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DISCLAIMER

HEAVY-ION POTENTIAL DERIVED FROM A VELO-
CITY-DEPENDENT NUCLEONIC INTERACTION[*]

J. Randrup[†]
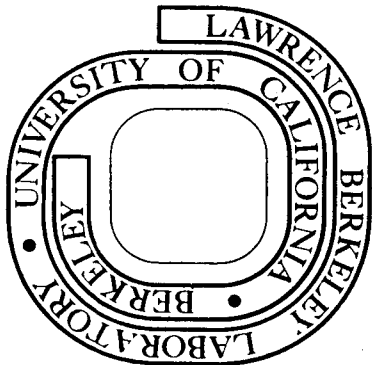
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ABSTRACT

A simple expression is derived for the elastic nuclear
interaction between two colliding heavy ions. It is based on
an effective velocity dependent nucleon-nucleon interaction.
An approximative treatment of the exclusion principle is
indicated. Numerical results are presented.

In this letter is reported the derivation of a very simple approximate expression for the elastic nucleon interaction between two colliding nuclei. The aim is to establish a simple semi-realistic tool for the discussion of the gross features associated with heavy-ion collisions, valid over a relatively wide energy range.

Recently [1] , a simple expression for the nuclear interaction potential has been derived on the basis of the proximity formula. In that treatment the matter distributions of the two interacting nuclear systems were superimposed without readjustments while the nucleonic momenta were redistributed in order to comply with the exclusion principle. However, when the two nuclei are in relative motion the importance of the exclusion principle is decreased and eventually, for velocities higher than twice the Fermi velocity (which corresponds to a kinetic energy of around 100 MeV per nucleon), it ceases to be in effect. In concert with this gradual disappearance of the exclusion principle, the velocity dependence of the basic two-nucleon interaction is expected to become of increased significance in the ion-ion potential.

In order to obtain a simple tool which may yield rough, but not too unrealistic, estimates of the expected dynamical implications of such velocity dependence the following approach has been taken. At first the exclusion principle is neglected all together (as stated above, this in strictly valid only for sufficiently high velocities) and the ion-ion interaction is derived on the basis of an effective nucleon-

nucleon interaction. It should be pointed out that the assumption of no dynamical readjustment of the colliding density distributions is not decisive for the present approach to be applicable. The essential point is the assumption that the two nuclei preserve their identity such that, at any stage, the situation may be described as an interaction between two distinct systems. The exclusion principle may be taken approximatively into account subsequently by an appropriate interpolation between this no-adjustment limit and the full-adjustment limit treated in ref. 1 .

The effective nucleon-nucleon interaction employed is of the following form

$$V_{12} = -C \, g\left(\frac{r_{12}}{a}\right)\left(1 - p_{12}^2 / b^2\right)$$

This type of momentum-dependent interaction was first introduced by Seyler and Blanchard [2] and has proved very useful for the discussion of gross nuclear properties, see for example ref. 3 . The parameter a governs the spatial range of the interaction while the parameter b is that 'critical' momentum at which the interaction changes from attractive to repulsive. The strength constant C is here assumed to be the same for all nucleon pairs, as is justified for systems with identical neutron and proton distribution; the extension to asymmetric systems is straightforward. An approximate account of the nuclear asymmetry may be taken by scaling the emerging potentials in proportion to the surface energy of the actual system

under study.

It can be shown [4] that the two-particle interaction energy density due to the above interaction may be written

$$w(\vec{r}', \vec{r}'') = -\tfrac{1}{2} C \ g(\tfrac{r}{a}) \ (\rho(\vec{r}') \ \rho(\vec{r}'')$$

$$+ 2 \vec{\pi}(\vec{r}') \cdot \vec{\pi}(\vec{r}'') - \rho(\vec{r}') \ \gamma(\vec{r}'') - \gamma(\vec{r}') \ \rho(\vec{r}''))$$

with $r = |\vec{r}' - \vec{r}''|$. Here ρ is the mass density (in units of the nucleon mass m), $\vec{\pi}$ the momentum density (in units of b), and γ the kinetic density (in units of $b^2/2m$). In order to derive the interaction between two moving systems it is necessary to know the Galilei transforms of these densities. They are

$$\rho_V = \rho_0$$

$$\vec{\pi}_V = \vec{\pi}_0 + \tfrac{m}{b} \vec{V} \rho_0$$

$$\gamma_V = \gamma_0 + 2 \tfrac{m}{b} \vec{V} \cdot \vec{\pi}_0 + \tfrac{m^2}{b^2} V^2 \rho_0$$

