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HIGH ENERGY PARTIVCLE DATA. Volume I: KINEMATICS OF PARTICLES AS A FUNCTION OF KINETIC ENERGY.

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HIGH-ENERGY PARTICLE DATA

Volume I

KINEMATICS OF PARTICLES AS A FUNCTION OF KINETIC ENERGY

Berkeley, California

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Berkeley, California

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Volume I

KINEMATICS OF PARTICLES AS A FUNCTION OF KINETIC ENERGY

W. Peter Trower,
Beverly Hill Willis, and Charles V. Stableford

July 1, 1963

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HIGH-ENERGY PARTICLE DATA
Volume I
KINEMATICS OF PARTICLES AS A FUNCTION OF KINETIC ENERGY

Contents

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Lawrence Radiation Laboratory
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INTRODUCTION

This collection of graphs is a complete revision of the 1956 edition of UCRL-2426. All the graphs have been redrawn with the current masses. Plots for π^0 and K^0 mesons have been added to the Kinematics section. Throughout the Dynamics section the quantities $\pi\lambda^2$ and ηM_p have been added. The energy range for all plots has been extended up to 100 BeV. Everywhere the quantity $(1-\beta)$ has replaced β as the latter approaches 1. The text has also been revised.

The first section of this volume displays the kinematic quantities P, $(\gamma - 1)$, $H\rho$, β or $(1-\beta)$, and $P\beta$ as a function of particle kinetic energy for the most common elementary particles (the notation is given below). The second part presents the dynamic quantities $\pi\lambda^2$, ηM_p, β or $(1-\beta)$, η, $(\gamma-1)$ and w as a function of incident particle kinetic energy for the most common elementary particles interacting with a proton. Σ^-, Σ^0, Ξ^0, and n were not plotted as they are so close in mass to one of the particles that has been included that they cannot be distinguished graphically.

Now a word about notation. In this publication, mass, energy, and momentum are expressed in terms of energy; M is an abbreviation for mc^2, and the momentum P is an abbreviation for Pc. The total energy (W) equals the kinetic energy (T) plus a rest energy (M). Capital letters refer to quantities relative to the laboratory system (LAB) and small letters refer to quantities relative to the center-of-mass (c.m.) system. Greek letters are used for the transformation quantities that relate motion of the c.m. system to the lab system. The numerical subscripts designate the particle to which the quantity refers.

LAB	Name	c. m.
T	Kinetic energy	t
P	Momentum	p
W	total energy	w
Θ	Angle	θ

We are concerned only with reactions of the form $M_1 + M_2 \rightarrow M_3 + M_4$, and of these possible cases we discuss only the one in which M_2 is at rest and M_1 bombards it with a momentum P_1; M_3 and M_4 represent the final state of the reaction.

W. P. Trower

ELLIPSE-PLOTTING METHOD FOR SOLUTION OF THE RELATIVISTIC TWO-BODY PROBLEM

A useful graphic solution for the relativistic two-body problem is known as the ellipse-plotting method.[1] One may wish to know, for a given P_1 in the LAB system, what the momenta P_3 and P_4 are that correspond to a θ_3 or θ_4 (where $\theta_3 = \pi - \theta_4$). We first perform the following calculations, recalling that the quantities described above in the Introduction can all be obtained from the graphs constituting the bulk of this volume:

(a) Find the total energy in both systems from

$$w = \left[M_1^2 + M_2^2 + 2M_2 \left(P_1^2 + M_1^2 \right)^{1/2} \right]^{1/2},$$

where w is the invariant that is plotted in the dynamics-of-collision plots, and

$$W = \left[P_1^2 - M_1^2 \right]^{1/2} + M_2.$$

(b) Find the transformation quantities from

$$\gamma = \frac{W}{w}, \quad \beta = \frac{P_1}{W}, \quad \text{and} \quad \eta = \beta\gamma = \frac{P_1}{w},$$

(c) Find the total energies of particles 3 and 4, in the c.m. system, from

$$w_3 = \frac{w^2 + M_3^2 - M_4^2}{2w}, \quad w_4 = w - w_3.$$

(d) Find the momentum, in the c.m. system, from

$$p = p_3 = p_4, \quad p = \left(w_3^2 - M_3^2 \right)^{1/2} = \left(w_4^2 - M_4^2 \right)^{1/2}.$$

(e) Finally, compute ηw_3, ηw_4, and γp.

---

[1] J. Blaton, On a Geometrical Interpretation of Energy and Momentum Conservation in Atomic Collisions and Disintegration Processes, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 24, No. 20 (1950); and M. L. Stevenson, Reaction Dynamics for Scanners, Lawrence Radiation Laboratory, University of California, Berkeley, Alvarez Group Memo 300 (unpublished).

The construction of the ellipse follows directly from the Lorentz transformation of the c. m. to the LAB quantities:

$$\begin{pmatrix} \gamma & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \eta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} p\cos\theta \\ p\sin\theta \\ 0 \\ w \end{pmatrix} = \begin{pmatrix} \gamma p\cos\theta + \eta w \\ p\sin\theta \\ 0 \\ \eta p\cos\theta + \gamma w \end{pmatrix} = \begin{pmatrix} P\cos\Theta \\ P\sin\Theta \\ 0 \\ W \end{pmatrix}.$$

Taking the quantities associated with particle 3 for our construction, we can write

$$P_x = P_3 \cos\Theta_3 = \gamma p \cos\theta_3 + \eta w_3,$$
$$P_y = P_3 \sin\Theta_3 = p \sin\theta_3,$$

which are simply the parametric equations of an ellipse. The term ($\gamma \cos\theta_3$) is a measure of the eccentricity, the degree to which the circle in the c. m. system has been dilated because of momentum P_1. The quantity ηw_3 represents the translation of the center of the coordinate origin in going from the LAB to the c. m. system owing to momentum P_1. When $\theta_3 = 0$, P_x is the semimajor axis, and when $\theta_3 = \pi/2$, P_y is the semiminor axis.

Having calculated all the necessary quantities, we construct a plot by performing the following geometrical operations (see Fig. 1):

(a) Construct a Cartesian base (y ordinate and x abscissa).

(b) Draw a circle of radius p with its center at the origin.

(c) Mark off an arc on the circle corresponding to the desired θ_3.

(d) Through this point on the circle, draw a line parallel to the x axis. We then have steps (c) and (d) represented in Fig. 1 by two dashed lines.

(e) Multiply the value for x at y by γ, and plot the new point (γx, y).

(f) Repeat (c) through (e), picking enough points to generate an ellipse.

(g) On the x axis, locate a point (3) at a distance ηw_3 to the left of the y axis, and a point (4) at a distance ηw_4 to the right of the y axis. The distance between points (3) and (4) equals the LAB momentum of the bombarding particle P_1. and is the vector sum of $\vec{P}_3$ and $\vec{P}_4$.

(h) Draw verticals through points (3) and (4), and locate points (3') and (4') at distance M_3 and M_4 above (3) and (4) respectively.

(i) From points (3) and (4) draw momentum vectors $\vec{P}_3$ and $\vec{P}_4$ to points on the ellipse corresponding to the c. m. angles desired.

(j) Measure LAB angles Θ_3 and Θ_4 with a protractor.

(k) If desired, $\vec{P}_3$ and $\vec{P}_4$ can be measured, and their values converted to T_3 and T_4 respectively, by reference to the graphs. Or alternatively, T_3 and T_4 can be ascertained by construction by applying Pythagoras' theorem as follows:

Place one tip of a pair of dividers on point (3), and extend the other tip a distance P_3 on the ellipse. Pivot the dividers about point (3) until the other tip touches the major axis of the ellipse, at point (a). Then pivot the dividers about point (a), and open them so that the other tip is on point (3'). The dividers now show a separation

$$W_3 = \left[P_3^{\,2} + M_3^{\,2}\right]^{1/2}.$$

Next, pivot the dividers about point (3') to where the other tip is vertically below points (3') and (3), at point (b). Point (b) then lies at a distance T_3 below point (3), since $T_3 = W_3 - M_3$. Now an energy scale plotted downward from point (3) permits a direct reading of T_3. This procedure, once mastered, is simple to perform. It is demonstrated in Fig. 2.

It is interesting to note that in the extreme relativistic limit a particle going backward in the c. m. system (i.e., $\theta_3 = \pi$) approaches a constant momentum in the LAB system; namely,

$$(M_3^{\,2} - M_2^{\,2}) / 2M_2.$$

Fig. 1

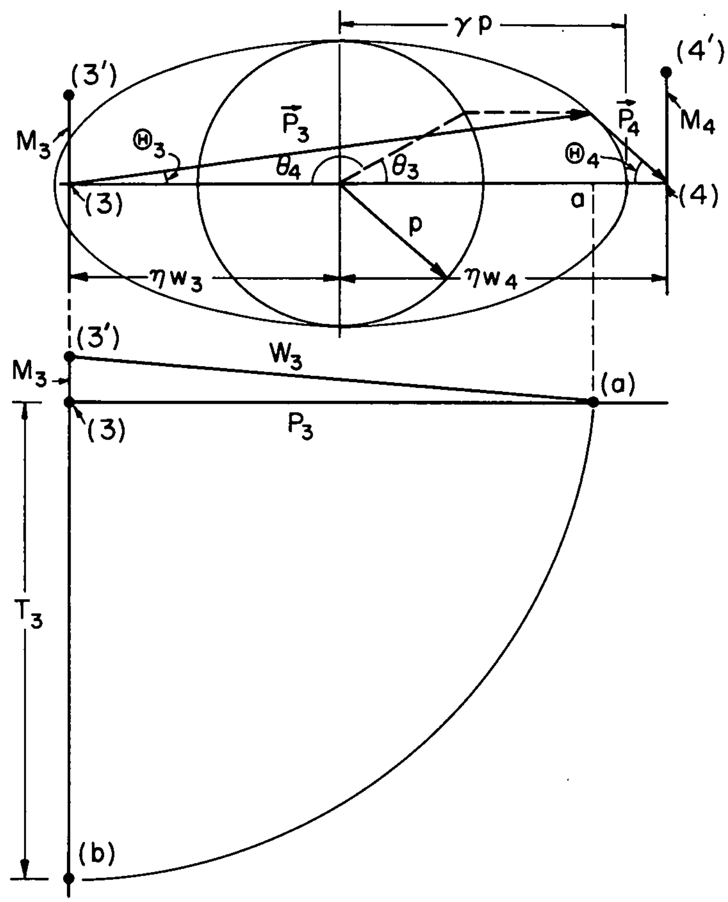


Fig. 2

MU-29966

Figs. 1-2. Typical diagram for a reaction of the type $\pi^- + p \rightarrow K^+ + \Sigma^-$. M_1 is the bombarding particle with kinetic energy T_1. M_3 and M_4 are the rest masses of the resulting particles.

ACKNOWLEDGMENTS

This collection of graphs was originally conceived by Dr. Luis W. Alvarez. Over the years Dr. Arthur H. Rosenfeld, Dr. M. Lynn Stevenson, and Dr. Robert D. Tripp have provided comments which have contributed to the collection's general usefulness.

For the patient art work of Mr. Robert Stevens the present author is particularly grateful.

OTHER VOLUMES IN THE UCRL-2426 SERIES

Vol. II (1963 Revision) Range-Energy (and Momentum) Plots of Passage of Particles Through Matter, October 1963.

Vol. III (1963 Edition), Kinematics of Particles as a Function of Momentum, July 1963.

UCRL-2426
Vol. I (1963 Rev.)

KINEMATICS OF ELEMENTARY PARTICLES

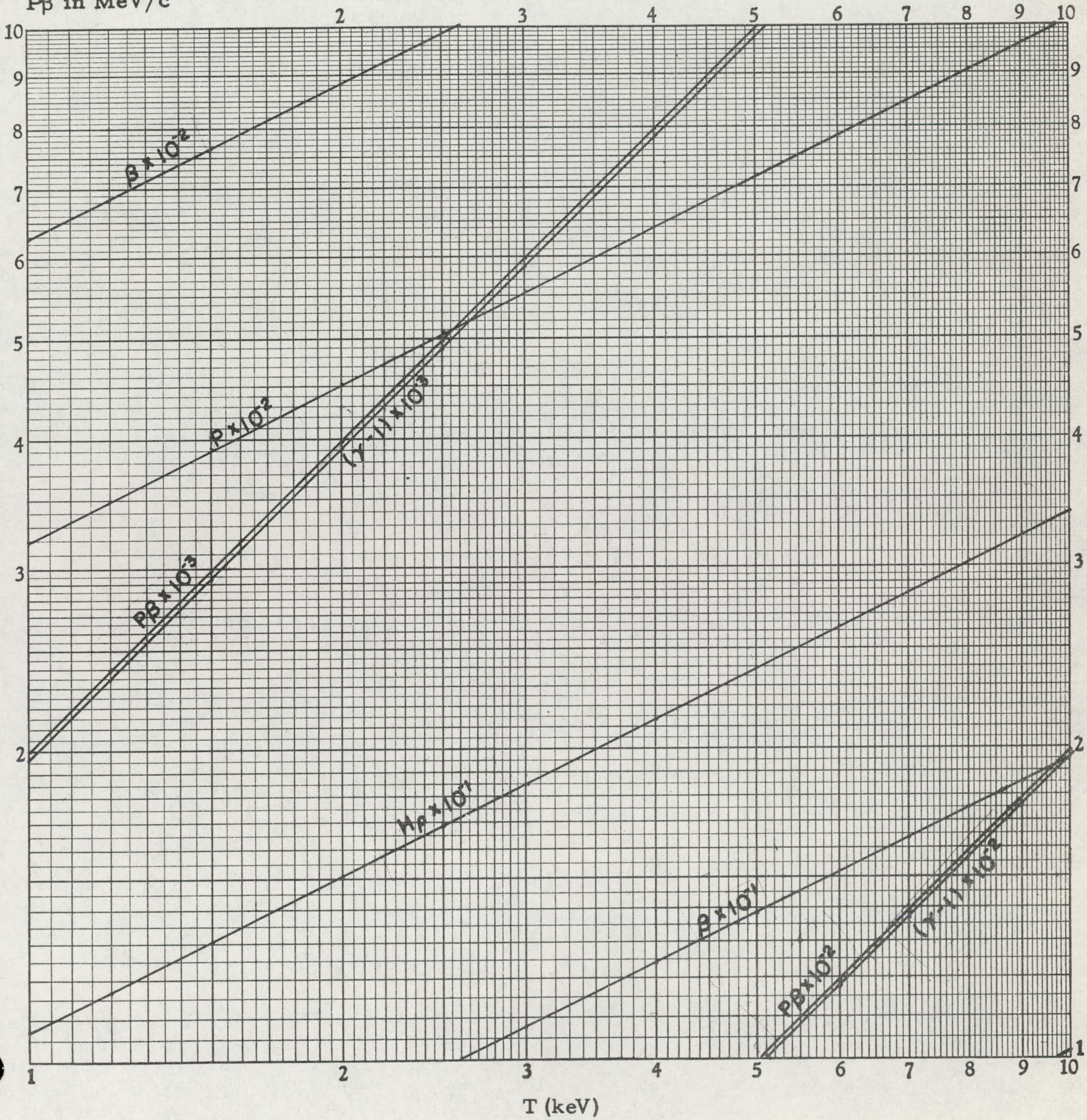
ELECTRONS

1 keV to 10 keV

β, (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_e = 0.510976 MeV
= 1 m

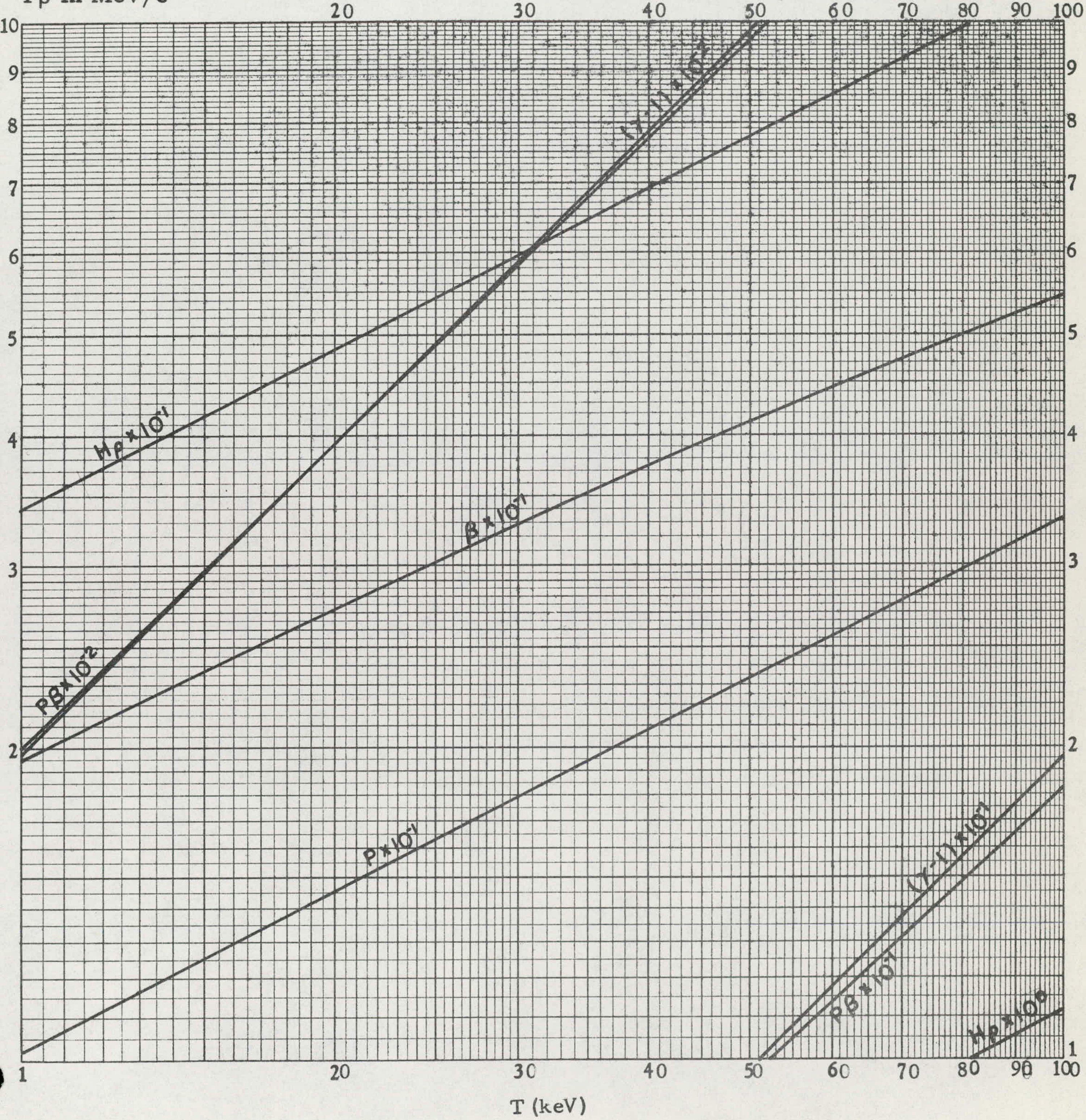


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 keV to 100 keV

β, (γ-1), P, Pβ, Hρ

M_e = 0.510976 MeV
= 1 m

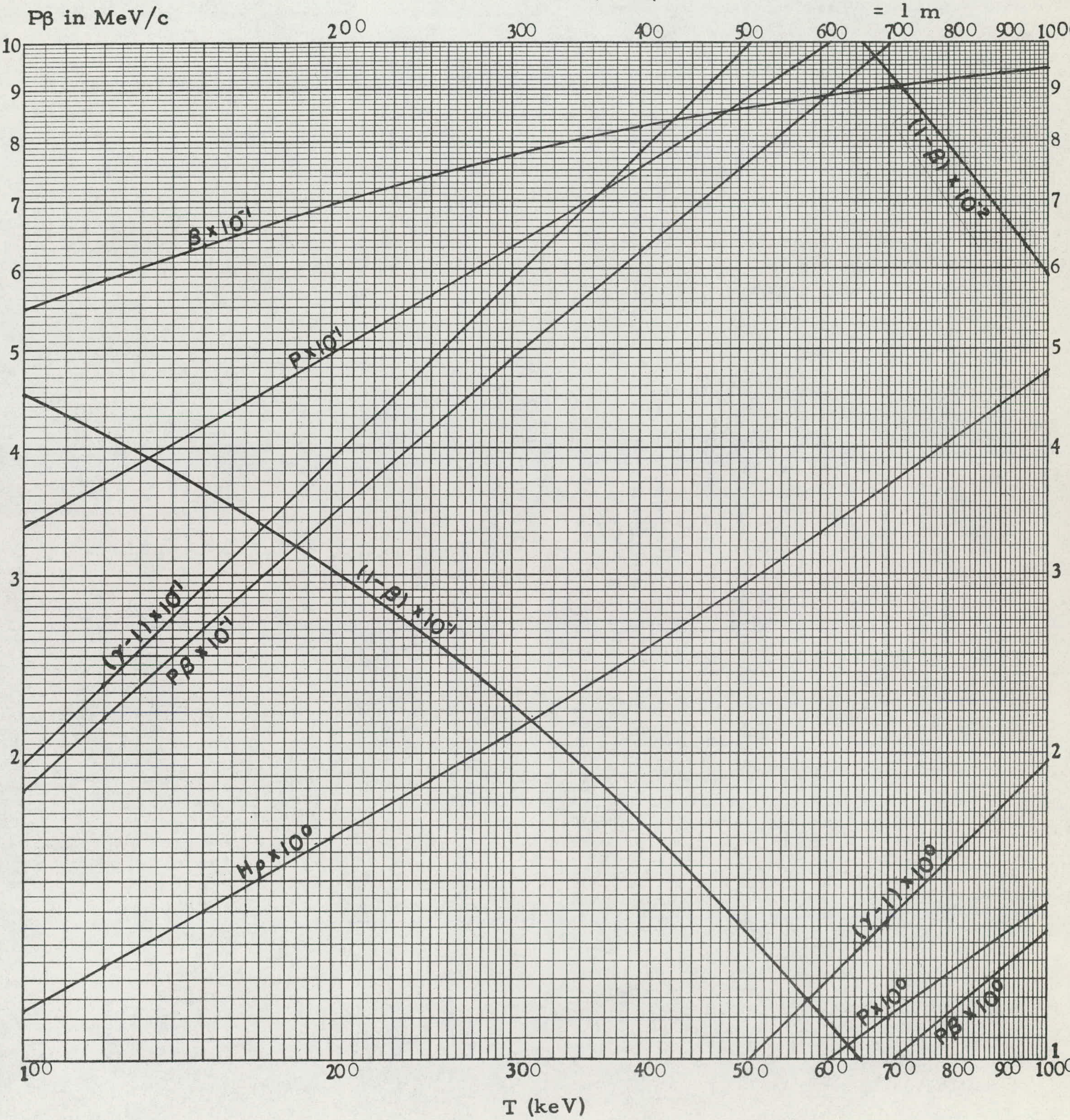


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

100 keV to 1 MeV

(1-β), (γ-1), P, Pβ, Hρ

M_e = 0.510976 MeV
= 1 m



ELECTRONS

1 MeV to 10 MeV

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P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

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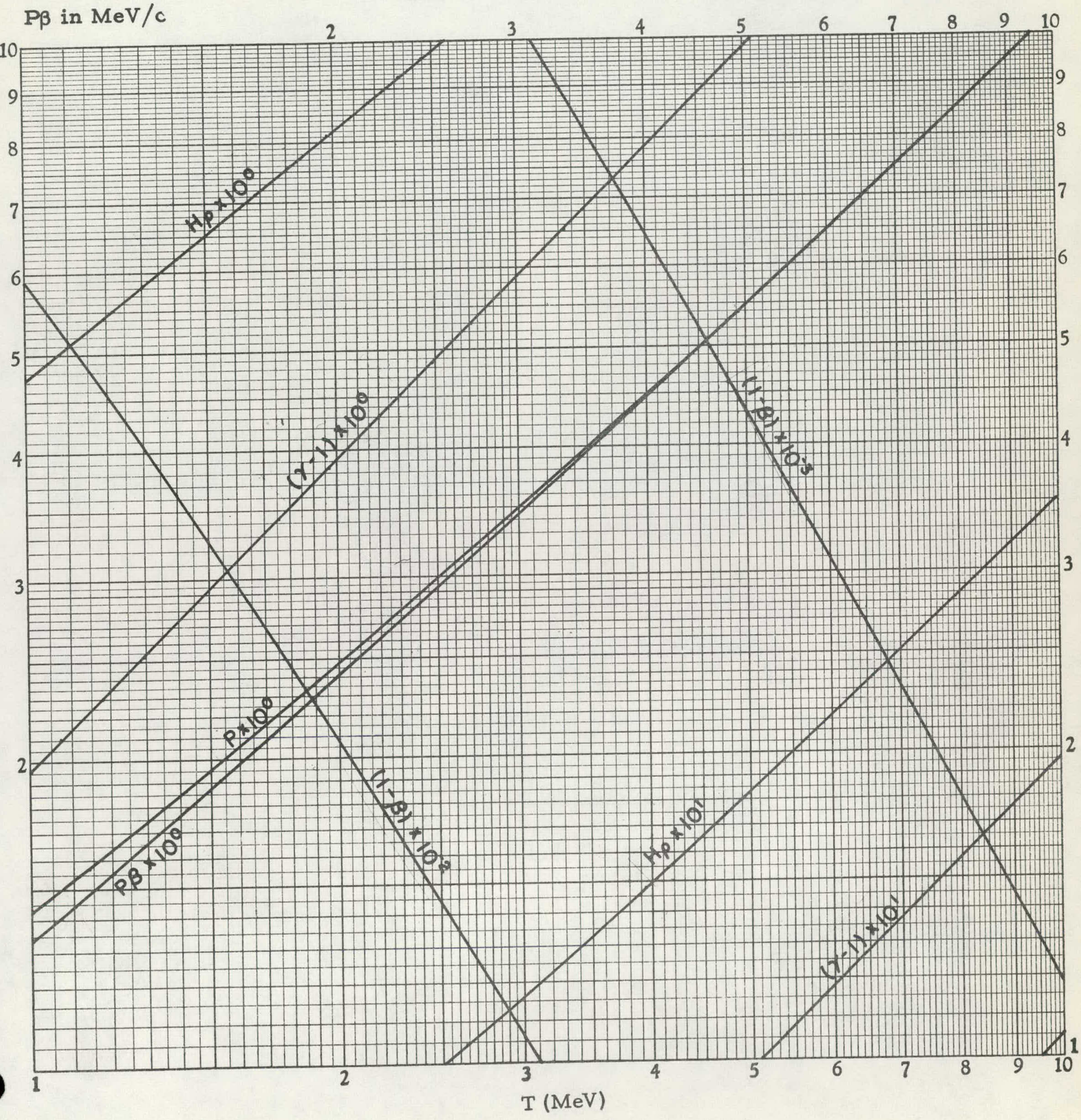
= 1 m

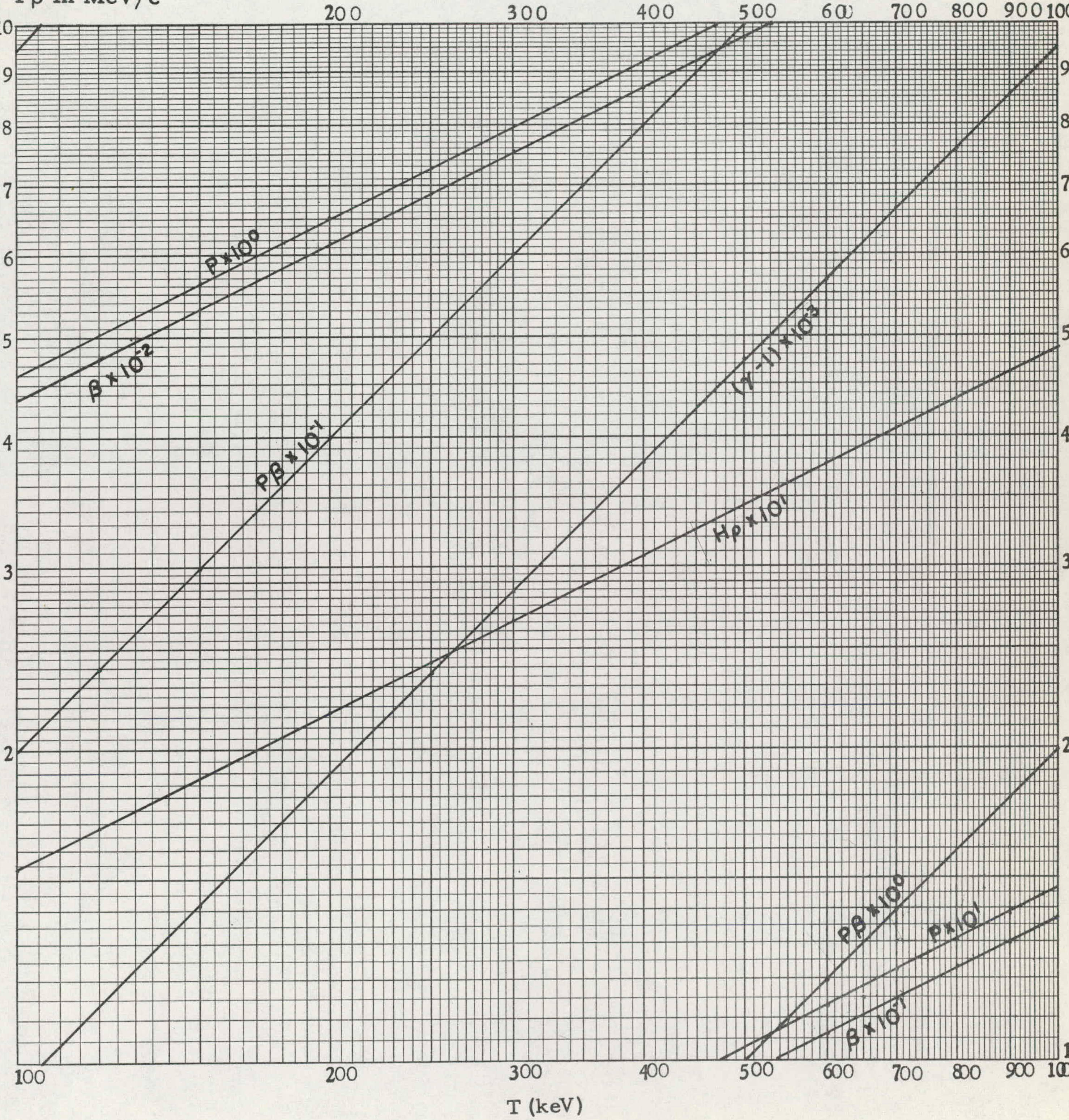


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

100 keV to 1 MeV

β, (γ-1), P, Pβ, Hρ

$M_{\mu^\pm}$ = 105.655 MeV
= 206.77 m



$\mu^{\pm}$ LEPTONS

1 MeV to 10 MeV

β, (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\mu^{\pm}}$ = 105.655 MeV
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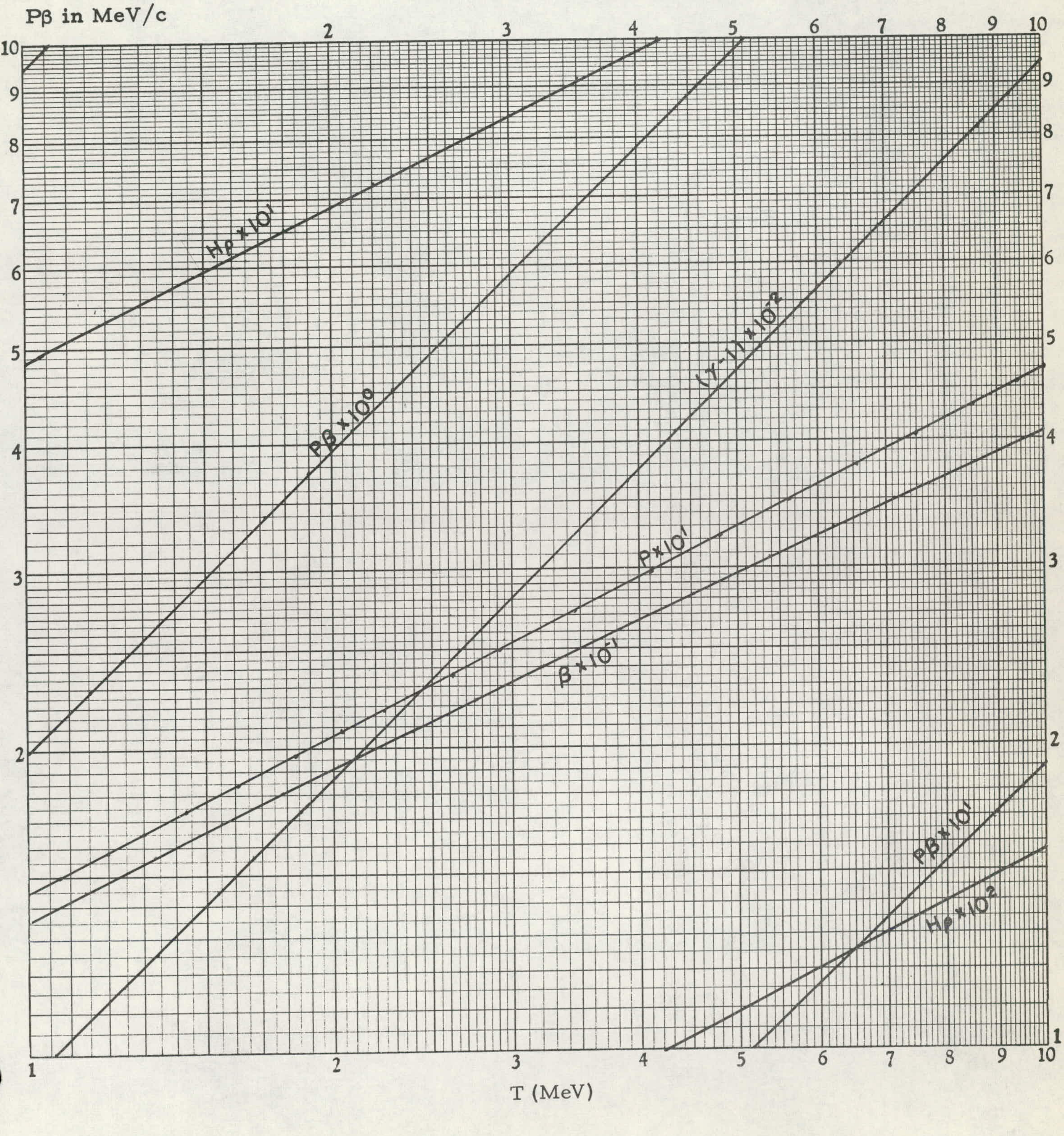


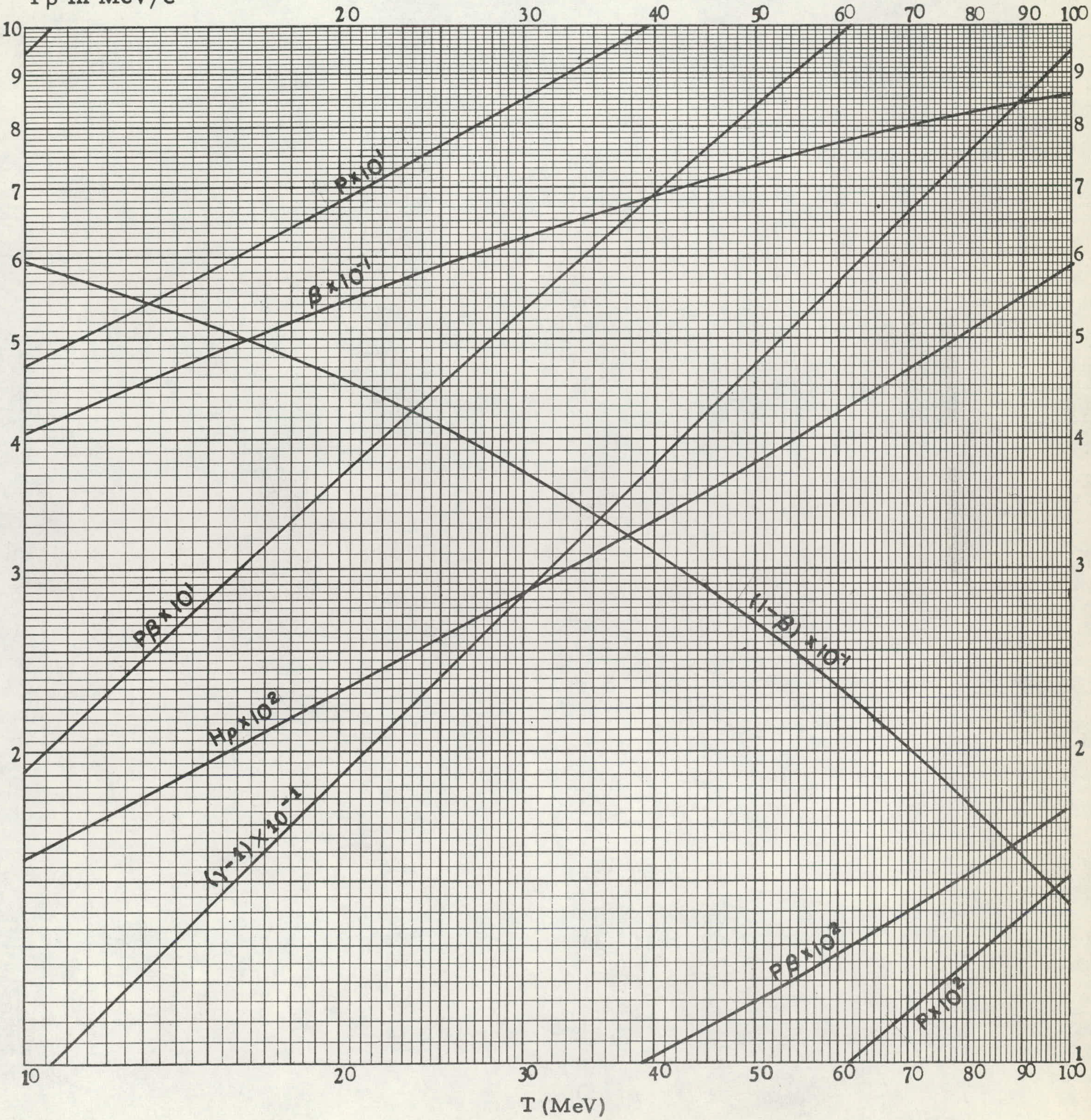
$\mu^{\pm}$ LEPTONS

10 MeV to 100 MeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\mu^{\pm}}$ = 105.655 MeV
= 206.77 m



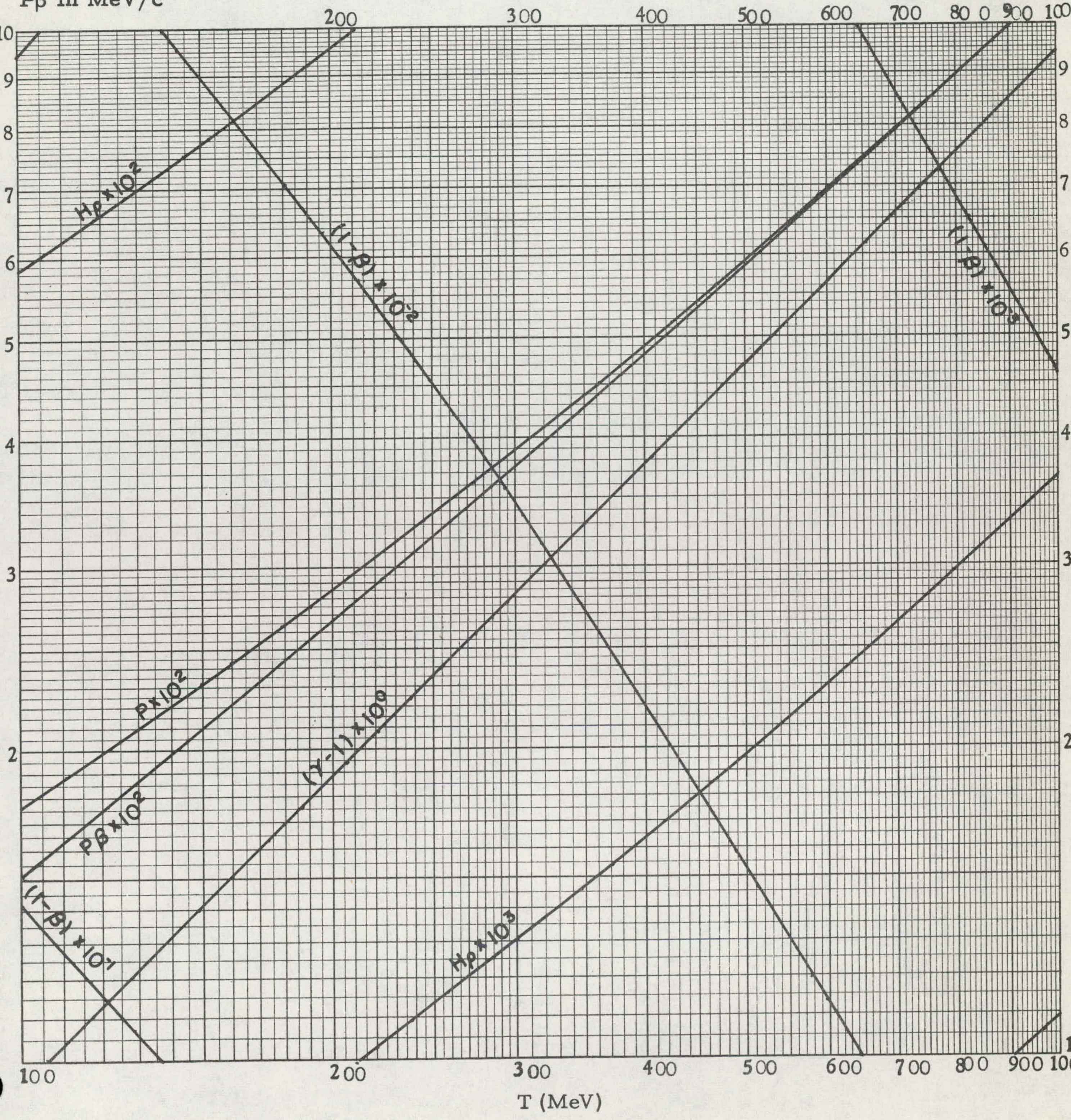
$\mu^{\pm}$ LEPTONS

100 MeV to 1 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\mu}\pm$ = 105.655 MeV
= 206.77 m



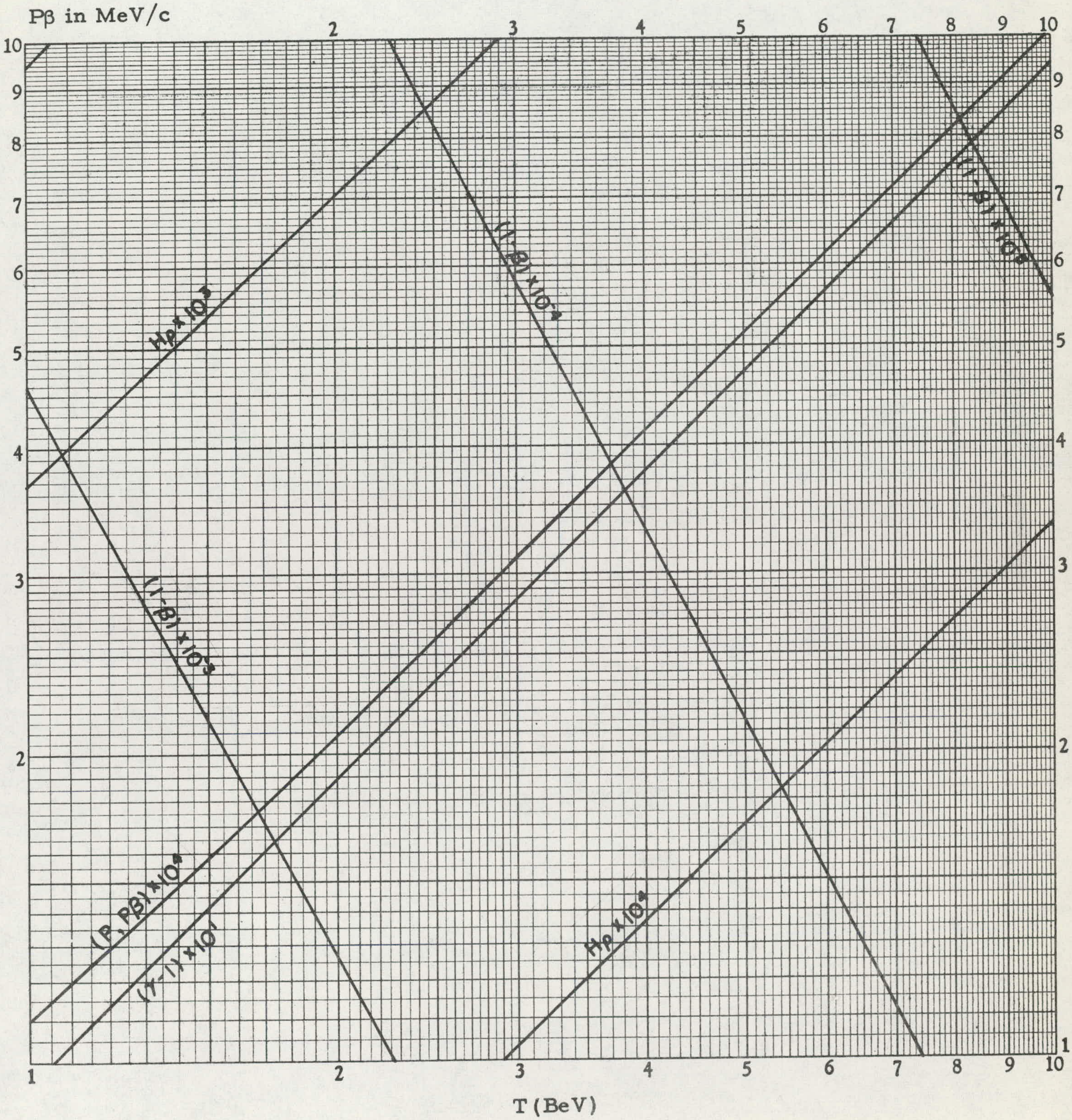
$\mu^{\pm}$ LEPTONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\mu}\pm$ = 105.655 MeV
= 206.77 m



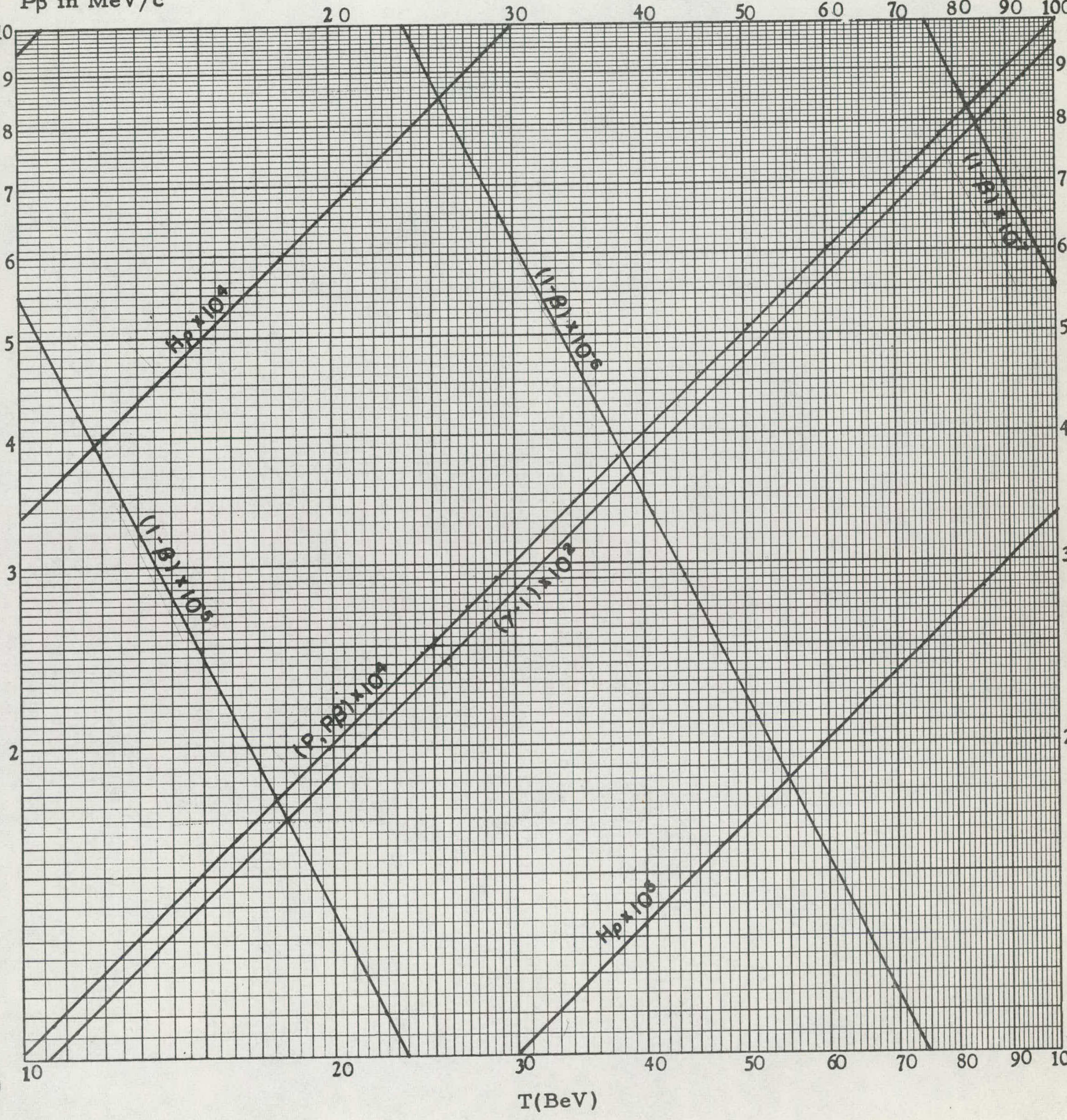
$\mu^{\pm}$ LEPTONS

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

$M_{\mu}\pm$ = 105.655 MeV
= 206.77 m

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c



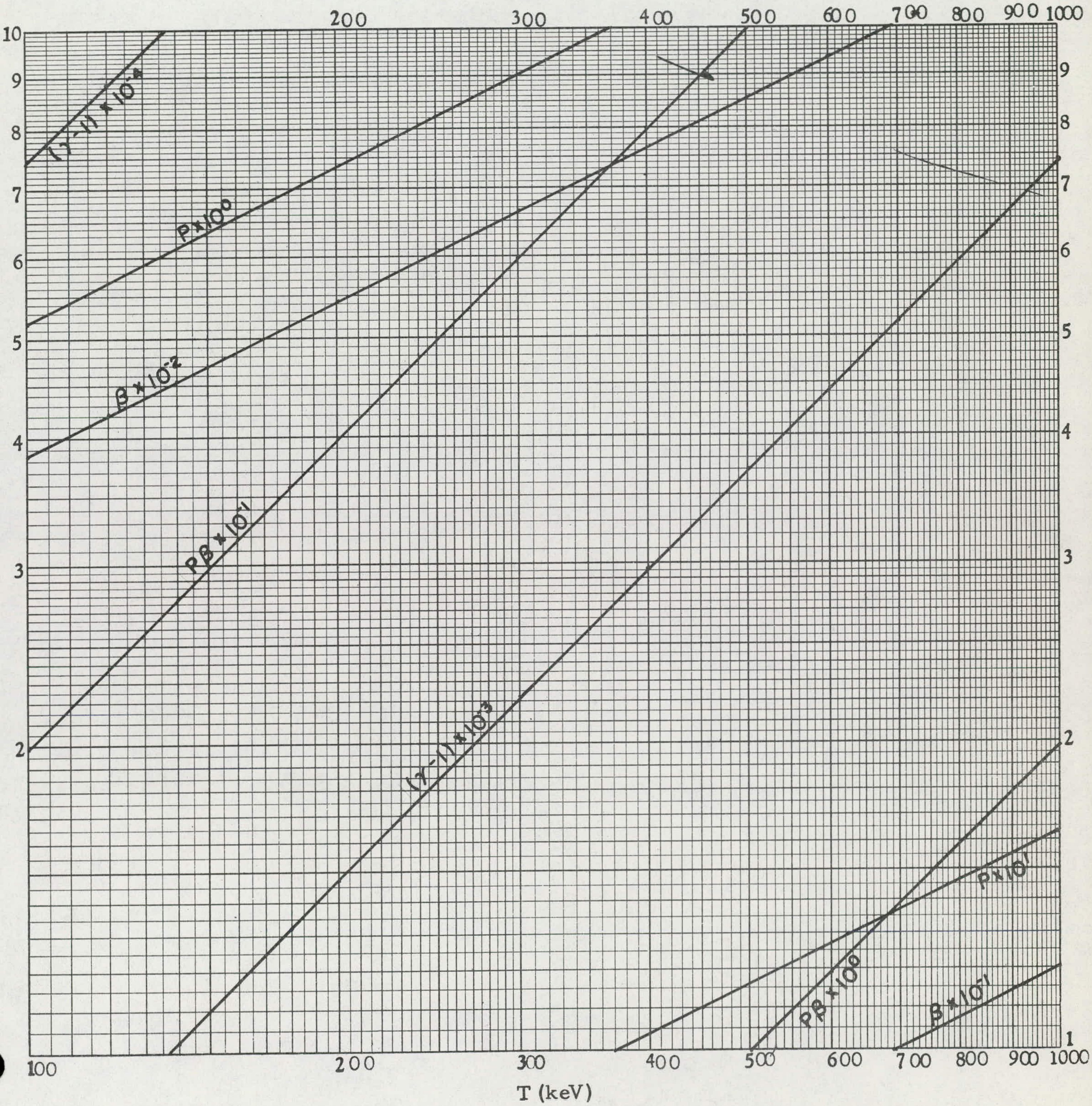
π^0 MESONS

100 keV to 1 MeV

β, $(\gamma-1)$, P, Pβ

M_{π^0} = 135.00 MeV
= 264.42 m

P and Pβ in MeV/c



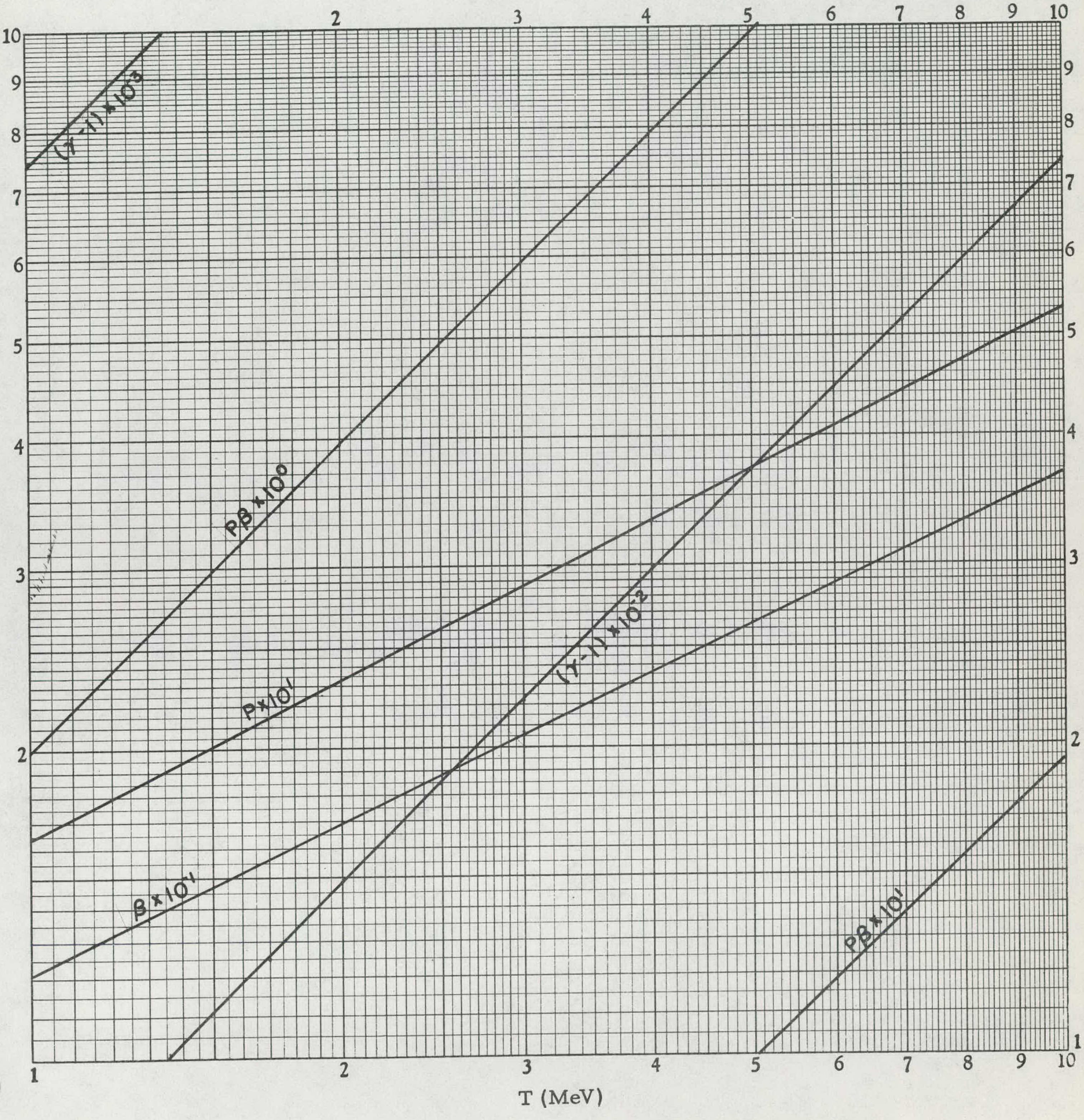
π^0 MESONS

1 MeV to 10 MeV

β, $(\gamma-1)$, P, Pβ

P and Pβ in MeV/c

M_{π^0} = 135.00 MeV
= 264.42 m



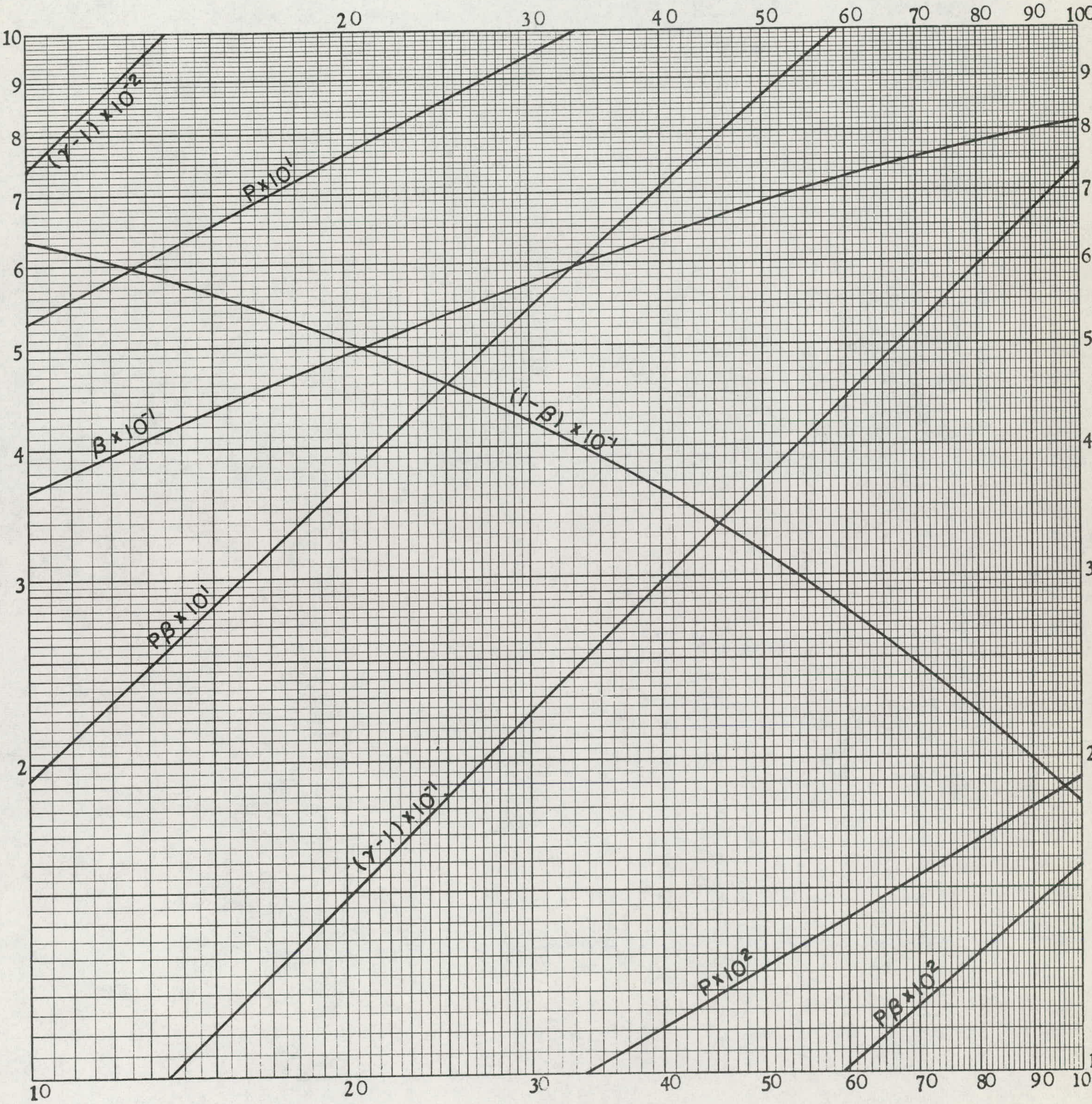
π^0 MESONS

10 MeV to 100 MeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ

M_{π^0} = 135.00 MeV
= 264.42 m

P and Pβ in MeV/c



T (MeV)

