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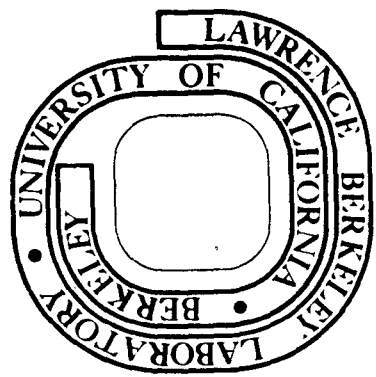
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Mordechai Bishari and Joel Koplik

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A PERTURBATIVE APPROACH TO THE POMERON

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ABSTRACT

The perturbative approach to the pomeron via the multi-fireball expansion proposed in the preceding paper is further analyzed in the context of a simplified model. A detailed picture of the pomeron is given, with special attention paid to the extent of factorization and the strength of the output two-pomeron cut. The role of various production mechanisms in building the output singularities is clearly exhibited. The energy dependence of total cross sections is discussed in the framework of the perturbative series and compared to the complementary interpretation in terms of output Regge poles. The possibility of a rising total cross section is carefully examined.

1. INTRODUCTION

A perturbative approach to the pomeron has been introduced recently [1] - [4], motivated by the familiar two-component model of multiparticle production processes. The perturbation series is an expansion in the number of diffractively produced fireballs or, in other words, in the number of pomerons exchanged in production amplitudes. The original two-component model is the leading term, and the smallness of multiple pomeron processes [7] ensures the rapid convergence of the expansion at feasible energies.

The general ingredients of our approach, such as the classification of various multiparticle final states and the distinction between small and large mass fireballs, have been discussed previously [2], [3] and will not be repeated in any detail. We shall present here a simplified model in which quantitative detail has been sacrificed in favor of clarity and tractability, but without omission of the physically significant aspects of the problem.

The zeroth order term, i.e., the original two-component model, has the following interpretation: the short-range correlation (SRC) component corresponds to a factorizable "bare" pomeron pole (denoted by \tilde{P}) and diffraction into low-mass states generates a generalized AFS cut. The "physical" pomeron (P) is obtained only after summing the higher order terms using unitarity, but the same physical pomeron governs elastic and other diffractive processes. This provides a consistency requirement on the pomeron--a point we shall return to. The result of the expansion is that the singularities of the two-component model are "renormalized" by the multifireball terms. High-mass fireballs, controlled by the PPP triple-Regge vertex, are particularly significant here.

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In section 2 we quote the complete solution for the absorptive part obtained in the preceding paper [3] (hereafter referred to as I), and then motivate and apply some simplifying assumptions. The general properties of the leading singularities, the pomeron pole and two-pomeron cut, are studied in section 3.

In section 4 the Regge singularities of the full amplitude are examined in detail. As is always the case in models with t-channel factorizability, these are controlled by a (Fredholm) denominator function, which bears an interesting relation to that of earlier multiperipheral models. We pay particular attention to the role of energy thresholds in generating complex poles [8].

Recent ISR data [12] has focused a great deal of theoretical interest on models which can accommodate rising cross sections. This subject will be discussed in the context of the present model in section 5. First, the possibility of an increase in the total cross section is discussed from the point of view of the perturbative series, which contains a small number of terms in the NAL-ISR energy regime. The significance of energy thresholds and the general characteristics of the energy dependence are given. Next, the alternative explanation in terms of complex Regge poles is discussed, and some of the physical implications of the results of section 4 clarified. We shall emphasize the importance of the use of a consistent theory of the pomeron for this subject, in order to avoid misleading results.

In section 6 we summarize our results and discuss some implications. A brief comparison of our treatment of rising cross sections with that of other authors is given. Some of the technicalities related to complex poles appear in two appendices.

2. FORMULATION OF A SIMPLIFIED MODEL

In I we have derived a rather formidable expression for the absorptive part of a scattering amplitude in terms of the parameters of the multifireball expansion. The key simplifying assumption we make in this paper is the replacement of the two-pomeron branch cut by an effective pole in the J-plane. It will be shown in section 3, however, that the residue of this effective pole does not factorize, in contrast to the usual behavior.* Since the pomeron has a finite slope, one expects that the effective cut is located to the left of the actual branch point. To clarify this somewhat, let α_c be the position of the effective cut with $\alpha_c < \alpha_c(0) \equiv 2\alpha_p(0) - 1$, and let $\delta = \alpha_c(0) - \alpha_c$. Then for small δ (which is the case if the pomeron slope is much smaller than the slope in t of its residues) and not too large s, the energy dependence of the effective cut is

$$\begin{aligned} s^{\alpha_c - 1} &= s^{\alpha_c(0) - 1 - \delta} \approx s^{\alpha_c(0) - 1} (1 - \delta \ln s) \\ &\approx \frac{s^{\alpha_c(0) - 1}}{1 + \delta \ln s} \end{aligned}$$

This is a familiar form for the contribution of a Regge cut to the cross section.

