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ANGULAR DISTRIBUTION OF UNDULATOR POWER FOR AN ARBITRARY DEFLECTION PARAMETER K

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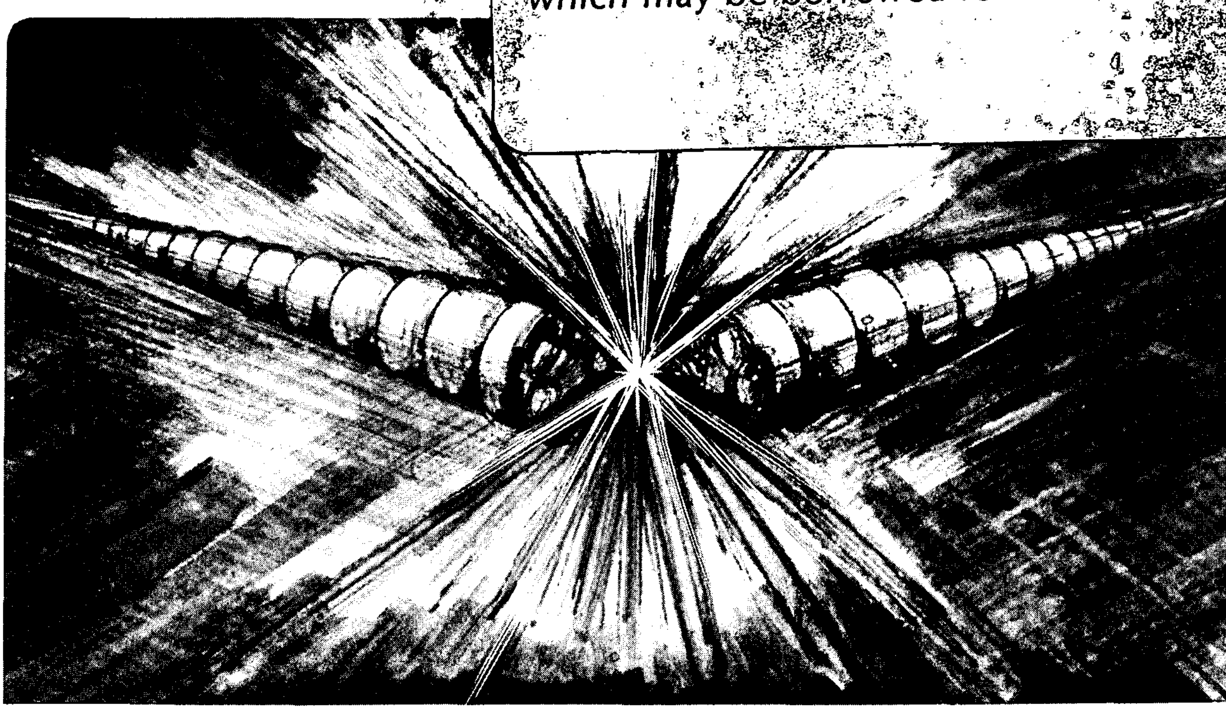

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ABSTRACT

A calculation of the angular distribution of power generated from an undulator, integrated over all frequencies, is presented. The result, valid for any arbitrary value of the deflection parameter K, reduces to the known expressions in the cases $K \to \infty$ and $K \to 0$.

*This work was supported by the Office of Basic Energy Sciences, U.S. Department of Energy, under Contract # DE-AC03-76SF00098.

One of the important considerations in the design of optical elements for future, advanced synchrotron radiation sources is the angular distribution of frequency-integrated power $dP/d\Omega$ from undulators. The expression for $dP/d\Omega$ is well-known in the limiting cases $K \to \infty$ (the wiggler limit) and $K \to 0$ (the weak undulator limit), where K is the usual deflection parameter given by

$$K = 0.934\ \lambda_u(\mathrm{cm})\ B_o(\mathrm{T}) \tag{1}$$

Here λ_u is the period of the undulator and B_o is the peak magnetic field. In this paper an exact expression for $dP/d\Omega$, valid for an arbitrary K, is derived and discussed.

