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Application of Genetic Programming to Flow Routing in Simple and Compound Channels

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Abstract: Hydraulic methods can model channel flow with high accuracy using data related to channel geometry and flow regime that render the computational effort burdensome. In contrast, hydrologic methods apply simplifying assumptions in their algorithms for flow routing. This paper implements genetic programming (GP) to calculate hydrographs in simple and compound channels. Predicted hydrographs for the simple and compound channels are compared with those predicted by a Muskingum model and a one-dimensional (1D) coupled characteristic-dissipative-Galerkin (CCDG-1D) procedure. Results show that the differences between predicted hydrographs by GP and modeled hydrographs by the Muskingum and CCDG-1D methods are similar in simple and compound channels. Moreover, GP yields acceptable predicted hydrographs with decreased computational burden. These results indicate that the proposed GP method is effective in the prediction of open-channel flow. **DOI:** 10.1061/(ASCE)IR.1943-4774.0001109. *© 2016 American Society of Civil Engineers.*

Author keywords: Hydrograph prediction; Open channels; Genetic programming; Muskingum routing.

Introduction

The Muskingum model is a hydrologic routing method that has been coupled with different optimization algorithms to calculate its optimal coefficients. Mohan (1997), Kim et al. (2001), Samani and Shamsipour (2004), Geem (2006), Chu and Chang (2009), and Geem (2011) applied the genetic algorithm (GA), transformed Powell's conjugate directions method, harmony search (HS), particle swarm optimization (PSO), and parameter-setting-free harmony search, respectively, to route downstream hydrographs in open channels. In the aforementioned investigations the channel was considered as a storage volume and the continuity equation was applied to calculate downstream flow hydrograph in a stream reach.

Among the wide range of statistical and optimization techniques reported in water resources publications (Ashofteh et al. 2015a, b, c; Beygi et al. 2014; Bozorg-Haddad et al. 2013, 2014; Bolouri-Yazdeli et al. 2014; Orouji et al. 2013, 2014; Shokri et al. 2013, 2014) genetic programming (GP) has been demonstrated to perform very well.

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Genetic programming is an artificial intelligence tool inspired by the theory of biological evolution, and is capable of determining an appropriate prediction relation between input and output data sets. It has been applied in various fields of water resources engineering. Savic et al. (1999), Khu et al. (2001), Rabunal et al. (2007), Sivapragasam et al. (2008), Guven and Gunal (2008), Kisi and Guven (2010), Guven and Kisi (2011), Izadifar and Elshorbagy (2010), Ghorbani et al. (2010), Azamathulla and Ghani (2011), Azamathulla et al. (2011), Fallah-Mehdipour et al. (2013), Hakimzadeh et al. (2014), Mehr et al. (2013, 2014), and Zaji and Bonakdari (2015) applied GP to flow prediction, runoff forecasting, determination of unit hydrograph in a typical urban basin, flood routing in natural channels, prediction of local scour downstream of hydraulic structures, evaporation estimation, determination of suspended sediment yield in natural rivers, actual evapotranspiration estimation, sea water level forecasting, prediction of longitudinal dispersion coefficients, stage-discharge development, simulation of dam breach hydrograph and peak outflow discharge, stage hydrograph routing, streamflow and successive-station monthly streamflow prediction, and estimation of a longitudinal velocity field in open-channel junctions, respectively.

This paper applies GP to predict flow hydrograph in open channels. Two simple and compound channels are considered as case studies to compare the capability of the GP approach with other hydrologic and hydraulic methods. Results are compared with those obtained from the Muskingum model and the St. Venant equations.

Muskingum Model

The Muskingum model is a hydrologic method based on the continuity and storage equations. It determines the variation of storage as the difference between inflow to and outflow from storage, as follows:

$$\frac{dS_t}{dt} = I_t - O_t \tag{1}$$

$$S_t = K[XI_t + (1 - X)O_t] \tag{2}$$

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where S_t, I_t, and O_t = storage, inflow, and outflow at the tth time step, respectively; K = storage-time constant for a channel reach, which has a value reasonably close to the flow travel time through the channel reach; and X = weighting factor usually ranging between 0 and 0.5 for storage volume, and between 0 and 0.3 for stream channels (Mohan 1997). The hydrograph predicted by GP is herein compared with those hydrographs obtained with the Muskingum model in a simple channel.