For two nuclei a and b moving with the relative velocity $\vec{V} = \vec{V}_a - \vec{V}_b$ the interaction is obtained by inserting the appropriate Galilei transforms into the general expression above and integrating over the variables $\vec{r}'$ and $\vec{r}''$. In the following it will for simplicity be assumed that there are no internal gross flows in the two nuclei, $\vec{\pi}_a = \vec{0} = \vec{\pi}_b$; it would not be difficult to relax this restriction. Now the interaction energy U_V of the two nuclei

can be written as

$$U_V = U + \tfrac{1}{2} W V^2$$

where the velocity-independent functions U and W are given by

$$U = -C \iint \left(\rho_a(\vec{r}') \rho_b(\vec{r}'') + 2\vec{n}_a(\vec{r}') \cdot \vec{n}_b(\vec{r}'') \right.$$

$$\left. - \rho_a(\vec{r}') \gamma_b(\vec{r}'') - \gamma_a(\vec{r}') \rho_b(\vec{r}'') \right) g(\tfrac{r}{a}) \, d^3\vec{r}' \, d^3\vec{r}''$$

and

$$W = m \frac{C}{b^2/2m} \iint \rho_a(\vec{r}') \rho_b(\vec{r}'') g(\tfrac{r}{a}) \, d^3\vec{r}' \, d^3\vec{r}''$$

It follows that the total energy of the two-ion system may be written on the form

$$E = E_{cM} + \tfrac{1}{2} B V^2 + U$$

where E_{cM} is the energy of the overall center-of-mass motion. Thus the relative motion is described in terms of a static potential U (in which the Coulomb contribution should be included) and an effective mass $B = \mu + W$ where μ is the usual reduced mass, $1/\mu = 1/M_a + 1/M_b$. The dynamical equation for the relative motion then takes on the simple form $B\dot{\vec{V}} = -\vec{\nabla} E$ where the differentiation is with respect to the relative position of the two ions.

As stated earlier, the potential U derived above applies to large relative velocities only. In the interme-

diate region a partial readjustment of the nucleonic momenta must take place because of the exclusion principle. This adds an implicit velocity dependence of the interaction to the explicit velocity dependence represented by the variable effective mass B. The question concerning the proper form of the interpolation between the two extremes is currently being pursued in a separate study [5] .

The general formulas above may be transformed into a practical tool for simple analyses by use of the proximity formula [1] . This formula expresses a short-range overlap integral between curved surfaces as a geometrical factor containing the actual geometry (such as size and deformation) times a "universal" function evaluated (once and for all) for a semi-infinite configuration. For a detailed discussion see ref. [1] . The application of the proximity formula leads to the following simple expressions,

$$u \approx 2\pi \bar{R} \, \tilde{u} \, (s)$$

$$w \approx 2\pi \bar{R} \, \tilde{w} \, (s)$$

Here the geometrical factor $\bar{R} = C_a C_b / (C_a + C_b)$ measures the radius of curvature of the gap between the surfaces. As discussed in ref. 1 , the effective surface locations C are given approximately by $C \approx R - b^2/R$ where the nuclear radius $R \approx 1.15 \, A^{1/3}$ fm and the surface diffuseness $b \approx 1$ fm . The variable s denotes the effective surface separation, $s = R - C_a - C_b$.

A numerical calculation of the "universal" functions $\tilde{u}$ and $\tilde{w}$ has been carried out. The calculation is based on the effective nucleonic interaction described earlier. The parameter values are those determined in ref. 3 :

a = 0.62567 fm, b = 372.48 MeV/c and C = 328.61 MeV. The
The results are displayed in figs. 1 and 2. Two parallel
sets of calculations have been performed, one based on the
Thomas-Fermi approximation and one based on the Hartree appro-
ximation (see ref. 4 for the formulation of the model in
the Hartree approximation). The overall shift towards larger
separation for the Hartree results is a consequence of the
extended density tails, which are not present in the Thomas-
Fermi treatment. Included in fig. 2 is the potential obtained
in ref. 1 based on a total momentum readjustment, $\tilde{v}_p$.