Schwinger [1] gave a general formula for $dP/d\Omega$ for an arbitrary electron trajectory. For electrons following a sinusoidal trajectory, as shown in Fig(1), his expression reduces to

$$\frac{dP}{d\Omega} \equiv \frac{d^2P}{d\theta d\psi} = P_T\,\frac{21\gamma^2}{16\pi K}\,G(K)f_K(\gamma\theta,\gamma\psi) \tag{2}$$

Here P_T is the total power (integrated over angles and frequencies) given by

$$P_T = \frac{N}{6}\,Z_o\,I\,\frac{2\pi ec}{\lambda_u}\,\gamma^2K^2 \tag{3}$$

$$= 0.633\ E^2(\mathrm{GeV})\ B_o^2(\mathrm{T})L(\mathrm{m})I(\mathrm{A})\ (\mathrm{kW}),$$

where

N = number of the undulator periods

Z_o = vacuum impedance (377 Ohms)

I = electron current

e = electron charge

c = velocity of light

γ = electron's relativistic energy/rest energy

E = electron's energy

L = length of the undulator

G(K) is a normalization factor given by

$$G(K) = K \frac{(K^6 + \frac{24}{7} K^4 + 4K^2 + \frac{16}{7})}{(1 + K^2)^{7/2}} \tag{4}$$

Finally $f_K(\gamma\theta,\gamma\psi)$ is a factor which gives the angular dependence as follows:

$$f_K(\gamma\theta,\ \gamma\psi) = \frac{16K}{7\pi G(K)} \int_{-\pi}^{\pi} d\alpha\ \sin^2\alpha \left(\frac{1}{D^3} - \frac{4(\gamma\theta - K\cos\alpha)^2}{D^5} \right), \tag{5}$$

where

$$D = 1 + (\gamma\psi)^2 + (\gamma\theta - K\cos\alpha)^2. \tag{6}$$

The function f_K is normalized so that

$$f_K\ (0,0) = 1 \tag{7}$$

The integral in Eq(5) can be evaluated using the contour integration technique. The result is complicated for the general case, and it is better to use numerical integration of Eq(5). For the special case $\theta = 0$, however, the contour integration method yields the following expression:

$$f_K(0,\gamma\psi) = \frac{1}{7G(K)(1+(\gamma\psi)^2)^{5/2}} \left[\frac{12 + 16\varepsilon}{(1+\varepsilon)^{3/2}} - \frac{5 + 16\varepsilon + 16\varepsilon^2}{(1+(\gamma\psi)^2)(1+\varepsilon)^{7/2}} \right], \tag{8}$$

where

$$\varepsilon = \frac{1 + (\gamma\psi)^2}{K^2} \tag{9}$$

When $K \to \infty$, Eq(5) becomes

$$K \to \infty:\ f_K(\gamma\theta,\gamma\psi) = \sqrt{1 - (\gamma\theta/K)^2} \left\{ \frac{1}{(1+(\gamma\psi)^2)^{5/2}} + \frac{5(\gamma\psi)^2}{7(1+(\gamma\psi)^2)^{7/2}} \right\}. \tag{10}$$

The factor in the curly bracket above is familiar from the expression for the angular distribution of power from a bending magnet [1], while the factor $\sqrt{1-(\gamma\theta/K)^2}$ arises because the magnetic field observed from a horizontal angle θ is reduced from B_o by the same factor. In the limit $K \to 0$, one can derive from Eq(5) that

$$K \to 0:\ f_K(\gamma\theta,\gamma\psi) = \frac{1+2\gamma^2(\psi^2-\theta^2)+(\gamma^2(\theta^2+\psi^2))^2}{(1+\gamma^2(\theta^2+\psi^2))^5} \tag{11}$$

This expression was first derived by Hofmann [2] by considering an oscillating dipole moving with relativistic velocity.

The functions $f_K(0,\gamma\psi)$ and $f_K(\gamma\theta,0)$ are plotted for values of K ranging between 0.25 and ∞ in Fig(2) and Fig(3). One observes in both cases that the curves for $K \geqslant 1$ quickly approach the curves for the limiting case corresponding to $K = \infty$.