Hydraulic Method [One-Dimensional Coupled Characteristic-Dissipative-Galerkin (CCDG- 1D) Procedure]

The CCDG-1D is a hydraulic routing method introduced by Tuitoek and Hicks (2001) for modeling unsteady flow in compound channels. They proposed a one-dimensional model that handled the river and floodplains as a compound channel while accounting for the flow interaction and mass transfer between the main channel and floodplain through the introduction of an apparent shear force. Interdependence between the channel and floodplains was established through mass and longitudinal momentum transfer functions. These momentum transfer functions included both convective momentum transport as well as the apparent shear force generated along the interfaces between the main channel and the floodplains (Tuitoek and Hicks 2001; Seckin et al. 2009). In the CCDG-1D, model a diffusive wave approximation facilitated the simulation of flow on a dry bed. Second, because of higher values of relative roughness on the floodplains, the magnitudes of the inertial term in the equations were very small in comparison with the pressure, slope, and friction terms. More information on the CCDG-1D approach is found in Tuitoek and Hicks (2001). This paper compares the results of the CCDG-1D application with GP results in a compound channel.

Genetic Programming

Genetic programming is a random-search evolutionary algorithm that searches the decision space with a tree-structured algorithm. The first report about the modern tree-structured GP was by Cramer (1985), and later expanded on by Koza (1992, 1994). The tree structure presents mathematical equations in a tree form that includes numerical and nonnumerical variables, arithmetic operators ($\pm$, $\times$, $\div$), mathematical functions (e.g., sin, cos), Boolean operators (e.g., and, or), logical expressions (e.g., if-then-else), and other user-defined functions. In this structure, all the variables and operators are assumed to be the terminal and functional sets, respectively. There is a random iterative process in GP in which a random set of trees is generated in the first iteration. In each iteration the trees are compared by considering the calculated fitness function. These trees are then selected based on their fitness values using selection techniques, such as roulette wheel, tournament, or ranking methods. The better trees have greater chances of being selected for passage to the next iteration. After selection, these trees are reproduced with some modifications performed by the genetic operators: crossover and mutation.

The GP crossover operator uses a two-point string crossover in which segments of random position and random length are selected in each parent and exchanged between them. If one of the resulting children exceeds the maximum length, crossover is abandoned and restarted by exchanging equalized segments (Brameier and Banzhaf 2001). Mutation is another efficient genetic operator, which randomly exchanges a node variable with another type of random variable. The new trees are then subjected to the same process of modification, and the searching process continues until the maximum number of iterations is reached, or the specified convergence criterion is achieved.

Methodology

The GP capability in predicting downstream hydrographs of simple and compound channels is tested in this work using a tool coded in the software package *MATLAB 8.0* and run on a PC/WindowsXP/256MB RAM/2.93GHz computer (Toshiba, New York). The number of trees was set equal to 50 as the population size, whereas the crossover and mutation probability were set equal to 0.4 and 0.3, respectively, based on a sensitivity analysis, with the search algorithm executing 1,000 iterations to achieve an accurate solution. In addition, four arithmetic operators, including $\pm$, $\times$, and $\div$, and two mathematical functions involving sin and cos were considered in the GP.

The GP performance was assessed with the minimization of the sum of the squared deviation (SSQ) between GP and observed outflows as the objective function, as follows:

$$\mathrm{SSQ} = \sum_{t=1}^{T} (Q_t - \hat{Q}_t)^2 \tag{3}$$

where T = number of time periods; Q_t = observed flow at period t; and $\hat{Q}_t$ = estimated flow by GP at period t.

Artificial intelligence tools are sensitive to the input data sets used to produce an output set. In this paper, the upstream observed discharge was considered as the input data. Observed discharge at the current period is the first category of input data. Observed discharges at the current and first previous periods were the second category of input data. Finally, observed discharges at the current, first, and second previous periods were the third data category. Downstream discharge constituted the predicted output data.