Now from eq. (5.6) of I, with the replacement

$$\int dt f(t) \frac{e^{\Delta(2\alpha_p(t) - 1 - J)}}{J - 2\alpha_p(t) + 1} \rightarrow \frac{e^{-\Delta(J - \alpha_c)}}{J - \alpha_c} \int dt f(t),$$

* In the usual case, a pole is a zero of a Fredholm denominator, which guarantees factorization. This effective pole is, so to speak, in the numerator.

we see that the J-plane projection of the forward a-b absorptive part is

$$\begin{aligned}
 A_{ab}(J) = & \lambda_{ab} \frac{e^{-\Delta(J-\alpha_c)}}{J-\alpha_c} + \frac{\mu_a \mu_b e^{-2\Delta(J-\alpha_c)}}{(J-\alpha_c)(J-\alpha_c-\eta_2 e^{-\Delta(J-\alpha_c)})} \\
 & + \left(\beta_{Pa}^{\sim} + v_a \frac{e^{-\Delta(J-\alpha_c)}}{J-\alpha_c} + \frac{\mu_a \eta_1 e^{-2\Delta(J-\alpha_c)}}{(J-\alpha_c)(J-\alpha_c-\eta_2 e^{-\Delta(J-\alpha_c)})} \right) \\
 & \times \frac{e^{-\tilde{\Delta}(J-\alpha_P)}}{J-\alpha_P} \\
 & \times \frac{1}{1 - \epsilon \frac{e^{-\tilde{\Delta}(J-\alpha_P)}}{J-\alpha_P} \frac{e^{-\Delta(J-\alpha_c)}}{J-\alpha_c} - \eta_1^2 \frac{e^{-\tilde{\Delta}(J-\alpha_P)}}{J-\alpha_P} \frac{e^{-2\Delta(J-\alpha_c)}}{(J-\alpha_c)(J-\alpha_c-\eta_2 e^{-\Delta(J-\alpha_c)})}} \\
 & \times \left(\beta_{Pb}^{\sim} + v_b \frac{e^{-\Delta(J-\alpha_c)}}{J-\alpha_c} + \frac{\mu_b \eta_1 e^{-2\Delta(J-\alpha_c)}}{(J-\alpha_c)(J-\alpha_c-\eta_2 e^{-\Delta(J-\alpha_c)})} \right)
 \end{aligned} \tag{1}$$

where

$$A_{ab}(J) = \int_1^{\infty} ds A_{ab}(s) s^{-J-1} \tag{2}$$

and

$$\alpha_T(s) = \frac{1}{s} A_{ab}(s). \tag{3}$$

(An energy scale of 1 GeV is used throughout this paper.) The various parameters appearing in eq. (1) are given by

$$\lambda_{ab} = \frac{1}{16\pi} \int dt B_{Pa}(t) B_{Pb}(t) |\xi_P(t)|^2 \tag{4.a}$$

$$\mu_a = \frac{1}{16\pi} \int dt B_{Pa}(t) v(t) |\xi_P(t)|^2 \tag{4.b}$$

$$v_a = \frac{1}{16\pi} \int dt B_{Pa}(t) g(t) |\xi_P(t)|^2 \tag{4.c}$$

$$\epsilon = \frac{1}{16\pi} \int dt g^2(t) |\xi_P(t)|^2 \tag{4.d}$$

$$\eta_1 = \frac{1}{16\pi} \int dt g(t) v(t) |\xi_P(t)|^2 \tag{4.e}$$

$$\eta_2 = \frac{1}{16\pi} \int dt v^2(t) |\xi_P(t)|^2 \tag{4.f}$$

where $B_{Pa}(t)$ measures the strength of low-mass diffractive excitation (including elastic scattering) of particle a, $v(t)$ is the P-P-particle coupling, and $g(t)$ is the PPP vertex. The precise definition of these vertices and a discussion of the rapidity gap parameters Δ and $\tilde{\Delta}$ is given in I. The original two-component model is obtained from (1) by setting $g = v = 0$, which then expresses $A_{ab}(J)$ in terms of bare singularities

$$\lambda_{ab} \frac{e^{-\Delta(J-\alpha_c)}}{J-\alpha_c} + \beta_{Pa}^{\sim} \frac{e^{-\tilde{\Delta}(J-\alpha_P)}}{J-\alpha_P} \beta_{Pb}^{\sim}. \tag{5}$$

