Title

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Publication Date

1982-10-01

LBL-14985

Conf-821214--6

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

EARTH SCIENCES DIVISION

Presented at the 8th Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford, CA, December 14-16, 1982

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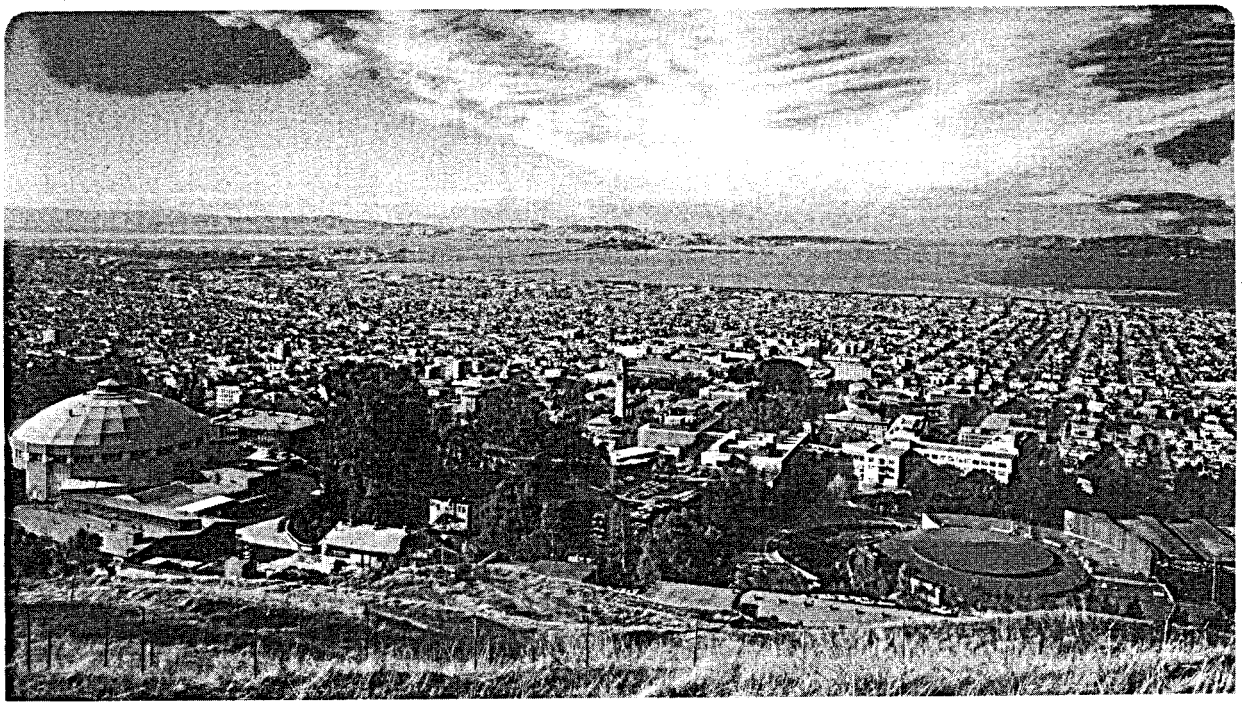

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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LBL-14985

Submitted to the Stanford
Geothermal Reservoir Engineering
Workshop, December 14-16, 1982

LBL--14985

DE83 005864

Proximity Functions for Modeling Fluid and Heat Flow in Reservoirs with Stochastic Fracture Distributions

by

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Most geothermal reservoirs are extensively fractured and have low matrix permeability. The fractures provide the principal conduits for fluid and heat flow.

Conventional approaches to geothermal reservoir modeling have employed a porous medium approximation, but recently methods have been developed which can take into account the different thermodynamic conditions in rock matrix and fractures. The multiple interacting continua method ("MINC") developed by Pruess and Narasimhan treats the thermal and hydraulic interaction between rock matrix and fractures in terms of a set of geometrical parameters. However, this approach was restricted to idealized fracture distributions with regularly shaped matrix blocks.

