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Heterotic road to the MSSM with *R* parity

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In a previous paper, referred to as a ''Mini-Landscape'' search, we explored a ''fertile patch'' of the heterotic landscape based on a $\mathbb{Z}_6 - \mathrm{II}$ orbifold with SO(10) and E_6 local grand unified theory structures. In the present paper we extend this analysis. We find many models with the minimal supersymmetric standard model spectra and an exact *R* parity. In all of these models, the vectorlike exotics decouple along *D*-flat directions. We present two benchmark models which satisfy many of the constraints of a realistic supersymmetric model, including nontrivial Yukawa matrices for three families of quarks and leptons and Majorana masses for right-handed neutrinos with nontrivial seesaw masses for the three light neutrinos. In an appendix we comment on the important issue of string selection rules and, in particular, the so-called ''gamma-rule''.

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I. INTRODUCTION

The string landscape denotes the space of string vacua [1–3]. It is believed that there are on the order of 10^{500} or more possible vacuum configurations. However, at the moment, only a paltry few such vacua have properties looking anything like our own, i.e. with three large space dimensions, the standard model gauge interactions and matter degrees of freedom, and a vanishingly small (in string units) cosmological constant. Much effort has gone into exploring the landscape in search of regions satisfying the latter requisite feature, while only a few groups have attempted to find the rest. If the string is to make contact with experiment, this situation must be inverted.

Can string theory with 10^{500} vacua make any predictions relevant for the LHC? It has been suggested that by exploring the *entire* string landscape one might obtain statistical data which could lead to probabilistic experimental statements [4,5]. Yet the clearest statement to date is that standard-like models are exceedingly rare. This has been demonstrated in the context of orientifolds of Gepner models [6,7] and in the context of intersecting *D*-branes in an orientifold background [8–11]. Nevertheless they may be more prevalent in the heterotic string because of the simple fact that grand unified theory (GUT) groups, such as SO(10) containing spinor representations, appear naturally. For explorations of the string landscape in the context of the free fermionic construction of the heterotic string, see [12–17]. However, even within the context of the heterotic string, standard model–like structure is not guaranteed. For example, it appears to be very difficult [18,19], if not impossible [20,21], to find standard model–like structure in the heterotic string compactified on a $\mathbb{Z}_3$ orbifold. To summarize, standard-like models are very rare; quite possibly a negligibly small set in the entire landscape. If so, what might we conclude from this statistic?

We suggest the following alternate strategy for obtaining low-energy predictions from string theory. One should introduce some *priors* into one's statistical analysis. The first prior is that the theory has only three large space dimensions. The second is that the string vacuum includes the standard model. Perhaps within this subset one may find some statistical correlations which can be useful. Indeed, it is also important to verify that the standard model actually sits in the string landscape. Of course, in order to make this analysis tractable, one may need to include additional priors. In particular, one may require that the theory is supersymmetric at the string scale and that below the string scale the spectrum is that of the minimal supersymmetric standard model (MSSM). Such theories typically have of order 100 moduli (geometric or others). The Yukawa and gauge couplings of the theory will generically be functions of these moduli. In the supersymmetric limit of the theory, one would hope to be able to tune the moduli in order to obtain acceptable low-energy physics. Of course, the problem of stabilizing moduli and supersymmetry breaking must be addressed. However, it is clear that if one *cannot* find an MSSM-like model with this caveat, then this class of theories can be ignored.[1] Finally, the cosmological constant problem would still need to be addressed. But perhaps the only role of the 10^{500} vacua is to resolve this problem.

[1]Note, a handful of heterotic string models with MSSM-like structure have been discussed in the literature [22–39].

In a previous paper, "Mini-Landscape" [ML] [40], we advocated this landscape philosophy. The present paper extends the previous search and also addresses some important phenomenological issues. We base our model scan on the heterotic $E_8 \times E_8$ string [41,42] compactified on an orbifold [43–49]. Our study is motivated by recent work on an orbifold GUT interpretation of heterotic string models [50–52]. We focus on the $\mathbb{Z}_6 - \text{II} \equiv \mathbb{Z}_3 \times \mathbb{Z}_2$ orbifold, which is described in detail in [50,52,53]. The search strategy is based on the concept of "local GUTs" [53–57] which inherits certain features of standard grand unification [58–61]. Local GUTs are specific to certain points in the compact space, while the 4D gauge symmetry is that of the standard model (SM). If matter fields are localized at such points, they form a complete GUT representation. This applies, in particular, to a **16**-plet of a local SO(10), which comprises one generation of the SM matter plus a right-handed neutrino [60,61],

$$\mathbf{16} = (\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_{1} + (\mathbf{1},\mathbf{1})_{0}, \tag{1.1}$$

where representations with respect to $SU(3)_C \times SU(2)_L$ are shown in parentheses and the subscript denotes hypercharge (with electric charge given by $Q = \mathsf{T}_{3L} + Y$). On the other hand, bulk fields are partially projected out and form incomplete GUT multiplets. This offers an intuitive explanation for the observed multiplet structure of the SM [53–56]. This framework is consistent with MSSM gauge coupling unification as long as the SM gauge group is embedded in a simple local GUT $G_{\text{local}} \supseteq SU(5)$, which leads to the standard hypercharge normalization.[2]

We find that the above search strategy, as opposed to a random scan, is successful and a considerable fraction of the models with SO(10) and E_6 local GUT structures pass our criteria. Out of about 3×10^4 inequivalent models which involve two Wilson lines, $\mathcal{O}(200)$ are phenomenologically attractive and can serve as an ultraviolet completion of the MSSM. In the present paper we extend our previous analysis in several ways.

(i) In ML [40], at the last step in our analysis of a theory, we evaluated the effective mass operators, for the vectorlike exotics, up to order 8 in fields. If all the exotics obtained mass, the model was retained. When calculating the rank of the mass matrices, we *assumed* that requiring the singlet configuration to respect supersymmetry would not change the result.

In this paper we explicitly demonstrate that the decoupling of the exotics is consistent with supersymmetry. We first find the $D = 0$ flat directions. If the exotics decouple along these directions, then, in particular models we check for $F = 0$. Then complexified gauge transformations allow us to satisfy $F = 0$ and $D = 0$ simultaneously.

(ii) In ML [40], we presented a model allowing for R parity. However, we did not perform a systematic search for R-parity invariant vacua. Dangerous R-parity violating dimension-4 operators can be forbidden by family reflection symmetry (FRS) (or *matter parity*) [63], i.e. a discrete $\mathbb{Z}_2$ subgroup of $U(1)_{B-L}$ (baryon minus lepton number). For other approaches see [21,64].

In this paper we evaluate $B - L$, searching for a "suitable" definition which has the accepted value on all standard model particles and gives most standard model singlets a value satisfying $3(B - L) = 0 \bmod 2$. This condition preserves a $\mathbb{Z}_2^{\mathcal{M}}$ subgroup of $B - L$ under which chiral matter superfields are odd and Higgs superfields are even. Singlets with $3(B - L) = 0 \bmod 2$ can obtain vacuum expectation values (VEVs) for decoupling exotics, as well as for giving effective quark and lepton Yukawa couplings. Some of these singlets give Majorana masses to right-handed neutrinos [65] (for earlier work see [19]), preserving R parity. Note, if singlets with $3(B - L) = 1 \bmod 2$ obtain VEVs, R parity is broken and dimension-4 baryon/lepton number violating operators are typically generated. In an Appendix, we also consider a possible $\mathbb{Z}_N$ generalization of FRS.

The paper is organized as follows. In Sec. II we review the search strategy defined in ML [40]. In Sec. III we present our results solely on the issue of decoupling of vectorlike exotics along D-flat directions. At this point we can compare our results to other MSSM searches in different regions of the string landscape. We show that we are extremely successful in finding models which have the characteristics of the MSSM. In the following sections we consider many of the phenomenological issues one must face on the road to the MSSM. In particular, in Sec. IV A we discuss the problem of obtaining one pair of light Higgs doublets, a heavy top and then the additional constraint for a conserved R parity/*family reflection symmetry*. In Sec. V we discuss two models which satisfy the aforementioned constraints in detail. In particular we consider the effective Yukawa couplings for quarks and leptons in the limit that exotics decouple. We also study the seesaw mechanism in these examples. Finally, in Sec. VI we summarize our results and discuss some remaining issues.

[2] Note that even if one relaxes this constraint as a prior it was shown that 90% of the MSSM-like models satisfying $\sin^2\theta_W = 3/8$ at the string scale necessarily satisfy this constraint [62]. Also, the discrepancy between the string scale, $\mathcal{O}(10^{17}$ GeV), and the 4D GUT scale, $\mathcal{O}(10^{16}$ GeV), can in principle be resolved by threshold corrections due to states near the string scale.

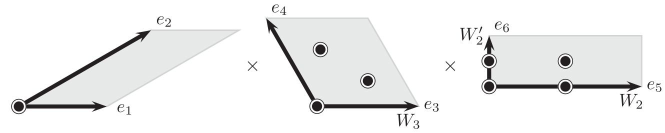

FIG. 1. $G_2 \times SU(3) \times SO(4)$ torus lattice of a $\mathbb{Z}_6$-II orbifold. Possible Wilson lines are denoted by W_3, W_2 and W_2'. The fixed points in the figure are those of the T_1 twisted sector.

II. "MINI-LANDSCAPE" SEARCH STRATEGY [40]: LOCAL GUTS

Our model search is carried out in the $\mathbb{Z}_6 - \text{II}$ orbifold compactification of the heterotic $E_8 \times E_8$ string with the twist vector

$$\vec{v} = \frac{1}{6}(1, 2, -3) \tag{2.1}$$

acting on an $G_2 \times SU(3) \times SO(4)$ torus (see Fig. 1; for details see [52,53]).

It is well known that with a suitable choice of Wilson lines it is not difficult to obtain the SM gauge group up to U(1) factors. The real challenge is to get the correct matter spectrum and the GUT hypercharge normalization. To this end, we base our strategy on the concept of local GUTs. An orbifold model is defined by the orbifold twist, the torus lattice and the gauge embedding of the orbifold action, i.e. the gauge shift V and the Wilson lines W_n. We consider only the gauge shifts V which allow for a local SO(10) or E_6 structure, i.e. including **16**- or **27**-plets in the T_1 twisted sector. For the twist (2.1), the SO(10) shifts are given by [66]

$$V^{SO(10),1} = \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0\right)\left(\frac{1}{3}, 0, 0, 0, 0, 0, 0, 0\right),$$
$$V^{SO(10),2} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0\right)\left(\frac{1}{6}, \frac{1}{6}, 0, 0, 0, 0, 0, 0\right), \tag{2.2}$$

while the E_6 shifts read

$$V^{E_6,1} = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 0, 0, 0, 0, 0\right)\left(0, 0, 0, 0, 0, 0, 0, 0\right),$$
$$V^{E_6,2} = \left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0\right)\left(\frac{1}{6}, \frac{1}{6}, 0, 0, 0, 0, 0, 0\right). \tag{2.3}$$

These gauge shifts are such that the left-moving momenta P (we use the standard notation, for details see e.g. [51–53]) satisfying

$$P \cdot V = 0 \bmod 1, \qquad P^2 = 2, \qquad P \in \Lambda_{E_8 \times E_8} \tag{2.4}$$

are roots of SO(10) or E_6 (up to extra group factors). In fact, this defines the "local" gauge symmetry in the T_1 sector, for states residing at the origin in the G_2 and SU(3) tori and at the two fixed points in the SO(4) torus which are unaffected by the W_2 Wilson line along the e_6 direction (see Fig. 1).[3] The massless states of the first twisted sector are guaranteed to contain **16**-plets of SO(10) at the fixed points with SO(10) symmetry or **27**-plets of E_6 at the fixed points with E_6 symmetry. This is established by considering the mass operator for left-movers,

$$\frac{1}{2}(P + V)^2 - 1 + \frac{1}{2}\sum_{i=1}^{3}|v_i|(1 - |v_i|) = 0, \tag{2.5}$$

with the shift vectors V (Eqs. (2.2) and (2.3)) and the twist vector $\vec{v}$ (Eq. (2.1)). For example, in the case of the two SO(10) shifts, the massless SO(10) spinor is given by

$$P = \frac{1}{2}(-1, -1, -1, \pm 1, \pm 1, \pm 1, \pm 1, \pm 1)(0^8) \tag{2.6}$$

with an even number of minus signs.

Since these massless states are automatically invariant under the orbifold action, they all survive in 4D and appear as complete GUT multiplets. In the case of SO(10), that gives two complete SM generations, while in the case of E_6 we have two **27**s with **27** = **16** + **10** + **1** under SO(10). It is thus necessary to decouple all (or part) of the **10**s from the low-energy theory. The third generation has to come from other twisted or untwisted sectors. The localized **16**- and **27**-plets are true GUT multiplets, whereas the third or "bulk" generation only has the SM quantum numbers of an additional **16**-plet.

The Wilson lines are chosen such that the standard model gauge group is embedded into the local GUT as

$$G_{SM} \subset SU(5) \subset SO(10) \quad \text{or} \quad E_6. \tag{2.7}$$

Moreover, hypercharge is that of standard GUTs and thus consistent with gauge coupling unification. The spectrum has certain features of traditional 4D GUTs, e.g. matter fields form complete GUT representations, yet there are important differences. In particular, interactions generally break GUT relations since different local GUTs are supported at different fixed points. Also, gauge coupling unification is due to the fact that the 10D (not 4D) theory is described by a single coupling.

Let us now recall the search strategy and results from the "Mini-Landscape" search [40]. Consider, for example, models with the SO(10) local structure. For each of the SO(10) shifts of Eq. (2.2), we follow the steps:

(1) Generate Wilson lines W_3 and W_2.
(2) Identify "inequivalent" models.
(3) Select models with $G_{SM} \subset SU(5) \subset SO(10)$.
(4) Select models with three net $(\mathbf{3}, \mathbf{2})$.
(5) Select models with nonanomalous $U(1)_Y \subset SU(5)$.
(6) Select models with net 3 SM families + Higgses + vectorlike.

[3]We assume that there are only two Wilson lines, W_3 in the SU(3) torus and W_2 in the SO(4) torus.

TABLE I. Statistics of $\mathbb{Z}_6$-II orbifolds based on the shifts $V^{\mathrm{SO}(10),1}$, $V^{\mathrm{SO}(10),2}$, $V^{\mathrm{E}_6,1}$, $V^{\mathrm{E}_6,2}$ with two Wilson lines.

Criterion	$V^{\mathrm{SO}(10),1}$	$V^{\mathrm{SO}(10),2}$	$V^{\mathrm{E}_6,1}$	$V^{\mathrm{E}_6,2}$
(2) inequivalent models with 2 Wilson lines	22 000	7800	680	1700
(3) SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or E_6)	3563	1163	27	63
(4) 3 net $(\mathbf{3}, \mathbf{2})$	1170	492	3	32
(5) nonanomalous $\mathrm{U}(1)_Y \subset$ SU(5)	528	234	3	22
(6) spectrum = 3 generations + vectorlike	128	90	3	2

The results are presented in Table I. The models with the chiral MSSM matter content are listed in [67].[4]

To show that the decoupling of exotics is consistent with string selection rules is a technically involved and time-consuming issue. We must select models in which the mass matrices for the exotics have a maximal rank such that no exotic states appear at low energies. We consider superpotential couplings up to order 6 in SM singlets. In our previous analysis, ML, we allowed any SM singlet to obtain a nonvanishing VEV. In the following section we refine our search and demand that all singlet VEVs be along D-flat directions. This requires solving the nontrivial D-flatness conditions. In this analysis we focus on the two SO(10) shifts. Note, there are 218 models in this sector after step 6. In the following section we consider the decoupling of exotics. We do this in two steps. In the first step we construct the effective mass operators for the exotics and check to see if the exotics decouple allowing arbitrary singlet VEVs. In the second step we only consider singlet VEVs along D-flat directions.

III. DECOUPLING EXOTICS

We evaluate all effective mass operators for the exotics x_i, $\bar{x}_j$ up to order 6 in SM singlet fields $\tilde{s}_i$,

$$W \supset x_i \bar{x}_j \langle \tilde{s}_1 \cdots \tilde{s}_N \rangle. \tag{3.1}$$

In general, $\tilde{s}$ transform nontrivially under the extra U(1)s and hidden sector gauge groups. To construct the mass operators (3.1), we find all monomials of the above form consistent with string selection rules. These rules have been discussed previously in the literature. They include space group and R-charge selection rules, in addition to the standard field theoretic requirement of gauge invariance. Complete details of these string selection rules are given in Appendix A. We should emphasize here that in previous analyses a γ selection rule has also been enforced [52,68,69]. We disagree with this additional γ rule and in Appendix A we give a general argument why this rule is *not* a selection rule.[5]

We consider the 218 models remaining after step 6 from the two SO(10) shifts (128 from $V^{\mathrm{SO}(10),1}$ and 90 from $V^{\mathrm{SO}(10),2}$), see Table I. If in a particular model all exotics decouple to order 6 in the product of $\tilde{s}$ fields, assuming arbitrary $\tilde{s}$ VEVs,[6] we retain the model. The number of models satisfying decoupling at this step is 191 (106 from $V^{\mathrm{SO}(10),1}$ and 85 from $V^{\mathrm{SO}(10),2}$). We now determine D-flat directions for all $\tilde{s}$ fields. Our procedure for determining D-flat directions is described in Appendix B. We then retain the subset of the 191 models for which the exotics decouple along D-flat directions to order 6 in the $\tilde{s}$ fields. We find 190 models remaining. Clearly, D-flatness does not impose an important constraint. Thus we are successful in $190/3 \cdot 10^4$ or 0.6% of the cases.

The results of our search may now be compared to many other searches in the literature. We have 218 models with the SM gauge group, three families and only vectorlike exotics from our two SO(10) shifts. Out of these we find 190 for which all exotics decouple along D-flat directions. In certain types of intersecting D-brane models, it was found that the probability of obtaining the SM gauge group and three generations of quarks and leptons, while allowing for chiral exotics, is less than 10^{-9} [8,9]. The criterion which comes closest to the requirements imposed in [8,9] is (4). We find that within our sample the corresponding probability is 6%. In [6,7], orientifolds of Gepner models were scanned for chiral MSSM matter spectra, and it was found that the fraction of such models is 4×10^{-14}. These constructions contain the MSSM matter spectrum plus, in general, vectorlike exotics. This is most similar to step (6) in our analysis where we find 218 models out of a total of 3×10^4 or 0.7%. In comparison, approximately 0.6% of our models have the MSSM spectrum at low energies with all vectorlike exotics decoupling (with exotic mass terms evaluated to order $\tilde{s}^6$) along D-flat directions. Note also

[4]In a recent paper [62], the two constraints, (3) SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or E_6) and (5) nonanomalous $\mathrm{U}(1)_Y \subset$ SU(5) were removed. This search has led to about 10 times more models. However the additional constraint that $\sin^2\theta_W = 3/8$ reduced this number by 90% so that there were only a handful of additional models. It suggests that in order to find the MSSM, one may need to require local GUTs.