The perturbations, represented by g and v , renormalize the bare pomeron \tilde{P} and two-pomeron cut in a manner given by eq. (1). Both residue and intercept of \tilde{P} are altered, the strength (but not the location) of the two pomeron cut is changed, and new singularities

are introduced. Aside from the effective cut at α_c , all other singularities arise from the vanishing of the denominator of the third term of (1). The apparent singularities at $J = \alpha_p$ and at the solution of

$$J - \alpha_c - \eta_2 e^{-\Delta(J-\alpha_c)} = 0$$

are in fact not present and decouple from the amplitude.

The solution (1) is still rather involved. It has been written here for completeness and for reference, and the bulk of an analysis will be given only in the simpler and more transparent case of $v \approx 0$ (see section 4).

3. THE LEADING SINGULARITIES

A basic assumption in our approach is that the output pomeron (P) is the same singularity that dominates the diffractive processes which appear in the unitarity sum. In other words we have a partial bootstrap condition for the pomeron. However such a condition is only approximate because the diffractive processes in the unitarity sum are assumed to be governed by only a pomeron pole whereas, in general, additional output singularities are generated in the absorptive part. Fortunately the degree of accuracy of the bootstrap condition can be explicitly investigated by studying the strength of the next important output singularity, the two-pomeron cut. As shown below, this cut is likely to be highly suppressed and hence, a posteriori, the assumption of exchanging only P in the unitarity sum is partly justified.

We now turn to eq. (1) from which the strength of the two-pomeron cut is obtained by letting J approach α_c :

$$A_{ab}(J) \xrightarrow{J \rightarrow \alpha_c} \frac{e^{-\Delta(J-\alpha_c)}}{J - \alpha_c} \left[\lambda_{ab} - \frac{\mu_a \mu_b}{\eta_2} + \left(v_a - \frac{\mu_a \eta_1}{\eta_2} \right) \times \frac{1}{-\epsilon + \frac{\eta_1}{\eta_2}} \left(v_b - \frac{\mu_b \eta_1}{\eta_2} \right) \right]. \quad (6)$$

Obviously eq. (6) will not lead, in general, to a factorized residue. However, with a little effort (6) can be cast into the familiar Gribov [15] form for the cut (with a positive sign, however):

$$A_{ab}(J) \xrightarrow{J \rightarrow \alpha_c} \frac{e^{-\Delta(J-\alpha_c)}}{J - \alpha_c} \int dt N_a(t) N_b(t) |\xi_p(t)|^2 \quad (7)$$

where the "fixed pole residues" are given by

$$N_a(t) = B_{Pa}(t) - \frac{\mu_a}{\eta_2} v(t) - \frac{\left(v_a - \frac{\mu_a \eta_1}{\eta_2} \right)}{\epsilon - \frac{\eta_1}{\eta_2}} \left(g(t) - \frac{\eta_1}{\eta_2} v(t) \right) \quad (8)$$

and similarly for $N_b(t)$. One can see from eqs. (4.a) - (4.f) and (8) that $N_a(t)$ can indeed be much smaller than $B_{Pa}(t)$. In the ideal case where $B_{Pa}(t)$, $g(t)$ and $v(t)$ are all proportional the fixed pole residues vanish identically. This suggests that it may be a reasonable approximation to neglect the exchange of the two-pomeron cut in the unitarity sum. Thus the interactions B_{Pa} , v and g cooperate so as to reduce the factorization breaking in the complete model significantly below the breaking implied by the two-component model (eq. (5)).

A similar interplay among the various mechanisms may result in an enhancement of the residues of the physical pomeron pole. This is almost apparent from eq. (1) where higher order terms have explicitly renormalized the SRC pole residue in a positive and factorized way. To be precise, the effect of the denominator function in (1) must also be included, as we shall do in the next section. It will turn out that the pomeron pole residue is renormalized in first order but the intercept only in second order, so that the corrections to the residue of \tilde{P} can be substantial while its intercept is quite close to that of the physical pomeron.

4. THE SINGULARITY STRUCTURE OF THE MODEL

It will be assumed here that the pomeron-pomeron-particle coupling, represented above by $v(t)$, is negligible. Recent experiments [16] indicate that v is small, and except for a brief discussion in section 6 our arguments will be restricted to the case $v = 0$. We feel that even in this approximation the essential physics is retained.

