0030	
$f^2\ i^2$						0.0018						-0.0008			-0.0002	
f^2 gd						0.0001	-0.0003				-0.0001	-0.0005	-0.0001	-0.0000	-0.0001	
f^2 id												-0.0002	-0.0000	-0.0000	-0.0001	
$f^2\ j^2$						-0.0002						0.0004			0.0001	
$f^2\ d^2$						-0.0001						-0.0001				
$i^2\ g^2$						-0.0017						-0.0012			-0.0007	-0.0024
i^2 gi	0.2970				0.0234	-0.0485	0.0408				0.0098	-0.0136	0.0106	0.0046	-0.0050	-0.0035
$i^2\ i^2$						-0.0023						0.0011			0.0003	0.0002
i^2 gd						0.0001	0.0005				0.0001	0.0008	0.0001	0.0000	0.0002	
i^2 id												0.0003	0.0001	0.0000	0.0002	0.0003
$i^2\ j^2$						0.0002						-0.0006			-0.0002	-0.0001
$i^2\ d^2$						0.0001						0.0001				

state	2_1											
(J_p,J_n)	(0,2)	(2,0)	(2,2)	(2,4)	(4,2)	(4,4)	(4,6)	(6,4)	(6,6)	(6,8)	(8,6)	(8,8)
$h^2\ g^2$	- 0.6443	- 0.4120	0.1854	- 0.2001	- 0.0844	0.0578	- 0.0904	- 0.0571	- 0.0039	- 0.0357	- 0.0223	- 0.0370
h^2 gi	0.0359		- 0.0144	0.0314	0.0182	- 0.0123	0.0032	0.0139	0.0001	0.0018	0.0008	0.0018
$h^2\ i^2$	- 0.1952	- 0.2157	0.0944	- 0.0133	- 0.0583	0.0041	- 0.0029	- 0.0042	- 0.0001	- 0.0007	- 0.0007	- 0.0008
h^2 gd	- 0.1146		- 0.0133	- 0.0162	- 0.0393	0.0049	- 0.0040	- 0.0050	- 0.0002		- 0.0010	
h^2 id				- 0.0034		0.0010	- 0.0018	- 0.0011	- 0.0001	- 0.0012	- 0.0004	- 0.0012
$h^2\ j^2$	0.0963	0.1481	- 0.0383	0.0084	0.0253	- 0.0026	0.0019	0.0026	0.0001	0.0005	0.0005	0.0005
$h^2\ d^2$	- 0.0342	- 0.0721	0.0071	- 0.0041	- 0.0076	0.0012		- 0.0012				
hf g^2		- 0.0206	0.0078	- 0.0120	- 0.0050	0.0034	- 0.0053	- 0.0043	- 0.0003	- 0.0027	- 0.0033	- 0.0055
hf gi			- 0.0032	0.0079	0.0011	- 0.0007	0.0002	0.0010	0.0000	0.0001	0.0001	0.0003
hf i^2		- 0.0097	0.0047	- 0.0013	- 0.0034	0.0002	- 0.0002	- 0.0003	- 0.0000	- 0.0001	- 0.0001	- 0.0001
hf gd			- 0.0060	- 0.0014	- 0.0023	0.0003	- 0.0002	- 0.0004	- 0.0000		- 0.0001	
hf id				- 0.0003		0.0001	- 0.0001	- 0.0001	- 0.0000	- 0.0001	- 0.0001	- 0.0002
hf j^2		0.0068	- 0.0016	0.0008	0.0015	- 0.0002	0.0001	0.0002	0.0000	0.0000	0.0001	0.0001
hf d^2		- 0.0035	- 0.0001	- 0.0003	- 0.0004	0.0001		- 0.0001				
$f^2\ g^2$	- 0.2110	- 0.0719	0.0416	- 0.0501	- 0.0038	0.0026	- 0.0041	- 0.0015	- 0.0001	- 0.0009		
f^2 gi	0.0117		- 0.0041	0.0055	0.0008	- 0.0006	0.0001	0.0004	0.0000	0.0000		
$f^2\ i^2$	- 0.0639	- 0.0212	0.0095	- 0.0031	- 0.0026	0.0002	- 0.0001	- 0.0001	- 0.0000	- 0.0000		
f^2 gd	- 0.0376		- 0.0058	- 0.0039	- 0.0018	0.0002	- 0.0002	- 0.0001	- 0.0000			
f^2 id				- 0.0008		0.0000	- 0.0001	- 0.0000	- 0.0000	- 0.0000		
$f^2\ j^2$	0.0316	0.0170	- 0.0045	0.0020	0.0011	- 0.0001	0.0001	0.0001	0.0000	0.0000		
