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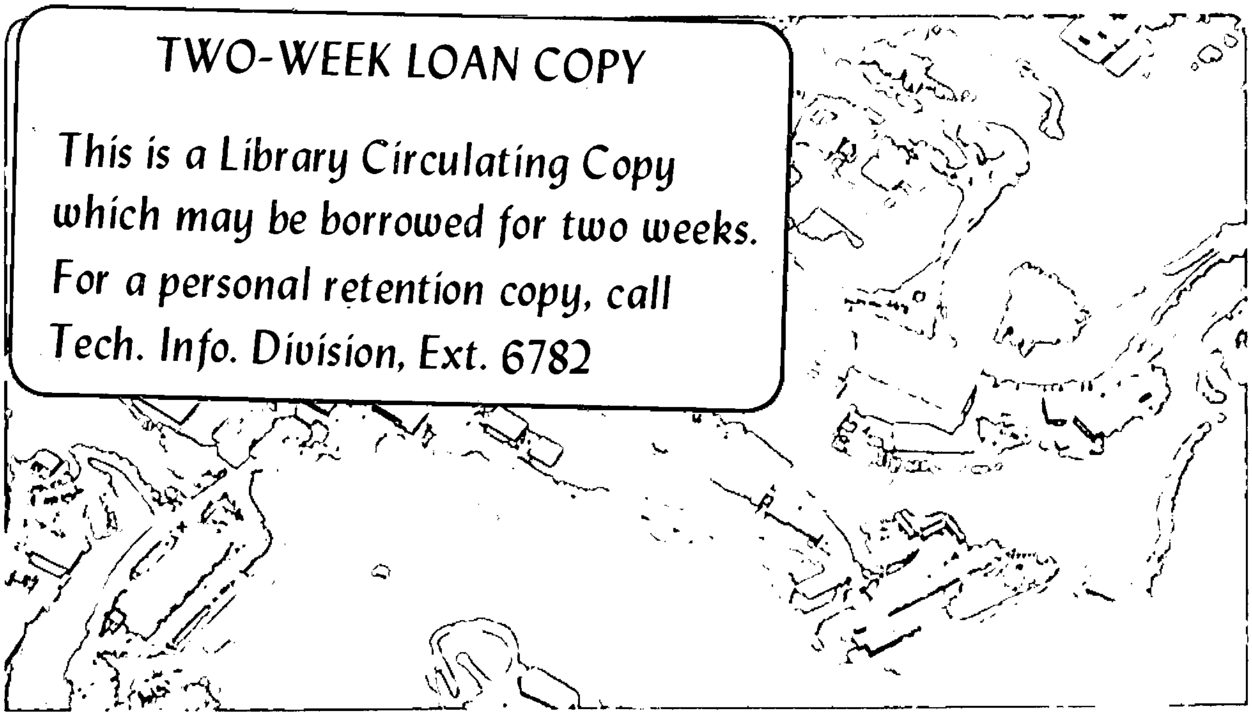

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A TEST OF INTEGRAL AND FRACTIONAL CHARGE QUARK MODELS*

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ABSTRACT

Properties of η(549) and η'(958) are used to measure the quark charges. The result strongly favors fractional charges.

It is very difficult to construct experimental tests of the integral charge quark model[1] if the color threshold E_c is assumed to be far above presently accessible energies. The radiative widths of η(549) and η'(958) may be the only presently measured quantities which provide a test independent of E_c. The η' width has recently been measured in two mutually consistent experiments[2] and for the first time a complete analysis of the problem is possible.

Previous analyses[3,4] require problematical assumptions: they assume equality of octet and singlet PCAC constants, $F_8 = F_1$, and/or they rely heavily on the PCAC extrapolation for η and η'. Here I present an analysis which is independent of the values of F_1 and F_8 and the η - η' mixing angle θ. For the fractional charge model the result depends only on vector meson dominance and flavor SU(3) symmetry (PCAC

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is <u>not</u> used) and is insensitive to a possible "glueball" component in the η'. For the integral charge model the result does depend on the PCAC extrapolation—but in a form which is less sensitive than earlier analyses--and would be affected by a large glueball component in η'.

I will define a quantity ξ^2 such that $\xi^2 = 1$ for fractional charges and $\xi^2 = 4$ for integral charges. The experimental value is $\xi^2 = 1.15 \pm .25$ which strongly favors the fractional charge model.