Setting $v = 0$, which implies $\mu = \eta_1 = \eta_2 = 0$, eq. (1) simplifies to

$$A_{ab}(J) = \lambda_{ab} \frac{e^{-\Delta(J-\alpha_c)}}{J - \alpha_c} + \left(\beta_{Pa}^{\sim} + \frac{v_a e^{-\Delta(J-\alpha_c)}}{J - \alpha_c} \right) \frac{e^{-\tilde{\Delta}(J-\alpha_P)}}{J - \alpha_P} \\ \times \frac{1}{1 - \epsilon \frac{e^{-\tilde{\Delta}(J-\alpha_P)}}{J - \alpha_P} \cdot \frac{e^{-\Delta(J-\alpha_c)}}{J - \alpha_c}} \cdot \left(\beta_{Pb}^{\sim} + \frac{v_b e^{-\Delta(J-\alpha_c)}}{J - \alpha_c} \right) \quad (9)$$

where ϵ is defined in terms of the PPP vertex $g(t)$ in (4.d).

The output two-pomeron cut has the same form as in eq. (7) but with the simpler "fixed pole residue"

$$N_a(t) = B_{Pa}(t) - \frac{v_a}{\epsilon} g(t)$$

and similarly for $N_b(t)$. Of course, as in the general case with $v \neq 0$, here also the two pomeron cut is suppressed.

In addition we will assume $\tilde{\Delta} \approx \Delta$. Notice that this together with the previous approximation leads to a (single-fireball) pomeron-pomeron cross section consisting of a negligibly small effective

low energy resonance plus a high energy tail starting at $s = e^\Delta$.

This may not be completely realistic, as discussed in section 2 of I, and we will return to this point in section 6.

The assumption $\alpha_p \approx \alpha_c$ will further simplify the analysis. The precise requirement is that $\Delta(\alpha_p - \alpha_c) \ll 1$, which is expected to hold because both α_p and α_c are expected to be very near α_0 .

Denoting

$$\alpha_p = \alpha_c = \alpha_0 \quad (11)$$

one can write eq. (9) as

$$A_{ab}(J) = \lambda_{ab} \frac{e^{-\Delta(J-\alpha_0)}}{J - \alpha_0} + F_a(J) \frac{e^{-\Delta(J-\alpha_0)}}{D(J)} F_b(J) \quad (12)$$

with

$$D(J) = (J - \alpha_0)^2 - \epsilon e^{-2\Delta(J-\alpha_0)} \quad (13)$$

$$F_a(J) = \beta_{Pa} + \nu_a \frac{e^{-\Delta(J-\alpha_0)}}{J - \alpha_0} \quad (14)$$

and similarly for $F_b(J)$

The quadratic nature of eq. (13) should be compared with still simpler models without leading cuts such as those of Chew and Pignotti ($\Delta = 0$) [18] and Chew and Snider ($\Delta \neq 0$) [11], where the corresponding denominator function is linear. In the present model the quadratic term in $D(J)$ is due to the two-pomeron cut induced by diffraction dissociation into high masses, as in the schizophrenic pomeron model [17], where $\Delta = 0$ however. It is the combination of quadratic and exponential in $D(J)$ which leads to the richer singularity structure of the present model.

For later purposes it is useful to decompose $D(J)$ into two functions

$$D(J) = D_1(J) D_2(J) \quad (15)$$

where

$$D_1(J) = J - \alpha_0 - \sqrt{\epsilon} e^{-\Delta(J-\alpha_0)} \quad (16)$$

and

$$D_2(J) = J - \alpha_0 + \sqrt{\epsilon} e^{-\Delta(J-\alpha_0)}. \quad (17)$$

The leading pole in the model, the pomeron (P), belongs to the D_1 family and its intercept α_p satisfies

$$\alpha_p = \alpha_0 + \sqrt{\epsilon} e^{-\Delta(\alpha_p - \alpha_0)} \quad (18)$$

If we choose $\alpha_p = 1$ then

$$1 - \alpha_0 = \sqrt{\epsilon} e^{-\Delta(1-\alpha_0)}, \quad (19)$$

which relates the bare pomeron intercept to the PPF coupling. Since $\Delta(\alpha_p - \alpha_0) \ll 1$, the approximate solutions to eqs. (18) and (19) are, respectively,

$$\alpha_p = \alpha_0 + \frac{\sqrt{\epsilon}}{1 + \Delta\sqrt{\epsilon}} \quad (18')$$

$$1 - \alpha_0 = \frac{\sqrt{\epsilon}}{1 + \Delta\sqrt{\epsilon}} \quad (19')$$

In addition to the pomeron the family D_1 contains an infinite sequence of complex poles in analogy to the model studied by Chew and Snider [11]. Indeed, $D_1(J)$ is formally identical to the corresponding denominator function there (with of course a different physical

interpretation for the couplings), which was shown to contain complex poles. As in ref. [11], the gap parameter Δ is responsible for the appearance of the complex poles in D_1 . These complex poles will be shown to generate a local rise in σ_T (section 5). In fact if $\Delta = 0$ then the whole family D_1 degenerates to the pomeron alone, and non-rising cross sections necessarily result.