The mathematical formulation of the aforementioned data is expressed as follows:

$$\hat{Q}(t) = f_1[Q(t)] \tag{4}$$

$$\hat{Q}(t) = f_2[Q(t-1), Q(t)] \tag{5}$$

$$\hat{Q}(t) = f_3[Q(t-2), Q(t-1), Q(t)] \tag{6}$$

where f_1, f_2, and f_3 = calculated functions by the GP for predicting the downstream hydrograph; and $Q(t-1)$ and $Q(t-2)$ = discharge in the first and second previous periods at the upstream station, respectively.

Silakhor River Case Study

The first case study was the Silakhor river simple natural channel in the Lorestan province of Iran (Fig. 1). This river was considered by Samani and Shamsipour (2004) and its hydrograph was calculated with the St. Venant and Muskingum models. Fig. 2 shows a cross section of the Silakhor river.

Genetic programming was herein used to predict downstream hydrographs of the Silakhor river. Figs. 3(a–d) present the SSQ variations corresponding to different values of depth and number of trees. It is seen in Fig. 3 that the minimum (best) SSQ is associated with the fourth and fifth depths and 45 and 50 trees, respectively. Results were compared with hydrographs predicted by the

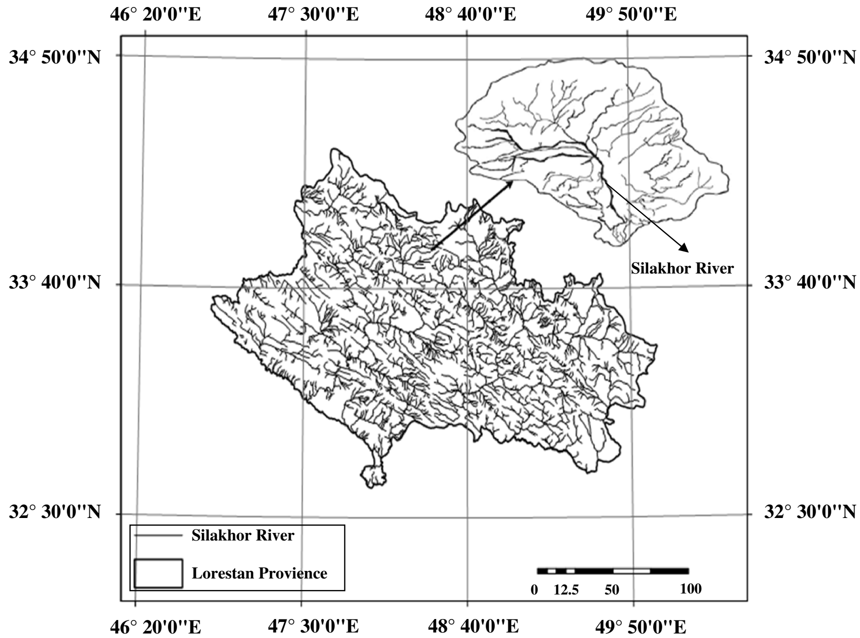

Fig. 1. Location of the Silakhor river in the Lorestan province

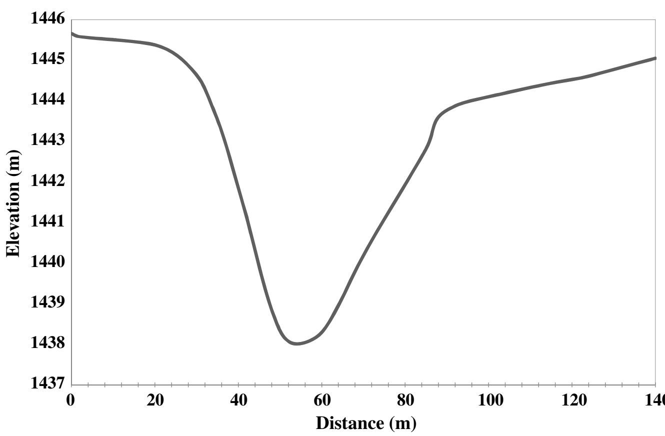

Fig. 2. Silakhor river cross section

Muskingum method. Genetic programming uses a random-based search process to determine an optimal solution. Thus, the *MATLAB 8.0* code was run five different times for each of the f_1, f_2, and f_3 functions. Table 1 lists results of those runs and their statistical measures. It is seen in Table 1 that the minimum, average, and maximum of the SSQ for five different runs used in the calculation of f_3 equal 54.79, 53.49, and 53.66% smaller (better), respectively, than the values for f_1. Thus, the current, first, and second previous periods data sets as the input data of GP yielded the best results compared with the other input data sets.

