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by: A. A. Shihab-Eldin, L. J. Jardine and J. O. Rasmussen

ERRATA

1. Right hand side of Eqs. 11, 12 and 13 on pages 8 and 9 should be preceded by:

$$\sum_{\beta_2'\beta_4'}$$

2. Right hand side of Eq. 14 on page 10 should be preceded by:

$$\sum_{\beta_1'\beta_2'\beta_3'\beta_4'}$$

3. The summation sign $\sum_{J_p J_\alpha} =$ that appears to the left of Eq. A-3 on page 16 should be deleted.

4. The summation sign $\sum$ that appears in the middle of Eq. A-4 on page 17 should be deleted.

5. Eqs. A-5, A-6 and A-7 on pages 17, 18 and 19 should be preceded by:

$$\sum_{J_p'}$$

A. Shihab-Eldin

EXTENSION OF RELATIVE ALPHA DECAY RATE THEORY TO SPHERICAL ODD-ODD NUCLEI (^{210}At)*

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ABSTRACT

The one-level R-matrix alpha decay rate theory has been extended to the alpha decay of odd-odd nuclei by carrying out the more involved recoupling and fractional parentage expansion. Graphical representations are employed to carry out the expansion which greatly simplifies the calculations. Relative alpha decay rates of ^{210}At were calculated assuming shell-model wave functions of the pure configurations $\{\pi(h_{9/2})\nu(f_{5/2})^{-1}\nu(p_{1/2})^{-2}\}$ and $\{\pi(h_{9/2})\nu(f_{5/2})^{-2}\nu(p_{1/2})^{-1}\}$ for the low-lying states in ^{206}Bi that are fed in the decay. The overall agreement with experiment is good except for decay to the 4_1^+, 5_1^+ and 5_2^+ states. It is shown that agreement for these states can be greatly improved if mixtures from the two pure configurations assumed above are admixed with the higher $\{\pi(h_{9/2})\nu(p_{3/2})^{-1}\nu(p_{1/2})^{-2}\}$ configuration in the description of the 4_1^+, 5_1^+, and 5_2^+ wave functions.

*Work done under the auspices of the U. S. Atomic Energy Commission.

I. INTRODUCTION

Theoretical calculations of relative alpha decay rates in the ^{208}Pb region, using the one-level formula in R-matrix theory with shell-model wave functions[1]) have been successfully carried out.[2,3,4]) A finite Gaussian internal wave function for the alpha particle was used by Mang[2]) and Harada.[3]) Later, Rasmussen[4]) showed that a simple delta-function approximation for the alpha particle internal wave function gave greatly simplified formulas and essential agreement with sophisticated theories, though it somewhat overestimates rate contributions from higher-j orbitals. Similar theoretical methods were also applied to calculate alpha decay rates in the deformed region.[5,6,7]) A coupled-channel formalism[4]) has been used to include electromagnetic and nuclear force coupling in the deformed region[8,9]) and for vibrational[10]) states. Moreover, configuration mixing[4]) and pairing effects[6,8]) have also been investigated within these theoretical frameworks.

A common defect in these calculations was their inability to reproduce absolute alpha decay rates. In some cases even the calculated relative alpha decay rates were not very satisfactory as compared to the measured values. This was mostly due to the arbitrary and sharp division of space into inner and outer regions which makes the barrier penetrability, relevant to alpha decay, very sensitive to the nuclear radius parameter, R_o, and also results in the usual mismatch problem of logarithmic derivatives at R_o of the alpha amplitudes in inner and outer regions.

Recently a new "Unified Theory of Alpha Decay"[11,12]) has been developed along the lines of Feshbach's "Unified Theory of Nuclear Reactions".[13]) This unified theory avoids the explicit use of nuclear-radius parameter to separate the partial alpha width, λ_L, into a reduced alpha width, γ_L, and a penetration factor, P_L. It thus has the added feature of permitting one to calculate the absolute alpha decay rates, whereas the one-level R-matrix alpha theory was limited in success to calculation of relative alpha decay rates. However, due to considerable ambiguities in values for the parameters of the α-nucleus potential, absolute alpha widths (decay rates) were not satisfactorily reproduced, even when the finite alpha size corrections were made.[14]) It was suggested that with more extensive analysis of the parameter values for the α-nucleus potential[11]), with inclusion of the finite size effects with Woods-Saxon shell-model wave functions and with a more sophisticated wave function for the alpha-particle,[15]) one might be able to achieve better agreement between theory and experiment for the absolute alpha decay rates.

For the alpha decay of spherical odd-odd nuclei around the ^{208}Pb region (such as ^{210}At, ^{212}At, etc.), low-lying states in the daughter nucleus might be described as members of pure shell model configurations. The relative reduced alpha width to states of different spin belonging to the same pure configuration will be independent of the nuclear-radius parameter in the one-level R-matrix alpha decay theory. Since the relative penetration factors, P_L, are not sensitive to the choice of the nuclear-radius parameter, R_o, then the relative alpha decay rates to members of one multiplet will also be independent of R_o. To test this possibility we have extended the one-level R-matrix alpha decay rate theory to the odd-odd ^{210}At nucleus by carrying out the more involved angular momentum recouplings and fractional parentage expansions.

II. OUTLINE OF THEORY

We write the L^{th} partial decay constant, λ_L, as:[4]

$$\lambda_L = \frac{2\gamma_L^{\,2}}{\hbar} \times \left(\frac{\rho}{G_L^{\,2} + F_L^{\,2}}\right)_{r\,=\,R_o} , \tag{1}$$

where $\left(\frac{\rho}{G_L^{\,2} + F_L^{\,2}}\right)_{r\,=\,R_o}$ is the penetration factor, and G_L and F_L are the irregular and regular Coulomb functions respectively; $\rho = kr$, where k denotes the wave number of the alpha particle at large distances, and γ_L is the partial reduced alpha width. γ_L is given by:[10]

$$\gamma_L = \int d\xi \; d\eta \; d\Omega_{rel} \left(\Phi^{M*}_{J_i} \; \chi_\alpha(\xi)\right) \left\{Y_L^{M'}(\underline{R}/R) \; \Psi_{J_d}^{M-M'}(\eta)\right\}_J^M \times \left[\binom{N}{2}\binom{Z}{2}\right]^{\frac{1}{2}} \tag{2}$$