$f^2\ d^2$	- 0.0112	- 0.0108	0.0010	- 0.0010	- 0.0003	0.0001		- 0.0000				
$i^2\ g^2$	0.2808	0.0573	- 0.0268	0.0296	0.0056	- 0.0038	0.0060	0.0025	0.0002	0.0015	0.0007	0.0012
i^2 gi	- 0.0156		0.0022	- 0.0046	- 0.0012	0.0008	- 0.0002	- 0.0006	- 0.0000	- 0.0001	- 0.0000	- 0.0001
$i^2\ i^2$	0.0851	0.0281	- 0.0124	0.0020	0.0039	- 0.0003	0.0002	0.0002	0.0000	0.0000	0.0000	0.0000
i^2 gd	0.0500		0.0024	0.0024	0.0026	- 0.0003	0.0003	0.0002	0.0000		0.0000	
i^2 id				0.0005		- 0.0001	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
$i^2\ j^2$	- 0.0420	- 0.0196	0.0051	- 0.0012	- 0.0017	0.0002	- 0.0001	- 0.0001	- 0.0000	- 0.0000	- 0.0000	- 0.0000
$i^2\ d^2$	0.0149	0.0098	- 0.0009	0.0006	0.0005	- 0.0001		0.0001				

state	$^2_2$											
(J_p,J_n)	(0,2)	(2,0)	(2,2)	(2,4)	(4,2)	(4,4)	(4,6)	(6,4)	(6,6)	(6,8)	(8,6)	(8,8)
$h^2\ g^2$	0.5053	-0.4416	0.0470	0.0824	-0.0403	0.0312	0.0383	-0.0226	0.0123	0.0197	-0.0083	0.0023
h^2 gi	0.0169		-0.0116	0.0231	0.0249	-0.0126	-0.0014	0.0070	-0.0004	-0.0010	0.0003	-0.0001
$h^2\ i^2$	-0.2450	-0.4539	0.1914	0.0023	-0.1053	0.0027	0.0012	-0.0018	0.0004	0.0004	-0.0003	0.0000
h^2 gd	0.0493		0.0849	0.0044	-0.0445	0.0030	0.0017	-0.0021	0.0005		-0.0004	
h^2 id				0.0007		0.0007	0.0008	-0.0005	0.0002	0.0006	-0.0002	0.0001
$h^2\ j^2$	0.0637	0.2785	-0.0721	-0.0019	0.0403	-0.0016	-0.0008	0.0011	-0.0003	-0.0003	0.0002	-0.0000
$h^2\ d^2$	0.0110	-0.1016	0.0145	0.0014	-0.0083	0.0007		-0.0005				
hf g^2		-0.0191	0.0018	0.0044	-0.0024	0.0018	0.0023	-0.0017	0.0009	0.0015	-0.0012	0.0003
hf gi			0.0034	0.0004	0.0015	-0.0007	-0.0001	0.0005	-0.0000	-0.0001	0.0000	-0.0000
hf i^2		-0.0176	0.0064	0.0002	-0.0062	0.0002	0.0001	-0.0001	0.0000	0.0000	-0.0000	0.0000
hf gd			0.0067	0.0003	-0.0026	0.0002	0.0001	-0.0002	0.0000		-0.0001	
hf id				0.0001		0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0000	0.0000
hf j^2		0.0110	-0.0026	-0.0001	0.0024	-0.0001	-0.0000	0.0001	-0.0000	-0.0000	0.0000	-0.0000
hf d^2		-0.0042	0.0008	0.0001	-0.0005	0.0000		-0.0000				
$f^2\ g^2$	0.1655	-0.0148	-0.0176	0.0343	-0.0018	0.0014	0.0017	-0.0006	0.0003	0.0005		
f^2 gi	0.0055		-0.0003	0.0020	0.0011	-0.0006	-0.0001	0.0002	-0.0000	-0.0000		
$f^2\ i^2$	-0.0802	-0.0322	0.0146	0.0016	-0.0047	0.0001	0.0001	-0.0000	0.0000	0.0000		
f^2 gd	0.0161		0.0104	0.0023	-0.0020	0.0001	0.0001	-0.0001	0.0000			
f^2 id				0.0004		0.0000	0.0000	-0.0000	0.0000	0.0000		
$f^2\ j^2$	0.0209	0.0185	-0.0050	-0.0011	0.0018	-0.0001	-0.0000	0.0000	-0.0000	-0.0000		