Fractures in geothermal reservoirs usually occur in nearly parallel sets with a certain scatter in orientation, and a stochastic distribution of spacings and apertures. We have extended the MINC-method to realistic fracture systems with stochastic distributions. The interaction between matrix and fractures is parameterized in terms of a "proximity function", which represents the volume of matrix rock as a function of distance from the fractures. We employ Monte Carlo techniques to compute proximity functions for a number of two-dimensional systems with regular or stochastic fracture distributions. It is shown how the proximity functions can be used to generate computational grids for modeling fluid and heat flow in fractured reservoirs.

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Technology. Division of Geothermal and Hydropower Technologies of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

Proximity Functions for Modeling Fluid and Heat Flow in Reservoirs with Stochastic Fracture Distributions

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1. Introduction

It is well established that most high-temperature geothermal reservoirs are extensively fractured. The fractures provide the principal conduits for fluid and heat flow. The rock matrix contains most of the fluid and heat reserves, but it usually has a very low permeability, perhaps in the microdarcy-range.

Conventional approaches to geothermal reservoir modeling have employed a porous medium approximation, although the validity of this approximation for naturally fractured reservoirs has never been demonstrated in detail. It appears that most researchers expected a porous medium approximation to work in cases with "not too large" fracture spacing. Recently it was shown by Pruess and Narasimhan (1982a), that in two-phase geothermal reservoirs strong discontinuities in vapor saturation can arise at matrix/fracture interfaces, due to an interplay between fluid convection and heat conduction. This suggests that fractured systems with two-phase fluid may behave quite differently than porous medium systems even in cases where fracture spacing is small in comparison to characteristic dimensions of the problem (e.g., reservoir size, well spacings, completion intervals).

In order to quantitatively model fractured reservoir behavior, Pruess and Narasimhan (1982b) developed a "multiple interacting continua" method ("MINC"), which is a generalization of the double porosity model of Barenblatt et al. (1960) and Warren and Root (1963). The classical double-porosity work employed a quasi-steady approximation for "interporosity" flow between rock matrix and fractures, which severely limits the range of systems and processes to which it is applicable. The MINC-method on the other hand, treats interporosity flow entirely by numerical methods. This makes possible a fully transient representation of interporosity flow, which is applicable to problems with coupled fluid and heat flow, and to multiphase fluids with large and varying compressibility, such as steam-water mixtures.

The work of Pruess and Narasimhan employed highly idealized regular fracture distributions, but the authors pointed out that the MINC-method can be extended to realistic (stochastic) fracture distributions as well. It is the purpose of the present paper to carry out the generalization to arbitrary irregular fracture distributions. After briefly reviewing the main assumptions of the MINC-method, we shall introduce the concept of a "proximity function" as the central geometrical quantity which defines the matrix-fracture interaction. Subsequently we shall consider proximity functions for regular or irregular fracture distributions, using Monte Carlo integration techniques.

2. Summary of the MINC - Method

The MINC-method follows the double-porosity approach in adopting a continuum treatment for both the fracture network and for the porous rock matrix. Global flow in the reservoir is assumed to occur only through the network of interconnected fractures, whereas fractures and rock matrix can exchange fluid and heat locally. In order to obtain a numerical description for interporosity flow, it is necessary to partition the flow domain into discrete volume elements, or grid blocks. The crucial point of the MINC-method is the partitioning (or discretization) procedure adopted for interporosity flow. It should be noted that the customary equations for mass- and energy-conservation, when written in integral form, hold for arbitrary reservoir subdomains (Narasimhan, 1982). However, discretized equations are only useful (solvable), when the flow terms between volume elements can be related to the accumulation of mass and heat within volume elements. Fluid and heat flow are driven by gradients of pressure and temperature, respectively, and these can be expressed in terms of average values of thermodynamic variables if (and only if) there is approximate thermodynamic equilibrium within each volume element at all times. In porous media, this requirement will usually be satisfied for any suitably "small" simply-connected subregion, as thermodynamic conditions generally vary continuously and smoothly with position.

The situation can be quite different in fractured media, where changes in thermodynamic conditions as a consequence of boiling or cold water injection may propagate rapidly in the fracture network, while migrating only slowly into the rock matrix. Thus, thermodynamic conditions may show strong variations as a function of position in the vicinity of the fractures. Because of the different response times, thermodynamic changes in the rock matrix will locally depend mainly upon the distance from the nearest fracture. Then, interporosity flow will be perpendicular to the fracture faces. This suggests partitioning (discretizing) of the rock matrix into sequences of nested volume elements, which are defined on the basis of distance from the fractures. Figure 1 illustrates this concept for the case of an idealized two-dimensional fracture distribution. In this case the geometric quantities governing the interporosity flow (element volumes, interface areas, and nodal distances) can be easily obtained in explicit analytical form (Pruess and Narasimhan, 1982b).