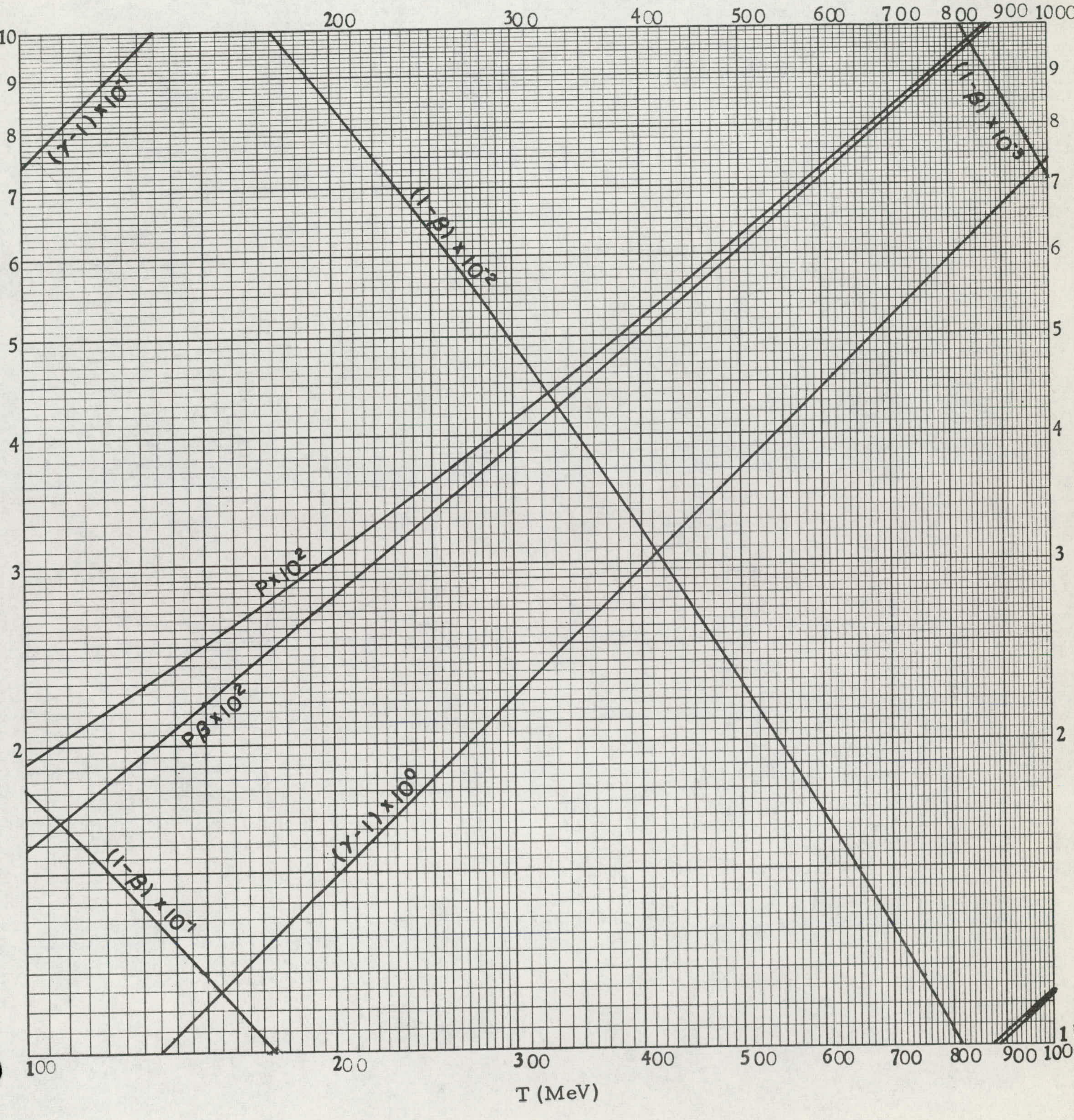
π^0 MESONS

100 MeV to 1 BeV

$(1-\beta)$, $(\gamma-1)$, P, Pβ

M_{π^0} = 135.00 MeV
= 264.42 m

P and Pβ in MeV/c



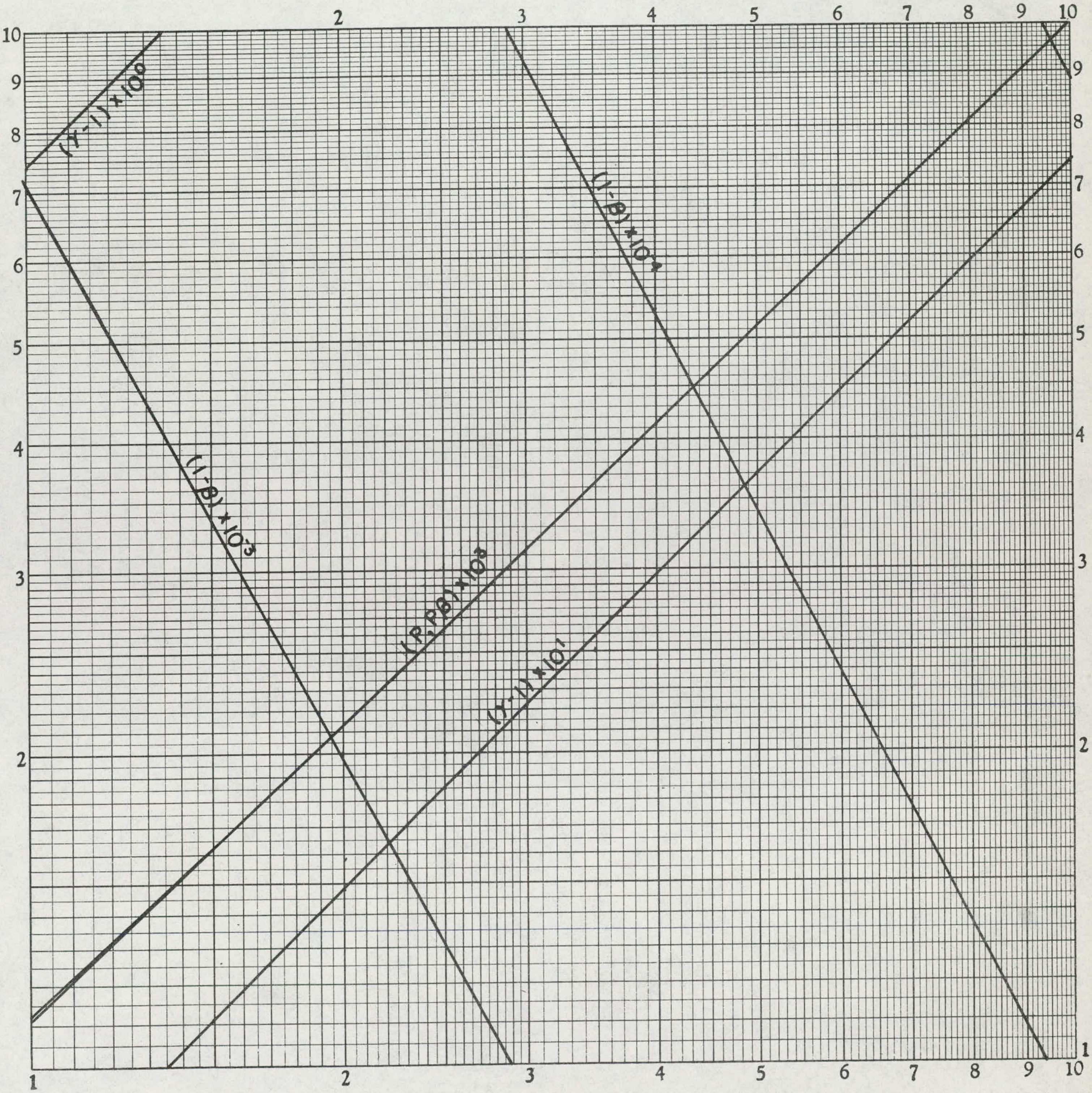
π^0 MESONS

1 BeV to 10 BeV

$(1-\beta)$, $(\gamma-1)$, P, Pβ

P and Pβ in MeV/c

M_{π^0} = 135.00 MeV
= 264.42 m



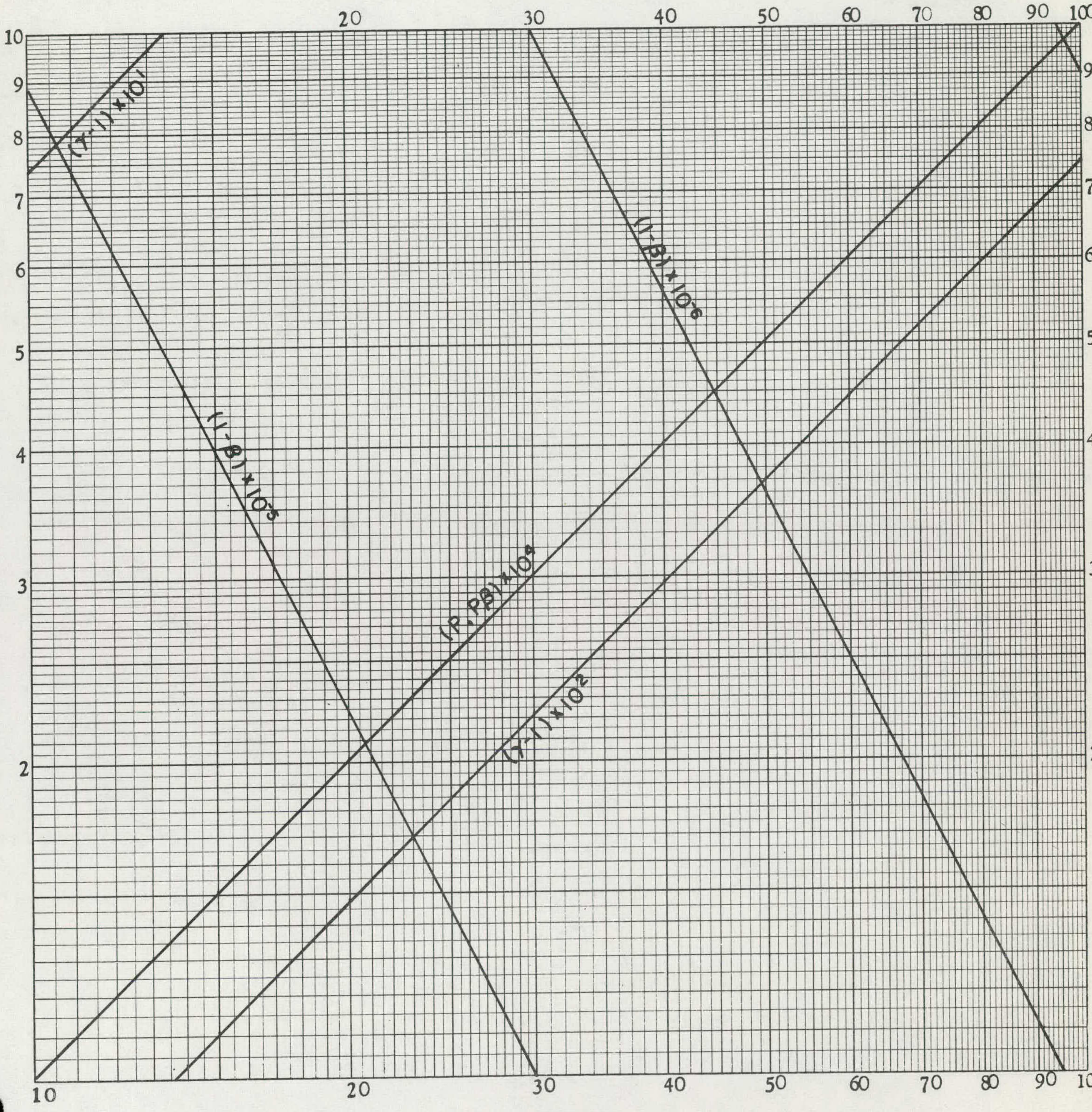
π^0 MESONS

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, P, Pβ

P and Pβ in MeV/c

M_{π^0} = 135.00 MeV
= 264.42 m



$\pi^{\pm}$ MESONS

100 keV to 1 MeV

β, $(\gamma-1)$, P, Pβ, Hρ

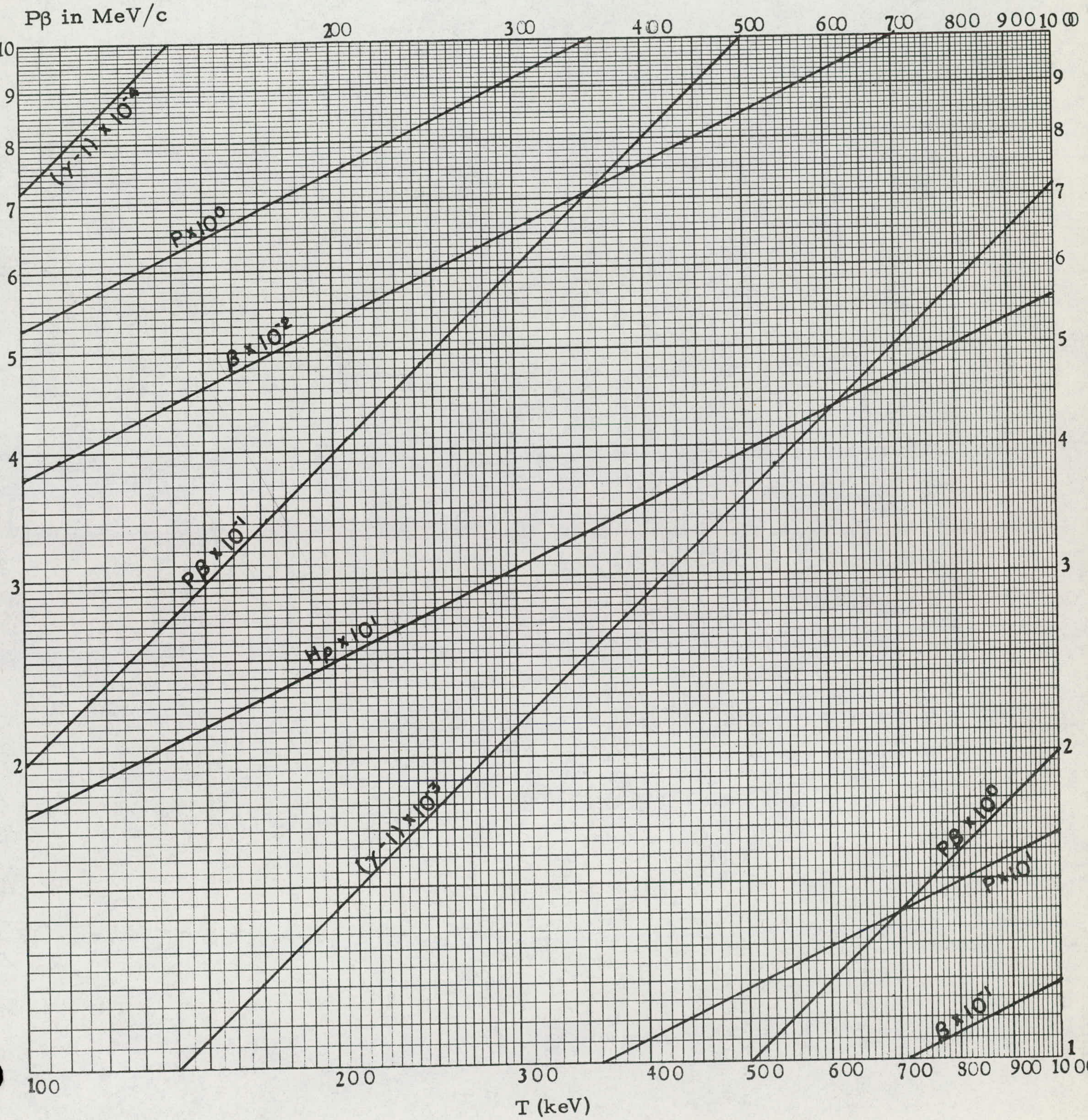
P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

$M_{\pi}\pm$ = 139.59 MeV

= 273.18 m



$\pi^{\pm}$ MESONS

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P in MeV/c

Hρ in kgauss-cm

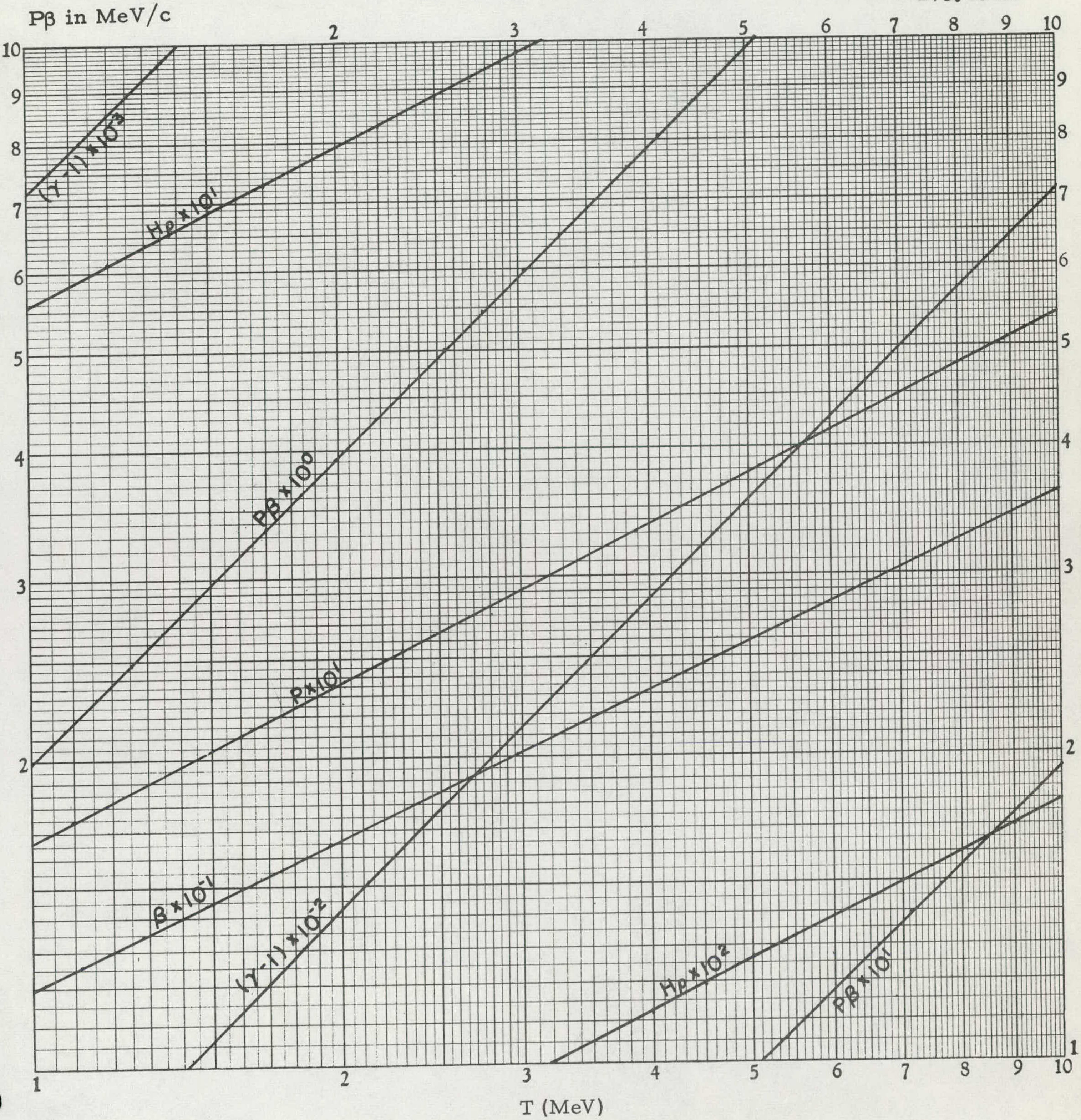
Pβ in MeV/c

1 MeV to 10 MeV

β, (γ-1), P, Pβ, Hρ

$M_{\pi}\pm$ = 139.59 MeV

= 273.18 m



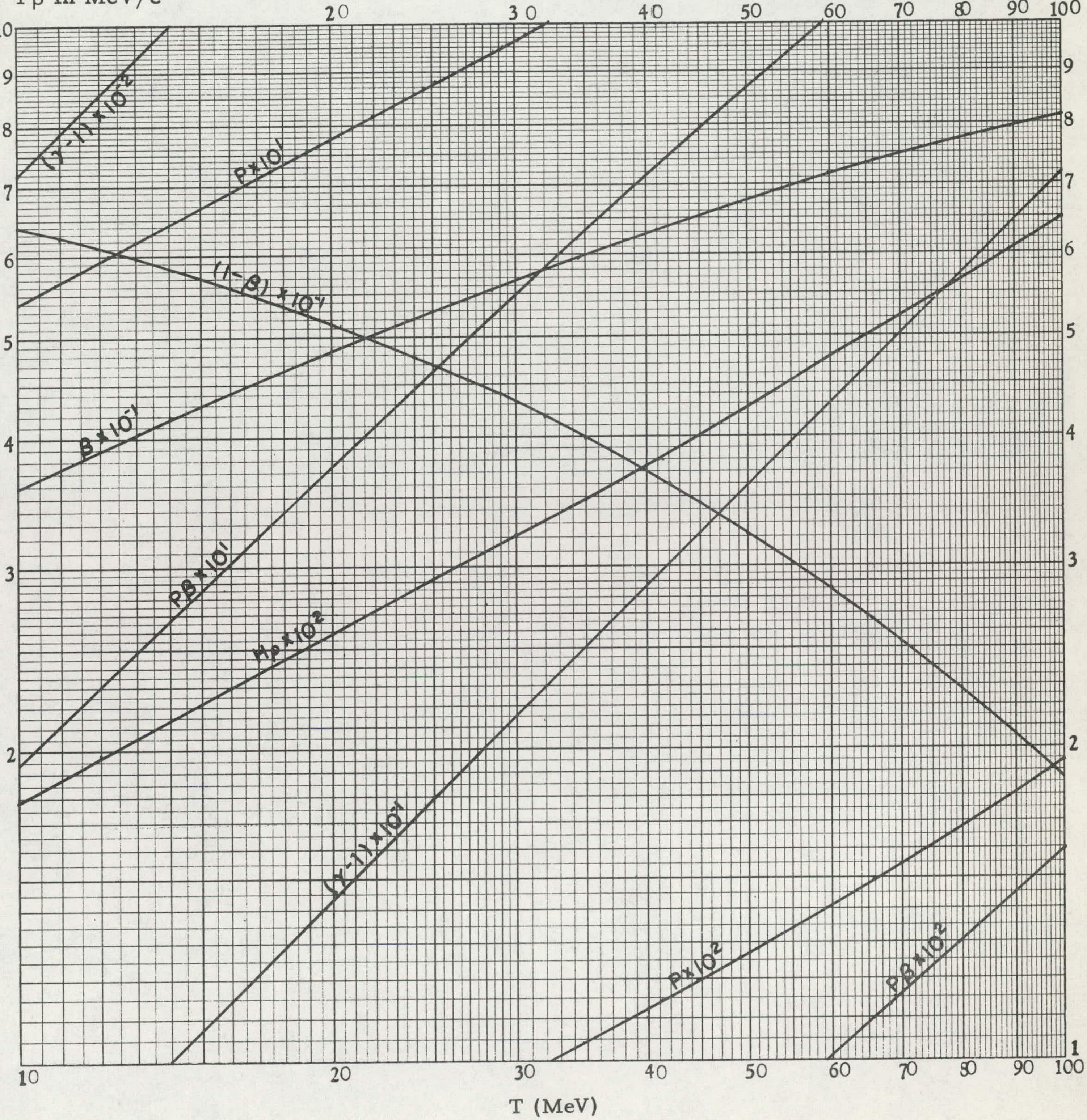
$\pi^{\pm}$ MESONS

10 MeV to 100 MeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\pi^{\pm}}$ = 139.59 MeV
= 273.18 m

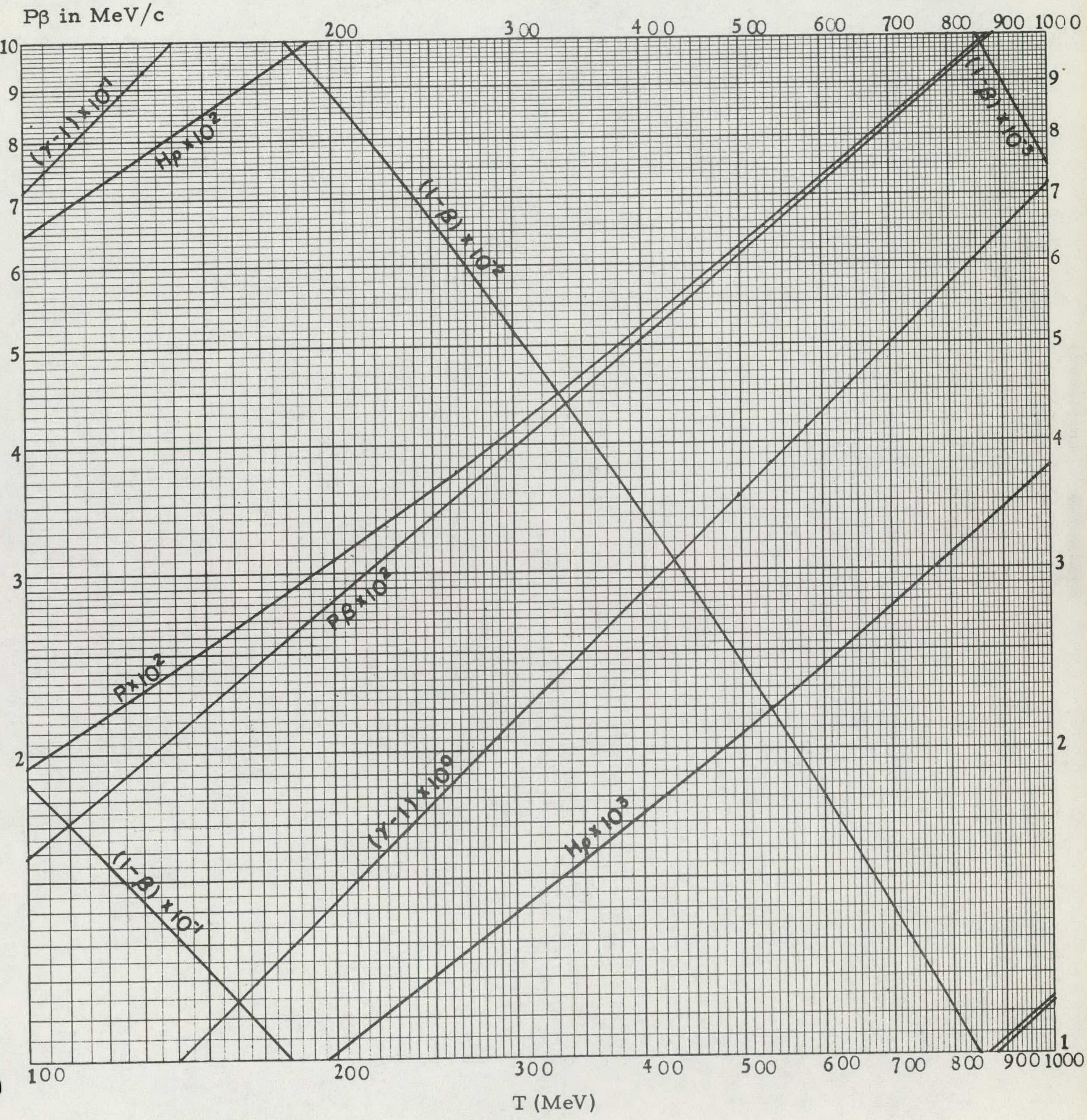


100 MeV to 1 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\pi}\pm$ = 139.59 MeV
= 273.18 m



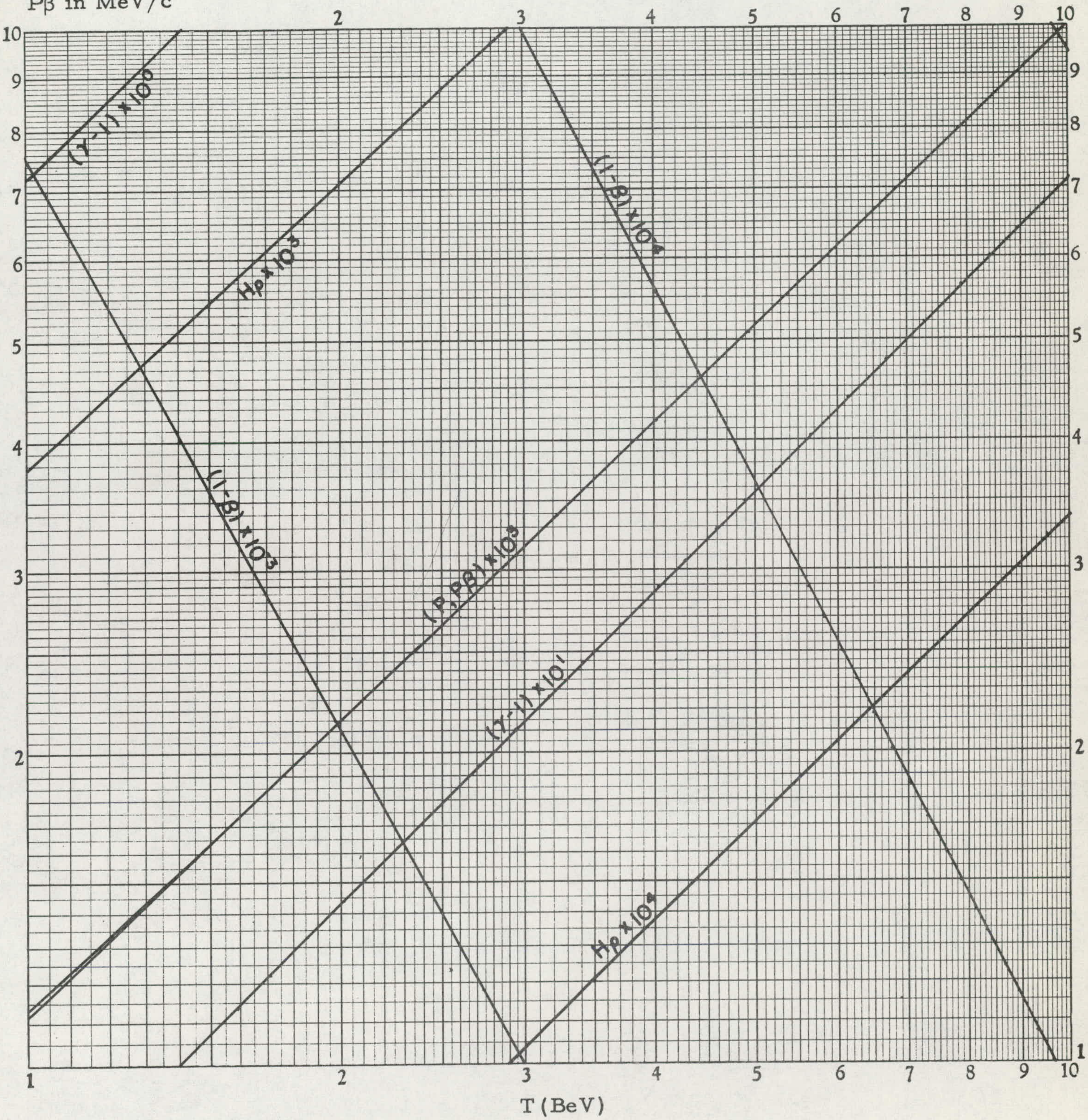
$\pi^{\pm}$ MESONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\pi^{\pm}}$ = 139.59 MeV
= 273.18 m



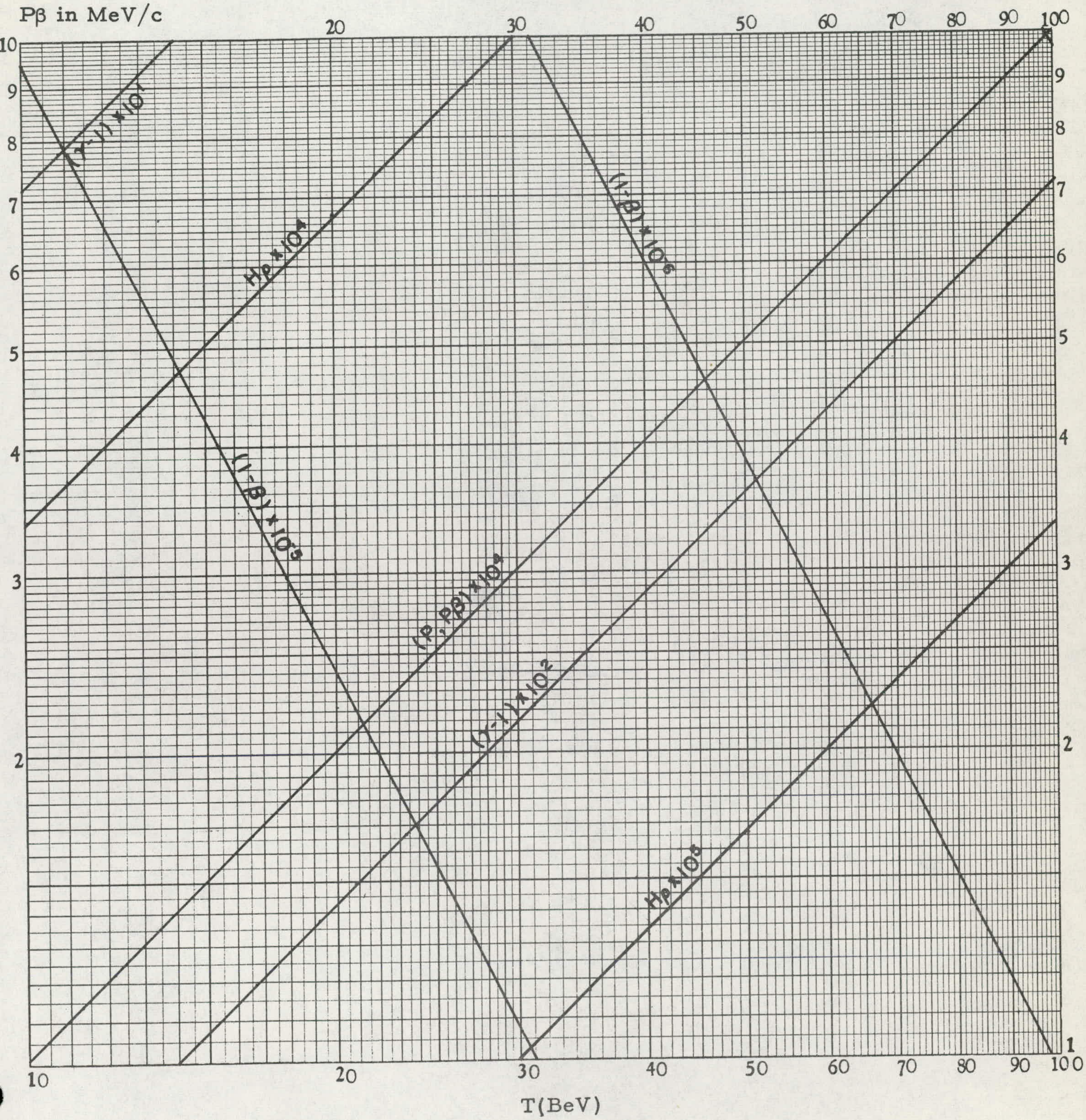
$\pi^{\pm}$ MESONS

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\pi^{\pm}}$ = 139.59 MeV
= 273.18 m



K$^{\pm}$ MESONS

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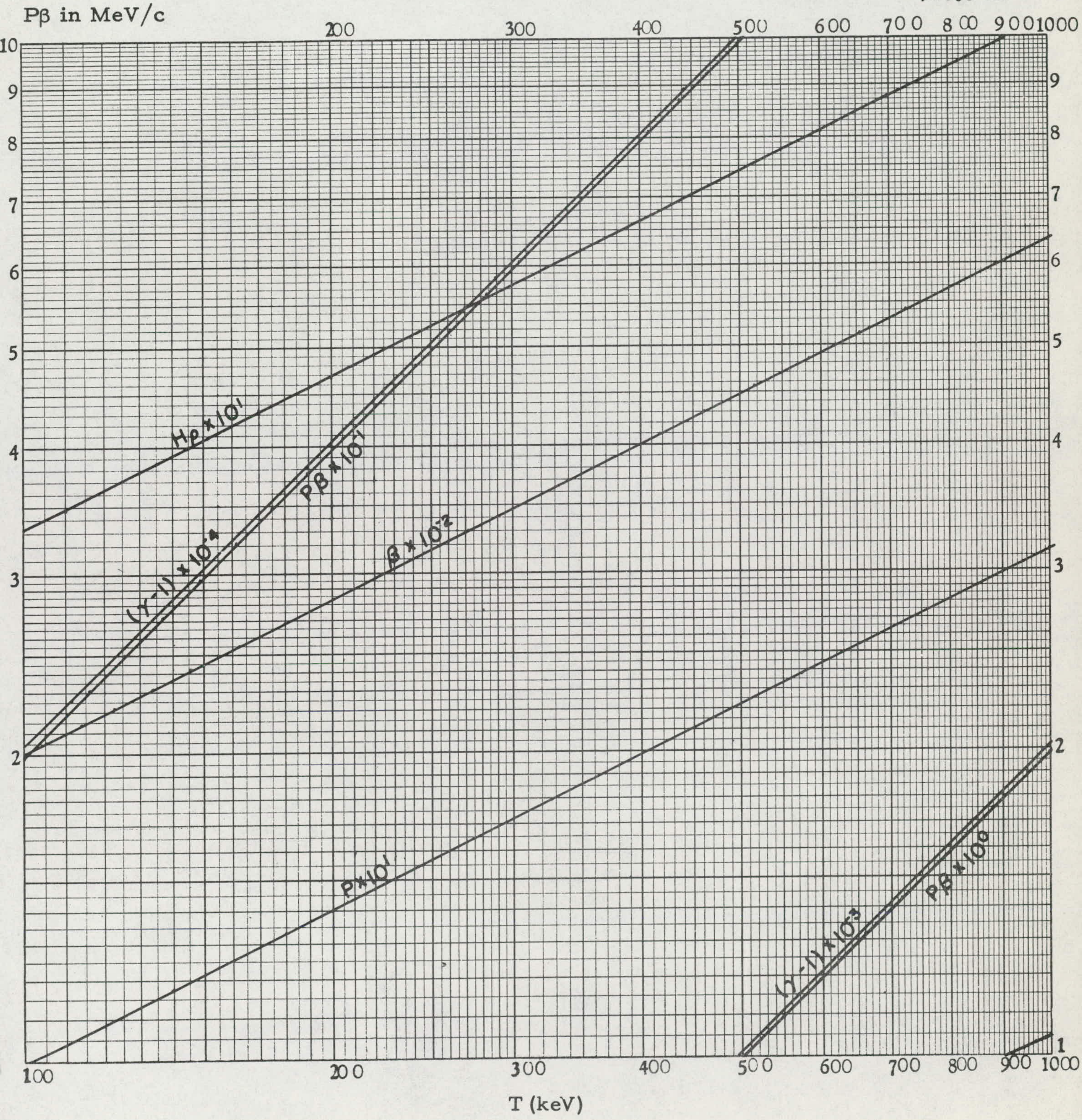
100 keV to 1 MeV

β, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{K^{\pm}}$ = 493.9 MeV
= 966.6 m



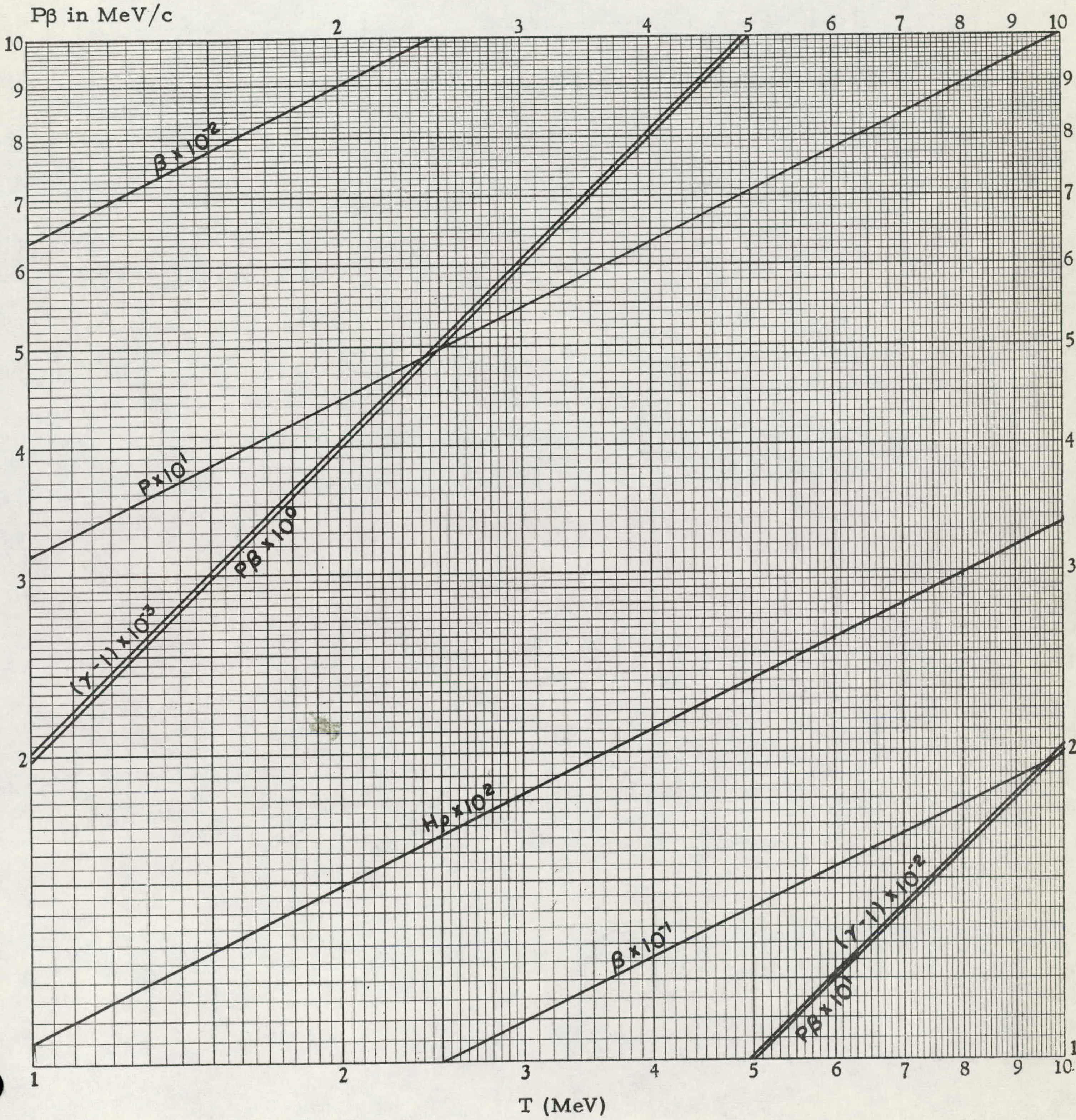
K$^{\pm}$ MESONS

1 MeV to 10 MeV

β, (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{K^{\pm}}$ = 493.9 MeV
= 966.6 m



$K^{\pm}$ MESONS

10 MeV to 100 MeV

β, (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{K^{\pm}}$ = 493.9 MeV
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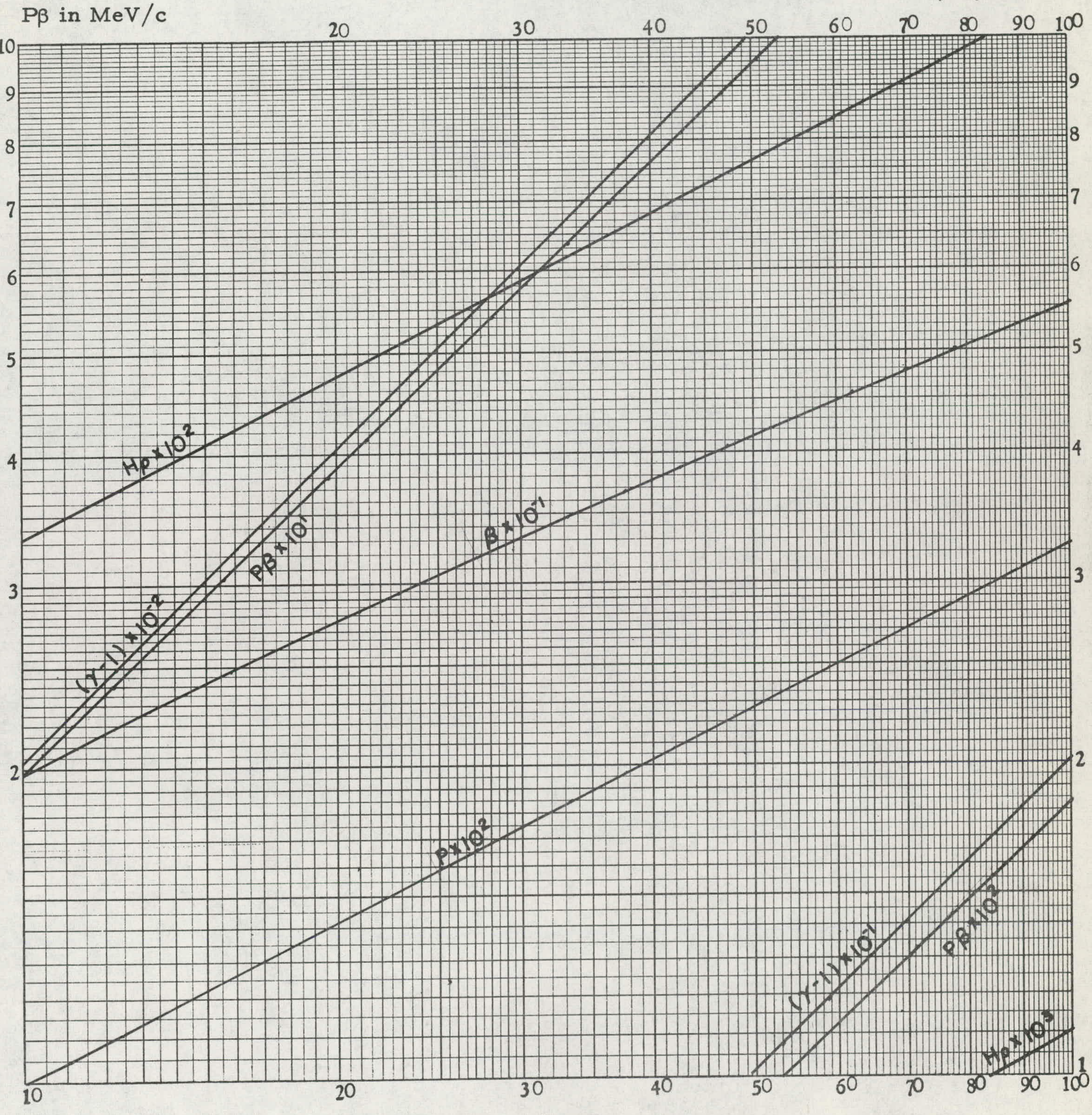