It was remarked that by using the Muskingum model, the obtained SSQ by Samani and Shamsipour (2004) was equal to 1,256.3. Accordingly, the best objective function value by GP in a 1-h time interval (358.29) was 71.48% better (smaller) than the value yielded by the Muskingum model. Fig. 4 shows calculated hydrographs by the St. Venant, Muskingum, and GP approaches for the Silakhor river. It is seen in Fig. 4 that the predicted hydrograph by GP is close to that obtained with the St. Venant hydraulic method, thus showing acceptable accuracy in flow modeling.

The time interval for calculating f_1, f_2, and f_3 was 1 h. Use of a larger time interval improves the prediction of the downstream hydrograph with less computational expense. A time interval equal to 2 h was implemented to test the GP performance in prediction of downstream hydrograph with fewer control points. In this case, the input data set considered current, first, and second previous periods data sets, which produced the best results for the time interval of 1 h. Fig. 5 depicts the predicted hydrograph for the 1- and 2-h time intervals. It is seen in Fig. 5 that there is no considerable difference in the hydrographs. A performance criterion to compare the two hydrographs is:

$$\mathrm{MRE} = \frac{1}{T}\sum_{t=1}^{T}\frac{|\hat{Q}_t - Q_t|}{Q_t} \tag{7}$$

where MRE = mean relative error.

Although the MRE for hydrographs with 1- and 2-h intervals are, respectively, 0.019 and 0.024, the difference between them is negligible. Thus, GP can yield acceptable results when predicting hydrographs using fewer number of measured flows.

Treske Channel Case Study

The capability of GP in predicting a hydrograph in a compound channel was tested and compared with the hydraulic flow model (CCDG-1D) using Treske's unsteady flow data as reported by Tuitoek and Hicks (2001). The Treske channel is a compound channel with a main channel with bed width and depth of 1.25 and 0.39 m, respectively. The left and right overbanks are 3 and 1.5 m wide, respectively. The bed slope is 0.019% and the Manning roughness coefficient for both main channel and floodplains is 0.012. The working length of the channel is 210 m. Fig. 6 shows a cross section of the Treske channel. In this channel, there are two upstream and downstream measurement stations within the 210-m distance. Fig. 7 graphs the observed discharge at the upstream and downstream stations.