The mesh design concept as shown in Figure 1 can be generalized, to make it more suitable for applications of practical interest. In reservoir regions where thermodynamic conditions vary slowly as a function of position, it is not necessary to have separate volume elements within each of the elementary units depicted in Figure 1. Instead, corresponding nested volumes in neighboring units, which are identified by an index number in Figure 1, can be lumped together into one computational volume element. Element volumes and interface areas scale proportional to the number of elementary units which are lumped together, whereas nodal distances remain unchanged. The scaling procedure can be further generalized by applying the same scaling law to grid blocks of arbitrary size or shape. Thus we arrive at a two-step procedure for defining a computational mesh for a fractured reservoir. The first step is to construct a mesh just as would be done for a porous-medium type system with small grid blocks near wells, etc. ("primary mesh"). The second step is to sub-partition each grid block into several continua, the respective volumes, interface areas, and nodal distances of which are obtained by appropriate scaling from the quantities pertaining to the basic fractured unit ("secondary mesh").

The concept of partitioning based on distance from the fractures can be readily extended to arbitrary irregular fracture distributions. Figure 2 illustrates this for a set of fractures of finite length. First it is necessary to eliminate the dead-end portions of the fractures, which do not participate in global flow within the fracture system (Figure 2b). The rock matrix can then be readily partitioned into several continua with increasing distance from the fractures (Figure 2c). While the general case of irregular fractures is straightforward from the conceptual point of view, it is not possible to obtain the geometrical parameters for the sub-continua in an explicit fashion. To accomplish this we introduce an auxiliary function, termed a "proximity function", which can be calculated for any given facture distribution, and which allows to completely define all geometric parameters for interporosity flow.

3. The Concept of Proximity Functions

For any given reservoir subdomain with known fracture distribution a function V(x) can be defined, which represents total matrix volume V within a distance x from the fracture faces. Note that the volume V will generally consist of a finite number of disjoint multiply-connected regions, representing a quite complex topological structure (see Figure 2c). If V_0 is the volume of the subdomain, and ϕ_1 is the volume fraction (average porosity) of the fracture system, the volume of the fracture continuum within V_0 is $V_1 = \phi_1 \cdot V_0$. It is convenient to introduce a "proximity function" PROX(x), which expresses, for a given reservoir subdomain V_0, the total fraction of matrix volume within a distance x from the fractures. Noting that the total matrix volume in domain V_0 is

$$V_m = (1-\phi_1)\, V_0 \tag{1}$$

we have

$$\text{PROX}(x) = \frac{V(x)}{V_m} = \frac{V(x)}{(1-\phi_1)\, V_0} \tag{2}$$

In the MINC-method, a discretization is adopted for the rock matrix (see Figure 3) whereby all matrix volume within a distance x_2 from the fracture faces will be lumped into one computational volume element (or subcontinuum) V_2; matrix volume within a distance larger than x_2 but less than x_3 will be lumped into V_3, etc. This is illustrated in Figure 3 for a regular fracture network, but it is evident that the same procedure can be applied to arbitrary irregular fracture distributions, see Figure 2c. To define flow towards or away from the fractures, it is necessary to specify interface areas and nodal distances between the matrix sub-continua. From the definition of the proximity function as given above, the interface area for flow at distance x is simply

$$A(x) = \frac{dV}{dx} = (1-\phi_1)\, V_0 \frac{d\ \text{PROX}(x)}{dx} \tag{3}$$

In conventional porous medium-type simulation methods with simply-connected grid blocks, the computational nodes are points, usually located at the center of a volume element. For the multiply connected volume elements of the MINC-method, the