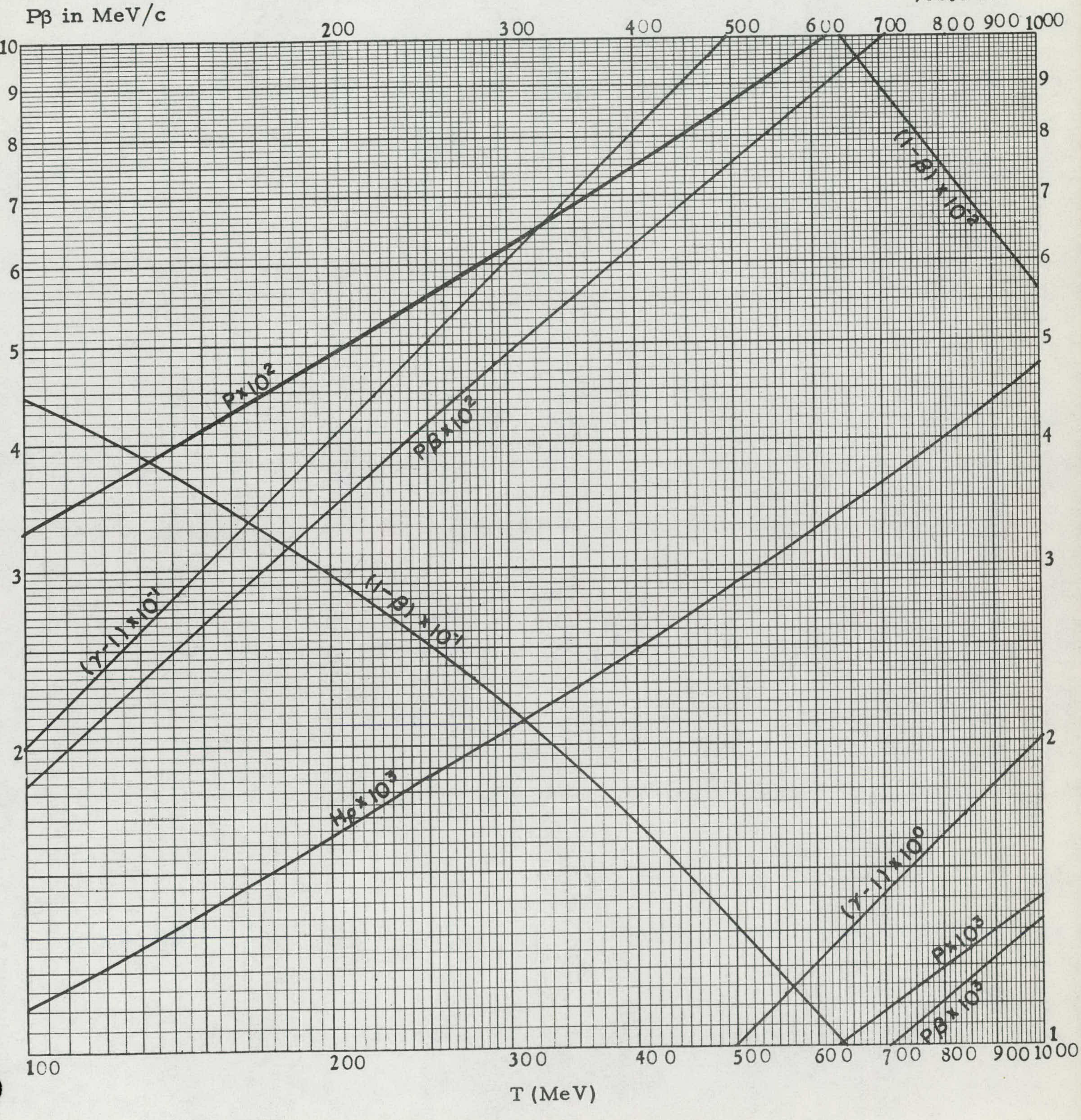
K$^{\pm}$ MESONS

100 MeV to 1 BeV

β, (1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{K^{\pm}}$ = 493.9 MeV
= 966.6 m



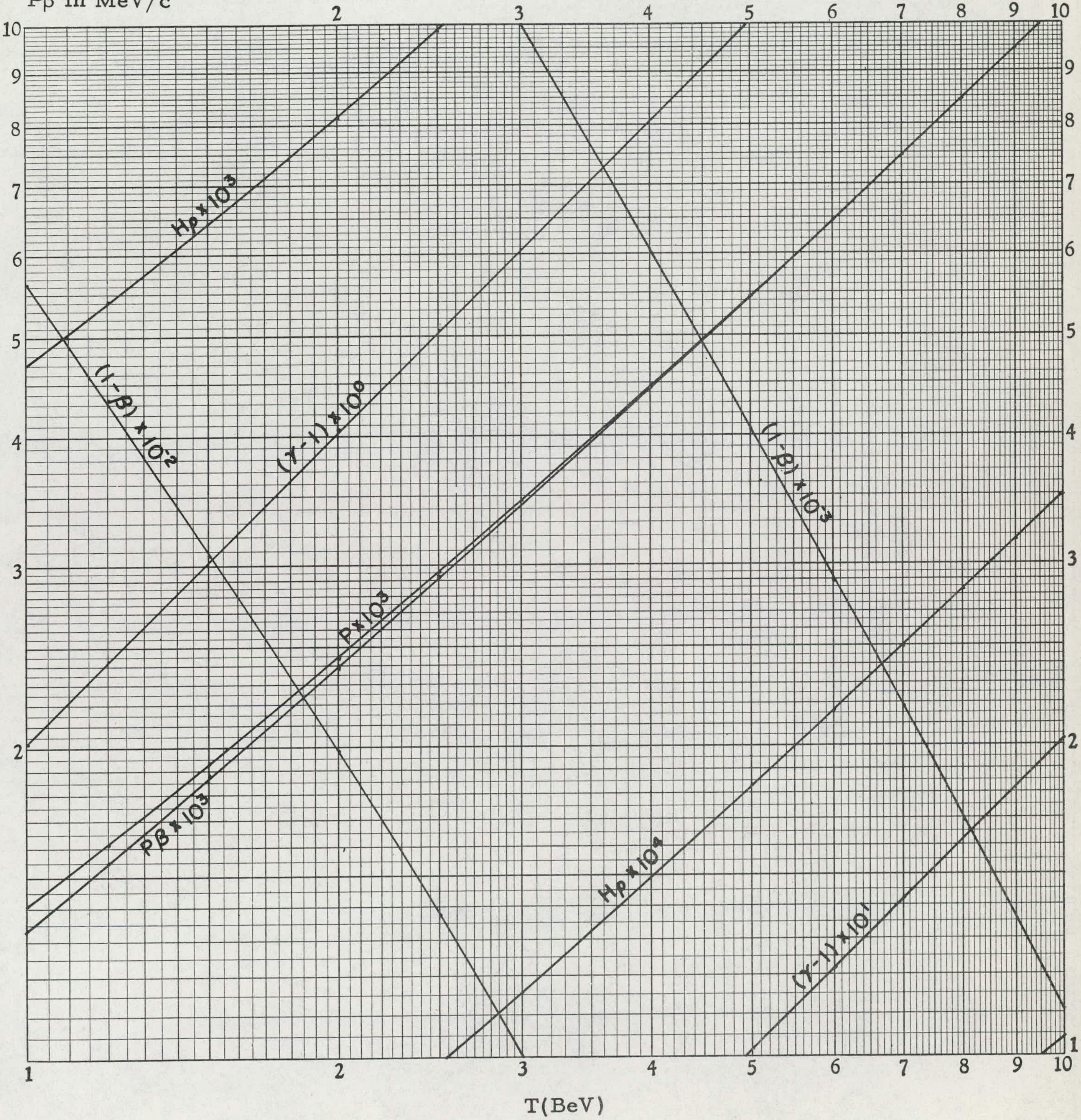
$K^{\pm}$ MESONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

$M_{K^{\pm}}$ = 493.9 MeV
= 966.6 m



K$^{\pm}$ MESONS

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

$M_{K^{\pm}}$ = 493.9 MeV
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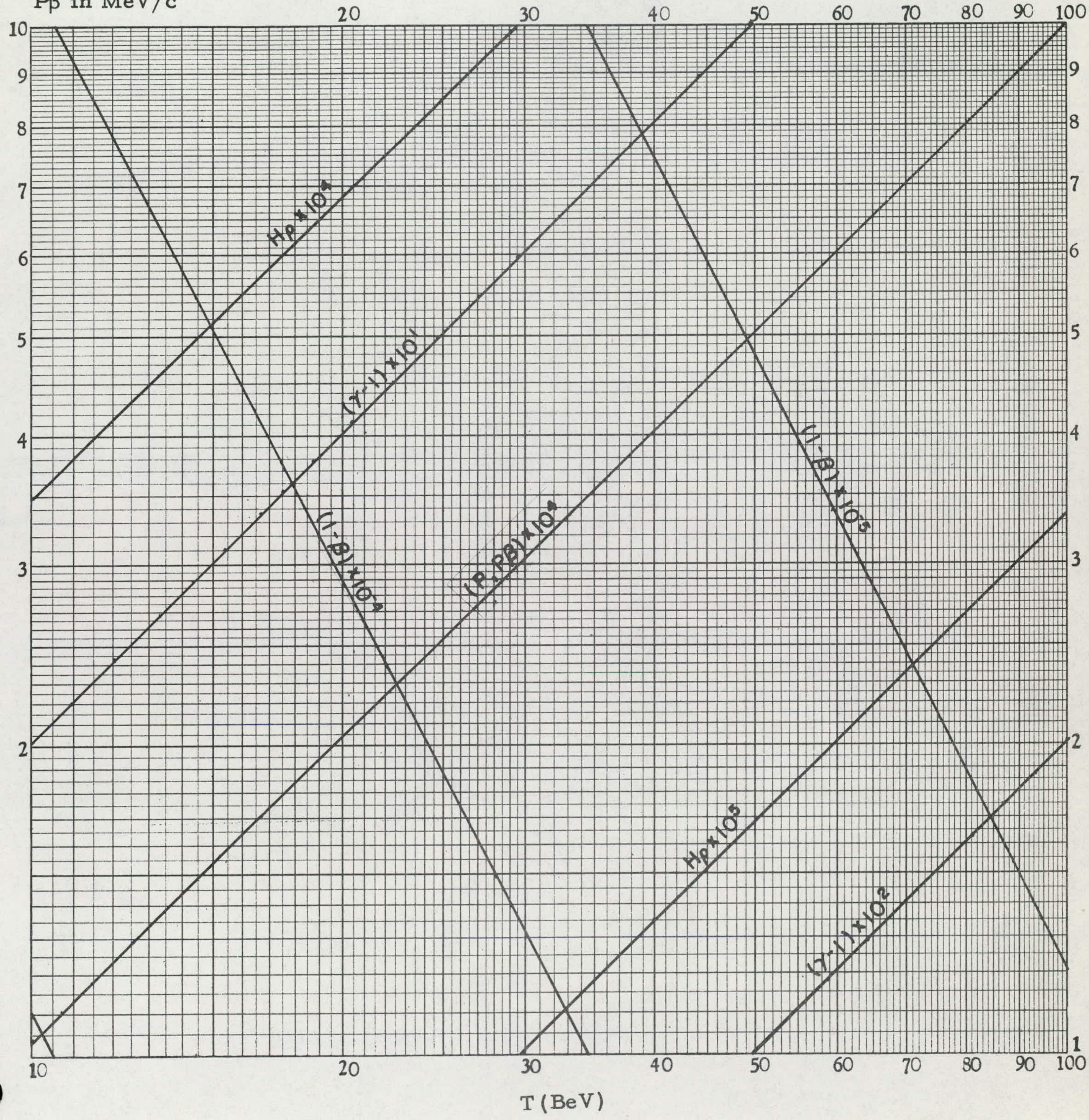
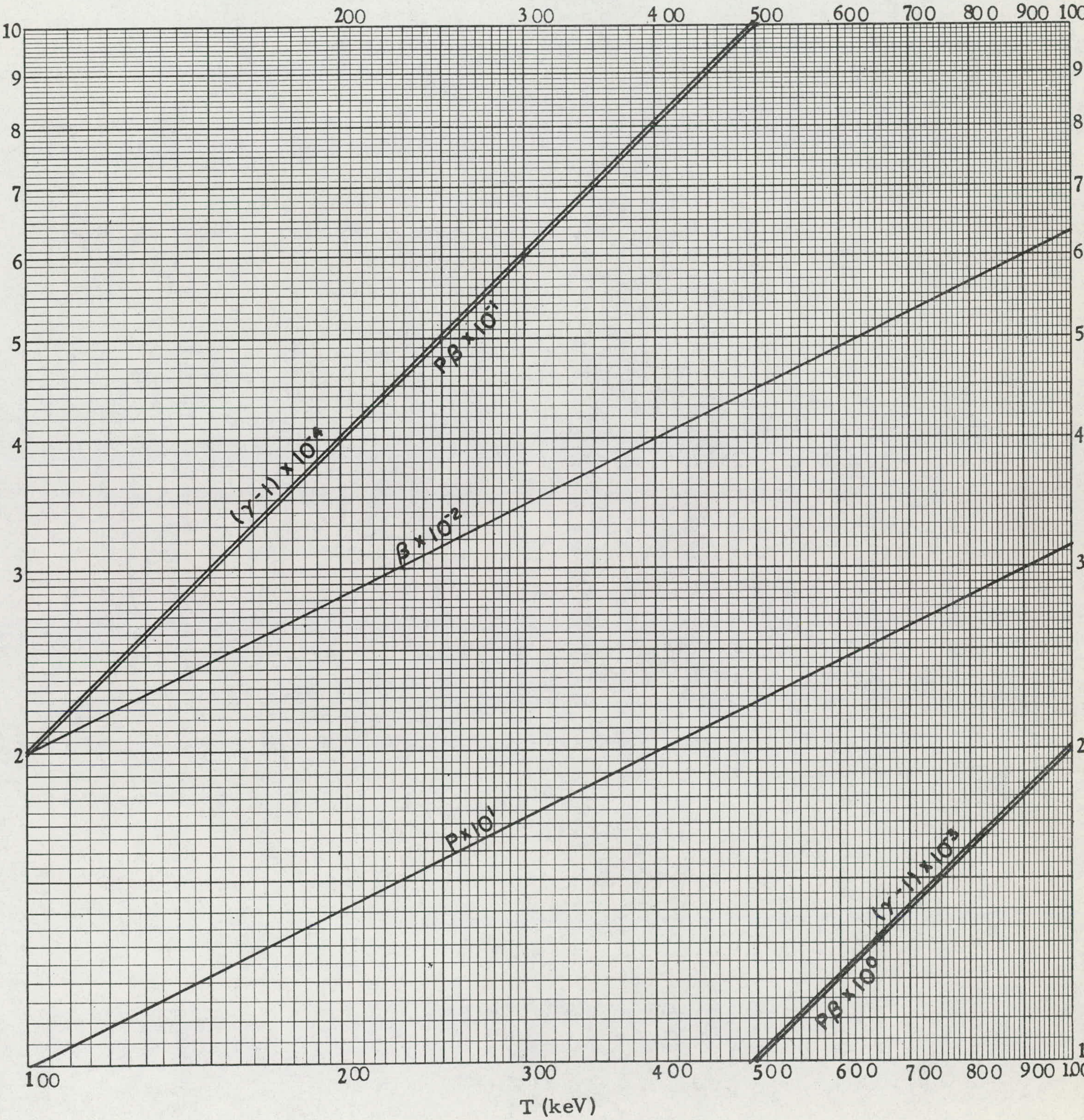


100 keV to 1 MeV

P and Pβ in MeV/c

β, (γ-1), P, Pβ

M_{K^0} = 497.8 MeV
= 974.4 m



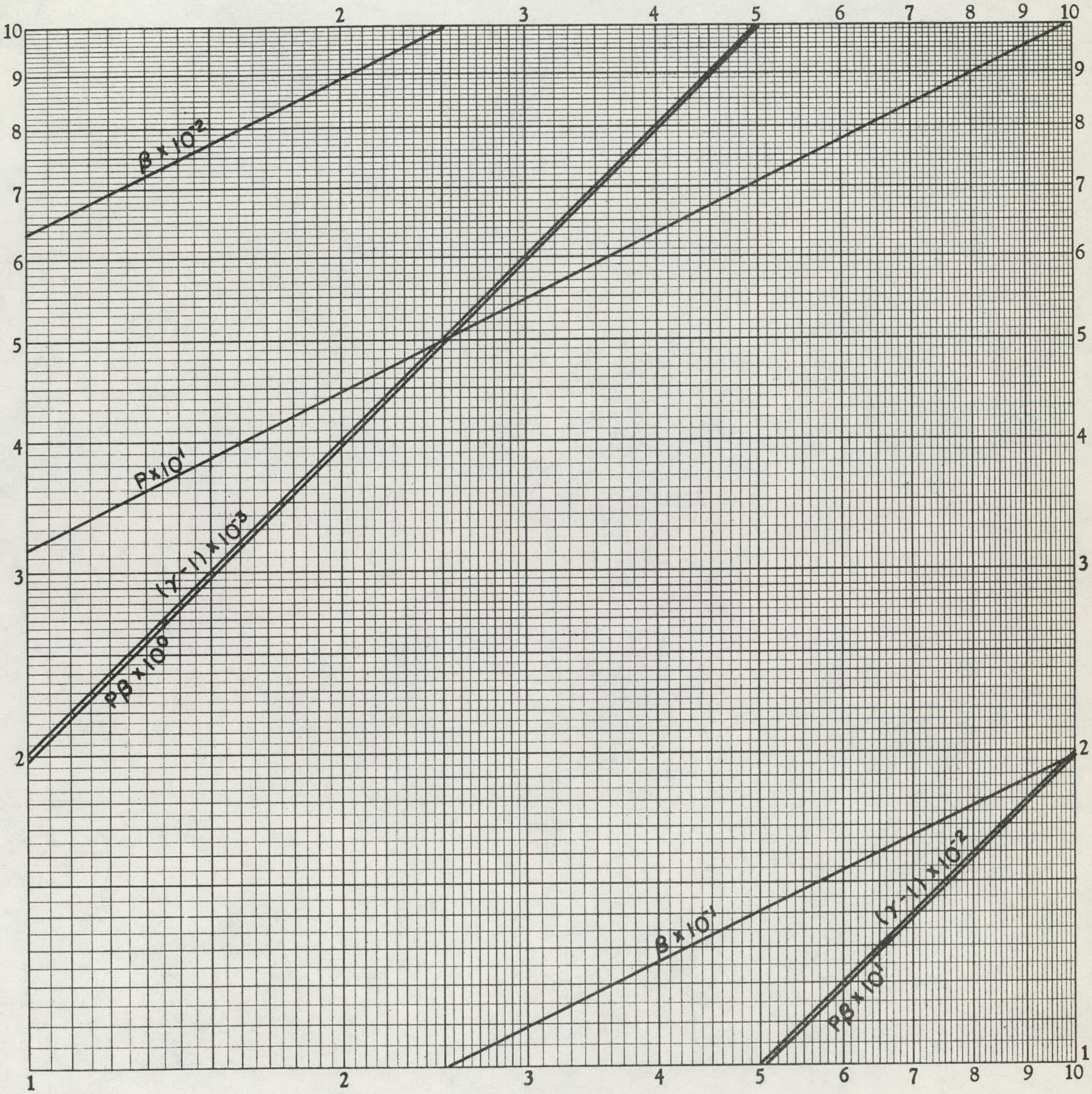
K^0 MESONS

1 MeV to 10 MeV

β, $(\gamma-1)$, P, Pβ

P and Pβ in MeV/c

M_{K^0} = 497.8 MeV
= 974.4 m



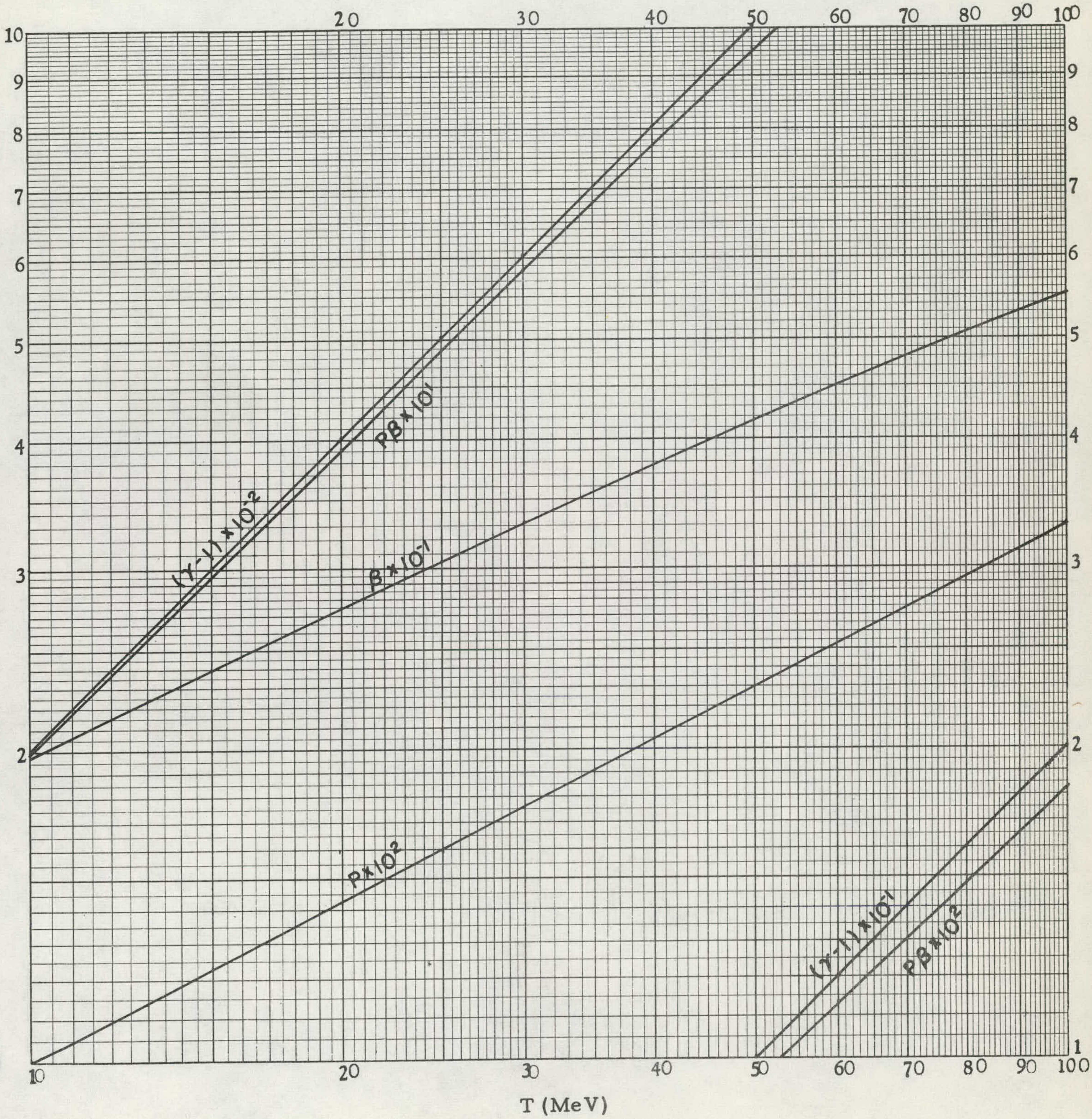
K^0 MESONS

10 MeV to 100 MeV

P and Pβ in MeV/c

β, (γ-1), P, Pβ

M_{K^0} = 497.8 MeV
= 974.4 m



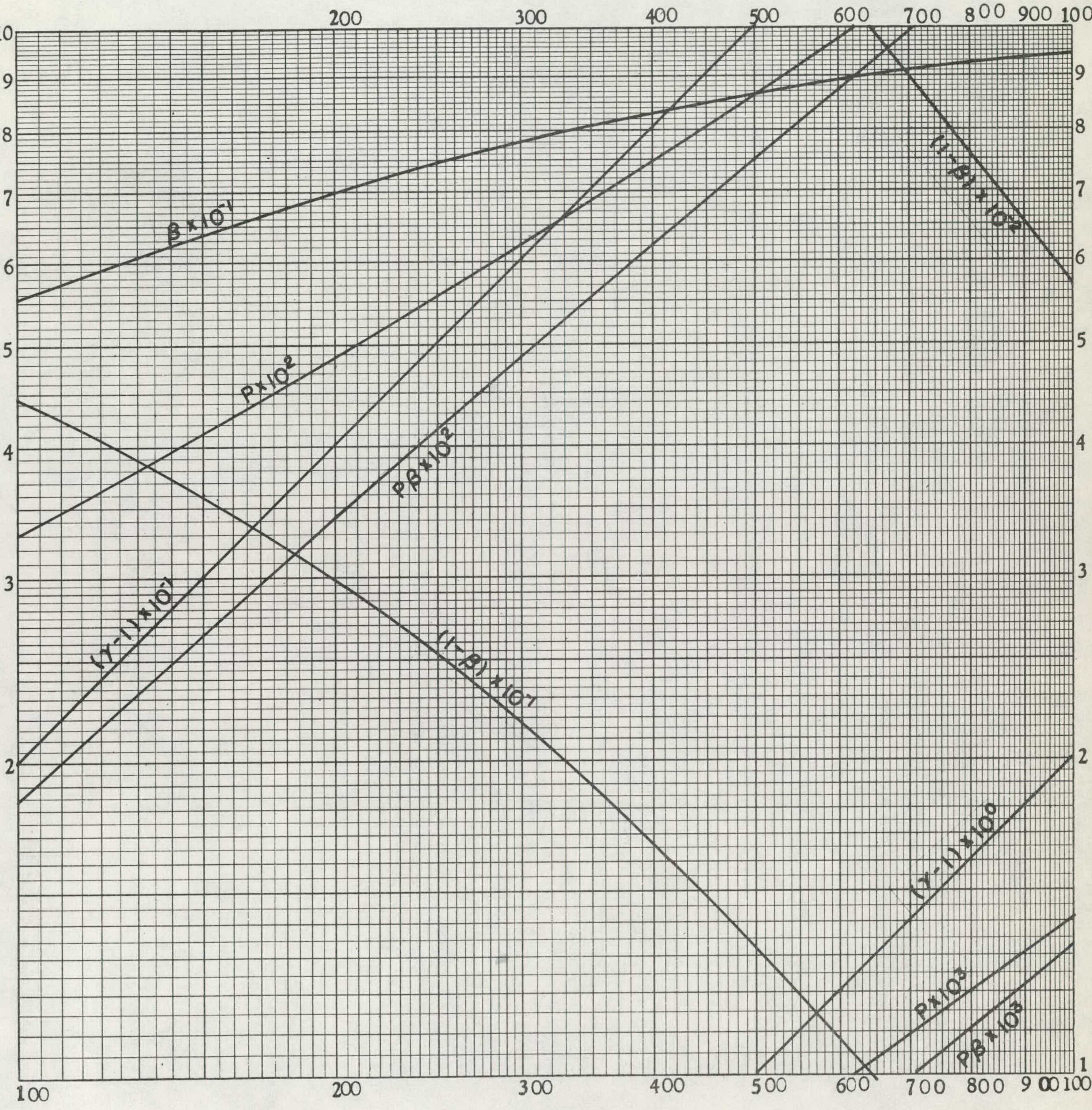
K^0 MESONS

100 MeV to 1 BeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ

P and Pβ in MeV/c

M_{K^0} = 497.8 MeV
= 974.4 m



T (MeV)

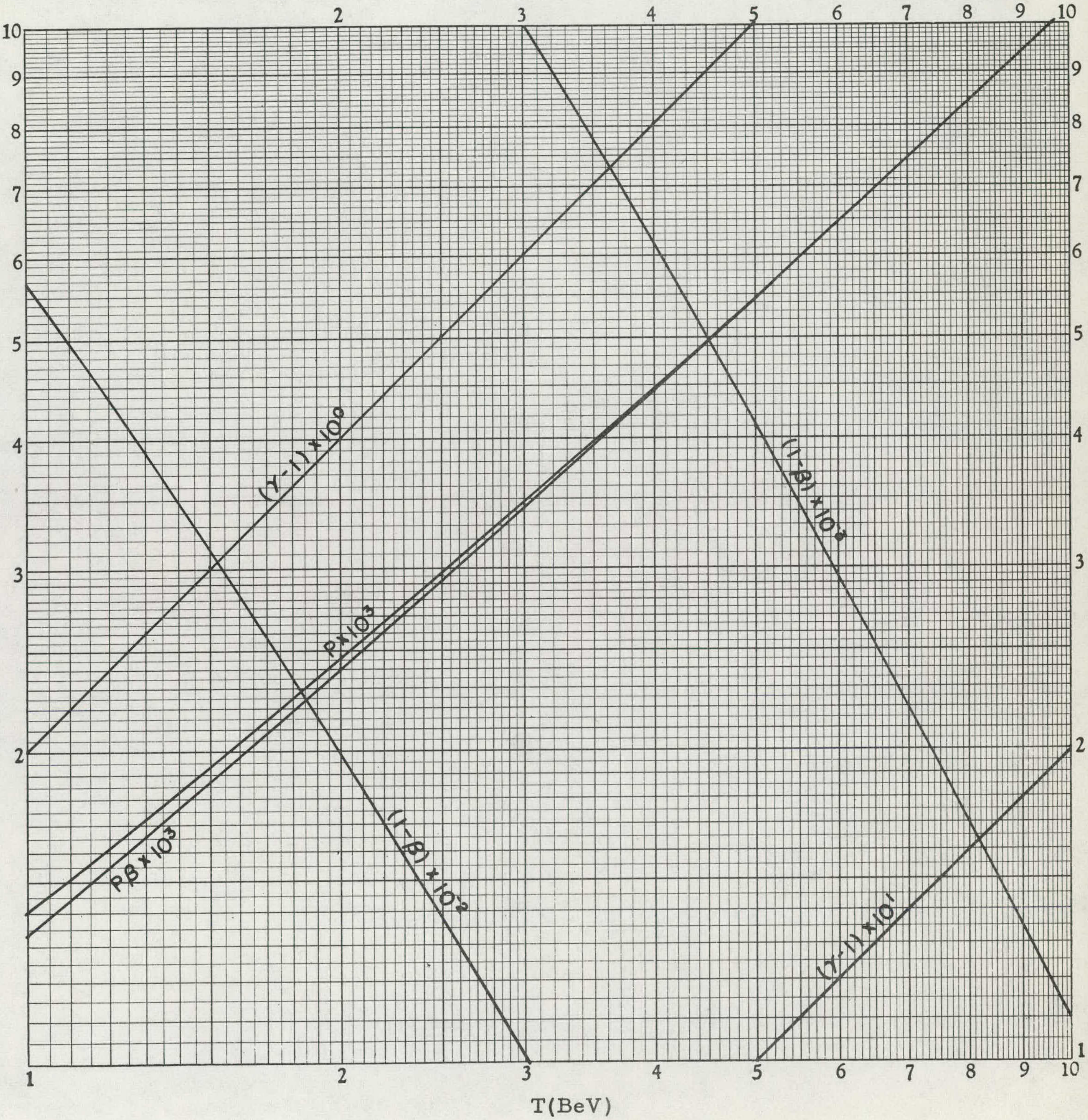
K^0 MESONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ

M_{K^0} = 497.8 MeV
= 974.4 m

P and Pβ in MeV/c

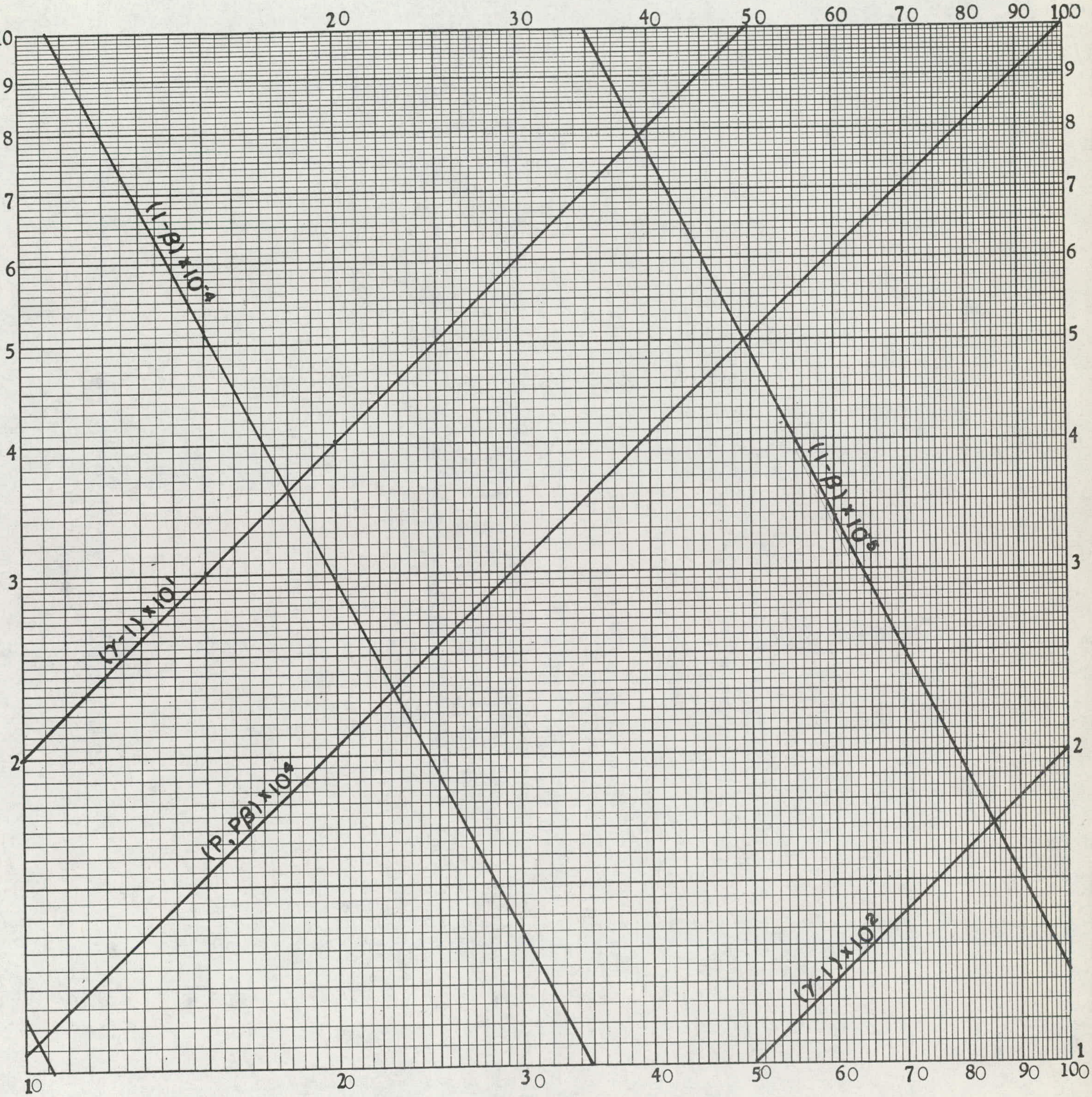


10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, P, Pβ

M_{K^0} = 497.8 MeV
= 974.4 m

P and Pβ in MeV/c



PROTONS

100 keV to 1 MeV

β, (γ-1), P, Pβ, Hρ

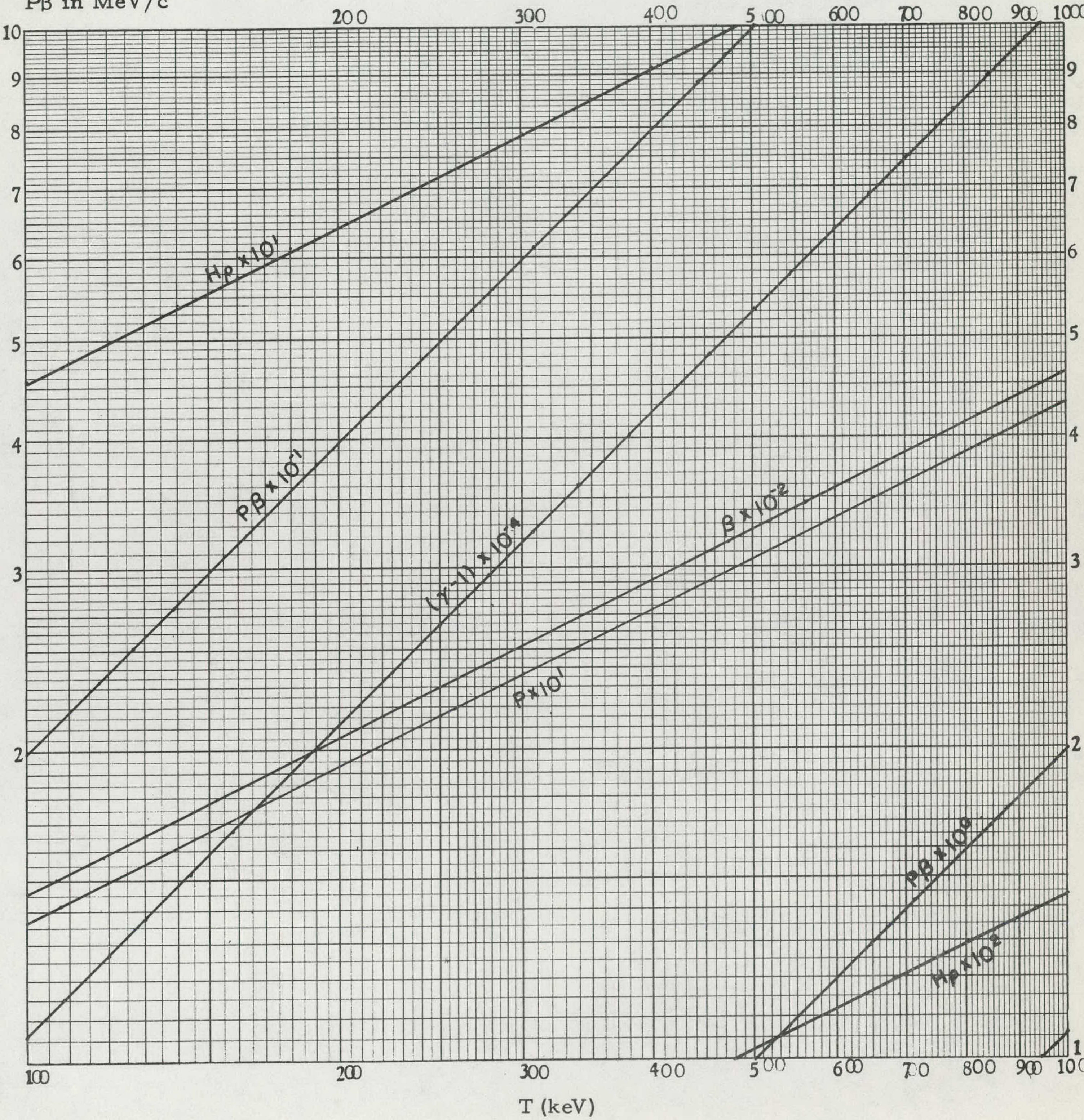
P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

M_p = 938.213 MeV

= 1836.12



PROTONS

1 MeV to 10 MeV

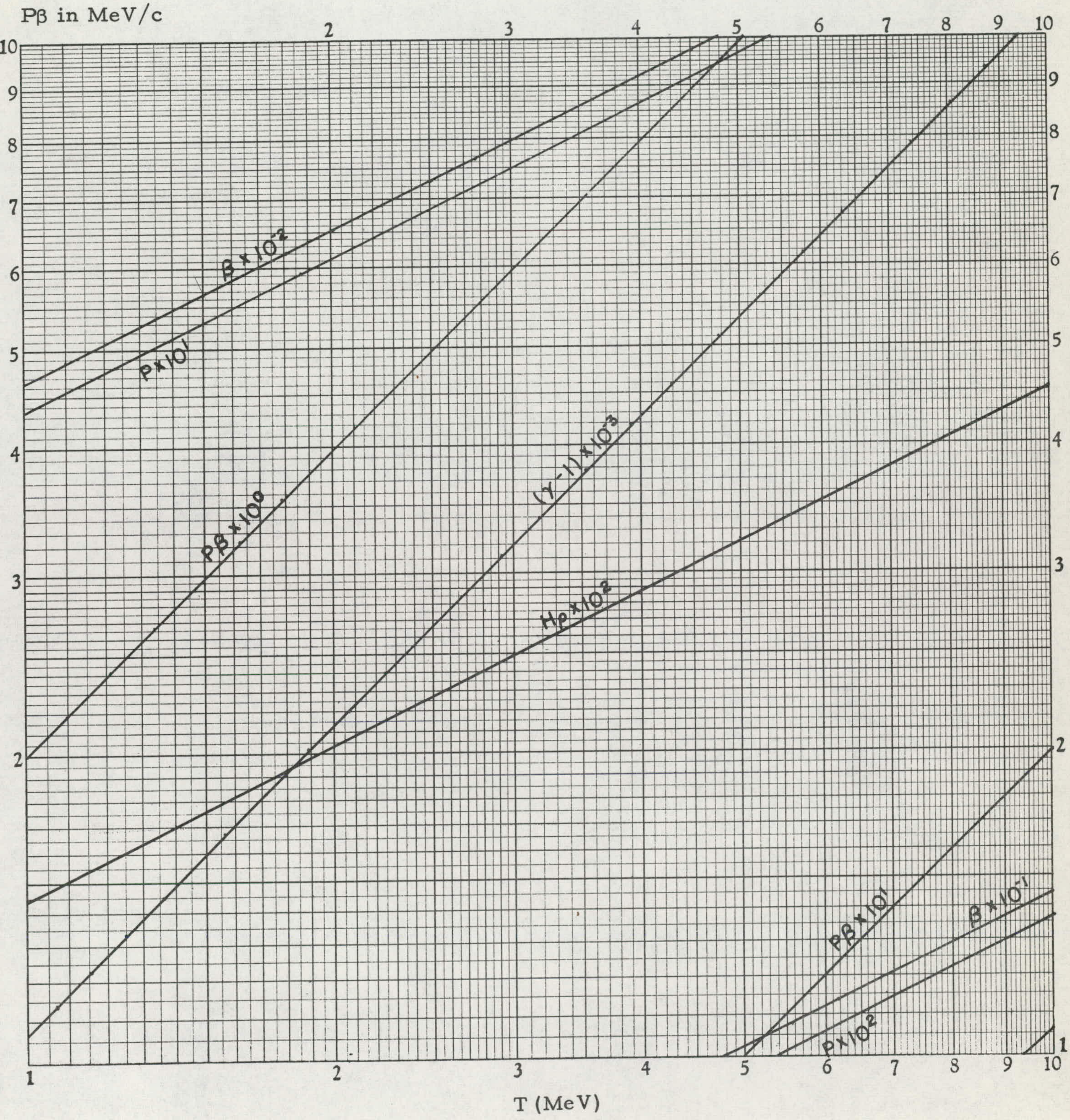
β, (γ-1), P, Pβ, Hρ

P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

M_p = 938.213 MeV
= 1836.12



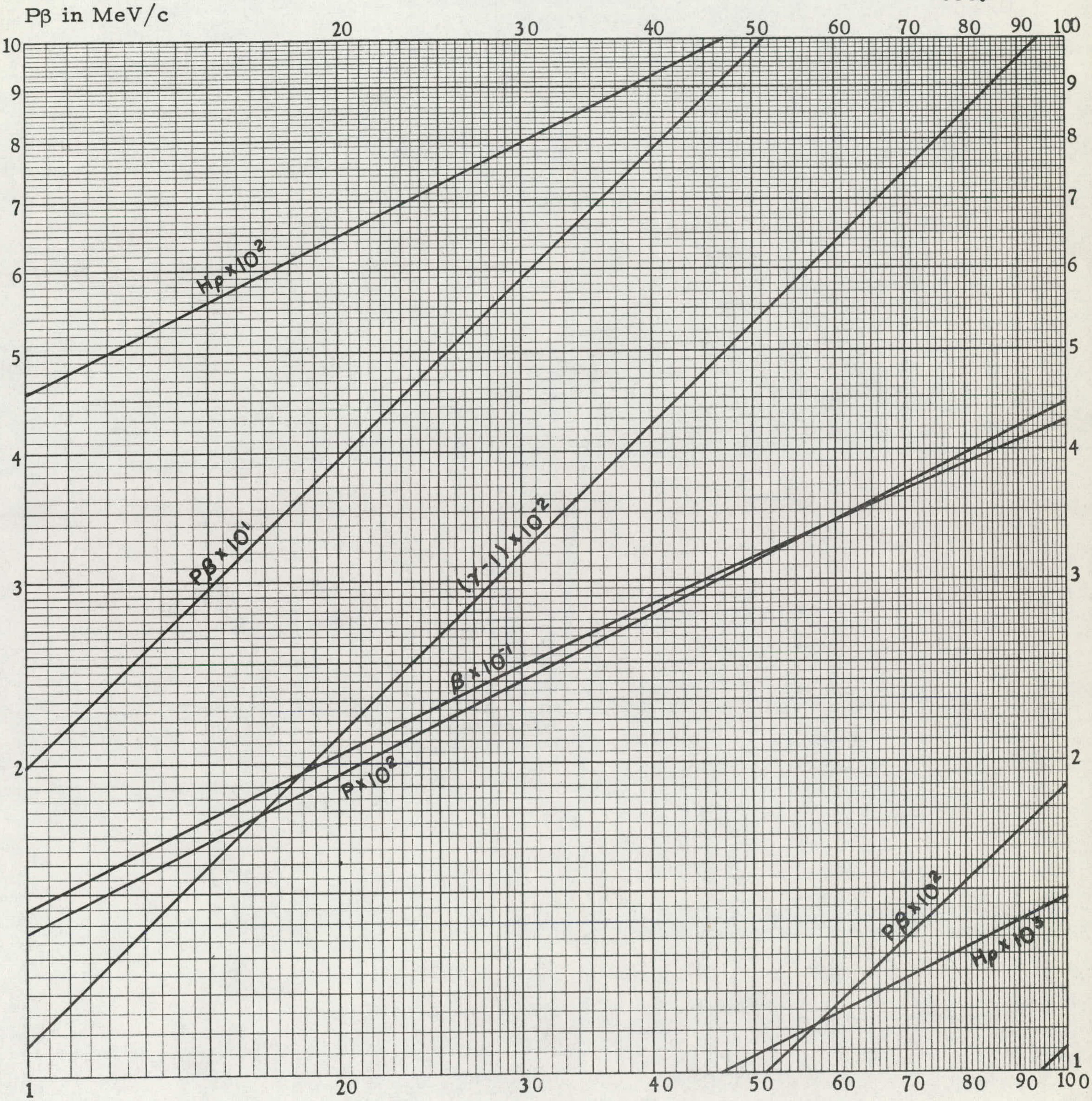
PROTONS

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV to 100 MeV

β, (γ-1), P, Pβ, Hρ

M_p = 938.213 MeV
= 1836.12



T (MeV)

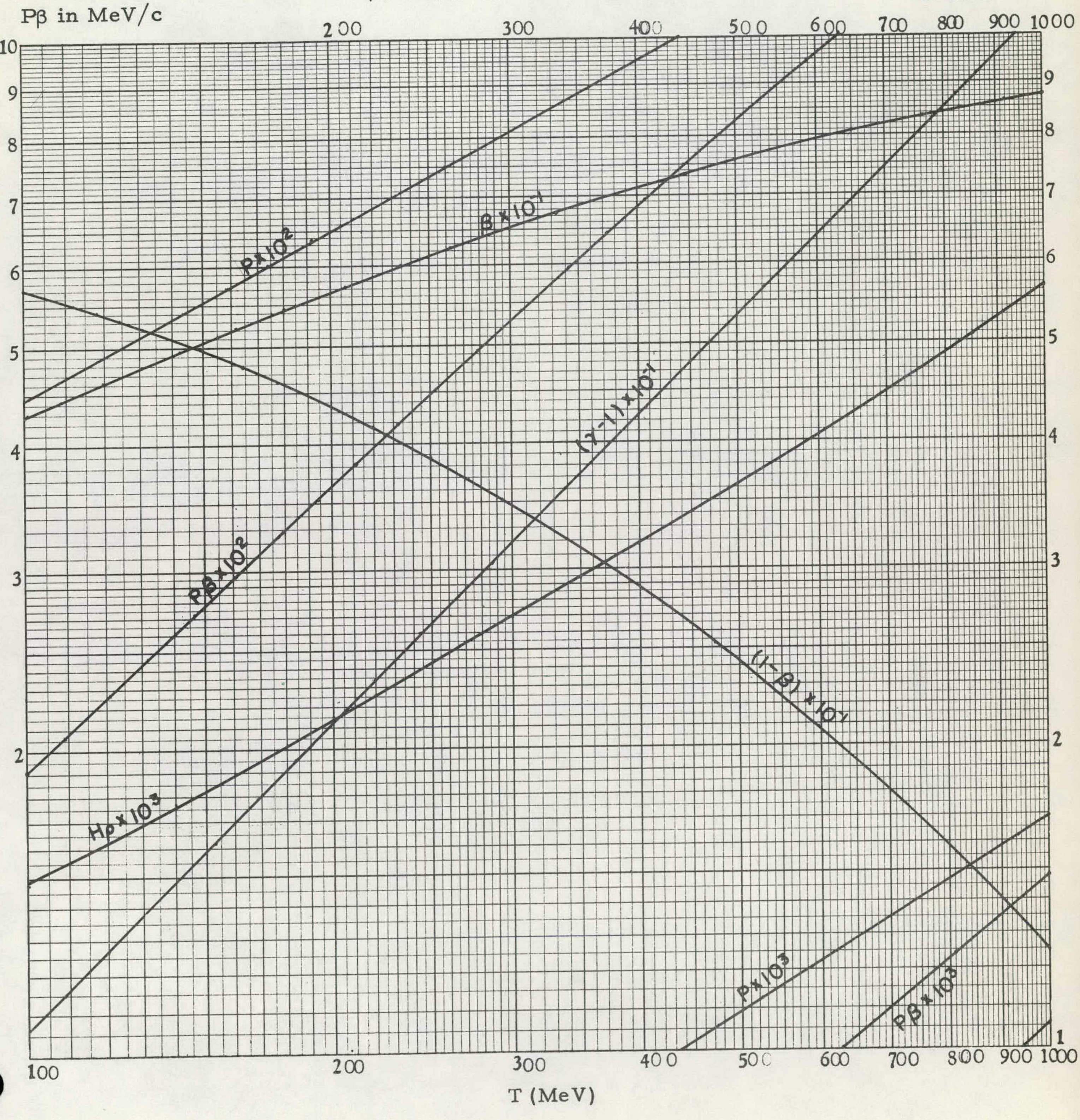
PROTONS

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

100 MeV to 1 BeV

β, (1-β), (γ-1), P, Pβ, Hρ

M_p = 938.213 MeV
= 1836.12



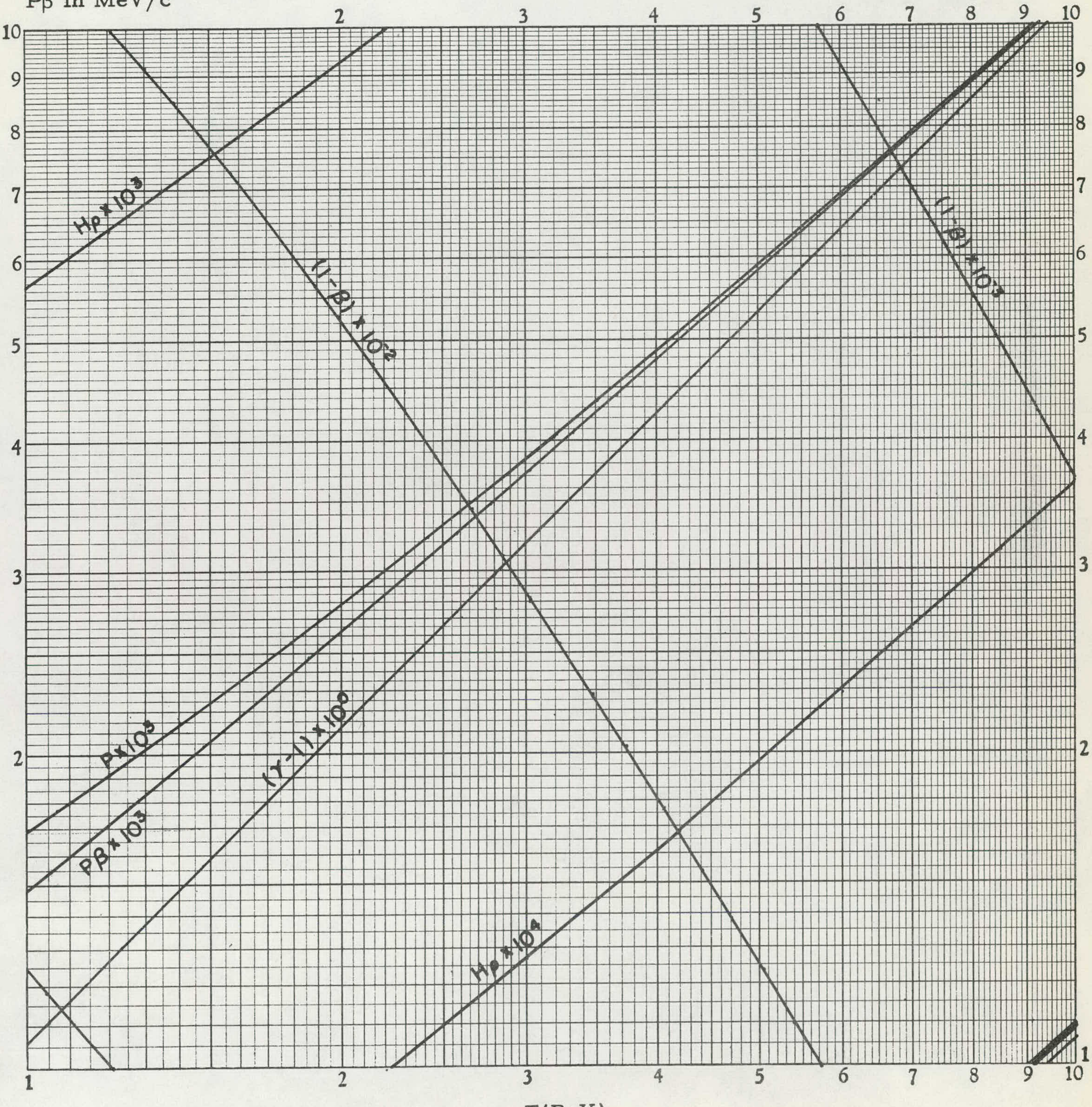
PROTONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_p = 938.213 MeV
= 1836.12



PROTONS

10 BeV to 100 BeV

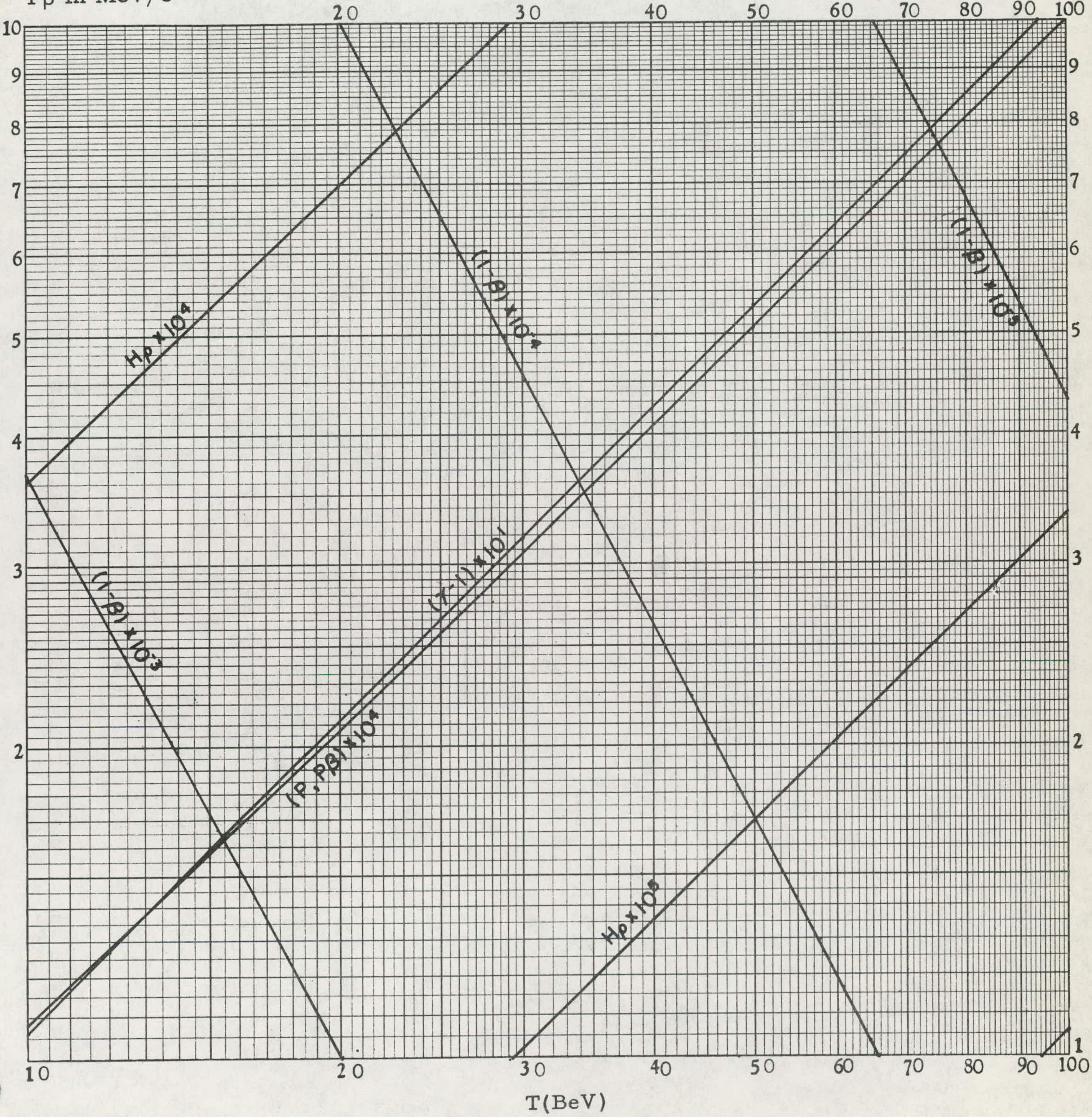
(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