$f^2\ d^2$	0.0036	-0.0053	0.0008	0.0006	-0.0004	0.0000		-0.0000				
$i^2\ g^2$	-0.2202	0.0544	-0.0036	-0.0137	0.0027	-0.0021	-0.0025	0.0010	-0.0005	-0.0009	0.0003	-0.0001
i^2 gi	-0.0073		0.0013	-0.0030	-0.0017	0.0008	0.0001	-0.0003	0.0000	0.0000	-0.0000	0.0000
$i^2\ i^2$	0.1068	0.0577	-0.0244	-0.0005	0.0070	-0.0002	-0.0001	0.0001	-0.0000	-0.0000	0.0000	-0.0000
i^2 gd	-0.0215		-0.0114	-0.0008	0.0030	-0.0002	-0.0001	0.0001	-0.0000		0.0000	
i^2 id				-0.0001		-0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0000	-0.0000
$i^2\ j^2$	-0.0277	-0.0353	0.0091	0.0004	-0.0027	0.0001	0.0001	-0.0000	0.0000	0.0000	-0.0000	0.0000
$i^2\ d^2$	0.0048	0.0127	-0.0018	-0.0002	0.0005	-0.0000		0.0000				

state	2_3											
(J_p,J_n)	(0,2)	(2,0)	(2,2)	(2,4)	(4,2)	(4,4)	(4,6)	(6,4)	(6,6)	(6,8)	(8,6)	(8,8)
$h^2\ g^2$	-0.0632	0.5251	-0.2471	0.0620	0.0607	-0.0885	0.0279	0.0465	-0.0206	0.0109	0.0224	0.0369
h^2 gi	0.1514		-0.0093	0.0624	0.0076	-0.0017	-0.0010	0.0016	0.0007	-0.0005	-0.0008	-0.0018
$h^2\ i^2$	-0.5897	-0.0975	0.1951	-0.0022	-0.0631	-0.0045	0.0009	0.0023	-0.0007	0.0002	0.0007	0.0008
h^2 gd	-0.1150		-0.0704	0.0004	0.0171	-0.0062	0.0012	0.0032	-0.0009		0.0010	
h^2 id				-0.0004		-0.0012	0.0005	0.0006	-0.0004	0.0004	0.0004	0.0012
$h^2\ j^2$	0.2125	0.0117	-0.0544	0.0006	0.0187	0.0030	-0.0006	-0.0016	0.0004	-0.0002	-0.0005	-0.0005
$h^2\ d^2$	-0.0304	0.0510	-0.0070	0.0006	0.0012	-0.0017		0.0009				
$hf\ g^2$		0.0041	-0.0047	0.0019	0.0036	-0.0052	0.0016	0.0035	-0.0015	0.0008	0.0033	0.0055
hf gi			0.0002	-0.0009	0.0004	-0.0001	-0.0001	0.0001	0.0001	-0.0000	-0.0001	-0.0003
$hf\ i^2$		-0.0166	0.0092	0.0002	-0.0037	-0.0003	0.0001	0.0002	-0.0000	0.0000	0.0001	0.0001
hf gd			-0.0101	0.0002	0.0010	-0.0004	0.0001	0.0002	-0.0001		0.0001	
hf id				0.0000		-0.0001	0.0000	0.0000	-0.0000	0.0000	0.0001	0.0002
$hf\ j^2$		0.0086	-0.0024	-0.0001	0.0011	0.0002	-0.0000	-0.0001	0.0000	-0.0000	-0.0001	-0.0001
$hf\ d^2$		-0.0013	-0.0007	0.0000	0.0001	-0.0001		0.0001				
$f^2\ g^2$	-0.0207	0.0546	-0.0229	0.0022	0.0027	-0.0040	0.0013	0.0012	-0.0005	0.0003		
f^2 gi	0.0496		-0.0005	0.0071	0.0003	-0.0001	-0.0000	0.0000	0.0000	-0.0000		
$f^2\ i^2$	-0.1932	-0.0204	0.0247	-0.0005	-0.0028	-0.0002	0.0000	0.0001	-0.0000	0.0000		
f^2 gd	-0.0377		-0.0098	-0.0003	0.0008	-0.0003	0.0001	0.0001	-0.0000			
f^2 id				-0.0001		-0.0001	0.0000	0.0000	-0.0000	0.0000		
$f^2\ j^2$	0.0696	0.0068	-0.0071	0.0002	0.0008	0.0001	-0.0000	-0.0000	0.0000	-0.0000		
$f^2\ d^2$	-0.0100	0.0042	-0.0007	-0.0000	0.0001	-0.0001		0.0000				