element nodes become nodal surfaces, which are located half-way between the inner and the outer surface of an element. The discretization procedure adopted in the MINC-method can now be described as follows. First, a "primary" mesh is specified in integral finite difference form by means of a set of volume elements $\{V_n,\ n=1, \ldots, N\}$, interface areas A_{nm}, and nodal distances d_{nm}. All primary "connections" (A_{nm}, d_{nm}) between volume elements are assigned to the fracture continuum. Each grid block V_n of the primary mesh is then partitioned into a sequence of interacting continua V_{nj} $(j=1, \ldots, J)$. The continua are specified by means of a set of volume fractions ϕ_j $(j=1, \ldots, J)$, where ϕ_1 is the average fracture porosity, and the $\phi_2, \ldots, \phi_J$ denote volume fractions in the matrix at increasing distance from the fractures. Obviously we must have

$$\sum_{j=1}^{J} \phi_j = 1 \tag{4}$$

Apart from this constraint, the ϕ_j $(j=2, \ldots, J)$ are arbitrary, but for best accuracy the volume fractions near the fractures $(\phi_2, \phi_3, \ldots)$ should be chosen not "too" large. The volumes of the sub-partitioning are simply

$$V_{nj} = \phi_j \cdot V_n \tag{5}$$

so that

$$\sum_{j=1}^{J} V_{nj} = V_n \tag{6}$$

In the "secondary" mesh $\{V_{nj};\ n=1, \ldots, N;\ j=1, \ldots, J\}$ each of the primary grid blocks V_{n1} (representing fractures) interacts with its neighbors through the fracture continuum, and with a one-dimensional string $V_{n2}, V_{n3}, \ldots, V_{nJ}$ of nested grid "blocks" in the matrix. The distances x_j to which the V_{nj} extend can be simply obtained by inverting the proximity function. We have

$$\mathrm{PROX}(x_j) = \sum_{j'=2}^{j} \frac{\phi_{j'}}{1-\phi_1} \tag{7}$$

The interface area between elements V_{nj} and V_{nj+1} is simply $A(x_j)$ as given by equation (3). Nodal distances are given by $(j=2, \ldots, J-2)$

$$d_{nj,\ nj+1} = \frac{x_{j+1} - x_j}{2} + \frac{x_j - x_{j-1}}{2} = \frac{1}{2}\,(x_{j+1} - x_{j-1}) \tag{8}$$

The fracture nodes are placed at the fracture-matrix interface, so that

$$d_{n1,\ n2} = \frac{x_2}{2} \tag{9}$$

The innermost nodal distance requires special consideration. Writing

$$d_{nJ-1,\ J} = \frac{x_{J-1} - x_{J-2}}{2} + D_J \tag{10}$$

we introduce the distance D_J of the nodal surface with index J from the innermost interface area, $A_{nJ-1,\ nJ}$. D_J should be chosen in such a way that the finite difference approximation for pressure - and temperature - gradients gives the most accurate estimate for the actual gradients at the interface $A_{nJ-1,\ nJ}$. In general, D_J may be taken in quasi-steady flow approximation. A good approximation for the quasi-steady nodal distance in many cases is (Warren and Root, 1963)

$$D_J = \frac{\left(V_{nJ}\right)^{1/3}}{10} \tag{11}$$

4. Examples of Proximity Functions

In the case of regularly shaped matrix blocks, analytical expressions can be written down for proximity functions. For example, for two-dimensional square matrix blocks with side length a the matrix volume within a distance x from the block faces is (per unit thickness)

$$V(x) = a^2 - (a-2x)^2 \tag{12}$$

so that, according to equation (2)

$$\mathrm{PROX}(x) = \frac{V(x)}{a^2} = 4\,\frac{x}{a} - 4\left(\frac{x}{a}\right)^2 \tag{13}$$

An interesting application of the methods presented here is for the Stanford large reservoir model, which has a loading of regularly shaped rocks. There are six layers, each of which has five parallelepiped blocks and four triangular blocks with side lengths a, b, c. The proximity function for a rectangular block is

$$P_r(x) = 8\,\frac{x^3}{ab^2} - \left(\frac{8}{ab} + \frac{4}{b^2}\right)x^2 + \left(\frac{4}{b} + \frac{2}{a}\right)x \tag{14}$$

and for a triangular block we have

$$P_t(x) = \left(3+2\sqrt{2}\right)\frac{4x^3}{ab^2} - \left[\frac{6+4\sqrt{2}}{b^2} + \frac{4\ 2+\sqrt{2}}{ab}\right]x^2 + \left(\frac{4+2\sqrt{2}}{b} + \frac{2}{a}\right)x \tag{15}$$

The averaged proximity function in each layer is

$$P_{rt}(x) = \frac{5}{7} P_r(x) + \frac{2}{7} P_t(x) \qquad (16)$$

These functions are illustrated in Figure 4. Fluid and heat flow calculations using these functions are reported in another paper presented at this workshop (Hunsbedt et al.,1982).