M_p = 938.213 MeV
= 1836.12

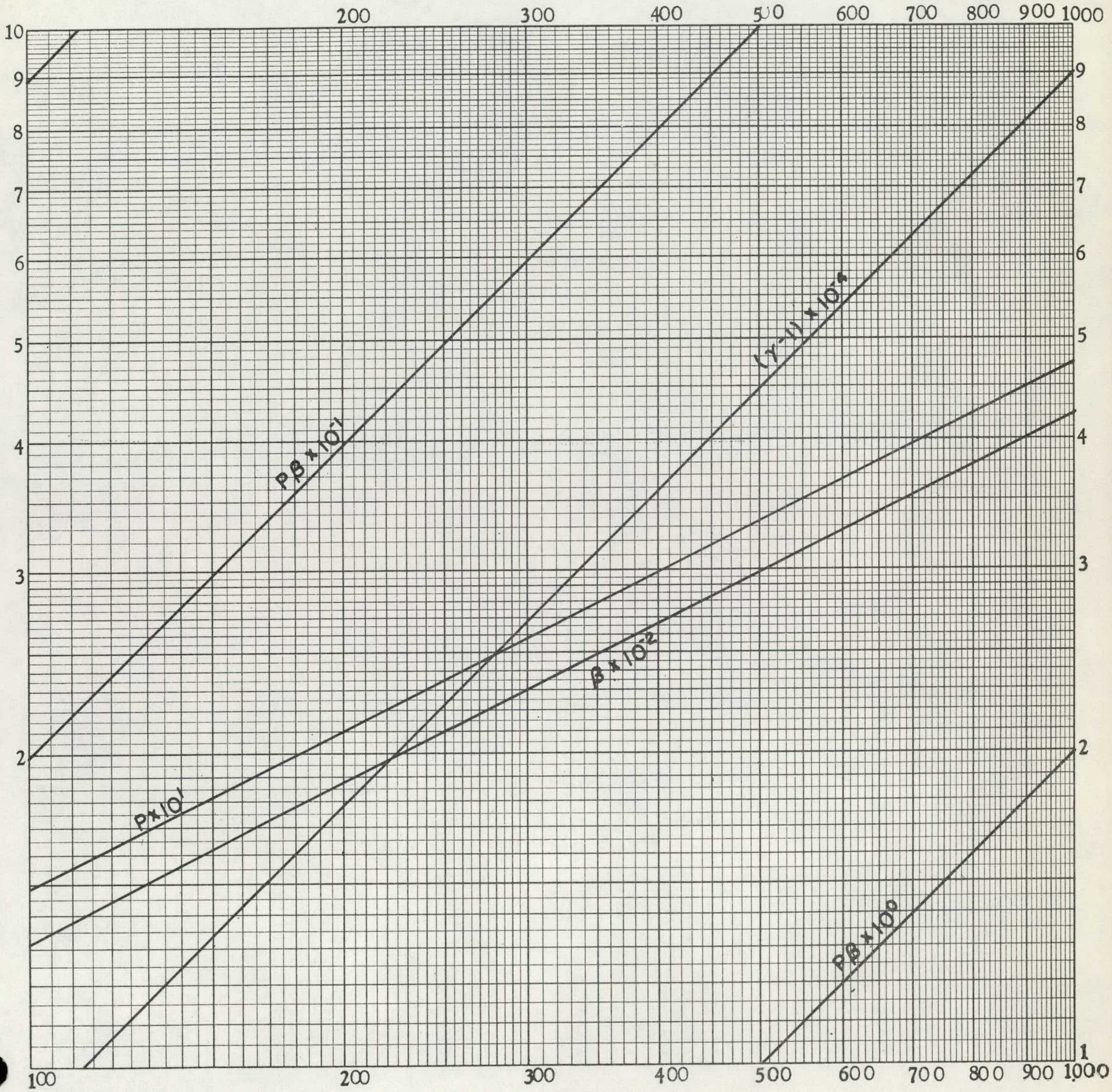


100 keV to 1 MeV

M_Λ = 1115.36 MeV
= 2182.80 m

P and Pβ in MeV/c

β, (γ-1), P, Pβ



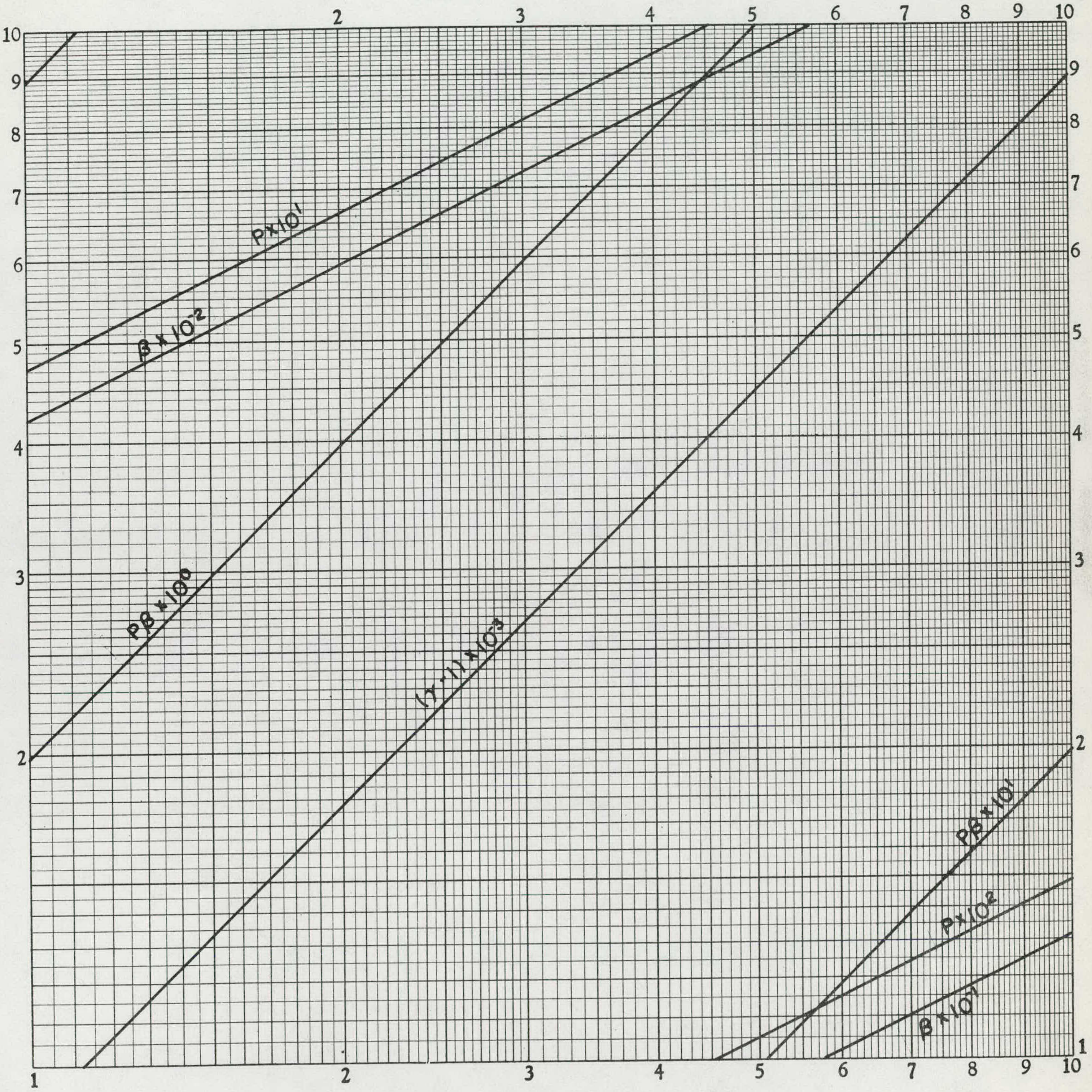
Λ HYPERONS

1 MeV to 10 MeV

β, (γ-1), P, Pβ

P and Pβ in MeV/c

M_Λ = 1115.36 MeV
= 2182.80 m

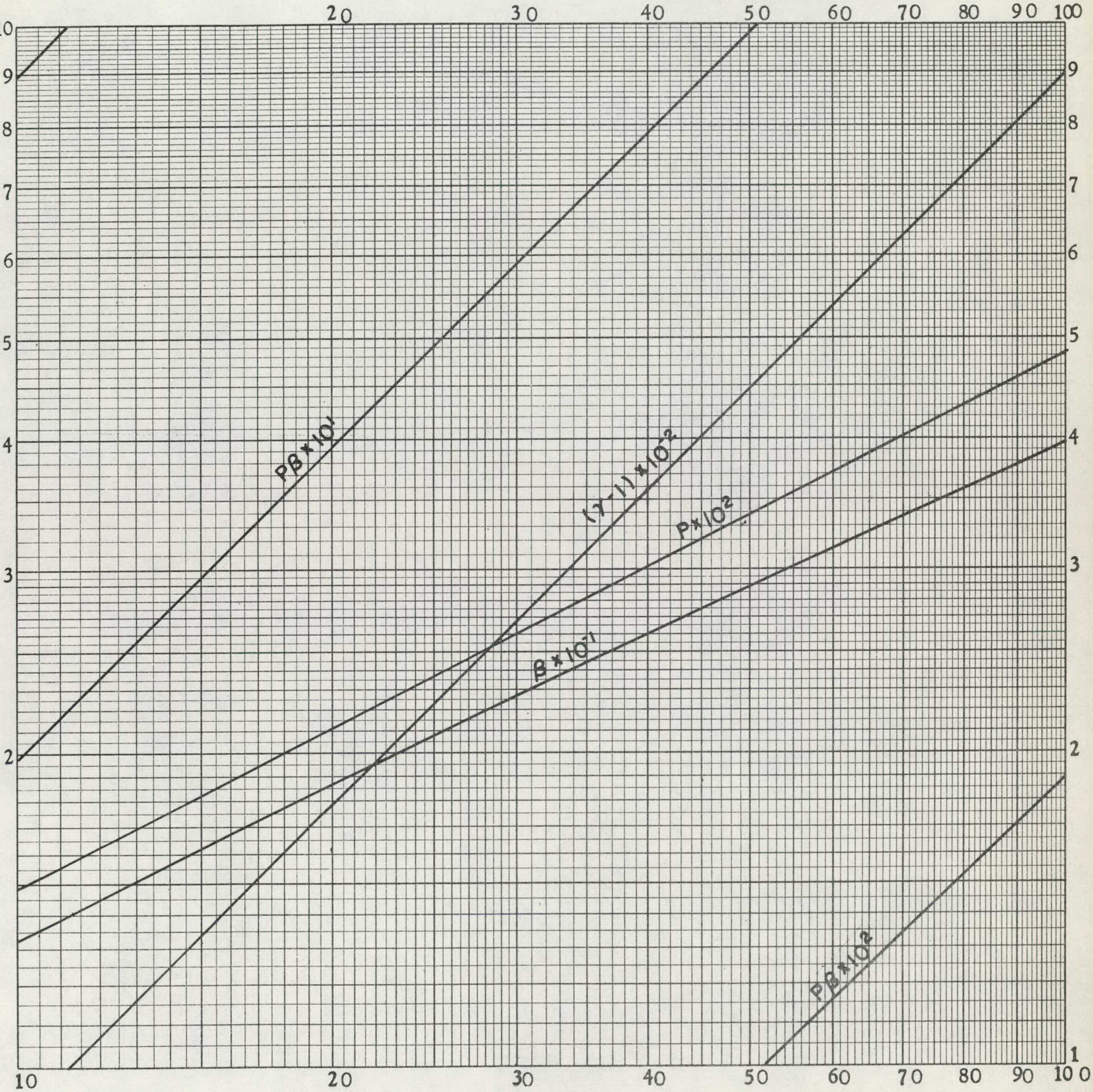


10 MeV to 100 MeV

P and Pβ in MeV/c

β, (γ-1), P, Pβ

M_Λ = 1115.36 MeV
= 2182.80 m

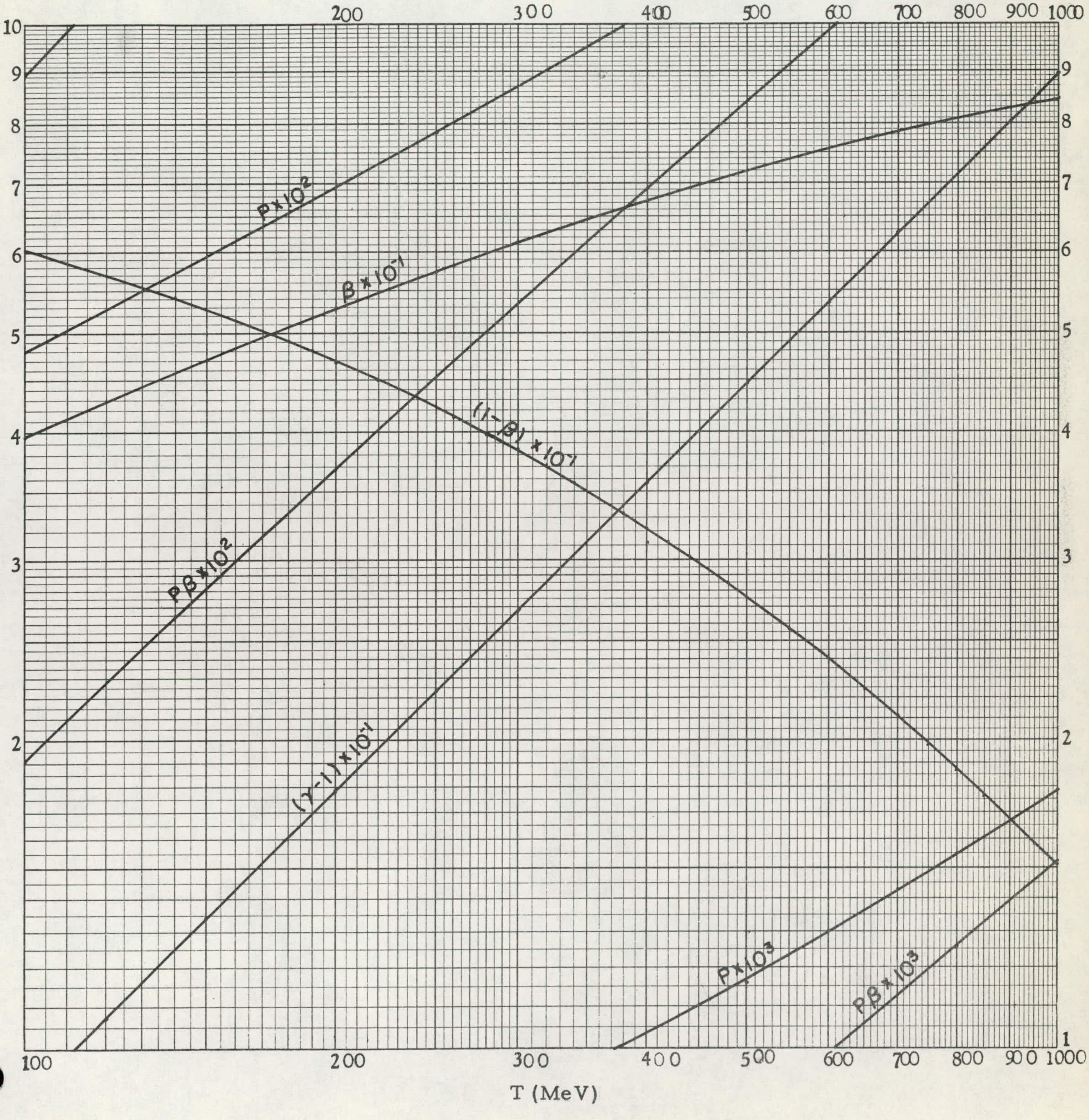


100 MeV to 1 BeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ

P and Pβ in MeV/c

M_Λ = 1115.36 MeV
= 2182.80 m

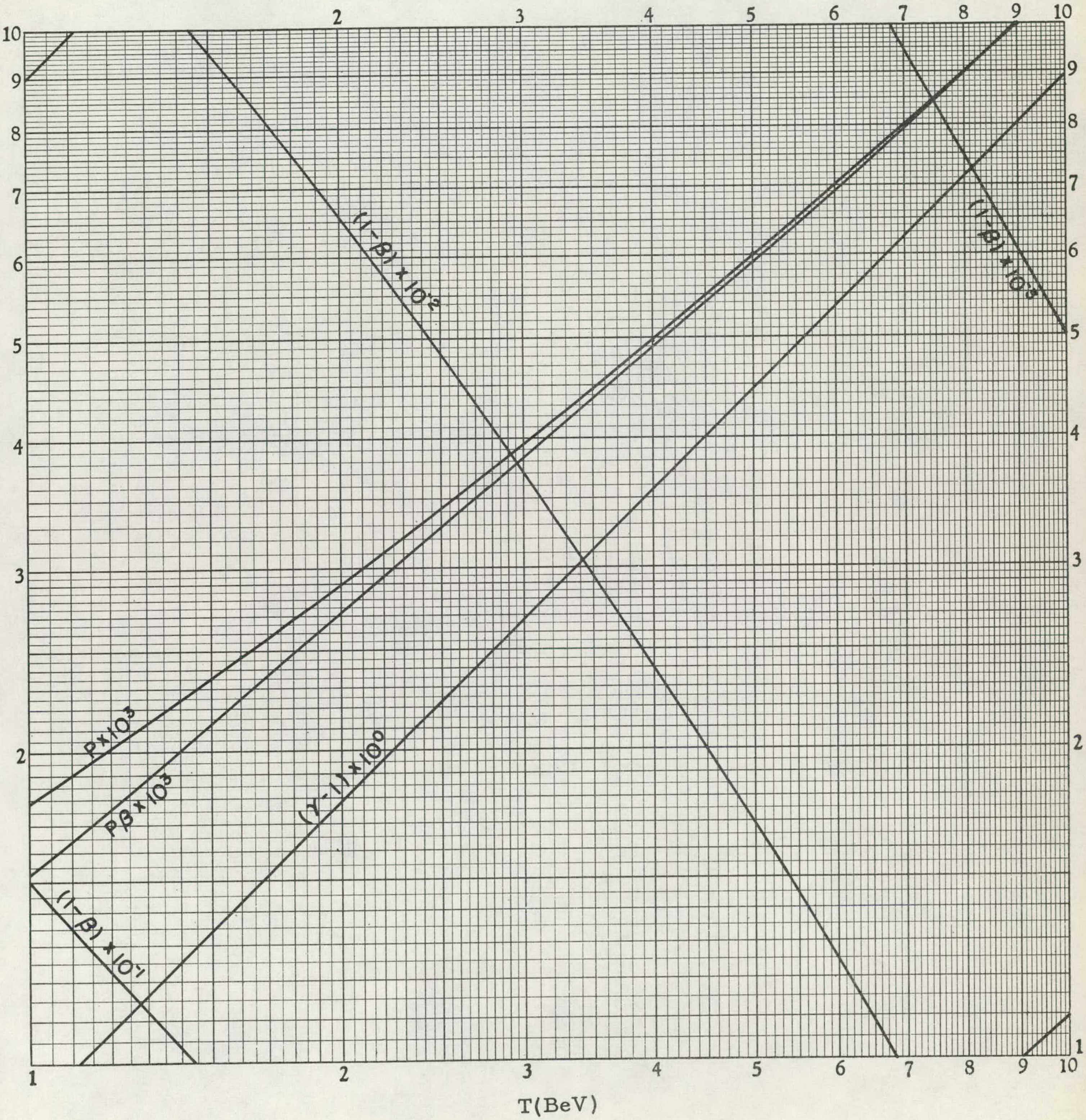


1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ

M_Λ = 1115.36 MeV
= 2182.80 m

P and Pβ in MeV/c



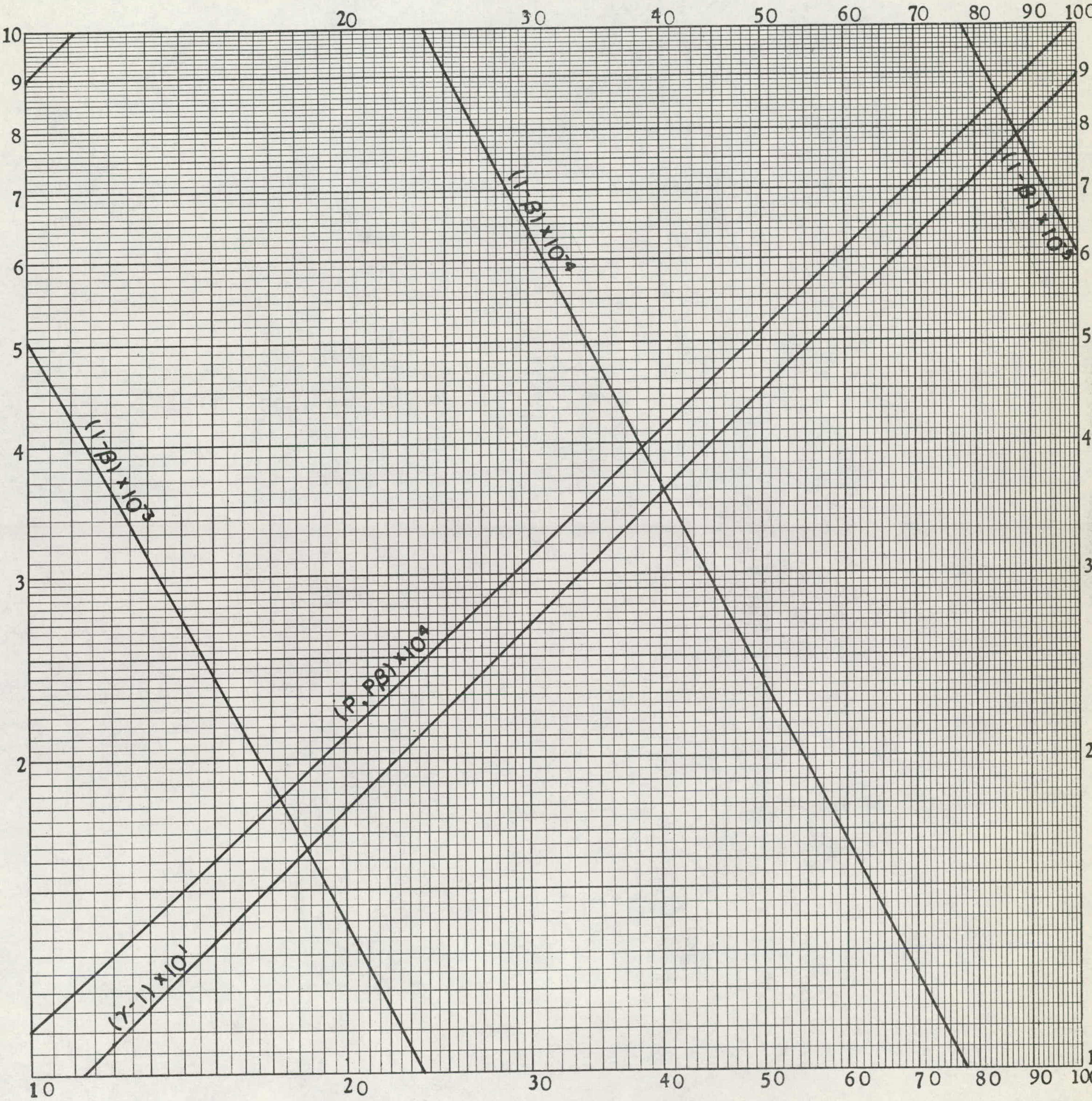
Λ HYPERONS

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ

M_Λ = 1115.36 MeV
= 2182.80 m

P and Pβ in MeV/c



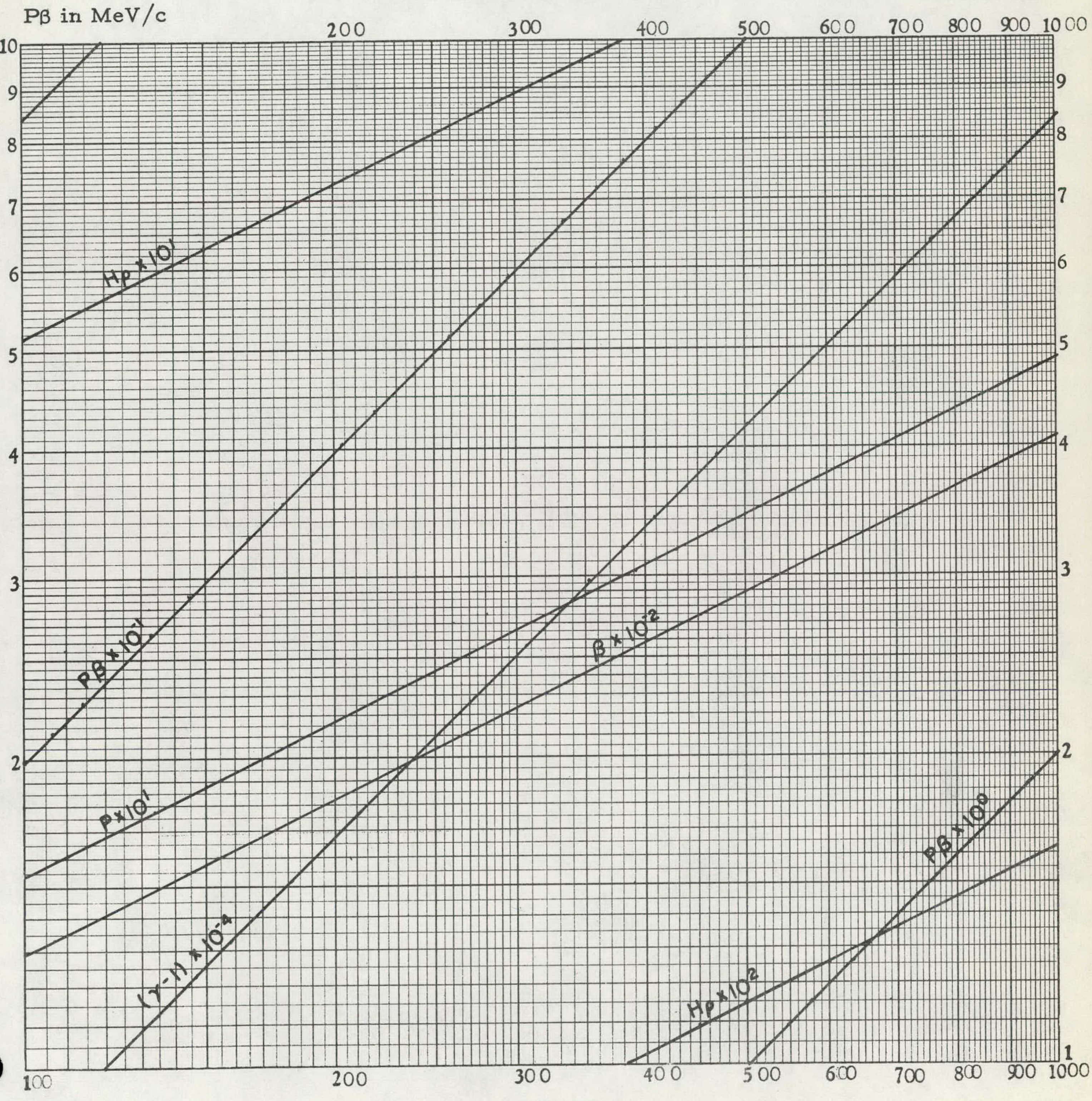
Σ^+ HYPERONS

100 keV to 1 MeV

β, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



T (keV)

Σ^+ HYPERONS

1 MeV to 10 MeV

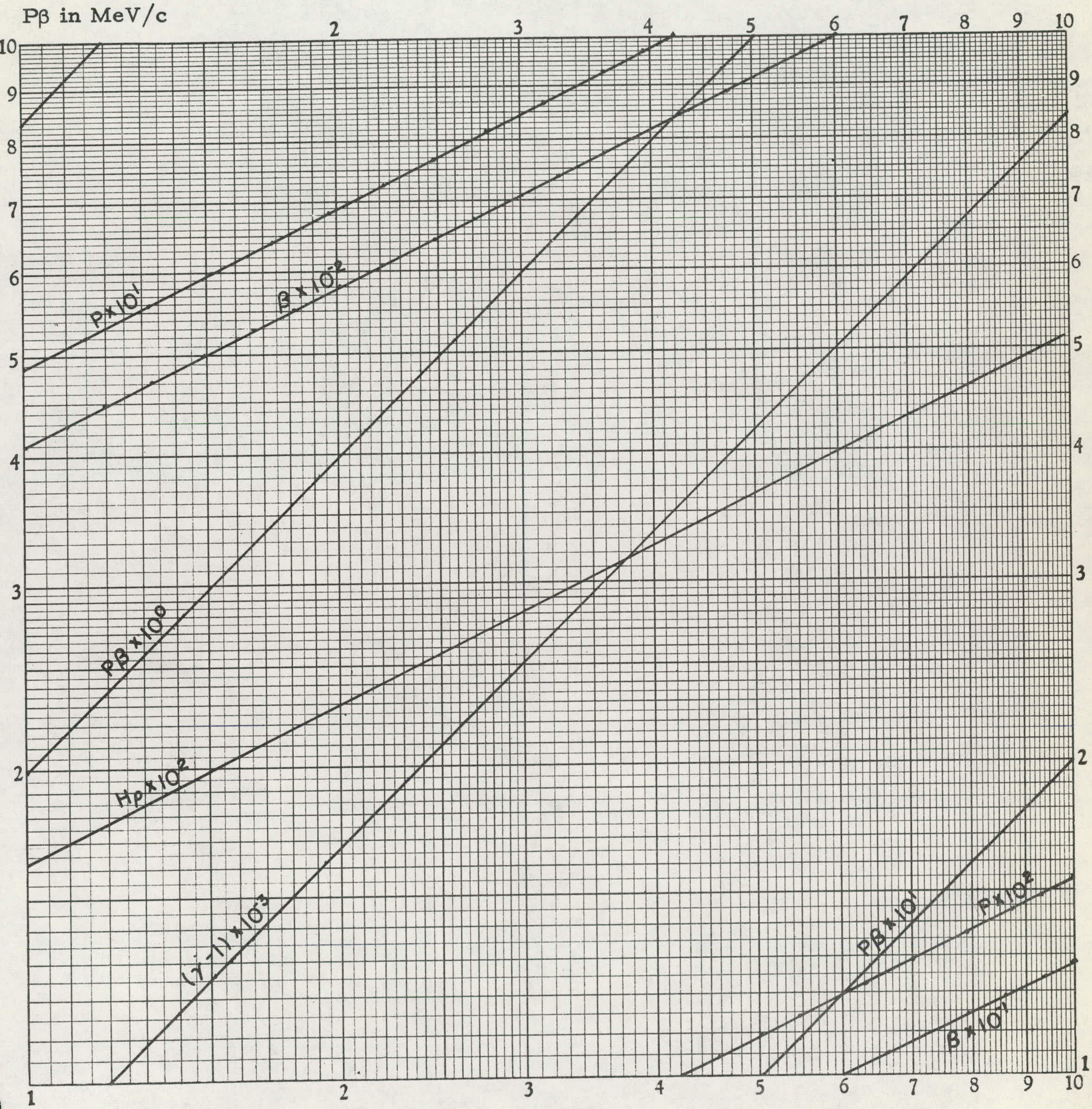
β, (γ-1), P, Pβ, Hρ

P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



Σ^+ HYPERONS

10 MeV to 100 MeV

β, (γ-1), P, Pβ, Hρ

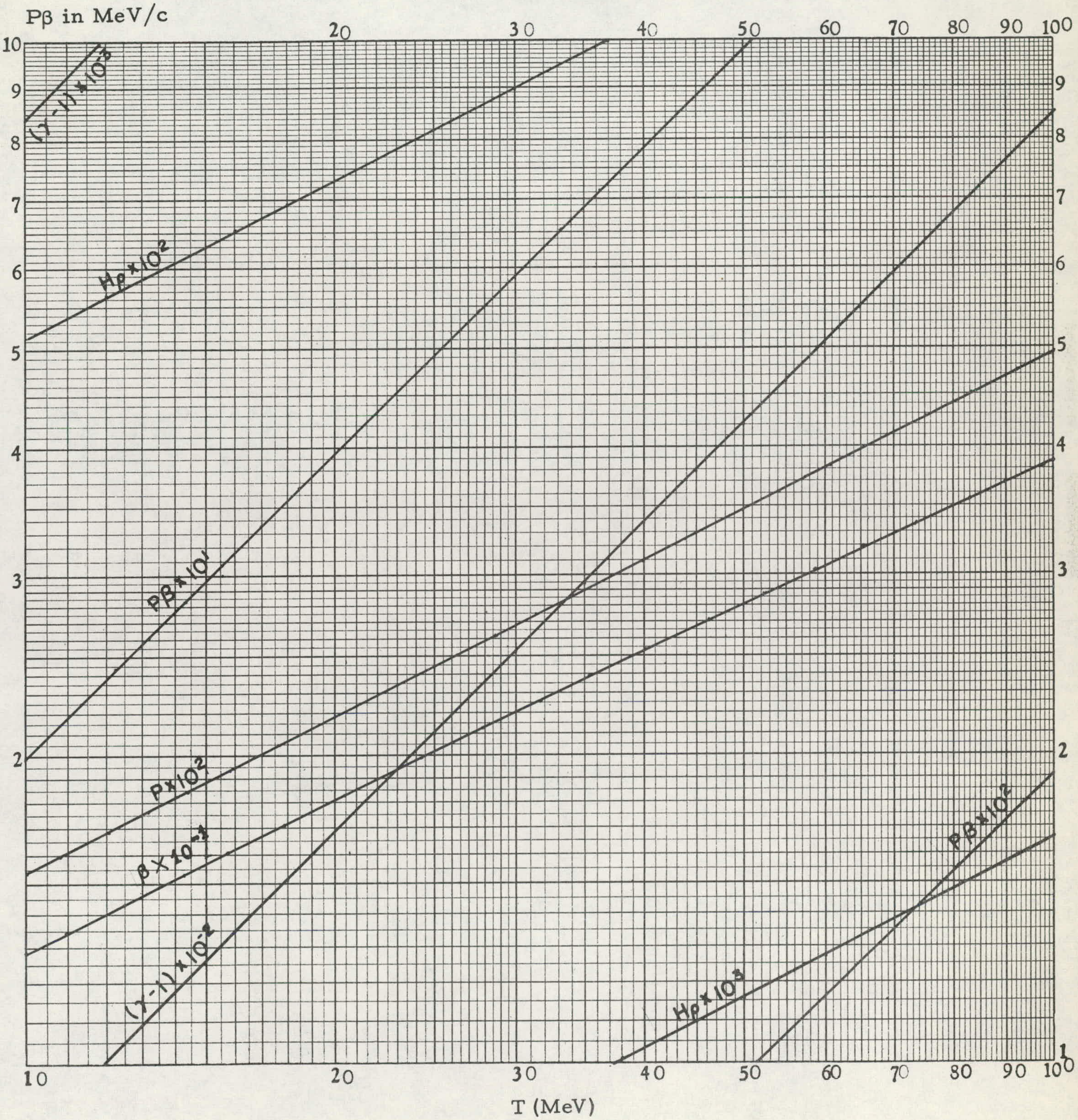
P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV

= 2321.83 m



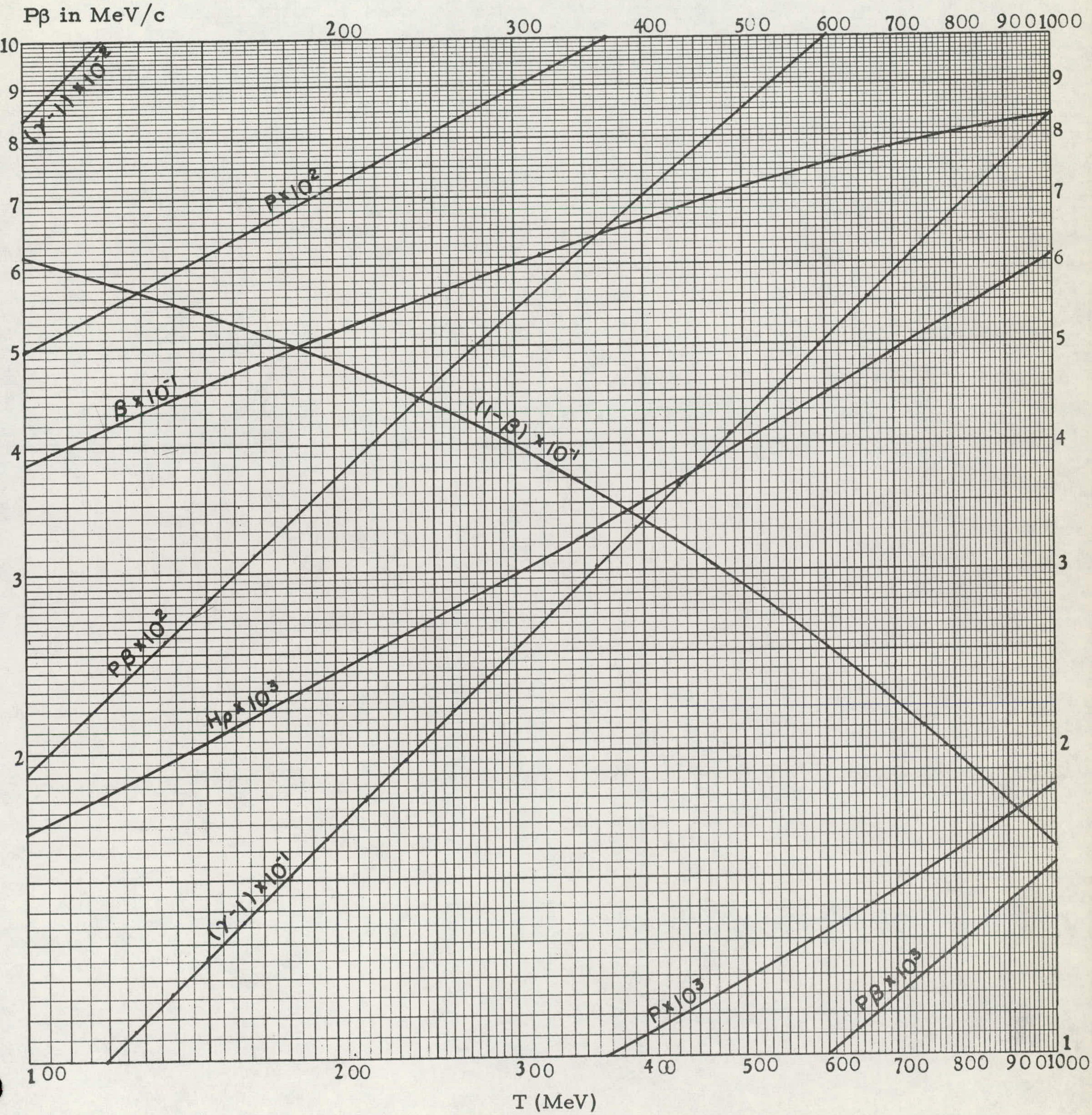
Σ^+ HYPERONS

100 MeV to 1 BeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



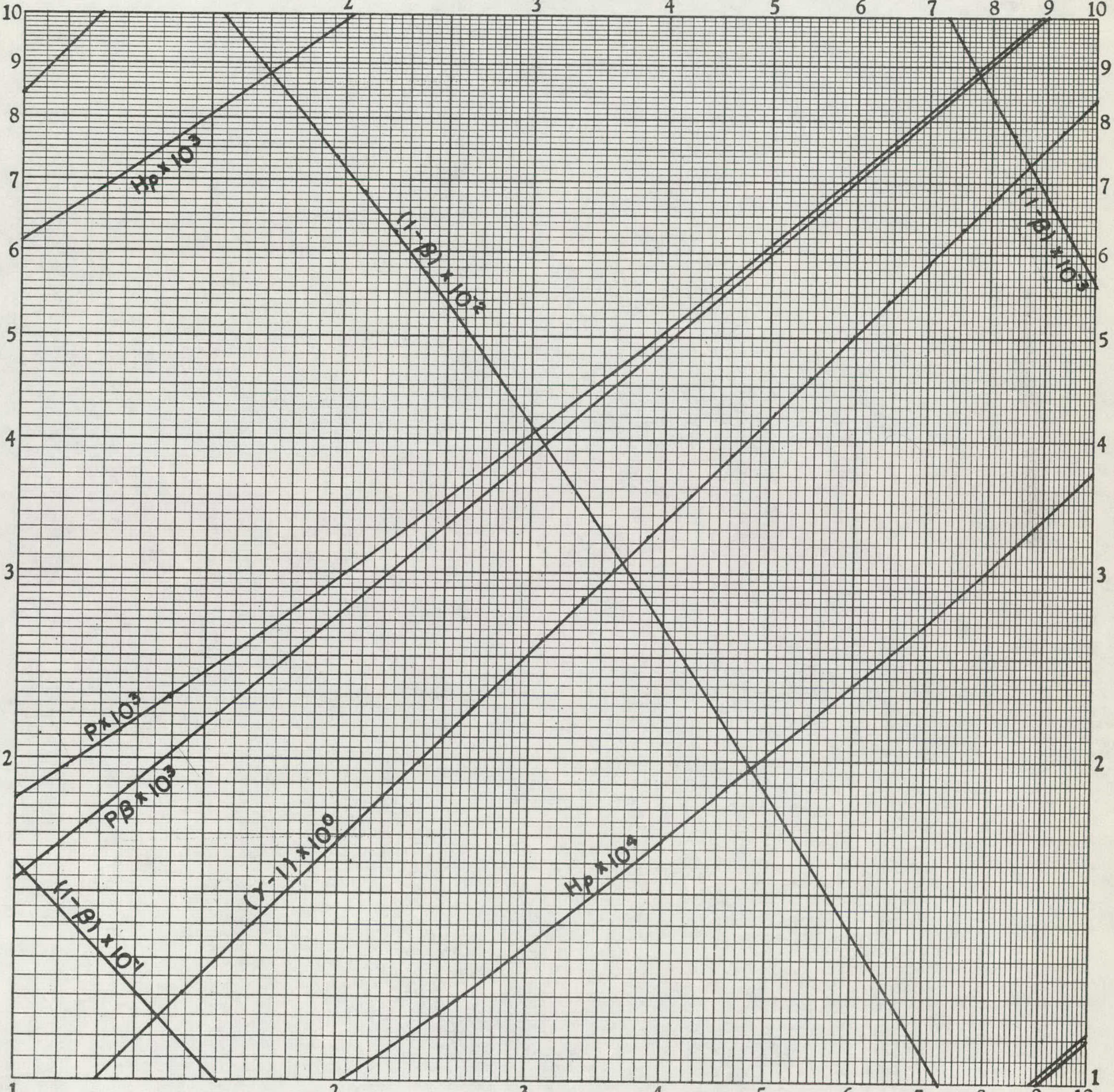
Σ^+ HYPERONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