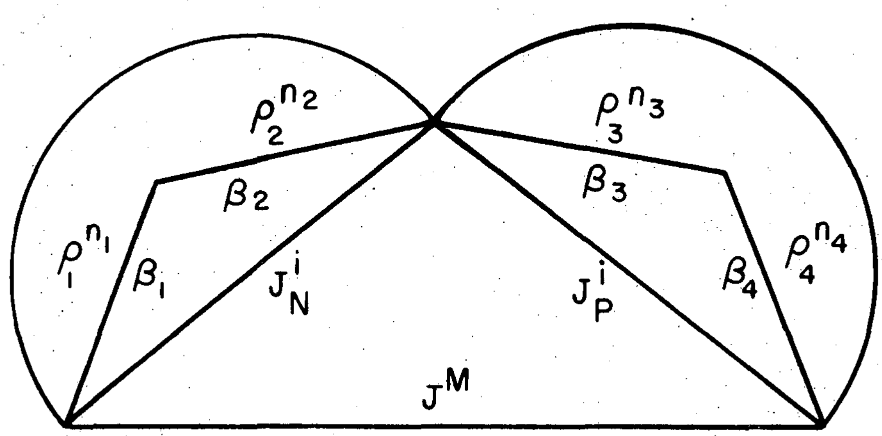
The binomial coefficient arises because the α-channel wave function,

$$\theta_J^M = \mathcal{A} \left\{\chi_\alpha \; \Psi_{J_d} \; \phi_L(\underline{R},\varepsilon) \; Y_L^{M'}(\underline{R}/R)\right\}_J^M , \tag{3}$$

is completely antisymmetrized for exchange of all nucleons, whereas the final state (α-particle + daughter) in Eq. (2) is not explicitly antisymmetrized for exchange of nucleons between the α-particle and daughter nucleus. $\Phi^M_{J_i}$, Ψ_{J_d}, and χ_α are the antisymmetrized wave function of the parent, daughter and α-particle, respectively; ψ_L and Y_L are the radial and angular parts of the α-particle relative-motion wave function. The overlap integral in Eq. (1) can be evaluated if we expand the parent-nucleus state vector (wave function) in terms of the daughter-nucleus state vector and the shell-model state vectors of the four nucleons that form the alpha particle. This can be carried out using the simple and elegant graphical representation technique of Macfarlane et al.[16]

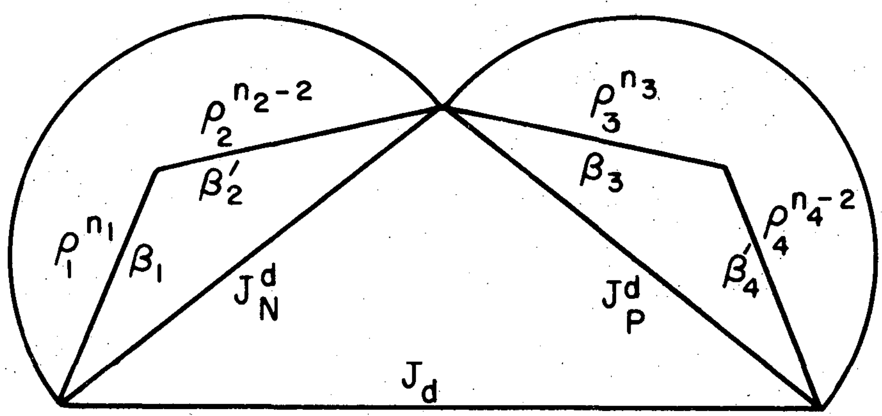
for the state vectors. This technique was used by Towner and Hardy[17]) to evaluate the similar problem of calculating the spectroscopic factors in two-nucleon transfer reactions.

The parent-nucleus state vector can be thought of as consisting of an inert core of nucleons that is common with daughter-nucleus state vector plus N neutrons and Z protons distributed among the active neutron orbitals ρ_1, $\rho_2, \ldots \rho_m$ and the active proton orbitals $\rho_{m+1}, \rho_{m+2}, \ldots \rho_k$. For the purposes of this paper it will be sufficient to limit the neutron and proton active orbitals to two each, i.e., ρ_1, ρ_2 (for neutrons), ρ_3 and ρ_4 (for protons). In the general case there will be n_1 neutrons in orbital ρ_1 (denoted $\rho_1^{n_1}$), n_2 in orbital ρ_2, and similarly n_3 and n_4 protons in orbital ρ_3 and ρ_4 respectively. β_1, β_2, etc. will designate a certain coupling scheme with all needed quantum numbers implicitly contained for simplicity. The β_1 and β_2 state vectors are coupled to a vector with J_N^i (angular momentum), and β_3 and β_4 coupled to J_P^i; and finally J_N^i and J_P^i are coupled to give the parent-nucleus state vector, referred to as $|J^M\rangle$. The neutron and proton state vectors, J_N^i and J_P^i are antisymmetric under exchange among all the neutrons and protons, respectively. In the graphical representation this antisymmetry is denoted by a semicircle surrounding the state vector for the nucleons involved. Thus $|J^M\rangle$ is represented by:



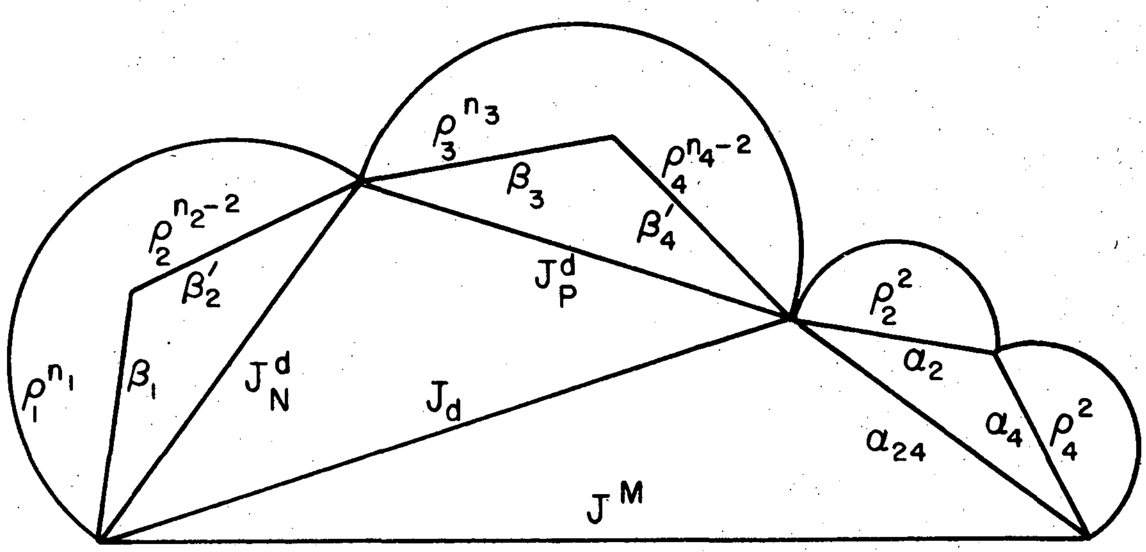
(4)