$i^2\ g^2$	0.0275	-0.0683	0.0319	-0.0076	-0.0040	0.0059	-0.0019	-0.0020	0.0009	-0.0005	-0.0007	-0.0012
i^2 gi	-0.0660		0.0012	-0.0081	-0.0005	0.0001	0.0001	-0.0001	-0.0000	0.0000	0.0000	0.0001
$i^2\ i^2$	0.2570	0.0143	-0.0261	0.0003	0.0042	0.0003	-0.0001	-0.0001	0.0000	-0.0000	-0.0000	-0.0000
i^2 gd	0.0510		0.0097	-0.0000	-0.0011	0.0004	-0.0001	-0.0001	0.0000		-0.0000	
i^2 id				0.0001		0.0001	-0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000
$i^2\ j^2$	-0.0926	-0.0024	0.0073	-0.0001	-0.0012	-0.0002	0.0000	0.0001	-0.0000	0.0000	0.0000	0.0000
$i^2\ d^2$	0.0132	-0.0065	0.0009	-0.0001	-0.0001	0.0001		-0.0000				

state	$^2_4$											
(J_p,J_n)	(0,2)	(2,0)	(2,2)	(2,4)	(4,2)	(4,4)	(4,6)	(6,4)	(6,6)	(6,8)	(8,6)	(8,8)
$h^2\ g^2$	-0.0436	0.0626	-0.0779	-0.0040	0.0126	-0.0569	0.0061	0.0285	-0.0451	0.0025	0.0289	-0.0516
h^2 gi	-0.8117		0.0917	-0.2622	0.0920	-0.0057	-0.0002	0.0701	0.0016	-0.0001	-0.0010	0.0025
$h^2\ i^2$	-0.1100	-0.0651	0.0436	0.0227	-0.0313	-0.0025	0.0002	-0.0046	-0.0015	0.0001	0.0009	-0.0011
h^2 gd	-0.0023		-0.0229	0.0166	-0.0031	-0.0037	0.0003	-0.0025	-0.0020		0.0013	
h^2 id				0.0051		-0.0007	0.0001	-0.0010	-0.0009	0.0001	0.0006	-0.0017
$h^2\ j^2$	0.0290	0.0302	-0.0098	-0.0114	0.0116	0.0017	-0.0001	0.0021	0.0010	-0.0000	-0.0006	0.0007
$h^2\ d^2$	-0.0011	0.0002	-0.0033	0.0024	-0.0013	-0.0010		-0.0001				
hf g^2		0.0118	-0.0072	0.0023	0.0007	-0.0033	0.0004	0.0021	-0.0034	0.0002	0.0043	-0.0076
hf gi			0.0032	-0.0164	0.0054	-0.0003	-0.0000	0.0052	0.0001	-0.0000	-0.0002	0.0004
hf i^2		0.0066	0.0014	0.0016	-0.0018	-0.0001	0.0000	-0.0003	-0.0001	0.0000	0.0001	-0.0002
hf gd			-0.0009	0.0012	-0.0002	-0.0002	0.0000	-0.0002	-0.0001		0.0002	
hf id				0.0004		-0.0000	0.0000	-0.0001	-0.0001	0.0000	0.0001	-0.0002
hf j^2		-0.0044	-0.0002	-0.0008	0.0007	0.0001	-0.0000	0.0002	0.0001	-0.0000	-0.0001	0.0001
hf d^2		0.0021	-0.0003	0.0002	-0.0001	-0.0001		-0.0000				
$f^2\ g^2$	-0.0143	0.0078	-0.0086	-0.0034	0.0006	-0.0026	0.0003	0.0008	-0.0012	0.0001		
f^2 gi	-0.2659		0.0212	-0.0497	0.0041	-0.0003	-0.0000	0.0019	0.0000	-0.0000		
$f^2\ i^2$	-0.0360	-0.0043	0.0040	0.0042	-0.0014	-0.0001	0.0000	-0.0001	-0.0000	0.0000		
f^2 gd	-0.0007		-0.0026	0.0030	-0.0001	-0.0002	0.0000	-0.0001	-0.0001			
f^2 id				0.0009		-0.0000	0.0000	-0.0000	-0.0000	0.0000		
$f^2\ j^2$	0.0095	0.0017	-0.0006	-0.0021	0.0005	0.0001	-0.0000	0.0001	0.0000	-0.0000		
$f^2\ d^2$	-0.0003	0.0004	-0.0004	0.0004	-0.0001	-0.0000		-0.0000				
$i^2\ g^2$	0.0190	-0.0086	0.0104	0.0008	-0.0008	0.0038	-0.0004	-0.0012	0.0020	-0.0001	-0.0010	0.0017