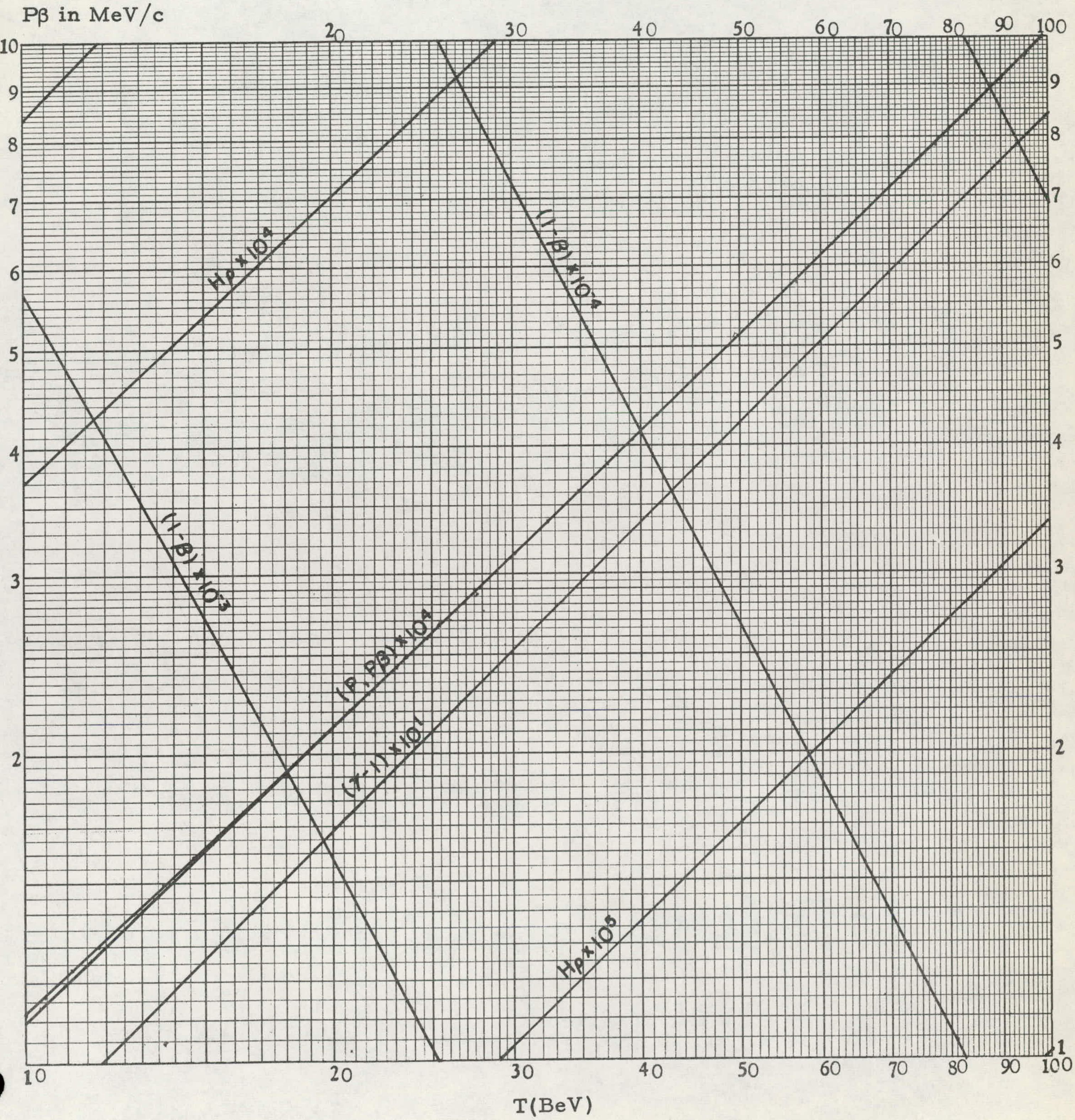
Σ^+ HYPERONS

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



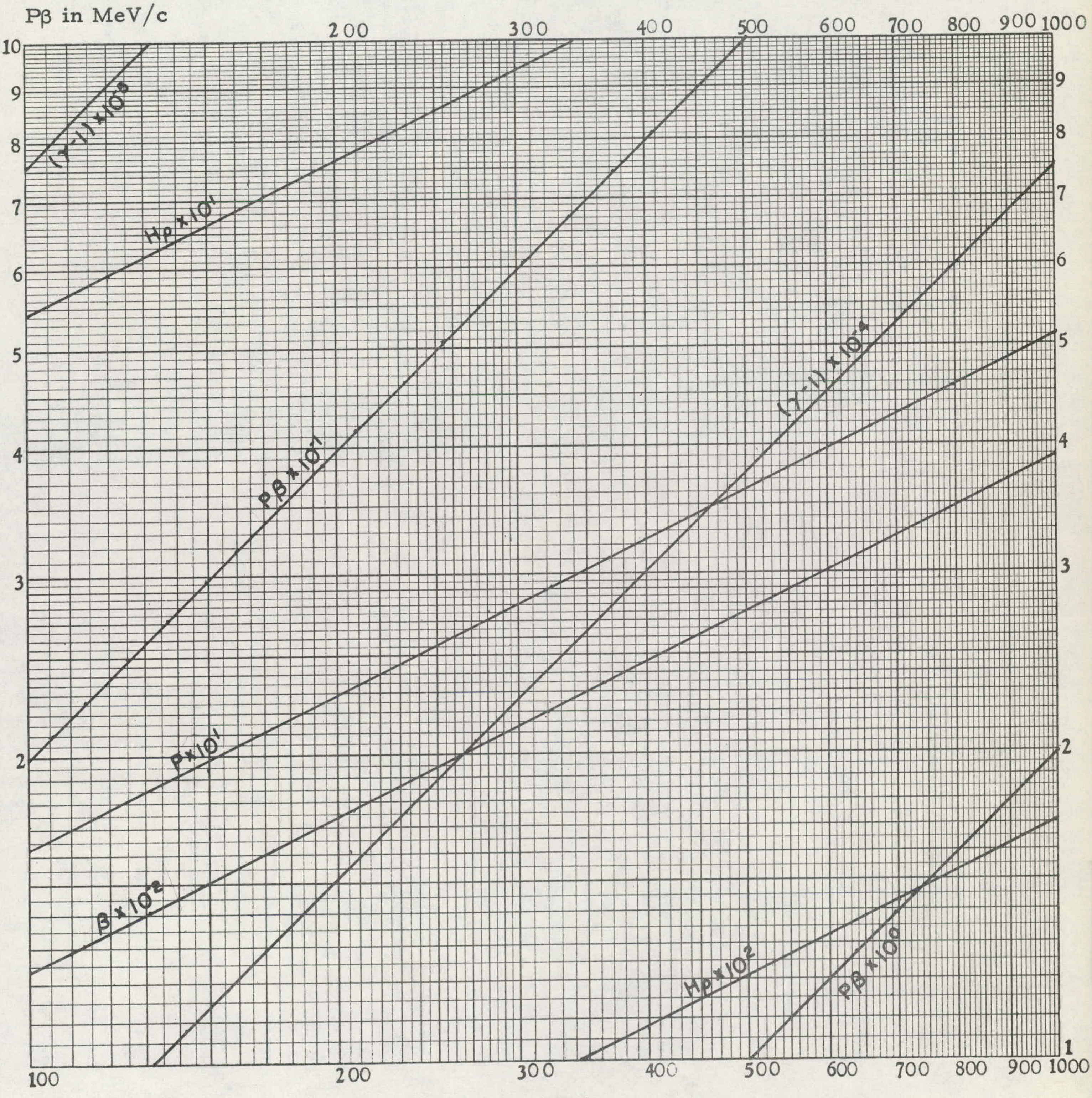
Ξ^- HYPERONS

100 keV to 1 MeV

β, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



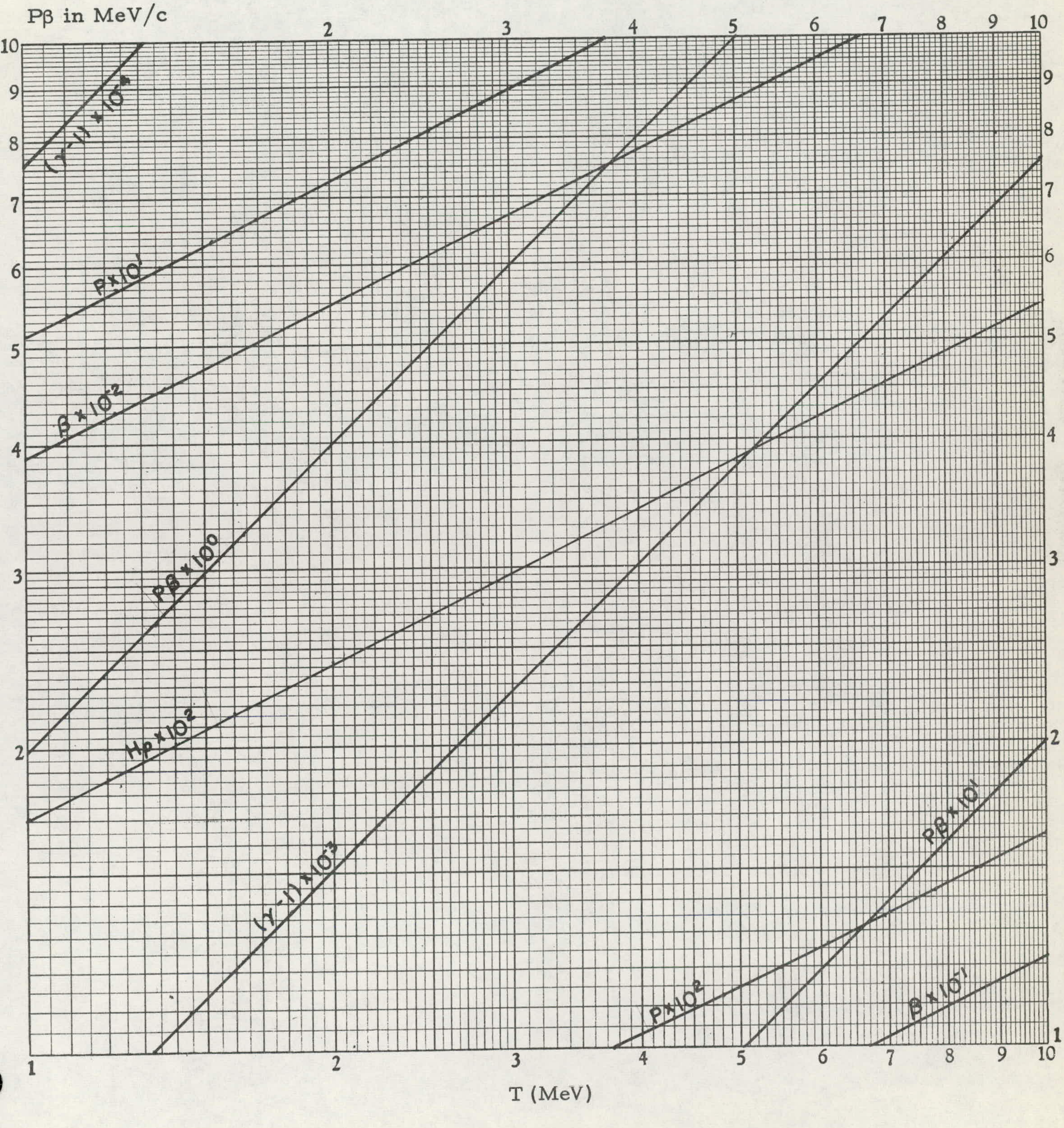
Ξ^- HYPERONS

1 MeV to 10 MeV

β, (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m

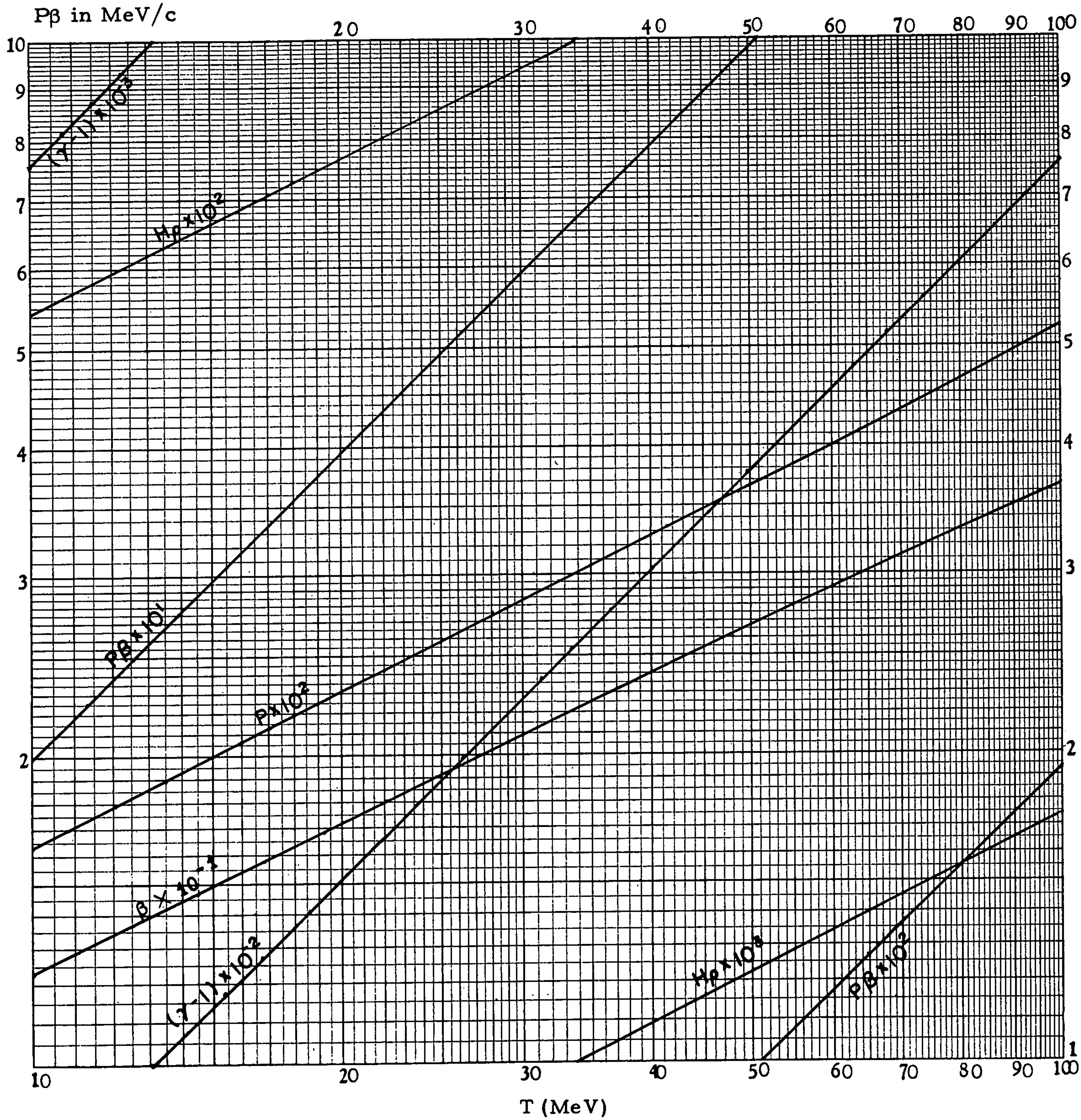


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV to 100 MeV

β, (γ-1), P, Pβ, Hρ

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



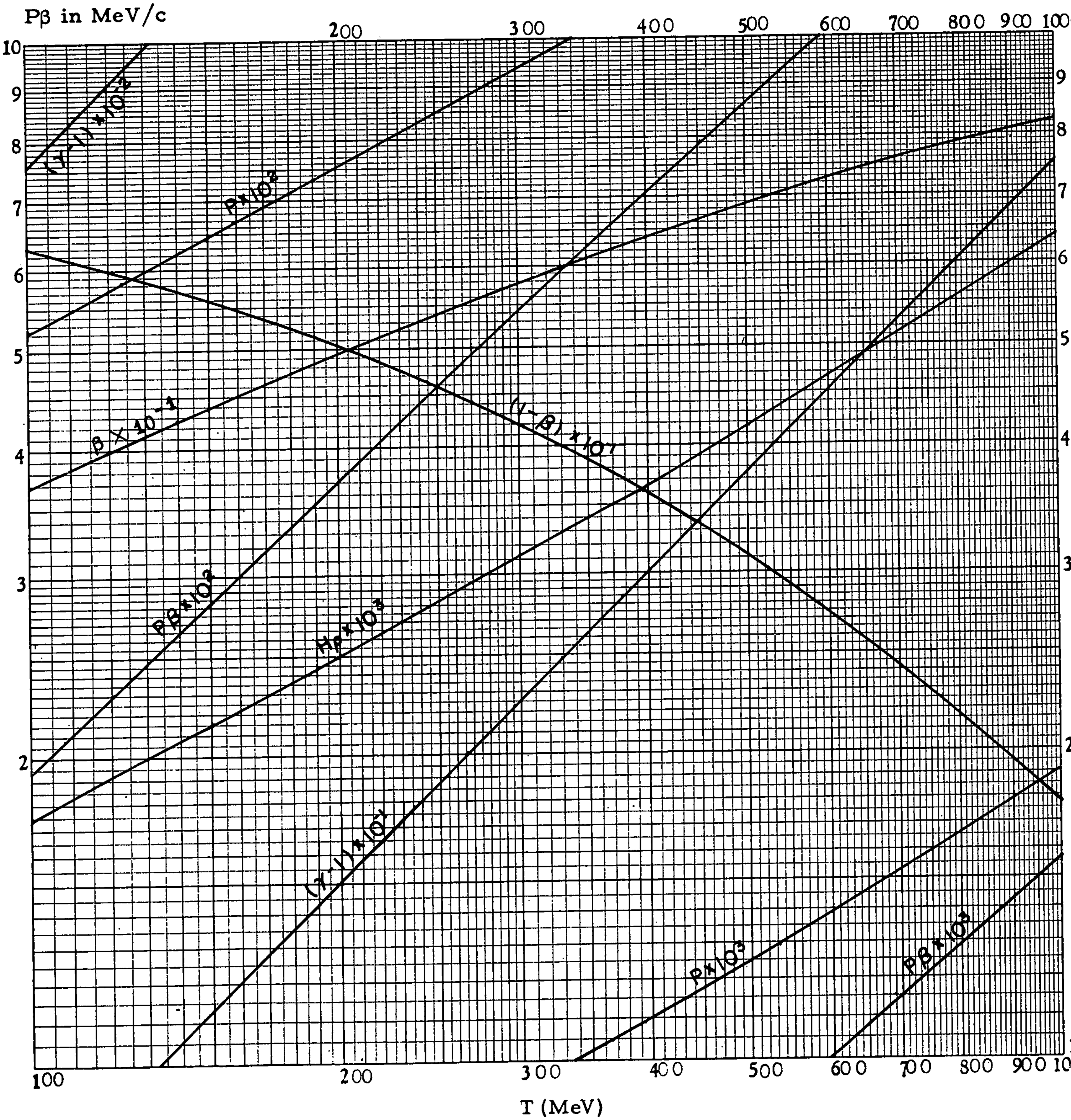
Ξ^- HYPERONS

100 MeV to 1 BeV

β, $(1-\beta)$, $(\gamma-1)$, P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



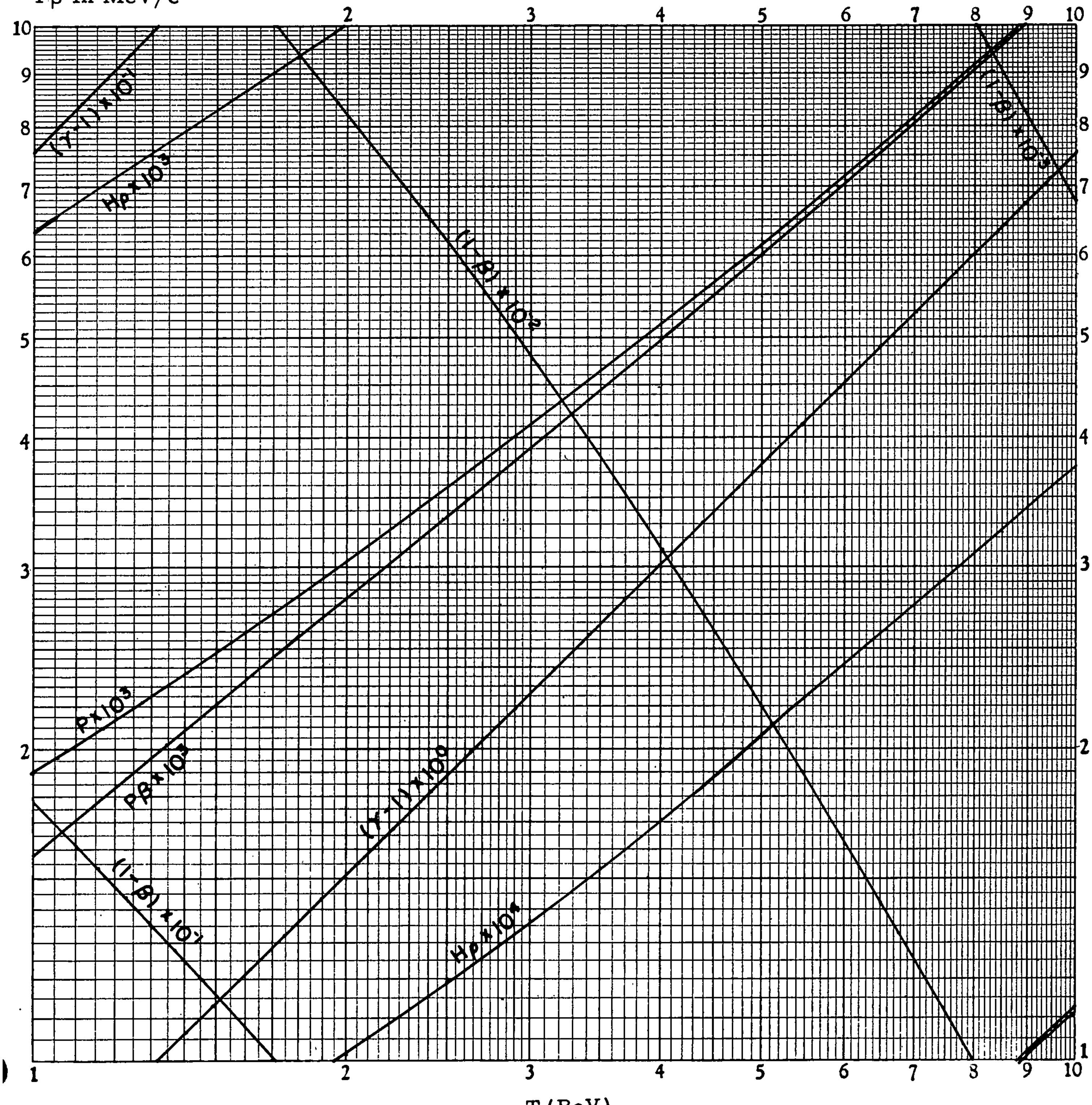
Ξ^- HYPERONS

1 BeV to 10 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



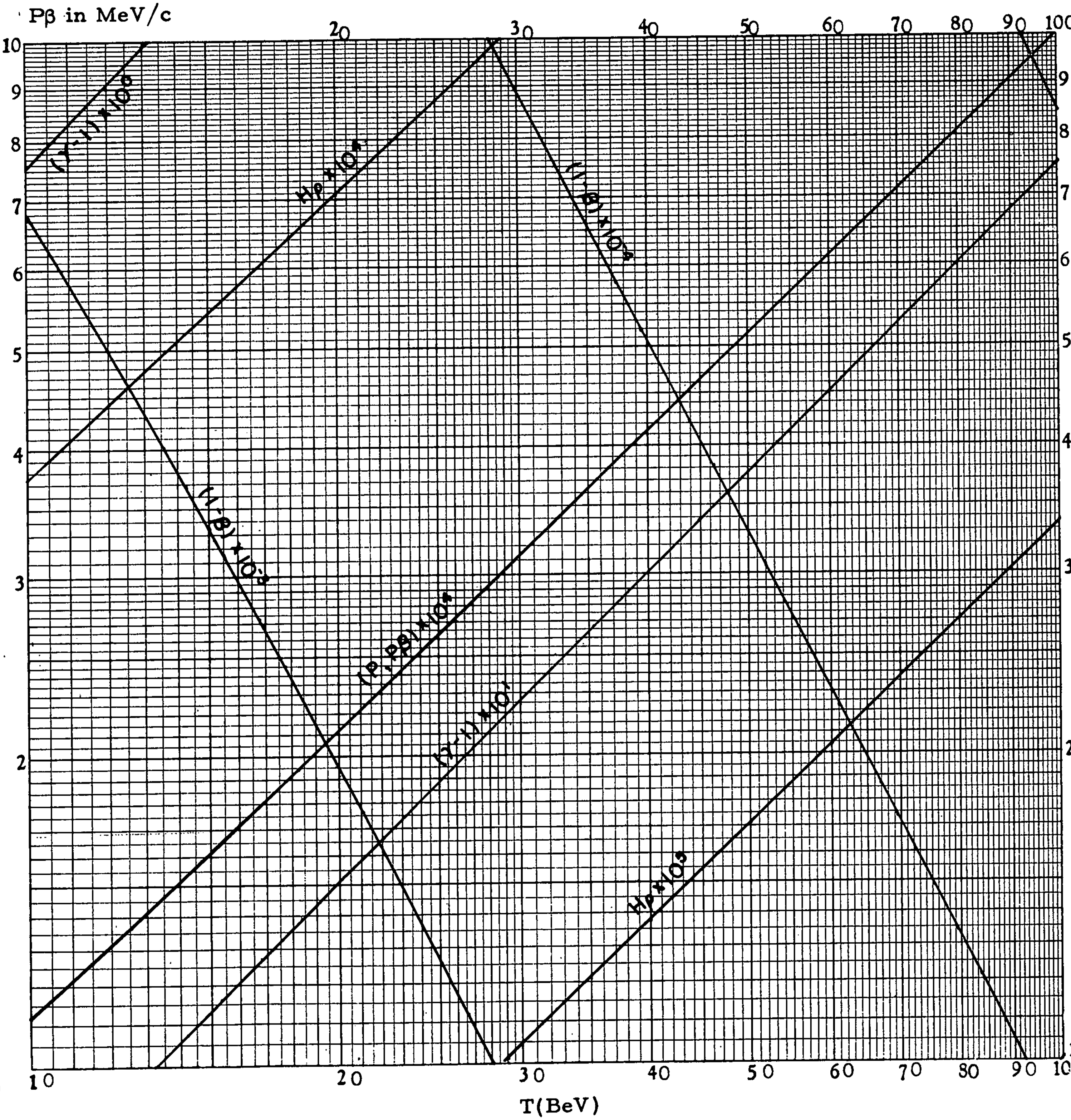
Ξ^- HYPERONS

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m

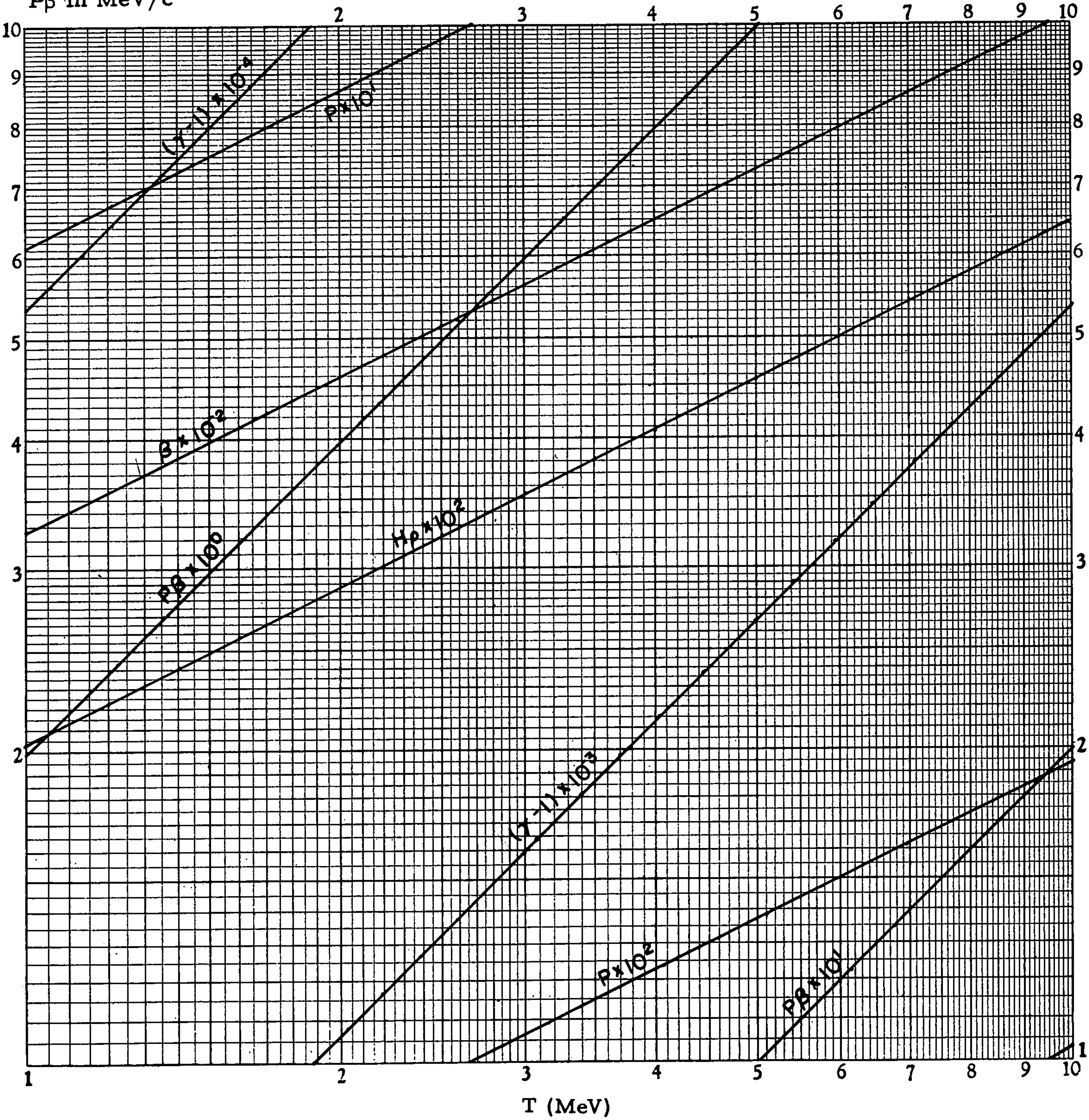


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

1 MeV to 10 MeV

β, (γ-1), P, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m

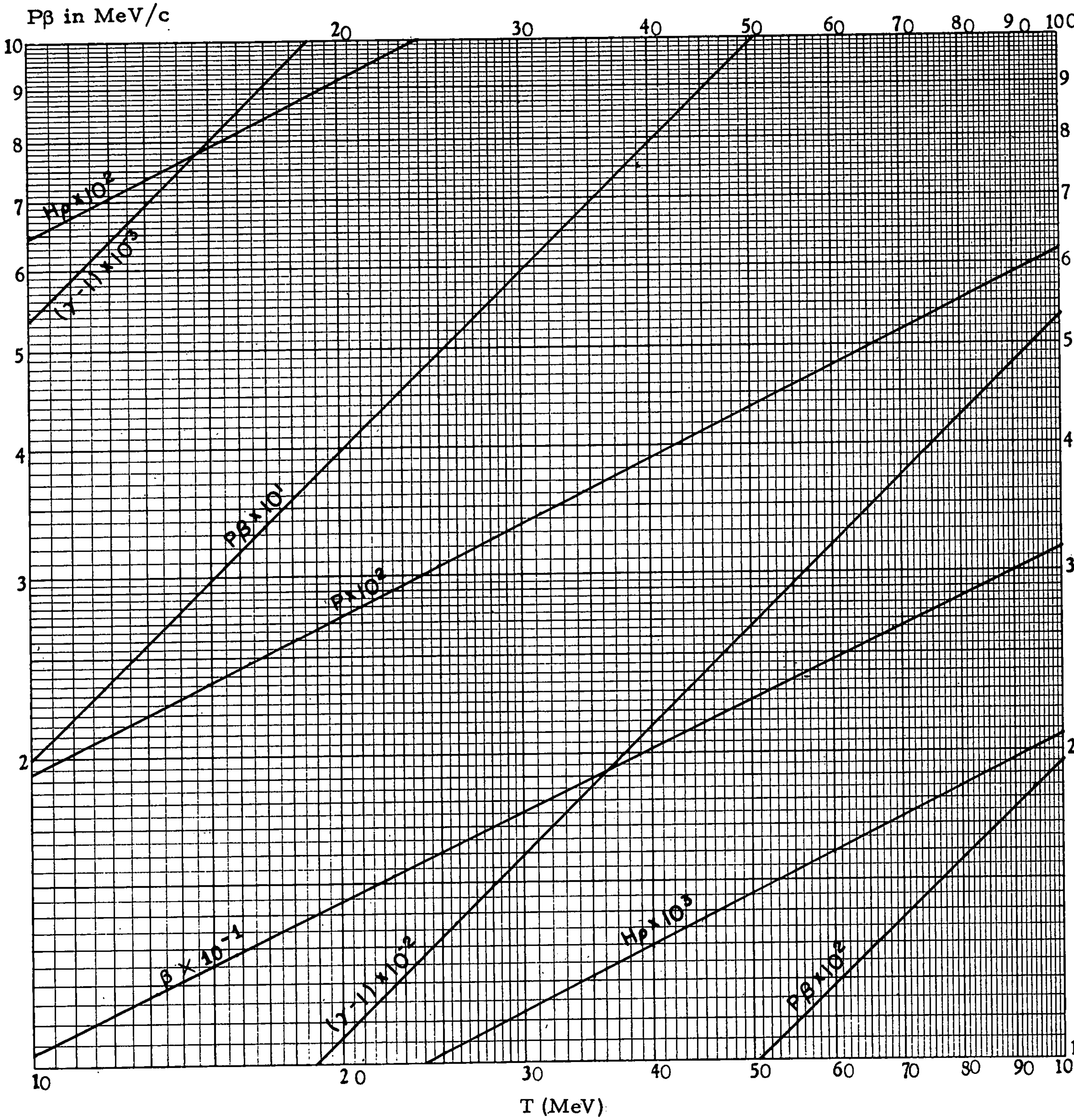


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV to 100 MeV

β, (γ-1), P, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m



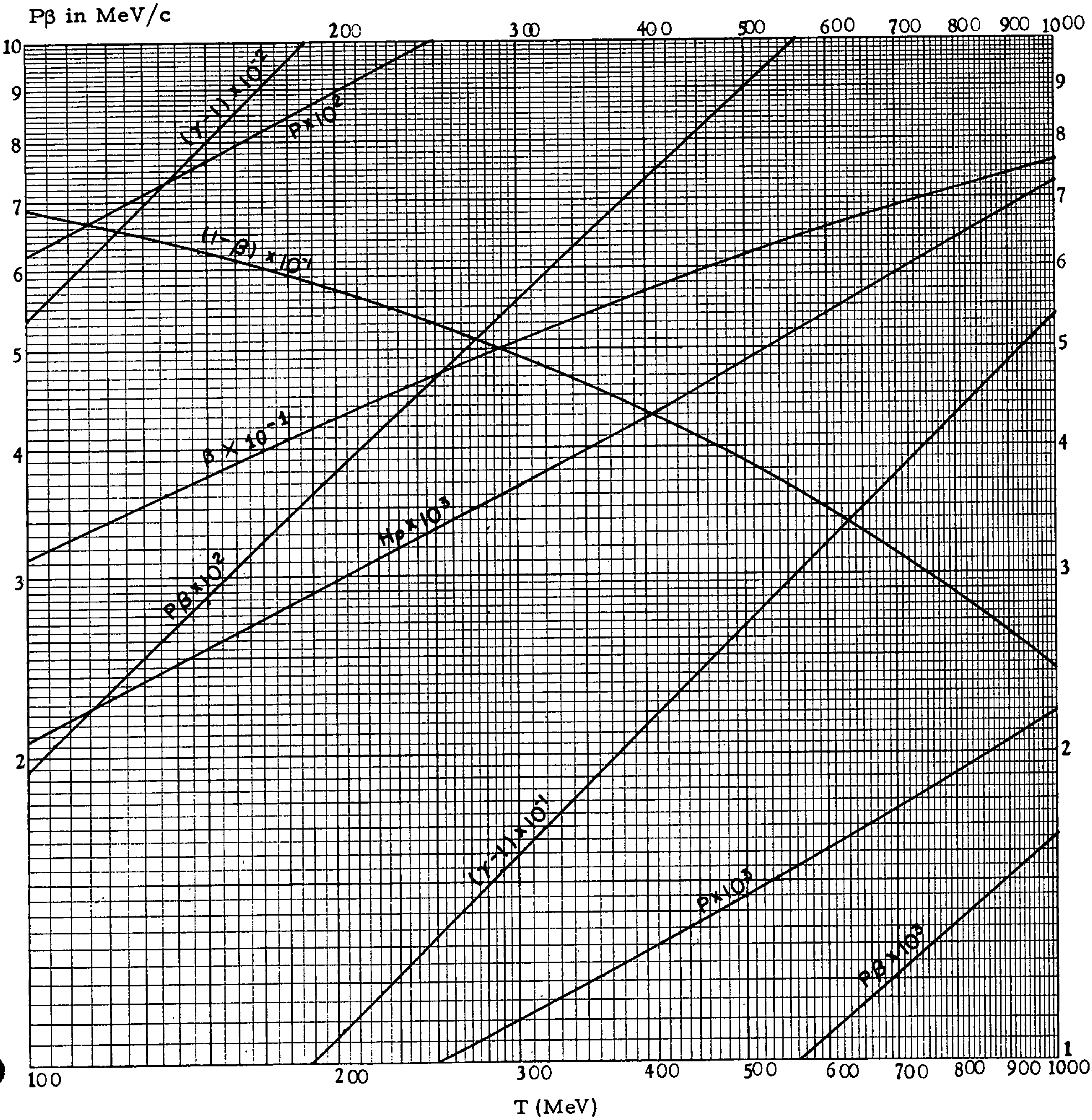
DEUTERONS

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

100 MeV to 1 BeV

β, (1-β), (γ-1), P, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m



1 BeV to 10 BeV

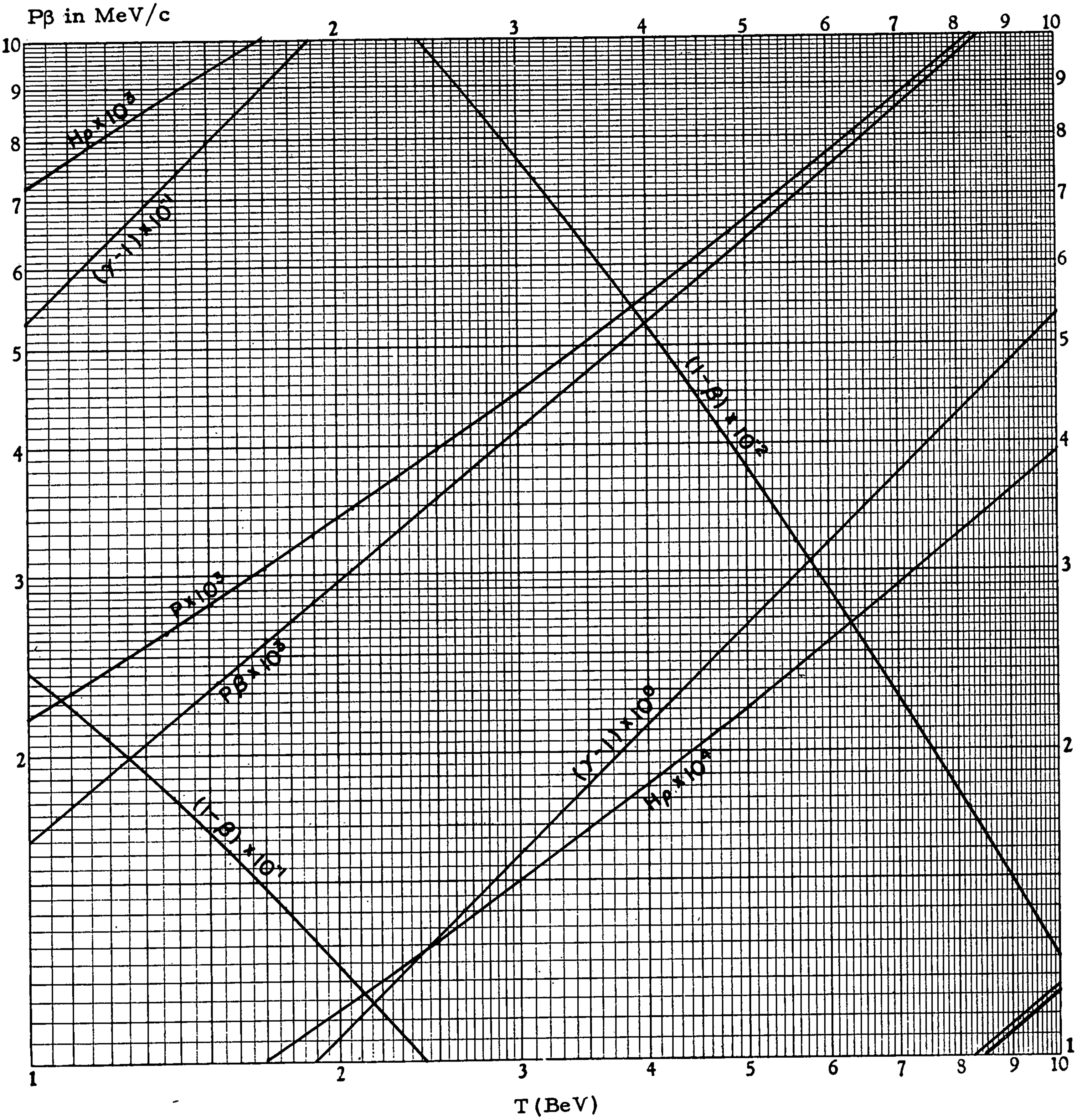
(1-β), (γ-1), P, Pβ, Hρ

P in MeV/c

Hρ in kgauss-cm

Pβ in MeV/c

M_d = 1875.49 MeV
= 3670.40 m

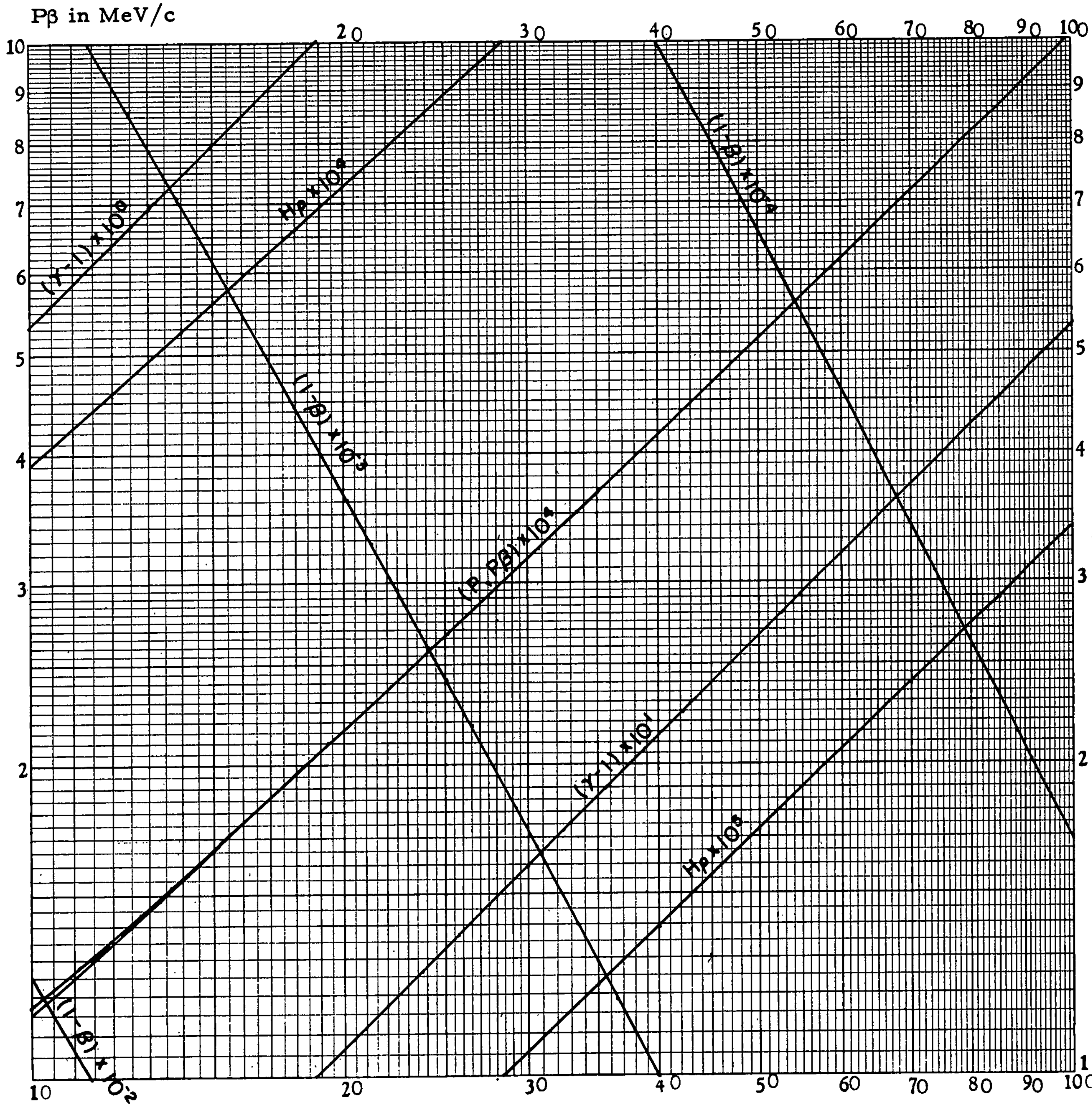


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m



P in MeV/c

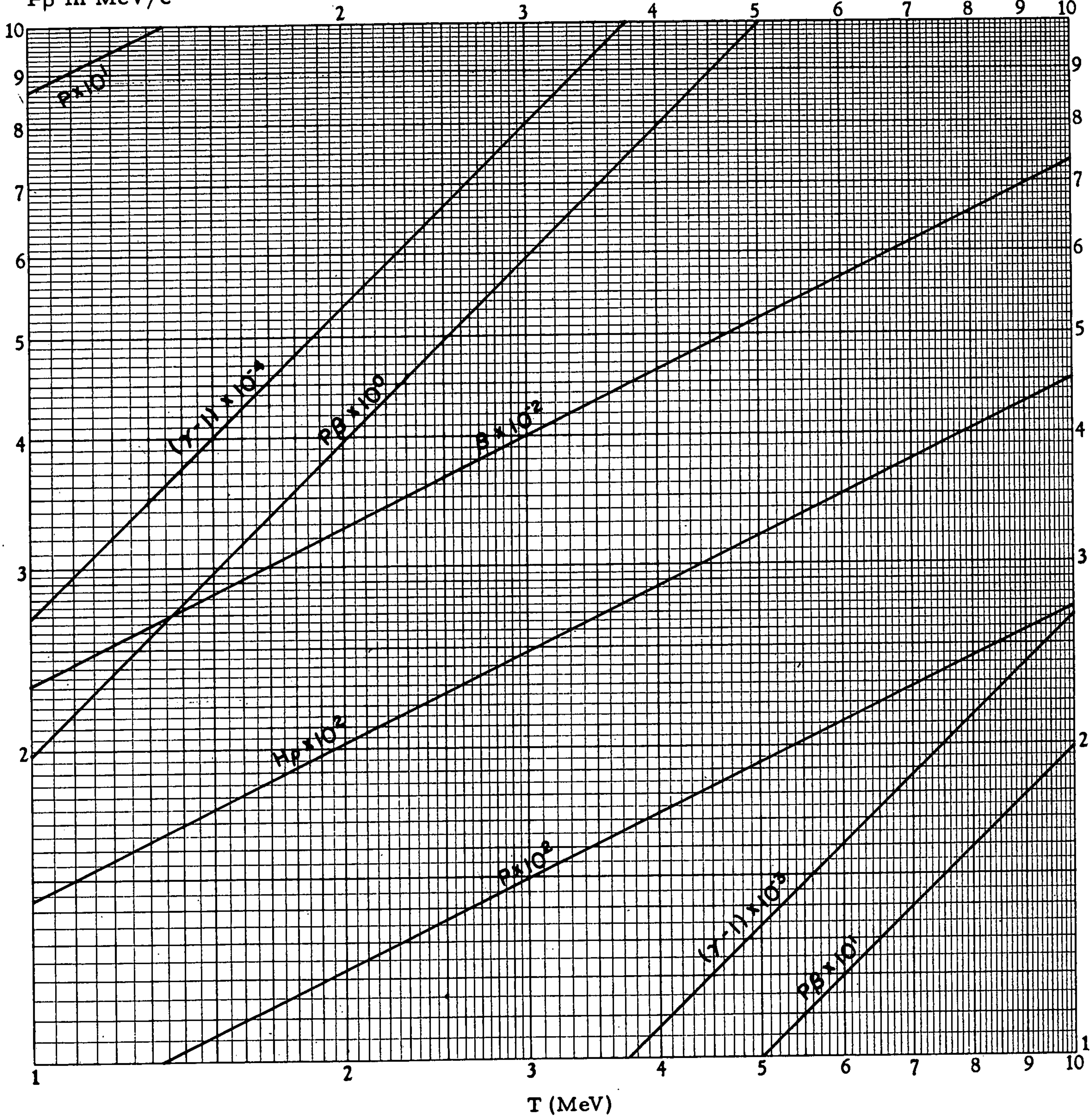
Hρ in kgauss-cm

Pβ in MeV/c

1 MeV to 10 MeV

β, (γ-1), P, Pβ, Hρ

M_α = 3727.23 MeV
= 7294.47 m



P in MeV/c

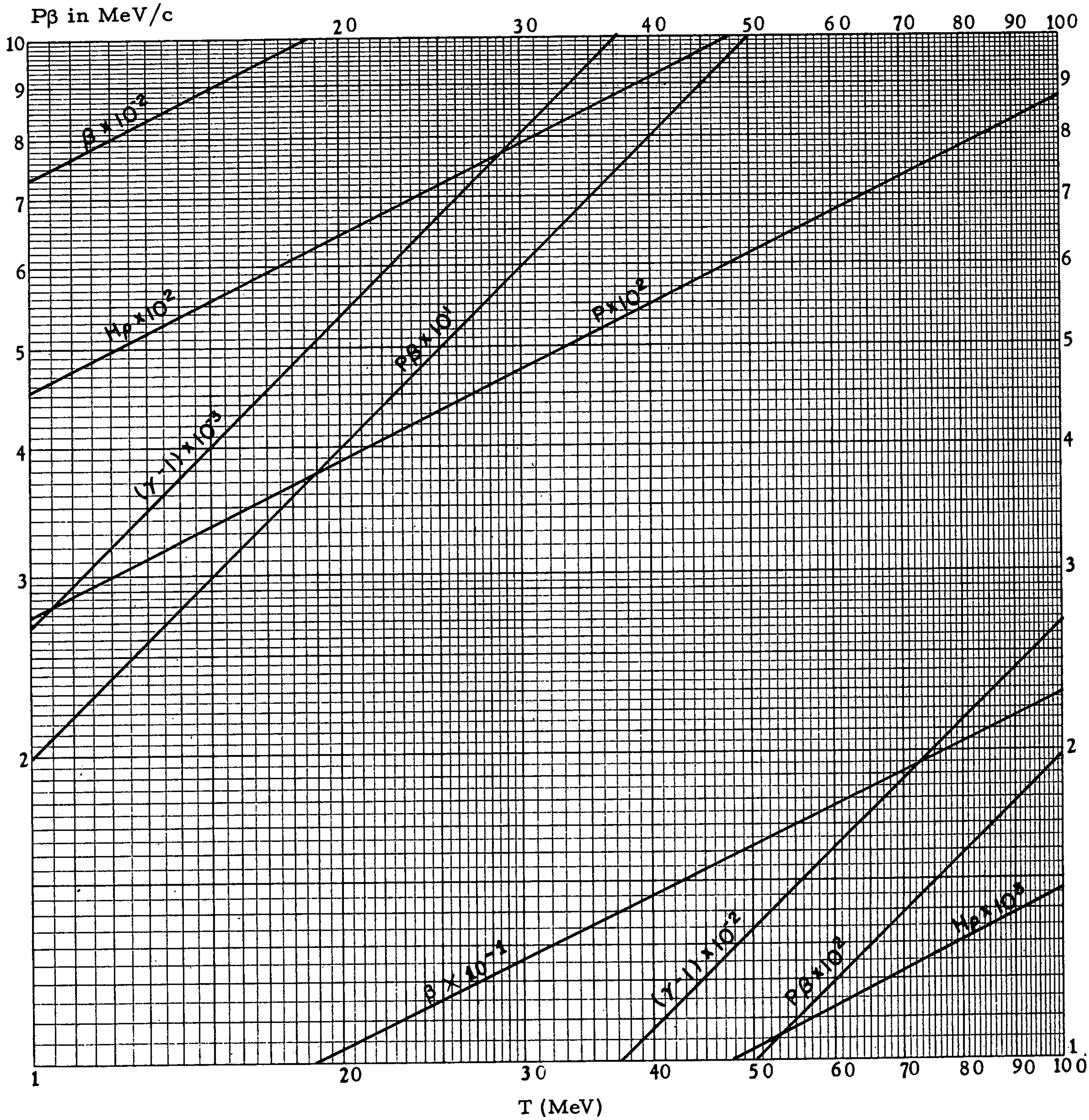
Hρ in kgauss-cm

Pβ in MeV/c

10 MeV to 100 MeV

β, (γ-1), P, Pβ, Hρ

M_a = 3727.23 MeV
= 7294.47 m



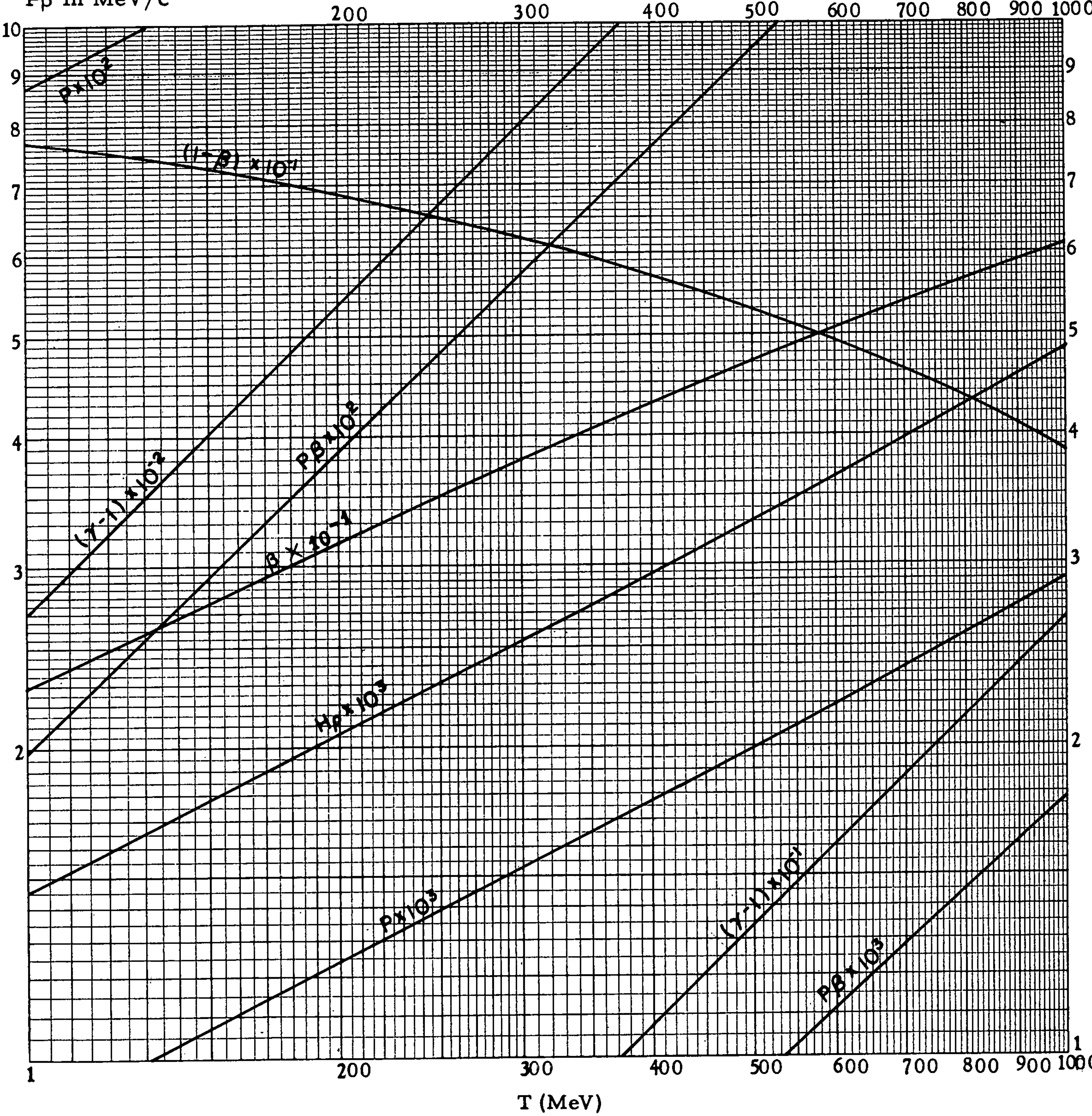
ALPHA PARTICLES

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

100 MeV to 1 BeV

β, (1-β), (γ-1), P, Pβ, Hρ

M_a = 3727.23 MeV
= 7294.47 m



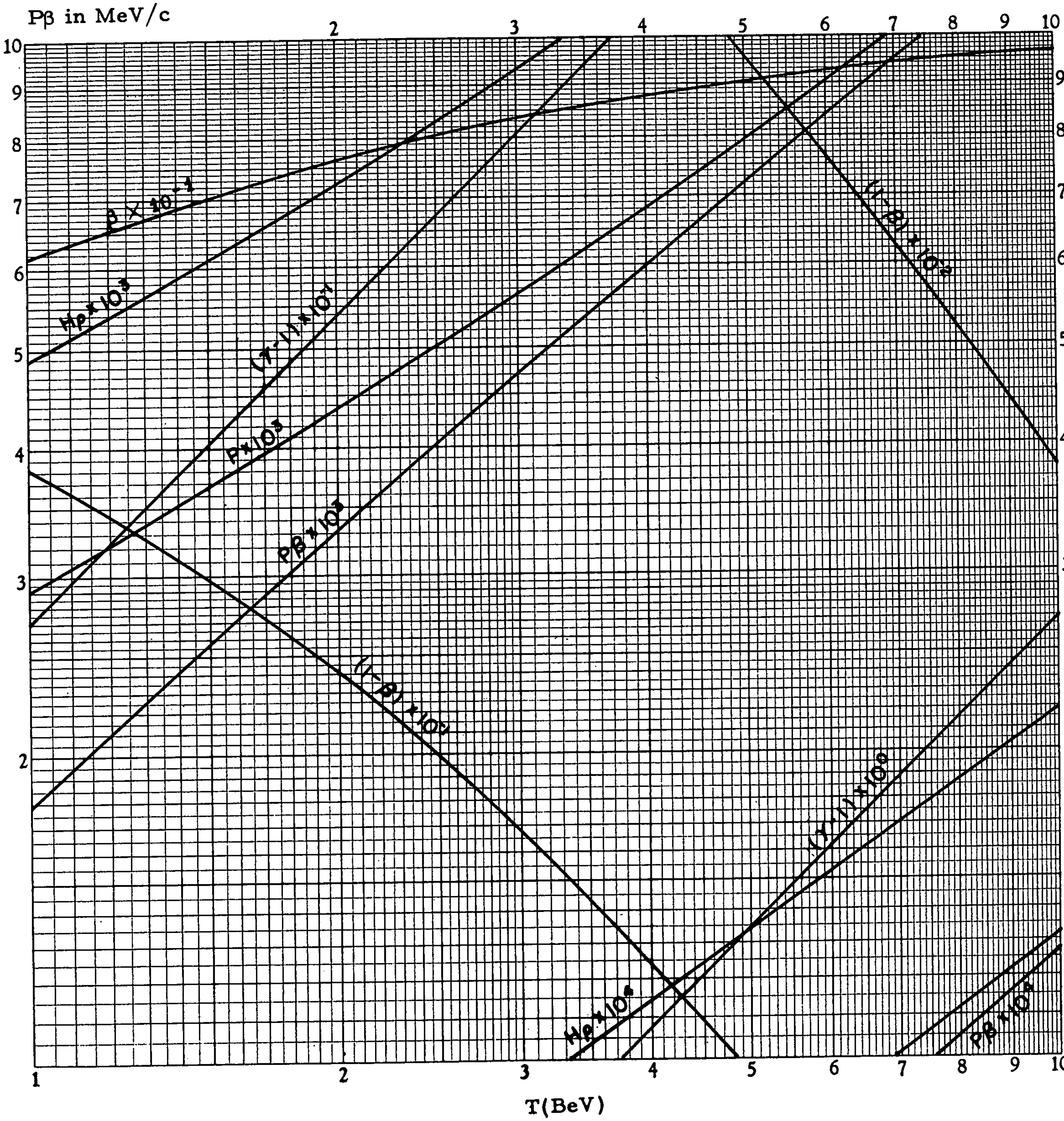
ALPHA PARTICLES

P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

1 BeV to 10 BeV

β, (1-β), (γ-1), P, Pβ, Hρ

M_{α} = 3727.23 MeV
= 7294.47 m

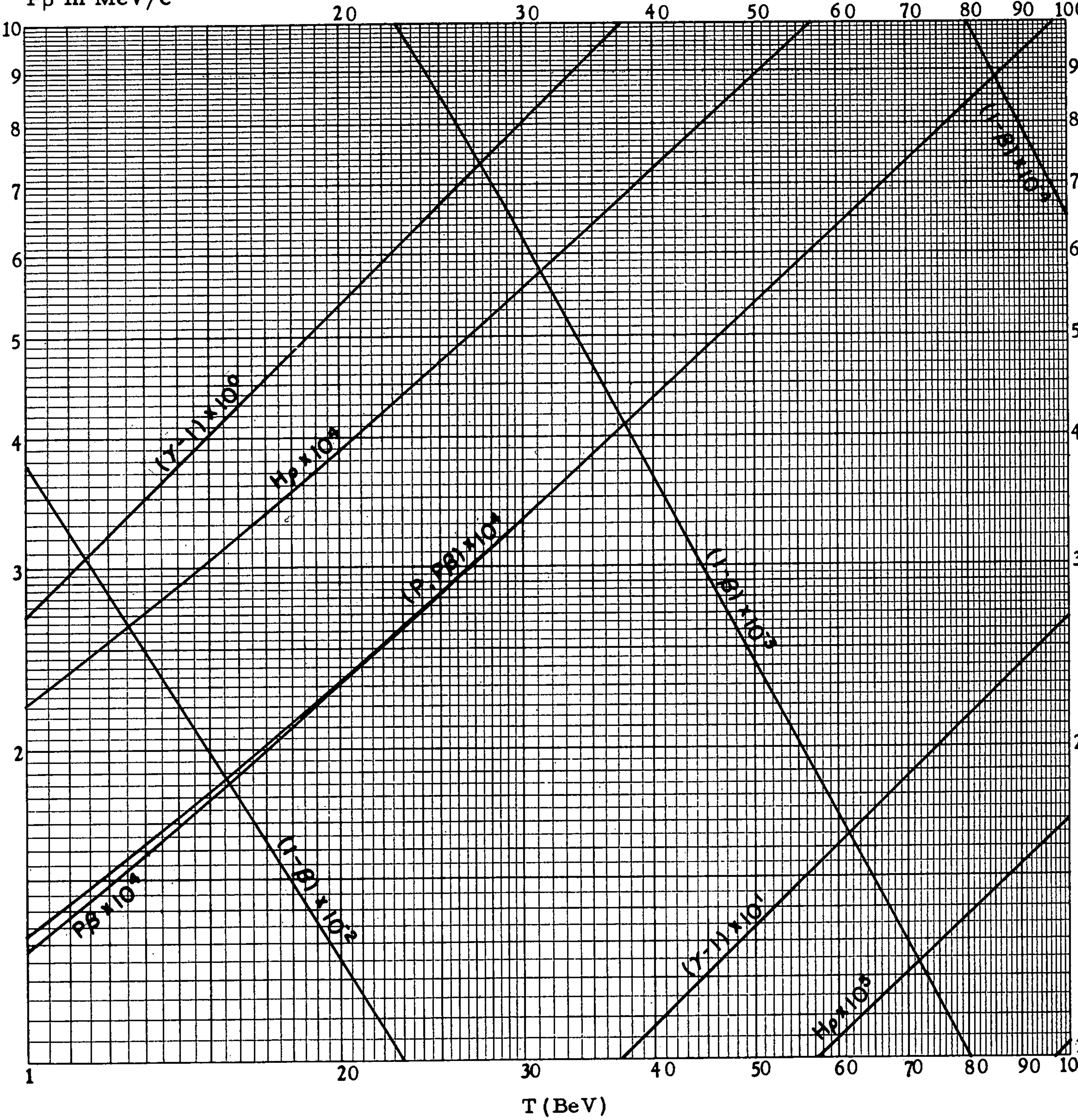


P in MeV/c
Hρ in kgauss-cm
Pβ in MeV/c

10 BeV to 100 BeV

(1-β), (γ-1), P, Pβ, Hρ

M_a = 3727.23 MeV
= 7294.47 m



UCRL-2426
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DYNAMICS OF COLLISIONS WITH A PROTON TARGET

w and ηM_p in MeV

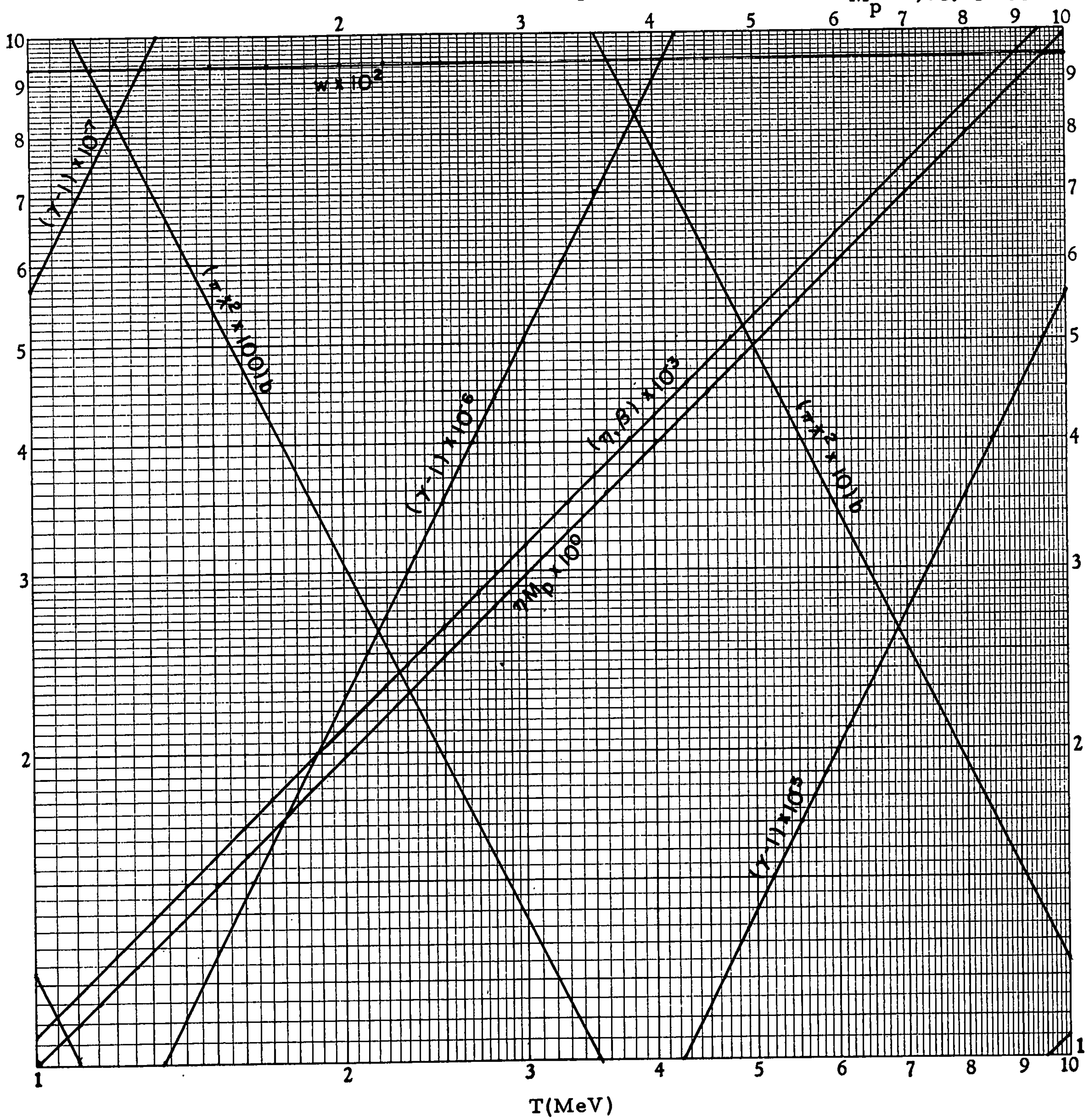
1b = 10^{-24} cm^2

1 MeV to 10 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_γ = 0 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