In the general case of arbitrary irregular fracture distributions, proximity functions can be computed by means of Monte Carlo - integration. A computer program was written which generates random points within a region V_0 with known fracture distribution. The minimum distance of each point from the fractures is computed, and all points are sorted in order of increasing distance. The fraction of points falling below a certain distance x is the value of the proximity function at x. This procedure, which is applicable to arbitrary fracture distributions, defines the proximity function at discrete points, subject to statistical fluctuations from the Monte Carlo - integration process. In order to be able to numerically compute derivatives of the proximity function, a smoothed curve is computed by fitting the discrete function with a succession of cubic splines. The accuracy of the Monte Carlo procedure was tested by computing proximity functions and their derivatives for cases where the results are known in analytical form.

Figures 5 and 6 show proximity functions and their derivatives for square matrix blocks. Note that the results of the Monte Carlo - integration give a close approximation to the analytical solution as given by equation (13) already for 5,000 integration points. However, small deviations are magnified when interface areas are computed by differentiation. When 50,000 integration points are used, a good approximation is obtained for interface areas, see Figure 6b.

Figure 7 shows a two-dimensional stochastic fracture pattern. This was generated with a computer program developed at LBL, according to a given distribution of orientations and lengths, with random locations (Long et al., 1982). The proximity function for this system, obtained by Monte Carlo - integration with 100,000 integration points, is shown in Figure 8, while Figure 9 gives the interface areas as obtained by numerical differentiation.

5. Summary

The proximity function quantifies, for a given fractured rock mass, the volume of rock matrix present in dependence upon the distance from the fractures. This function and its first derivative are sufficient to completely define the geometric parameters for interporosity flow between rock matrix and fractures, as required by the method of "multiple interacting continua" (MINC; Pruess and Narasimhan, 1982b). For regularly shaped matrix blocks, proximity functions can be written down in analytical form, while for stochastic fracture distributions they are obtained by means of Monte Carlo-integration. We are currently studying the dependence of proximity functions upon the parameters of fracture distributions, and upon sample size and specific realization of a stochastic distribution. Also, we have begun simulations of fluid and heat flow in geothermal reservoirs with realistic fracture distributions.

It should be emphasized that for modeling of flow in fractured rock masses, the proximity function of the flow system can be computed once and for all, ahead of actual flow simulations. A pre-processor program has been written (Pruess, 1982), which generates all geometric parameters for interporosity flow in a format compatible with Lawrence Berkeley Laboratory's geothermal simulators SHAFT79 and MULKOM. The preprocessor can also interface with other integral finite difference simulators, such as TRUST (saturated-unsaturated flow), FT (single-phase non-isothermal flow), and TRUMP (advective-diffusive heat and chemical transport). With the methods outlined in this paper, modeling of fluid and heat flow in naturally fractured reservoirs is no more difficult than simulations for porous media.

Acknowledgement

The authors thank Drs. T. N. Narasimhan, C. R. Wilson, and Jane Long for reviewing the manuscript. This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Technology, Division of Geothermal and Hydropower Technologies of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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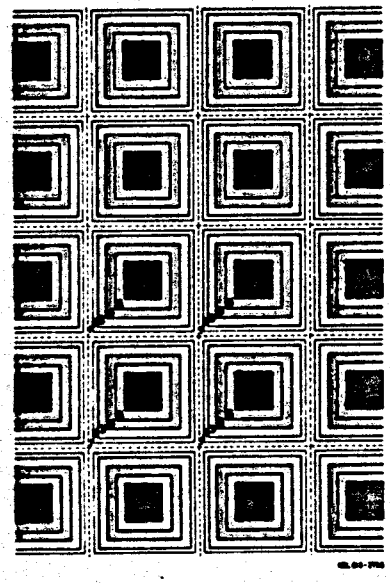

Figure 1. Basic computational mesh for fractured porous medium, shown here for a 2-D case. The fractures enclose matrix blocks of low permeability, which are subdivided into sequences of nested volume elements.