From Eq(2) and Eq(7), the power density in the forward direction is

$$\frac{dP}{d\Omega}(0,0) = P_T \frac{21\ \gamma^2}{16\pi K} G(K) \tag{12}$$

$$= 10.84\ B_o(T)E^4(\text{GeV})I(A)NG(K)\ \ (\text{W}/(\text{mr})^2)$$

The function G(K), which is given by Eq(4), is plotted in Fig(4). Its limiting behavior is

$$\begin{aligned} G(K) &\rightarrow 1 \quad (K \rightarrow \infty) \\ &\rightarrow \frac{16}{7} K \quad (K \rightarrow 0) \quad . \end{aligned} \tag{13}$$

If one approximates $dP/d\Omega$ by a rectangular distribution, then

$$\frac{dP}{d\Omega} = \frac{P_T}{(\Delta\psi)(\Delta\theta)}, \quad -\frac{\Delta\psi}{2} \leqslant \psi \leqslant \frac{\Delta\psi}{2}, \quad -\frac{\Delta\theta}{2} \leqslant \theta \leqslant \frac{\Delta\theta}{2} \tag{14}$$

$$= 0, \text{ otherwise.}$$

From Fig(2), one finds that $\Delta\psi \sim \gamma^{-1}$. The magnitude of $\Delta\theta$ can be obtained from Eqs(12), (13) and (14). The result is

$$\begin{aligned} \Delta\psi &\sim \frac{1}{\gamma} \quad (\text{all } K) \\ \Delta\theta &\sim \frac{1}{\gamma} \quad (K \rightarrow 0) \\ &\sim \frac{2K}{\gamma} \quad (K \rightarrow \infty) \quad . \end{aligned} \tag{15}$$

$\Delta\psi$ and $\Delta\theta$ in the above can roughly be identified as the FWHM angular widths of the power distribution.

At a given frequency, the angular distribution of power per unit frequency interval $d^2P/d\omega d\Omega$ is concentrated in narrow rings in angular space [3]. The radii of these rings change as the frequency changes. Thus, $dP/d\Omega$, which is obtained by integrating $d^2P/d\omega d\Omega$ over ω, is a smooth function of θ and ψ. This is shown in Fig(5).

Of particular importance is the behavior of $d^2P/d\omega d\Omega$ near $\omega = n\omega_1$, where ω_1 is the fundamental frequency of undulator radiation given by

$$\omega_1 = \frac{2\pi c}{\lambda_u} \frac{1+K^2/2}{2\gamma^2} \quad . \tag{16}$$

At these frequencies, $d^2P/d\omega d\Omega$ develops a peak around the forward direction $\theta = \psi = 0$. For n=1, the rms angular widths of the central cone are

$$\begin{aligned} (\Delta\psi)_c &= \sqrt{\frac{1+K^2/2}{2N\gamma^2} + \sigma_{y'}^2}, \\ (\Delta\theta)_c &= \sqrt{\frac{1+K^2/2}{2N\gamma^2} + \sigma_{x'}^2} \end{aligned} \tag{17}$$

Here $\sigma_{x'}$ and $\sigma_{y'}$ are the rms angular divergences of the electron beam in the horizontal and vertical directions, respectively. In an undulator beam line, a pinhole will select out the central cone, which contains the useful photon flux. The power transmitted through the central cone is

$$P_{cen} = \frac{dP}{d\Omega}(0,0) \times 2\pi(\Delta\psi)_c(\Delta\theta)_c \tag{18}$$

For synchrotron radiation sources, $\sigma_{x'}$ and $\sigma_{y'}$ are usually smaller than γ^{-1}, and the effects of the beam divergence on $dP/d\Omega$ are negligible. On the other hand, these angles are comparable to the central radiation cone of the fundamental radiation of an undulator, and thus must be taken into account.

Power generated from undulators for various synchrotron radiation sources, as well as other useful data, can be found in Reference [4].