The daughter state vector will be assumed to consist of an identical inert core plus N-2 neutrons and Z-2 protons distributed in some manner among the active neutron and proton orbitals. We will consider first the case of favored α decay involving removal of two neutrons from one orbital, say ρ_2, and two protons from orbital ρ_4. The daughter-nucleus state vector $|\Psi_{J_d}\rangle$ can then be represented by:



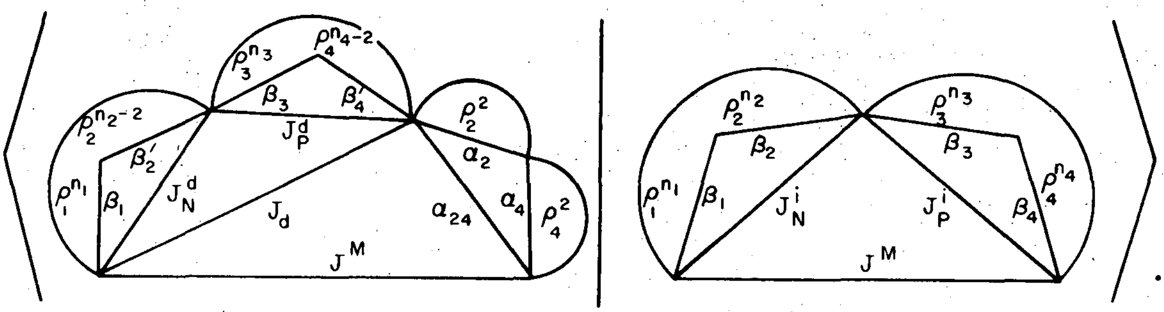
, (5)

where J_N^d and J_P^d are the neutron and proton intermediate state vector, respectively, and J_d represents the daughter-nucleus state vector denoted by its total angular momentum, $|J_d\rangle$. We can now project the parent-nucleus state vector onto a product subspace of the daughter-nucleus state vector and the shell-model state vectors of the four nucleons that form the α particle. A typical basis vector of such a subspace looks as follows:



. (6)

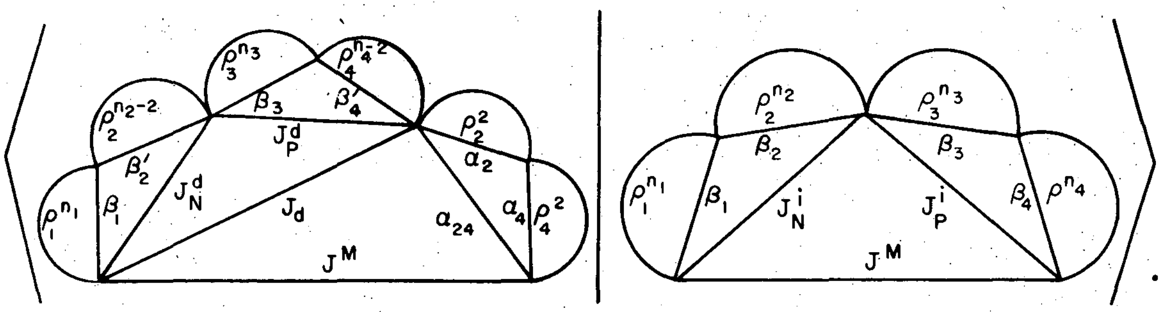
Thus the projection amplitude of the parent-nucleus state vector onto such a basis state vector can be represented by:



. (7)

According to Towner and Hardy[17]), such a term is equal to:

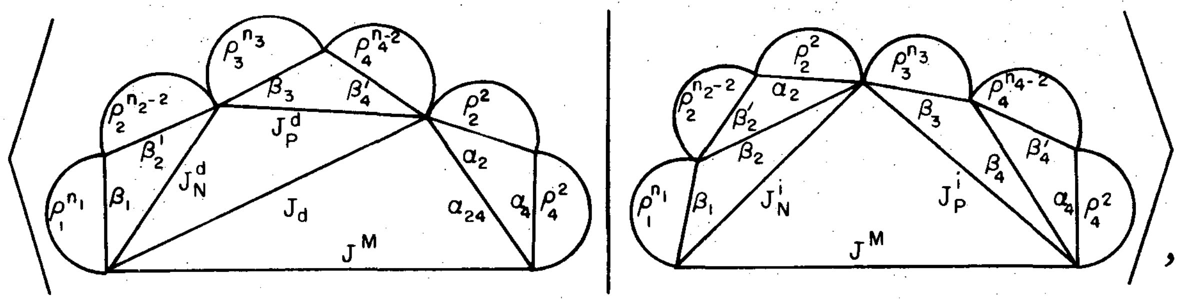
$$\binom{n_2}{2}^{\frac{1}{2}} \binom{n_1 + n_2}{2}^{-\frac{1}{2}} \binom{n_4}{2}^{\frac{1}{2}} \binom{n_3 + n_4}{2}^{-\frac{1}{2}} \times$$



. (8)

The second and fourth binomial coefficients in the product appearing in Eq. (8) will cancel out the binomial coefficient factors appearing in front of Eq. (2) ($n_1 + n_2 = N$, and $n_3 + n_4 = Z$), and henceforth will be dropped. We now expand $|\rho_2^{n_2}(\beta_2)\rangle$ and $|\rho_4^{n_4}(\beta_4)\rangle$ in the parent-nucleus state vector in terms of $|\rho_2^{n_2-2}(\beta_2')\rangle \otimes |\rho_2^2(\alpha_2)\rangle$ and $|\rho_4^{n_4-2}(\beta_4')\rangle \otimes |\rho_4^2(\alpha_4)\rangle$ using two-particle cfp's (coefficient of fractional parentage). The projection amplitude will thus become equal to:

$$\sum_{\beta_2',\,\beta_4'} b^{n_2}_{\beta_2\ \beta_2'\ \alpha_2} \times b^{n_4}_{\beta_4\ \beta_4'\ \alpha_4} \times \binom{n_2}{2}^{\frac{1}{2}} \binom{n_4}{2}^{\frac{1}{2}} \times$$