[5]In fact, we have shown that all exotics decouple in model A1 in Ref. [52] if one eliminates the γ rule.

[6]Note that giving VEVs to the $\tilde{s}$ fields can often be interpreted as blowing up the orbifold singularities (for recent developments in this direction see [70–73]).

that, in all of our models, hypercharge is normalized as in standard GUTs and thus consistent with gauge coupling unification.

IV. ROAD TO THE MSSM

In this section we consider other phenomenological hurdles which must be overcome in order to reach the MSSM. These hurdles include finding supersymmetric minima with proton stability, an exactly conserved R parity, a μ term for the light Higgs doublets of order the weak scale, a top quark Yukawa coupling of order 1, gauge coupling unification, and more.

A. Constraints

1. R parity conservation

One of the most formidable obstacles in string constructions is obtaining a conserved R parity. In this paper we propose one possible route, i.e. obtaining a "family reflection symmetry" or "matter parity." In this regard, we evaluate $B - L$, searching for a suitable definition which has the accepted value on all standard model particles, is vectorlike on all exotics and produces a number of SM singlets with even and zero $3(B - L)$ charge.

Giving such singlets VEVs preserves a $\mathbb{Z}_2^{\mathcal{M}}$ subgroup of $B - L$, denoted family reflection symmetry or matter parity, under which chiral matter superfields are odd and Higgs superfields are even. We find that the exotics can be decoupled and the right-handed neutrinos can be given Majorana masses consistent with this symmetry. In Appendix C, we show that it is possible to allow any $\tilde{s}$ field to obtain a VEV as long as it has $B - L$ eigenvalue $f = 0, \pm 2/(2\mathbb{Z} + 1)$. This will leave invariant $\mathbb{Z}_2^{\mathcal{M}}$.

To apply the above strategy, we must first give a suitable definition of $B - L$. A possible algorithm to identify the corresponding generators is discussed in Appendix D. Upon defining $B - L$, we must verify D-flatness for the subset of SM singlets with $B - L$ charges $f = 0$, $\pm 2/(2\mathbb{Z} + 1)$ and check that all exotics decouple. This is a tedious task, requiring much computer time. In order to minimize the amount of time, we focus our attention on a subset of the 190 models which have renormalizable top quark Yukawa couplings.

2. Light Higgs doublets

The Higgs doublets of the MSSM are vectorlike and generically in our analysis all the Higgs doublets decouple. Retaining one pair of light Higgs doublets in the MSSM is the μ problem, and we must now face this issue. We look for vacuum configurations in which the μ term vanishes to a certain order in the $\tilde{s}$ fields. At the same time we require that all the exotics decouple. Of course, it would be nice to have a symmetry argument for a small μ.

3. Order 1 top quark Yukawa coupling

The top quark Yukawa coupling is necessarily of order 1. Hence it is natural (although perhaps not absolutely necessary) to require that for the top quark we have a renormalizable $\mathcal{O}(1)$ Yukawa coupling $(\mathbf{3}, \mathbf{2})_{1/6}(\bar{\mathbf{3}}, \mathbf{1})_{-2/3}(\mathbf{1}, \mathbf{2})_{1/2}$, i.e. one of the following types

$$UUU, \qquad UTT, \qquad TTT, \tag{4.1}$$

where U and T denote generic untwisted and twisted fields, respectively. The UUU coupling is given by the gauge coupling, UTT is a local coupling and thus is unsuppressed, while the TTT coupling is significant only when the twisted fields are localized at the same fixed point. We discard models in which the above couplings are absent or suppressed. In ML we required that the top quark have a Yukawa coupling at tree level, i.e. cubic order in the fields, in addition to decoupling of all exotics, albeit assuming arbitrary VEVs for the SM singlets. Of the 190 models which decouple along D-flat directions we have 105 (85) coming from the first (second) SO(10) shift. Out of these there are 55 (32) with "heavy top" and 50 (53) with "no heavy top." We thus find 87 models which decouple along D-flat directions and have a "heavy top." Note, this is just one less than discussed in ML at step 8.

4. R parity invariant models with cubic top Yukawa coupling

We find a suitable definition of $B - L$ for 34 of the 55 (5 of the 32) models of the first (second) SO(10) shifts. Note however that for each case there are several possible inequivalent choices. This is because of two ambiguities which need to be resolved.

(1) In many cases there are vectorlike exotics with SM gauge charges identical to those of quarks, leptons and Higgs doublets. Thus there are different ways to choose which of these states have standard $B - L$ charges. Each choice can lead to a different definition of $B - L$.

(2) For each choice of SM particles above, there may be more than one $B - L$ definition. In some cases there are continuous families of solutions.

Including all of these possibilities we find 3447 (144) suitable $B - L$ generators from the first (second) SO(10) shifts, which also lead to the presence of SM singlets with charges $B - L = 0, \pm 2, \pm 2/3, \pm 2/5, \ldots, \pm 6/7$. We find, however, that these lead to 85 (8) inequivalent models. Requiring the absence of extra unbroken U(1)s reduces this set to 42 (0) acceptable models. Finally, demanding that all exotics decouple along D-flat directions leads to 15 (0) acceptable solutions with an exact low-energy R parity. This result is specific to our $(B - L)$-based strategy and we expect, in general, more acceptable models to exist.

B. Approaching the MSSM

Further issues to be addressed are as follows.

(1) We must check that quarks and leptons obtain nontrivial masses. For neutrinos, this includes an analysis of Majorana masses and the seesaw mechanism.

(2) We must also consider dimension-5 baryon and lepton number violating operators [63,74,75]. These operators are not forbidden by R parity and are typically generated. Their coefficients must necessarily be suppressed in order to be consistent with proton decay experiments [76,77].

(3) Precision gauge coupling unification should be addressed [20,52,78]. This includes a calculation of the string threshold corrections [79–81].

(4) Finally, $F = 0$ has to be verified. This constraint guarantees that our vacua are indeed supersymmetric. In general, $F = 0$ solutions exist. Some of them can be found numerically by truncating the superpotential and solving polynomial equations. Once they are found, $F = 0$ and $D = 0$ can be satisfied simultaneously using complexified gauge transformations [82] (for a detailed discussion see [53]).

All of these checks are clearly time-consuming, and we have not performed an inclusive analysis. We have however found many vacua with R parity. In the next section we discuss our results for two particular examples. In these examples we have demanded that

(i) all exotics are massive,
(ii) there is one pair of massless Higgses,
(iii) the mass matrix for the right-handed neutrinos has full rank,
(iv) no extra U(1) factors remain,
(v) hidden sector gaugino condensation is possible,
(vi) R parity is unbroken.

V. TWO "BENCHMARK" MODELS WITH *R* PARITY

We now discuss two particular benchmark models. In Model 1 we also consider two different vacuum configurations and show how the phenomenology depends on the different choices of vacua.

A. Model 1

The model is defined by the shifts and Wilson lines given in Appendix E. The gauge group after compactification is

$$G_{\rm orbifold} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times [\mathrm{SU}(4) \times \mathrm{SU}(2)'] \times \mathrm{U}(1)^9. \tag{5.1}$$

The resulting massless spectrum includes three SM generations plus vectorlike exotics with respect to the SM gauge group.

The model allows us to define a suitable $B - L$ generator,

TABLE II. Spectrum. The quantum numbers under $\mathrm{SU}(3) \times \mathrm{SU}(2) \times [\mathrm{SU}(4) \times \mathrm{SU}(2)']$ are shown in boldface; hypercharge and $B - L$ charge appear as subscripts. Note that the states $s_i^{\pm}$, f_i, $\bar{f}_i$ and m_i have different $B - L$ charges for different i, which we do not explicitly list.

#	Representation	Label	#	Representation	Label
3	$(\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/6,1/3)}$	q_i	3	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1,1)}$	$\bar{e}_i$	8	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(0,*)}$	m_i
4	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$	1	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/3,1/3)}$	d_i
4	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(-1/2,-1)}$	ℓ_i	1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(-1/2,0)}$	ϕ_i	1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
6	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$	6	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/3,-2/3)}$	δ_i
14	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/2,*)}$	s_i^+	14	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/2,*)}$	s_i^-
16	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,1)}$	$\bar{n}_i$	13	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,-1)}$	n_i
5	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0,1)}$	$\bar{\eta}_i$	5	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0,-1)}$	η_i
10	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0,0)}$	h_i	2	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})_{(0,0)}$	y_i
6	$(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1})_{(0,*)}$	f_i	6	$(\mathbf{1}, \mathbf{1}; \bar{\mathbf{4}}, \mathbf{1})_{(0,*)}$	$\bar{f}_i$
2	$(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1})_{(-1/2,-1)}$	f_i^-	2	$(\mathbf{1}, \mathbf{1}; \bar{\mathbf{4}}, \mathbf{1})_{(1/2,1)}$	$\bar{f}_i^+$
4	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,\pm 2)}$	χ_i	32	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,0)}$	s_i^0
2	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/6,2/3)}$	$\bar{v}_i$	2	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/6,-2/3)}$	v_i

$$\mathsf{t}_{B-L} = \left(0, 0, 0, 0, 0, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}\right)(0, 0, 0, 0, 0, 2, 0, 0), \tag{5.2}$$

with two essential properties (cf. Table II):

(i) the spectrum includes three generations of quarks and leptons plus vectorlike exotics with respect to $G_{\rm SM} \times \mathrm{U}(1)_{B-L}$, and
(ii) there are SM singlets with $B - L$ charge ± 2.

In the following discussion we consider two different vacuum configurations to illustrate the dependence on the particular vacuum class, i.e. the set of SM singlets with nonzero supersymmetric VEVs.

1. Model 1, vacuum configuration A

Consider a vacuum configuration where the fields

$$\{\tilde{s}_i\} = \{\chi_1, \chi_2, \chi_3, \chi_4, h_1, h_2, h_3, h_4, h_5, h_6, h_9, h_{10}, s_1^0, s_4^0, s_5^0, s_6^0, s_9^0, s_{11}^0, s_{13}^0, s_{15}^0, s_{16}^0, s_{17}^0, s_{18}^0, s_{20}^0, s_{21}^0, s_{22}^0, s_{23}^0, s_{25}^0, s_{26}^0, s_{27}^0, s_{30}^0, s_{31}^0\} \tag{5.3}$$

develop a VEV while the expectation values of all other fields vanish. In this vacuum configuration we set 14 of the original 46 SM and hidden SU(4) singlets to zero. The emerging effective theory has the following properties:

(1) the unbroken gauge symmetries are

$$G_{\rm SM} \times G_{\rm hid}, \tag{5.4}$$

where $G_{\rm hid} = \mathrm{SU}(4)$.

(2) since $B - L$ is broken by two units, there is an effective matter parity $\mathbb{Z}_2^{\mathcal{M}}$.

(3) there is only one pair of Higgs candidates, ϕ_1 and $\bar{\phi}_1$; the μ-term

$$\mu = \frac{\partial^2 W}{\partial \phi_1 \partial \bar{\phi}_1}\bigg|_{\phi_1=\bar{\phi}_1=0} \tag{5.5}$$

vanishes up to order $\tilde{s}^6$, at which we work. That is, there is one pair of massless Higgs doublets.

(4) we check that the solution satisfies $F = 0$ for all fields (cf. **F-flatness** below) and in addition, switching on $\{\tilde{s}_i\}$-fields allows us to cancel the Fayet-Iliopoulos (FI) term without inducing D-terms (cf. Appendix B). Thus we obtain a supersymmetric vacuum configuration, $F = D = 0$.

(5) all vectorlike exotics decouple (cf. Appendix E 2).

(6) neutrino masses are suppressed via the seesaw mechanism.

That is, we have obtained a supersymmetric vacuum with the precise matter content of the MSSM *with an exact* R parity. This has to be contrasted to [53] where R parity was approximate, and to [33] where R parity exists only at the classical level, and where Y_e and Y_d vanish at the same level. Our model also does not suffer from the problem encountered in [37], where it was found that you can either decouple all exotics or have R parity but never both.

2. Charged fermion Yukawa matrices

The charged fermion Yukawa matrices are[7]

$$Y_u = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 1 \end{pmatrix}, \qquad Y_d = \begin{pmatrix} 0 & \tilde{s}^5 & 0 \\ \tilde{s}^5 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 \end{pmatrix}, \qquad Y_e = \begin{pmatrix} 0 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^5 & 0 & 0 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}. \tag{5.6}$$

Here, $\tilde{s}^n$ for n a non-negative integer, represents the smallest value of n for which this term appears in the matrix; thus giving the dominant contribution. Each term is in fact a sum of monomials containing several different SM and hidden SU(4) singlets. The up-type quark Yukawa matrix is given directly in terms of the coupling of the up-type Higgs to the three q and $\bar{u}$ fields. The down-type quark and charged lepton Yukawa matrices are obtained by integrating out a pair of vectorlike d- and $\bar{d}$-quarks and ℓ- and $\bar{\ell}$-fields, respectively. We find that the up and charged lepton Yukawa matrices have rank 3, while the down quark Yukawa matrix has only rank 2, at this order in $\tilde{s}$ singlets. However, we have checked that at order 8 in $\tilde{s}$ fields Y_d has rank 3.

[7]Quark and lepton doublets multiply the Yukawa matrices on the left.

3. Neutrino matrices

In our vacua SU(2)$'$ is broken such that the SU(2)$'$ doublets η_i and $\bar{\eta}_i$ correspond to SM singlets with $q_{B-L} = \pm 1$,

$$\bar{\eta}_1 = \begin{pmatrix} \bar{n}_{17} \\ \bar{n}_{18} \end{pmatrix}, \dots \bar{\eta}_5 = \begin{pmatrix} \bar{n}_{25} \\ \bar{n}_{26} \end{pmatrix} \quad \text{and} \quad \eta_1 = \begin{pmatrix} n_{14} \\ n_{15} \end{pmatrix}, \dots \eta_5 = \begin{pmatrix} n_{22} \\ n_{23} \end{pmatrix}. \tag{5.7}$$

The dimensions of the "right-handed" neutrino mass matrices are

$$\mathcal{M}_{nn} = 23 \times 23, \qquad \mathcal{M}_{n\bar{n}} = 23 \times 26, \qquad \mathcal{M}_{\bar{n}\bar{n}} = 26 \times 26, \tag{5.8}$$

with the complete $\bar{\nu} - \bar{\nu}$ mass matrix given by

$$\mathcal{M}_{\bar{\nu}\bar{\nu}} = \begin{pmatrix} \mathcal{M}_{nn} & \mathcal{M}_{n\bar{n}} \\ \mathcal{M}_{n\bar{n}}^T & \mathcal{M}_{\bar{n}\bar{n}} \end{pmatrix}. \tag{5.9}$$

We have checked that it indeed has full rank. For more details, see the Web site [67].

The Dirac neutrino Yukawa couplings have dimensions

$$Y_n = 4 \times 23, \qquad Y_{\bar{n}} = 4 \times 26. \tag{5.10}$$

The effective light neutrino mass operator emerges from

$$\kappa = Y_\nu \mathcal{M}_{\bar{\nu}\bar{\nu}}^{-1} Y_\nu^T, \tag{5.11}$$

where $Y_\nu = (Y_n, Y_{\bar{n}})$, by integrating out the pair of heavy leptons $\bar{\ell}_1$ and ℓ' where ℓ' is a linear combination of the ℓ_i. We have checked that the light neutrinos all obtain a small mass. The large dimension of the matrices effectively reduces the seesaw scale [65]. Also, neutrino phenomenology works differently in the presence of many "right-handed" neutrinos [83,84].

4. Dimension-5 baryon and lepton number violating operators

We further analyzed the question of dimension-5 proton decay operators. We find that both $qqq\ell$ and $\bar{u}\,\bar{u}\,\bar{d}\,\bar{e}$ appear at order $\tilde{s}^6$. They are also generated by integrating out the heavy exotics. For example, the following couplings exist:

$$\begin{array}{cccc} q_1\ell_1\bar{\delta}_4, & q_1\ell_1\bar{\delta}_5, & q_2\ell_2\bar{\delta}_4, & q_2\ell_2\bar{\delta}_5, \\ q_1q_1\delta_4, & q_1q_1\delta_5, & q_2q_2\delta_4, & q_2q_2\delta_5. \end{array} \tag{5.12}$$

Hence integrating out the states $\bar{\delta}_i$, δ_i produces dangerous dimension-5 operators. These must be sufficiently suppressed to be consistent with present bounds on proton decay [76,77]. We have verified that, for some particular $\tilde{s}$ VEVs, it is possible to suppress the $qqq\ell$ operators induced by the trilinear couplings (5.12). However, higher-order couplings also introduce baryon and lepton number violating operators. We have not been able to identify a suppression mechanism for such operators yet.

5. μ-term and Minkowski space

Since our singlet configuration satisfies $F = D = 0$, the vacuum energy is zero in the global SUSY limit. In supergravity, one should include nonperturbative moduli potentials which would be responsible for spontaneous SUSY breaking. In fact, in the context of gaugino condensation [85–88], the SU(4) subgroup of the second E_8 gives rise to TeV soft masses, which is a common feature of our "fertile" patch of the landscape [89]. A specific realization of SUSY breaking via gaugino condensation in heterotic string compactifications is given by Kähler stabilization [90–92]. Requiring a Minkowski vacuum puts a constraint on the total superpotential which includes contributions from charged matter and moduli.

An interesting feature of our singlet configuration is that the charged matter superpotential vanishes at order 6 in singlet fields,

$$\langle W(\tilde{s})\rangle = 0. \tag{5.13}$$

In fact the superpotential is a polynomial in $\tilde{s}$ fields and in this vacuum configuration each monomial term in W vanishes independently. Therefore, the total superpotential is given solely by its nonperturbative part. This is expected to be very small and thus a small gravitino mass and a small cosmological constant can in principle be achieved.

In this model we also find an intriguing correlation between the μ-term and $W(\tilde{s})$. Since the Higgs doublets are untwisted and the combination $\phi_1\bar{\phi}_1$ has vacuum quantum numbers, requiring that each monomial appearing in μ vanishes also implies $\langle W(\tilde{s})\rangle = 0$.[8] This means that the μ-term is of the order of the expectation value of W, i.e. the gravitino mass.