References

[1] J. Schwinger, Phys. Rev. 75 (1949)1912

[2] A. Hofmann, Nucl. Inst. Methods 152 (1978)17

[3] See, for example, D. Attwood, K. Halbach and K-J. Kim, Science 228 (1985)1265

[4] D. Attwood and K-J. Kim, these proceedings

Figure Captions

Fig(1) The electron trajectory and coordinate system. The electron moves on a sinusoidal trajectory in horizontal plane x–z. θ and ψ are the angles of observation in the horizontal and vertical directions, respectively.

Fig(2) The behavior of the function $f_K(0,\gamma\psi)$

Fig(3) The behavior of the function $f_K(\gamma\theta,0)$

Fig(4) The behavior of the function $G(K)$

Fig(5) Illustration of how $dP/d\omega d\Omega$ adds up to $dP/d\Omega$.

Coordinate system

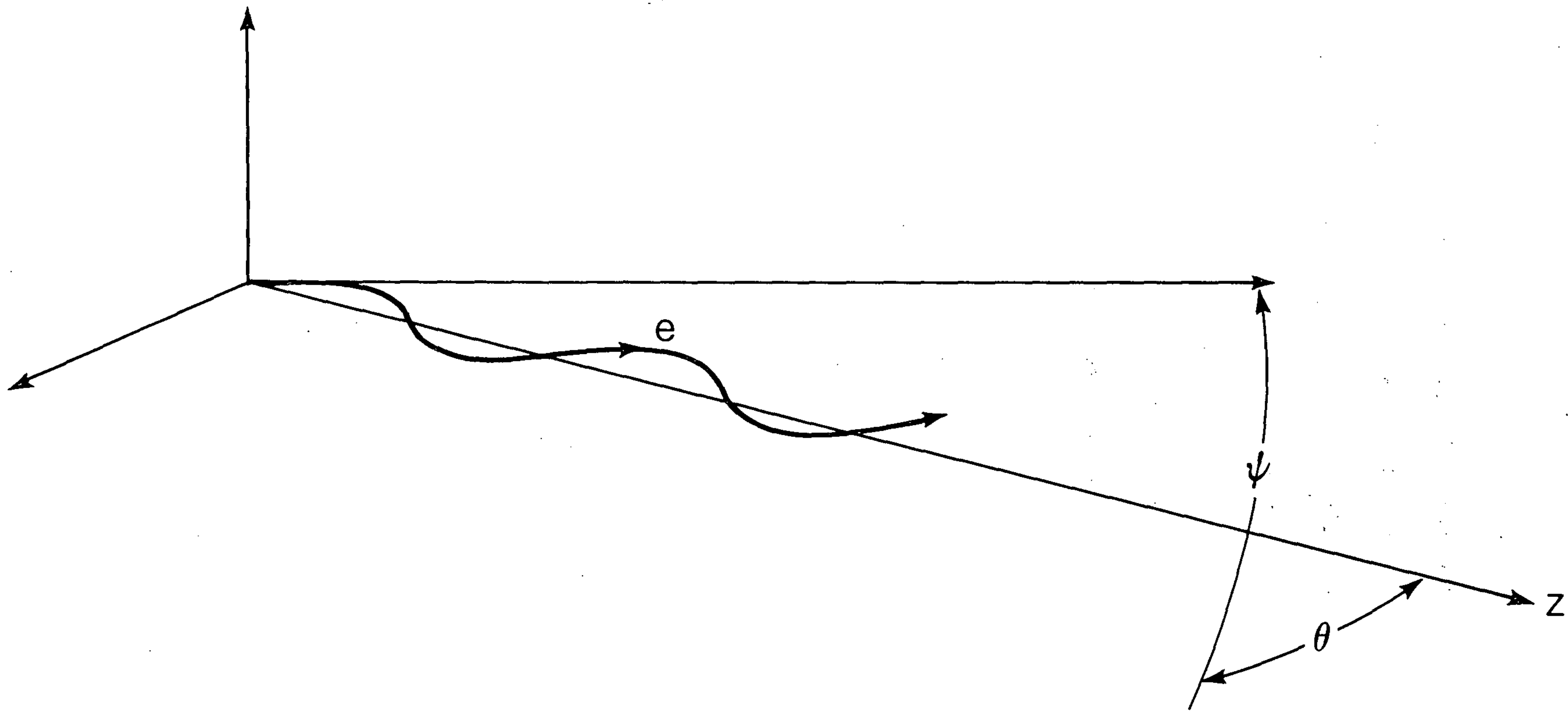


Figure 1

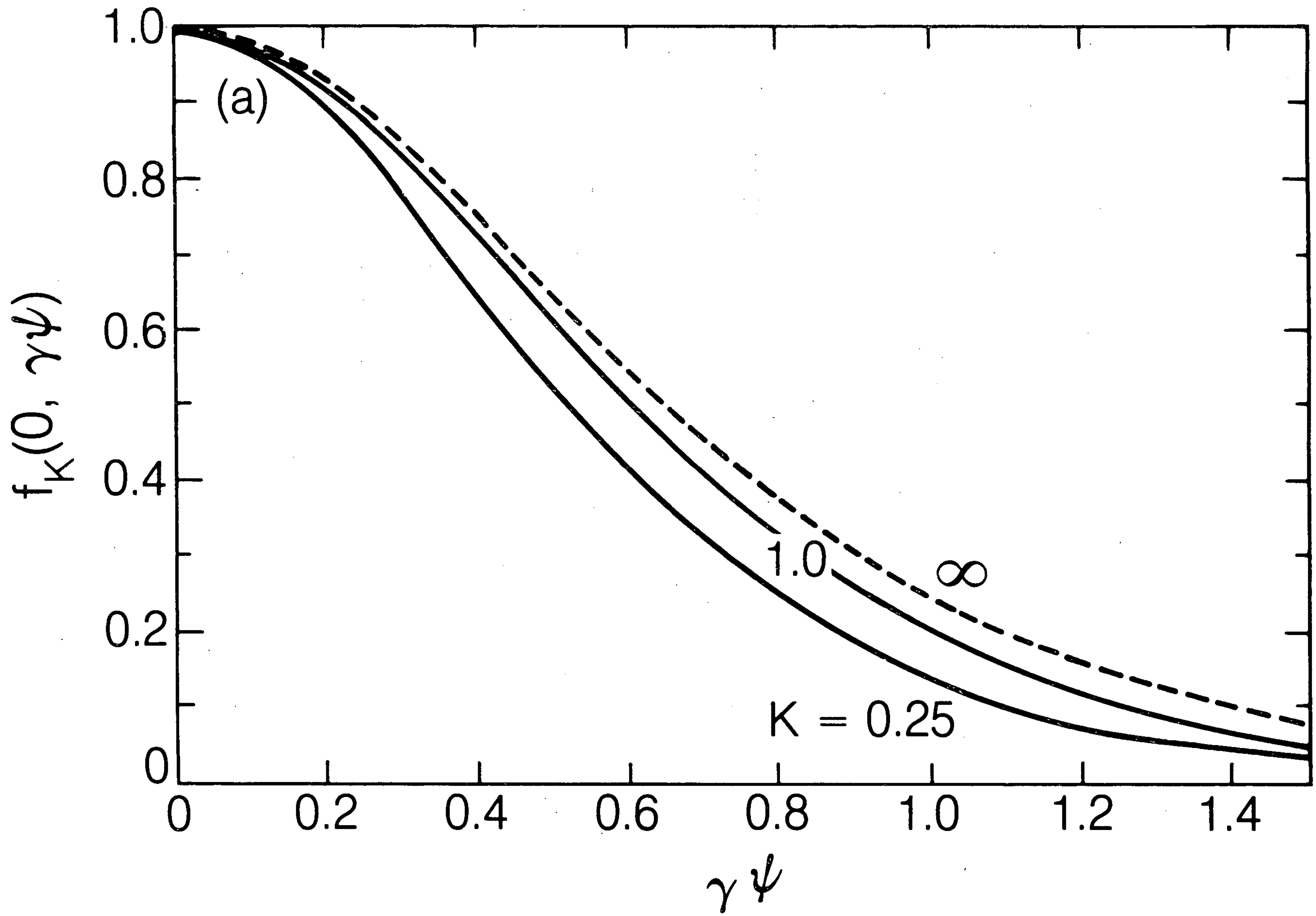
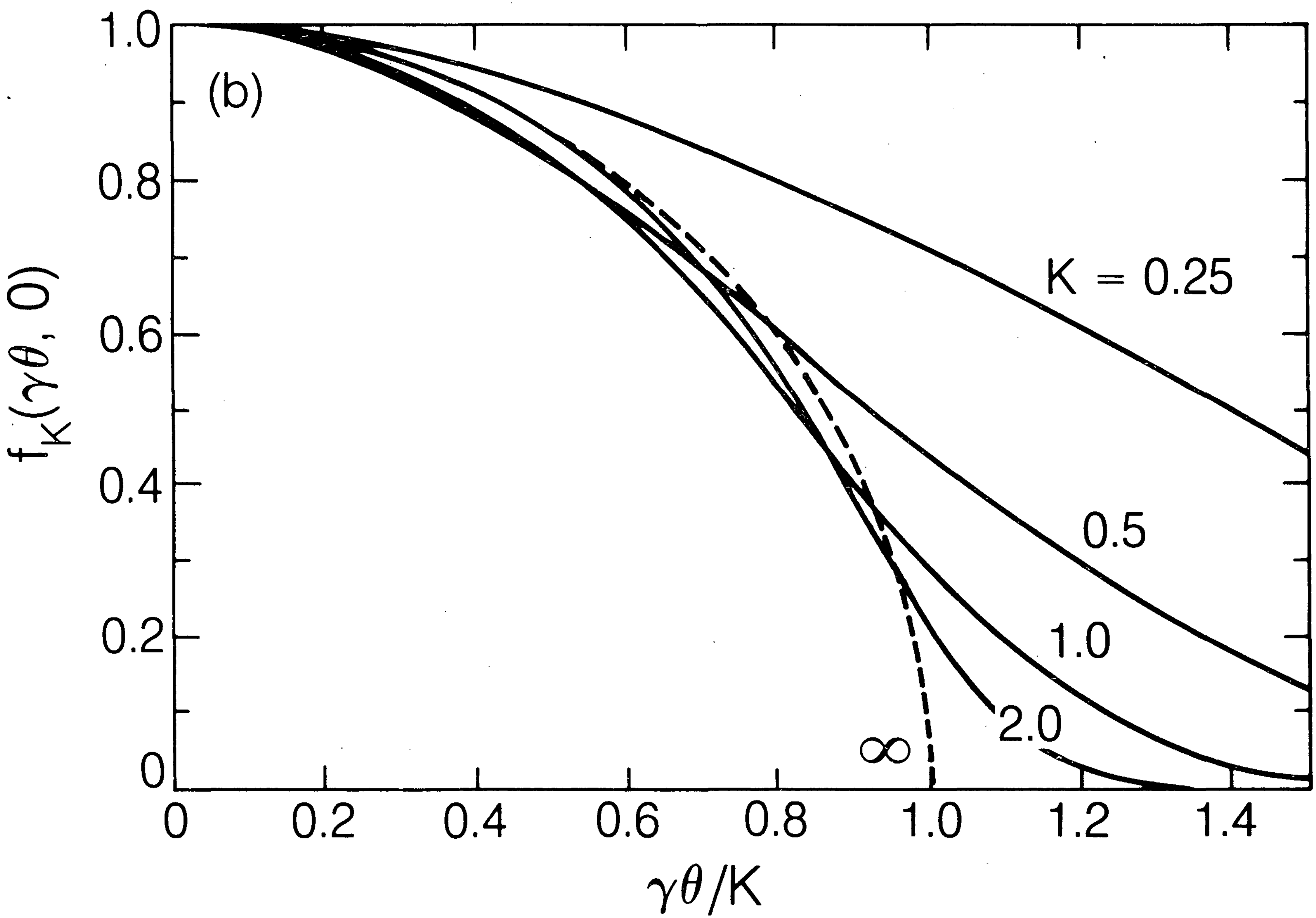