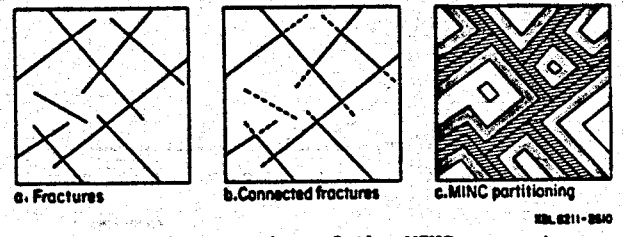

Figure 2. Illustration of the MINC-concept for an arbitrary two-dimensional fracture distribution.

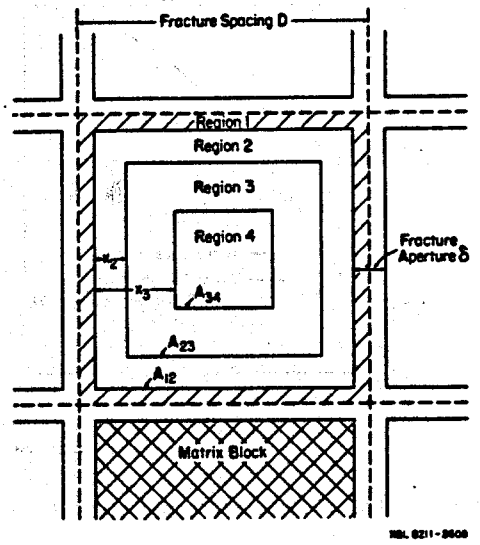

Figure 3. MINC-partitioning for an idealized fracture system.

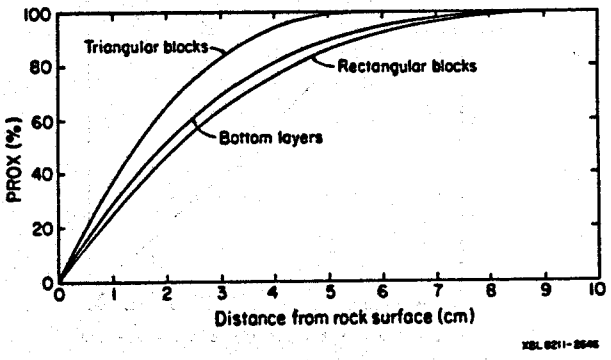

Figure 4. Proximity functions for Stanford large reservoir model.

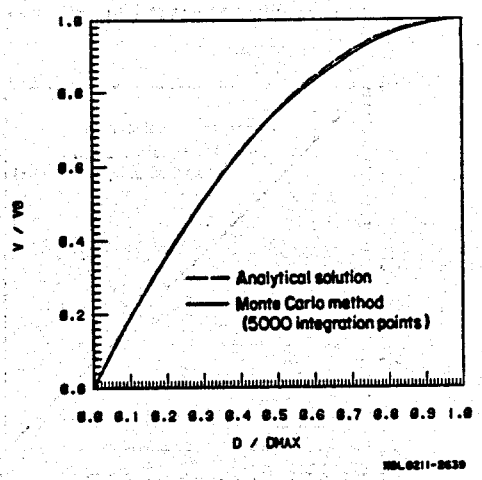

Figure 5. Proximity function for 2-D square matrix blocks (maximum distance from fractures is DMAX = a/2). (a) 5,000 integration points.

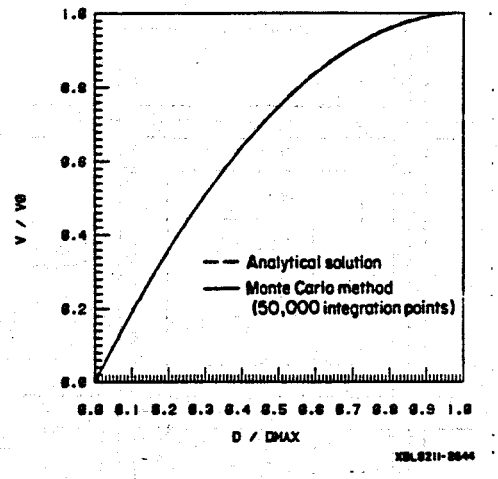

Figure 5. (b) 50,000 integration points.