The family D_2 is a new feature of the model. Because the coupling $\sqrt{\epsilon}$ appears in $D_2(J)$ with sign opposite to that in $D_1(J)$, the family D_2 differs in its structure and strength from D_1 . In fact as shown in appendix A there exist a critical coupling, $\epsilon_c = 1/\Delta^2 e^2$, such that the leading singularities in D_2 are either two real poles with residues of opposite sign ($\epsilon < \epsilon_c$), a dipole ($\epsilon = \epsilon_c$), or a pair of conjugate complex poles ($\epsilon > \epsilon_c$). The poles in D_2 are not simply related to rising cross sections, in contrast to the complex poles from D_1 . We shall show in section 5 below that a certain condition on the relative importance of SRC and low mass diffraction will precisely decouple the entire family D_2 .

We now wish to calculate the respective strengths with which D_1 and D_2 couple to the amplitude. If α_1 is a pole from D_1 then, using $D_1(\alpha_1) = 0$ and eqs. (12) - (13), its residue is

$$e^{-\Delta(\alpha_1 - \alpha_0)} \frac{(\beta_{Pa}^{\sim} + v_a/\sqrt{\epsilon})(\beta_{Pb}^{\sim} + v_b/\sqrt{\epsilon})}{2(1 + \Delta(\alpha_1 - \alpha_0))} \quad (20)$$

For a pole α_2 belonging to D_2 the residue is (now using

$$D_2(\alpha_2) = 0) \\ e^{-\Delta(\alpha_2 - \alpha_0)} \frac{(\beta_{Pa}^{\sim} - v_a/\sqrt{\epsilon})(\beta_{Pb}^{\sim} - v_b/\sqrt{\epsilon})}{2(1 + \Delta(\alpha_2 - \alpha_0))} \quad (21)$$

(The dipole case is obtained when the denominator in eq. (21) vanishes; this is considered in appendix A.) Thus the SRC and diffraction mechanisms are adding in the residue of α_1 and subtracting in the residue of α_2 . The entire family D_2 decouples if the following relation between the SRC and low mass diffraction cross sections holds for every particle a :

$$\beta_{Pa}^{\sim} = \frac{v_a}{\sqrt{\epsilon}} \quad (22)$$

To understand the significance of this condition, let us first assume B_{Pa} and g have the same t -dependence. The low mass diffraction cross section is then (from eq. (4))

$$\sigma_D^{ab} = \frac{v_a v_b}{\epsilon} s^{\alpha_0 - 1},$$

while the SRC cross section is

$$\sigma_M^{ab} = \beta_{Pa}^{\sim} \beta_{Pb}^{\sim} s^{\alpha_0 - 1},$$

so that

$$\beta_{Pa}^{\sim} \pm \frac{v_a}{\sqrt{\epsilon}} \propto \sqrt{\sigma_M^{aa}} \pm \sqrt{\sigma_D^{aa}} \quad (23)$$

In p - p scattering this would lead to a substantial reduction in importance for D_2 relative to D_1 by a factor of ~ 27 . If, however, we take into account the difference in t -dependence and write $B_{Pa}(t) = B_{Pa}(0) e^{b_1 t}$, $g(t) = g(0) e^{b_2 t}$ then

$$\beta_{Pa}^{\sim} \pm \frac{v_a}{\sqrt{\epsilon}} \propto \sqrt{\sigma_M^{aa}} \pm \frac{2b_2}{b_1 + b_2} \sqrt{\sigma_D^{aa}} \quad (23')$$

Since experiment suggests b_1 may be much larger than b_2 , the suppression of D_2 may not be so pronounced.

From the work of ref. [17] it is tempting to conjecture that D_2 is related to the P' Regge trajectory. Unfortunately, the results of appendix A indicate that D_2 is more complicated and contains at least two leading poles (or a dipole). Furthermore, it is known that this "schizophrenic pomeron" interpretation requires a large triple pomeron coupling (to produce the observed splitting between P and P'), whereas the philosophy of our approach and preliminary experimental indications suggest this coupling is small.