1.0
0.8
0.6
0.4
0.2
0
(a)
$f_K(0, \gamma\psi)$
1.0
∞
K = 0.25
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\gamma\psi$

Figure 2



1.0
0.8
0.6
0.4
0.2
0
(b)
$f_K(\gamma\theta, 0)$
K = 0.25
0.5
1.0
2.0
∞
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\gamma\theta/K$

Figure 3

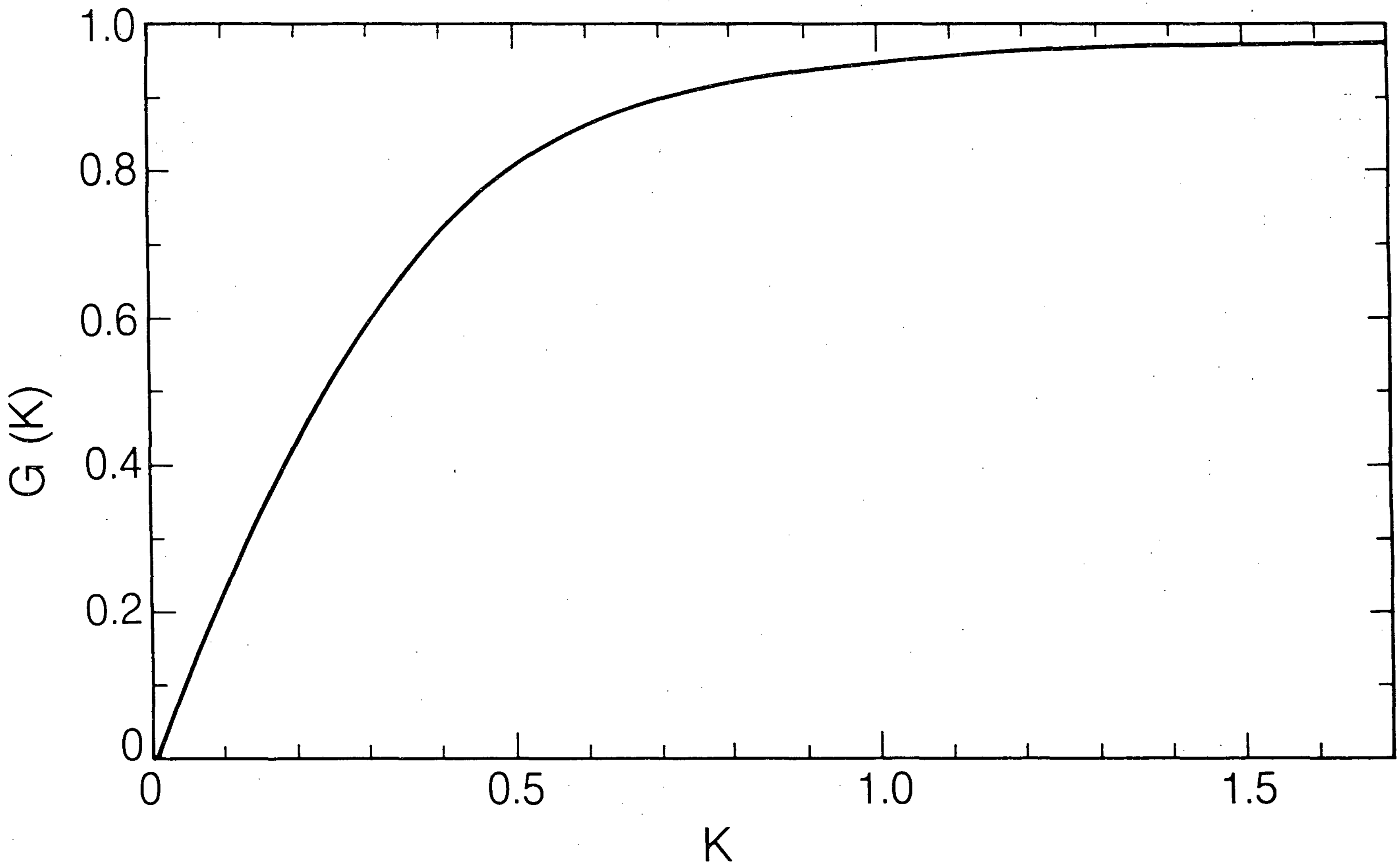


Figure 4

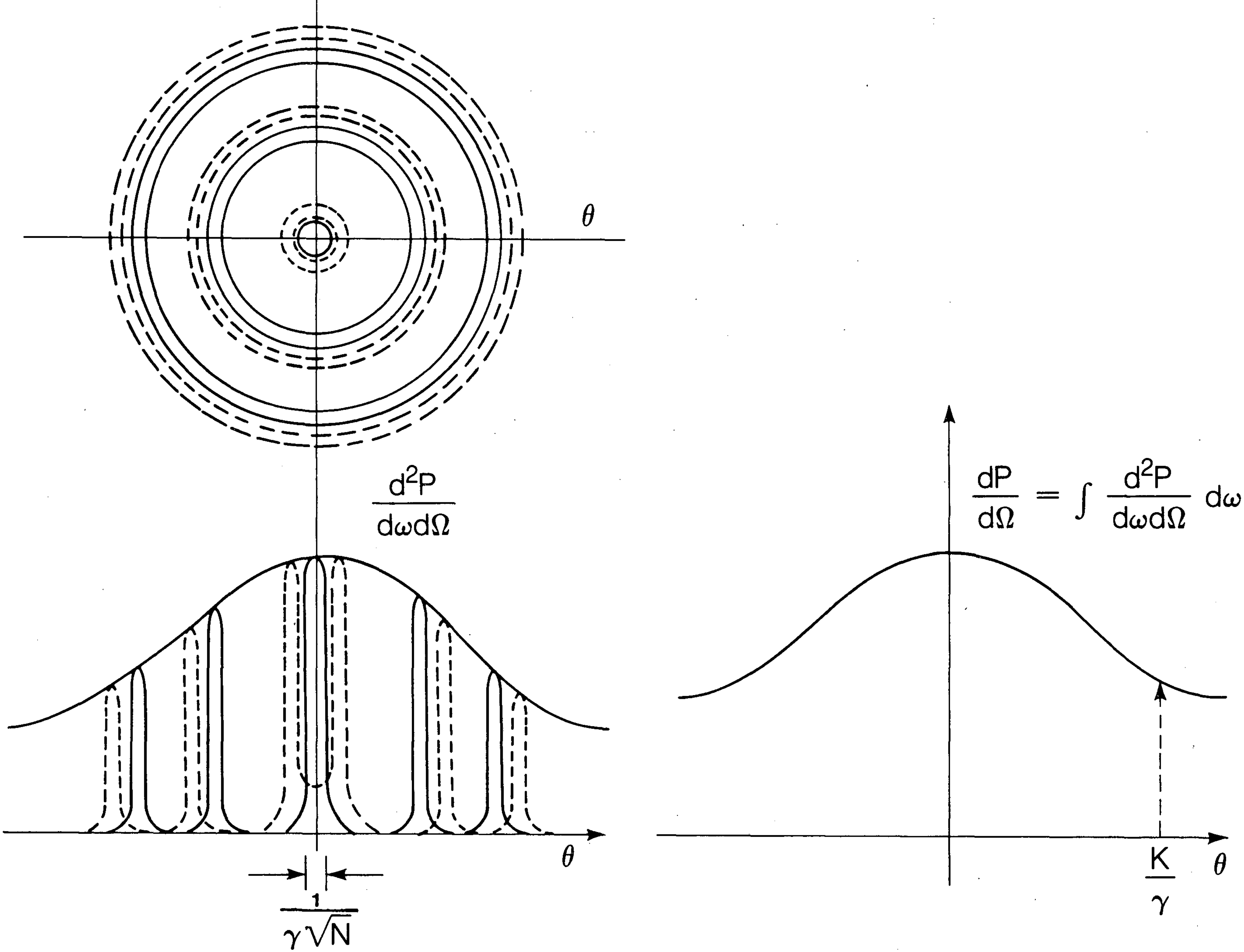

θ
$\frac{d^2P}{d\omega d\Omega}$
θ
$\frac{1}{\gamma\sqrt{N}}$
$\frac{dP}{d\Omega} = \int \frac{d^2P}{d\omega d\Omega}\, d\omega$
$\frac{K}{\gamma}$
θ

Figure 5

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