, (9)

where $b^{n_i}_{\beta_i\ \beta_i'\ \alpha_i}$ represents the two-particle cfp:[23]

$$b^{n_i}_{\beta_i\ \beta_i'\ \alpha_i} = \left\langle \rho_i^{n_i}(\beta_i) \left\{ \middle| \rho_i^{n_i-2}(\beta_i'),\ \rho_i^2(\alpha_i) \right. \right\rangle . \tag{10}$$

The overlap bracket in Eq. (9) represents a $\{15\text{-}j\}$-symbol recoupling coefficient involving six basic angular momentum vectors, β_1, β_2', α_2, β_3, β_4' and α_4 with two different coupling schemes to arrive at the total angular momentum vector J^M. For favored α decay of ^{210}At we show in the Appendix that this $\{15\text{-}j\}$-symbol recoupling coefficient reduces to a $\{6\text{-}j\}$-symbol recoupling (Racah) coefficient, thus Eq. (2) becomes:

$$\gamma_L = \{15\text{-}j\} \binom{n_2}{2}^{\frac{1}{2}} \binom{n_4}{2}^{\frac{1}{2}} \times b^{n_2}_{\beta_2\ \beta_2'\ \alpha_2} \times b^{n_4}_{\beta_4\ \beta_4'\ \alpha_4} \times \int d\xi\ d\eta\ d\Omega \ \times$$

$$\left\{ \Psi^*_{J_d},\ \Psi^*_{\alpha_{24}}(1,2,3,4) \right\}^M_J \times \chi_\alpha(\xi) \sum C(L\ J_d\ M\ M\text{-}M' \mid J\ M)\, Y_L^{M'}(\underline{R}/R) \times \Psi^{M-M'}_{J_d}(\eta)\ . \tag{11}$$

Upon integration over the relative coordinates of the four nucleons of the alpha particle,[4] using the delta-function approximation for the spatial part of the α-particle wave function, we get:

$$\gamma_L = C\{15\text{-}j\}\binom{n_2}{2}^{\frac{1}{2}}\binom{n_4}{2}^{\frac{1}{2}} b^{n_2}_{\beta_2\ \beta_2'\ \alpha_2}\ b^{n_4}_{\beta_4\ \beta_4'\ \alpha_4}\ R_1R_2R_3R_4\ \times$$

$$\int d\Omega\ d\eta\ \left\{\Psi_{J_d}(\eta)\ Y_L(\Omega)\right\}^M_J \times \pi_s(1,2)\ \pi_s(3,4) \times \left\{\Psi_{J_d} \times\right.$$

$$\left\{\left\{\chi_{n_2-1}(\Omega,\sigma_1),\ \chi_{n_2}(\Omega,\sigma_2)\right\}_{\alpha_2}\left\{\chi_{n_4-1}(\Omega,\sigma_3),\ \chi_{n_4}(\Omega,\sigma_4)\right\}_{\alpha_4}\right\}_{\alpha_{24}}\Big\}^M_J . \tag{12}$$

The χ_k function refers to the spin and angular parts of the k^{th} nucleon wave function, R is the value of the nuclear radial wave function evaluated at R_o. Definitions and meanings of other symbols and factors can be found in Ref. 4. In analogous manner to Ref. 4, Eq. (11) can be further reduced to give the following:

$$\gamma_L = C\{15\text{-}j\}\binom{n_2}{2}^{\frac{1}{2}}\binom{n_4}{2}^{\frac{1}{2}} b^{n_2}_{\beta_2\ \beta_2'\ \alpha_2}\ b^{n_4}_{\beta_4\ \beta_4'\ \alpha_4} \times R_1R_2R_3R_4\ \times$$

$$F_N(j_1,j_2,j_N)\ F_P(j_3,j_4,j_P)\ G(J_P\ J_N\ J_L)\ , \tag{13}$$

where

$$C = \left(\frac{\hbar\ R_o}{2M}\right)^{\frac{1}{2}}\left(\frac{4\pi S_\alpha^3}{3}\right)^{3/2} ;$$

$$F_N = (-1)^{\ell_1}\ (2j_1+1)^{\frac{1}{2}}\ (J_N\ j_1\ 0\ -1/2\ |\ j_2\ -1/2)\ B_N\ ,$$

$$F_P = (-1)^{\ell_3}\ (2j_3+1)^{\frac{1}{2}}\ (J_P\ j_3\ 0\ -1/2\ |\ j_4\ -1/2)\ B_P\ ,$$

and $G(J_P\ J_N\ L) = (2J_P+1)^{\frac{1}{2}}\ (L\ \ J_P\ 0\ 0\ |\ J_N\ \ 0)$.

We have used j_1, j_2, j_3 and j_4 to indicate the angular momentum of the two neutrons and two protons involved in α decay (in the previous notation $j_1 = j_2 = j(\rho_2)$ and $j_3 = j_4 = j(\rho_4)$).

For the most general unfavored alpha decay, we would have to extract one neutron each from the ρ_1 and ρ_2 orbital and one proton each from the ρ_3 and ρ_4 orbitals. The derivation is basically the same and the results are similar to that of Eq. (13). However, we would end up with four one-particle cfp's, a {21-j}-symbol recoupling coefficient, and different binomial coefficients, i.e.:

$$\gamma_L = C\{21\text{-}j\}\binom{n_1}{1}^{\frac{1}{2}}\binom{n_2}{1}^{\frac{1}{2}}\binom{n_3}{1}^{\frac{1}{2}}\binom{n_4}{1}^{\frac{1}{2}} b^{n_1}_{\beta_1\ \beta'_1\ \alpha_1}\ b^{n_2}_{\beta_2\ \beta'_2\ \alpha_2}\ b^{n_3}_{\beta_3\ \beta'_3\ \alpha_3} \times$$

$$b^{n_4}_{\beta_4\ \beta'_4\ \alpha_4} \times R_1R_2R_3R_4 \times F_NF_PG\ , \qquad (14)$$

where $b^{n_i}_{\beta_i\ \beta'_i\ \alpha_i}$ represents the one-particle cfp:[23)]

$$b^{n_i}_{\beta_i\ \beta'_i\ \alpha_i} = \left\langle \rho_i^{n_i}(\beta_i) \right\} \left| \rho_i^{n_i-1}(\beta'_i),\ \rho_i(\alpha_i) \right\rangle . \qquad (15)$$

For ^{210}At unfavored decay the {21-j} symbol reduces to a product of {6-j} and {9-j} symbols because the protons in this case are taken from the same orbital (see Appendix).