With a very large value of E_c the integral charge model might approximately resemble Q. C. D. in the presently accessible physics of the color singlet sector. The electric current of the integral charge model is the sum of color singlet and octet components

$$J_I = J_1 + J_8 \tag{1}$$

where J_1 is precisely the current of the fractional charge model

$$J_F = J_1 \tag{2}$$

Therefore in the color singlet sector first order electromagnetic amplitudes are equal in the two models.

For second order amplitudes between color singlet states we have

$$\langle J_I\, J_I \rangle_1 - \langle J_F\, J_F \rangle_1 = \langle J_8\, J_8 \rangle_1 \tag{3}$$

The product of two octets contains a singlet projection so the right hand side may be nonvanishing and it may be possible to distinguish between the two models. But more than group theory is involved. If the right hand side is of the same order as the terms on the left hand side it means a "low" energy amplitude probes the quanta of an arbitrarily high energy scale. If we imagine writing a dispersion relation for $\langle J_8\, J_8 \rangle_1$ the integral over the absorptive part would be suppressed by powers of E_c and only a purely real contribution, like a fixed pole, could survive. In configuration space we require that the two currents interact instantaneously before the color and electric charge can fluctuate.[5] No convincing arguments have been presented[6] that these requirements are met in the parton model for $\gamma N \to \gamma X$ or $\gamma^* \gamma \to X$.

The chiral anomaly[7] probes the ultraviolet limit of the theory[8] and is one context in which we can be sure that the right side of eq. (3) is not suppressed. Since J_8 is a flavor singlet we must consider the two photon decay of a flavor SU(3) singlet pseudoscalar meson, $X_1 \to \gamma\gamma$. In quark model folklore the η and η' are mixtures of singlet and octet $\bar{q}q$ states

$$\eta = \cos\theta \; X_8 - \sin\theta \; X_1$$
$$\eta' = \sin\theta \; X_8 + \cos\theta \; X_1 \qquad (4)$$

hence our interest in their radiative decays.

It is generally presumed that X_1 is not a Goldstone boson in the massless quark limit. But ∂A_1, the singlet axial divergence, is an acceptable interpolating field for X_1 and we can certainly use the standard current algebra techniques to compute $X_1 \to \gamma\gamma$ at the off-shell point $m_{X_1} = 0$. The U(1) problem arises in (1) whether we can trust the extrapolation to the physical amplitudes and (2) whether η and η' contain a significant pure gluon component in addition to the $\bar{q}q$ states X_1 and X_8. Since $m_{\eta'} = 958$ MeV., it is clear that the extrapolation is potentially dangerous, regardless of the nature of the chiral limit. And the apparent smallness of the mixing angle ($\theta = -11^{\circ}$ from the naive mass formula) suggests that η' may contain a substantial purely gluonic component.

In the standard current algebra formulation[3,4] these problems are ignored. The low energy theorems for $X_i \to \gamma\gamma$ are

$$\mathcal{F}_8(0) = -\frac{\alpha}{\sqrt{3}\pi}\frac{1}{F_8} \qquad (5)$$

$$\mathcal{F}_1(0) = 2\sqrt{2}\xi \frac{F_8}{F_1} \mathcal{F}_8(0) \qquad (6)$$

The factor ξ in eq. (6) is given by $\xi = 1$ for fractional charges and $\xi = 2$ for integral charges. (The factor 2 is due to the right side of eq. (3): in the integral charge model

$$\mathcal{F}_1(0) \propto \langle X_1 | J_1 J_1 | 0 \rangle + \langle X_1 | J_8 J_8 | 0 \rangle = 2 \langle X_1 | J_1 J_1 | 0 \rangle .)$$

The η and η' amplitudes are then obtained from (5) and (6) using the mixing (4). If one assumes $\theta = -11^{\circ}$ and $F_1 = F_8 = F_\pi$ (so-called "nonet symmetry") then for the fractional (integral) charge model the result is $\Gamma(\eta \to \gamma\gamma) = 390$ (720) eV. and $\Gamma(\eta' \to \gamma\gamma) = 6.1$ (26) keV. The experimental values[9] are $\Gamma(\eta \to \gamma\gamma) = 324 \pm 46$ eV. and $\Gamma(\eta' \to \gamma\gamma) = 5.9 \pm 1.6$ keV.