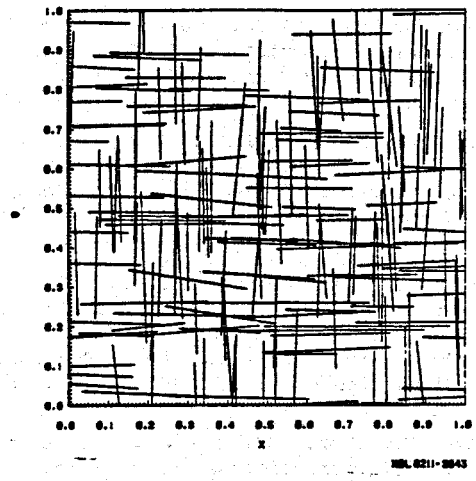

Figure 7. Two-dimensional stochastic fracture distribution.

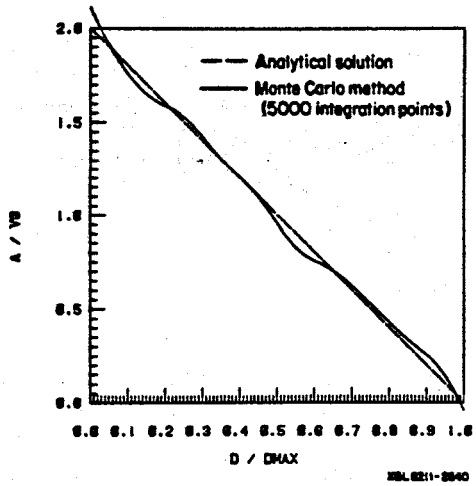

Figure 6. Derivative of proximity function for two-dimensional square matrix blocks (DMAX = 1). (a) 5,000 integration points.

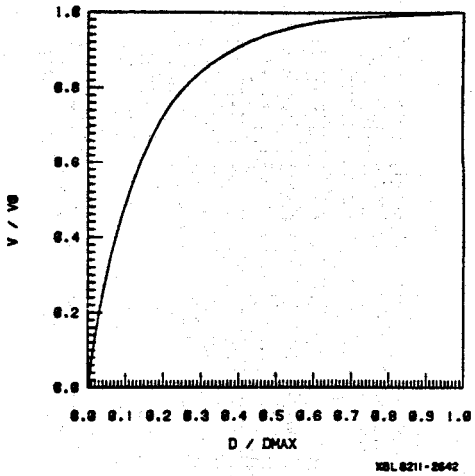

Figure 8. Proximity function for stochastic fracture distribution.

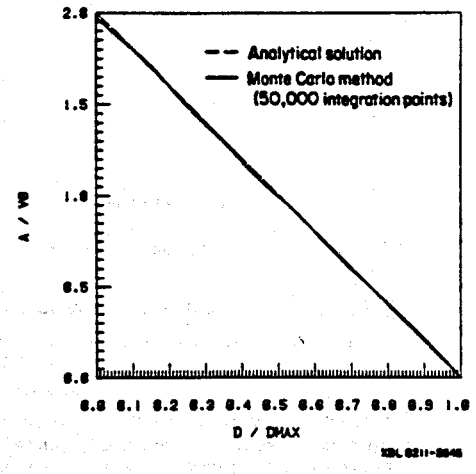

Figure 6. (b) 50,000 integration points.

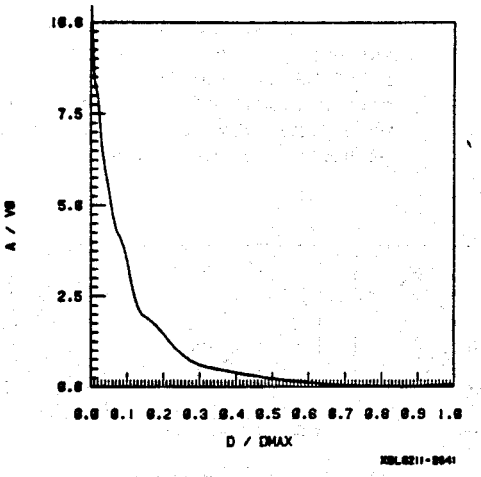

Figure 9. Derivative of proximity function for stochastic fracture distribution.

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