In conclusion the following can be stated. A simple
approximate representation of the elastic ion-ion interac-
tion has been suggested. The interaction contains an explicit
velocity dependence in the form of a variable reduced mass;
this is due to the velocity dependence of the basic nucleo-
nic interaction. In addition an implicit velocity dependence
appears because of the exclusion principle. A simple approxi-
mative treatment of this effect is suggested and a study of
the detailed form is in progress [5] . A major simplification
is accomplished by employing the proximity formula for the
evaluation of the overlap integrals entering. The corresponding
"universal" mass and potential functions have been calculated
numerically. The very simple form of the final formulas
makes the model a practical tool for discussing the gross
features associated with nuclear collisions. A study of heavy-
ion dynamics based on this model is currently being carried
out [5].

FOOTNOTES AND REFERENCES

† On leave from University of Aarhus, Aarhus, Denmark. Present address: The Niels Bohr Institute, Copenhagen, Denmark.

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FIGURE CAPTIONS

Fig. 1   The "universal" mass-renormalization function $\tilde{w}$

as calculated on the basis of the velocity-de-

pendent effective interaction in the Thomas-Fermi

(full curve) and Hartree (dashed curve) approximations.

Fig. 2   The "universal" static-potential function $\tilde{u}$.

Similar to fig. 1. The proximity function $\tilde{v}_P$
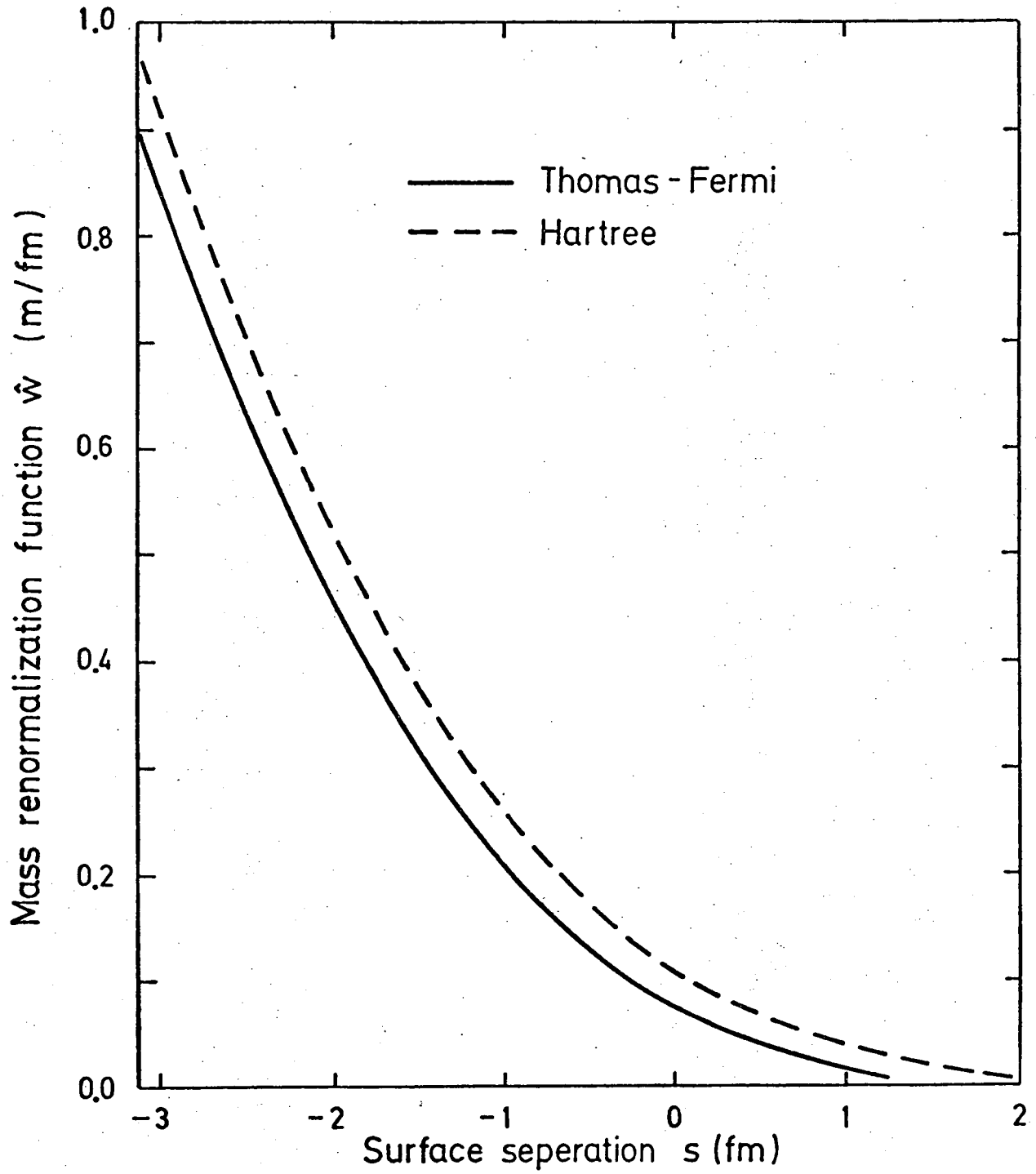
derived in ref. 1   is indicated by the dot-dashed
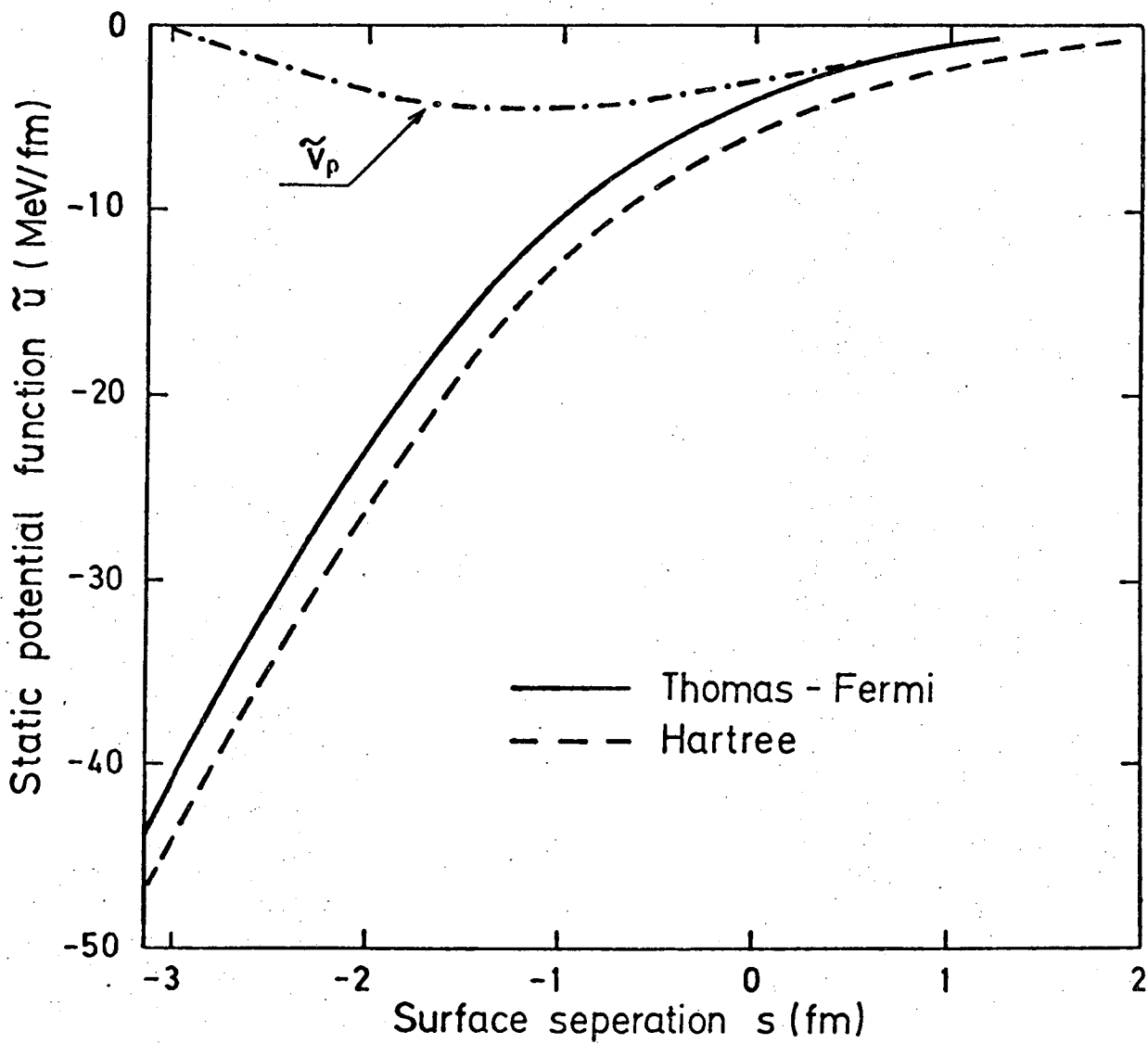
curve.

Fig. 1

Fig. 2