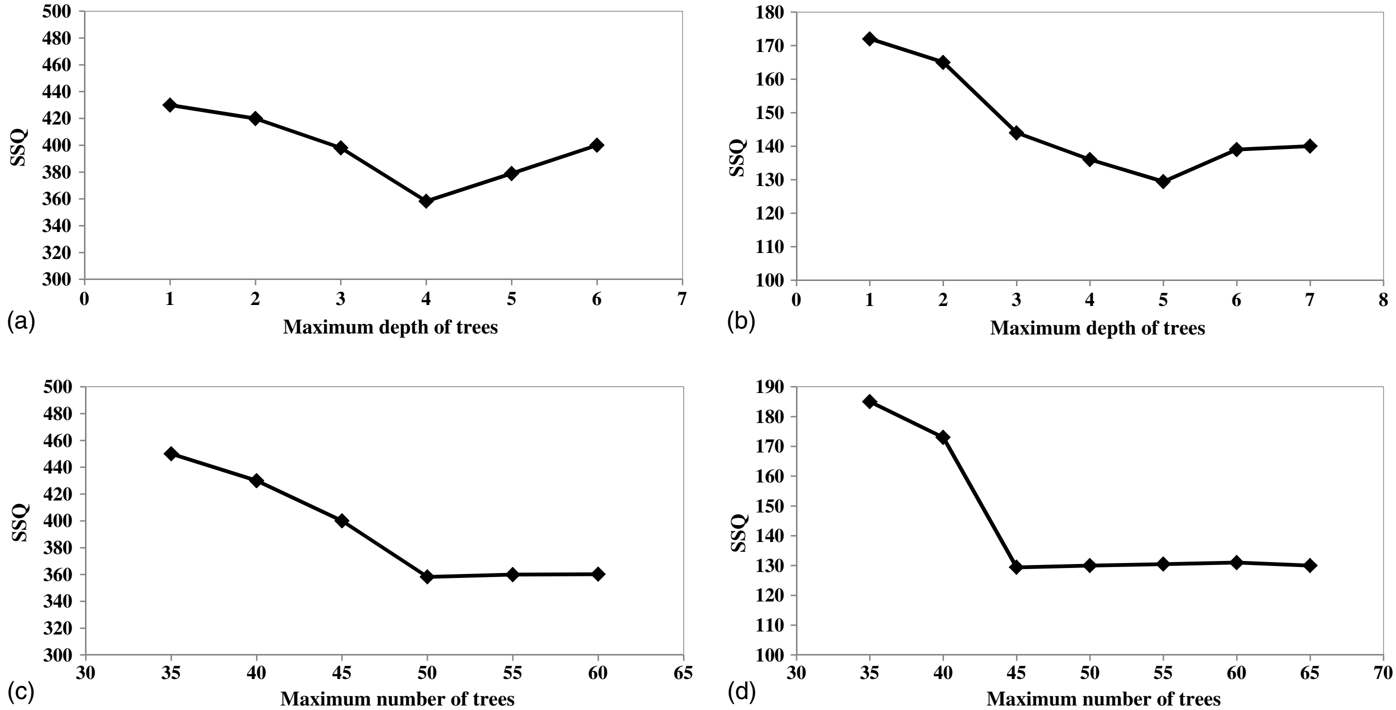

Fig. 3. Sensitivity analysis of the GP parameters for time interval: (a and b) 1 h; (c and d) 2 h for the Silakhor river

Table 1. SSQ Results for Various Runs in the Silakhor River

		Number of runs					Statistical measure				
Time interval (hour)	SSQ	1	2	3	4	5	Minimum	Average	Maximum	Standard deviation	Coefficient of variation
1	f_1	1,023.05	945.32	1,112.65	1,232.98	983.20	945.32	1,059.44	1,232.98	115.23	0.11
	f_2	939.01	823.50	792.58	942.00	846.79	792.58	868.78	942.00	68.25	0.08
	f_3	436.50	393.65	410.36	358.29	421.50	358.29	404.06	436.50	30.00	0.07
2	f_3	156.26	129.42	159.31	147.53	139.45	129.42	146.39	159.31	12.27	0.08