6. F-flatness

We analyze the F-terms in the configuration defined by (5.3). The only nonvanishing F-terms are

$$F_i = \frac{\partial W}{\partial s_i^0} \quad \text{where } s_i^0 \in \{s_3^0, s_7^0, s_{10}^0, s_{14}^0, s_{19}^0\}. \tag{5.14}$$

They read

$$F_3 = \alpha_{27}s_{26}^0 s_4^0 + \alpha_{28}s_{15}^0(s_{26}^0)^2 s_4^0 + \alpha_{29}s_{16}^0(s_{26}^0)^2 s_4^0, \tag{5.15a}$$

$$F_7 = \alpha_{41}h_1h_{10}s_{30}^0 s_4^0 + \alpha_{44}h_2h_9s_{30}^0 s_4^0 + \alpha_{46}s_{17}^0 s_{25}^0 s_{30}^0 s_4^0 + \alpha_{47}s_1^0 s_{18}^0 s_{26}^0 s_{30}^0 s_4^0 + \alpha_{50}s_{18}^0 s_{27}^0 s_{30}^0 s_4^0 + \alpha_{55}s_{15}^0(s_{30}^0)^2 s_4^0 + \alpha_{56}s_{16}^0(s_{30}^0)^2 s_4^0 + \alpha_{57}s_{20}^0 s_{30}^0 s_{31}^0 s_4^0 + \alpha_{58}s_{21}^0 s_{30}^0 s_{31}^0 s_4^0, \tag{5.15b}$$

$$F_{10} = \alpha_1h_1h_{10}s_{13}^0 s_{30}^0 + \alpha_4h_2h_9s_{13}^0 s_{30}^0 + \alpha_{12}s_{13}^0 s_{17}^0 s_{25}^0 s_{30}^0 + \alpha_{13}s_1^0 s_{13}^0 s_{18}^0 s_{26}^0 s_{30}^0 + \alpha_{16}s_{13}^0 s_{18}^0 s_{27}^0 s_{30}^0 + \alpha_{21}s_{13}^0 s_{15}^0(s_{30}^0)^2 + \alpha_{22}s_{13}^0 s_{16}^0(s_{30}^0)^2 + \alpha_{23}s_{13}^0 s_{20}^0 s_{30}^0 s_{31}^0 + \alpha_{24}s_{13}^0 s_{21}^0 s_{30}^0 s_{31}^0 + \alpha_{81}s_{26}^0 s_9^0 + \alpha_{82}s_{15}^0(s_{26}^0)^2 s_9^0 + \alpha_{83}s_{16}^0(s_{26}^0)^2 s_9^0, \tag{5.15c}$$

$$F_{14} = \alpha_{89}h_1h_{10}s_{30}^0 s_9^0 + \alpha_{92}h_2h_9s_{30}^0 s_9^0 + \alpha_{94}s_{17}^0 s_{25}^0 s_{30}^0 s_9^0 + \alpha_{95}s_1^0 s_{18}^0 s_{26}^0 s_{30}^0 s_9^0 + \alpha_{98}s_{18}^0 s_{27}^0 s_{30}^0 s_9^0 + \alpha_{103}s_{15}^0(s_{30}^0)^2 s_9^0 + \alpha_{104}s_{16}^0(s_{30}^0)^2 s_9^0 + \alpha_{105}s_{20}^0 s_{30}^0 s_{31}^0 s_9^0 + \alpha_{106}s_{21}^0 s_{30}^0 s_{31}^0 s_9^0, \tag{5.15d}$$

$$F_{19} = \alpha_{35}h_1h_2s_{30}^0 s_5^0 s_6^0 + \alpha_{36}s_{17}^0 s_{18}^0 s_{30}^0 s_5^0 s_6^0 + \alpha_{37}s_{20}^0 s_{22}^0 s_{30}^0 s_5^0 s_6^0 + \alpha_{38}s_{21}^0 s_{22}^0 s_{30}^0 s_5^0 s_6^0 + \alpha_{39}s_{20}^0 s_{23}^0 s_{30}^0 s_5^0 s_6^0 + \alpha_{40}s_{21}^0 s_{23}^0 s_{30}^0 s_5^0 s_6^0. \tag{5.15e}$$

Here α_i denote superpotential coefficients. The F-term equations, $F_i = 0$, have trivial and nontrivial solutions. For instance, (5.15a) has the trivial solutions

$$s_{26}^0 = 0 \quad \text{or} \quad s_4^0 = 0,$$

as well as the nontrivial solution relating various VEVs,

$$s_{16}^0 = \frac{-\alpha_{27} - \alpha_{28}s_{15}^0 s_{26}^0}{\alpha_{29}s_{26}^0}. \tag{5.16}$$

The strategy is now to take the nontrivial solution and insert it into the other equations. By doing so, one can trade the F-term constraints for relations between the VEVs. We find that this strategy is successful and we can satisfy all equations with nontrivial $\tilde{s}$ VEVs.

7. Model 1, vacuum configuration B

Now consider the "vacuum" configuration where the fields

$$\{\tilde{s}_i\} = \{\chi_1, \chi_2, \chi_3, \chi_4, h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, s_1^0, s_2^0, s_3^0, s_4^0, s_5^0, s_6^0, s_7^0, s_8^0, s_9^0, s_{10}^0, s_{11}^0, s_{12}^0, s_{13}^0, s_{14}^0, s_{15}^0, s_{16}^0, s_{17}^0, s_{18}^0, s_{20}^0, s_{21}^0, s_{22}^0, s_{23}^0, s_{24}^0, s_{25}^0, s_{26}^0, s_{27}^0, s_{28}^0, s_{29}^0, s_{30}^0, s_{31}^0, s_{32}^0\} \tag{5.17}$$

develop a VEV while the expectation values of all other fields vanish. Hence, in this vacuum configuration only one SM singlet VEV is set to zero, i.e. $\langle s_{19}^0\rangle = 0$. The emerging effective theory has most properties identical to those in Model 1A:

(1) the unbroken gauge symmetries are

$$G_{SM} \times G_{hid}, \tag{5.18}$$

where $G_{hid} = SU(4)$.

[8] This applies to the untwisted Higgs pairs in many models of our Mini-Landscape, for instance also to the model presented in [53,55].

(2) since $B - L$ is broken by two units, there is an effective matter parity $\mathbb{Z}_2^{\mathcal{M}}$.
(3) there is only one pair of Higgs candidates, ϕ_1 and $\bar{\phi}_1$; the μ-term

$$\mu = \frac{\partial^2 W}{\partial \phi_1 \partial \bar{\phi}_1} \bigg|_{\phi_1 = \bar{\phi}_1 = 0} \tag{5.19}$$

vanishes up to order $\tilde{s}^6$, at which we work. That is, there is one pair of massless Higgs doublets.
(4) we check that the solution satisfies $F = 0$ for all fields and in addition, switching on $\{\tilde{s}_i\}$-fields allows us to cancel the FI term without inducing D-terms.
(5) all vectorlike exotics decouple (cf. Appendix E 3).
(6) neutrino masses are suppressed via the seesaw mechanism.

However, the detailed form of the Yukawa and exotic mass matrices has changed.

8. Charged fermion Yukawa matrices

The charged fermion Yukawa matrices are

$$Y_u = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^6 & \tilde{s}^6 & 1 \end{pmatrix}, \qquad Y_d = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & 0 \\ \tilde{s}^5 & \tilde{s}^5 & 0 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}, \qquad Y_e = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}. \tag{5.20}$$

The up-type quark Yukawa matrix is given directly in terms of the coupling of the up-type Higgs to the three q and $\bar{u}$ fields. The down-type quark and charged lepton Yukawa matrices are obtained by integrating out a pair of vectorlike d- and $\bar{d}$-quarks and ℓ- and $\bar{\ell}$-fields, respectively. We find that (just as in Model 1A) the up and charged lepton Yukawa matrices have rank 3, while the down quark Yukawa matrix has only rank 2, at this order in $\tilde{s}$ singlets. In fact, to this order in SM singlet fields, the superpotential does not couple two right-handed down quarks, $\bar{d}_{3,4}$, to the quark doublets. This is because $\bar{d}_{3,4}$ are in the T_4 twisted sector. However, we have verified that some of the zeros in Y_d get filled in at higher orders and at order 8 Y_d has rank 3. Note that in this vacuum configuration the Yukawa matrices retain a form consistent with the underlying D_4 family symmetry.[9]

9. Neutrino matrices

We have checked that all right-handed neutrinos obtain mass in this vacuum configuration. Thus the seesaw mechanism works exactly as in Model 1A, although the detailed forms of the matrices differ. For more details, see the Web site [67].

10. Dimension-5 baryon and lepton number violating operators

We further analyzed the question of dimension-5 proton decay operators. We find that both $qqq\ell$ and $\bar{u}\,\bar{u}\,\bar{d}\,\bar{e}$ appear at order $\tilde{s}^6$. They are also generated by integrating out the heavy exotics. For example, the following couplings exist:

$$\begin{matrix} q_1\ell_1\bar{\delta}_4, & q_1\ell_1\bar{\delta}_5, & q_2\ell_2\bar{\delta}_4, & q_2\ell_2\bar{\delta}_5, \\ q_1q_1\delta_4, & q_1q_1\delta_5, & q_2q_2\delta_4, & q_2q_2\delta_5. \end{matrix} \tag{5.21}$$

Hence integrating out the states $\bar{\delta}_i$, δ_i produces dangerous dimension-5 operators. These must be sufficiently suppressed to be consistent with present bounds on proton decay [76,77]. We have verified that, for some particular $\tilde{s}$ VEVs, it is possible to suppress the $qqq\ell$ operators induced by the trilinear couplings (5.21). However, higher-order couplings also introduce baryon and lepton number violating operators. We have not been able to identify a suppression mechanism for such operators yet.

11. μ-term and Minkowski space

As in Model 1A, requiring a Minkowski vacuum puts a constraint on the total superpotential which includes contributions from charged matter and moduli. An interesting feature of the present vacuum configuration is that the SM matter singlet superpotential to order $\tilde{s}^6$ is of the form,

$$W(\tilde{s}) = \sum_i P_i(\tilde{D})\tilde{P}_i(\tilde{s}), \tag{5.22}$$

where P_i are polynomials in SM singlet fields (the index i labels a particular polynomial) which are either D_4 doublets, which we now relabel as $\tilde{D}$, or SM and D_4 singlets, $\tilde{s}$. In particular, the D_4 doublets which enter $W(\tilde{s})$ are

$$\begin{matrix} \tilde{D}_1 = (s_3^0, s_9^0) & \tilde{D}_2 = (s_4^0, s_{10}^0) & \tilde{D}_3 = (s_5^0, s_{11}^0) \\ \tilde{D}_4 = (s_6^0, s_{12}^0) & \tilde{D}_5 = (s_7^0, s_{13}^0) & \tilde{D}_6 = (s_8^0, s_{14}^0). \end{matrix} \tag{5.23}$$

The polynomial in D_4 doublets is, to this order, quadratic in doublets and is given by the trivial D_4 singlet scalar product, for example,

$$\tilde{D}_1 \cdot \tilde{D}_2 = (s_3^0 s_4^0 + s_9^0 s_{10}^0). \tag{5.24}$$

We then find (up to calculable dimensionful coefficients in units of the string scale)

[9] The D_4 family symmetry is a consequence of the space group selection rules and the geometry of the SO(4) torus (Fig. 1) [52,93]. States sitting at the two vertical fixed points on the SO(4) torus transform as doublets under D_4.

$$W = (\tilde{D}_1 \cdot \tilde{D}_2)(s^0_{26} + s^0_{29} + (s^0_{26}s^0_{26} + s^0_{26}s^0_{29} + s^0_{29}s^0_{29})(s^0_{15} + s^0_{16})) + (\tilde{D}_1 \cdot \tilde{D}_6 + \tilde{D}_2 \cdot \tilde{D}_5)s^0_{30}[s^0_{30}(s^0_{15} + s^0_{16}) + s^0_{17}(s^0_{25} + s^0_{28}) + s^0_{18}(s^0_{24} + s^0_{27}) + s^0_{31}(s^0_{20} + s^0_{21}) + s^0_{32}(s^0_{22} + s^0_{23}) + (s^0_{19} + s^0_1 s^0_{18} + s^0_2 s^0_{17})(s^0_{26} + s^0_{29}) + h_1(h_8 + h_{10}) + h_2(h_7 + h_9)] + (\tilde{D}_3 \cdot \tilde{D}_4)s^0_{19}s^0_{30}(s^0_{17}s^0_{18} + h_1 h_2 + (s^0_{20} + s^0_{21})(s^0_{22} + s^0_{23})). \tag{5.25}$$

Thus, to order 6 in SM and hidden SU(4) singlets, the polynomials $P_i(\tilde{D})$ are completely determined by the D_4 symmetry, while the polynomials $\tilde{P}_i(\tilde{s})$ are nontrivial for all i. One particular $F = D = 0$ solution is given by the roots of $\langle P_i(\tilde{D})\rangle = \langle \tilde{P}_i(\tilde{s})\rangle = 0$ for all polynomials i. Hence, once again,

$$\langle W(\tilde{s})\rangle = 0. \tag{5.26}$$

Therefore, the total superpotential is given solely by its nonperturbative part. This is expected to be very small and thus a small gravitino mass and a small cosmological constant can in principle be achieved.

In addition, just as in Model 1A, the μ-term contains all terms present in $W(\tilde{s})$ and to order 6 in SM singlets we have $\mu = 0$, when $W = 0$. This means that the μ-term is of the order of the expectation value of W, i.e. the gravitino mass.

B. Model 2

The model is defined by the shift and Wilson lines given in Appendix F. It was already included as an example in ML [40,65].

The gauge group after compactification is

$$G = [\mathrm{SU}(3) \times \mathrm{SU}(2)] \times [\mathrm{SO}(8) \times \mathrm{SU}(2)'] \times \mathrm{U}(1)^8. \tag{5.27}$$

As before, we are able to define a suitable $B - L$ generator,

$$\mathsf{t}_{B-L} = \left(1, 1, 0, 0, 0, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}\right)\left(\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, 0, 0, 0\right) \tag{5.28}$$

with two important properties (cf. Table III):

(i) the spectrum includes three generations of quarks and leptons plus vectorlike exotics with respect to $G_{\mathrm{SM}} \times \mathrm{U}(1)_{B-L}$, and

(ii) there are SM singlets with $B - L$ charge ± 2.

Consider a "vacuum" configuration where the fields

$$\{\tilde{s}_i\} = \{\chi_1, \chi_2, s^0_3, s^0_5, s^0_8, s^0_9, s^0_{12}, s^0_{15}, s^0_{16}, s^0_{22}, s^0_{24}, s^0_{35}, s^0_{41}, s^0_{43}, s^0_{46}, h_2, h_3, h_5, h_9, h_{13}, h_{14}, h_{20}, h_{21}, h_{22}\} \tag{5.29}$$

develop a VEV while the expectation values of all other fields vanish. The emerging effective theory has the following properties:

(1) the unbroken gauge symmetries are

$$G_{\mathrm{SM}} \times G_{\mathrm{hid}}, \tag{5.30}$$

where $G_{\mathrm{hid}} = \mathrm{SO}(8)$.

(2) since $B - L$ is broken by two units, there is an effective matter parity $\mathbb{Z}_2^{\mathcal{M}}$.

(3) the Higgs mass terms are

$$\bar{\phi}_i(\mathcal{M}_{\bar{\phi}\phi})_{ij}\phi_j,$$

$$\text{where } \mathcal{M}_{\bar{\phi}\phi} = \begin{pmatrix} \tilde{s}^4 & 0 & 0 & \tilde{s} \\ \tilde{s} & \tilde{s}^3 & \tilde{s}^3 & \tilde{s}^6 \\ \tilde{s}^5 & 0 & 0 & \tilde{s}^3 \\ \tilde{s} & 0 & 0 & \tilde{s}^3 \end{pmatrix}. \tag{5.31}$$

The up-type Higgs h_u is a linear combination of $\bar{\phi}_1$, $\bar{\phi}_3$ and $\bar{\phi}_4$,

$$h_u \sim \tilde{s}^2\bar{\phi}_1 + \bar{\phi}_3 + \tilde{s}^4\bar{\phi}_4, \tag{5.32}$$

while the down-type Higgs is composed out of ϕ_2 and ϕ_3,

$$h_d \sim \phi_2 + \phi_3. \tag{5.33}$$

The vacuum configuration is chosen such that the μ-term, being defined as the smallest eigenvalue of $\mathcal{M}_{\bar{\phi}\phi}$,

$$\mu = \frac{\partial^2 W}{\partial h_d \partial h_u}\bigg|_{h_u = h_d = 0}, \tag{5.34}$$

vanishes up to order $\tilde{s}^6$, at which we work.

TABLE III. Spectrum. The quantum numbers under SU(3) × SU(2) × [SO(8) × SU(2)′] are shown in boldface; hypercharge and $B - L$ charge appear as subscript. Note that the states $s_i^{\pm}$, m_i and v_i have different $B - L$ charges for different i, which we do not explicitly list.

#	Representation	Label	#	Representation	Label
3	$(\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/6,1/3)}$	q_i	3	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1,1)}$	$\bar{e}_i$	4	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(0,*)}$	m_i
4	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$	1	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/3,1/3)}$	d_i
4	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(-1/2,-1)}$	ℓ_i	1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
4	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(-1/2,0)}$	ϕ_i	4	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
3	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$	3	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/2,*)}$	s_i^+	20	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/2,*)}$	s_i^-
15	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,1)}$	$\bar{n}_i$	12	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,-1)}$	n_i
3	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0,1)}$	$\bar{\eta}_i$	3	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0,-1)}$	η_i
20	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0,0)}$	h_i	2	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})_{(0,0)}$	y_i
2	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(1/2,1)}$	x_i^+	2	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(-1/2,-1)}$	x_i^-
2	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,\pm 2)}$	χ_i	18	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0,0)}$	s_i^0
4	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/6,*)}$	$\bar{v}_i$	4	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/6,*)}$	v_i
2	$(\mathbf{1}, \mathbf{1}; \mathbf{8}, \mathbf{1})_{(0,-1/2)}$	f_i	2	$(\mathbf{1}, \mathbf{1}; \mathbf{8}, \mathbf{1})_{(0,1/2)}$	$\bar{f}_i$
5	$(\mathbf{1}, \mathbf{1}; \mathbf{8}, \mathbf{1})_{(0,0)}$	w_i			

(4) we check that switching on $\{\tilde{s}_i\}$-fields allows us to cancel the FI term without inducing D-terms (cf. Appendix B).
(5) all exotics decouple (cf. Appendix F 2).
(6) neutrino masses are suppressed via the seesaw mechanism.

Thus, again we have obtained a supersymmetric vacuum with the precise matter content of the MSSM and R parity.

