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Author
Meldner, Heiner.

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Heiner Meldner

August 29, 1967

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NUCLEAR ORBITAL REARRANGEMENT ENERGY

Heiner Meldner

August 29, 1967

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Heiner Meldner **

Lawrence Radiation Laboratory and Department of Physics

University of California

Berkeley, California

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ABSTRACT

On the basis of self-consistent calculations orbital rearrangement energies are estimated to be comparable to level differences in nuclear potential wells. Hence it is an unjustified simplification to identify such level spacings with the mass differences observed e.g. in nucleon transfer reactions.

Currently it seems to be a common procedure to interpret the spectra of nucleon transfer reactions in terms of energy eigenvalues in static Woods Saxon or oscillator potentials.[1,2] Apparently the corresponding wave functions are good enough to allow reasonable estimates of spin assignments (via DWBA calculations with adjusted

energies). Despite this fact, agreement with the spectra is probably of dubious significance, since mass differences can be compared quantitatively with the energy eigenvalues in a given well only if further simplifications are possible. For example, one must assume that orbital rearrangement energies are small compared to the observed mass differences. The purpose of this note is to show that these rearrangement energies can be large and can be quite different for different orbitals.

In single nucleon transfer reactions one determines total energy differences, which can be written as

$$E_A - E_{A^*} = C - C^* + \epsilon_A - \epsilon_{A^*} \qquad (1)$$

where the binding energy E_A of a nucleus with A nucleons is split into a "core" term

$$C = \sum_{i=1}^{A-1} t_i + \tfrac{1}{2} \sum_{i,j=1}^{A-1} v_{ij}$$

and the energy of a single "valence" particle or hole

$$\epsilon_A = t_A + \sum_{i=1}^{A-1} v_{iA} \quad ,$$

abbreviating in an obvious notation[3] kinetic and potential energies $t_i = \langle i|t|i\rangle$ and $v_{ij} = \langle ij|v|ij\rangle - \langle ji|v|ij\rangle$. The asterisks in (1) indicate that such matrix elements are calculated with wave functions of the rearranged self-consistent field where one particle or hole is in a state $A^* \neq A$.

Now the usual assumption $C^* = C$ is investigated here. This equality is found to hold only up to order $0.1A^{-1}$ in self-consistent models. Such a slight non-inertness of the core, however, means rearrangement effects of the order of MeV. The estimates of orbital rearrangement were inferred from calculations using the following equation for self-consistent nuclear single particle wave functions:

$$\left[\frac{\hbar^2}{2M}\frac{\partial^2}{\partial \vec{r}^2} - \epsilon_{\nu,m_t}\right]\varphi_{\nu,m_t}(\vec{r}) = \int d\vec{r}\,' K_{m_t}(\vec{r},\vec{r}\,')\varphi_{\nu,m_t}(\vec{r}\,') \quad . \qquad (2)$$

With the simple ansatz that K is essentially of the classic Van Vleck form,[4,5] namely for spherical nuclei

$$K_{m_t}(\vec{r},\vec{r}') = u(|\vec{r}-\vec{r}'|)\left[1-(\rho_{m_t}(x)/\rho_1)^{2/3}\right]\left[1-\frac{\sigma}{x}\vec{S}\vec{l}\frac{d}{dx}\right]\rho_{m_t}(x)$$

$$+ \delta(\vec{r}-\vec{r}')\left[\tfrac{1}{2}-m_t\right]V_C(x) \quad , \qquad (3)$$

with $x=\frac{r+r'}{2}$, $\rho_{m_t}=\left[\rho_{(-m_t)}+\tau\rho_{(m_t)}\right]\frac{1}{1+\tau}$, and $\rho_{(m_t)}=\sum_{\nu}^{\nu_F}|\varphi_{\nu m_t}|^2$.

ν denotes all quantum numbers specifying a bound nucleon except the third component m_t of its i-spin, and V_C is the static Coulomb potential corresponding to the proton density $\rho_{(-\frac{1}{2})}$. A suitable choice for the parameter ρ_1 [6] leads to self-consistent nucleon density distributions saturating with the observed rms radii and surface thicknesses. The parameter σ is chosen to give the shell model spin assignments, whereas the effect of the parameter τ is similar to the strength of the usual isospin or (N-Z) term in nuclear potentials. A sum of two Yukawa functions was used for u, i.e.

$$u(y) = \sum_{i=1}^{2} a_i v_i \exp(-y/a_i)/y$$

One fixed set of these parameters allows a reasonable fit of all gross

nuclear data that are expected to be given from a realistic nuclear single particle Hamiltonian.[4,5] Hence the examples presented in the figure should allow quantitative estimates of orbital rearrangements. For the first hole states in Ca^{47}, Pb^{207} and Tl^{207}, the level spacings are respectively 5.6, 2.1 and 1.2 MeV in the ground state and 4.0, 1.9 and 0.4 MeV in the excited state, whereas the calculated mass differences are 0.8, 0.9 and 1.6 MeV. The experimental mass differences are 2.6, 0.57 and 0.35 MeV.[7] This kind of discrepancy indicates that a reasonable calculation has to take into account residual two-body interactions. Hence level distances in single particle potentials, rearrangement energies, and level shifts due to residual two-body forces all seem to be of the same order of magnitude.