These results favor the fractional charge model. But they are not compelling, not only for the reasons associated with the U(1) problem but also because they rely too strongly on the assumption that $F_1 = F_8$. If instead we let $F_1 \cong 2F_8$, the factor $\xi = 2$ in eq. (6) is cancelled and the results favor the integral charge model. $F_1 = F_8$ is not a consequence of any symmetry but depends on dynamics. In particular in the naive quark model it would follow from the equality of the X_1 and X_8 wave functions, which would be plausible if η and η' were ideally mixed like ω and ϕ. But η and η' are far from being ideally mixed: while $(m_\omega^2 + m_\phi^2)/2m_{k^*}^2 = 1.04$ we have $(m_\eta^2 + m_{\eta'}^2)/2m_k^2 = 2.48$. The deviation from one measures the deviation from ideal mixing. For η and η' the deviation is of order one. In the naive quark model it means that the difference in the X_1 and X_8 binding energies is large, of the same order as the masses. So the wave functions may be very different as may F_8 and F_1. To decide between the two quark charge assignments we want a method which does not require the dubious "nonet symmetry" assumption.

One possibility[4] is to use the low energy theorems for $\eta \to \pi\pi\gamma$ and $\eta' \to \pi\pi\gamma$, also determined by the chiral anomaly, to determine F_8/F_1. But the dipion in both decays is in the ρ channel and strongly dominated by the ρ. There is no unambiguous prescription for how to include the effect of the ρ on the extrapolation. Different choices give quite different results and none give a satisfactory description of $\Gamma(\eta' \to \pi\pi\gamma)/\Gamma(\eta' \to \gamma\gamma)$. A large component of $\eta' \to \pi\pi\gamma$ might be "elementary" $\rho\gamma$ production and be unrelated to the low energy theorem for $\eta' \to \pi\pi\gamma$.

The use of PCAC for η and η' is at best on the empirical footing of vector meson dominance. ρ, ω, and ϕ dominance succeed at the $\lesssim 25\%$ level.[10] It should be more reliable to use the well-tested vector dominance approximation to relate $\rho \to \eta\gamma$ and $\eta' \to \rho\gamma$ with $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ than to extrapolate the four low energy theorems for $\eta/\eta' \to \gamma\gamma/\pi\pi\gamma$. The analogous relationship[11] between $\omega \to \pi_0\gamma$ and $\pi_0 \to \gamma\gamma$ is accurate to 7% in the rate.

Saturating with ρ, ω, and ϕ, using SU(3) symmetry, and taking ϕ and ω to be ideally mixed (just as in ref. (11)), we find

$$\mathcal{F}_1 = \xi \sum_{V=\rho,\omega,\phi} \frac{e}{f_V} \mathcal{M}(X_1 \to V\gamma)$$

$$= \xi \cdot \frac{4}{3}\frac{e}{f_\rho} \mathcal{M}(X_1 \to \rho\gamma) \qquad (7)$$

The important feature is the factor ξ. For fractional charges $\xi = 1$ and eq. (7) makes no use of current algebra or PCAC. But for integral charges $\xi = 2$ and eq. (7) incorporates part of the content of the low

energy theorem (6). As explained below eq. (6) the factor $\xi = 2$ occurs because $\mathcal{F}_1(0)$ receives equal contributions from $\langle X_1 | J_1 J_1 | 0 \rangle$ and $\langle X_1 | J_8 J_8 | 0 \rangle$ while only the former is dominated by ρ, ω, and ϕ. The only use of PCAC is to assume that the ratio $\langle X_1 | J_1 J_1 | 0 \rangle / \langle X_1 | J_8 J_8 | 0 \rangle = 1$ extrapolates smoothly from the low energy point. This is clearly a more mild assumption than is required by the current algebra analysis: there is no extrapolation of the magnitudes of the amplitudes (5) and (6) and no assumption about F_1, F_8, or θ. It is easy to turn eq. (7) into an experimental determination of ξ. We use $\mathcal{F}_1^2 = \mathcal{F}_\eta^2 + \mathcal{F}_{\eta'}^2 - \mathcal{F}_8^2$ and the analagous relationship for $\mathcal{M}(X_1 \to \rho\gamma)$. By SU(3) symmetry $\mathcal{F}_8^2 = \frac{1}{3}\mathcal{F}_{\pi_0}^2$ and $\mathcal{M}(X_8 \to \gamma\rho)^2 = \frac{1}{3}\mathcal{M}(\omega \to \pi_0\gamma)^2$. Finally then[12,13]