1. Charged fermion Yukawa matrices

The up-Higgs Yukawa couplings decompose into

$$W_{\text{Yukawa}} \supset \sum_{k=1}^{4} (Y_u)_{ij}^{(k)} q_i \bar{u}_j \bar{\phi}_k, \tag{5.35}$$

where

$$Y_u^{(1)} = \begin{pmatrix} 0 & 0 & \tilde{s}^6 \\ 0 & 0 & \tilde{s}^6 \\ \tilde{s}^3 & \tilde{s}^3 & 1 \end{pmatrix}, \qquad Y_u^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix},$$
$$Y_u^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix}, \qquad Y_u^{(4)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix}. \tag{5.36a}$$

Thus, the physical 3×3 up-Higgs Yukawa matrix is

$$Y_u \sim \tilde{s}^2 Y_u^{(1)} + Y_u^{(3)} + \tilde{s}^4 Y_u^{(4)} = \begin{pmatrix} 0 & 0 & \tilde{s}^8 \\ 0 & 0 & \tilde{s}^8 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^2 \end{pmatrix}. \tag{5.37}$$

Note that due to the Higgs mixing the top quark Yukawa coupling for this vacuum configuration is given by $\tilde{s}^2$. Thus the corresponding $\tilde{s}$ VEVs are required to be quite large.

The down-Higgs Yukawa couplings decompose into

$$W_{\text{Yukawa}} \supset \sum_{k=1}^{4} (Y_d)_{ij}^{(k)} q_i \bar{d}_j \phi_k, \tag{5.38}$$

where

$$Y_d^{(1)} = \begin{pmatrix} \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}, \qquad Y_d^{(2)} = \begin{pmatrix} 1 & \tilde{s}^4 & 0 & 0 \\ \tilde{s}^4 & 1 & 0 & 0 \\ \tilde{s} & \tilde{s} & 0 & 0 \end{pmatrix},$$
$$Y_d^{(3)} = \begin{pmatrix} 1 & \tilde{s}^4 & 0 & 0 \\ \tilde{s}^4 & 1 & 0 & 0 \\ \tilde{s} & \tilde{s} & 0 & 0 \end{pmatrix}, \qquad Y_d^{(4)} = 0. \tag{5.39a}$$

The physical 3×3 down-Higgs Yukawa matrix emerges by integrating out a pair of vectorlike $d-$ and $\bar{d}-$ quarks,

$$Y_d = \begin{pmatrix} 1 & \tilde{s}^3 & 0 \\ 1 & \tilde{s}^3 & 0 \\ \tilde{s} & \tilde{s}^4 & 0 \end{pmatrix}. \tag{5.40}$$

We note that both the up and down quarks are massless at order 6 in SM singlets. However, we have checked that the up quark becomes massive at order 7 and the down quark gets a mass at order 8.

The charged lepton Yukawa couplings decompose into

$$W_{\text{Yukawa}} \supset \sum_{k=1}^{4} (Y_e)_{ij}^{(k)} \ell_i \bar{e}_j \phi_k, \tag{5.41}$$

where

$$Y_e^{(1)} = \begin{pmatrix} \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 \\ \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad Y_e^{(2)} = \begin{pmatrix} 1 & \tilde{s}^4 & \tilde{s} \\ \tilde{s}^4 & 1 & \tilde{s} \\ 0 & 0 & \tilde{s}^6 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix},$$
$$Y_e^{(3)} = \begin{pmatrix} 1 & \tilde{s}^4 & \tilde{s} \\ \tilde{s}^4 & 1 & \tilde{s} \\ 0 & 0 & \tilde{s}^6 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix}, \qquad Y_e^{(4)} = \begin{pmatrix} 0 & 0 & \tilde{s}^5 \\ 0 & 0 & \tilde{s}^5 \\ 0 & 0 & \tilde{s}^6 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix}. \tag{5.42a}$$

The physical 3×3 matrix emerges by integrating out a pair of vectorlike $\ell-$ and $\bar{\ell}-$ leptons,

$$Y_e = \begin{pmatrix} 1 & 1 & \tilde{s} \\ \tilde{s} & \tilde{s} & \tilde{s}^2 \\ 0 & 0 & \tilde{s}^6 \end{pmatrix}. \tag{5.43}$$

2. Neutrino masses

We consider vacua where $SU(2)'$ is broken. This means that the η_i and $\bar{\eta}_i$ give rise to further SM singlets with $q_{B-L} = \pm 1$,

$$\bar{\eta}_1 = \begin{pmatrix} \bar{n}_{16} \\ \bar{n}_{17} \end{pmatrix}, \ldots \bar{\eta}_3 = \begin{pmatrix} \bar{n}_{20} \\ \bar{n}_{21} \end{pmatrix} \quad \text{and}$$
$$\eta_1 = \begin{pmatrix} n_{13} \\ n_{14} \end{pmatrix}, \ldots \eta_3 = \begin{pmatrix} n_{17} \\ n_{18} \end{pmatrix}. \tag{5.44}$$

The dimensions of the "right-handed" neutrino mass matrices are

$$\mathcal{M}_{nn} = 18 \times 18, \tag{5.45}$$

$$\mathcal{M}_{n\bar{n}} = 18 \times 21, \tag{5.46}$$

$$\mathcal{M}_{\bar{n}\bar{n}} = 21 \times 21, \tag{5.47}$$

with the neutrino mass matrix given by

$$\mathcal{M}_{\bar{\nu}\bar{\nu}} = \begin{pmatrix} \mathcal{M}_{\bar{n}\bar{n}} & \mathcal{M}_{n\bar{n}}^T \\ \mathcal{M}_{n\bar{n}} & \mathcal{M}_{nn} \end{pmatrix}. \tag{5.48}$$

We have checked that it has full rank.

The neutrino Yukawa couplings decompose into

$$W_{\text{Yukawa}} \supset \sum_{k=1}^{4} (Y_n)_{ij}^{(k)} \ell_i n_j \bar{\phi}_k + (Y_{\bar{n}})_{ij}^{(k)} \ell_i \bar{n}_j \bar{\phi}_k, \quad (5.49)$$

where $Y_n^{(1)}$, $Y_n^{(2)}$ are nonvanishing 4×18 matrices; $Y_n^{(k>2)} = 0$ and $Y_{\bar{n}}^{(1)}$, $Y_{\bar{n}}^{(2)}$, $Y_{\bar{n}}^{(3)}$, $Y_{\bar{n}}^{(4)}$ are nonvanishing 4×21 matrices. The effective neutrino mass matrix obtained as

$$\kappa = Y_\nu \mathcal{M}_{\bar{\nu}\bar{\nu}}^{-1} Y_\nu^T, \quad (5.50)$$

where $Y_\nu = (Y_n, Y_{\bar{n}})$ and κ has nonzero determinant. See the Web site [67] for details.

3. Dimension-5 baryon and lepton number violating operators

We have looked for effective dimension-5 baryon and lepton number violating operators in this model. We find that to order $\tilde{s}^6$ no such operators exist. However, these operators can be generated once the exotics δ_i, $\bar{\delta}_i$ are integrated out. Fortunately, a clever choice of VEVs for the fields $\{\tilde{s}_i\}$ can guarantee sufficient suppression of all induced $qqq\ell$ operators, consistent with current bounds on proton decay [76,77].

4. μ-term and Minkowski space

Unlike in the previous model, there is no relation between the μ-term and $W(\tilde{s})$. This is because the Higgs doublets do not come entirely from the untwisted sector. Requiring spontaneous SUSY breaking in a Minkowski vacuum puts a constraint on the moduli VEVs. Fine-tuning is likely to be necessary to obtain a realistic gravitino mass as well as a small cosmological constant.

VI. CONCLUSIONS AND DISCUSSION

In this paper we have described the construction of heterotic MSSMs with R parity. Our setup is based on a particular $\mathbb{Z}_6$-II orbifold with an SO(10) local GUT structure. In the first part of the paper we have obtained 218 models with the MSSM gauge group structure, 3 light families and vectorlike exotics. We show that all the vectorlike exotics can decouple along D-flat directions for 190 of these models.[10] The total number of inequivalent models with SO(10) shifts and two Wilson lines is $3 \cdot 10^4$. Hence 0.6% of our total model set are MSSM candidates. This can be compared with D-brane constructions where the probability of getting MSSM-like models is much less than 10^{-9} or Gepner orientifold constructions where this probability is 10^{-14}.

[10] In this analysis, we have taken into account superpotential terms up to order 6 in SM singlets. At higher orders, we expect more models to be retained.

In the second part of the paper we go further down the road towards the MSSM. We define a successful strategy for obtaining models with an exact R parity. We find 87 models which have a renormalizable top Yukawa coupling. We identify 15 models with an exact R parity, no light exotics or U(1) gauge bosons and an order 1 top quark Yukawa coupling.[11]

We present two explicit benchmark examples satisfying the following criteria:

(i) MSSM spectrum below the string scale—
all exotics decouple;
one pair of light Higgs doublets;
top quark Yukawa coupling of order 1;
nontrivial Yukawa matrices for charged fermions;
seesaw mechanism for neutrinos;
(ii) an exact R parity.

The two examples have different phenomenological properties such as different structures of the Yukawa coupling matrices and dimension-5 operators. In particular, the Yukawa matrices Y_d and Y_e have more nonvanishing entries in Model 1B than in Model 1A. In both Models 1A/B the lightest down-type quark is massless at order 6 in SM singlets and becomes massive at order 8. The top Yukawa coupling is order 1 in Models 1A/B, while it is order $\tilde{s}^2$ in Model 2. This is due to Higgs doublet mixing in the latter. In Model 2, both the up and down quarks are massless at order 6 in SM singlets. However, the up quark becomes massive at order 7 and the down quark gets a mass at order 8.

An interesting feature of Model 1 is that there is a correlation between the μ term and the expectation value of the superpotential. In fact the pair of Higgs fields are the only vectorlike fields whose mass is correlated with the expectation value of the superpotential, while all exotics can consistently get mass with $W = 0$. This provides a novel, stringy solution to the MSSM μ problem. Indeed, in Models 1A/B the vacuum expectation value of the superpotential and μ both vanish at order 6 in SM singlets. Thus, neglecting nonperturbative effects, this model leads to a supersymmetric Minkowski vacuum with $\mu = 0$. One expects that when nonperturbative effects (hidden sector gaugino condensation) are taken into account, supersymmetry is broken at a hierarchically small scale and, because of the correlation between μ and $\langle W \rangle$, μ is of order the gravitino mass. In Model 2, on the other hand, the superpotential does not vanish in this limit and inclusion of nonperturbative contributions to the superpotential is necessary.

[11] The number 15 is a lower bound, since our search is based on a specific strategy related to $B - L$ symmetry. Furthermore, more models are retained if we do not insist on having a renormalizable top Yukawa coupling. Also one can drop the strict constraint that exotics be vectorlike with respect to $B - L$. For example, two exotics x, $\bar{x}$ with $B - L$ charge -1 can get mass from a SM singlet VEV with charge $+2$.

Dimension-5 baryon and lepton number violating operators come from two sources. They are generated in the superpotential to some order in SM singlets. They may also be generated when integrating out heavy exotics. In Models 1A/B the direct dimension-5 operators appear at order $\tilde{s}^6$, while in Model 2 they do not appear at this order. In addition, in Models 1A/B and Model 2 dimension-5 operators appear when integrating out heavy exotics. In Model 2 these can be sufficiently suppressed with some fine-tuning.

There are some phenomenological issues that we have not addressed in this paper. In particular, we have not studied precision gauge coupling unification. Although hypercharge is normalized as in 4D GUTs thus allowing gauge coupling unification in the first approximation, there are various corrections that can be important. First, a detailed analysis would require the calculation of string threshold corrections in the presence of discrete Wilson lines. However in specific cases these corrections are known to be small [94]. Second, there are corrections from the vectorlike exotic states. It is possible that precision gauge coupling unification may require anisotropic compactifications, leading to an effective orbifold GUT [20,52,78,95].

Another issue concerns proton stability. The examples we studied are challenged by the presence of dimension-5 proton decay operators. Their suppression may require additional (discrete) symmetries. There are also dimension-6 operators, generated by GUT gauge boson exchange, which we have not discussed.

Finally, there are the usual questions of moduli stabilization and supersymmetry breakdown in a Minkowski vacuum. Some of them we discussed previously in [89]. We have not addressed all of these issues here. On the other hand, it is clear that if given the freedom of arbitrarily tuning moduli VEVs we are not able to find the MSSM, the whole approach would be futile. However, with a number of MSSM candidates in this fertile patch of the landscape, it is now imperative to tackle the hard problems just mentioned.

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APPENDIX A: PHYSICAL STATES AND STRING SELECTION RULES

In this appendix, we discuss how to build consistent physical states. Furthermore, we list the string selection rules used in this work. Finally, we comment on an additional selection rule present in the literature: the γ rule. We find that our construction of physical states is useful in order to apply the γ rule correctly. It turns out that, in contrast to previous statements, the γ rule does not further constrain allowed couplings.

1. Physical states

An element of the *space group* $g = (\theta^k, n_\alpha e_\alpha) \in S$, where θ is the twist and e_α are the lattice basis vectors, corresponds to a boundary condition of a closed string [43,44]. For $k = 0$ ($k \neq 0$), the string is named *untwisted string* (*twisted string*). Focusing on its bosonic degrees of freedom in the six extra dimensions, the boundary condition reads

$$X(\tau, \sigma + 2\pi) = gX(\tau, \sigma), \qquad \text{(A1)}$$

where g is called the *constructing element* of the closed string. For each constructing element g, there exists a corresponding Hilbert space $\mathcal{H}_g$ of physical states. Using a mode expansion for $X(\tau, \sigma)$, the general solutions of the string equation of motion with boundary condition Eq. (A1) can be written down. From these solutions one finds that twisted strings are localized at the *fixed-point* $f_g \in \mathbb{R}^6$ corresponding to g (i.e. $\theta^k f_g + n_\alpha e_\alpha = f_g$). Furthermore, their quantization leads to the mass equation for left-movers (and the mass equation for right-movers is derived analogously). Focusing on the massless case, the solutions are denoted by[12]

$$|q_{\rm sh}\rangle_{\rm R} \otimes |P_{\rm sh}\rangle_{\rm L}, \qquad \text{(A2)}$$

with *shifted momenta* $q_{\rm sh} \equiv q + v_g$ and $P_{\rm sh} \equiv P + V_g$, where q and P lie in the SO(8) weight lattice and $E_8 \times E_8$ root lattice, respectively. The local twist and shift corresponding to the space group element $g = (\theta^k, n_\alpha e_\alpha)$ are defined by $v_g \equiv kv$ and $V_g \equiv kV + n_\alpha W_\alpha$, respectively. Since the string is completely specified by its constructing element g and its left- and right-moving shifted momenta $P_{\rm sh}$ and $q_{\rm sh}$, we write down a first ansatz for a physical state:

$$|{\rm phys}\rangle \sim |q_{\rm sh}\rangle_{\rm R} \otimes |P_{\rm sh}\rangle_{\rm L} \otimes |g\rangle. \qquad \text{(A3)}$$

[12] In this discussion we disregard oscillator states. Their inclusion is straightforward and does not change our conclusions.

Up to now it is not guaranteed that a physical state is actually compatible with the orbifold. To ensure this compatibility, invariance of $|\text{phys}\rangle$ under the action of all elements of the orbifold group $O \subset S \otimes G$ must be imposed (G is the embedding of S into the gauge degrees of freedom and is called the *gauge twisting group*). To do so, Eq. (A1) is multiplied by an arbitrary element $h = (\theta^l, m_\alpha e_\alpha) \in S$:

$$hX(\tau, \sigma + 2\pi) = hgX(\tau, \sigma) \tag{A4}$$

$$\Leftrightarrow hX(\tau, \sigma + 2\pi) = hgh^{-1}hX(\tau, \sigma). \tag{A5}$$

Furthermore, the transformation properties of left- and right-movers under h are

$$|q_{\text{sh}}\rangle_{\text{R}} \otimes |P_{\text{sh}}\rangle_{\text{L}} \xrightarrow{h} \Phi |q_{\text{sh}}\rangle_{\text{R}} \otimes |P_{\text{sh}}\rangle_{\text{L}}, \tag{A6}$$

where[13]

$$\Phi \equiv e^{2\pi i[q_{\text{sh}}\cdot v_h - P_{\text{sh}}\cdot V_h]}. \tag{A7}$$

Now, we can distinguish two cases:

Commuting elements: $[h, g] = 0$

First, let us consider the transformation property of $|\text{phys}\rangle$ with respect to a commuting element h. In this case, Eq. (A5) yields

$$hX(\tau, \sigma + 2\pi) = ghX(\tau, \sigma), \tag{A8}$$

i.e. the constructing element g is invariant under the action of h,

$$|g\rangle \xrightarrow{h} |hgh^{-1}\rangle = |g\rangle. \tag{A9}$$

hX closes under the same constructing element g as X. Thus, both give rise to the same Hilbert space $\mathcal{H}_g \xrightarrow{h} \mathcal{H}_{hgh^{-1}} = \mathcal{H}_g$. Furthermore, on the orbifold space $\mathbb{R}^6/S$ the string coordinates hX and X are identified. Thus, hX and X describe the same physical state.