If daughter-nucleus state vectors are of mixed configuration nature, they can be represented by the expansion: $|\psi_d\rangle = \sum_i a_i |\psi_d^i\rangle$, where i refers to different configurations. Then Eq. (1) for the partial alpha decay rate becomes:

$$\lambda_L = \left|\sum_i a_i\ \gamma_L^i\right|^2 P_L\ . \qquad (16)$$

III. RESULTS AND DISCUSSION

1. Relative Alpha Decay Rates Using Pure Shell-Model Configurations

Using the formulation of the previous section, we have calculated the relative alpha decay rates from the ^{210}At 5+ ground state, to the lowest, 6_1^+,

4_1^+, 3_1^+, 5_1^+, 7_1^+, and 2_1^+ states in ^{206}Bi. First we assumed these to consist of a pure configuration $(\pi(h_{9/2})\nu(f_{5/2})^{-1}_{5/2}\nu(p_{1/2})^{-2}_{0})$ (hereafter denoted configuration {1}). We also calculated the relative alpha decay rates to the second 5_2^+ and 4_2^+ states in ^{206}Bi, which we assumed to belong to the pure configuration $(\pi(h_{9/2})\nu(f_{5/2})^{-2}_{0}\nu(p_{1/2})^{-1})$ (hereafter denoted configuration {2}). In Table 1 we show these calculated relative alpha decay rates (normalized to 100 for the $\lambda_\alpha(6_1^+)$) and compare them to the experimental relative intensities of the alpha groups measured by Golovkov et al.[18]) The radial nucleon wave functions were taken from Blomqvist and Wahlborn tables[19]) evaluated at $R_o = 8$ fm, as suggested by Ref. 4.

There is generally fair agreement between the experimental and theoretical relative alpha decay rates; the agreement is exceptionally good for the relative alpha group intensities to the 6_1^+ and 7_1^+. Also impressive is the fact that theory predicts small relative alpha group intensities to the 3_1^+, 2_1^+, and 4_2^+ states that are below the intensity limits set from experiment for such possible alpha groups. However, with the present pure configuration assumption, the theory fails to reproduce the experimental relative alpha group intensities leading to the 4_1^+ and 5_1^+ states and slightly overestimates the intensity to the second 5_2^+ state. This suggests that the pure configuration assumption for the 4_1^+, 5_1^+, and 5_2^+ and 4_2^+ states in ^{206}Bi is not valid. No detailed shell-model wave functions, with configuration mixing, are yet available for states of ^{206}Bi. However, we have attempted to use some simple trial wave functions for the 4_1^+, 5_1^+, 5_2^+, and 4_2^+, with configuration mixing limited to the two configurations, {1} and {2}, mentioned earlier. Before proceeding further with the calculation, it should be noted that the relative partial reduced alpha widths, γ_L^i, are dependent on the square of the ratio $[(R_1R_2R_3R_4)_i/(R_1R_2R_3R_4)_j]$, denoted R_{ij}, evaluated at R_o, where i and j refer to configuration {i} and {j} in the daughter nucleus. The ratio R_{ij} is sensitive to R_o. Table II shows that R_{12} can vary by a factor of 1.5 over a range from 7 to 10 fm for R_o. Thus the results of the calculation including configuration mixing, carried out at any arbitrary value of R_o (8 fm), should not be accepted without caution.

III.2. Relative Alpha Decay Rates with Configuration Mixings

By limiting the trial wave functions for 4_1^+, 5_1^+, 5_2^+, and 4_2^+ to mixing of configurations $\{1\}$ and $\{2\}$ only, we were able to arrive at two sets of trial wave functions that gave improved agreement between the calculated alpha decay rates to the 5_1^+ and 5_2^+ and the experimentally measured values; the two sets and the relative alpha decay rates they yield are given in Table III. The large experimental value for the alpha group leading to the 4_1^+ (26 ± 2%) could not be reproduced with any value of mixing in the wave function of the 4_1^+ due to the smallness of the relative partial reduced widths to the 4^+ component of configuration $\{2\}$. This is illustrated in Table IV, where we show the relative partial reduced widths to the 4^+ and 5^+ components for configurations $\{1\}$, $\{2\}$ and the $(\pi(h_{9/2})\nu(p_{3/2})^{-1}\nu(p_{1/2})^{-2})$ configuration (the latter will be denoted configuration $\{3\}$ hereafter). The large value of $\gamma_2(\{3\}_{4+})$ immediately suggests the possibility of improving the agreement with experiment for the 4_1^+ state by including some admixture of configuration $\{3\}$ in the 4_1^+ wave function. We can achieve exact agreement with experiment if we choose the following mixture for the 4_1^+ wave function:

$$|4_1^+\rangle = \sqrt{0.92}\ |\{1\}_{4^+}\rangle - \sqrt{0.08}\ |\{3\}_{4^+}\rangle\ . \tag{17}$$

Of course such a simple description for the 4_1^+ is not realistic as some mixture from the $|\{2\}_{4^+}\rangle$ has to be included, but this will not alter things by much, as $\gamma_2(\{2\}_{4^+})$ is small compared to $\gamma_2(\{1\}_{4^+})$ and $\gamma_2(\{3\}_{4^+})$.