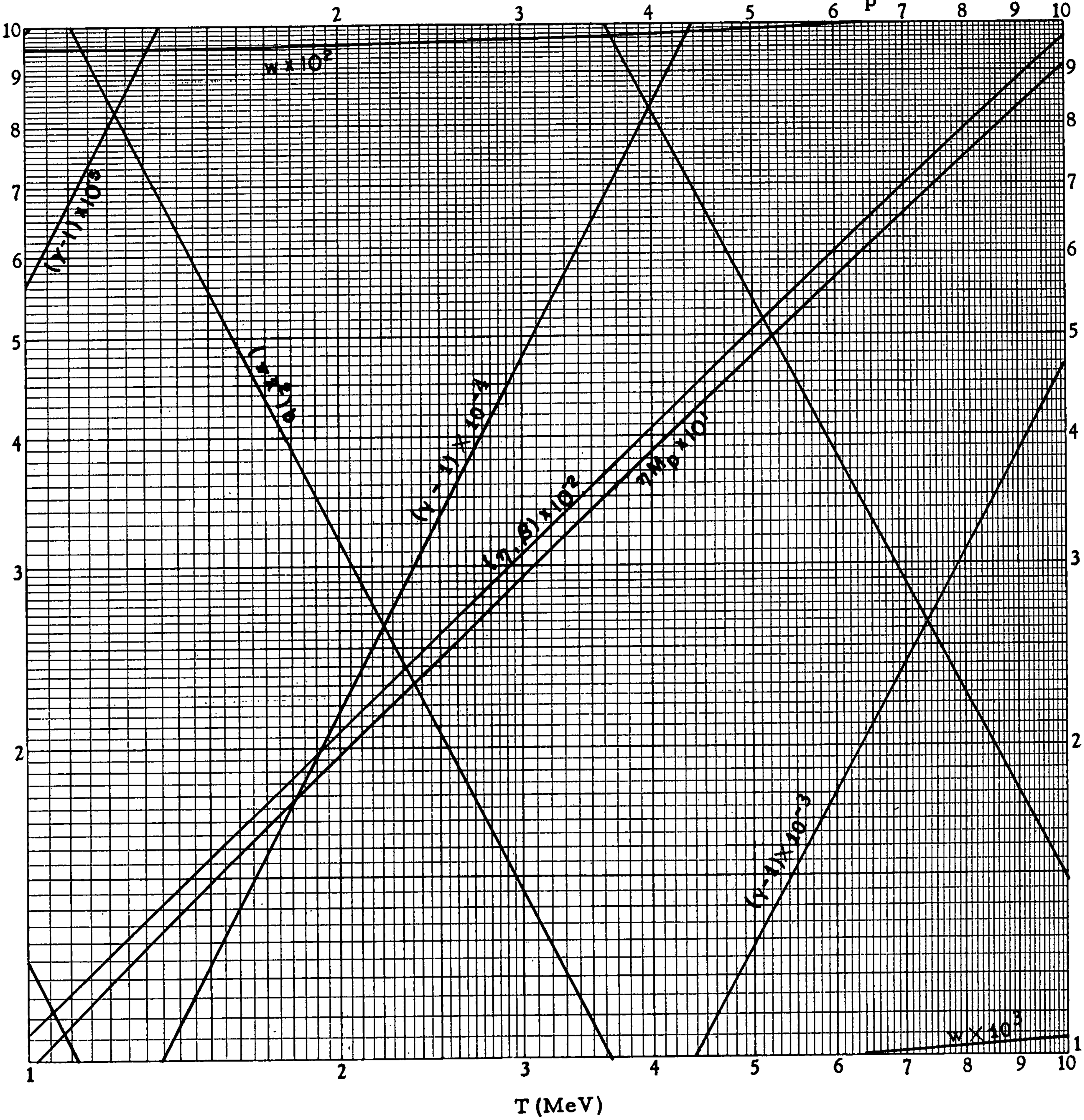
1b = 10^{-24} cm^2

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda^2$

M_γ = 0 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

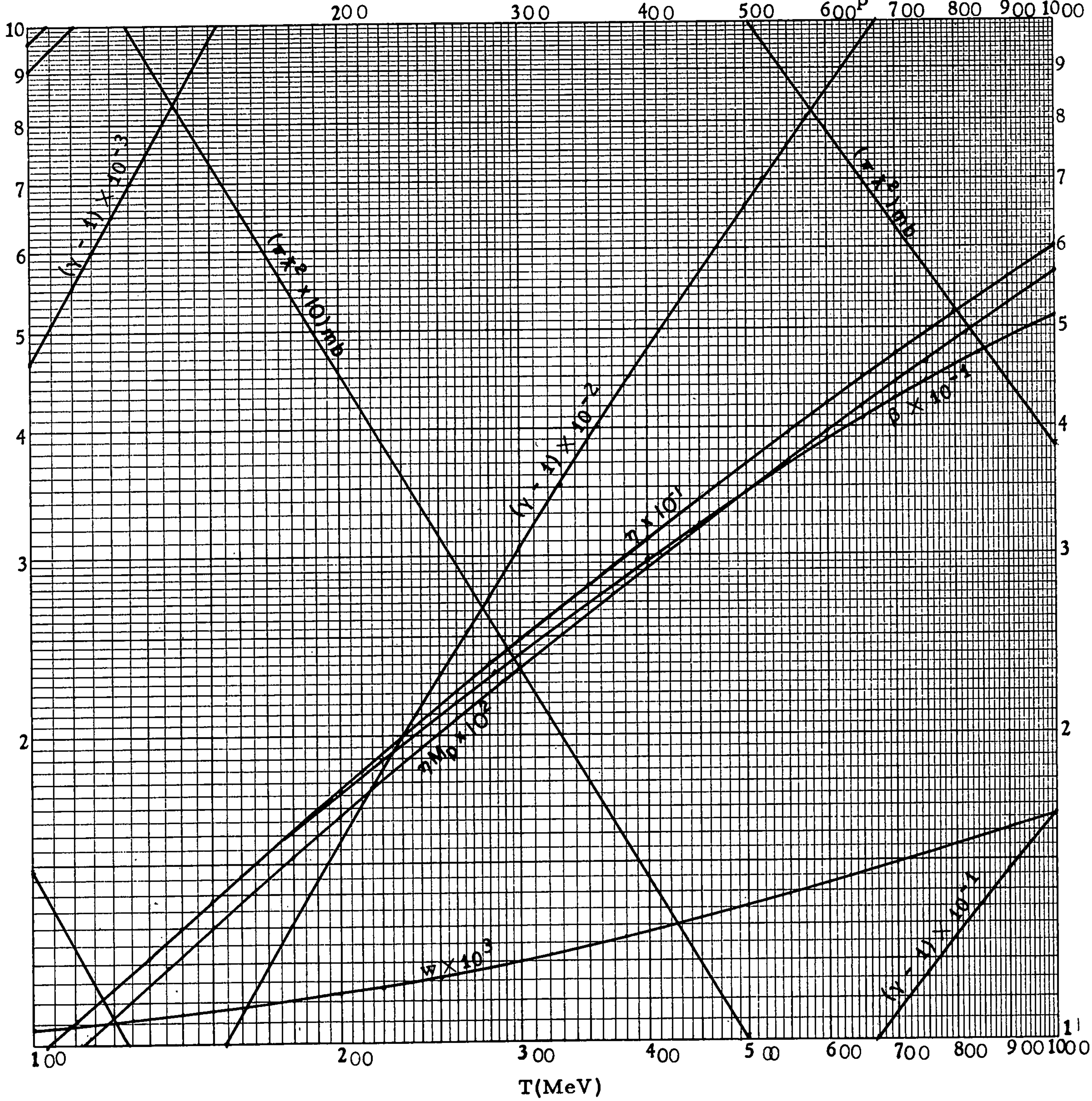
1b = 10^{-24} cm^2

100 MeV to 1 BeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda^2$

M_γ = 0 MeV

M_p = 938.213 MeV



PHOTONS: $\gamma + p$

w and ηM_p in MeV

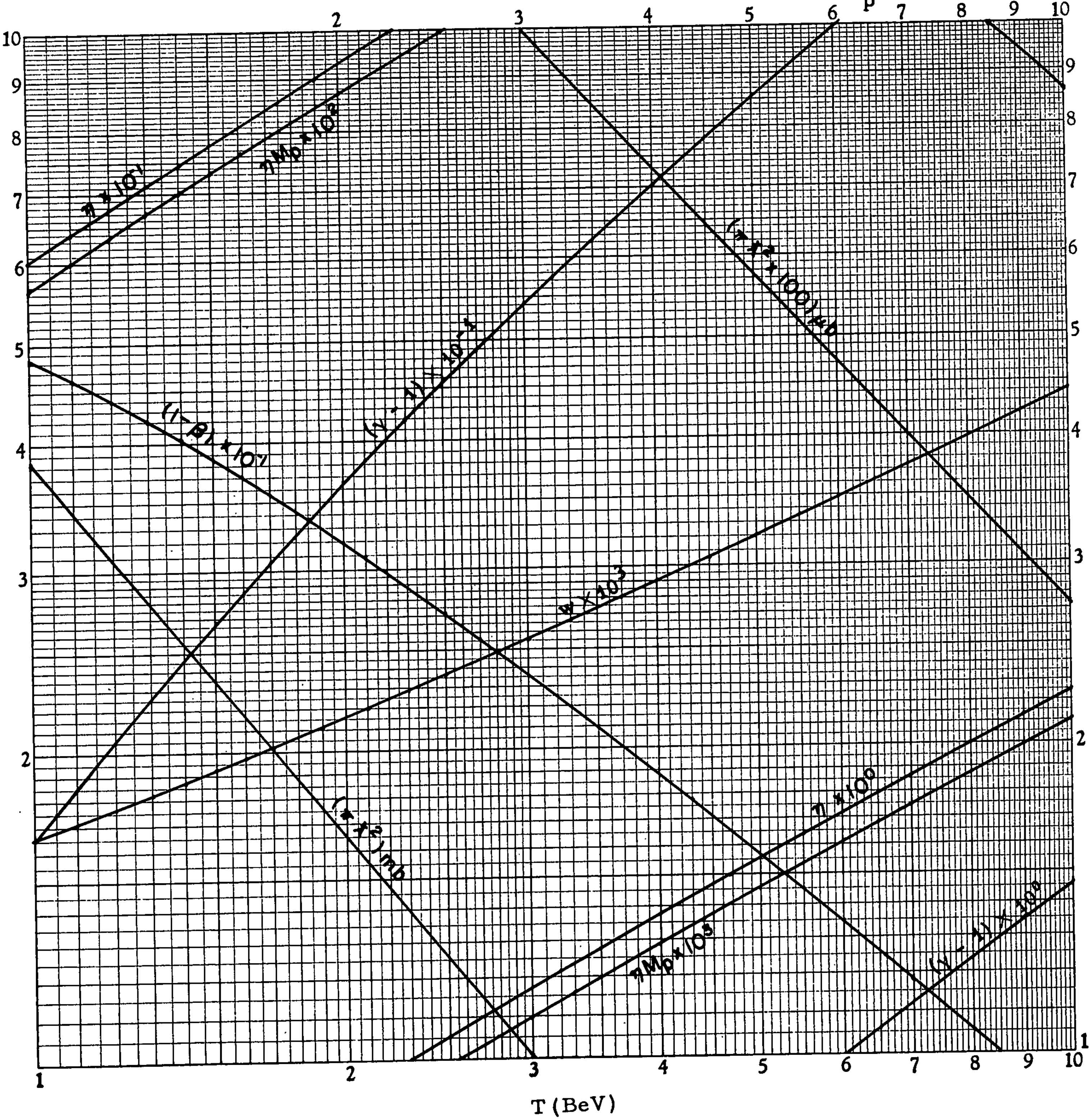
1 BeV to 10 BeV

$1 b = 10^{-24} cm^2$

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_\gamma = 0$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

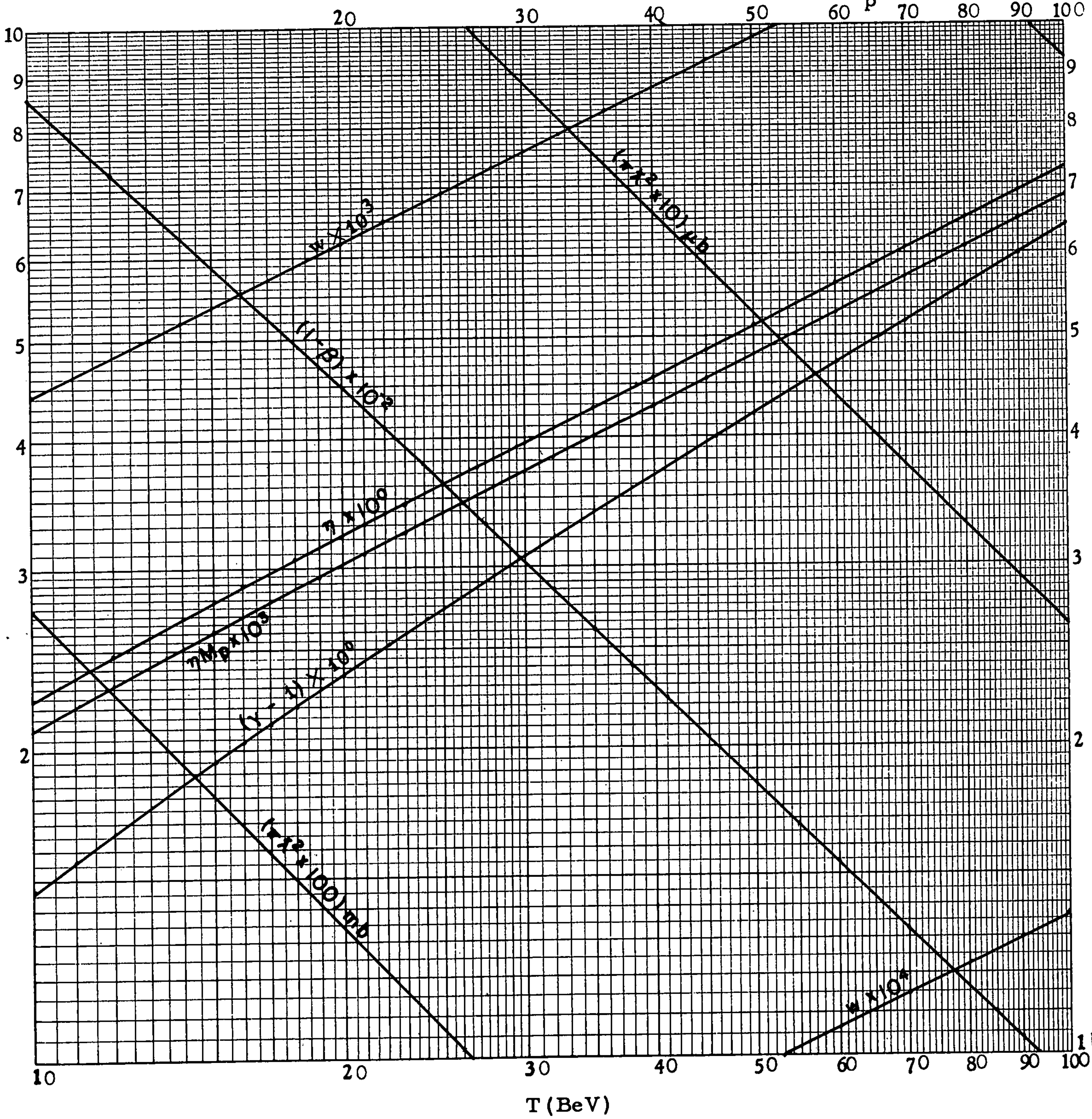
$1b = 10^{-24}\ cm^2$

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_\gamma = 0$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

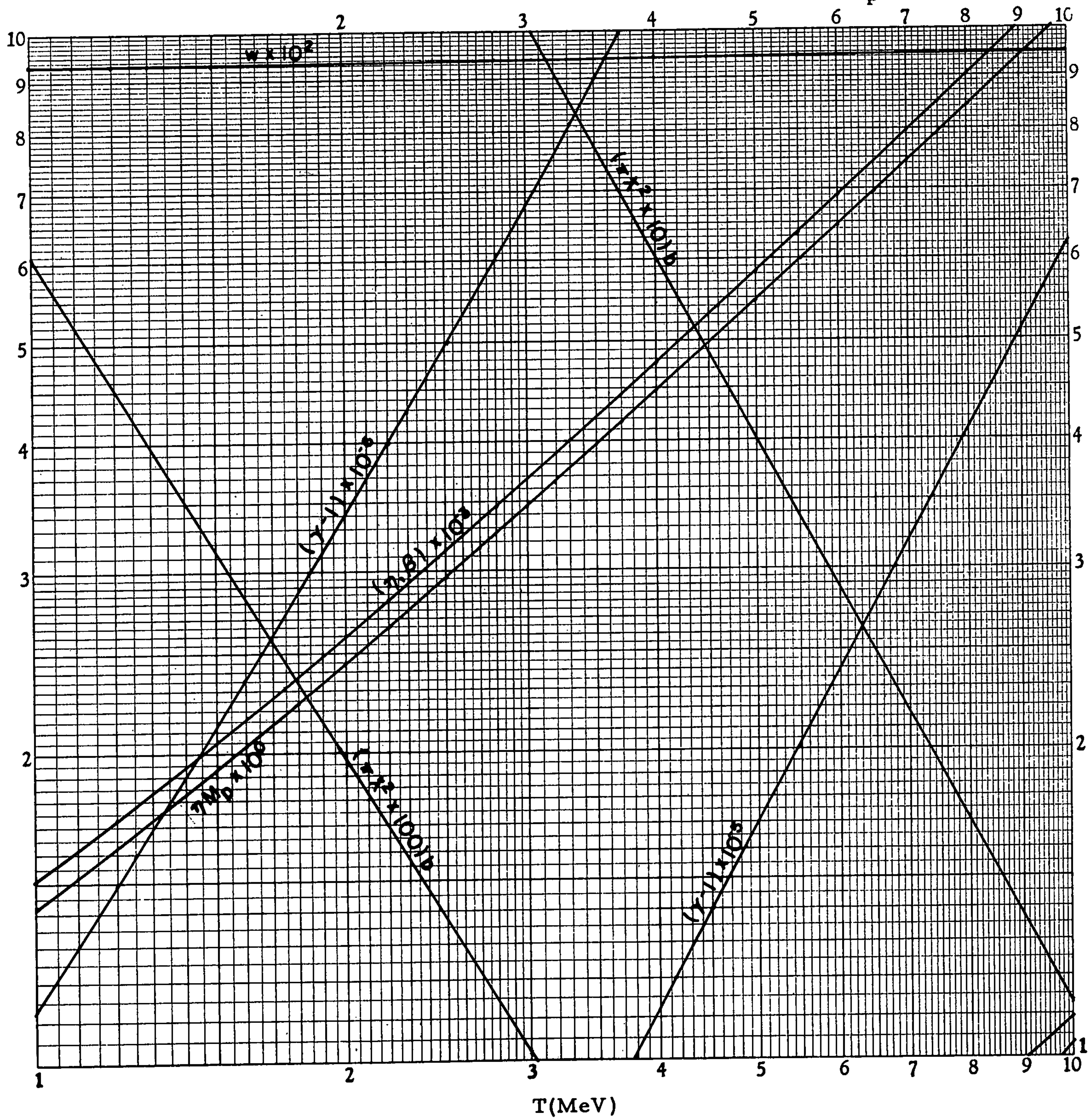
1 MeV to 10 MeV

$M_{e^{\pm}}$ = 0.510976 MeV

1b = 10^{-24} cm^2

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda\!\!\!^{-2}$

M_p = 938.213 MeV



w and ηM_p in MeV

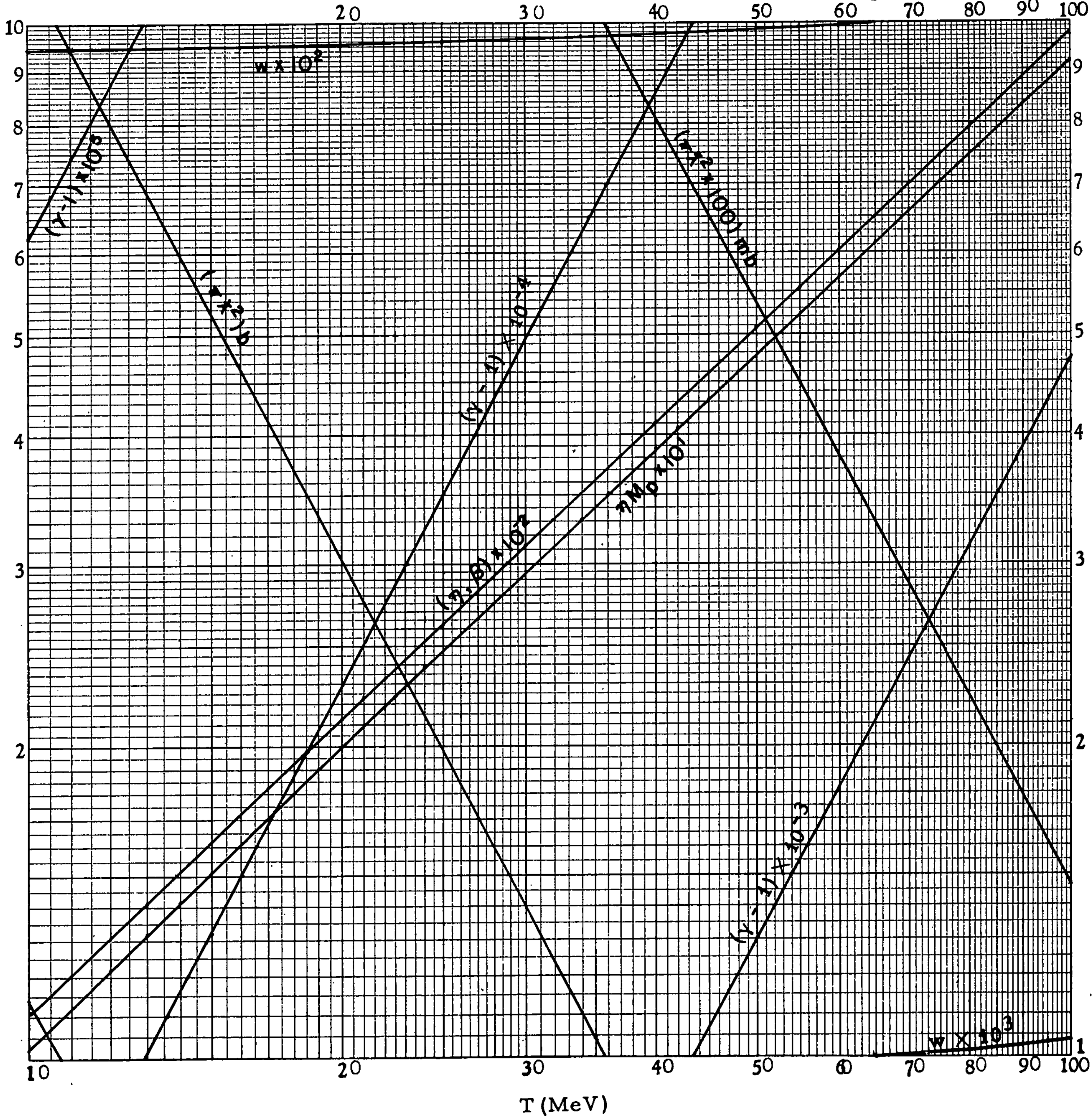
$1b = 10^{-24} cm^2$

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{e\pm} = 0.510976$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

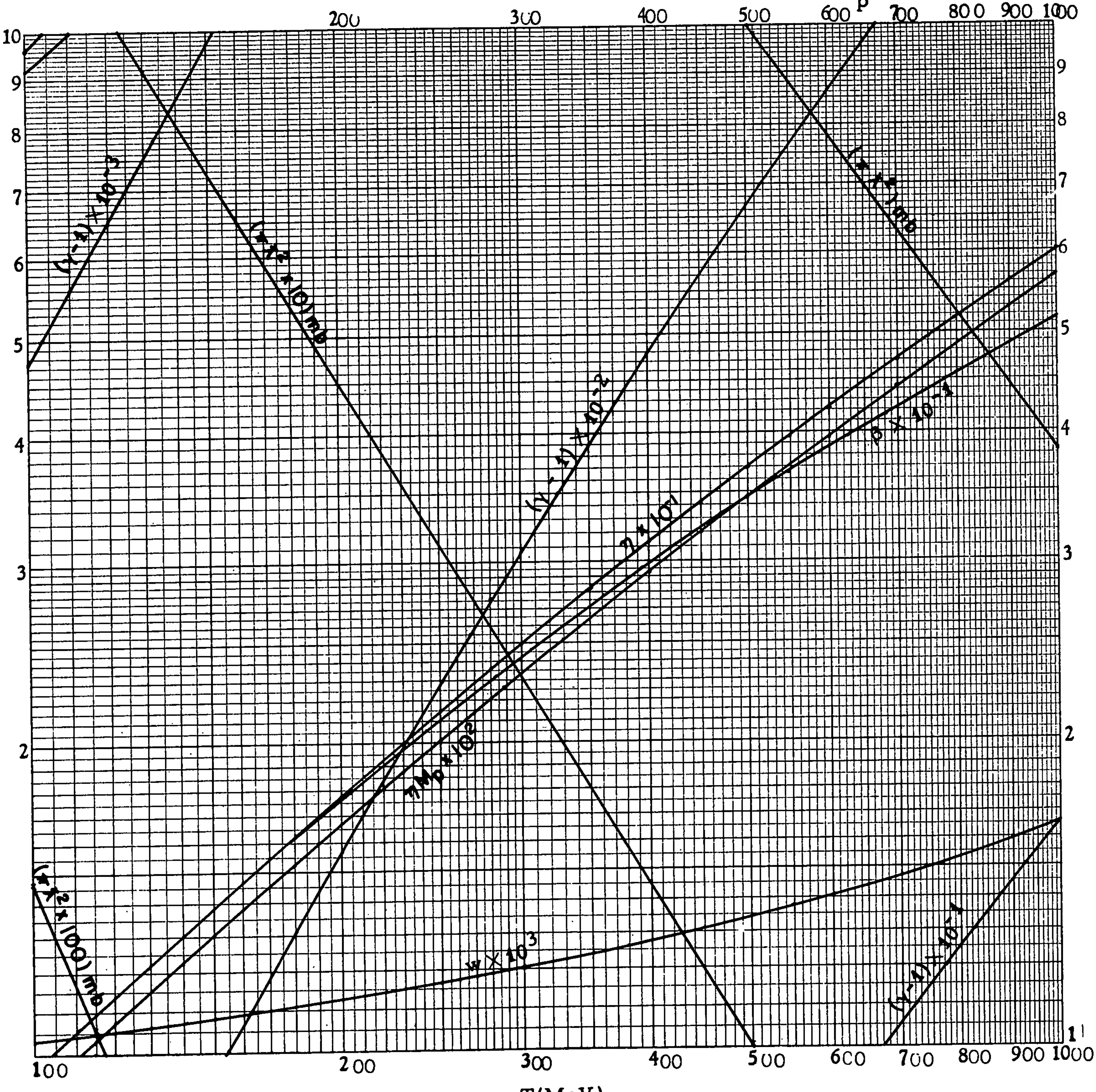
1b = 10^{-24} cm^2

100 MeV to 1 BeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda^2$

$M_{e^\pm}$ = 0.510976 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

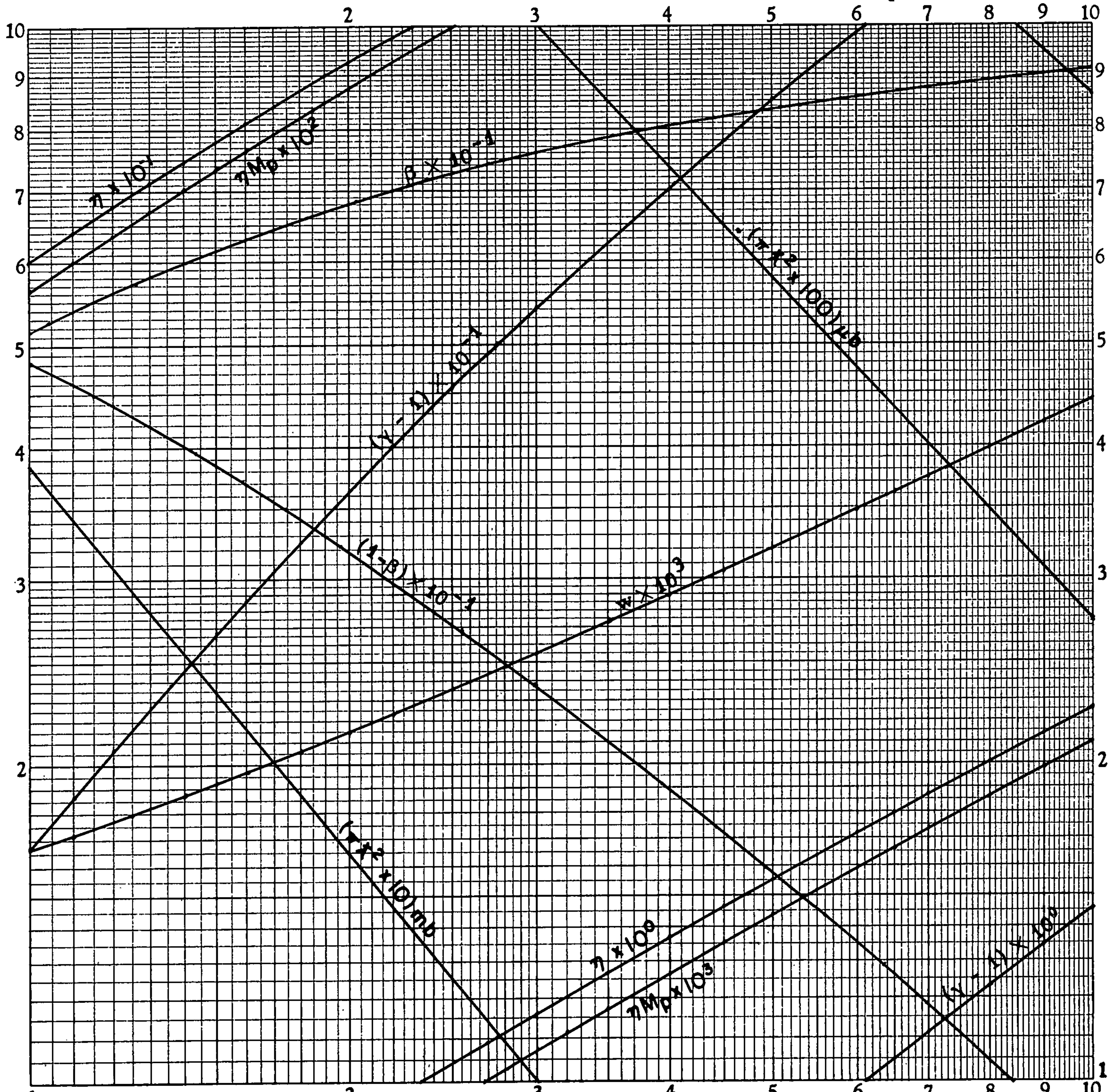
1 BeV to 10 BeV

$M_{e^{\pm}}$ = 0.510976 MeV

1b = 10^{-24} cm^2

$(1-\beta)$, β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_p = 938.213 MeV



T(BeV)

w and ηM_p in MeV

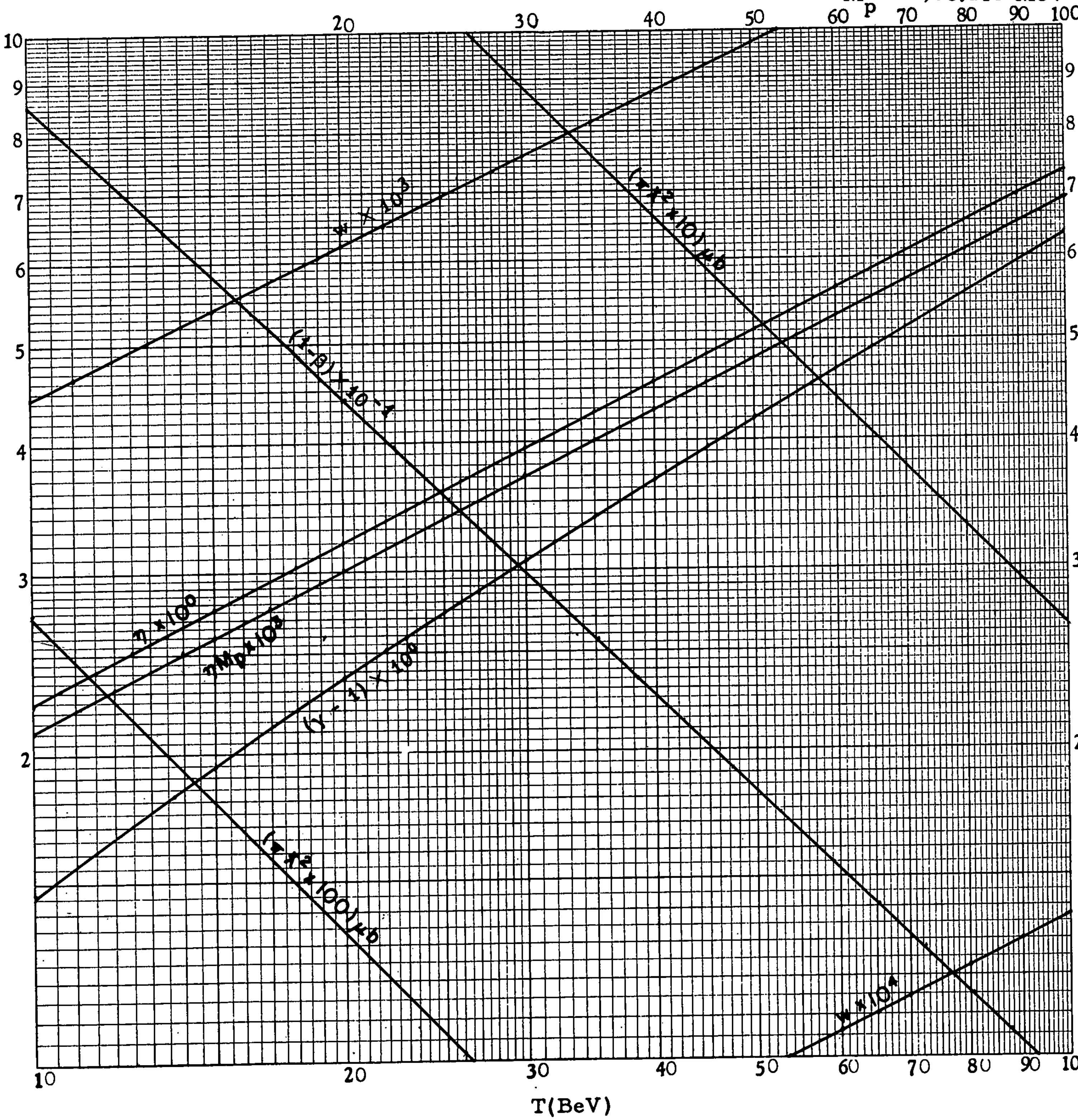
$1 b = 10^{-24}\ cm^2$

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda^2$

$M_{e\pm} = 0.510976$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

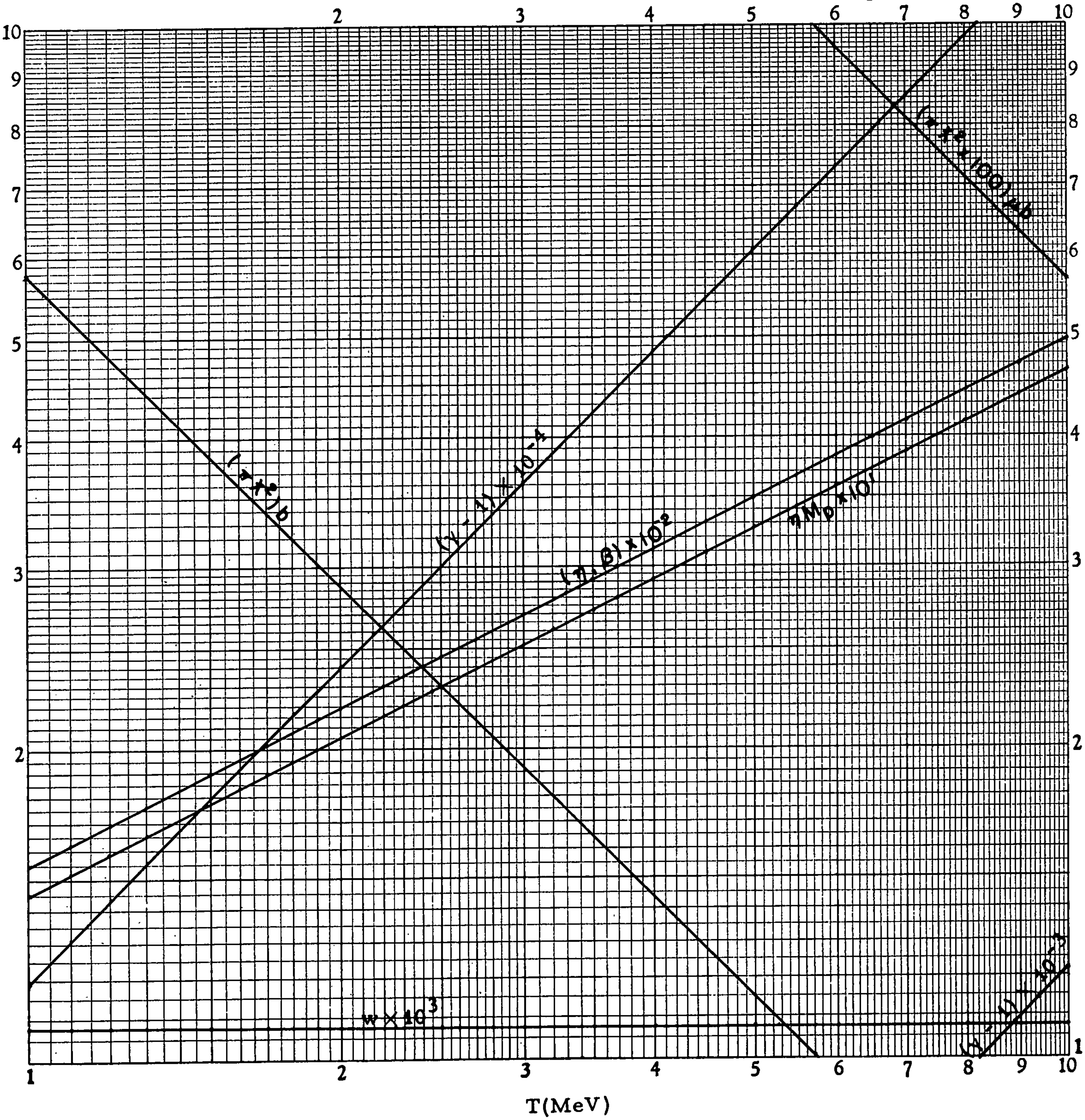
$1b = 10^{-24}\ cm^2$

1 MeV to 10 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{\pi^{\pm}}$ = 139.59 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

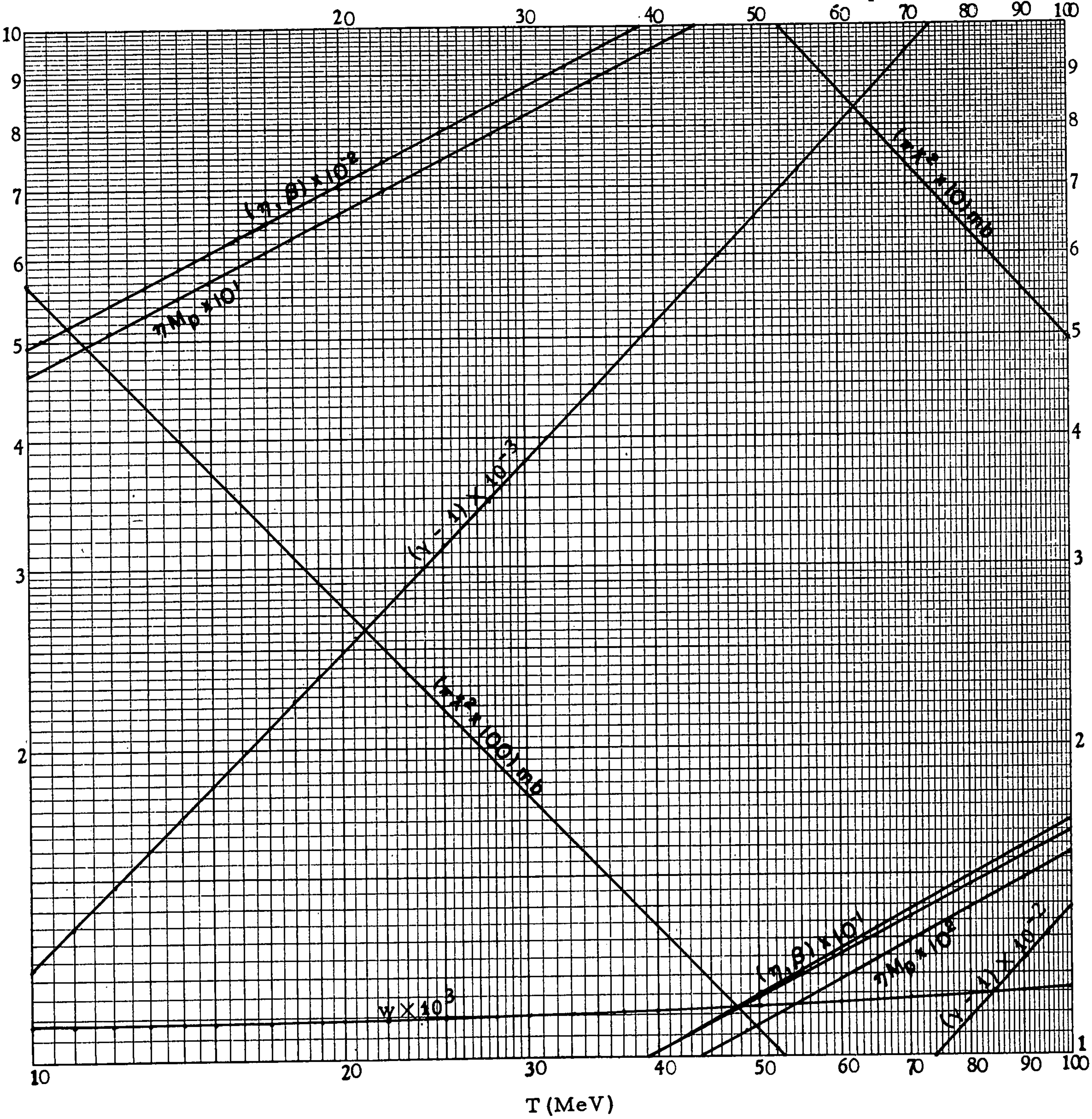
$1b = 10^{-24}$ cm^2

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{\pi^{\pm}} = 139.59$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

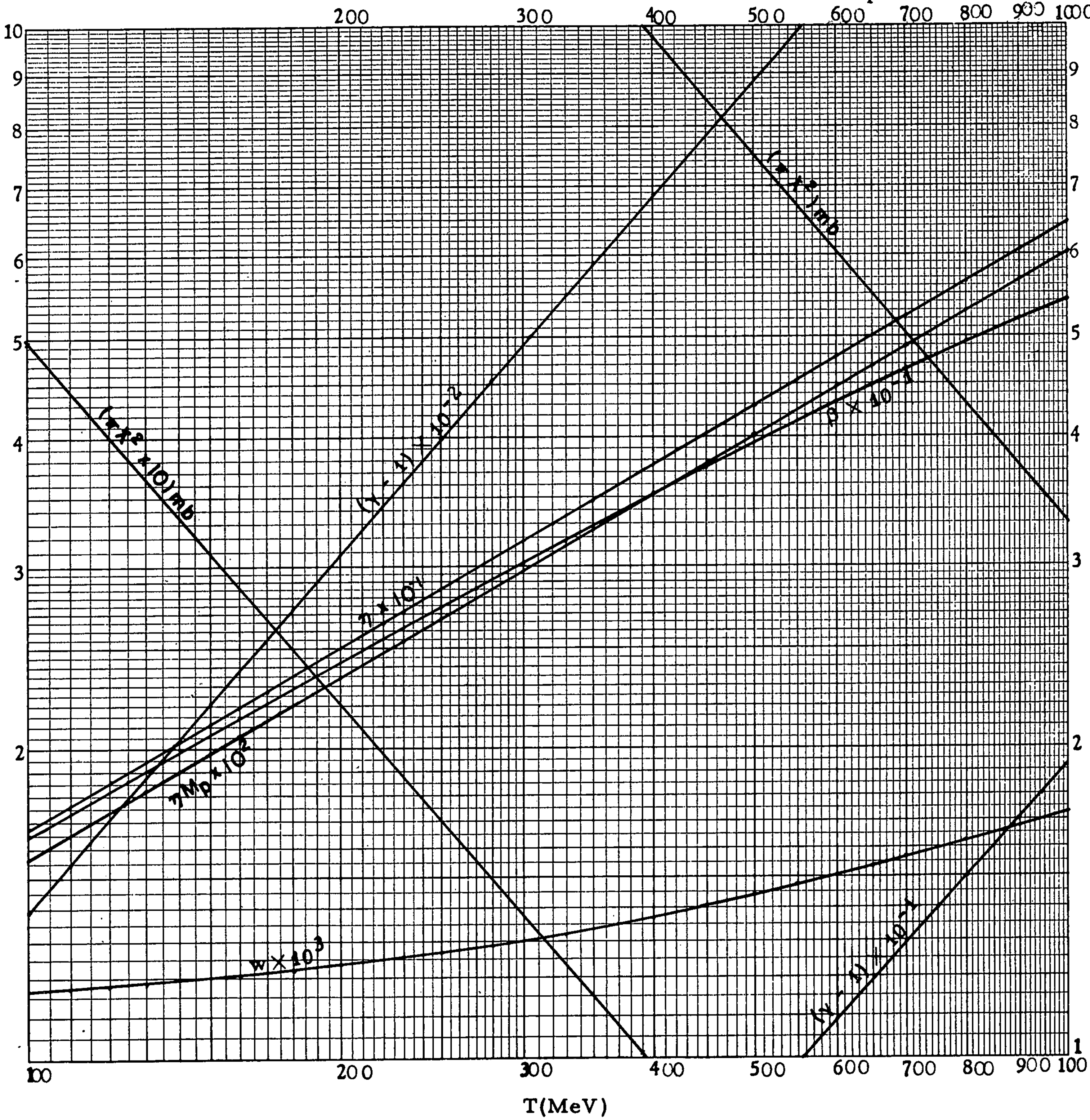
100 MeV to 1 BeV

$M_{\pi^{\pm}} = 139.59$ MeV

$1b = 10^{-24}$ cm^2

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_p = 938.213$ MeV



w and ηM_p in MeV

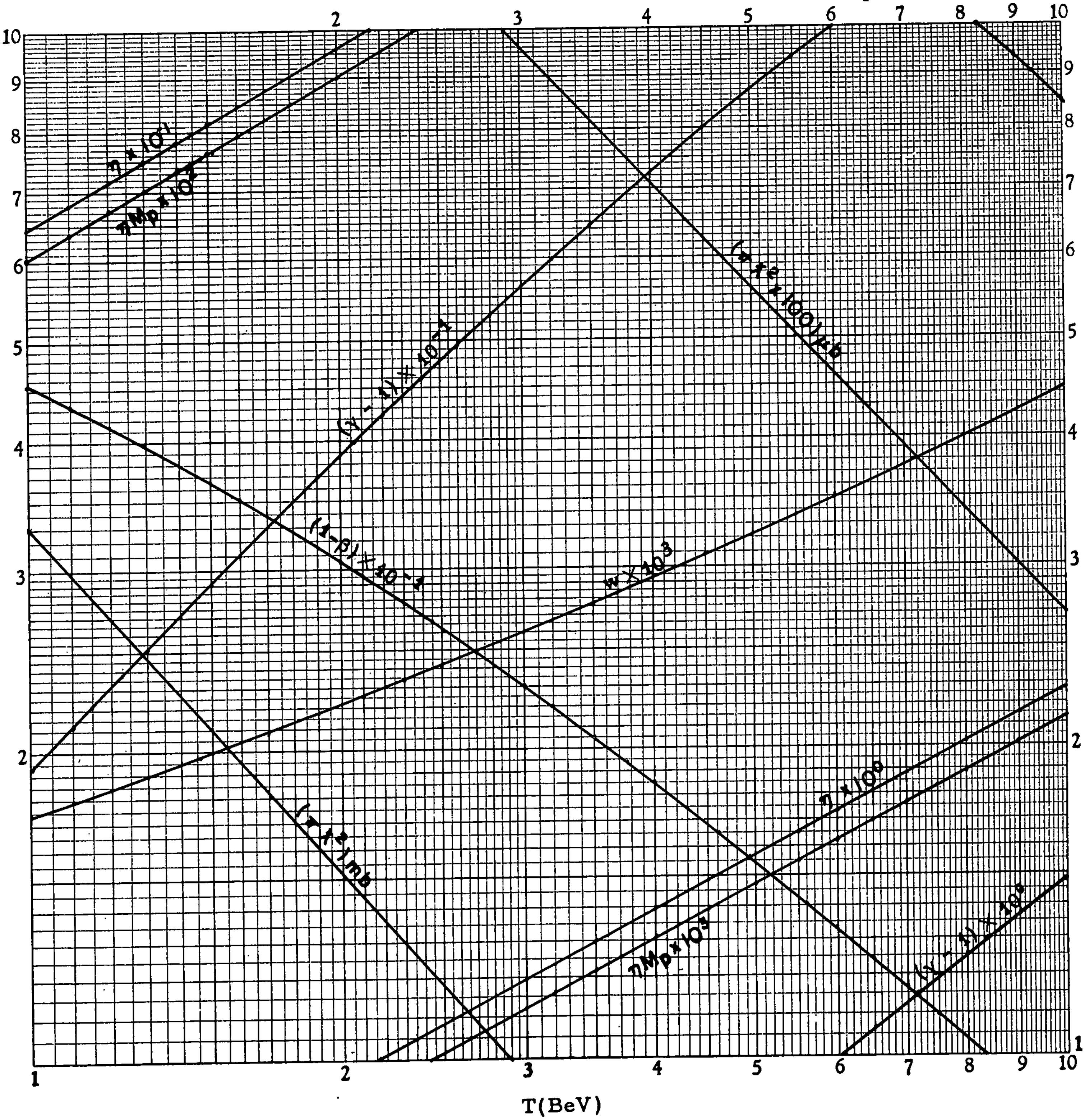
1b = 10^{-24} cm^2

1 BeV to 10 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{\pi^\pm}$ = 139.59 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