In summary, provided a constructing element g, we have shown that for commuting elements h, hX and X give rise to the same physical state from the same Hilbert space. Since h has to act as the identity on $|\text{phys}\rangle$, the following condition follows using Eqs. (A3), (A6), and (A9):

$$q_{\text{sh}} \cdot v_h - P_{\text{sh}} \cdot V_h \stackrel{!}{=} 0 \bmod 1. \tag{A10}$$

Noncommuting elements: $[h, g] \neq 0$

Next, considering a noncommuting element h in Eq. (A5) yields

$$hX(\tau, \sigma + 2\pi) = (hgh^{-1})hX(\tau, \sigma), \tag{A11}$$

i.e. the constructing element g is not invariant under the action of h,

$$|g\rangle \xrightarrow{h} |hgh^{-1}\rangle \neq |g\rangle. \tag{A12}$$

In the upstairs picture, i.e. in the covering space $\mathbb{R}^6$ of the orbifold $\mathbb{R}^6/S$, one has different Hilbert spaces for the states with boundary conditions g and hgh^{-1}. In this picture, Eq. (A12) says that h maps states from a given Hilbert space $\mathcal{H}_g$ onto a different Hilbert space $\mathcal{H}_{hgh^{-1}}$. Subsequent application of h then leads to the sequence[14]

$$\mathcal{H}_g \xrightarrow{h} \mathcal{H}_{hgh^{-1}} \xrightarrow{h} \mathcal{H}_{h^2gh^{-2}} \xrightarrow{h} \mathcal{H}_{h^3gh^{-3}} \xrightarrow{h} \ldots. \tag{A13}$$

The crucial point is now that on the orbifold hX and X are identified. This means that, on the orbifold, the different Hilbert spaces $\mathcal{H}_{h^ngh^{-n}}$ of the upstairs picture are to be combined into a single orbifold Hilbert space. Invariant states are then linear combinations of states from all $\mathcal{H}_{h^ngh^{-n}}$. Such linear combinations do, in general, involve relative phase factors (often called *gamma-phase* γ). So, the new ansatz for a physical state reads

$$\begin{aligned}|\text{phys}\rangle &\sim \sum_n (e^{-2\pi i n\gamma}|q_{\text{sh}}\rangle_{\text{R}} \otimes |P_{\text{sh}}\rangle_{\text{L}} \otimes |h^ngh^{-n}\rangle)\\ &= |q_{\text{sh}}\rangle_{\text{R}} \otimes |P_{\text{sh}}\rangle_{\text{L}} \otimes \left(\sum_n e^{-2\pi i n\gamma}|h^ngh^{-n}\rangle\right),\end{aligned} \tag{A14}$$

where $\gamma = \text{integer}/N$, N being the order of the orbifold. The geometrical part of the linear combination transforms nontrivially under h

$$\sum_n e^{-2\pi i n\gamma}|h^ngh^{-n}\rangle \xrightarrow{h} e^{2\pi i\gamma}\sum_n e^{-2\pi i n\gamma}|h^ngh^{-n}\rangle. \tag{A15}$$

Since h has to act as the identity on $|\text{phys}\rangle$, the following condition follows using Eqs. (A6), (A14), and (A15) for noncommuting elements:

$$q_{\text{sh}} \cdot v_h - P_{\text{sh}} \cdot V_h + \gamma \stackrel{!}{=} 0 \bmod 1. \tag{A16}$$

Notice that γ depends on h. Thus we can always choose $\gamma(h)$ such that this condition is satisfied.[15] In principle, these steps have to be repeated for all noncommuting elements in order to ensure invariance of the physical state under the action of the whole orbifold group $O \subset S \otimes G$. The result for $|\text{phys}\rangle$ reads

$$\begin{aligned}|\text{phys}\rangle = {}& |q_{\text{sh}}\rangle_{\text{R}} \otimes |P_{\text{sh}}\rangle_{\text{L}}\\ &\otimes \left(\sum_{h=\mathbb{1} \text{ or } [h,g]\neq 0} e^{-2\pi i\gamma(h)}|hgh^{-1}\rangle\right),\end{aligned} \tag{A17}$$

where the summation over h is such that each term $|hgh^{-1}\rangle$ appears only once. Note that the summation over h can be understood as a summation over all elements of the conjugacy class of g.

[13] Here, we set $\Phi_{\text{vac}} = 1$ as discussed in [96].

[14] Note that in all $\mathcal{H}_{h^ngh^{-n}}$ the left-moving momenta P_{sh} of equivalent states are identical. The same holds for q_{sh}.

[15] In this sense, building linear combinations and computing the γ phase is not a projection condition. Note that $\gamma(h)$ is well-defined: if $h_1gh_1^{-1} = h_2gh_2^{-1}$ then $\gamma(h_1) = \gamma(h_2)$.

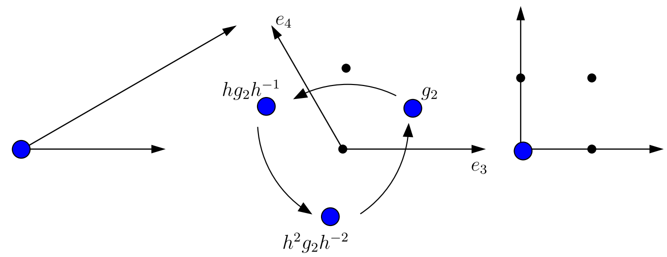

FIG. 2 (color online). Illustration of the γ-factor. The fixed point associated with the space group element $g_2 = (\theta, e_3)$ is invariant under (θ, e_3), but transforms into equivalent fixed points outside the fundamental domain under $h = (\theta, 0)$. To form an eigenstate of $(\theta, 0)$, one needs to build linear combinations of the equivalent fixed points. The corresponding eigenvalues can be 1, $e^{\pm 2\pi i/3}$.

Example

To illustrate the construction of physical states, let us consider an example in the first twisted sector of the $\mathbb{Z}_6$-II orbifold. In the SU(3) lattice spanned by e_3 and e_4, there are three inequivalent fixed points associated to the constructing elements $g_1 = (\theta, 0)$, $g_2 = (\theta, e_3)$ and $g_3 = (\theta, e_3 + e_4)$, or analogously $g_i = (\theta, a_i e_3 + b_i e_4)$ for $i = 1, 2, 3$ with $a_i = (0, 1, 1)$ and $b_i = (0, 0, 1)$. Then, restricting to the SU(3) lattice, the geometrical part of a physical state can be written as

$$\sum_{n,m} e^{-2\pi i(n+m)\gamma} |(\theta, (n + m + a_i)e_3 + (2m - n + b_i)e_4)\rangle. \tag{A18}$$

Since the action of θ in the SU(3) lattice has order 3, the only possible θ-eigenvalues of Eq. (A18) have $\gamma = 0, \pm\frac{1}{3}$. In the case of $\gamma = 0$, Eq. (A18) is invariant under all rotations and translations for all three g_i. However, if $\gamma = \pm\frac{1}{3}$, the eigenvalue of Eq. (A18) depends on g_i: for the fixed point at the origin associated to g_1, Eq. (A18) is invariant under θ, but has an eigenvalue $e^{2\pi i\gamma(k+l)}$ under $(\mathbb{1}, ke_3 + le_4)$. Similarly, for the fixed points away from the origin, corresponding to g_i ($i \neq 1$), Eq. (A18) picks up a phase $e^{-2\pi i\gamma(a_i+b_i)}$ under θ (see Fig. 2). It can be shown that for physical states $\gamma \neq 0$ is only possible in the presence of a Wilson line in the e_3 and e_4 directions.

2. String selection rules

Consider the n-point correlation function of two fermions and $n - 2$ bosons [97,98]

$$\langle \mathrm{FFB} \ldots \mathrm{B} \rangle. \tag{A19}$$

The corresponding physical states shall be denoted by Ψ_i, $i = 1, \ldots, n$. Then, in the field theory limit, a nonvanishing correlation function induces the following term in the superpotential:

$$W \supset \Psi_1 \Psi_2 \Psi_3 \ldots \Psi_n. \tag{A20}$$

A complete evaluation of Eq. (A19) has only been performed for 3-point couplings and yields a moduli dependent coupling strength [68,97–99].

On the other hand, symmetries of Eq. (A19) give rise to the so-called string selection rules. These rules determine whether a given coupling vanishes or not. We use the following notation: the constructing elements of Ψ_i are denoted by $g_i \in S$ and their left- and right-moving shifted momenta, by $P_{\mathrm{sh},i}$ and $q_{\mathrm{sh},i}$, respectively. Then, the string selection rules read

(1) *Gauge invariance*
The sum over all left-moving shifted momenta $P_{\mathrm{sh},i}$ must vanish:

$$\sum_i P_{\mathrm{sh},i} = 0. \tag{A21}$$

This translates to the field theoretic requirement of gauge invariance for allowed terms in the superpotential.

(2) *Conservation of R-charge*
R-charge is defined by

$$R_i^a = q_{\mathrm{sh},i}^a - N_i^a + N_i^{*a} \quad \text{for } a = 0, \ldots, 3, \tag{A22}$$

where N_i^a and N_i^{*a} are integer oscillator numbers, counting the number of excitations with oscillators $\tilde{\alpha}^a$ and $\tilde{\alpha}^{\bar{a}}$, respectively. Then the conditions [52]

$$\sum_i R_i^a = 0 \bmod N^a \quad \text{for } a = 1, 2, 3 \tag{A23}$$

have to be imposed, where N^a denotes the order of the twist component v^a in the ath complex plane, i.e. $N^a v^a \in \mathbb{Z}$ (no summation). Here, two of the R_i come from fermions and the rest from bosons. For computational purposes, it is more convenient to use the purely bosonic notation, where Eq. (A23) becomes $\sum_i R_i^a = -1 \bmod N^a$.
This condition can be understood as a remnant of 10-dimensional Lorentz invariance.

(3) *Space group selection rule*
The product of constructing elements g_i must be the identity:

$$\prod_i g_i = (\mathbb{1}, 0). \tag{A24}$$

In terms of conjugate elements $h_i g_i h_i^{-1}$ of g_i, this condition can be reformulated as $\prod_i h_i g_i h_i^{-1} = (\mathbb{1}, v)$ with $v \in \sum_i (\mathbb{1} - \theta^{k_i})\Lambda$ [100].
This selection rule can be visualized as the geometrical ability of twisted strings to join.

3. On the need for a γ selection rule

In the literature, there exists an additional selection rule, here referred to as the γ rule. In our notation, it reads [52,68]

$$\sum_i \gamma_i = 0 \bmod 1, \tag{A25}$$

where γ_i denotes the gamma-phase of Ψ_i. In this section, we argue that, in contrast to previous statements, a fully consistent approach yields to automatic fulfillment of the γ rule.

The correlation function corresponding to the coupling

$$\Psi_1\Psi_2\ldots\Psi_n \tag{A26}$$

should be invariant under the action of the full space group. Let us assume first that the states Ψ_i corresponded to linear combinations of equivalent fixed points within the fundamental domain of the torus (see e.g. [52,68,101]). For example, in the case of the $\mathbb{Z}_6$-II orbifold only fixed points in the G_2 lattice could form linear combinations. Under this assumption, different states Ψ_i would be eigenstates with respect to *different* space group elements. So one could not transform the coupling Eq. (A26) with a given $h = (\theta^l, m_\alpha e_\alpha)$. Thus the fully consistent approach for building invariant linear combinations, as presented in Appendix A 1, is necessary. In this case, we can compute the gamma-phase for all states Ψ_i from Eq. (A16), i.e. $\gamma_i = \gamma_i(h)$ for arbitrary $h = (\theta^l, m_\alpha e_\alpha)$. But since allowed couplings already fulfill the selection rules Eqs. (A21) and (A23), the γ rule is satisfied trivially[16]:

$$\gamma_i(h) = P_{\text{sh},i}\cdot V_h - q_{\text{sh},i}\cdot v_h, \tag{A27}$$

$$\Rightarrow \sum_i \gamma_i(h) = \underbrace{\Big(\sum_i P_{\text{sh},i}\Big)}_{=0 \text{ see Eq. (A.21)}}\cdot V_h - \underbrace{\Big(\sum_i q_{\text{sh},i}\Big)}_{\sim 0 \text{ see Eq. (A.23)}}\cdot v_h, \\ = 0 \bmod 1. \tag{A28}$$

Thus, the γ rule in the fully consistent approach is *not* a selection rule. It is a consequence of other selection rules and invariance of the states. We therefore conclude that the coupling must only satisfy gauge invariance, R-charge conservation and the space group selection rule.

This has important consequences. For example, in the model A1 of [52], there is no mass term for the exotics $\bar{q}_2 q_2$ up to order 9 in singlets. However, we find that the coupling $\bar{q}_2 q_2 S_9 S_{15} S_{22} S_{33}$ is allowed by the selection rules of Appendix A 2. Further, using the prescription of Appendix A 1, the gamma-phases of the corresponding physical states are $\gamma_i = (\frac{1}{2}, 0, 0, \frac{5}{6}, \frac{2}{3}, 0)$ for $h = (\theta, 0)$, which sum up to 2. This is in contrast to [52], where $\gamma_i = (0, 0, 0, \frac{1}{2}, \frac{2}{3}, 0)$ and linear combinations were built differently.

[16]Also in the presence of oscillators, the γ rule is satisfied automatically.

APPENDIX B: D-FLATNESS

In this appendix, a simple method is reviewed that allows to analyze D-flatness. It also provides a simple test whether it is possible to cancel the FI term with a given set of fields.

Let us start by briefly reviewing the issue of D-flatness and cancellation of the FI term [102–106]. In supersymmetric theories, there is the so-called D-term potential. In the case of a U(1) gauge theory it is given by

$$V_D \propto \Big[\sum_i q_i|\phi_i|^2\Big]^2. \tag{B1}$$

Consider as a first example a U(1) gauge theory with two fields $\phi_\pm$ carrying the charges ± 1. Clearly, as long as $|\phi_+| = |\phi_-|$, V_D vanishes. That is, one has a D-flat direction, parametrized by $x = |\phi_+| = |\phi_-|$.

Consider now a theory with one field (ϕ_1) with charge 2 and two fields (ϕ_2, ϕ_3) with charges -1.[17] Then we have many flat directions, described by the roots of the equation $2|\phi_1|^2 - |\phi_2|^2 - |\phi_3|^2 = 0$. It is convenient to associate these directions to the (holomorphic) monomials

$$\phi_1\phi_2^2, \qquad \phi_1\phi_3^2, \qquad \phi_1\phi_2\phi_3,$$

respectively. That is, a monomial $\phi_1^{n_1}\phi_2^{n_2}\cdots\phi_k^{n_k}$ represents a flat direction, defined by the relation

$$\frac{|\phi_1|}{\sqrt{n_1}} = \frac{|\phi_2|}{\sqrt{n_2}} = \ldots = \frac{|\phi_k|}{\sqrt{n_k}} \quad \text{and} \quad |\phi_j| = 0 \quad \text{for } n_j = 0.$$

The crucial feature of such monomials is that they are (obviously) gauge invariant. More precisely, every holomorphic gauge invariant monomial represents a D-flat direction [102].

It is, however, clear that there is only a finite number of linearly independent D-flat directions. In the previous example, the third direction is not independent of the other two. In other words, the requirement $V_D = 0$ poses only one constraint on the three real variables ($|\phi_i|^2$) entering (B1). The space of absolute values $|\phi_i|$ is 2-dimensional. The power of using the monomials is that checking whether certain monomials are linearly independent or not is fairly simple: identify with each monomial the vector of exponents, $v = (n_1, n_2\ldots)$. The directions are independent if and only if the vectors are linearly independent. In the previous example one would get the vectors (1, 2, 0), (1, 0, 2), and (1, 1, 1), out of which only two are linearly independent.

It is also clear how to obtain these vectors: all of them are orthogonal to the vector of charges $q = (q_1, q_2\ldots)$. That is, the problem of finding the above monomials (and thus the D-flat directions) is reduced to the problem of finding vectors v with the following properties:

(1) $q\cdot v = 0$,

[17]For the moment, we ignore anomalies.

(2) $v_i \in \mathbb{N}_0$.

The property that the v_i be integer-valued does not pose a constraint in our models: since the charges are rational, one can rescale any v having the first property such as to have integer entries. However, the requirement that the entries be non-negative, which reflects that the monomials ought to be holomorphic, is a constraint.

The discussion so far can easily be extended to $\mathrm{U}(1)^n$ theories. Here the D-term potential is

$$V_D \propto \sum_{j=1}^{n}\Big[\sum_i q_i^{(j)}|\phi_i|^2\Big]^2, \tag{B2}$$

where $q_i^{(j)}$ is the charge of the field ϕ_i under the jth U(1) factor. Now a D-flat direction has to satisfy the above constraints for each U(1) factor separately. Again, it is advantageous to represent D-flat directions by holomorphic gauge invariant monomials (dubbed "HIMs" in the literature [106]). Then the vector v of exponents has to be orthogonal to every charge vector $q^{(j)} = (q_1^{(j)}, q_2^{(j)}, \ldots)$. In other words, v has to be in the kernel of the charge matrix Q,

$$Q \cdot v = 0, \quad \text{with } Q = \begin{pmatrix} q_1^{(1)} & q_2^{(1)} & \cdots \\ q_1^{(2)} & q_2^{(2)} & \cdots \\ \vdots & \vdots & \vdots \\ q_1^{(n)} & q_2^{(n)} & \cdots \end{pmatrix}. \tag{B3}$$

Hence, the problem of finding the D-flat directions of a $\mathrm{U}(1)^n$ gauge theory is reduced to the task of calculating the kernel of the charge matrix Q, and to forming linear combinations of elements of this kernel in such a way that the entries are non-negative integers. The maximal linear independent set of such linear combinations is in one-to-one correspondence with the independent D-flat directions.

Next, let us comment on what happens if there are non-Abelian gauge factors. Then the D-term potential is to be amended by

$$V_D^{\text{non-Abelian}} \propto \sum_a\Big[\sum_i \phi_i^\dagger \mathsf{T}_a \phi_i\Big]^2, \tag{B4}$$

with T_a denoting the group generators. It is straightforward to see that the results obtained so far generalize to the non-Abelian case [102]: the D-flat directions are again in correspondence with holomorphic gauge invariant monomials. That is, one can amend the monomials discussed so far such as to include fields transforming nontrivially under non-Abelian gauge factors, as long as these fields are contracted in such a way that the monomials are gauge invariant.

Finally, let us review the issue of cancelling the FI term. For an "anomalous" U(1), the D-term potential (B1) gets modified to

$$V_D^{\text{anom}} \propto \Big[\sum q_i^{\text{anom}}|\phi_i|^2 + \xi\Big]^2, \tag{B5}$$

where in our convention $\xi > 0$. To cancel the FI term one thus has to find a holomorphic monomial,

$$I = \phi_1^{n_1}\phi_2^{n_2}\ldots, \tag{B6}$$

with net negative charge under $\mathrm{U}(1)_{\text{anom}}$, i.e.

$$\sum_i n_i q_i^{\text{anom}} < 0. \tag{B7}$$

To summarize, the D-flat directions are in one-to-one correspondence with holomorphic gauge invariant monomials. In the Abelian case, such monomials can be identified with elements of the kernel of the charge matrix Q with non-negative integer entries. Cancellation of the FI term requires the existence of a holomorphic monomial with net negative charge under $\mathrm{U}(1)_{\text{anom}}$, which is gauge invariant with respect to all other group factors.

APPENDIX C: FAMILY REFLECTION SYMMETRY AND MATTER PARITY, $\mathbb{Z}_2^{\mathcal{M}}$

We would like to define an effective low-energy theory which preserves R parity. This has the advantage of greatly reducing the number of arbitrary parameters in the superpotential, forbidding dimension-3 and -4 baryon or lepton number violating operators, and preserving a viable dark matter candidate, i.e. the lightest supersymmetric particle. Our strategy for accomplishing this is, in principle, quite simple. We make use of "family reflection symmetry" or "matter parity" defined as a discrete subgroup of $\mathrm{U}(1)_{B-L}$. This is a global $\mathbb{Z}_2^{\mathcal{M}}$ symmetry (commuting with supersymmetry) which is *even* on the Higgs doublets and *odd* on all SM quark and lepton fields. It forbids the following dangerous baryon or lepton number violating operators,

$$\bar{u}\,\bar{d}\,\bar{d}, \qquad q\bar{d}\ell, \qquad \ell\ell\bar{e} \quad \text{and} \quad \ell h_u. \tag{C1}$$

On the other hand, it allows quark and lepton Yukawa couplings as well as the Majorana neutrino mass operator $\bar{\nu}\,\bar{\nu}$.