Relative intensities for the 5_1^+ and 5_2^+ states can also be brought into a better agreement with experimental values if admixtures from $|\{3\}_{5^+}\rangle$ are included in the description for 5_1^+ and 5_2^+ state vectors. Quantitative evaluation of the $\{1\}$, $\{2\}$ and $\{3\}$ configuration amplitudes in $|5_1^+\rangle$ and $|5_2^+\rangle$ wave functions by trial and error are not unique nor meaningful in this more complex situation, and will not be attempted. However, we can assert qualitatively, that matching of the relative alpha decay rates to the 4_1^+, 5_1^+, and 5_2^+ in ^{206}Bi must require wave functions with appreciable admixture from configuration $\{3\}$, which has a center of gravity of about 400 keV above the configuration $\{1\}$, in addition to admixtures from configurations $\{1\}$ and $\{2\}$.

It is interesting to note that the same conclusions about the qualitative nature of the 4_1^+, 5_1^+, and 5_2^+ states in ^{206}Bi were arrived at when Jardine and Shihab-Eldin[22]) attempted to reproduce the electromagnetic transition rates (γ-ray branching ratios) in ^{206}Bi following ^{210}At alpha decay. The set of wave functions given in Table 5 of Ref. [22]), which incorporate admixtures from configurations $\{1\}$, $\{2\}$ and $\{3\}$ for the description of the 4_1^+ and 5_2^+ states, gives similar values for the relative alpha decay rates that are in good agreement with experiment.

III.3. Theoretical L-Mixing Ratios for Alpha Decay

The calculations also reveal that $L=2$ alpha waves (λ_2) are almost invariably the dominant mode of decay for each transition, even in the case of $J_i = 5^+ \rightarrow 5_1^+$ which has an allowed $L=0$ mode. An exception to this is the $J_i = 5^+ \rightarrow 5_2^+$ (favored decay) where the $L=0$ mode is the dominant one. In Table 5 we compare the relative contributions (in arbitrary units for each transition) from the various λ_L modes for some selected transitions to the two ^{206}Bi multiplets (configurations $\{1\}$ and $\{2\}$). The theoretical L-mixing ratios of Table 5 might be subjected to experimental tests by various alpha angular correlation experiments.

IV. CONCLUSIONS

The relative alpha decay rates for spherical odd-odd nuclei in the lead region can be calculated systematically, as outlined in this paper, when wave functions of the parent- and daughter-nucleus state vectors become available. Calculation of the relative alpha decay rates for ^{210}At were shown to be rather sensitive to variations in the configuration admixtures that describe the state vectors. Thus to verify the mixed-configuration shell-model wave functions for the parent- and daughter-nucleus state vectors generated by theoretical calculations, one can use the comparison between experimental and theoretical relative alpha decay rates (and the L-mixing ratio, if experimentally measured), as well as the electromagnetic transition rates (and mixing ratios) in the daughter-nucleus.

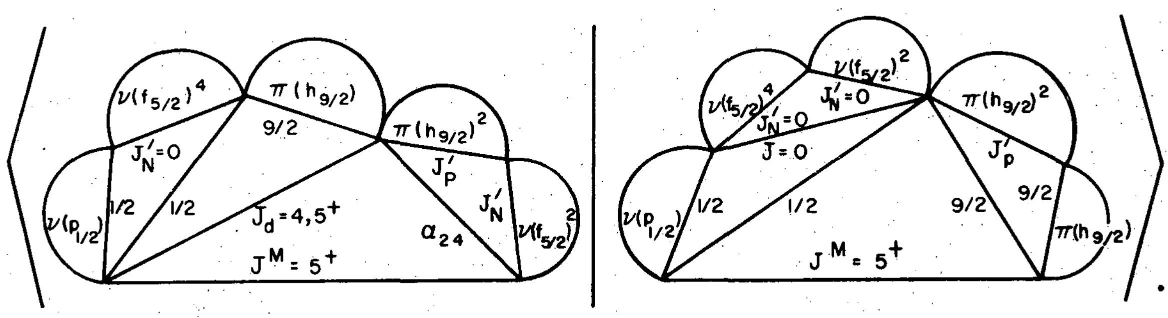
APPENDIX

EVALUATION OF RECOUPLING AND FRACTIONAL PARENTAGE EXPANSION COEFFICIENTS FOR ALPHA DECAY OF ^{210}At TO STATES WITH CONFIGURATIONS {1} AND {2} IN ^{206}Bi

A.I. Favored Decay to the Configuration $\{\pi(h_{9/2})\nu(f_{5/2})^{-2}\nu(p_{1/2})^{-1}\}$

The reduction of {15-j}-symbol recoupling coefficient into the familiar Racah or {6-j}-symbol recoupling coefficients is outlined below. The alpha decaying ground state of ^{210}At is always assumed to be of a pure $\{\pi(h_{9/2})^3_{9/2}\nu(p_{1/2})^{-1}\}_{5^+}$ configuration. Favored alpha decay to configuration {2} in ^{206}Bi proceeds via removal of a proton pair from the $\pi(h_{9/2})$ orbital and a pair of neutrons from the filled $\nu(f_{5/2})$ orbital. The projection amplitude of Eq. (9) in the text can be explicitly written out for this case as:

$$\binom{6}{2}^{\frac{1}{2}}\binom{3}{2}^{\frac{1}{2}}\ \overset{6}{b}_{0\ J_N'\ J_N'}\ \overset{3}{b}_{9/2\ 9/2\ J_P'}\ \times$$

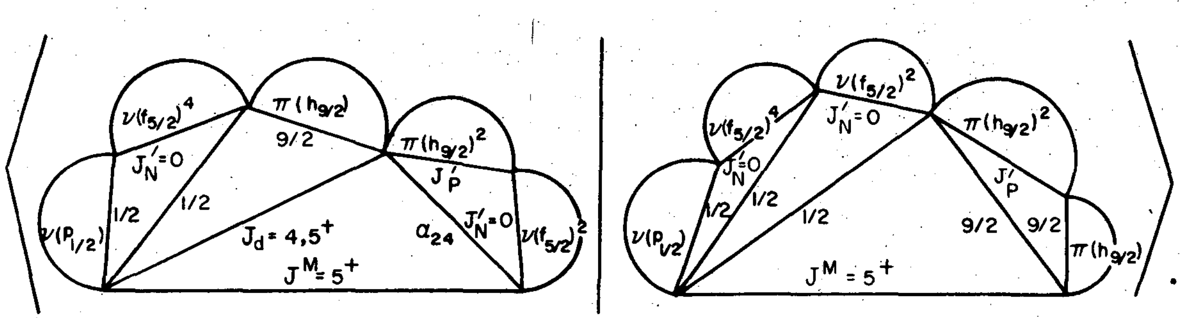


. (A-1)

We have limited J_N' to zero since the daughter state is assumed to have seniority one for the neutrons. The ket vector can be expanded in terms of another complete set of basis vectors that involves a different coupling scheme for the three neutron vectors $|\nu(p_{1/2})_{1/2}\rangle$, $|\nu(f_{5/2})^4_{J_N'(=0)}\rangle$ and $|\nu(f_{5/2})^2_{J_N'(=0)}\rangle$.