$$\xi^2 = \left(\frac{3f_\rho}{4e}\right)^2 \frac{\mathcal{F}_\eta^2 + \mathcal{F}_{\eta'}^2 - \frac{1}{3}\mathcal{F}_{\pi_0}^2}{\mathcal{M}_{\eta'\rho\gamma}^2 + \mathcal{M}_{\rho\eta\gamma}^2 - \frac{1}{3}\mathcal{M}_{\omega\pi_0\gamma}^2} = 1.15 \pm .25 \qquad (8)$$

to be compared with

$$\xi^2 = \begin{cases} 1 & \text{Fractional Charge} \\ 4 & \text{Integral Charge} \end{cases} \qquad (9)$$

The $\pm .25$ uncertainty is due primarily to the experimental uncertainty[14] in $\Gamma(\rho \to \eta\gamma)$. The substantial uncertainties in the η' rates tend to cancel in eq. (8). The agreement with the fractional charge model is similar to the success of the relationship between $\Gamma(\pi_0 \to \gamma\gamma)$ and $\Gamma(\omega \to \pi_0\gamma)$ and is at a plausible level for an exercise in SU(3) symmetry and vector dominance.

Eq. (8) does not take account of a possible "glueball" admixture in η and η'. Let G_0 be the hypothetical pure glueball $J^P = 0^-$ state. Then the physical states η, η', and G would be mixtures of

η_8, η_1, and G_0. The state G presumably remains to be identified at a mass above the η'. Since $m_\eta \cong m_{\eta_8} = \sqrt{\frac{4}{3} m_K^2 + \frac{1}{3} m_\pi^2} = 580$ MeV, η is probably strongly dominated by η_8. η' would then contain most of the small remaining η_8 component. The heavier state G would be made almost completely of G_0 and η_1.

Neglecting the very small η_8 component in G, the mixing would be described by two angles

$$\begin{aligned} \eta &= c_1 \eta_8 - s_1 c_2 \eta_1 + s_1 s_2 G_0 \\ \eta' &= s_1 \eta_8 + c_1 c_2 \eta_1 - c_1 s_2 G_0 \\ G &= s_2 \eta_1 + c_2 G_0 \end{aligned} \qquad (10)$$

where $c_i \equiv \cos \theta_i$ and $s_i \equiv \sin \theta_i$.
In the fractional charge model the unmixed gluon state G_0 would have no coupling to photons,[15] $\mathcal{F}_{G_0} = \mathcal{M}_{G_0} = 0$, in which case we find

$$c_2^2 \mathcal{F}_1^2 = \mathcal{F}_\eta^2 + \mathcal{F}_{\eta'}^2 - \mathcal{F}_8^2 \quad \text{and} \quad c_2^2 \mathcal{M}_1^2 = \mathcal{M}_\eta^2 + \mathcal{M}_{\eta'}^2 - \mathcal{M}_8^2 .$$

Taking the ratio of these equations the factor c_2^2 cancels and the result is again eq. (8). So in the fractional charge model eq. (8) is a plausible result even if η' does have a large glueball component.

In the integral charge model gluons carry electric charge so that $\mathcal{F}_{G_0}$ and $\mathcal{M}_{G_0}$ need not vanish. A large value of θ_2 in eq. (10) might induce large corrections in eq. (8). For this reason and because of the need to extrapolate the ratio $\langle X_1 | J_1 J_1 | 0 \rangle / \langle X_1 | J_8 J_8 | 0 \rangle = 1$ to the η and η' mass shells, the derivation of eq. (8) is on a less secure footing for the integral charge model than for the fractional charge model. But there is no reason to expect these effects to cause the integral charge model to agree with the experimental result recorded in eq. (8). That result

is rather compelling evidence in favor of the fractional charge assignment.

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13) According to ref. (10) the effect of finite width corrections is to renormalize the $\rho - \gamma$ coupling at $k^2 = 0$ to $f^2_{\rho\pi\pi}/4\pi = 2.95 \pm .06$. I am grateful to D. Yennie for correspondence on this point.

14) D. E. Andrews et. al., op. cit. They cite a phase ambiguity which I have resolved by choosing the solution consistent with SU(3) symmetry. For this solution $\Gamma(\rho \to \eta\gamma) = 50 \pm 13$ keV. (The other solution would give $\xi^2 = 0.8 \pm 0.2$).

15) I am grateful to Tom Weiler for emphasizing this to me.

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