i^2 gi	0.3537		-0.0133	0.0370	-0.0061	0.0004	0.0000	-0.0030	-0.0001	0.0000	0.0000	-0.0001
$i^2\ i^2$	0.0479	0.0080	-0.0056	-0.0032	0.0021	0.0002	-0.0000	0.0002	0.0001	-0.0000	-0.0000	0.0000
i^2 gd	0.0010		0.0030	-0.0023	0.0002	0.0002	-0.0000	0.0001	0.0001		-0.0000	
i^2 id				-0.0007		0.0000	-0.0000	0.0000	0.0000	-0.0000	-0.0000	0.0001
$i^2\ j^2$	-0.0126	-0.0036	0.0012	0.0016	-0.0008	-0.0001	0.0000	-0.0001	-0.0000	0.0000	0.0000	-0.0000
$i^2\ d^2$	0.0005	-0.0001	0.0004	-0.0003	0.0001	0.0001		0.0000				

state	$^2_5$											
(J_p,J_n)	(0,2)	(2,0)	(2,2)	(2,4)	(4,2)	(4,4)	(4,6)	(6,4)	(6,6)	(6,8)	(8,6)	(8,8)
$h^2\ g^2$	0.0122	-0.2429	-0.6690	-0.0280	-0.0014	-0.3445	-0.0540	-0.0442	-0.1673	-0.0442	-0.0282	-0.0794
h^2 gi	0.0484		0.0299	0.0056	-0.0049	0.0323	0.0019	0.0021	0.0060	0.0022	0.0010	0.0039
$h^2\ i^2$	0.1376	-0.0725	-0.1614	-0.0020	0.0051	-0.0210	-0.0018	-0.0025	-0.0054	-0.0009	-0.0009	-0.0016
h^2 gd	-0.3819		-0.1941	-0.0024	-0.0550	-0.0266	-0.0024	-0.0033	-0.0074		-0.0012	
h^2 id				-0.0005		-0.0054	-0.0011	-0.0007	-0.0033	-0.0014	-0.0006	-0.0026
$h^2\ j^2$	-0.0259	0.0579	0.0899	0.0012	0.0013	0.0135	0.0012	0.0016	0.0036	0.0006	0.0006	0.0011
$h^2\ d^2$	-0.0283	-0.0365	-0.0407	-0.0006	-0.0046	-0.0069		-0.0009				
hf g^2		-0.0359	-0.0340	0.0016	-0.0001	-0.0203	-0.0032	-0.0033	-0.0125	-0.0033	-0.0042	-0.0118
hf gi			0.0179	0.0016	-0.0003	0.0019	0.0001	0.0002	0.0004	0.0002	0.0001	0.0006
hf i^2		-0.0102	-0.0090	-0.0001	0.0003	-0.0012	-0.0001	-0.0002	-0.0004	-0.0001	-0.0001	-0.0002
hf gd			-0.0074	0.0000	-0.0032	-0.0016	-0.0001	-0.0002	-0.0006		-0.0002	
hf id				-0.0000		-0.0003	-0.0001	-0.0000	-0.0002	-0.0001	-0.0001	-0.0004
hf j^2		0.0083	0.0049	0.0000	0.0001	0.0008	0.0001	0.0001	0.0003	0.0000	0.0001	0.0002
hf d^2		-0.0053	-0.0020	0.0000	-0.0003	-0.0004		-0.0001				
$f^2\ g^2$	0.0040	-0.0253	-0.0766	-0.0040	-0.0001	-0.0115	-0.0024	-0.0012	-0.0044	-0.0012		
f^2 gi	0.0159		0.0024	0.0020	-0.0002	0.0015	0.0001	0.0001	0.0002	0.0001		
$f^2\ i^2$	0.0451	-0.0046	-0.0152	-0.0004	0.0002	-0.0009	-0.0001	-0.0001	-0.0001	-0.0000		
f^2 gd	-0.1251		-0.0316	-0.0004	-0.0025	-0.0012	-0.0001	-0.0001	-0.0002			
f^2 id				-0.0001		-0.0002	-0.0000	-0.0000	-0.0001	-0.0000		
$f^2\ j^2$	-0.0085	0.0044	0.0097	0.0002	0.0001	0.0006	0.0001	0.0000	0.0001	0.0000		
$f^2\ d^2$	-0.0093	-0.0035	-0.0054	-0.0001	-0.0002	-0.0003		-0.0000				
$i^2\ g^2$	-0.0053	0.0326	0.0886	0.0037	0.0001	0.0229	0.0036	0.0019	0.0073	0.0019	0.0009	0.0027
i^2 gi	-0.0211		-0.0043	-0.0009	0.0003	-0.0021	-0.0001	-0.0001	-0.0003	-0.0001	-0.0000	-0.0001