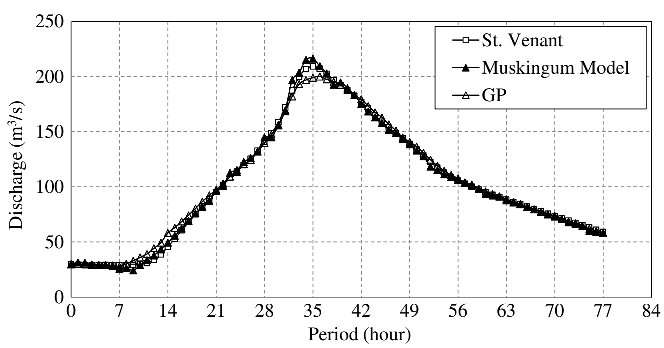

Fig. 4. Calculated downstream hydrographs by hydraulic and hydrologic methods

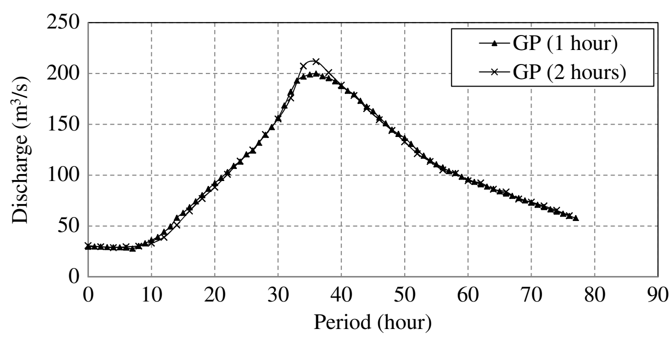

Fig. 5. Calculated downstream hydrograph by GP for various time intervals

Figs. 8(a–d) present the SSQ analysis by considering different values of depth and number of trees. The minimum SSQ corresponds to the fourth and fifth depths and 50 and 55 trees. To compare the capability of GP as a random-based tool with that of the CCDG-1D as a hydraulic method in flow modeling, five runs of GP were performed. Table 2 lists the SSQ of these runs for the f_1, f_2, and f_3 functions. It can be seen that the minimum, average, and maximum of the SSQ for five different runs implemented in the calculation of f_3 were less than the corresponding values for f_1 and f_2, respectively. Thus, the third combination of input data sets (current, first, and second previous periods) that yielded the best results compared with the other input combinations was selected as the best function.

Note that the larger the number of runs conducted, the higher the probability of obtaining a better solution. The coefficient of

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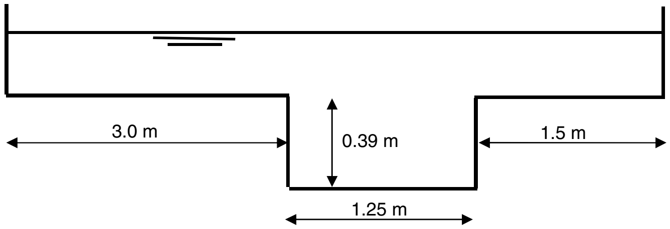

Fig. 6. Cross section of Treske channel's straight compound channel

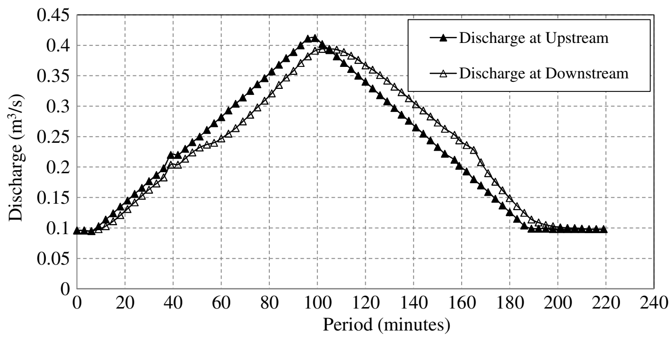

Fig. 7. Treske channel's discharge at upstream and downstream stations

variation is a dimensionless metric of variation. Smaller coefficients of variations of obtained objectives mean a higher probability of an accurate solution. According to the results, the coefficients of variation of all combinations were acceptable (small value), indicating the high probability of obtaining an accurate solution even with a single run.

Fig. 9 shows the downstream hydrograph obtained from the application of CCDG-1D and GP, indicating that the difference between calculated and observed hydrographs is negligible. According to the results, those differences in the peak of the hydrograph for the CCDG-1D and GP are in the range of 4 to 7% and 3 to 6%, respectively.

The time interval of 3 min for the flow model time step was extended to 6 min in an attempt to compare the GP capability in using different time intervals. In this instance, the number of control points decreased to determine the best appropriate mathematical function in the downstream hydrograph. Table 2 lists the SSQ and statistical measures for this time interval. Fig. 10 presents the best predicted hydrographs by GP with different time intervals, indicating that the two hydrographs are very similar to each other.