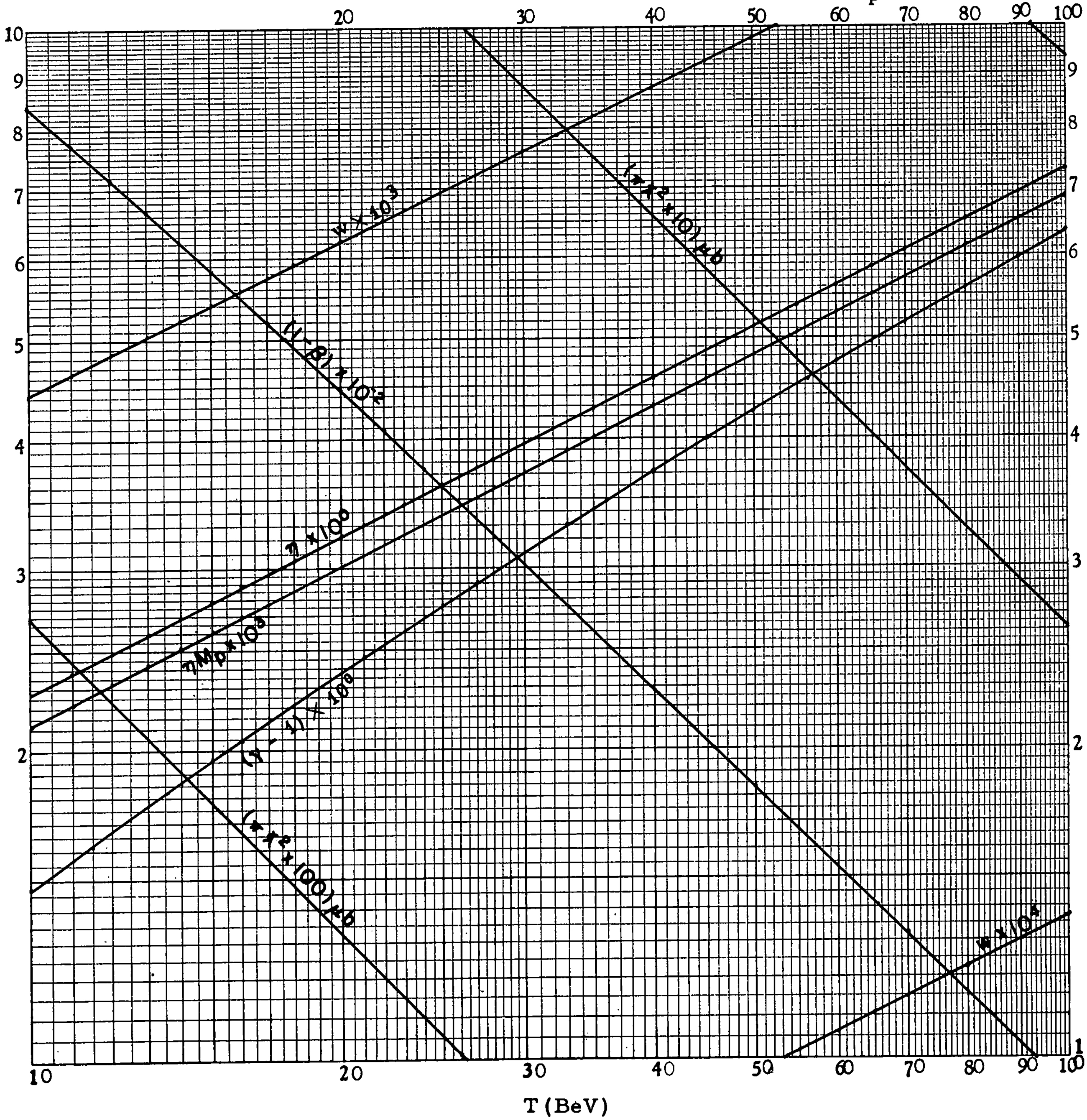
1b = 10^{-24} cm^2

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{\pi^\pm}$ = 139.59 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

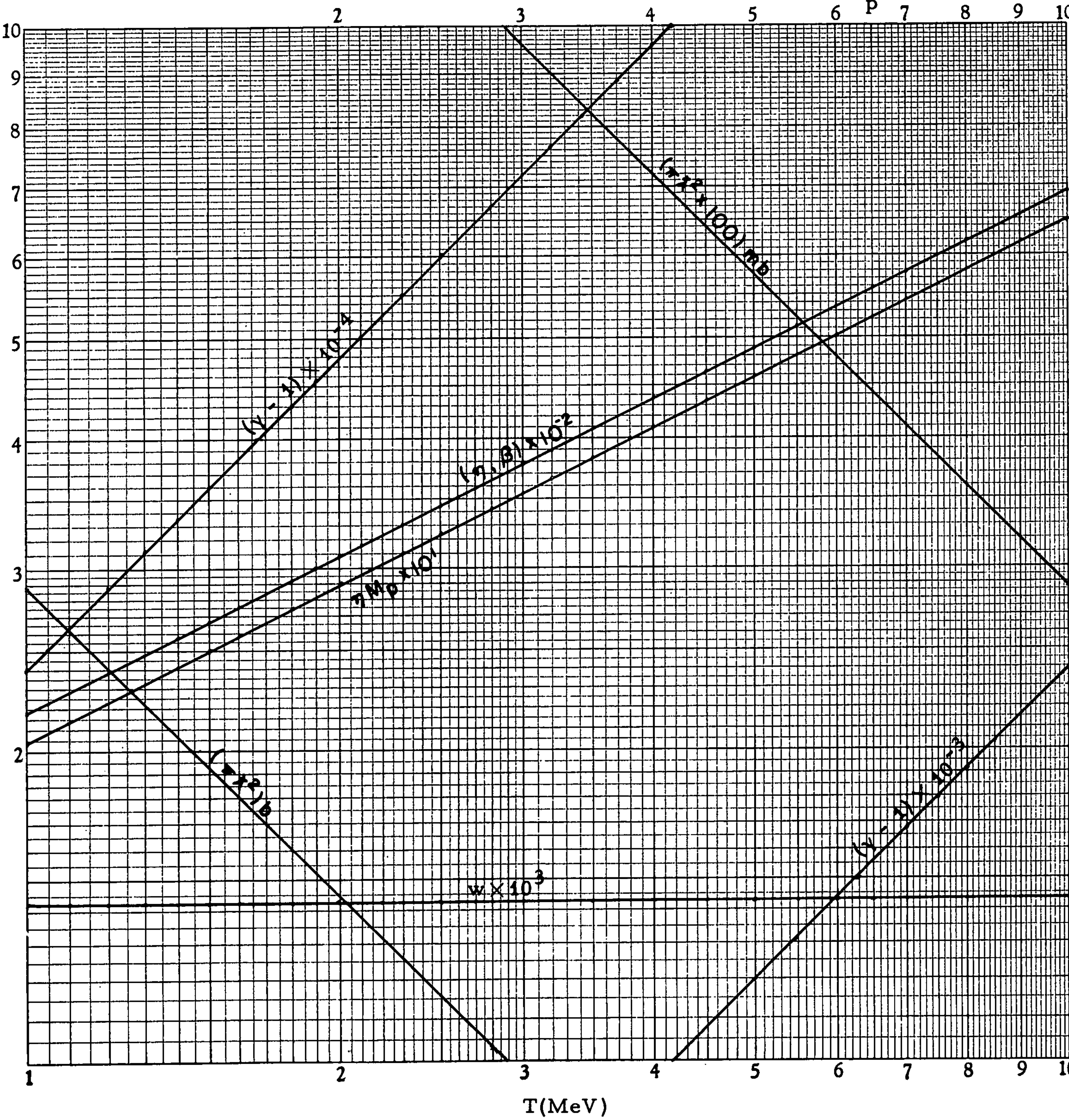
1b = 10^{-24} cm^2

1 MeV to 10 MeV

β, $(\gamma - 1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{K^{\pm}}$ = 493.9 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

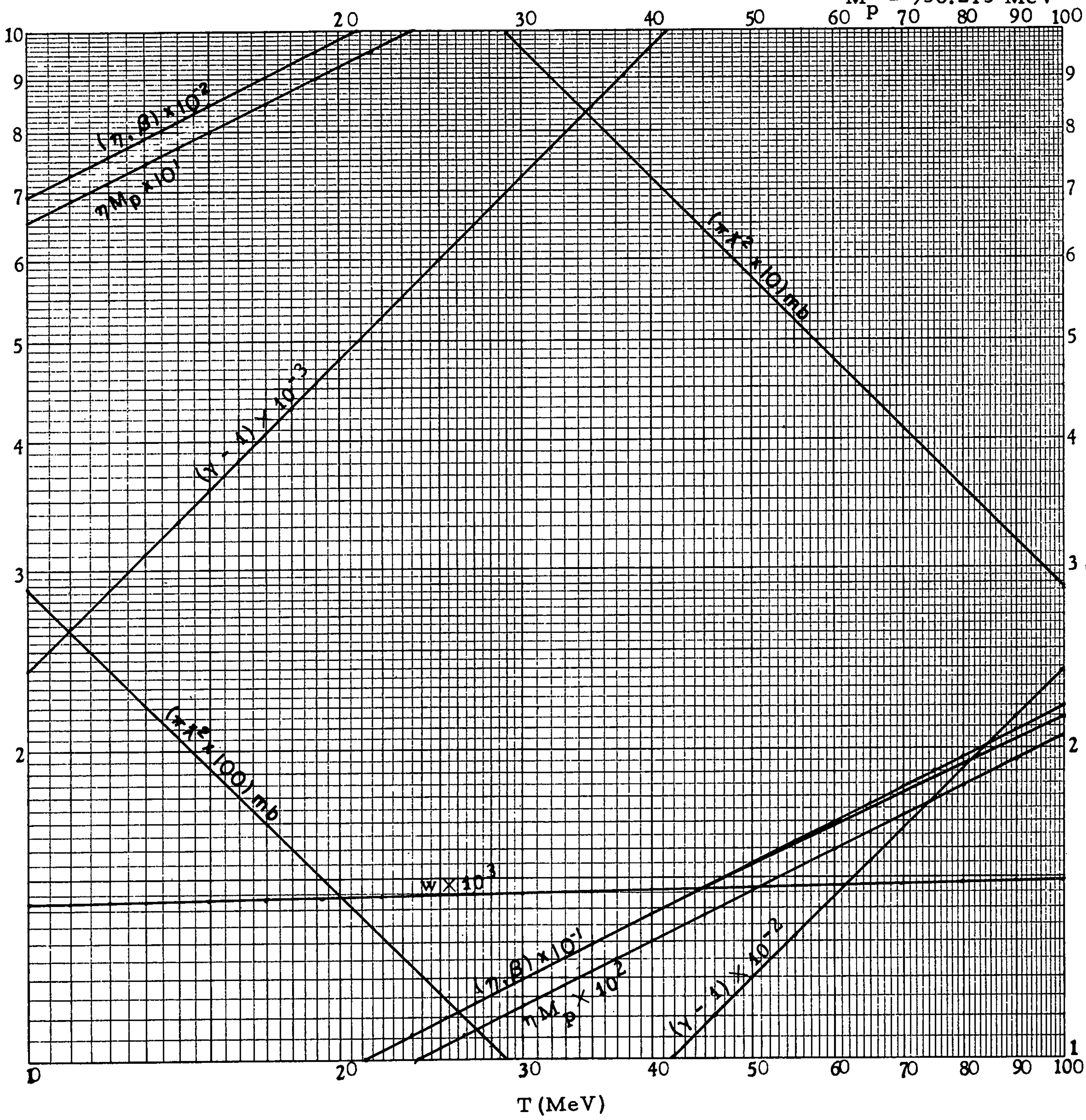
$1b = 10^{-24}$ cm^2

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{K^{\pm}}$ = 493.9 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

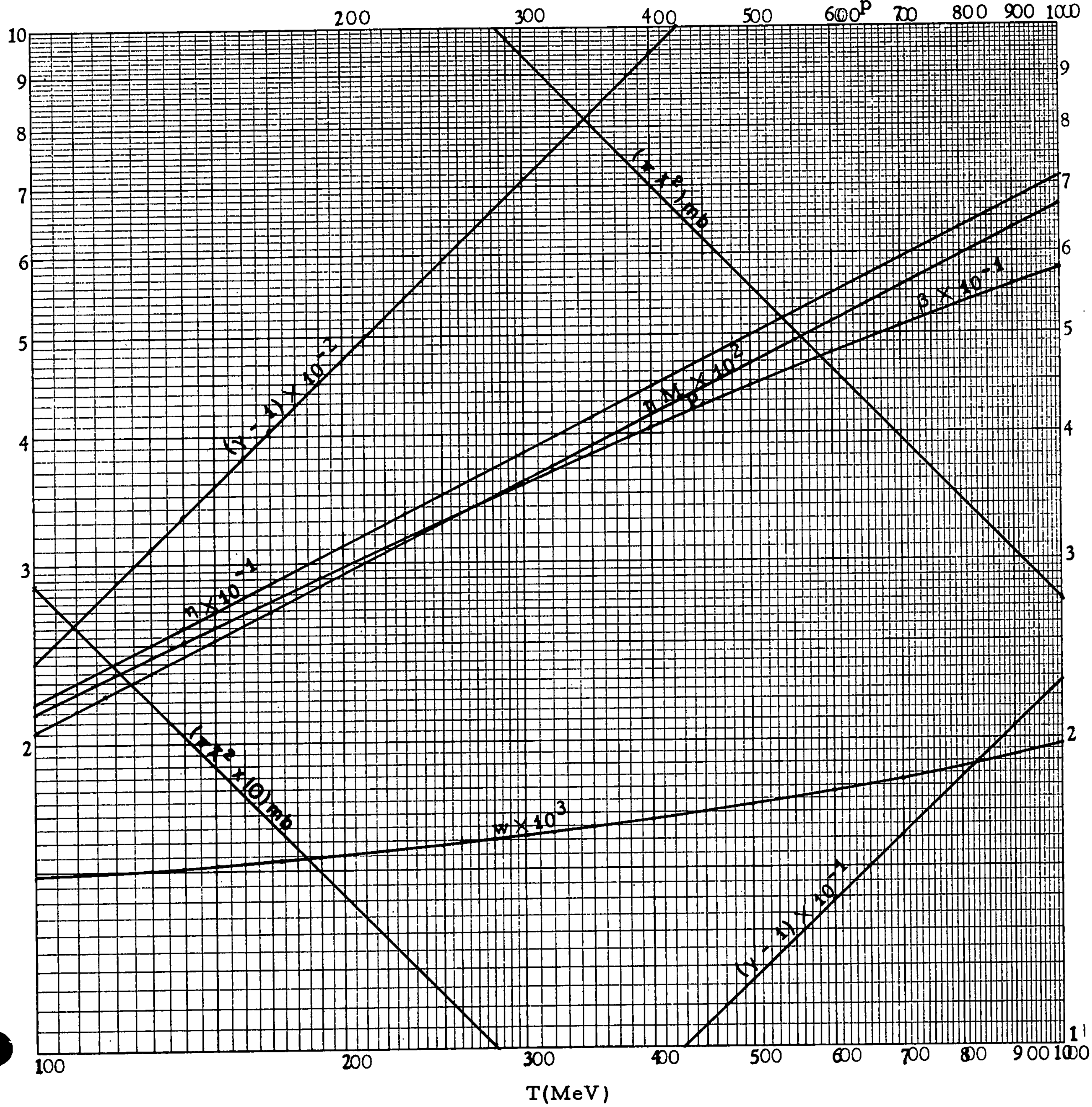
$1b = 10^{-24}$ cm^2

100 MeV to 1 BeV

β, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda^2$

$M_{K^{\pm}} = 493.9$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

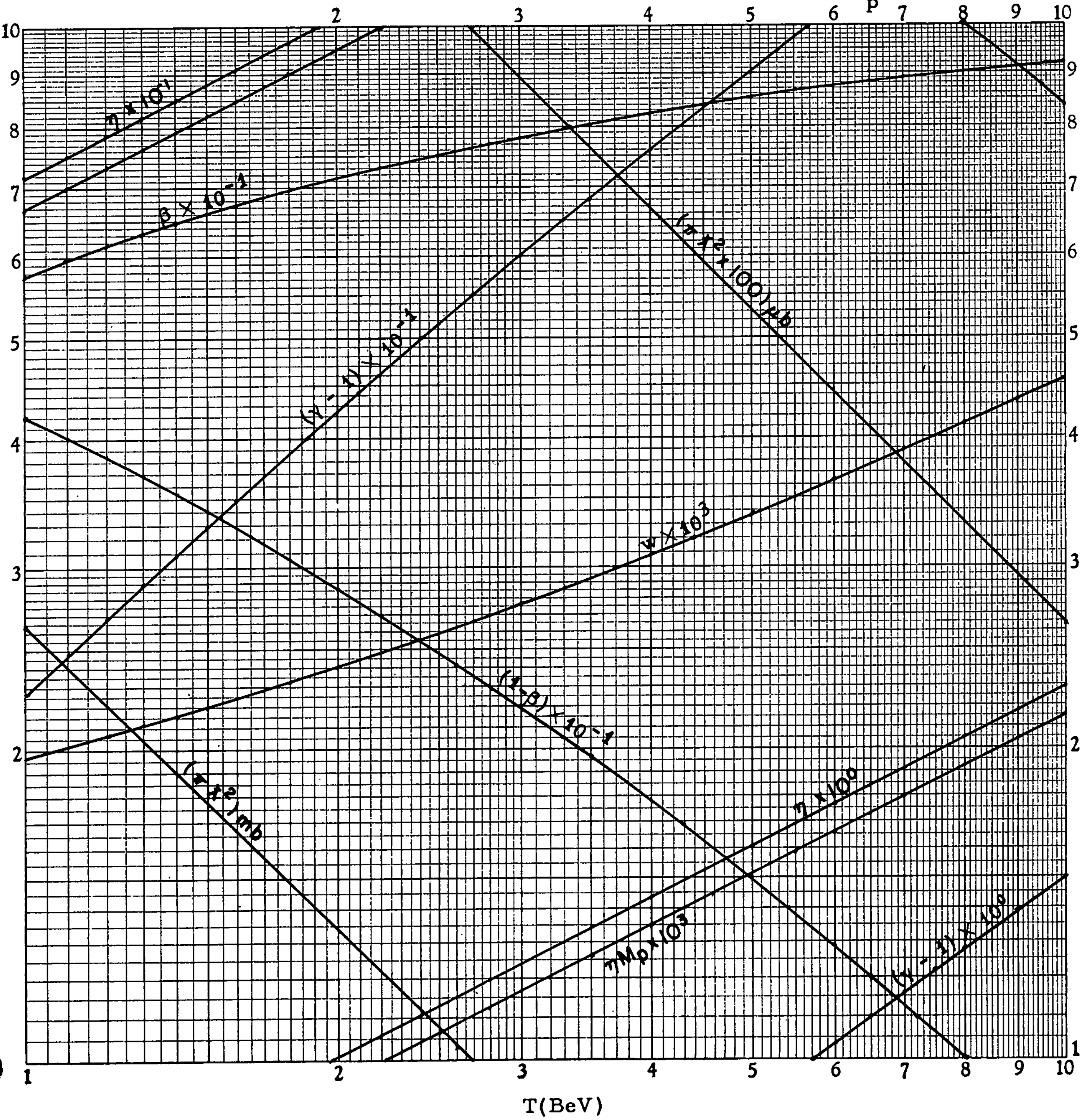
$1b = 10^{-24}\ cm^2$

1 BeV to 10 BeV

$(1-\beta)$, β, $(\gamma-1)$, η, ηM_p, w, $\pi ƛ^2$

$M_{K^{\pm}} = 493.9$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

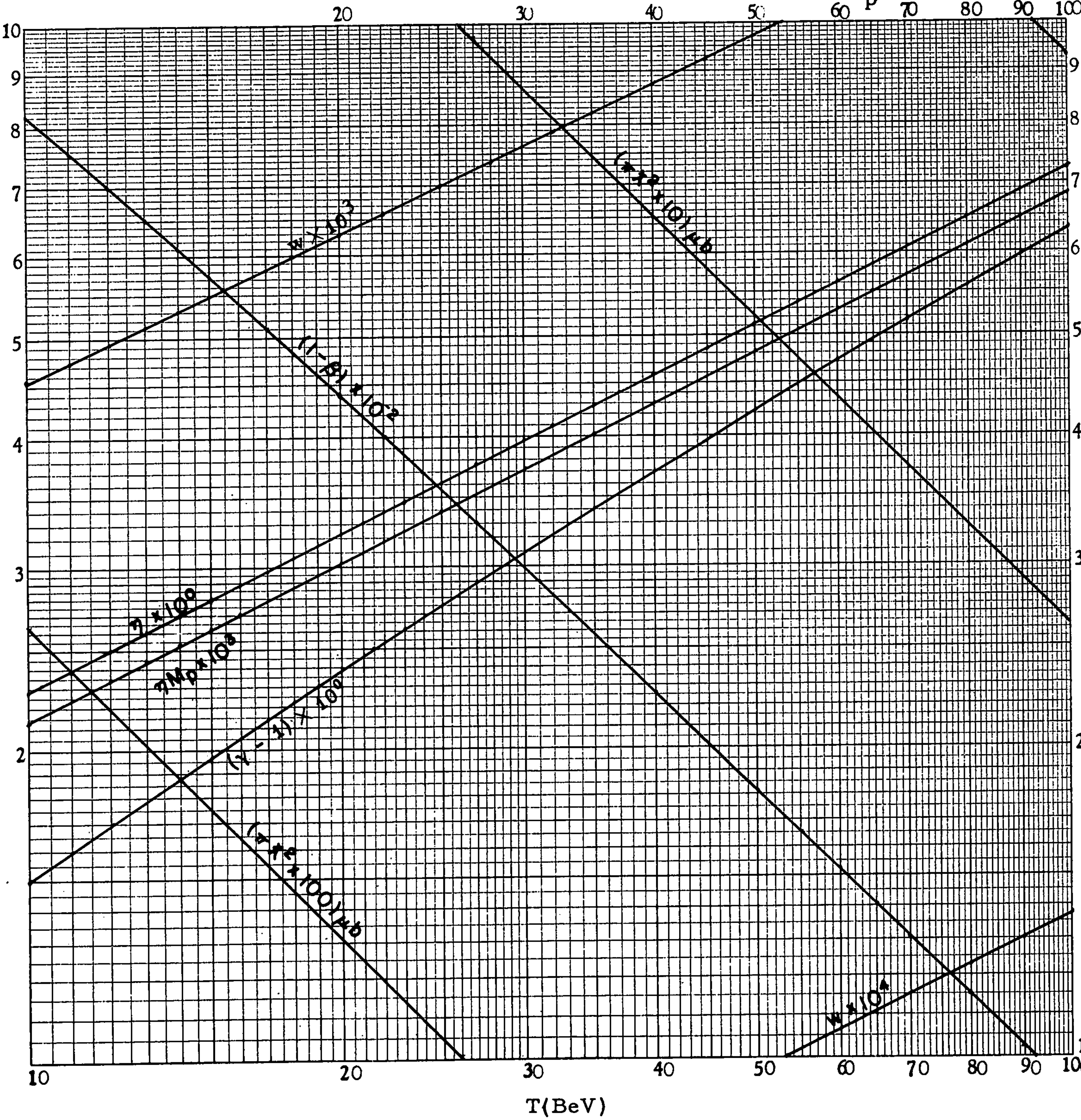
1b = 10^{-24} cm^2

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{K^{\pm}}$ = 493.9 MeV

M_p = 938.213 MeV



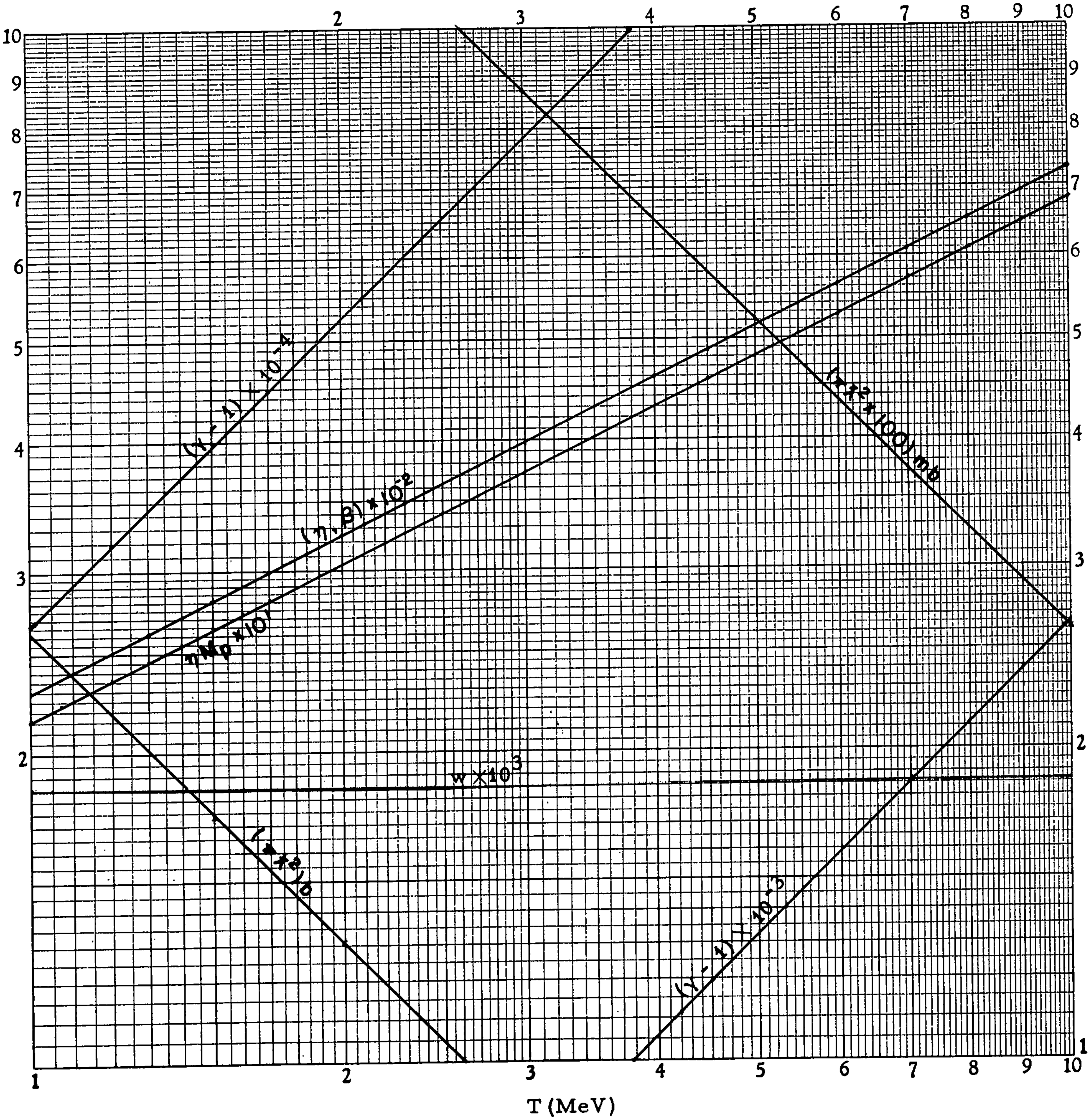
w and ηM_p in MeV

1b = 10^{-24} cm^2

1 MeV to 10 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_p = 938.213 MeV



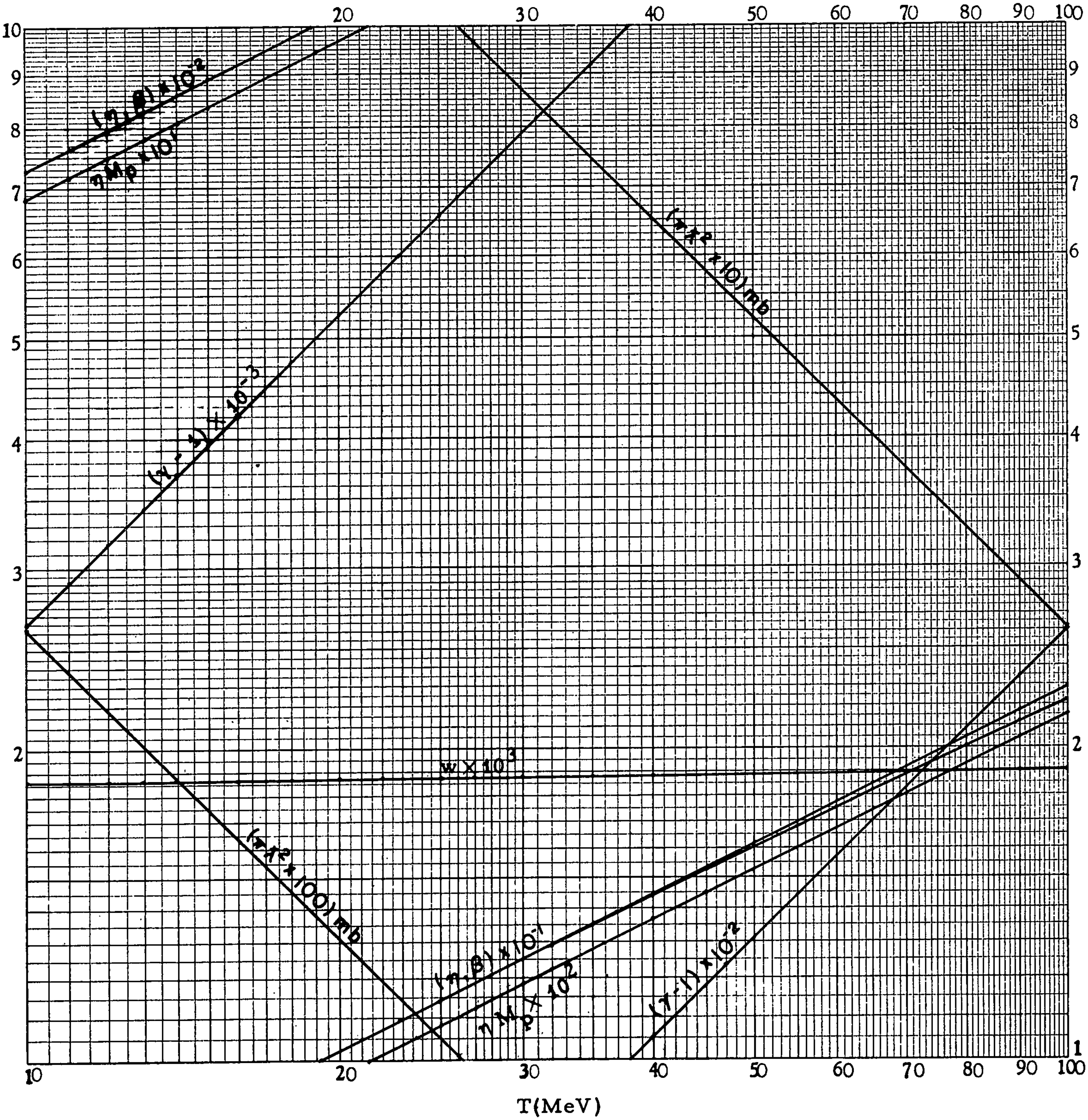
w and ηM_p in MeV

$1b = 10^{-24} cm^2$

10 MeV to 100 MeV

β, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda^2$

M_p = 938.213 MeV



PROTONS: p + p

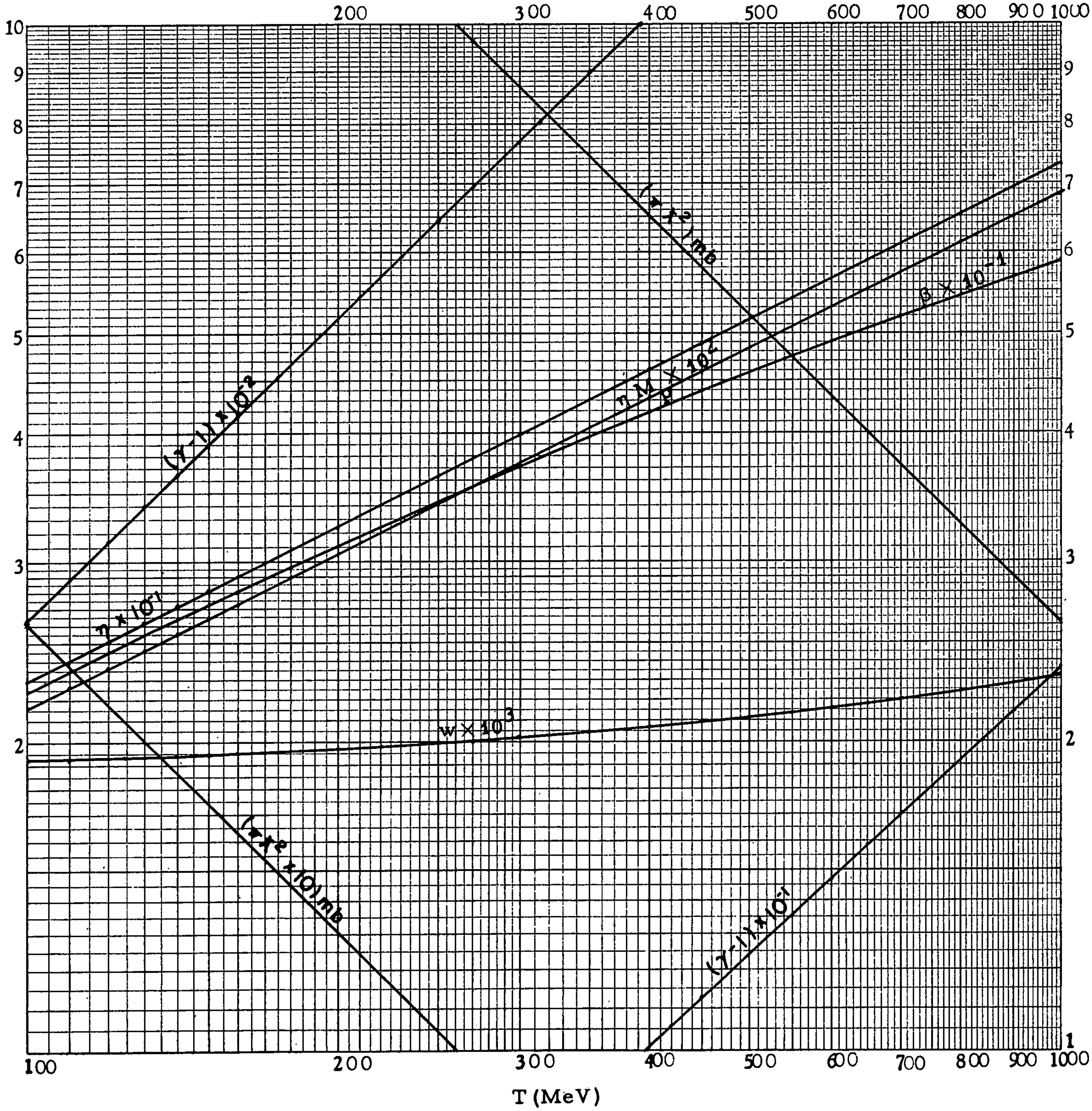
w and ηM_p in MeV

$1b = 10^{-24}$ cm^2

100 MeV to 1 BeV

M_p = 938.213 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda\!\!\!^{-2}$



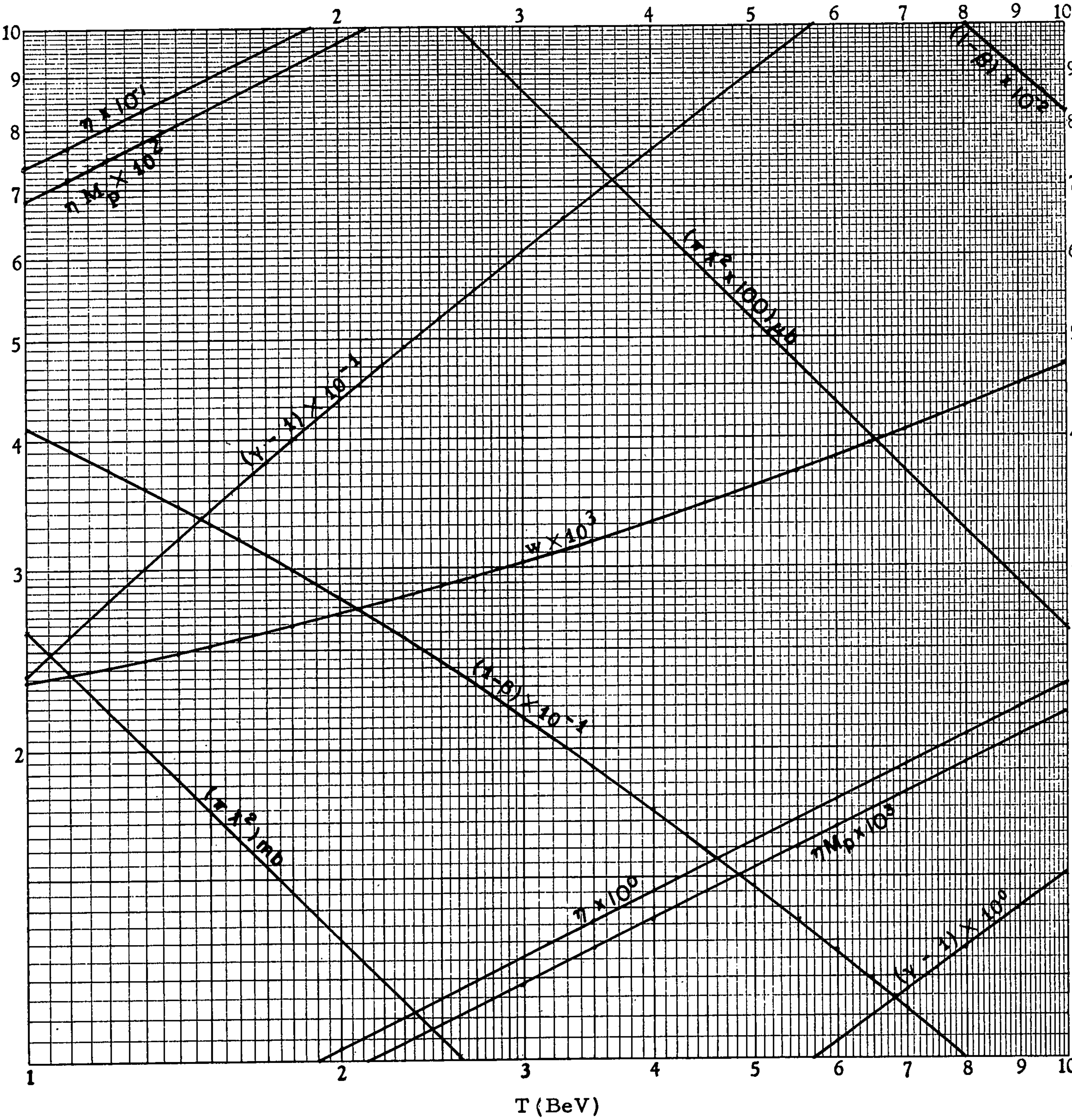
w and ηM_p in MeV

1b = 10^{-24} cm^2

1 BeV to 10 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_p = 938.213 MeV



PROTONS: p + p

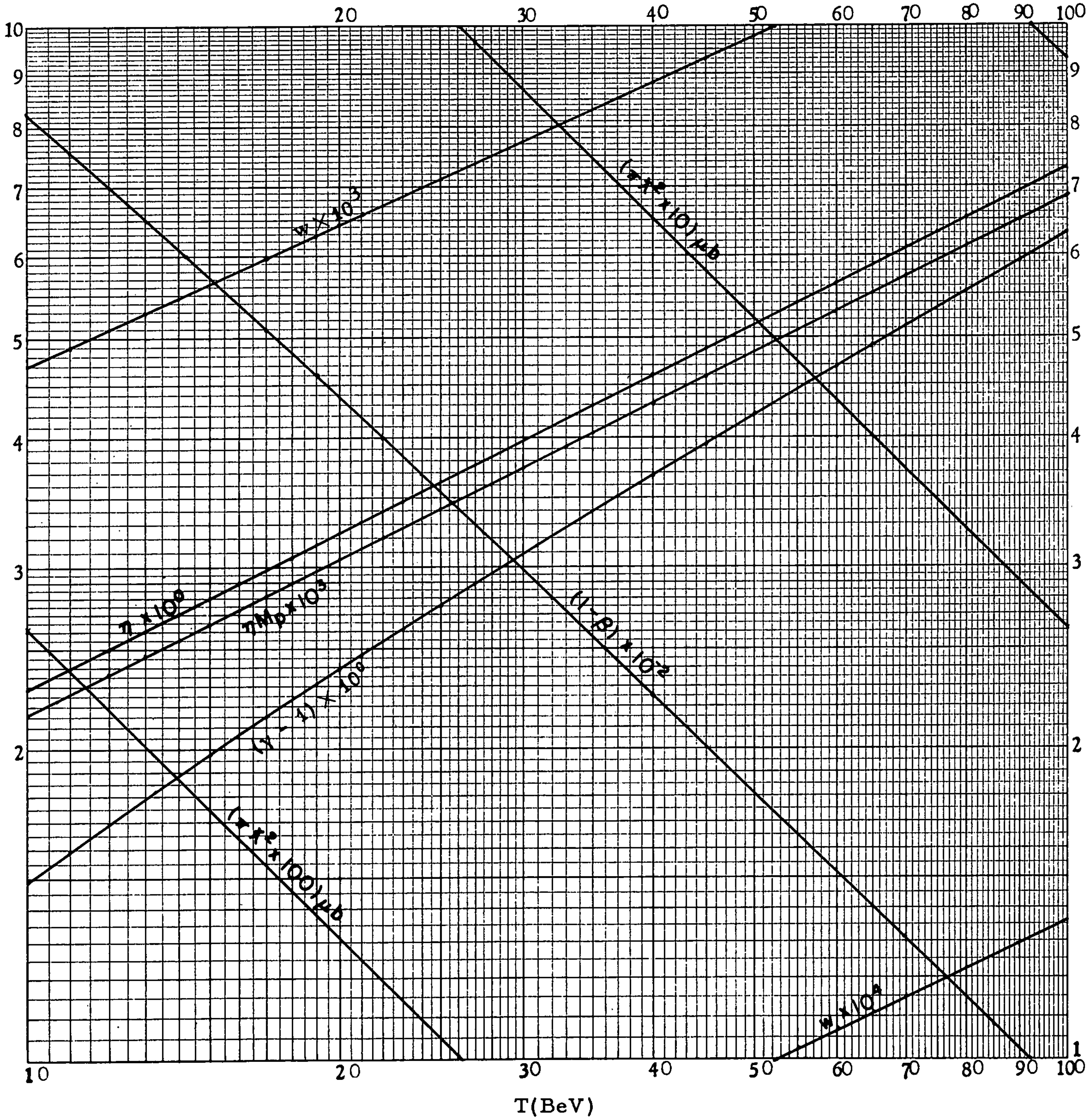
w and ηM_p in MeV

$1b = 10^{-24}$ cm^2

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_p = 938.213 MeV



w and ηM_p in MeV

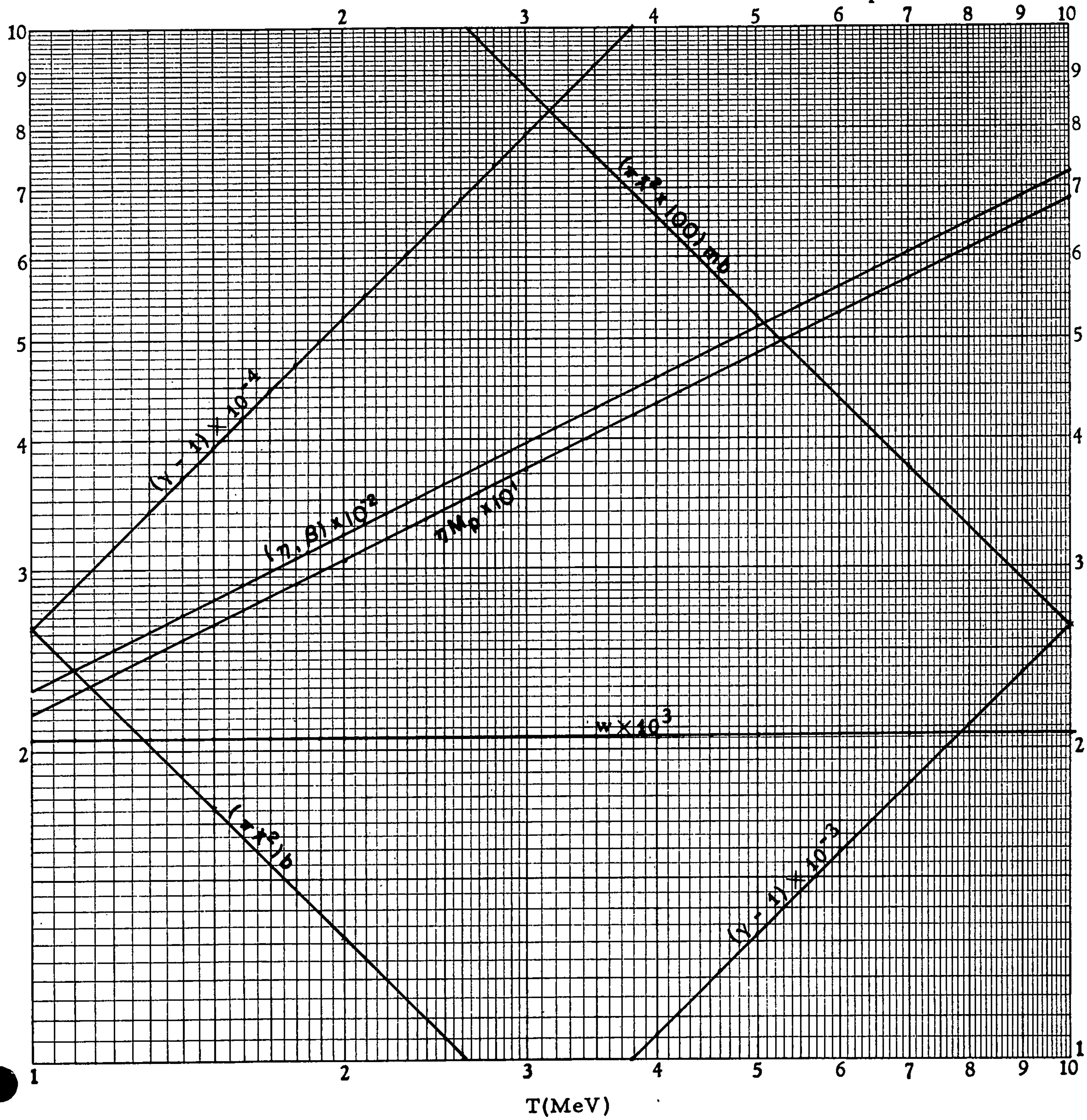
1b = 10^{-24} cm^2

1 MeV to 10 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_Λ = 1115.36 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

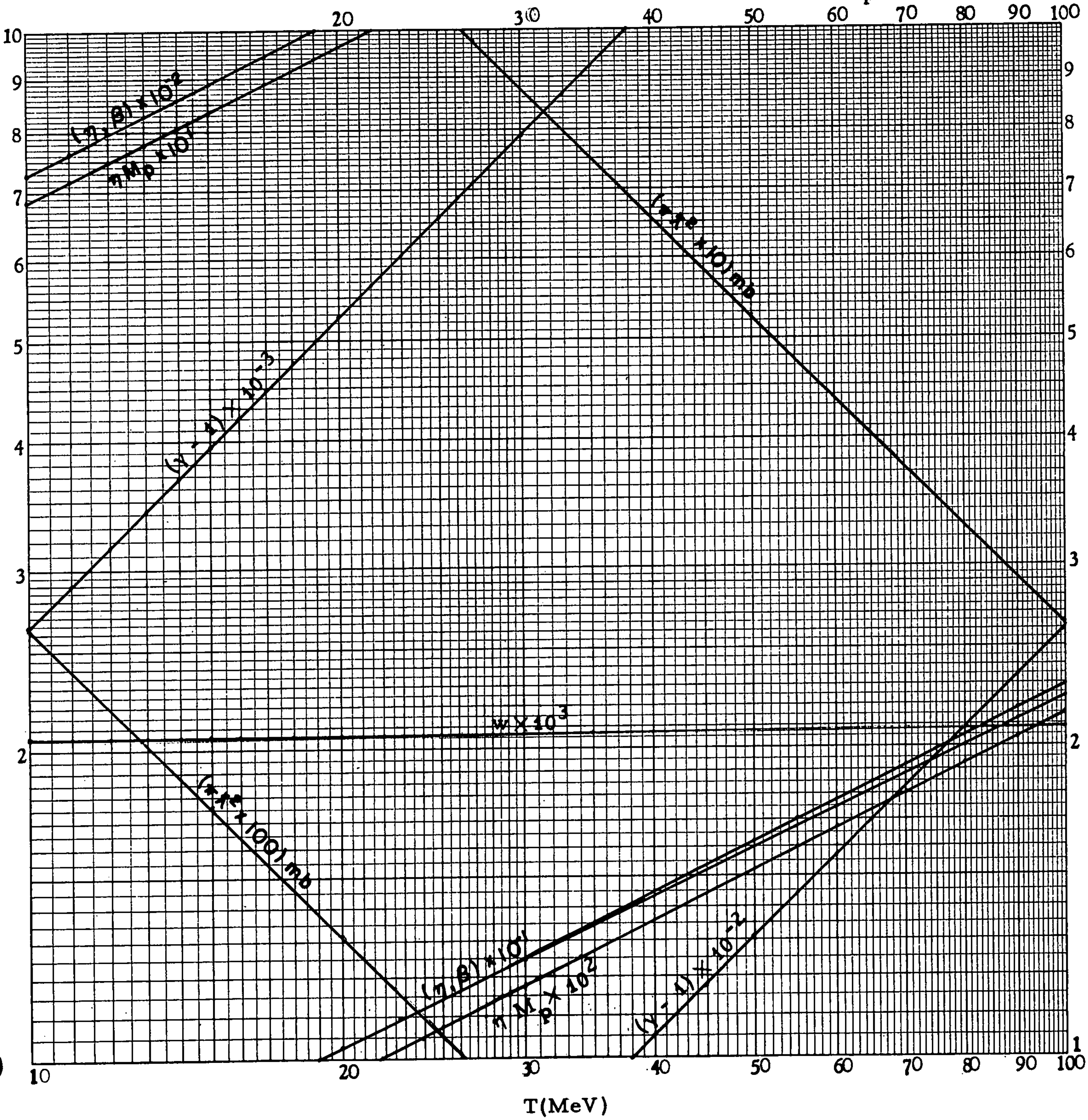
$1b = 10^{-24}$ cm^2

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda\!\!\!^{-2}$

M_Λ = 1115.36 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

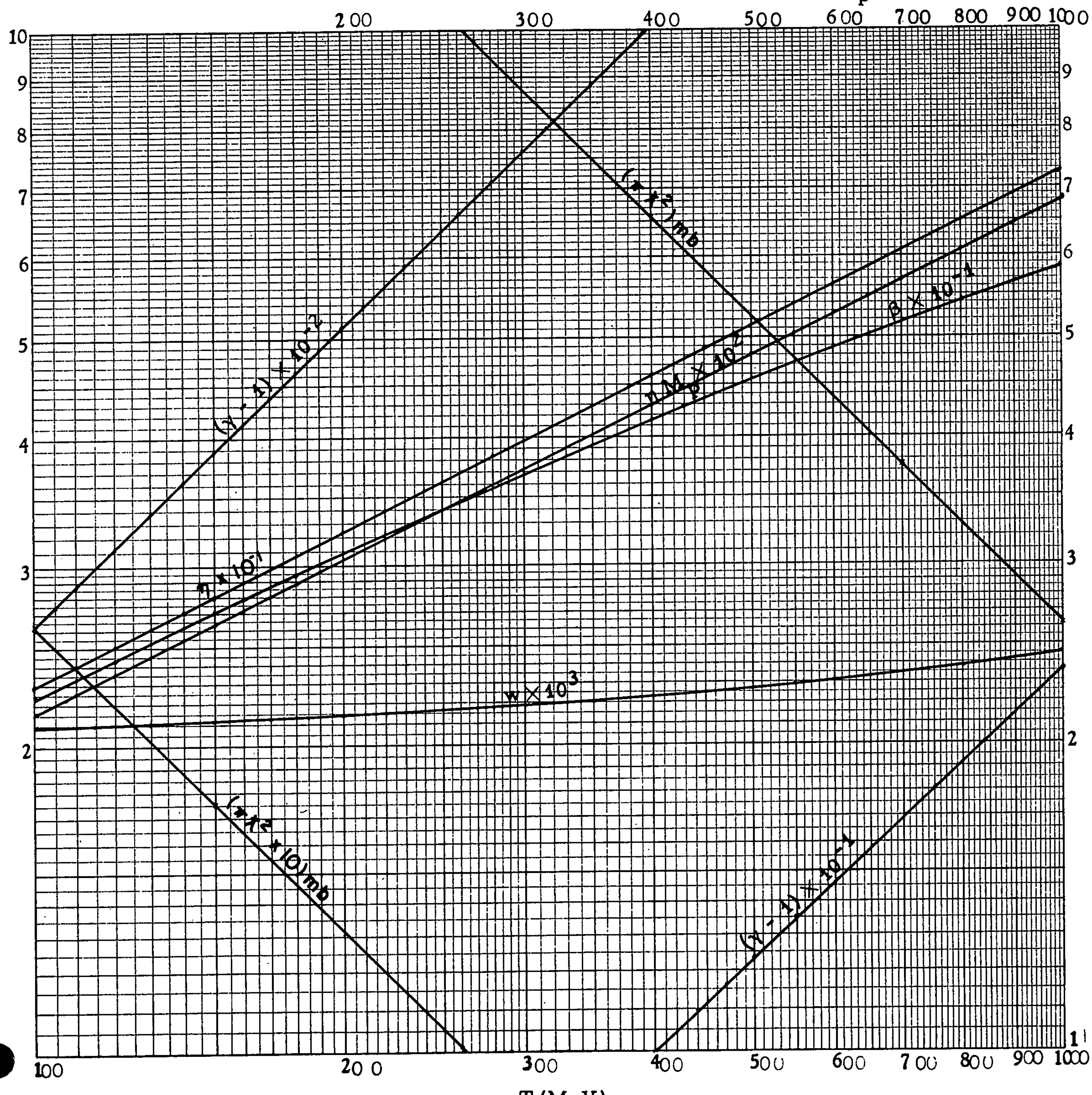
1b = 10^{-24} cm^2

100 MeV to 1 BeV

β, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda^2$

M_Λ = 1115.36 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

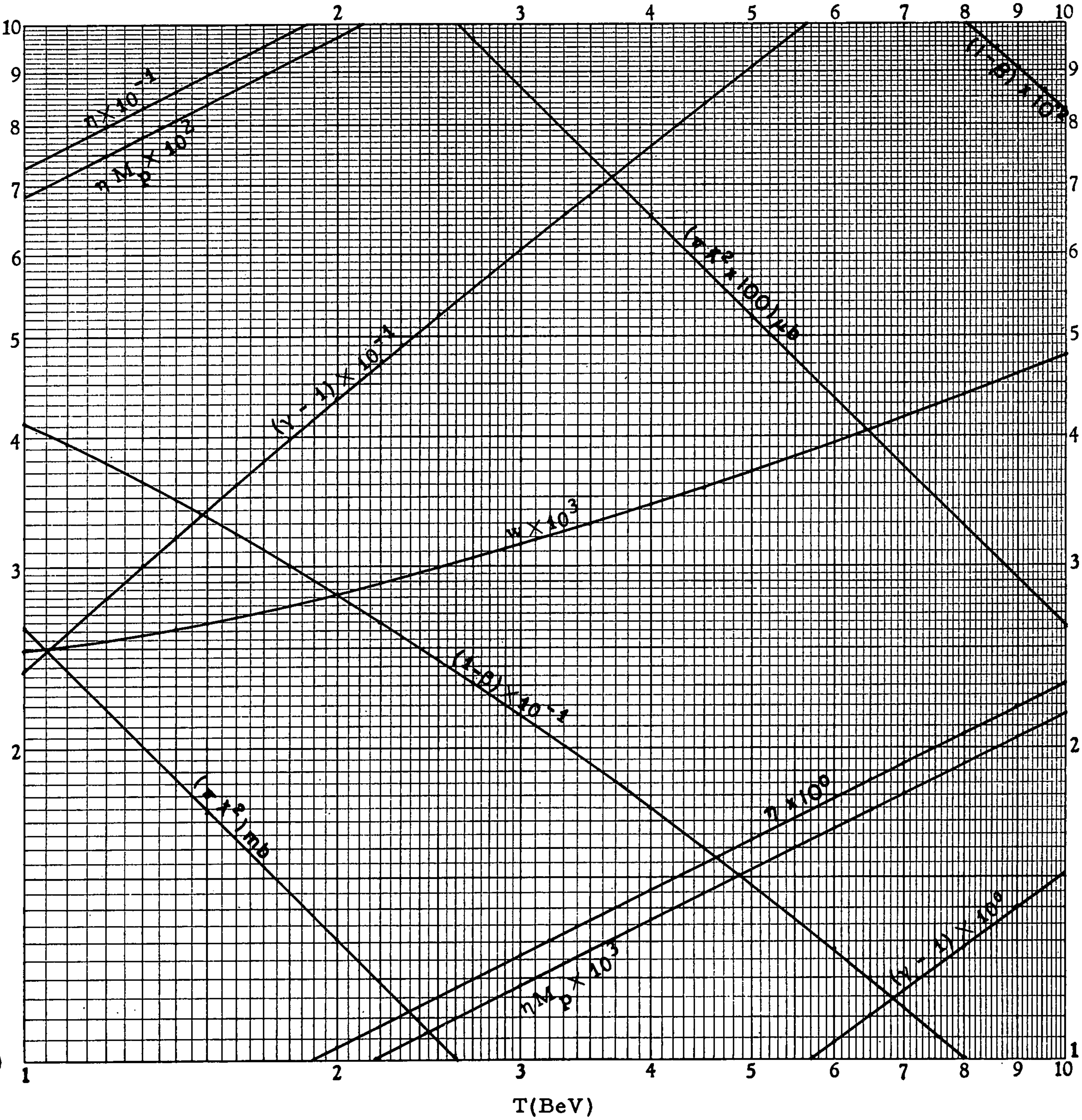
1b = 10^{-24} cm^2

1 BeV to 10 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_Λ = 1115.36 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