$i^2\ i^2$	-0.0600	0.0094	0.0210	0.0003	-0.0003	0.0014	0.0001	0.0001	0.0002	0.0000	0.0000	0.0001
i^2 gd	0.1664		0.0267	0.0003	0.0036	0.0018	0.0002	0.0001	0.0003		0.0000	
i^2 id				0.0001		0.0004	0.0001	0.0000	0.0001	0.0001	0.0000	0.0001
$i^2\ j^2$	0.0113	-0.0076	-0.0019	-0.0002	-0.0001	-0.0009	-0.0001	-0.0001	-0.0002	-0.0000	-0.0000	-0.0000
$i^2\ d^2$	0.0123	0.0049	0.0055	0.0001	0.0003	0.0005		0.0000				

state	4_1															
(J_p,J_n)	(0,4)	(2,2)	(2,4)	(2,6)	(4,0)	(4,2)	(4,4)	(4,6)	(4,8)	(6,2)	(6,4)	(6,6)	(6,8)	(8,4)	(8,6)	(8,8)
$h^2\ g^2$	-0.7204	-0.3277	0.1113	-0.1568	-0.3175	0.0862	-0.0035	-0.0258	-0.0447	-0.0535	-0.0287	0.0392	-0.0541	-0.0195	-0.0448	-0.0007
h^2 gi	0.0414	0.0109	-0.0133	0.0056		-0.0029	0.0031	0.0009	0.0022	0.0018	-0.0019	-0.0014	0.0027	0.0042	0.0016	0.0000
$h^2\ i^2$	-0.0822	-0.0436	0.0296	-0.0051	-0.1181	0.0115	-0.0063	-0.0008	-0.0009	-0.0071	0.0050	0.0013	-0.0011	-0.0064	-0.0015	-0.0000
h^2 gd	-0.0774	-0.0570	-0.0228	-0.0069		0.0150	0.0149	-0.0011		-0.0093	-0.0168	0.0017		-0.0002	-0.0020	
h^2 id	-0.0122		0.0012	-0.0031			0.0003	-0.0005	-0.0015		-0.0007	0.0008	-0.0018	-0.0004	-0.0009	-0.0000
$h^2\ j^2$	0.0357	0.0297	-0.0074	0.0033	0.0882	-0.0078	0.0007	0.0006	0.0006	0.0049	0.0004	-0.0008	0.0008	0.0017	0.0010	0.0000
$h^2\ d^2$	-0.0158	-0.0156	0.0013		-0.0503	0.0041	0.0004			-0.0025	-0.0010			-0.0004		
$hf\ g^2$		-0.0150	0.0051	-0.0072	-0.0279	0.0053	0.0022	-0.0035	-0.0049	-0.0040	-0.0021	0.0029	-0.0040	-0.0029	-0.0066	-0.0001
hf gi		0.0005	-0.0006	0.0003		-0.0002	0.0065	0.0001	0.0002	0.0001	-0.0001	-0.0001	0.0002	0.0006	0.0002	0.0000
$hf\ i^2$		-0.0020	0.0014	-0.0002	-0.0099	0.0007	0.0018	-0.0001	-0.0001	-0.0005	0.0004	0.0001	-0.0001	-0.0009	-0.0002	-0.0000
hf gd		-0.0026	-0.0010	-0.0003		0.0009	0.0131	-0.0002		-0.0007	-0.0013	0.0001		-0.0000	-0.0003	
hf id			0.0001	-0.0001			0.0003	-0.0001	-0.0002		-0.0001	0.0001	-0.0001	-0.0001	-0.0001	-0.0000
$hf\ j^2$		0.0014	-0.0003	0.0002	0.0075	-0.0005	-0.0008	0.0001	0.0001	0.0004	0.0000	-0.0001	0.0001	0.0003	0.0001	0.0000
$hf\ d^2$		-0.0007	0.0001		-0.0044	0.0002	0.0006			-0.0002	-0.0001			-0.0001		
$f^2\ g^2$	-0.2360	-0.0291	0.0099	-0.0139	-0.0146	0.0039	-0.0001	-0.0012	-0.0021	-0.0014	-0.0008	0.0010	-0.0014			
f^2 gi	0.0136	0.0010	-0.0012	0.0005		-0.0001	0.0004	0.0000	0.0001	0.0000	-0.0000	-0.0000	0.0001			
$f^2\ i^2$	-0.0269	-0.0039	0.0026	-0.0005	-0.0054	0.0005	-0.0002	-0.0000	-0.0000	-0.0002	0.0001	0.0000	-0.0000			
f^2 gd	-0.0254	-0.0051	-0.0020	-0.0006		0.0007	0.0011	-0.0001		-0.0002	-0.0004	0.0000				