This is achieved by the familiar recoupling techniques for three angular momentum vectors using Racah coefficients[23] ({6-j} symbol). However, since two of the vectors have zero angular momentum, the expansion reduces to an identity. Thus projection coefficient (A-1) can be rewritten as:

$$\binom{6}{2}^{\frac{1}{2}} \binom{3}{2}^{\frac{1}{2}} \; {}^{6}b_{0\, J_N'\, J_N'} \; {}^{3}b_{9/2\; 9/2\; J_P'} \times$$



. (A-2)

It is possible now to expand the ket vector in terms of the bra vector coupling scheme for the four angular momentum vectors, J_P', 9/2, J_N' (=0), and J_N^d (=1/2). This involves the {9-j}-symbol recoupling coefficient defined by de-Shalit and Talmi.[23] Thus the projection amplitude becomes:

$$\sum_{J_P J_\alpha} = \sqrt{\hat{9/2}\ \hat{1/2}\ \hat{J}_\alpha\ \hat{J}_d} \begin{Bmatrix} 1/2 & J_N'(=0) & 1/2 \\ 9/2 & J_P' & 9/2 \\ J_d & J_\alpha & J \end{Bmatrix} \times \binom{6}{2}^{\frac{1}{2}} \binom{3}{2}^{\frac{1}{2}} \times$$

$${}^{6}b_{0\, J_N'\, J_N'} \; {}^{3}b_{9/2\; 9/2\; J_P'} \quad , \qquad \text{(A-3)}$$

where we have substituted the more meaningful J_α symbol for α_{24}.

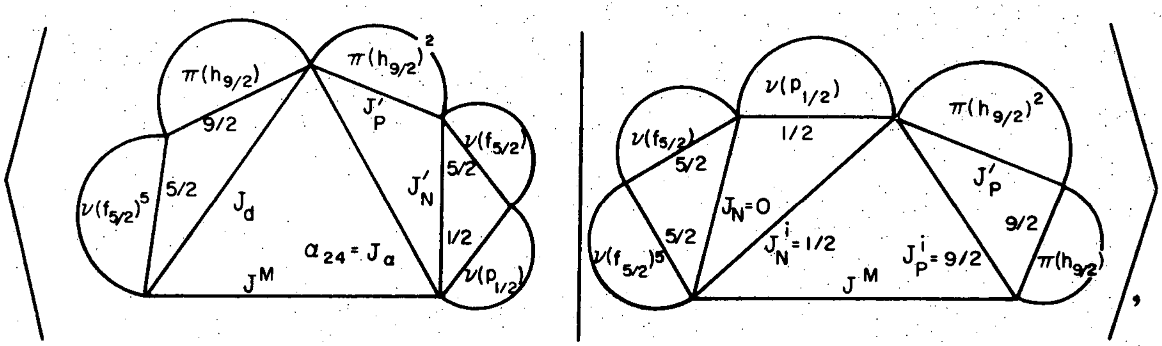
The $\{9\text{-}j\}$ symbol that appears in Eq. (A-3) can be reduced to a $\{6\text{-}j\}$ symbol due to the presence of two pairs of identical angular momenta plus one term that is zero (J_N'). Thus we finally get the following for the projection amplitude:

$$= \binom{6}{2}^{\frac{1}{2}} \binom{3}{2}^{\frac{1}{2}} \; \overset{6}{b}_{0\; J_N'\; J_N'} \; \overset{3}{b}_{9/2\; 9/2\; J_P'} \; \sum \sqrt{\hat{9/2}\; \hat{J_d}} \times \begin{Bmatrix} J_d & J & J_\alpha = J_P' \\ 9/2 & 9/2 & 1/2 \end{Bmatrix} . \tag{A-4}$$

A.II. Unfavored Decay to the Configuration $\{\pi(h_{9/2})\nu(f_{5/2})^{-1}\nu(p_{1/2})^{-2}\}$

In a similar fashion we show below the reduction of the $\{21\text{-}j\}$ symbol that appears in Eq. (14) into a product of a $\{6\text{-}j\}$ and $\{9\text{-}j\}$ symbols. Here the neutron pair in the alpha particle are taken, one each, from the $\nu(p_{1/2})$ and the filled $\nu(f_{5/2})$ orbitals. The proton pair is again taken from the $\pi(h_{9/2})$ orbital. The projection amplitude of Eq. (9) for this decay is written as:

$$\binom{3}{2}^{\frac{1}{2}} \binom{6}{1}^{\frac{1}{2}} \binom{1}{1}^{\frac{1}{2}} \; \overset{6}{b}_{0\; 5/2\; 5/2} \; \overset{3}{b}_{9/2\; 9/2\; J_P'} \; \times$$

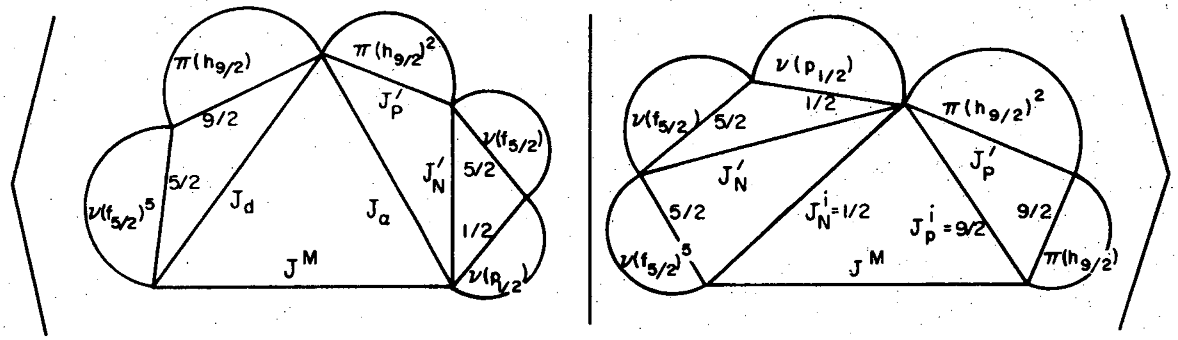


, (A-5)

where $\overset{6}{b}_{0\; 5/2\; 5/2}$ is identically equal to one.