Commonly, the peak of the hydrograph with the maximum discharge plays a main role in flow prediction. In this paper, the ranges of difference between observed and predicted hydrographs at the peak point are (0.09, 4.83%) and (−2.28, 0.1%) in the Silakhor river for 1- and 2-h time intervals, respectively. Ranges are (−10.39, −4.79%) and (−4.30, −2.45%) in the Treske channel for 3- and 6-min time intervals, respectively. In flow prediction by GP, the depth and number of trees are the parameters that mainly affect the calculated objective function. Accordingly, the ranges of the obtained objective function's SSQ differences are (358, 430) and (129, 172) in the Silakhor river for 1- and 2-h time intervals, respectively. Ranges are (0.0069, 0.0099%) and (0.0071, 0.0099%) in the Treske channel for 3- and 6-min time intervals, respectively.

Concluding Remarks

Many water resource investigations have shown the efficiency of GP in developing an appropriate relation between input and output data sets. In this paper, the GP was applied to estimate the best-predicted downstream hydrograph of simple and compound

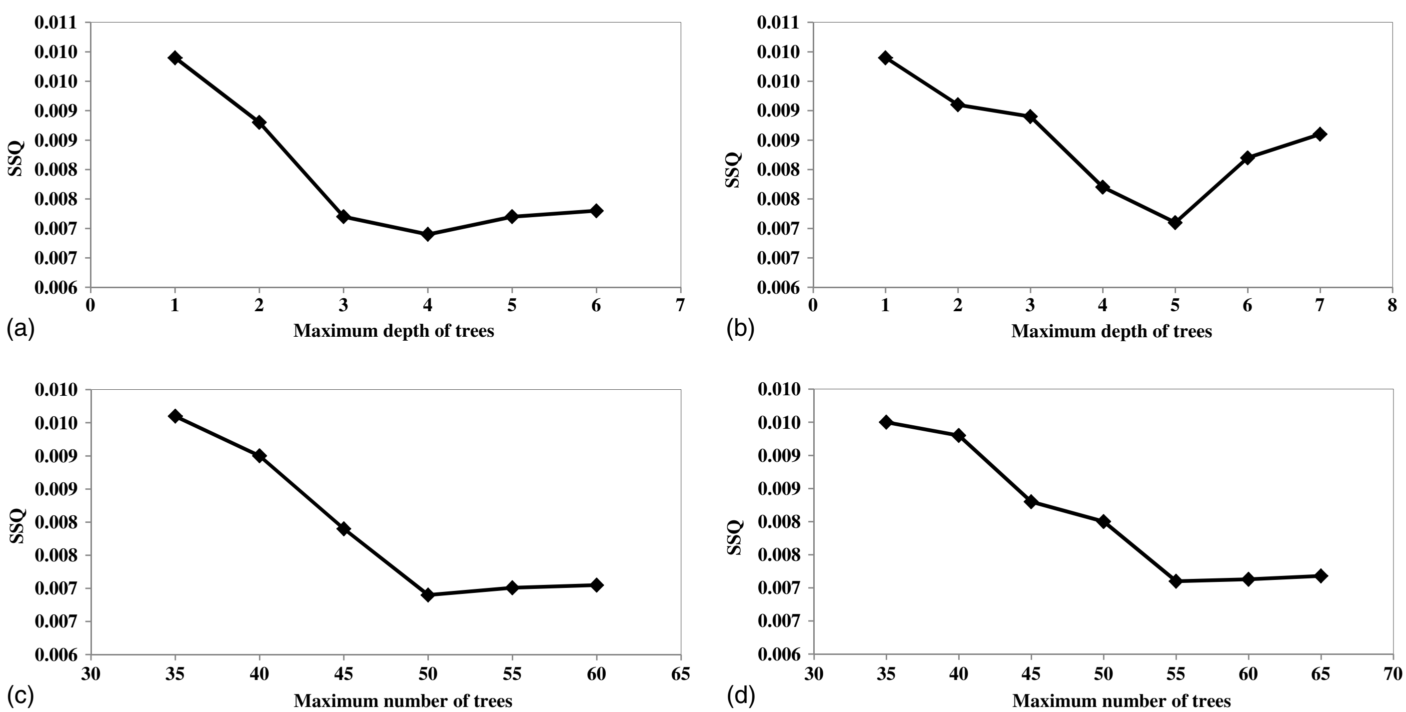

Fig. 8. Sensitivity analysis on GP parameters for time interval: (a and b) 3 min; (c and d) 6 min for the Treske channel