f^2 id	-0.0040		0.0001	-0.0003			0.0000	-0.0000	-0.0001		-0.0000	0.0000	-0.0000			
$f^2\ j^2$	0.0117	0.0026	-0.0007	0.0003	0.0040	-0.0004	0.0000	0.0000	0.0000	0.0001	0.0000	-0.0000	0.0000			
$f^2\ d^2$	-0.0052	-0.0014	0.0001		-0.0023	0.0002	0.0000			-0.0001	-0.0000					
$i^2\ g^2$	0.3139	0.0422	-0.0143	0.0202	0.0214	-0.0057	0.0001	0.0018	0.0031	0.0023	0.0012	-0.0017	0.0023	0.0007	0.0015	0.0000
i^2 gi	-0.0180	-0.0014	0.0017	-0.0007		0.0002	-0.0004	-0.0001	-0.0002	-0.0001	0.0001	0.0001	-0.0001	-0.0001	-0.0001	-0.0000
$i^2\ i^2$	0.0358	0.0056	-0.0038	0.0007	0.0079	-0.0008	0.0003	0.0001	0.0001	0.0003	-0.0002	-0.0001	0.0000	0.0002	0.0000	0.0000
i^2 gd	0.0337	0.0073	0.0029	0.0009		-0.0010	-0.0014	0.0001		0.0004	0.0007	-0.0001		0.0000	0.0001	
i^2 id	0.0053		-0.0002	0.0004			-0.0000	0.0000	0.0001		0.0000	-0.0000	0.0001	0.0000	0.0000	0.0000
$i^2\ j^2$	-0.0156	-0.0038	0.0009	-0.0004	-0.0059	0.0005	-0.0000	-0.0000	-0.0000	-0.0002	-0.0000	0.0000	-0.0000	-0.0001	-0.0000	-0.0000
$i^2\ d^2$	0.0069	0.0020	-0.0002		0.0034	-0.0003	-0.0000			0.0001	0.0000			0.0000		

state	$^{10}_{1}$														
(J_p,J_n)	(0,10)	(2,8)	(2,10)	(4,6)	(4,8)	(4,10)	(6,4)	(6,6)	(6,8)	(6,10)	(8,2)	(8,4)	(8,6)	(8,8)	(8,10)
$h^2\ g^2$		-0.0110		-0.0049	-0.0163		-0.0034	-0.0149	-0.0168		-0.0025	-0.0108	-0.0152	-0.0083	
h^2 gi	0.7029	0.0886	0.4868	0.0002	0.1349	0.2154	0.0001	0.0005	0.1094	0.0807	0.0001	0.0004	0.0005	0.0496	0.0203
$h^2\ i^2$	-0.0546	-0.0060	-0.0369	-0.0002	-0.0091	-0.0178	-0.0002	-0.0005	-0.0074	-0.0101	-0.0003	-0.0006	-0.0005	-0.0034	-0.0027
h^2 gd				-0.0002			-0.0002	-0.0007			-0.0004	-0.0008	-0.0007		
h^2 id		-0.0095		-0.0001	-0.0144		-0.0000	-0.0003	-0.0118			-0.0002	-0.0003	-0.0054	
$h^2\ j^2$	0.0335	0.0037	0.0231	0.0001	0.0057	0.0103	0.0001	0.0003	0.0047	0.0040	0.0002	0.0004	0.0003	0.0021	0.0010
$h^2\ d^2$							-0.0001				-0.0001	-0.0002			
hf g^2		-0.0005		-0.0003	-0.0010		-0.0003	-0.0011	-0.0012		-0.0165	-0.0426	-0.0560	-0.0342	
hf gi		0.0041	0.0223	0.0000	0.0079	0.0127	0.0000	0.0000	0.0082	0.0060	0.0005	0.0014	0.0020	0.0524	0.0214
hf i^2		-0.0003	-0.0017	-0.0000	-0.0005	-0.0010	-0.0000	-0.0000	-0.0006	-0.0008	-0.0022	-0.0024	-0.0018	-0.0040	-0.0348
hf gd				-0.0000			-0.0000	-0.0000			-0.0029	-0.0031	-0.0025		
hf id		-0.0004		-0.0000	-0.0008		-0.0000	-0.0000	-0.0009			-0.0006	-0.0011	-0.0064	
hf j^2		0.0002	0.0011	0.0000	0.0003	0.0006	0.0000	0.0000	0.0003	0.0003	0.0015	0.0016	0.0012	0.0025	0.0028
hf d^2							-0.0000				-0.0008	-0.0008			