Consider the effective operators

$$\mathcal{O}\langle s_1 \ldots s_n\rangle \quad \text{with } \mathcal{O} \text{ in (C.1)}, \tag{C2}$$

$$\bar{\nu}\,\bar{\nu}\langle s_1' \ldots s_n'\rangle. \tag{C3}$$

We want to forbid the dangerous proton decay operators (C2), while allowing for Majorana neutrino masses (C3). This puts a constraint on the $B - L$ charges of the SM singlets which get nonzero VEVs. In particular, it requires

$$-1 + \sum_i q_i \neq 0 \tag{C4}$$

for any set of singlets with nonzero VEVs, where q_i are the $B - L$ charges. In addition,

$$2 + \sum_i q_i' = 0 \tag{C5}$$

must be satisfied for at least one singlet configuration with $B-L$ charges q_i'. In our theory, the singlets come in pairs with opposite $B-L$ charges and these charges are rational. Then the relevant solution to the above equations is

$$q_i = \pm \frac{2k_i}{2l_i + 1} \tag{C6}$$

for integer k_i and l_i, with the additional condition that adding/subtracting the numerators of q_i can yield 2, i.e.

$$\sum_i k_i N_i = 1, \tag{C7}$$

for some $N_i \in \mathbb{Z}$. For example, the numerators can differ by 2. If there are only two fields with charges $\pm q_a$, the corresponding constraint is $q_a = \pm 2/(2l+1)$.

The above singlet VEVs break $U(1)_{B-L}$ to a discrete subgroup. Consider an element of $U(1)_{B-L}$ defined by

$$\tilde{R}(\alpha) = e^{i\pi\alpha \mathsf{t}_{B-L}}. \tag{C8}$$

The $B-L$ and $\tilde{R}(\alpha)$ charges of SM particles are given in Table IV. The choice $\alpha = 3$ corresponds to family reflection symmetry. If only the singlets satisfying

$$q_{B-L}(\tilde{s}) = \pm \frac{2}{3}\mathbb{Z} \tag{C9}$$

obtain VEVs and there is at least one singlet for which $\mathbb{Z} \neq 0$, then $U(1)_{B-L}$ is broken to $\tilde{R}(3) \equiv$ FRS. Clearly, products of these singlets can contribute to Yukawa couplings for quarks and leptons. Further, products of singlets with $B-L$ charge $-2/3$ (or, more generally, those with $B-L$ charge $2/3$ and $-4/3$, etc.) can generate Majorana neutrino masses. On the other hand, the proton decay operators are forbidden.

It is possible to generalize FRS to a $\mathbb{Z}_N$ group. In general as long as $\alpha \neq 2\mathbb{Z}$, $\tilde{R}(\alpha)$ will forbid the dangerous operators, Eq. (C1), and allow all Yukawa couplings. Consider a field $\phi \subset \{\tilde{s}\}$ with $q_{B-L}(\phi) = f$. Such a field breaks $U(1)_{B-L}$ to the subgroup $\tilde{R}(\alpha)$ with $\alpha = 2/f$. The effective Majorana neutrino mass operator $\bar{\nu}\,\bar{\nu}\,\phi^n$ is allowed for $[2+nf]/f = \mathbb{Z}$ or $\alpha \equiv 2/f = \mathbb{Z}$. Hence, for odd α, we can both forbid the dangerous operators, Eq. (C1), and obtain nonzero Majorana neutrino mass. The corresponding constraint on f is then $f = \pm 2/(2\mathbb{Z}+1)$, as expected. For example,

(1) $f = \pm 2$, $\alpha = 1$ gives $\tilde{R} = e^{i\pi q_{B-L}} \in \mathbb{Z}_6$,
(2) $f = \pm 2/3$, $\alpha = 3$ gives $\tilde{R} = e^{3i\pi q_{B-L}} \in \mathbb{Z}_2$,
(3) $f = \pm 2/5$, $\alpha = 5$ gives $\tilde{R} = e^{5i\pi q_{B-L}} \in \mathbb{Z}_6$,
(4) $f = \pm 2/7$, $\alpha = 7$ gives $\tilde{R} = e^{7i\pi q_{B-L}} \in \mathbb{Z}_6$,
(5) $f = \pm 2/9$, $\alpha = 9$ gives $\tilde{R} = e^{9i\pi q_{B-L}} \in \mathbb{Z}_2$.

This is easily generalized to configurations with many different singlets getting VEVs (with the constraints given in (C6) and (C7)). These conserve matter parity, $\mathbb{Z}_2^{\mathcal{M}}$.

TABLE IV. $B-L$ and $\tilde{R}$ charges for SM particles, with opposite $B-L$ charges for antiparticles, and for baryon and lepton number violating operators with dimension ≤ 4.

	Quarks	Leptons	h_u	h_d	$\bar{u}\,\bar{d}\,\bar{d}$	$q\bar{d}\ell$	$\ell\ell\bar{e}$	ℓh_u
$B-L$	$1/3$	-1	0	0	-1	-1	-1	-1
$\tilde{R}$	$e^{i\alpha\pi/3}$	$e^{-i\alpha\pi}$	1	1	$e^{-i\alpha\pi}$	$e^{-i\alpha\pi}$	$e^{-i\alpha\pi}$	$e^{-i\alpha\pi}$

APPENDIX D: SEARCH FOR $B-L$ AND R PARITY

Our search for $U(1)_{B-L}$ is based on the methods developed in Ref. [62] (for an earlier discussion of $U(1)_{B-L}$ and its applications see [40,53]). In Table V, we list the standard model particle content with their hypercharge and $B-L$ charges.

The choice for $U(1)_{B-L}$ depends on the choice of hypercharge in the first place. In this publication, we do not take the most general approach, but assume that hypercharge is given by $SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)_Y$. Furthermore, we demand that the first and second families come from **16**-plets localized in the first twisted sector, whereas the multiplets of the third family may come from any sector of the theory.

To find a suitable $U(1)_{B-L}$, we proceed as follows. In general, the shift and Wilson lines break the gauge group in 10 dimensions,

$$E_8 \times E_8' \quad \rightarrow \quad \text{non}-\text{Abelian} \times U(1)^n. \tag{D1}$$

The $U(1)$ generators are n linearly independent directions t_i in the root lattice of $E_8 \times E_8'$ that are orthogonal to the simple roots of the unbroken non-Abelian gauge group. For the $B-L$ direction, we make the general ansatz

$$\mathsf{t}_{B-L} = x_1\mathsf{t}_1 + x_2\mathsf{t}_2 + \ldots + x_n\mathsf{t}_n. \tag{D2}$$

The $B-L$ charge of a particular representation is given by the scalar product of its highest weight and $B-L$. We denote the highest weights of the left-handed quark doublets and of the right-handed quark singlets by Λ_i, $i = 1, \ldots, 9$. Note that the first two families are fixed, and we loop over all representations which have the right quantum numbers to be the quarks of the third generation. For each such choice, we have

$$\Lambda_i \cdot \mathsf{t}_{B-L} = x_1\Lambda_i \cdot \mathsf{t}_1 + x_2\Lambda_i \cdot \mathsf{t}_2 + \ldots + x_n\Lambda_i \cdot \mathsf{t}_n, \tag{D3}$$

which is a system of nine linear equations. Although one may think that these nine equations severely constrain the

TABLE V. Matter content of the standard model, where the subscripts denote hypercharge and $B-L$, respectively. In our conventions, $Q = T_{3L} + Y$.

q	$(\mathbf{3},\mathbf{2})_{1/6,1/3}$	ℓ	$(\mathbf{1},\mathbf{2})_{-1/2,-1}$	h_u	$(\mathbf{1},\mathbf{2})_{1/2,0}$
$\bar{u}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1/3}$	$\bar{e}$	$(\mathbf{1},\mathbf{1})_{1,1}$	h_d	$(\mathbf{1},\mathbf{2})_{-1/2,0}$
$\bar{d}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3,-1/3}$	$\bar{\nu}$	$(\mathbf{1},\mathbf{1})_{0,1}$		

values of x_i, this is not true. In general, the system will be under-determined, since the quarks may differ by localization, but not necessarily by the highest weights of their gauge representations. In order to account for the $B-L$ charges of the leptons and Higgses, and for the absence of chiral exotics, we set up necessary, but in general *not sufficient*, linear constraints:

$$\sum_{(\mathbf{3},\mathbf{2}),(\bar{\mathbf{3}},\mathbf{2})} q_{B-L} = 1, \qquad \sum_{(\mathbf{3},\mathbf{1}),(\bar{\mathbf{3}},\mathbf{1})} q_{B-L} = -2,$$
$$\sum_{(\mathbf{1},\mathbf{2})} q_{B-L} = -3, \qquad \sum_{(\mathbf{1},\mathbf{1}),Y\neq 0} q_{B-L} = 3. \tag{D4}$$

For readability, we use q_{B-L} in the above equations as a shorthand for $\Lambda_k \cdot \mathsf{t}_{B-L}$, where Λ_k runs over the highest weights of the representations in the sum. Note that the sum over e.g. the $(\mathbf{3}, \mathbf{1})$ and $(\bar{\mathbf{3}}, \mathbf{1})$ representations reduces to that over the right-handed quarks $\bar{u}$ and $\bar{d}$ alone, since we assume pairs of exotic particles to carry $B-L$ charge assignments that are equal in magnitude but opposite in sign. Another linear constraint comes from the requirement that $\mathrm{U}(1)_{B-L}$ be nonanomalous. If the model has an anomalous U(1) direction that we will denote by $\mathsf{t}_{\mathrm{anom}}$, we demand that it be orthogonal to $B-L$:

$$(x_1\mathsf{t}_1 + x_2\mathsf{t}_2 + \ldots + x_n\mathsf{t}_n) \cdot \mathsf{t}_{\mathrm{anom}} = 0. \tag{D5}$$

In principle, we could write down many more linear conditions, e.g. we could demand that the sum of the $B-L$ charges of u-type quarks like $(\bar{\mathbf{3}}, \mathbf{1})_{-2/3,-1/3}$ and $(\mathbf{3}, \mathbf{1})_{2/3,1/3}$ alone gives -1, which is more constraining than Eq. (D4). However, our experience shows that it is more practical to drop these conditions, since the nonlinear equations to be introduced below are constraining enough.

The linear equations by no means exhaust the constraints we may require to be fulfilled by a vectorlike spectrum. In particular, there are the cubic,

$$\sum_{(\mathbf{3},\mathbf{2}),(\bar{\mathbf{3}},\mathbf{2})} (q_{B-L})^3 = \frac{1}{9}, \qquad \sum_{(\mathbf{3},\mathbf{1}),(\bar{\mathbf{3}},\mathbf{1})} (q_{B-L})^3 = -\frac{2}{9},$$
$$\sum_{(\mathbf{1},\mathbf{2})} (q_{B-L})^3 = -3, \qquad \sum_{(\mathbf{1},\mathbf{1}),Y\neq 0} (q_{B-L})^3 = 3, \tag{D6}$$

and the quintic,

$$\sum_{(\mathbf{3},\mathbf{2}),(\bar{\mathbf{3}},\mathbf{2})} (q_{B-L})^5 = \frac{1}{81}, \qquad \sum_{(\mathbf{3},\mathbf{1}),(\bar{\mathbf{3}},\mathbf{1})} (q_{B-L})^5 = -\tfrac{2}{81},$$
$$\sum_{(\mathbf{1},\mathbf{2})} (q_{B-L})^5 = -3, \qquad \sum_{(\mathbf{1},\mathbf{1}),Y\neq 0} (q_{B-L})^5 = 3, \tag{D7}$$

constraints. As before, the sum over all $(\mathbf{1}, \mathbf{1})$ representations with nonvanishing hypercharge reduces to that of the right-handed electrons that carry $B-L$ charge $+1$, since the exotic particles come in vectorlike pairs so that their contribution to the sum vanishes.

This leaves us with a set of highly nonlinear equations to be solved. To this end, we used the computer algebra system SINGULAR [107]. The following cases need to be distinguished. (i) The number of solutions is finite. (ii) The solutions are given by continuous parameters, and the relations intertwining these parameters are linear. (iii) The solutions are given by continuous parameters, but this time, the relations intertwining the parameters are nonlinear. $B-L$ directions which lead to irrational charges for the exotics are discarded in all three cases.

In the first case, we calculate the spectra for the $B-L$ generators and check that they are really vectorlike. A given model is particularly interesting if its spectrum contains standard model singlets with $B-L$ charges $2n/(2m+1)$ for $n, m \in \mathbb{N}$ whose VEVs break the continuous $\mathrm{U}(1)_{B-L}$ symmetry to a discrete subgroup which may play the role of a generalized R parity. In the second case, we use the parametric freedom we have at hand to assign $B-L$ charges of $2n/(2m+1)$ for $m, n = 0, 1, 2, 3$ in all possible combinations to standard model singlets and thus generate a list of $B-L$ directions. In the third case, we specialize to a numerical value and check whether the spectrum satisfies our criteria. We do not perform a complete search, because these cases are rare and the analysis more complicated, thus not very rewarding.

APPENDIX E: DETAILS OF MODEL 1

The model is defined by the shifts and Wilson lines

$$V = (\tfrac{1}{3}, -\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0, 0, 0, 0)(\tfrac{1}{2}, -\tfrac{1}{6}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}), \tag{E1a}$$

$$W_2 = (0, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, 0, 0, 0) \times (4, -3, -\tfrac{7}{2}, -4, -3, -\tfrac{7}{2}, -\tfrac{9}{2}, \tfrac{7}{2}), \tag{E1b}$$

$$W_3 = (-\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6})(\tfrac{1}{3}, 0, 0, \tfrac{2}{3}, 0, \tfrac{5}{3}, -2, 0). \tag{E1c}$$

A possible second order 2 Wilson line is set to zero. The gauge group after compactification is

$$G_{\mathrm{orbifold}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times [\mathrm{SU}(4) \times \mathrm{SU}(2)'] \times \mathrm{U}(1)^9. \tag{E2}$$

The U(1) generators can be chosen as

$$\mathsf{t}_1 = \mathsf{t}_Y = (0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3})(0, 0, 0, 0, 0, 0, 0, 0), \tag{E3a}$$

$$\mathsf{t}_2 = (1, 0, 0, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0), \tag{E3b}$$

$$\mathsf{t}_3 = (0, 1, 0, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0), \tag{E3c}$$

$$\mathsf{t}_4 = (0, 0, 1, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0), \tag{E3d}$$

$$\mathsf{t}_5 = (0, 0, 0, 1, 1, 1, 1, 1)(0, 0, 0, 0, 0, 0, 0, 0), \tag{E3e}$$

$$\mathsf{t}_6 = (0, 0, 0, 0, 0, 0, 0, 0)(0, 1, 0, 0, 0, 0, 0, 0), \tag{E3f}$$

$$\mathsf{t}_7 = (0, 0, 0, 0, 0, 0, 0, 0)(-1, 0, 0, 1, 0, 0, 0, 0), \tag{E3g}$$

$$\mathsf{t}_8 = (0, 0, 0, 0, 0, 0, 0, 0)(0, 0, 0, 0, 1, 0, 0, 0), \tag{E3h}$$

$$\mathsf{t}_9 = (0, 0, 0, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 1, 0, 0). \tag{E3i}$$

The "anomalous" U(1) is generated by

$$\mathsf{t}_{\text{anom}} = \sum_{i=1}^{9} \alpha_i \mathsf{t}_i, \tag{E4}$$

where $\{\alpha_i\} = \{0, \frac{2}{3}, 0, -\frac{5}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, 2, \frac{1}{3}\}$.

The sum of anomalous charges is

$$\operatorname{tr} \mathsf{t}_{\text{anom}} = \frac{296}{3} > 0. \tag{E5}$$

1. Spectrum

The detailed spectrum is provided in Table VI.

2. Mass matrices for exotics in Model 1A

Here we show that all exotics can be made massive. The exotic's mass terms are

$$x_i (\mathcal{M}_{x\bar{x}})_{ij} \bar{x}_j. \tag{E6}$$

In the following, we list the structure of the corresponding mass matrices.

TABLE VI. The spectrum of model 1 in terms of *left-chiral* states. The U(1) charges refer to the basis of generators (E3). R_i denote R-charges. Note, the symbol * means the state is not localized in the corresponding directions.