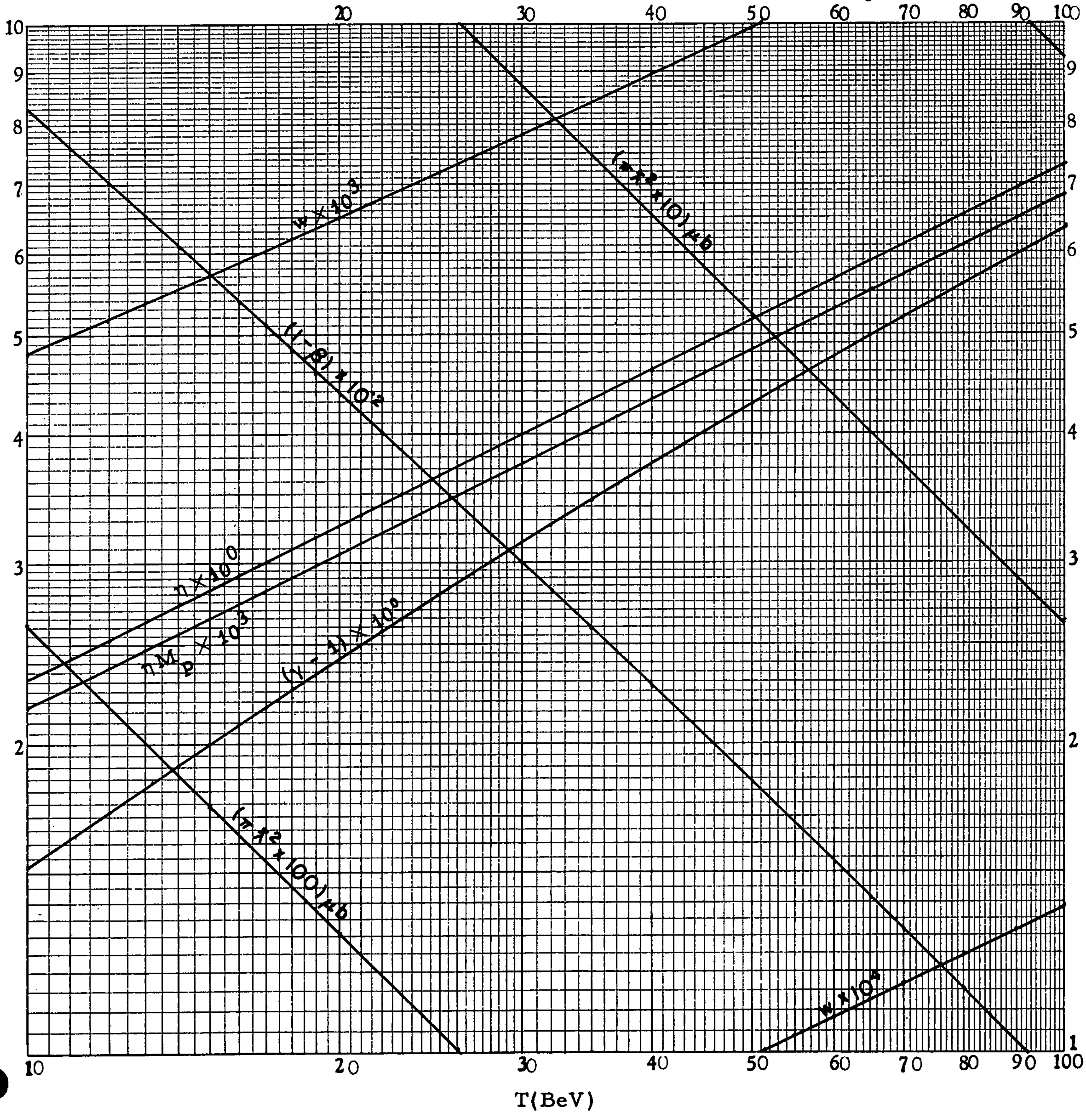
1b = 10^{-24} cm^2

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda\!\!\!^{-2}$

M_Λ = 1115.36 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

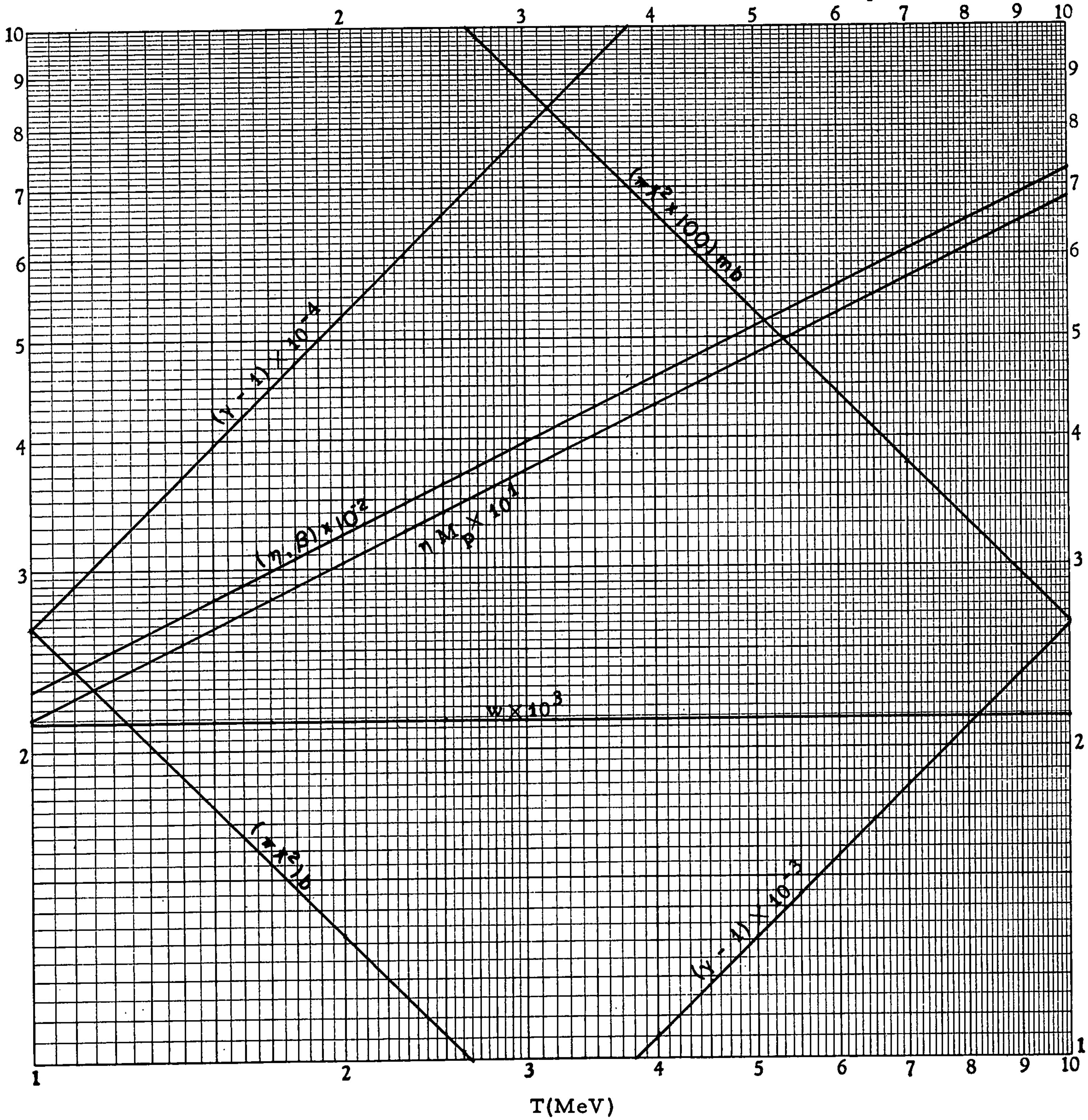
1b = 10^{-24} cm^2

1 MeV to 10 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Σ^+} = 1189.40 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

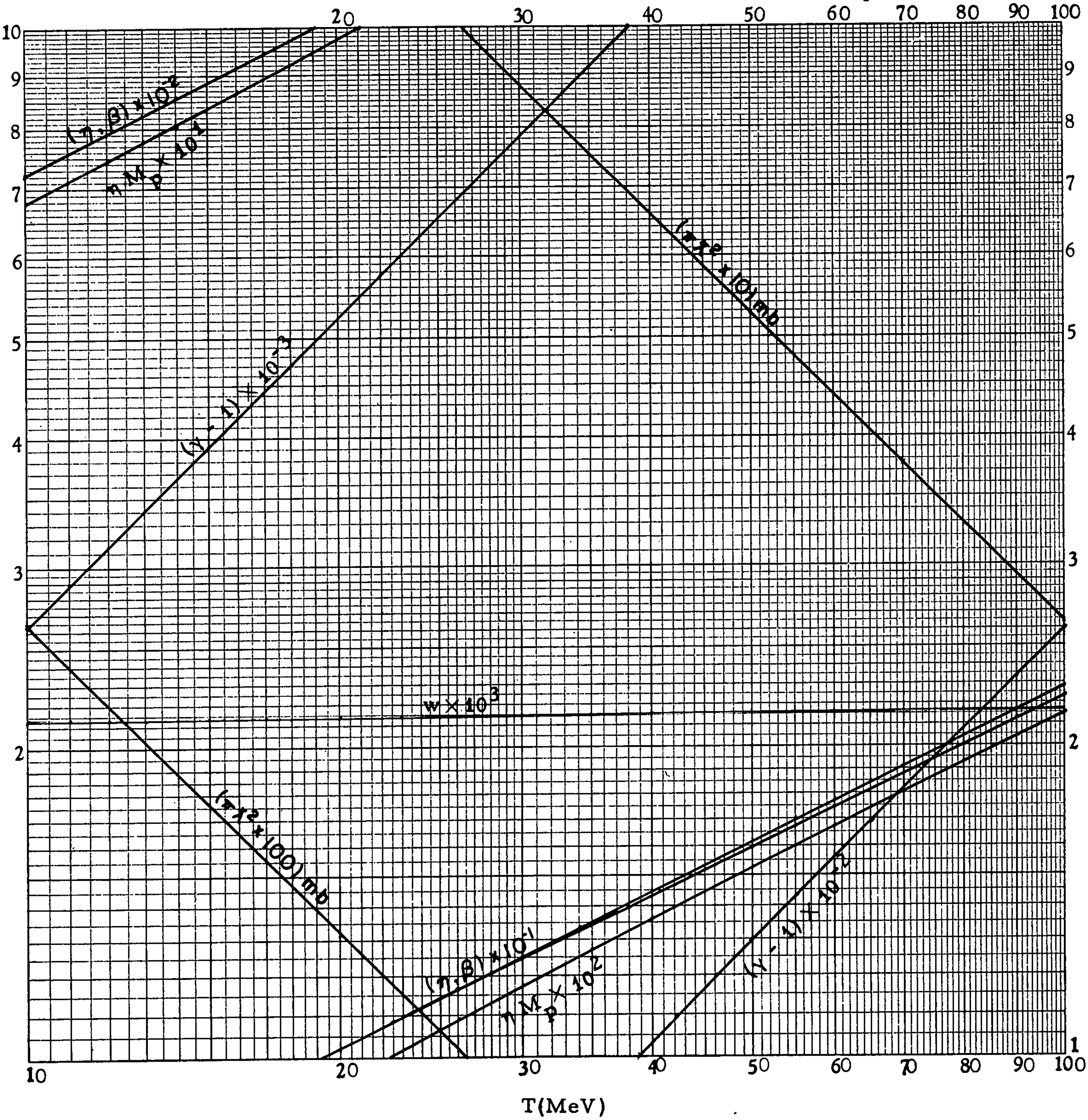
$1b = 10^{-24}\ cm^2$

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

$M_{\Sigma^+} = 1189.40$ MeV

$M_p = 938.213$ MeV



w and ηM_p in MeV

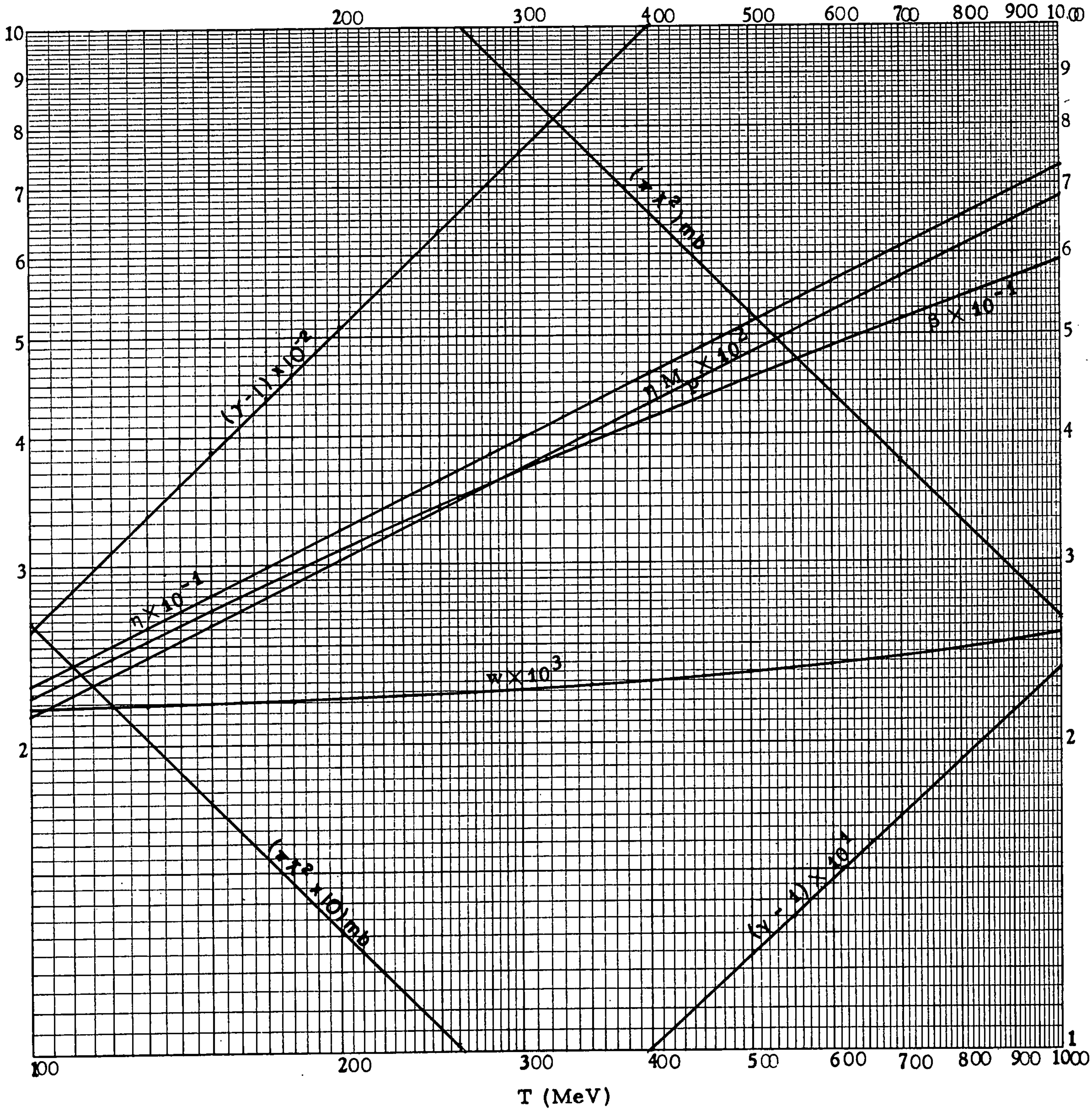
$1b = 10^{-24}$ cm^2

100 MeV to 1 BeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda^2$

M_{Σ^+} = 1189.40 MeV

M_p = 938.213 MeV



Σ HYPERONS: $\Sigma^+ + p$

w and ηM_p in MeV

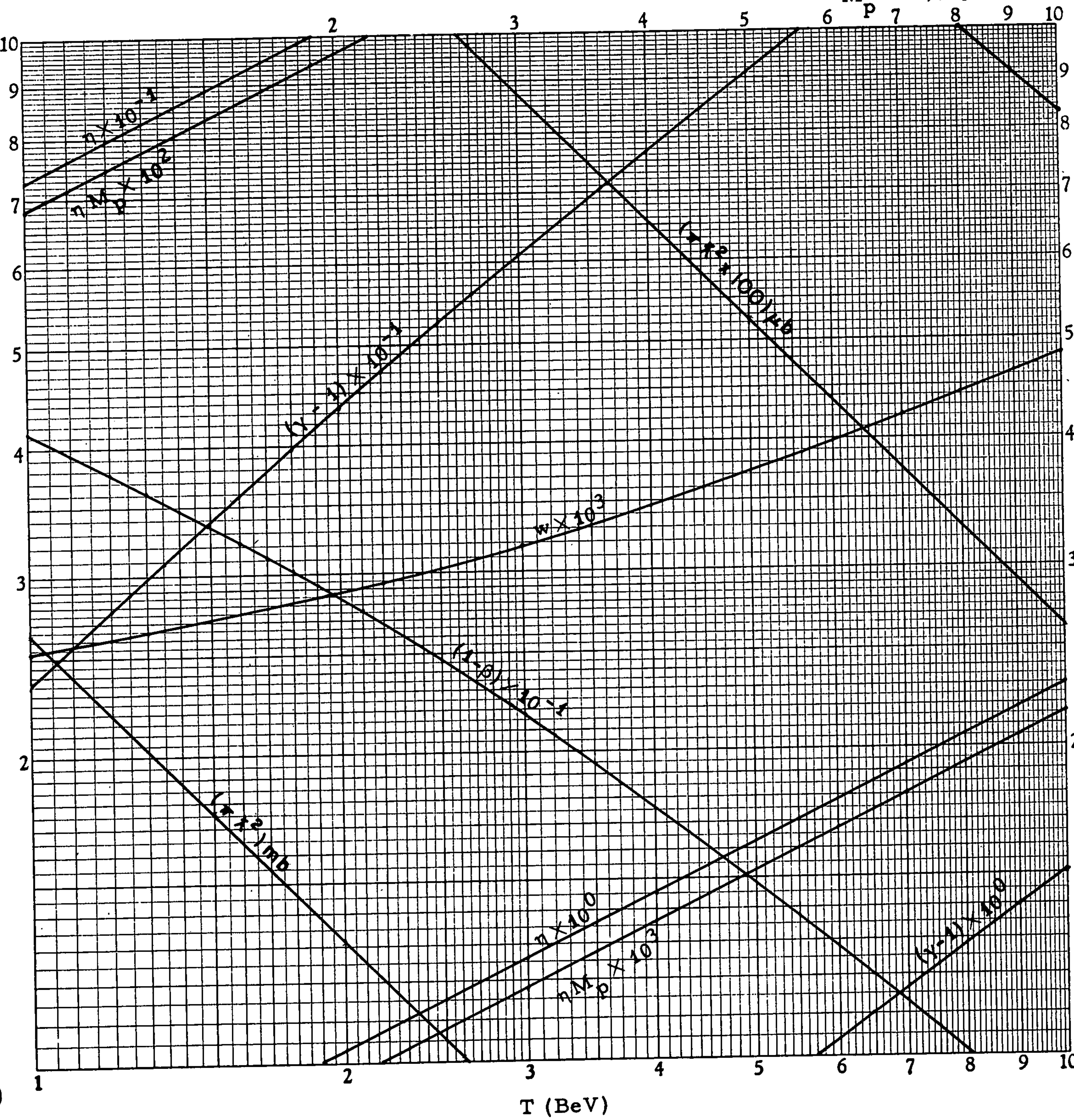
1b = 10^{-24} cm^2

1 BeV to 10 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Σ^+} = 1189.40 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

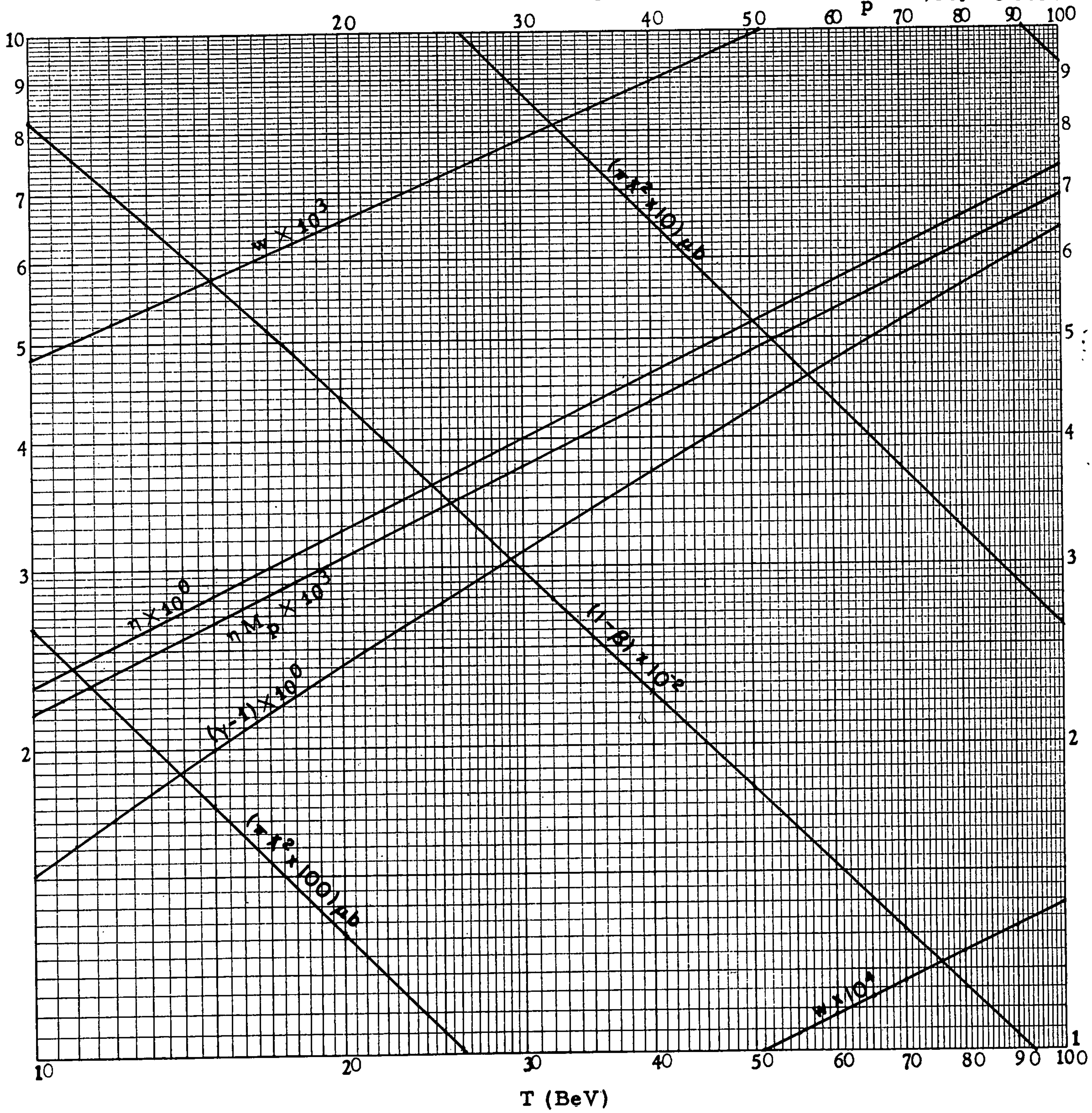
1b = 10^{-24} cm^2

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Σ^+} = 1189.40 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

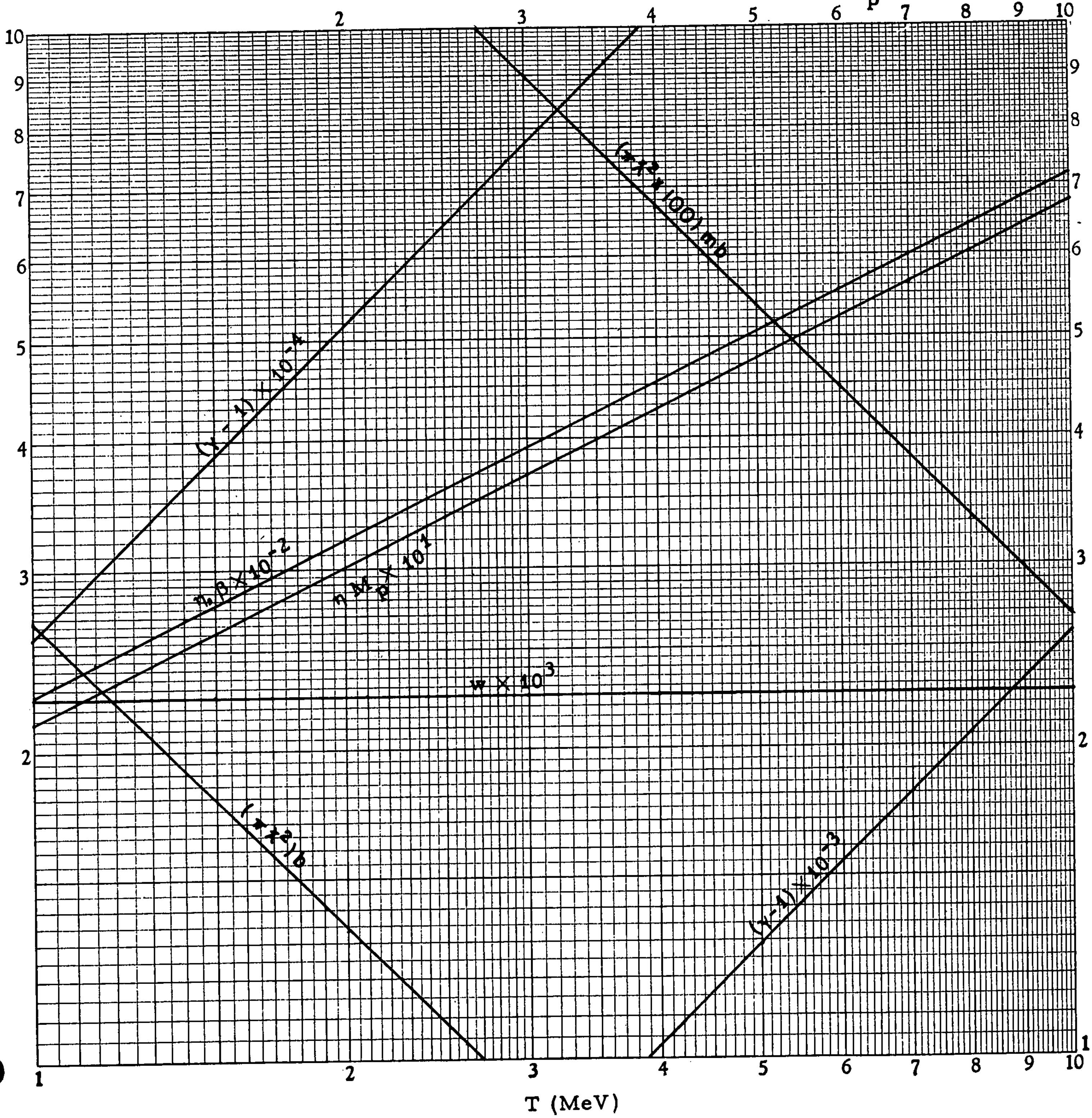
1b = 10^{-24} cm^2

1 MeV to 10 MeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Ξ^-} = 1318.4 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

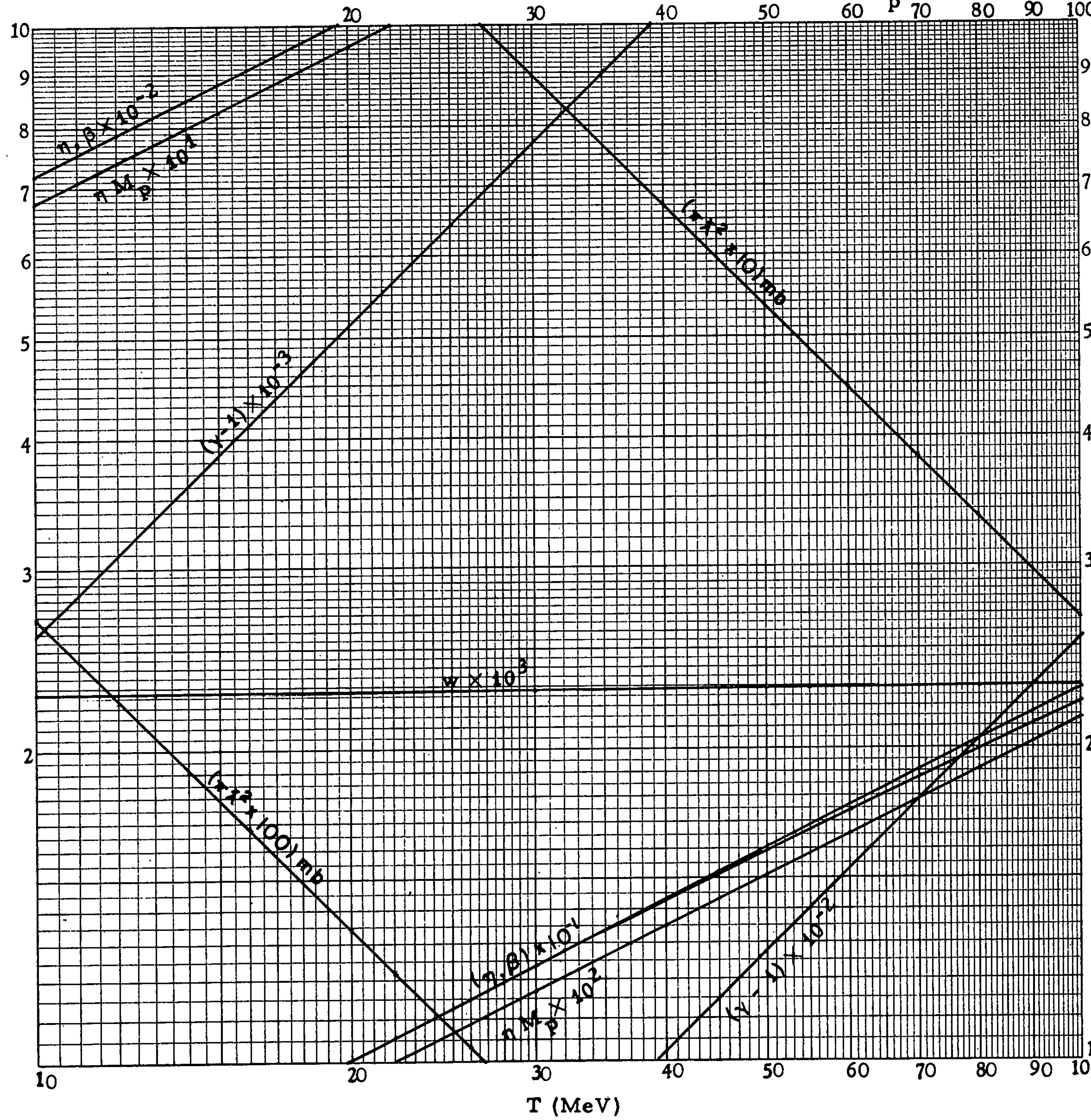
1b = 10^{-24} cm^2

10 MeV to 100 MeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Ξ^-} = 1318.4 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

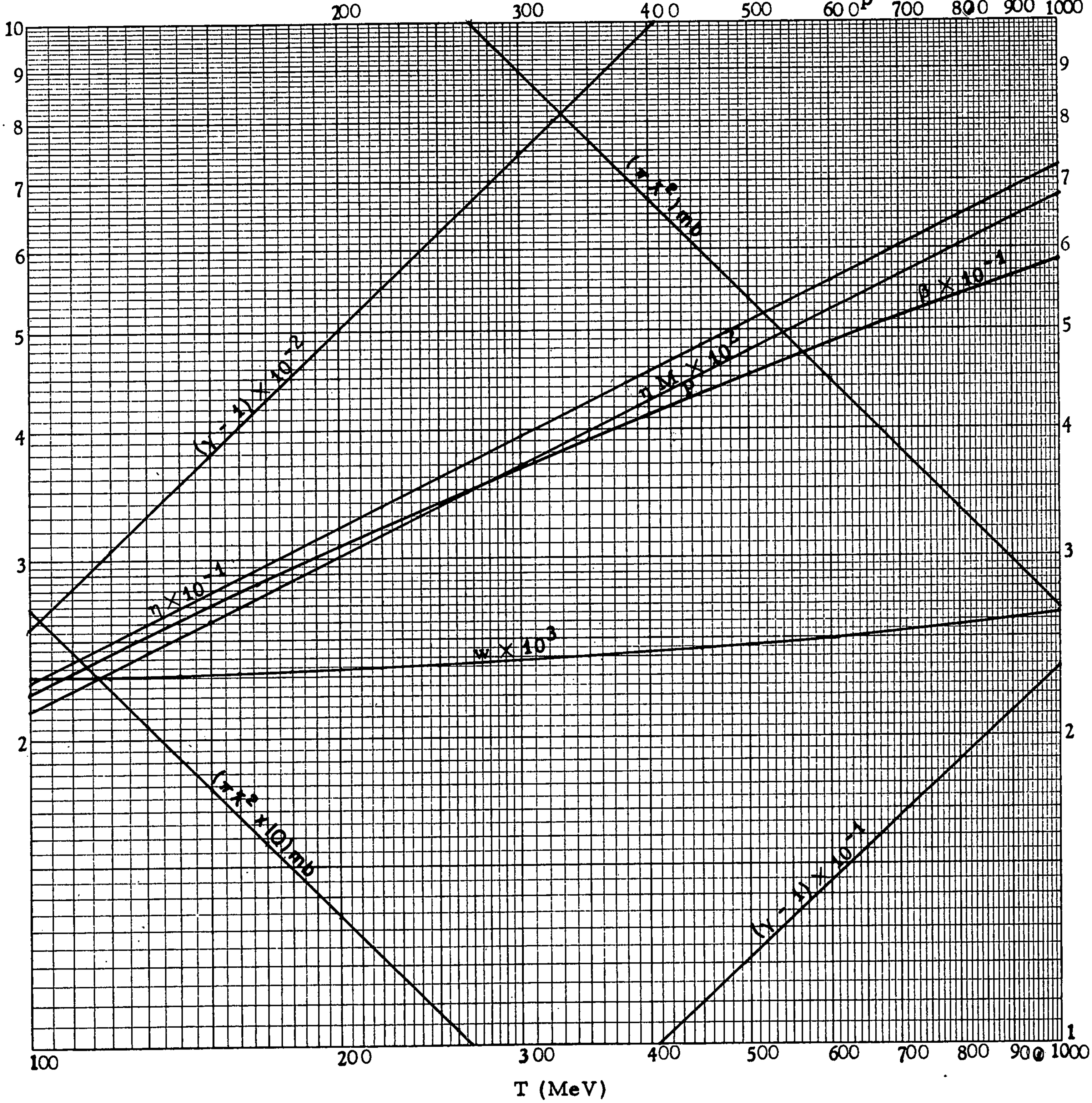
1b = 10^{-24} cm^2

100 MeV to 1 BeV

β, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Ξ^-} = 1318.4 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

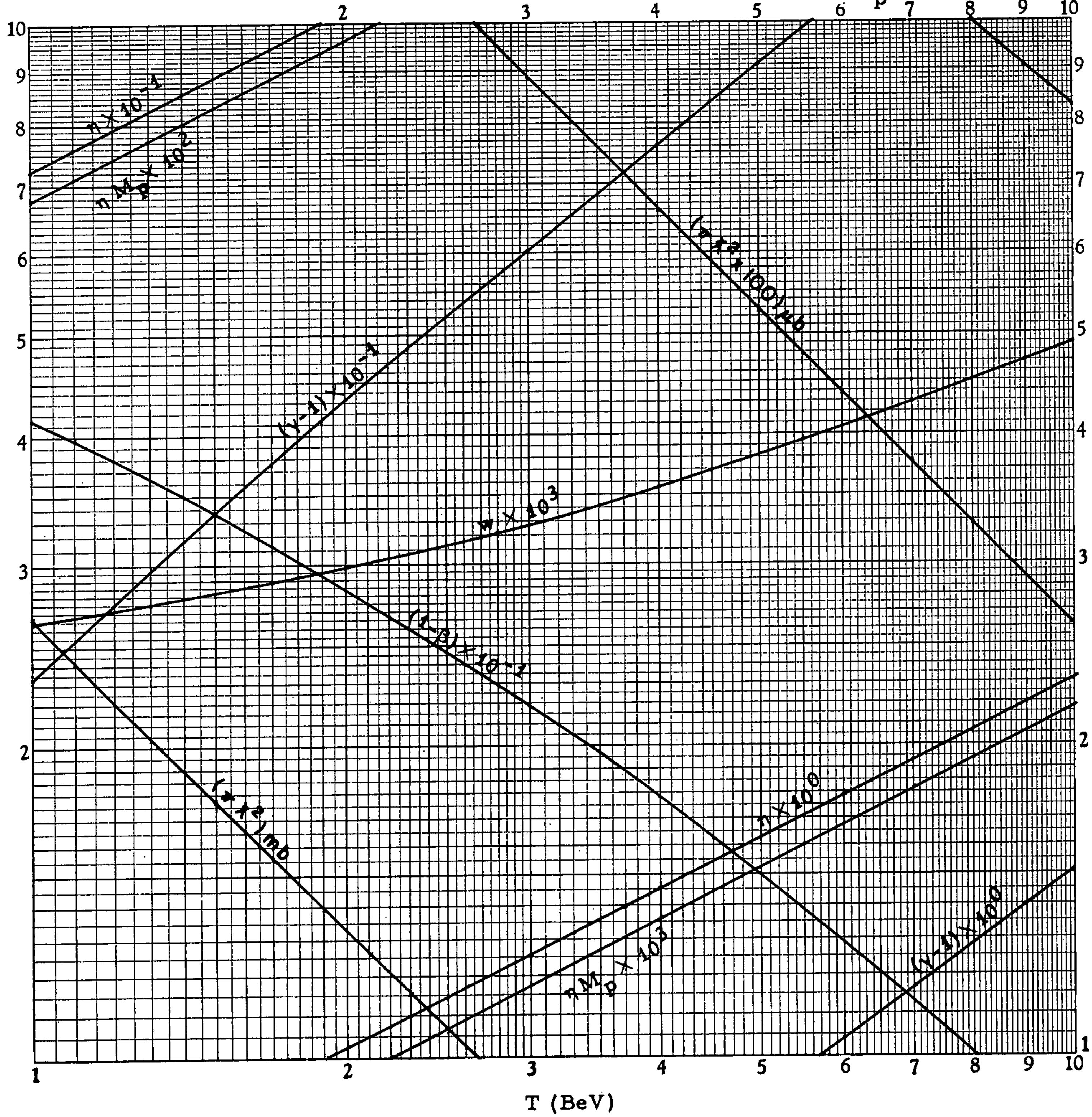
1b = 10^{-24} cm^2

1 BeV to 10 BeV

(1-β), (γ-1), η, ηM_p, w, $\pi \lambda^2$

M_{Ξ^-} = 1318.4 MeV

M_p = 938.213 MeV



w and ηM_p in MeV

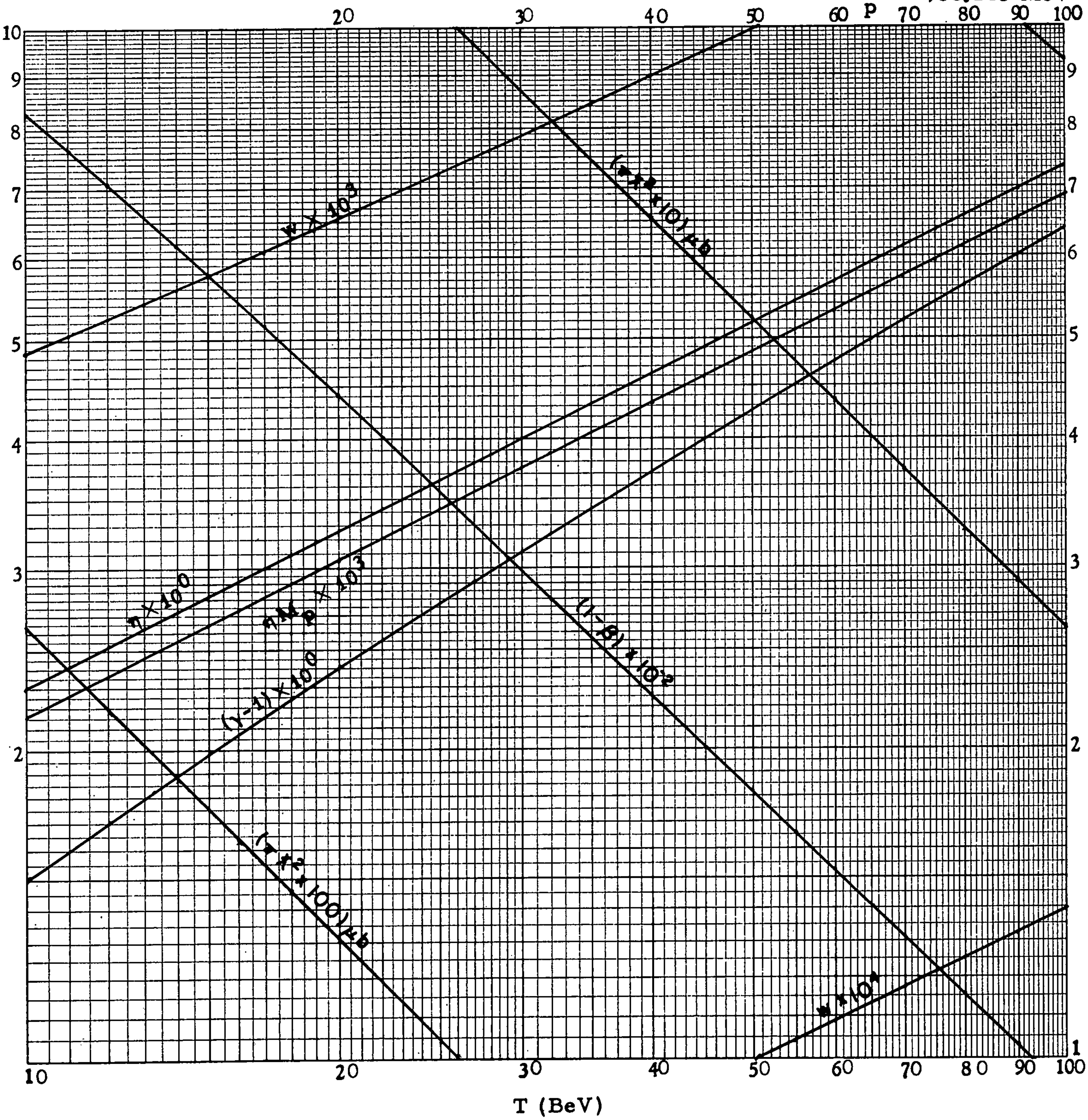
1b = 10^{-24} cm^2

10 BeV to 100 BeV

$(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi\lambda^2$

M_{Ξ^-} = 1318.4 MeV

M_p = 938.213 MeV



MNEMONIC DEVICE FOR RELATIVISTIC KINEMATIC FORMULAS[2]

Frank S. Crawford, Jr.

In order to obtain slide-rule accuracy for kinetic energies, one should use formulae expressed in terms of kinetic energies, because if one uses the total energy, the rest mass often uses up the first few significant figures.

The following mnemonic device enables one to write down the exact relativistic (R) formula, if one remembers the nonrelativistic (NR) one:

(a) Write the correct NR formula.

(b) To the rest energy of each moving particle add one-half the total kinetic energy in the center-of-mass (c.m.) system.

Example 1:

A particle of rest mass m_1 and LAB system kinetic energy T_1^0 is incident on a stationary particle of rest mass m_2. Letting c = 1, what is the total kinetic energy T in the c.m. system?

$$\text{(a) NR:}\quad T = T_1^0 \left(\frac{m_2}{m_2+m_1}\right)$$

$$\text{(b) R:}\quad T = T_1^0 \left(\frac{m_2}{m_2+m_1+(T/2)}\right)$$

To solve this quadratic expression numerically for T, it is easier and faster to consider this as a recursion formula, and use a slide rule, rather than to rewrite and solve by radicals. For example, a 600-MeV proton on a proton gives

$$T_{(n+1)} = 600 \left(\frac{938}{938+938+T_{(n)}/2}\right) = (0),\ \underset{\uparrow \text{NR}}{300},\ 278,\ \underset{\uparrow \text{R}}{279}, \cdots \text{MeV}.$$

Example 2:

Two particles of rest mass m_1 and m_2 share the kinetic energy $T = T_1 + T_2$ in their c.m. system. How do they divide-up T?

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2. From Am. J. Phys. 26, 376 (1958).

$$\text{(a) NR:} \quad T_1 = T\left(\frac{m_2}{m_2+m_1}\right);$$

$$\text{(b) R:} \quad T_1 = T\left(\frac{m_2+T/2}{m_2+t/2+m_1+T/2}\right) = T\left(\frac{m_2+T/2}{m_1+m_2+T}\right)$$

For instance, what is the kinetic energy of the μ meson (rest mass 106 MeV) in the decay of a π meson (rest mass 140 MeV), $\pi \rightarrow \mu + \nu$?

$$T_\mu = 34\,\frac{(0+34/2)}{(0+106+34)} = 4.13 \text{ MeV.}$$

Example 3:

A single particle m_1 moves relative to the LAB system origin, where an infinite mass is located (so that the laboratory and c.m. systems are equivalent). Express the particle's kinetic energy in terms of its momentum and rest energy.

$$\text{(a) NR:} \quad T = \frac{p^2}{2m};$$

$$\text{(b) R:} \quad T = \frac{p^2}{2(m+(T/2)}.$$

This example exposes the underlying root of the "mnemonic," in the exact relativistic formulae. We added an infinite mass at the origin so that we could use the "mnemonic" without modifying the phrase "in the c.m. system." One could perhaps say that the infinite mass provides the inertial frame in which special relatively is true.

SIMPLE RULES OF RELATIVISTIC KINEMATICS

Frank T. Solmitz

I. Energy-Angular Distributions

In many problems in particle physics one has to transform an energy-angular distribution from one frame of reference to another (say, from the LAB system to the c.m. systems of a reaction). This transformation is given simply by

$$\frac{1}{P}\frac{d^2n}{dW\,d\Omega} = \frac{1}{P'}\frac{d^2n}{dW'\,d\Omega'},$$

where P, W, and Ω are the momentum, total energy, and solid angle in one frame, and the corresponding primed quantities refer to the other frame. In other words, $(P\,dW\,d\Omega)$ is Lorentz-invariant.

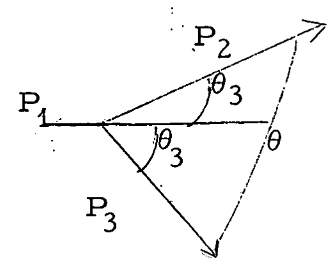
II. Two-Body Decays

Consider particle 1 decaying into particles 2 and 3, with respective masses and momenta M_1, M_2, M_3, P_1, P_2, P_3; let the angle between the directions of particles 2 and 1 be θ_2, and that between 3 and 1 be θ_3, and let $\theta \equiv 0_2 + \theta_3$. Given two of the momenta and one angle, or both angles and one momentum, one can identify the type of decay process with the help of kinematics tables and graphs.[3] However, one can often rule out certain possible identifications, even if only two quantities are measured, by the use of simple inequalities:

$$P_2 \sin\theta_2 = P_3 \sin\theta_3 \leq P_{c.m.}, \qquad \text{(a)}$$

$$P_2 \sin\theta_2 = P_3 \sin\theta \leq (M_1/M_2)\, P_{c.m.}, \qquad \text{(b)}$$

$$P_1 \sin\theta_3 = P_2 \sin\theta \leq (M_1/M_3)\, P_{c.m.}, \qquad \text{(c)}$$



Here $P_{c.m.}$ denotes the momentum of particle 2 (or 3) in the rest frame of particle 1:

$$P_{c.m.} = \frac{1}{2M_1}\left[(M_1+M_2+M_3)(M_1-M_2-M_3)(M_1+M_2-M_3)(M_1-M_2+M_3)\right]^{1/2}.$$

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[3] See, for example, Howard S. White, Identification Curves for Heavy-Meson and Hyperon Decays, University of California Radiation Laboratory Report UCRL-3514, September (unpublished).

Equation (a) is a consequence of the invariance of the momentum components transverse to the direction of flight of particle 1. Similarly, Eq. (b) follows from consideration of the momentum components transverse to the direction of flight of particle 2, and from noting that the momentum of particle 1 (or 3), in the rest frame of particle 2, is $(M_1/M_2)\ P_{c.m.}$.

SOLID ANGLE SUBTENDED BY A FINITE RECTANGULAR COUNTER

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A geometry problem that arises in particle detection is the calculation of the solid angle Ω subtended by a "finite" detector at a source of particles. For a rectangular detector and a point source, a simple formula can be obtained for the integrated solid angle. First consider the special case in which the point source P is located a distance c perpendicularly above a corner of a rectangle of length a and width b (see Fig. 3a). Then we have

$$\tan \Omega = \frac{ab}{r^2_{eff}} ,$$

where ab = area of rectangle,

$r_{eff} = \sqrt{cd}$ = geometric mean of smallest and largest distances from P to the rectangle,

$d = (c^2 + a^2 + b^2)^{1/2}$.

Thus the finite solid-angle formula is obtained from that of an infinitesimal detector by replacing r^2 by r^2_{eff}, and Ω by $\tan \Omega$.

The above holds only for the special case in which the perpendicular from P to the plane of the detector intersects one corner of the detector. We can now use this result to obtain the solid angle subtended by a rectangle oriented arbitrarily with respect to P. Let σ be the intersection with the plane of the rectangle of the perpendicular from P to the plane of the rectangle. If σ lies inside the rectangle (Fig. 3b), it implies the four sub-rectangles A, B, C, and D; we simply apply the formula to them and add the results. If σ lies outside the rectangle (Fig. 3c), then we apply the formula to the four rectangles A + B + C + D, B + C, C + D, and C, and find the desired quantity from A = (A+B+C+D) - (C+D) - (B+C) + C.

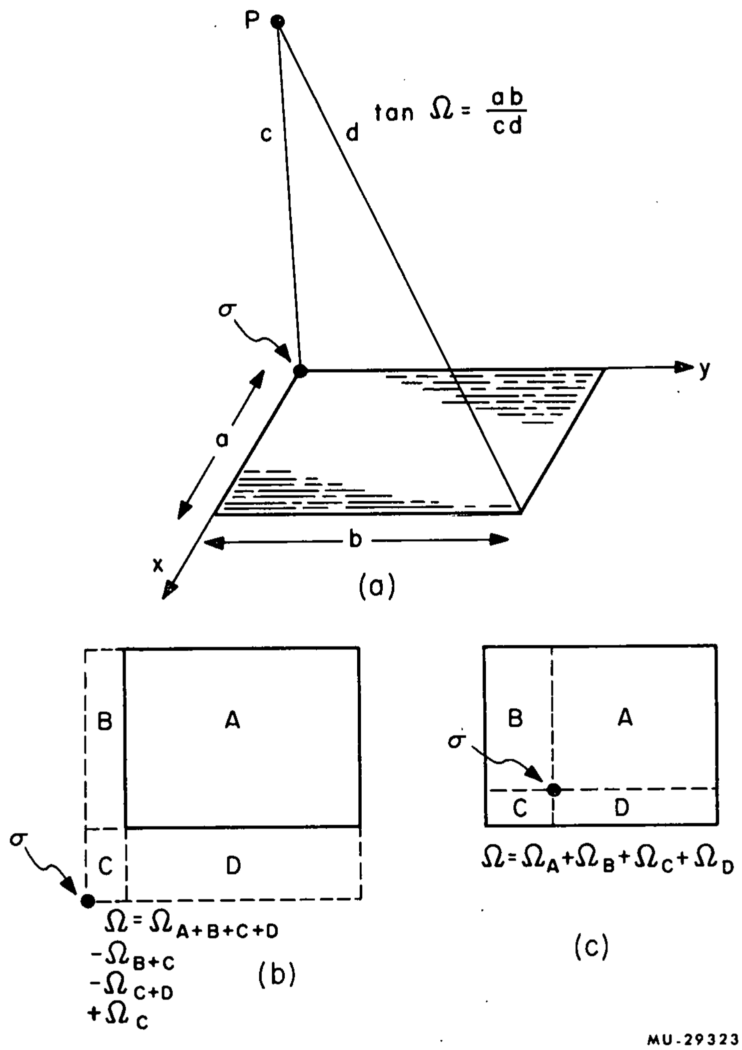

Fig. 3. Geometrical solution for the solid angle subtended by a finite rectangular counter.