$f^2\ g^2$		-0.0010		-0.0002	-0.0007		-0.0001	-0.0004	-0.0004						
f^2 gi	0.2302	0.0079	0.0432	0.0000	0.0061	0.0097	0.0000	0.0000	0.0029	0.0021					
$f^2\ i^2$	-0.0179	-0.0005	-0.0033	-0.0000	-0.0004	-0.0008	-0.0000	-0.0000	-0.0002	-0.0003					
f^2 gd				-0.0000			-0.0000	-0.0000							
f^2 id		-0.0008		-0.0000	-0.0006		-0.0000	-0.0000	-0.0003						
$f^2\ j^2$	0.0110	0.0003	0.0021	0.0000	0.0003	0.0005	0.0000	0.0000	0.0001	0.0001					
$f^2\ d^2$							-0.0000								
$i^2\ g^2$		0.0014		0.0003	0.0011		0.0001	0.0006	0.0007		0.0009	0.0025	0.0033	0.0020	
i^2 gi	-0.3063	-0.0115	-0.0630	-0.0000	-0.0090	-0.0143	-0.0000	-0.0000	-0.0048	-0.0035	-0.0000	-0.0001	-0.0001	-0.0040	-0.0016
$i^3\ i^2$	0.0238	0.0008	0.0048	0.0000	0.0006	0.0012	0.0000	0.0000	0.0003	0.0004	0.0001	0.0001	0.0001	0.0003	0.0019
i^2 gd				0.0000			0.0000	0.0000			0.0002	0.0002	0.0001		
i^2 id		0.0012		0.0000	0.0010		0.0000	0.0000	0.0005			0.0000	0.0001	0.0005	
$i^2\ j^2$	-0.0146	-0.0005	-0.0030	-0.0000	-0.0004	-0.0007	-0.0000	-0.0000	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001	-0.0002	-0.0002
$i^2\ d^2$							0.0000				0.0000	0.0000			

state	16_1					
(J_p,J_n)	(6,10)	(7,9)	(7,10)	(8,8)	(8,9)	(8,10)
$h^2\ g^2$				-0.1826		
h^2 gi	0.7135			0.3070	-0.0346	0.4748
$h^2\ i^2$	-0.0916			-0.0782		-0.0439
h^2 gd						
h^2 id				-0.1120		
$h^2\ j^2$	0.0359			0.0246		0.0230
$h^2\ d^2$						
hf g^2				-0.2104		
hf gi	0.1138	-0.0202	0.0013	0.1347	0.0036	0.1412
hf i^2	-0.0174		0.0091	-0.0457		-0.0371
hf gd						
hf id				-0.0537		
hf j^2	0.0059		-0.0004	0.0132		0.0081
hf d^2						
$f^2\ g^2$						
f^2 gi	0.0262					
$f^2\ i^2$	-0.0036					
f^2 gd						
f^2 id						
$f^2\ j^2$	0.0013					
$f^2\ d^2$						
$i^2\ g^2$				0.0126		
i^2 gi	-0.0336			-0.0120	-0.0002	-0.0184
$i^2\ i^2$	0.0044			0.0033		0.0026
i^2 gd						
i^2 id				0.0043		
$i^2\ j^2$	-0.0017			-0.0010		-0.0009
$i^2\ d^2$						

state	18_1					
(J_p,J_n)	(4,14)	(6,12)	(6,14)	(8,10)	(8,12)	(8,14)
$h^2\ g^2$						
h^2 gi				-0.8104		
$h^2\ i^2$				0.1043		
h^2 gd						
h^2 id						
$h^2\ j^2$	-0.0010	-0.0041	-0.0009	-0.0301	-0.0002	0.0003
$h^2\ d^2$						
hf g^2						
hf gi				-0.5702		
hf i^2				0.0699		
hf gd						
hf id						
hf j^2	-0.0001	-0.0003	-0.0001	-0.0176	-0.0002	-0.0009
hf d^2						
$f^2\ g^2$						
f^2 gi						
$f^2\ i^2$						
f^2 gd						
f^2 id						
$f^2\ j^2$	-0.0000	-0.0001	-0.0000			
$f^2\ d^2$						
$i^2\ g^2$						
i^2 gi				0.0351		
$i^2\ i^2$				-0.0040		
i^2 gd						
i^2 id						
$i^2\ j^2$	0.0001	0.0002	0.0000	0.0015	0.0000	0.0000
$i^2\ d^2$