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	Representation	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{B-L}	q_{anom}
$\bar{n}_3$	0	*	*	*	0	-1	0	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$	0	0	0	0	1	$-\frac{1}{3}$
$\bar{e}_3$	0	*	*	*	0	-1	0	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	1	$\frac{2}{3}$
$\bar{f}_1$	0	*	*	*	0	-1	0	0	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	0	0	0	0	0	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{5}{3}$
f_1	0	*	*	*	0	-1	0	0	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	0	0	0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{2}{3}$
$\bar{u}_3$	0	*	*	*	0	-1	0	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{3}$	$\frac{2}{3}$
s_1^0	0	*	*	*	0	0	-1	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	0	0	0	1	0	-1	0	0	$\frac{7}{3}$
s_2^0	0	*	*	*	0	0	-1	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	0	0	0	1	0	1	0	0	$-\frac{5}{3}$
q_3	0	*	*	*	0	0	-1	0	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$\frac{1}{3}$	$\frac{4}{3}$
ϕ_1	0	*	*	*	0	0	0	-1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	1	-1	0	0	0	0	0	2
$\bar{\phi}_1$	0	*	*	*	0	0	0	-1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	-1	1	0	0	0	0	0	-2
$\bar{n}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	0	$-\frac{5}{2}$	$\frac{1}{3}$	0	0	0	1	$\frac{19}{18}$
$\bar{e}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	1	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	1	$\frac{7}{18}$
ℓ_2	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	$\frac{1}{3}$	0	0	0	-1	$-\frac{5}{18}$
$\bar{u}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	$-\frac{1}{3}$	$\frac{7}{18}$
$\bar{d}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	$\frac{1}{3}$	0	0	0	$-\frac{1}{3}$	$-\frac{5}{18}$
q_2	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	$\frac{1}{3}$	$\frac{7}{18}$
s_3^0	1	0	0	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$-\frac{17}{18}$
s_4^0	1	0	0	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{13}{18}$
s_5^0	1	0	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{25}{18}$
s_6^0	1	0	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$-\frac{5}{18}$
s_7^0	1	0	0	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$-\frac{17}{18}$
s_8^0	1	0	0	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{13}{18}$
$\bar{n}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	0	$-\frac{5}{2}$	$\frac{1}{3}$	0	0	0	1	$\frac{19}{18}$
$\bar{e}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	1	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	1	$\frac{7}{18}$
ℓ_1	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	$\frac{1}{3}$	0	0	0	-1	$-\frac{5}{18}$
$\bar{u}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	$-\frac{1}{3}$	$\frac{7}{18}$
$\bar{d}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	$\frac{1}{3}$	0	0	0	$-\frac{1}{3}$	$-\frac{5}{18}$
q_1	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	$\frac{1}{3}$	$\frac{7}{18}$
s_9^0	1	0	0	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$-\frac{17}{18}$
s_{10}^0	1	0	0	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{13}{18}$
s_{11}^0	1	0	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{25}{18}$
s_{12}^0	1	0	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$-\frac{5}{18}$
s_{13}^0	1	0	0	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$-\frac{17}{18}$
s_{14}^0	1	0	0	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{13}{18}$

TABLE VI. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	Representation	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{B-L}	q_{anom}
m_1	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	0	0	$-\frac{1}{6}$	-1	$\frac{1}{2}$	0	0	$-\frac{17}{18}$
m_2	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	0	0	$-\frac{1}{6}$	1	$\frac{1}{2}$	0	0	$-\frac{29}{18}$
y_1	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$	0	$\frac{1}{3}$	0	0	0	$-\frac{1}{6}$	0	$-\frac{1}{2}$	0	0	$\frac{13}{18}$
m_3	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	0	0	$-\frac{1}{6}$	-1	$\frac{1}{2}$	0	0	$-\frac{17}{18}$
m_4	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	0	0	$-\frac{1}{6}$	1	$\frac{1}{2}$	0	0	$-\frac{29}{18}$
y_2	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$	0	$\frac{1}{3}$	0	0	0	$-\frac{1}{6}$	0	$-\frac{1}{2}$	0	0	$\frac{13}{18}$
n_1	1	1	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{19}{18}$
$\bar{n}_4$	1	1	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	1	$\frac{19}{18}$
η_1	1	1	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	-1	$\frac{19}{18}$
n_2	1	1	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{19}{18}$
$\bar{n}_5$	1	1	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	1	$\frac{19}{18}$
η_2	1	1	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	-1	$\frac{19}{18}$
s_1^-	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$-\frac{17}{18}$
s_1^+	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{19}{18}$
s_2^+	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{37}{18}$
m_5	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	-1	$\frac{19}{18}$
v_1	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{18}$
s_2^-	1	1	1	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	-1	$\frac{25}{18}$
s_3^-	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$-\frac{17}{18}$
s_3^+	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{19}{18}$
s_4^+	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{37}{18}$
m_6	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	-1	$\frac{19}{18}$
v_2	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{18}$
s_4^-	1	1	1	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	-1	$\frac{25}{18}$
$\bar{f}_2$	1	2	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{37}{18}$
f_2	1	2	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{18}$
$\bar{n}_6$	1	2	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$\frac{7}{18}$
n_3	1	2	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	-1	$\frac{19}{18}$
$\bar{\eta}_1$	1	2	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	$\frac{19}{18}$
$\bar{f}_3$	1	2	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{37}{18}$
f_3	1	2	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{18}$
$\bar{n}_7$	1	2	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$\frac{7}{18}$
n_4	1	2	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	-1	$\frac{19}{18}$
$\bar{\eta}_2$	1	2	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	$\frac{19}{18}$
s_5^-	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{31}{18}$
s_6^-	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{11}{18}$
s_5^+	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{2}{3}$	-1	$-\frac{17}{18}$
m_7	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{19}{18}$
$\bar{v}_1$	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{25}{18}$
s_6^+	1	2	1	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{13}{18}$
s_7^-	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{31}{18}$
s_8^-	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{11}{18}$
s_7^+	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{2}{3}$	-1	$-\frac{17}{18}$
m_8	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{19}{18}$

TABLE VI. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	Representation	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{B-L}	$q_{\rm anom}$
$\bar{v}_2$	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{25}{18}$
s_8^+	1	2	1	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{13}{18}$
s_{15}^0	2	0	*	*	0	$\frac{2}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	0	0	$\frac{2}{3}$	0	0	0	0	$-\frac{2}{9}$
s_{16}^0	2	0	*	*	1	$\frac{2}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	0	0	$\frac{2}{3}$	0	0	0	0	$-\frac{2}{9}$
s_{17}^0	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	0	0	$-\frac{1}{3}$	0	-1	0	0	$\frac{13}{9}$
s_{18}^0	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	0	0	$-\frac{1}{3}$	0	1	0	0	$-\frac{23}{9}$
h_1	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{2}{3}$	0	0	0	$-\frac{1}{3}$	-1	0	0	0	$-\frac{2}{9}$
h_2	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{2}{3}$	0	0	0	$-\frac{1}{3}$	1	0	0	0	$-\frac{8}{9}$
$\bar{\delta}_1$	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	-1	$\frac{2}{3}$	0	0	0	$\frac{2}{3}$	$\frac{7}{9}$
δ_1	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0	1	$\frac{2}{3}$	0	0	0	$-\frac{2}{3}$	$\frac{1}{9}$
s_{19}^0	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{5}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	0	0	$\frac{2}{3}$	0	0	0	0	$-\frac{2}{9}$
n_5	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	-1	$\frac{1}{9}$
s_{20}^0	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	0	$-\frac{8}{9}$
η_3	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{2}{3}$	-1	$\frac{1}{9}$
$\bar{\delta}_2$	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{9}$
n_6	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	-1	$\frac{1}{9}$
s_{21}^0	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	0	$-\frac{8}{9}$
η_4	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{2}{3}$	-1	$\frac{1}{9}$
$\bar{\delta}_3$	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{9}$
$\bar{n}_8$	2	1	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{5}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$\frac{1}{9}$
f_4	2	1	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{8}{9}$
d_1	2	1	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{8}{9}$
s_{22}^0	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{5}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	0	$-\frac{2}{9}$
f_5	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	$\frac{7}{9}$
ℓ_3	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{4}{9}$
δ_2	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{9}$
n_7	2	2	*	*	0	$\frac{2}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{1}{9}$
s_{23}^0	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{5}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	0	$-\frac{2}{9}$
f_6	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	$\frac{7}{9}$
ℓ_4	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{4}{9}$
δ_3	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{9}$
n_8	2	2	*	*	1	$\frac{2}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{1}{9}$
n_9	2	2	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{3}$	$\frac{4}{3}$	0	$-\frac{1}{3}$	-1	$-\frac{5}{9}$
n_{10}	2	2	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{5}{6}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$-\frac{14}{9}$
η_5	2	2	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	-1	$\frac{1}{9}$
n_{11}	2	2	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{5}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$\frac{1}{9}$
χ_1	3	*	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	1	2	$\frac{1}{2}$
χ_2	3	*	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	-1	-2	$-\frac{1}{2}$
h_3	3	*	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0	0	0	$\frac{1}{2}$
h_4	3	*	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	0	0	$-\frac{1}{2}$
χ_3	3	*	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	1	2	$\frac{1}{2}$
χ_4	3	*	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	-1	-2	$-\frac{1}{2}$
h_5	3	*	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0	0	0	$\frac{1}{2}$
h_6	3	*	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	0	0	$-\frac{1}{2}$
$\bar{f}_1^+$	3	*	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}$
f_1^-	3	*	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	0	0	0	$-\frac{1}{2}$	-1	$\frac{1}{2}$

TABLE VI. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	Representation	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{B-L}	q_{anom}
s_9^-	3	*	1	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_9^+	3	*	1	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{10}^-	3	*	1	0	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{10}^+	3	*	1	0	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{11}^-	3	*	1	0	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$-\frac{5}{6}$
s_{11}^+	3	*	1	0	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$-\frac{5}{6}$
$\bar{f}_2^+$	3	*	1	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}$
f_2^-	3	*	1	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	0	0	0	$-\frac{1}{2}$	-1	$\frac{1}{2}$
s_{12}^-	3	*	1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{12}^+	3	*	1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{13}^-	3	*	1	1	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{13}^+	3	*	1	1	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{5}{6}$
s_{14}^-	3	*	1	1	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	-1	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$-\frac{5}{6}$
s_{14}^+	3	*	1	1	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	0	0	0	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$-\frac{5}{6}$
s_{24}^0	4	0	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	0	-1	0	0	$\frac{23}{9}$
s_{25}^0	4	0	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	0	1	0	0	$-\frac{13}{9}$
h_7	4	0	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	-1	0	0	0	$\frac{8}{9}$
h_8	4	0	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	1	0	0	0	$\frac{2}{9}$
$\bar{\delta}_4$	4	0	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	-1	$-\frac{2}{3}$	0	0	0	$\frac{2}{3}$	$-\frac{1}{9}$
δ_4	4	0	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	1	$-\frac{2}{3}$	0	0	0	$-\frac{2}{3}$	$-\frac{7}{9}$
s_{26}^0	4	0	*	*	0	$-\frac{2}{3}$	$\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$-\frac{2}{3}$	0	0	0	0	$\frac{2}{9}$
s_{27}^0	4	0	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	0	-1	0	0	$\frac{23}{9}$
s_{28}^0	4	0	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	0	1	0	0	$-\frac{13}{9}$
h_9	4	0	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	-1	0	0	0	$\frac{8}{9}$
h_{10}	4	0	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	$\frac{1}{3}$	1	0	0	0	$\frac{2}{9}$
$\bar{\delta}_5$	4	0	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	-1	$-\frac{2}{3}$	0	0	0	$\frac{2}{3}$	$-\frac{1}{9}$
δ_5	4	0	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	1	$-\frac{2}{3}$	0	0	0	$-\frac{2}{3}$	$-\frac{7}{9}$
s_{29}^0	4	0	*	*	1	$-\frac{2}{3}$	$\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$-\frac{2}{3}$	0	0	0	0	$\frac{2}{9}$
s_{30}^0	4	0	*	*	$\frac{1}{2}$	$-\frac{5}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	$-\frac{2}{3}$	0	0	0	0	$\frac{2}{9}$
n_{12}	4	1	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{5}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$-\frac{1}{9}$
$\bar{f}_4$	4	1	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	$\frac{8}{9}$
$\bar{d}_3$	4	1	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{8}{9}$
n_{13}	4	1	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{5}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	-1	$-\frac{1}{9}$
$\bar{f}_5$	4	1	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	$\frac{8}{9}$
$\bar{d}_4$	4	1	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{8}{9}$
s_{31}^0	4	1	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{5}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	0	$\frac{8}{9}$
$\bar{n}_9$	4	1	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	1	$-\frac{1}{9}$
$\bar{\eta}_3$	4	1	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	1	$-\frac{1}{9}$
δ_6	4	1	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{9}$
$\bar{n}_{10}$	4	2	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{5}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$\frac{14}{9}$
$\bar{n}_{11}$	4	2	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$-\frac{4}{3}$	0	$\frac{1}{3}$	1	$\frac{5}{9}$
$\bar{\eta}_4$	4	2	*	*	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{9}$
$\bar{n}_{12}$	4	2	*	*	0	$-\frac{2}{3}$	$\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{9}$
$\bar{n}_{13}$	4	2	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{5}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$\frac{14}{9}$
$\bar{n}_{14}$	4	2	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$-\frac{4}{3}$	0	$\frac{1}{3}$	1	$\frac{5}{9}$
$\bar{\eta}_5$	4	2	*	*	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{9}$

TABLE VI. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	Representation	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{B-L}	$q_{\rm anom}$
$\bar{n}_{15}$	4	2	*	*	1	$-\frac{2}{3}$	$\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{9}$
s_{32}^0	4	2	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	0	$\frac{2}{9}$
$\bar{f}_6$	4	2	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{7}{9}$
$\bar{\ell}_1$	4	2	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$-\frac{4}{9}$
$\bar{\delta}_6$	4	2	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{9}$
$\bar{n}_{16}$	4	2	*	*	$\frac{1}{2}$	$-\frac{5}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{9}$

$$\mathcal{M}_{\bar{\ell}\ell} = \begin{pmatrix} 0 & 0 & \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}, \tag{E7a}$$

$$\mathcal{M}_{d\bar{d}} = \begin{pmatrix} 0 & 0 & \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}, \tag{E7b}$$

$$\mathcal{M}_{\delta\bar{\delta}} = \begin{pmatrix} 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \\ 0 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 & \tilde{s}^6 \\ 0 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 & \tilde{s}^6 \\ \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \\ \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \\ 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & 0 & 0 \end{pmatrix}, \tag{E7c}$$

$$\mathcal{M}_{yy} = \begin{pmatrix} \tilde{s} & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s} \end{pmatrix}, \tag{E7d}$$

$$\mathcal{M}_{v\bar{v}} = \begin{pmatrix} \tilde{s} & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s} \end{pmatrix}, \tag{E7e}$$

$$\mathcal{M}_{f\bar{f}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 \\ 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 \\ 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\ 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\ 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}, \tag{E7f}$$

$$\mathcal{M}_{\bar{f}^+f^-} = \begin{pmatrix} \tilde{s}^6 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}. \tag{E7g}$$

$$\mathcal{M}_{mm} = \begin{pmatrix} 0 & \tilde{s} & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 \\ \tilde{s} & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 & \tilde{s} & 0 & 0 & 0 & 0 \\ \tilde{s}^6 & 0 & \tilde{s} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 \end{pmatrix}, \tag{E7h}$$

$$\mathcal{M}_{s^+s^-} = \begin{pmatrix} 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s} & \tilde{s}^5 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^5 & \tilde{s} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^2 & \tilde{s}^2 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^2 & \tilde{s}^2 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^2 & \tilde{s}^2 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^2 & \tilde{s}^2 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}, \tag{E7i}$$

3. Mass matrices for exotics in Model 1B

Here we show that all exotics can be made massive. The exotic's mass terms are

$$x_i(\mathcal{M}_{x\bar{x}})_{ij}\bar{x}_j. \tag{E8}$$

In the following, we list the structure of the corresponding mass matrices.

$$\mathcal{M}_{\bar{\ell}\ell} = \begin{pmatrix} 0 & 0 & \tilde{s}^3 & \tilde{s}^3 \end{pmatrix}, \tag{E9a}$$

$$\mathcal{M}_{d\bar{d}} = \begin{pmatrix} 0 & 0 & \tilde{s}^3 & \tilde{s}^3 \end{pmatrix}, \tag{E9b}$$

$$\mathcal{M}_{mm} = \begin{pmatrix} 0 & \tilde{s} & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 \\ \tilde{s} & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 & \tilde{s} & 0 & 0 & 0 & 0 \\ \tilde{s}^6 & 0 & \tilde{s} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 \end{pmatrix}, \tag{E9c}$$

$$\mathcal{M}_{\delta\bar{\delta}} = \begin{pmatrix} \tilde{s}^6 & 0 & 0 & \tilde{s}^3 & \tilde{s}^3 & 0 \\ 0 & \tilde{s}^4 & \tilde{s}^4 & 0 & 0 & \tilde{s}^3 \\ 0 & \tilde{s}^4 & \tilde{s}^4 & 0 & 0 & \tilde{s}^3 \\ \tilde{s}^3 & 0 & 0 & \tilde{s}^4 & \tilde{s}^4 & 0 \\ \tilde{s}^3 & 0 & 0 & \tilde{s}^4 & \tilde{s}^4 & 0 \\ 0 & \tilde{s}^3 & \tilde{s}^3 & 0 & 0 & 0 \end{pmatrix}, \tag{E9d}$$

$$\mathcal{M}_{yy} = \begin{pmatrix} \tilde{s} & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s} \end{pmatrix}, \tag{E9e}$$

$$\mathcal{M}_{v\bar{v}} = \begin{pmatrix} \tilde{s} & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s} \end{pmatrix}, \tag{E9f}$$

$$\mathcal{M}_{f\bar{f}} = \begin{pmatrix} \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 \\ 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 \\ 0 & 0 & 0 & \tilde{s}^3 & \tilde{s}^3 & \tilde{s}^5 \\ 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^3 \\ 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^3 \end{pmatrix}, \tag{E9g}$$

$$\mathcal{M}_{\bar{f}^+f^-} = \begin{pmatrix} \tilde{s}^6 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}. \tag{E9h}$$

$$\mathcal{M}_{s^+s^-} = \begin{pmatrix} 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s} & \tilde{s}^5 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^6 & \tilde{s}^5 & \tilde{s} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s} & \tilde{s}^2 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^2 & \tilde{s} & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s} & \tilde{s}^2 & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^2 & \tilde{s} & \tilde{s}^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}, \tag{E9i}$$

APPENDIX F: DETAILS OF MODEL 2

The model is defined by the shift and Wilson lines [40]

$$V = \left(\tfrac{1}{3}, -\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0, 0, 0, 0\right)\left(\tfrac{1}{2}, -\tfrac{1}{6}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}\right), \tag{F1a}$$

$$W_2 = \left(\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}\right) \times \left(1, -1, -\tfrac{5}{2}, -\tfrac{3}{2}, -\tfrac{1}{2}, -\tfrac{5}{2}, -\tfrac{3}{2}, \tfrac{3}{2}\right), \tag{F1b}$$

$$W_3 = \left(-\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}\right)\left(\tfrac{10}{3}, 0, -6, -\tfrac{7}{3}, -\tfrac{4}{3}, -5, -3, 3\right). \tag{F1c}$$

We use the U(1) generators

$$\mathsf{t}_1 = \mathsf{t}_Y = \left(0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}\right)(0, 0, 0, 0, 0, 0, 0, 0), \tag{F2a}$$

$$\mathsf{t}_2 = (1, 0, 0, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0), \tag{F2b}$$

$$\mathsf{t}_3 = (0, 1, 0, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0), \tag{F2c}$$

$$\mathsf{t}_4 = (0, 0, 1, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0), \tag{F2d}$$

$$\mathsf{t}_5 = (0, 0, 0, 1, 1, 1, 1, 1)(0, 0, 0, 0, 0, 0, 0, 0), \tag{F2e}$$

$$\mathsf{t}_6 = (0, 0, 0, 0, 0, 0, 0, 0)(1, 0, 0, 0, 0, 0, 0, 0), \tag{F2f}$$

$$\mathsf{t}_7 = (0, 0, 0, 0, 0, 0, 0, 0)(0, 1, 0, 0, 0, 0, 0, 0), \tag{F2g}$$

$$\mathsf{t}_8 = (0, 0, 0, 0, 0, 0, 0, 0)(0, 0, 0, 1, 1, 0, 0, 0). \tag{F2h}$$

The anomalous U(1) generator can be expressed through a linear combination of all U(1) generators,

$$\mathsf{t}_{\text{anom}} = -\sum c_i \mathsf{t}_i, \quad \text{where } c_i = \left(0, \frac{7}{3}, -1, -\frac{5}{3}, \frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}\right). \tag{F3}$$

The sum of anomalous charges is

$$\operatorname{tr} \mathsf{t}_{\text{anom}} = \frac{416}{3} > 0. \tag{F4}$$

1. Spectrum

The detailed spectrum is provided in Table VII.