5. A LOCAL RISE IN CROSS SECTIONS AND ITS INTERPRETATION

Much attention has been recently directed to models which predict increasing cross sections, largely due to the findings from cosmic-ray [19] and ISR [12] experiments. A number of authors [4], [13], [14], [20] have attributed this effect to the onset of high mass diffraction, and we now turn to this question.

Our main concern here is to investigate rising cross sections consistently in the framework of our model of the pomeron, and to discuss the mechanisms responsible for the rise. Needless to say, the results derived from this simplified model should not be taken too literally; however, the simple model is useful in clarifying some basic aspects of theories of rising cross sections.

The behavior of $\sigma_T(s)$ is first studied from the perturbation expansion, taking into account the pomeron constraint (eq. (19)) which must be considered in a consistent approach. The results obtained from the expansion are then interpreted in terms of the singularities of the complete solution.

According to the perturbative approach, only a few terms in the series are important in the NAL-ISR energy range. As in section 4 of I, we assume the value of Δ is such that in the energy range of interest to us the total cross section is

$$\begin{aligned} \sigma_T(s) \approx & \left(\beta_{Pa}^2 s^{\alpha_0-1} + \frac{1}{16\pi} \int dt B_{Pa}(t) B_{Pa}(t) |\xi_P(t)|^2 s^{\alpha_0-1} \right) \\ & \times \theta(\ln s - \Delta) + 2 \beta_{Pa}^2 v_a s^{\alpha_0-1} (\ln s - 2\Delta) \theta(\ln s - 2\Delta). \end{aligned} \quad (25)$$

For simplicity we have assumed identical incident particles. The first and the second terms represent, respectively, the SRC and low mass

diffraction components. The third term is generated from events where one of the colliding particles is diffractively excited into low mass and the other into high mass, and vice versa. This term is proportional to the weak coupling $g(t)$ (see eq. (4.c)) and is the first order correction to the two-component model. Even if further terms are kinematically possible, the smallness of g ensures that they will make a small contribution.

Let us denote by σ_M and σ_D the SRC and low mass diffraction cross sections at $\ln s = 2\Delta$, namely

$$\sigma_M = \beta_{Pa}^2 e^{2\Delta(\alpha_0-1)} \quad (26)$$

$$\sigma_D = \frac{1}{16\pi} \int dt B_{Pa}(t) B_{Pa}(t) |\xi_p(t)|^2 e^{2\Delta(\alpha_0-1)} \quad (27)$$

Assuming that $g(t)$ and $B_{Pa}(t)$ have the same slope in t , one can rewrite eq. (25), for $\ln s > 2\Delta$, as

$$\sigma_T(s) \approx \sigma_M \left(s/e^{2\Delta}\right)^{\alpha_0-1} + \sigma_D \left(s/e^{2\Delta}\right)^{\alpha_0-1} + 2\sqrt{\epsilon} \sqrt{\sigma_M \sigma_D} \left(s/e^{2\Delta}\right)^{\alpha_0-1} \times (\ln s - 2\Delta) \quad (28)$$

The derivative at $\ln s = 2\Delta$ is

$$\left. \frac{d\sigma_T(s)}{d \ln s} \right|_{\ln s=2\Delta} = (\alpha_0 - 1)(\sigma_M + \sigma_D) + 2\sqrt{\sigma_M \sigma_D} \sqrt{\epsilon} \quad (29)$$

Since ϵ is nonvanishing, (19) implies $\alpha_0 < 1$ and thus the SRC component must decrease with s . This consistency condition is often ignored in treatments of rising cross sections, although (as in appendix C of I) its effects can be critical.

Taking eq. (19) into account, we can write (29) as

$$\left. \frac{d\sigma_T(s)}{d \ln s} \right|_{\ln s=2\Delta} = -\sqrt{\epsilon} e^{-\Delta(1-\alpha_0)} \left(\sqrt{\sigma_M} - \sqrt{\sigma_D}\right)^2 + 2\sqrt{\sigma_M \sigma_D} \sqrt{\epsilon} \left(1 - e^{-\Delta(1-\alpha_0)}\right) \quad (30)$$

We can now clearly see the role of the gap parameter Δ . If $\Delta = 0$ then the second term is absent and σ_T must decrease, although slowly because of cancellations between σ_M and σ_D in the first term. If Δ is large enough (30) can indeed be positive. Another way to see this is to note that if $\Delta = 0$ there are no complex poles. The remaining singularities are then the pomeron pole, a pole from D_2 whose residue in (21) is positive, and a cut whose discontinuity must also be positive, and a decreasing cross section results. Note also that since one of the Δ 's involved originated with the pomeron-particle absorptive part, it may depend on the incident particle, and different reactions may begin to rise at different energies.