Therefore, it would seem futile for example[8] to infer an anomalous effective mass around the Fermi level by simply identifying level spacings with mass differences, i.e. completely neglecting rearrangements and residual interactions.

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** NATO Fellow. On leave from Freie Universität Berlin, West-Berlin.

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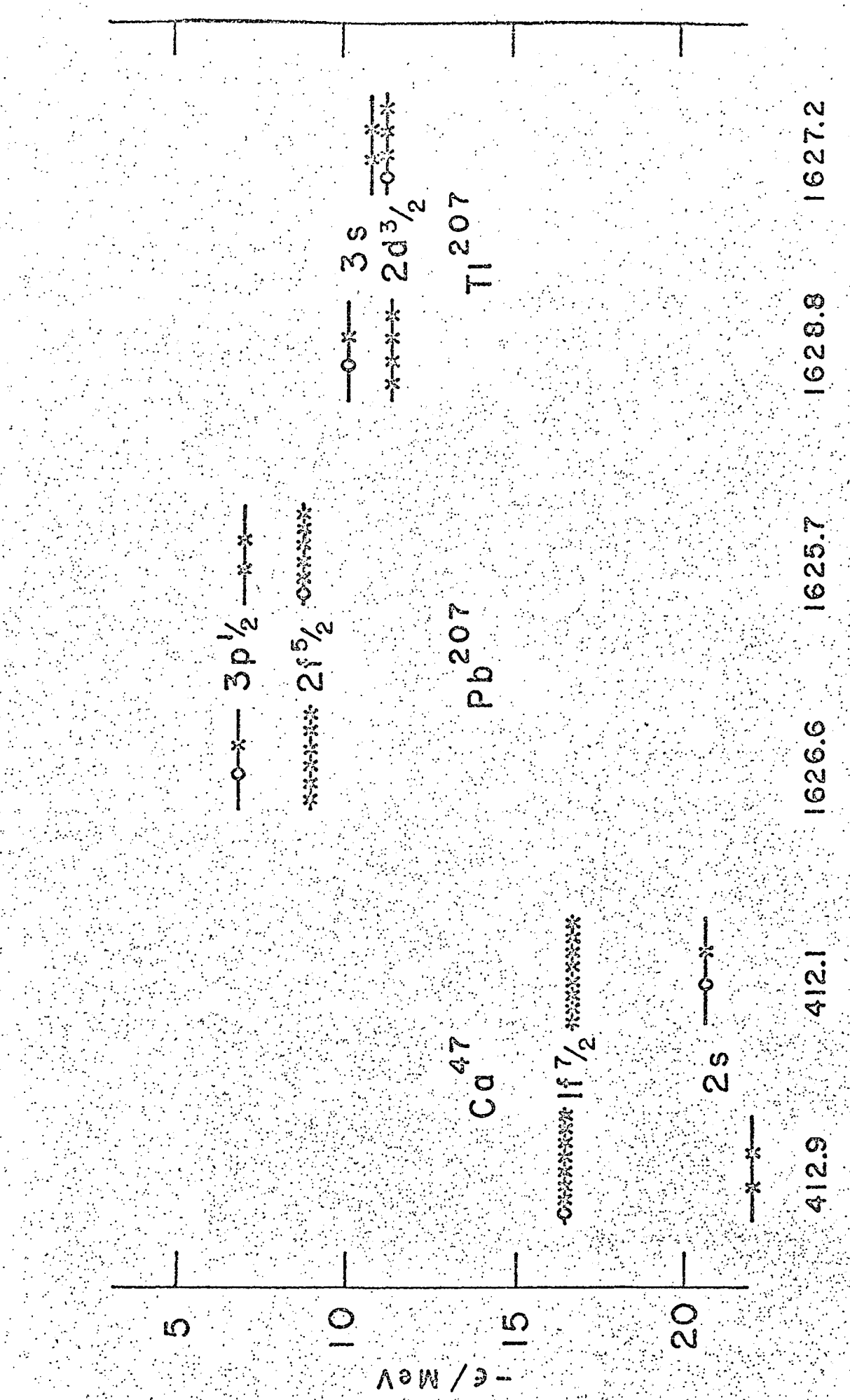
FIGURE CAPTION

Binding energies and single particle levels for the ground state and the first excited one-hole state in three doubly magic nuclei calculated with eqs. (2), (3) and the parameters: ρ_1=0.3 fm^{-3}, σ=0.55 fm^2, τ=0.3, v_1=220 MeV, v_2=3.1 MeV, a_1=0.3 fm, a_2=1.7 fm. For medium and heavy nuclei this parameter set gives a surprisingly good fit to rms radii, ls-proton levels as observed in (e,e'p) reactions, and total binding energies. The above examples are expected to estimate lower limits, since orbital rearrangement is likely to be least for nuclei with one hole in a closed shell [9].


-ε / MeV
5
10
15
20
3p 1/2
2f 5/2
3s
2d 3/2
Tl 207
Pb 207
Cd 47
1f 7/2
2s
412.9
412.1
1626.6
1625.7
1628.8
1627.2
Binding Energy / MeV

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