2. Mass matrices

Here we show that all exotics can be made massive. The exotic's mass terms are

$$x_i (\mathcal{M}_{x\bar{x}})_{ij} \bar{x}_j. \tag{F5}$$

In the following, we list the structure of the corresponding mass matrices.

$$\mathcal{M}_{\bar{\ell}\ell} = (\tilde{s}^2 \quad \tilde{s}^2 \quad \tilde{s}^3 \quad \tilde{s}^3), \tag{F6a}$$

$$\mathcal{M}_{d\bar{d}} = (\tilde{s}^6 \quad \tilde{s}^6 \quad \tilde{s}^3 \quad \tilde{s}^3), \tag{F6b}$$

$$\mathcal{M}_{mm} = \begin{pmatrix} 0 & 0 & \tilde{s}^6 & \tilde{s}^6 \\ 0 & 0 & \tilde{s}^6 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}, \tag{F6c}$$

$$\mathcal{M}_{\delta\bar{\delta}} = \begin{pmatrix} \tilde{s}^3 & \tilde{s}^3 & \tilde{s}^3 \\ \tilde{s}^3 & \tilde{s}^3 & \tilde{s}^3 \\ 0 & \tilde{s}^3 & \tilde{s}^3 \end{pmatrix}, \tag{F6d}$$

$$\mathcal{M}_{yy} = \begin{pmatrix} \tilde{s}^1 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^1 \end{pmatrix}, \tag{F6e}$$

$$\mathcal{M}_{v\bar{v}} = \begin{pmatrix} \tilde{s} & \tilde{s}^5 & 0 & 0 \\ \tilde{s}^5 & \tilde{s} & 0 & 0 \\ 0 & 0 & \tilde{s}^5 & \tilde{s}^5 \\ 0 & 0 & \tilde{s}^5 & \tilde{s}^5 \end{pmatrix}, \tag{F6f}$$

$$\mathcal{M}_{x^+x^-} = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 \end{pmatrix}, \tag{F6g}$$

$$\mathcal{M}_{f\bar{f}} = \begin{pmatrix} 0 & \tilde{s}^3 \\ 0 & \tilde{s}^3 \end{pmatrix}, \tag{F6h}$$

$$\mathcal{M}_{ww} = \begin{pmatrix} \tilde{s} & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s} & 0 & \tilde{s}^5 & \tilde{s}^5 \\ 0 & 0 & 0 & \tilde{s}^3 & \tilde{s}^3 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^3 & \tilde{s}^6 & \tilde{s}^6 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^3 & \tilde{s}^6 & \tilde{s}^6 \end{pmatrix}, \tag{F6i}$$

TABLE VII. The spectrum of model 2 in terms of *left-chiral* states. The U(1) charges refer to the basis of generators (F2). R_i denote R-charges. Note, the symbol $*$ means the state is not localized in the corresponding directions.

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	irrep	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_{B-L}	q_{anom}
q_3	0	$*$	$*$	$*$	0	-1	0	0	$(\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1})$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{3}$	$-\frac{2}{3}$
χ_1	0	$*$	$*$	$*$	0	-1	0	0	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	-1	-1	0	0	0	0	0	-2	$\frac{4}{3}$
χ_2	0	$*$	$*$	$*$	0	-1	0	0	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$	0	0	0	2	$-\frac{2}{3}$
$\bar{f}_1$	0	$*$	$*$	$*$	0	-1	0	0	$(\mathbf{1}, \mathbf{1}; \bar{\mathbf{8}}, \mathbf{1})$	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{3}$
$\bar{e}_3$	0	$*$	$*$	$*$	0	0	-1	0	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	1	$\frac{8}{3}$
$\bar{u}_3$	0	$*$	$*$	$*$	0	0	-1	0	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{3}$	$\frac{8}{3}$
s_3^0	0	$*$	$*$	$*$	0	0	-1	0	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$	0	0	0	0	$\frac{2}{3}$
ϕ_1	0	$*$	$*$	$*$	0	0	0	-1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})$	$-\frac{1}{2}$	0	0	1	-1	0	0	0	0	2
$\bar{\phi}_1$	0	$*$	$*$	$*$	0	0	0	-1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})$	$\frac{1}{2}$	0	0	-1	1	0	0	0	0	-2
$\bar{e}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	1	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	1	$\frac{7}{9}$
ℓ_2	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	0	$\frac{1}{3}$	0	-1	$\frac{1}{9}$
$\bar{u}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	$-\frac{2}{3}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{7}{9}$
$\bar{d}_2$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{9}$
q_2	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{7}{9}$
$\bar{n}_1$	1	0	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$-\frac{1}{6}$	0	0	$-\frac{5}{2}$	0	$\frac{1}{3}$	0	1	$\frac{13}{9}$
s_5^0	1	0	0	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{3}$	0	0	$-\frac{17}{9}$
$\bar{n}_2$	1	0	0	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{3}$	0	1	$\frac{7}{9}$
n_1	1	0	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{3}$	0	-1	$\frac{19}{9}$
s_8^0	1	0	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{3}$	0	0	$\frac{13}{9}$

TABLE VII. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	irrep	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_{B-L}	q_{anom}
s_9^0	1	0	0	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{3}$	0	0	$-\frac{17}{9}$
$\bar{n}_3$	1	0	0	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{3}$	0	1	$\frac{7}{9}$
$\bar{e}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	1	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	1	$\frac{7}{9}$
ℓ_1	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	0	$\frac{1}{3}$	0	-1	$\frac{1}{9}$
$\bar{u}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{2}{3}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{7}{9}$
$\bar{d}_1$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{6}$	0	0	$\frac{3}{2}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{9}$
q_1	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{7}{9}$
$\bar{n}_4$	1	0	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	0	$-\frac{5}{2}$	0	$\frac{1}{3}$	0	1	$\frac{13}{9}$
s_{12}^0	1	0	0	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{3}$	0	0	$-\frac{17}{9}$
$\bar{n}_5$	1	0	0	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{3}$	0	1	$\frac{7}{9}$
n_2	1	0	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{3}$	0	-1	$\frac{19}{9}$
s_{15}^0	1	0	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{3}$	0	0	$\frac{13}{9}$
s_{16}^0	1	0	0	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{3}$	0	0	$-\frac{17}{9}$
$\bar{n}_6$	1	0	0	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{3}$	0	1	$\frac{7}{9}$
s_1^-	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{5}{12}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	-2	$\frac{7}{9}$
s_2^-	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{5}{12}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	-1	$\frac{1}{9}$
s_1^+	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{6}$	0	1	$-\frac{5}{9}$
m_1	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$-\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	-1	$\frac{13}{9}$
$\bar{v}_1$	1	0	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	$-\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{1}{3}$	$\frac{19}{9}$
s_2^+	1	0	1	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$-\frac{5}{9}$
s_3^-	1	0	1	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$-\frac{5}{9}$
s_3^+	1	0	1	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$-\frac{5}{9}$
s_4^-	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{5}{12}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	-2	$\frac{7}{9}$
s_5^-	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{5}{12}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	-1	$\frac{1}{9}$
s_4^+	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{6}$	0	1	$-\frac{5}{9}$
m_2	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$-\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	-1	$\frac{13}{9}$
$\bar{v}_2$	1	0	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	$-\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{1}{3}$	$\frac{19}{9}$
s_5^+	1	0	1	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$-\frac{5}{9}$
s_6^-	1	0	1	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$-\frac{5}{9}$
s_6^+	1	0	1	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	0	$-\frac{5}{9}$
n_3	1	1	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	$\frac{13}{9}$
n_4	1	1	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	-1	$\frac{1}{9}$
h_1	1	1	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{13}{9}$
n_5	1	1	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	$\frac{13}{9}$
n_6	1	1	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	-1	$\frac{1}{9}$
h_2	1	1	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{13}{9}$
x_1^-	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{5}{12}$	$\frac{13}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	-1	$\frac{1}{9}$
x_1^+	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{4}$	$-\frac{1}{12}$	$\frac{7}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	1	$-\frac{11}{9}$
y_1	1	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$	0	$-\frac{5}{12}$	$\frac{1}{4}$	$-\frac{1}{12}$	$-\frac{5}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{13}{9}$
x_2^-	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{5}{12}$	$\frac{13}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	-1	$\frac{1}{9}$
x_2^+	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{4}$	$-\frac{1}{12}$	$\frac{7}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	1	$-\frac{11}{9}$
y_2	1	1	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$	0	$-\frac{5}{12}$	$\frac{1}{4}$	$-\frac{1}{12}$	$-\frac{5}{12}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{13}{9}$
w_1	1	2	0	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	0	$\frac{7}{9}$
s_{22}^0	1	2	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{13}{9}$
$\bar{n}_7$	1	2	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{13}{9}$
h_3	1	2	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{13}{9}$

TABLE VII. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	irrep	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_{B-L}	q_{anom}
w_2	1	2	0	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	0	$\frac{7}{9}$
s_{24}^0	1	2	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{13}{9}$
$\bar{n}_8$	1	2	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{13}{9}$
h_4	1	2	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{13}{9}$
s_7^-	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$-\frac{7}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{4}{3}$	-1	$-\frac{11}{9}$
s_7^+	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{5}{12}$	$\frac{1}{4}$	$\frac{7}{12}$	$-\frac{13}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	1	$\frac{25}{9}$
s_8^+	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{4}$	$-\frac{5}{12}$	$-\frac{13}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	2	$-\frac{11}{9}$
m_3	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$\frac{1}{12}$	$\frac{3}{4}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	1	$\frac{7}{9}$
v_1	1	2	1	0	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{5}{12}$	$\frac{1}{4}$	$-\frac{5}{12}$	$-\frac{1}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{7}{9}$
s_8^-	1	2	1	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$-\frac{7}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	0	$\frac{1}{9}$
s_9^+	1	2	1	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{17}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	0	$-\frac{5}{9}$
s_9^-	1	2	1	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$-\frac{7}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	0	$\frac{1}{9}$
s_{10}^-	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$-\frac{7}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{4}{3}$	-1	$-\frac{11}{9}$
s_{10}^+	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{5}{12}$	$\frac{1}{4}$	$\frac{7}{12}$	$-\frac{13}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	1	$\frac{25}{9}$
s_{11}^+	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{4}$	$-\frac{5}{12}$	$-\frac{13}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	2	$-\frac{11}{9}$
m_4	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	0	$\frac{1}{12}$	$\frac{3}{4}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	1	$\frac{7}{9}$
v_2	1	2	1	1	0	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{5}{12}$	$\frac{1}{4}$	$-\frac{5}{12}$	$-\frac{1}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{7}{9}$
s_{11}^-	1	2	1	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$-\frac{7}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	0	$\frac{1}{9}$
s_{12}^+	1	2	1	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{17}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	0	$-\frac{5}{9}$
s_{12}^-	1	2	1	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$-\frac{7}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	0	$\frac{1}{9}$
$\bar{\phi}_2$	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{3}$	0	0	1	0	$\frac{2}{3}$	0	0	$\frac{8}{9}$
$\bar{\delta}_1$	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	-1	0	$\frac{2}{3}$	0	$\frac{2}{3}$	$\frac{14}{9}$
s_{26}^0	2	0	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	0	1	0	0	$\frac{2}{3}$	0	0	$\frac{26}{9}$
$\bar{n}_9$	2	0	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	1	0	0	0	$\frac{2}{3}$	0	1	$\frac{20}{9}$
s_{28}^0	2	0	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	0	1	0	0	$\frac{2}{3}$	0	0	$\frac{26}{9}$
$\bar{n}_{10}$	2	0	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	1	0	0	0	$\frac{2}{3}$	0	1	$\frac{20}{9}$
h_5	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{2}{3}$	0	0	0	0	$-\frac{1}{3}$	-1	0	$-\frac{22}{9}$
$\bar{\eta}_1$	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{2}{3}$	0	0	0	0	$-\frac{1}{3}$	1	1	$-\frac{10}{9}$
$\bar{f}_3$	2	0	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$\frac{2}{3}$	0	0	0	0	$-\frac{1}{3}$	0	$\frac{1}{2}$	$-\frac{16}{9}$
$\bar{d}_3$	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{14}{9}$
$\bar{d}_4$	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{14}{9}$
$\bar{\ell}_1$	2	1	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{2}{9}$
$\bar{n}_{11}$	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{20}{9}$
h_7	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	0	$-\frac{4}{9}$
h_8	2	1	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{16}{9}$
$\bar{n}_{12}$	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{20}{9}$
h_9	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	0	$-\frac{4}{9}$
h_{10}	2	1	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{16}{9}$
s_{32}^0	2	1	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{5}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{26}{9}$
w_3	2	1	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	$-\frac{10}{9}$
$\bar{\phi}_3$	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{8}{9}$
$\bar{\delta}_2$	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$-\frac{4}{9}$
$\bar{\phi}_4$	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{4}{9}$
$\bar{\delta}_3$	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$-\frac{4}{9}$
n_7	2	2	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	-1	$\frac{14}{9}$
n_8	2	2	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	-1	$\frac{14}{9}$
s_{35}^0	2	2	*	*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{5}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{14}{9}$

TABLE VII. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	irrep	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_{B-L}	q_{anom}
η_1	2	2	$*$	$*$	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	-1	$-\frac{4}{9}$
h_{12}	2	2	$*$	$*$	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{4}{9}$
n_9	3	$*$	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	-1	0	0	-1	1
$\bar{n}_{13}$	3	$*$	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	0	1	-1
h_{13}	3	$*$	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	1	0	1
h_{14}	3	$*$	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	-1	0	-1
n_{10}	3	$*$	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	-1	0	0	-1	1
$\bar{n}_{14}$	3	$*$	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	0	1	-1
h_{15}	3	$*$	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	1	0	1
h_{16}	3	$*$	0	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	-1	0	-1
s_{13}^-	3	$*$	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	-1
s_{13}^+	3	$*$	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	1
$\bar{v}_3$	3	$*$	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{2}{3}$	1
v_3	3	$*$	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{2}{3}$	-1
s_{14}^-	3	$*$	1	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$\frac{5}{3}$
s_{14}^+	3	$*$	1	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{5}{3}$
s_{15}^-	3	$*$	1	0	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$\frac{5}{3}$
s_{15}^+	3	$*$	1	0	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{5}{3}$
s_{16}^-	3	$*$	1	0	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$-\frac{5}{3}$
s_{16}^+	3	$*$	1	0	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$-\frac{5}{3}$
s_{17}^-	3	$*$	1	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	-1
s_{17}^+	3	$*$	1	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	1
$\bar{v}_4$	3	$*$	1	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{2}{3}$	1
v_4	3	$*$	1	1	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{2}{3}$	-1
s_{18}^-	3	$*$	1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$\frac{5}{3}$
s_{18}^+	3	$*$	1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{5}{3}$
s_{19}^-	3	$*$	1	1	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$\frac{5}{3}$
s_{19}^+	3	$*$	1	1	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{5}{3}$
s_{20}^-	3	$*$	1	1	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$-\frac{5}{3}$
s_{20}^+	3	$*$	1	1	$\frac{1}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	$\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$-\frac{5}{3}$
ϕ_2	4	0	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	-1	0	$-\frac{2}{3}$	0	0	$-\frac{8}{9}$
δ_1	4	0	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	1	0	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{14}{9}$
ϕ_3	4	0	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	-1	0	$-\frac{2}{3}$	0	0	$-\frac{8}{9}$
δ_2	4	0	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	1	0	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{14}{9}$
η_2	4	0	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	0	$\frac{1}{3}$	-1	-1	$\frac{10}{9}$
h_{18}	4	0	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	0	$\frac{1}{3}$	1	0	$\frac{22}{9}$
f_1	4	0	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	0	$\frac{1}{3}$	0	$-\frac{1}{2}$	$\frac{16}{9}$
η_3	4	0	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	0	$\frac{1}{3}$	-1	-1	$\frac{10}{9}$
h_{20}	4	0	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{2}{3}$	0	0	0	0	$\frac{1}{3}$	1	0	$\frac{22}{9}$
f_2	4	0	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$-\frac{2}{3}$	0	0	0	0	$\frac{1}{3}$	0	$-\frac{1}{2}$	$\frac{16}{9}$
n_{11}	4	0	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	-1	0	0	0	$-\frac{2}{3}$	0	-1	$-\frac{20}{9}$
s_{41}^0	4	0	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	-1	0	0	$-\frac{2}{3}$	0	0	$-\frac{26}{9}$
ℓ_3	4	1	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	$-\frac{2}{9}$
ℓ_4	4	1	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	$-\frac{2}{9}$
d_1	4	1	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{14}{9}$
s_{42}^0	4	1	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{5}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{26}{9}$

TABLE VII. (*Continued*)

	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	irrep	q_Y	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_{B-L}	q_{anom}
w_4	4	1	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{10}{9}$
s^0_{43}	4	1	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{5}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{26}{9}$
w_5	4	1	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{10}{9}$
n_{12}	4	1	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	$-\frac{20}{9}$
h_{21}	4	1	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{16}{9}$
h_{22}	4	1	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{4}{9}$
ϕ_4	4	2	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{4}{9}$
δ_3	4	2	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})$	$-\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{4}{9}$
s^0_{45}	4	2	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{14}{9}$
h_{23}	4	2	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{4}{9}$
$\bar{\eta}_2$	4	2	$*$	$*$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	1	$\frac{4}{9}$
s^0_{46}	4	2	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{14}{9}$
h_{25}	4	2	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{4}{9}$
$\bar{\eta}_3$	4	2	$*$	$*$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	1	$\frac{4}{9}$
$\bar{n}_{15}$	4	2	$*$	$*$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$	0	$\frac{5}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{14}{9}$

$$\mathcal{M}_{s^+s^-} = \left(\begin{array}{cccccccccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & \tilde{s} & 0 & 0 & \tilde{s}^5 & 0 & \tilde{s} & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 \\
\tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & \tilde{s}^5 & 0 & 0 & \tilde{s} & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s} & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 \\
\tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5
\end{array}\right). \tag{F6j}$$

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