It is an important question whether the rise of $\sigma_T(s)$, at $\ln s = 2\Delta$, is determined by $\sqrt{\epsilon}$ or by ϵ . The two possibilities will have very different physical implications because ϵ is expected to be very small. Suppose, first, one approaches the problem ignoring the consistency condition of the model. Then one is tempted to assume $\alpha_0 \approx 1$ in eq. (29) resulting in

$$\left. \frac{d\sigma_T}{d \ln s} \right|_{\ln s=2\Delta} \approx 2\sigma_M \sqrt{\epsilon} \quad (31)$$

where for the sake of the argument we have taken $\sigma_M \approx \sigma_D$. For ϵ as small as 0.0025 an increase of 2 mb per unit of $\ln s$ is easily obtained.

However in a consistent treatment the increase is given by eq. (30) which reads (again with $\sigma_M \approx \sigma_D$)

$$\left. \frac{d\sigma_T}{d \ln s} \right|_{\ln s = 2\Delta} = 2 \sigma_M \sqrt{\epsilon} \left(1 - e^{-\Delta(1-\alpha_0)} \right) \quad (32)$$

since $\Delta(1 - \alpha_0)$ is small we obtain, using eq. (19'),

$$\left. \frac{d\sigma_T}{d \ln s} \right|_{\ln s = 2\Delta} \approx 2 \sigma_M \frac{\epsilon \Delta}{1 + \Delta \sqrt{\epsilon}} \quad (32')$$

and now the increase is determined by ϵ and not $\sqrt{\epsilon}$. Therefore if ϵ is very small the derivative of $\sigma_T(s)$ will be also small, e.g., 0.3 mb per unit of $\ln s$ with $\Delta \approx 3$ and ϵ as above.

Of course one can have a substantial increase in a consistent theory if ϵ is sufficiently large. However in such a case the separation of α_0 from 1 will accordingly increase because of eq. (19). This implies, using eq. (25), that $\sigma_T(s)$ will decrease for $\ln s < 2\Delta$ in a corresponding manner. In other words in the consistent approach a considerable increase for $\ln s \geq 2\Delta$ requires a comparable decrease for $\ln s < 2\Delta$.

The moral is that in the discussion of the behavior of $\sigma_T(s)$, one must take into account the consistency requirement, or misleading physical results can be obtained.

It will be instructive to consider in more detail the physical relevance of the parameters Δ and ϵ in determining the structure

of $\sigma_T(s)$. For this purpose we calculate from eq. (28) the following derivative (for $2\Delta < \ln s < 3\Delta$):

$$\frac{d\sigma_T(s)}{d \ln s} = \left(s/e^{2\Delta} \right)^{\alpha_0 - 1} \left[(\alpha_0 - 1)(\sigma_M + \sigma_D) + 2 \sqrt{\sigma_M \sigma_D} \sqrt{\epsilon} \times \left((\alpha_0 - 1)(\ln s - 2\Delta) + 1 \right) \right] \quad (33)$$

A maximum of $\sigma_T(s)$ occurs at

$$\ln s_1 - 2\Delta = \frac{1}{1 - \alpha_0} \left[1 - e^{-\Delta(1-\alpha_0)} \frac{(\sigma_M + \sigma_D)}{2 \sqrt{\sigma_M \sigma_D}} \right] \approx \Delta \quad (34)$$

where eq. (19) has again been used. Hence the gap parameter Δ measures the distance (in rapidity) between the points where the cross section starts to rise and begins to fall. The cross section at the maximum is

$$\sigma_T(s_1) \approx (\sigma_M + \sigma_D) \left(1 + \frac{\Delta^2 \epsilon}{2(1 + \Delta \sqrt{\epsilon})^2} \right) \quad (35)$$

and the increase, relative to the cross section at $\ln s = 2\Delta$, is proportional to ϵ (and not $\sqrt{\epsilon}$), as can be anticipated from eq. (32').

We now turn to the interpretation, of the rise at $\ln s = 2\Delta$ and the maximum at $\ln s \approx 3\Delta$, in terms of Regge poles generated by the unitarity sum [8]. It has been demonstrated in section 4 that apart from the cut all the output singularities fall into two distinct families, D_1 and D_2 . As shown explicitly in appendix A the family D_2 tends to complicate the Regge representation of σ_T . A complete decoupling of D_2 is ensured by the relation given in eq. (23) which is translated into $\sigma_M = \sigma_D$ if $B_{Pb}(t)$ and $g(t)$ have the same slope