By a different recoupling of the three neutron angular momentum vectors, $\nu(f_{5/2})^5_{5/2}$, $\nu(f_{5/2})_{5/2}$, $\nu(p_{1/2})_{1/2}$ in the ket vector, the projection amplitude becomes:

$$\binom{3}{2}^{\frac{1}{2}}\binom{6}{1}^{\frac{1}{2}}\binom{1}{1}^{\frac{1}{2}}\; b^{3}_{9/2\;9/2\;J'_P} \times \sqrt{\hat{J}'_N \cdot \hat{0}}\; \begin{Bmatrix} 5/2 & 1/2 & J'_N \\ 1/2 & 5/2 & 0 \end{Bmatrix} \times$$



. (A-6)

It should be noted that the selection rule of $G(J_PJ_NL_\alpha)$ (i.e., $L_\alpha + J_N + J_P$ must be even) coupled with the additional restrictions that L_α (J_α above) are even (no parity change in $5^+ \to \{1\}^+_J$ and the two protons are taken from the same orbital) will restrict J'_N to even values only. The only allowed even value for the $\{1/2, 5/2\}$ coupling is 2. Therefore no summation over J'_N appears in front of Eqs. (A-5) and (A-6). The four angular momentum vectors of the ket vector in Eq. (A-6) can now be recoupled as in the bra vector in the same equation using a $\{9\text{-}j\}$-symbol recoupling coefficient. The projection amplitude will then be given by:

$$\binom{3}{2}^{\frac{1}{2}}\binom{6}{2}^{\frac{1}{2}}\binom{1}{1}^{\frac{1}{2}} \; b^{3}_{9/2\;9/2\;J'_P} \times \sqrt{\hat{J}'_N\;(=2)\cdot\hat{0}} \begin{Bmatrix} 5/2 & 1/2 & J'_N\;(=2) \\ 1/2 & 5/2 & 0 \end{Bmatrix} \times$$

$$\sqrt{\hat{9/2}\cdot\hat{1/2}\cdot\hat{J}_\alpha\cdot\hat{J}_d} \begin{Bmatrix} 5/2 & 9/2 & J_d \\ J'_N & J'_P & J_\alpha \\ 1/2 & 9/2 & J \end{Bmatrix} \tag{A-7}$$

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Table I. A comparison of the experimental and theoretical relative alpha intensities in the decay of ^{210}At (for pure configuration wave functions).

^{206}Bi Configurations	^{206}Bi States[a] J^{π}	E(keV)	Relative α intensity[b] Experimental[c]	Calculated[d]
$(\pi(h_{9/2})\nu(f_{5/2})^{-1}\nu(P_{1/2})^{-2})_{J^{\pi}}$	6_1^+	0	100	100
	4_1^+	60	26 ± 2	14
	3_1^+	70	n. o.[e]	0.3
	5_1^+	83	95 ± 6	11
	7_1^+	140	14 ± 2	15
	2_1^+	409	n. o.[e]	0.005
$(\pi(h_{9/2})\nu(P_{1/2})^{-1}\nu(f_{5/2})^{-2})_{J^{\pi}}$	5_2^+	167	83 ± 6	95.4
	4_2^+	200	n. o.[e]	0.13

[a]Spin and energy assignments were taken from Refs. 20, 21 and 22.

[b]Calculated and experimental alpha group intensities are relative to the g.s. (6_1^+) group.

[c]Experimental relative intensities were taken from Ref. 18.

[d]The value of the parameter R_o used was 8 fm.

[e]Alpha groups not observed by Ref. 18 can be given an upper limit of ≤ 5.

Table II. The y(r)* radial wave functions for neutron hole states in ^{208}Pb.

r (fm)	$10^3 \cdot y(r)$			
	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$\lvert y(r, p_{1/2})/y(r, f_{5/2}) \rvert$
7	477	-457	477	1.04
8	428	-369	432	1.16
9	272	-202	272	1.35
10	154	-103	150	1.46

*y(r) = rR(r), taken from tables of ref. 19.

Table III. Comparison of theoretical and experimental relative alpha intensities using mixed-configuration wave functions for the 5_1^+ and 5_2^+ states

State	Wave function		Relative α intensity[(a)]		
			Calc.		Expt.
	Set I	Set II	Set I	Set II	
$\|5_1^+\rangle$	$\sqrt{0.78}\{1\}+\sqrt{0.22}\{2\}$	$\sqrt{0.67}\{1\}-\sqrt{0.33}\{2\}$	83	83	95 ± 6
$\|5_2^+\rangle$	$\sqrt{0.22}\{1\}-\sqrt{0.78}\{2\}$	$\sqrt{0.33}\{1\}+\sqrt{0.67}\{2\}$	74	76	83 ± 6

[(a)] Calculated and experimental alpha-group intensities are relative to that of the ground state (6_1^+) group.

Table IV. The relative reduced α-widths, γ_L.

State		Configuration		
J^{π}	L	{1}	{2}	{3}
4+	2	+0.65	+0.154	-0.88
5+	0	+0.24	-2.39	-0.145
	2	-0.64	+0.64	-0.73

Table V. Relative intensity strength λ_L, for various orbital angular momentum groups in α-transitions, in arbitrary units for each transition.

Orbital angular momentum	Transitions $5^+\rightarrow6^+_1$	$5^+\rightarrow5^+_1$	$5^+\rightarrow7^+_1$	$5^+\rightarrow5^+_2$	$5^+\rightarrow4^+_2$
0	forbidden	8.55(-31)	forbidden	1.15(-25)	forbidden
2	1.64(-28)†	1.75(-29)	2.29(-28)	4.56(-27)	1.22(-28)
4	1.85(-32)	5.85(-31)	8.43(-31)	1.04(-27)	1.04(-28)
6	4.91(-33)	4.56(-32)	8.16(-32)	8.96(-29)	2.50(-29)
8	4.86(-34)	8.48(-34)	3.36(-33)	forbidden	forbidden

†Numbers in parentheses refer to power of 10 that multiplies preceding numbers.