Table 2. SSQ Results for Various Runs in the Treske Channel

Time interval (minute)	SSQ	Number of runs 1	2	3	4	5	Statistical measure Minimum	Average	Maximum	Standard deviation	Coefficient of variation
3	f_1	0.0092	0.0085	0.0077	0.0093	0.0080	0.0077	0.0085	0.0093	0.0007	0.0830
	f_2	0.0096	0.0085	0.0088	0.0083	0.0086	0.0083	0.0086	0.0088	0.0002	0.0243
	f_3	0.0083	0.0072	0.0091	0.0070	0.0069	0.0069	0.0077	0.0091	0.0010	0.1249
6	f_2	0.0071	0.0091	0.0077	0.0073	0.0098	0.0071	0.0082	0.0098	0.0012	0.1464

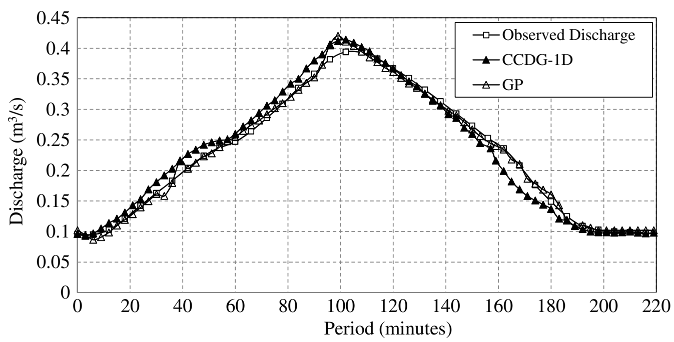

Fig. 9. Observed and estimated downstream hydrographs for the Treske channel

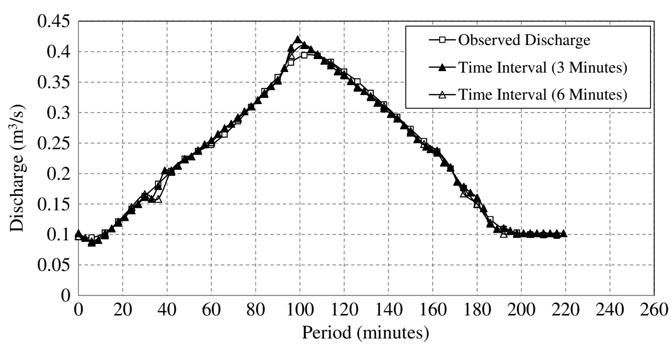

Fig. 10. Observed and estimated downstream hydrographs for various time intervals in the Treske channel

channels. The Muskingum and CCDG-1D approaches, being hydrologic and hydraulic methods, respectively, were implemented to test the GP results and model downstream flow by using upstream conditions and channel specifications for simple and compound channels, respectively. Three different data sets of an upstream hydrograph were used to analyze the sensitivity of input data to predict a downstream hydrograph, including the observed discharge at the upstream station corresponding to (1) the current period, (2) the current and first previous periods, and (3) the current, first, and second previous periods. Our results demonstrated that the SSQ of the observed discharge at the upstream current period and first and second previous periods yielded the best solution with a 50 and 10% improvement (decrease) compared with two other input data sets which included (1) current periods and (2) current and first previous period.

This paper conducted a sensitivity analysis to investigate the effect of the time interval in the predicted hydrographs. Time steps of 1- and 2-h duration in the simple channel and 3- and 6-min duration in the compound channel were used with GP modeling. This paper's results show that these two choices of set of time intervals produced a 3–6% difference from the peak of the observed hydrograph in the compound channel, indicating the efficiency of GP in modeling flow in compound channels. Moreover, there is no considerable difference in the MRE for simple and compound channels with the aforementioned time intervals.

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