in t . In this case the output cut is also decoupled, as one can see from eqs. (24) and (10). Thus we are left with only the family D_1 which contains the pomeron P and an infinite sequence of complex Regge poles. The most important contributions to $\sigma_T(s)$ come from the pomeron, $\alpha_1=1$, and the first pair of complex poles at $J = \alpha$ and $J = \alpha^*$. Denoting the real and imaginary parts of α by α_R and α_I , respectively, the description of $\sigma_T(s)$ in terms of the pomeron and the first pair of complex poles is found from eq. (20):

$$\sigma_T(s) \approx 2 \sigma_M e^{\Delta(1-\alpha_0)} \left[\frac{1}{1 + \Delta(1 - \alpha_0)} + \frac{2}{\left[(1 + \Delta(\alpha_R - \alpha_0))^2 + (\Delta\alpha_I)^2 \right]^{1/2}} \left(\frac{s}{e^\Delta} \right)^{\alpha_R - 1} \times \cos [\alpha_I (\ln s - \Delta) + \theta] \right] \quad (37)$$

where σ_M is defined in eq. (26) and the phase θ is given by

$$\theta = \text{tg}^{-1} \frac{(-\Delta\alpha_I)}{1 + \Delta(\alpha_R - \alpha_0)}. \quad (38)$$

Now (37) should be compared with the perturbative series given in (28) (taking $\sigma_M \approx \sigma_D$) if one considers energies such that $\ln s < 3 \Delta$. Both representations of $\sigma_T(s)$ should agree if indeed the pair of complex poles explain the features exhibited by the series expansion. This has been verified for a mathematically identical situation by Chew and Snider [11] and need not be repeated here. Note that our remark above on the possible reaction dependence of the energy at which a rise may occur is equivalent to the statement that the phase of a complex pole's residue is reaction-dependent. The period, of course, is universal.

VI. SUMMARY AND DISCUSSION

This work has been strongly motivated by the phenomenological two-component model of multiparticle production processes which has had a considerable success in describing recent data. Our approach is a perturbative one where, within the present set of approximations, the expansion parameter is provided by the weak $PP\tilde{P}$ vertex. In this perturbative scheme the zeroth order term is precisely the two-component model where the SRC (or multiperipheral) part generates the factorizable "bare" pomeron \tilde{P} and low-mass diffractive processes give the input generalized AFS cut. We have shown that the higher order processes renormalize the "bare" singularities and that the renormalization effects may be rather important in some respects. Explicitly, the output cut may be considerably smaller [2] than the diffraction component which a posteriori justifies the assumption of including only the pomeron pole in the diffractive processes which appear in the unitarity sum. Such a suppression of the output cut may be compensated by an increase of the factorizable piece, i.e., an enhancement of the residue of \tilde{P} .^{*} Therefore the factorization breaking in the complete model is expected to be smaller than the breaking in lowest order (the two-component model). We remark that while the residues of \tilde{P} and P can differ appreciably, nevertheless their intercepts are very close which ensures that the SRC part will not vary strongly with energy.

Our simple model clearly illustrates the manner in which the various mechanisms renormalize the bare singularities and in generating new output singularities. Apart from the output two-pomeron cut, all the other singularities are associated with two families
^{*}Strictly speaking this depends on the details of the parameters involved.

D_1 and D_2 . The D_1 family contains the pomeron pole and an infinite number of complex poles, the latter disappearing for a zero gap parameter ($\Delta = 0$). The second family D_2 , which is studied further in appendix A, is a new feature for which we lack a satisfactory interpretation at present. Although at first glance D_2 appears to be associated with the P' as in the schizophrenic pomeron model, there are the above-mentioned obstacles to this identification. While it is formally possible to decouple D_2 entirely if (22) holds, this is probably not realized in nature except perhaps approximately. We hope to return to this question in the future.

We have examined the possibility of increasing total cross sections in some detail. Within the context of this model we have found that a substantial rise is possible for sufficiently large triple-pomeron (really PPP) coupling, but only at the expense of a corresponding decrease at lower energies. This followed from the necessity of using a bare pomeron of lower intercept in our (consistent) approach. Indeed, using the somewhat different set of approximations in appendix C of I a strictly decreasing cross section results. An essential ingredient in obtaining a rise is a strong threshold constraint to delay the energy at which triple-pomeron behavior appears. In other words, a rise requires large rapidity gap parameters to provide important complex poles. In the absence of this threshold effect, an increase in $\sigma_T(s)$ cannot be associated with the triple-pomeron coupling. We emphasize that a complete and consistent model of the cross section is required before its energy dependence can be sensibly discussed.

Obviously the approach adopted in this work provides an iterative scheme for calculating the eventual output singularities. Here we have presented the results from the first iteration only, but higher order iterations may lead to further renormalization effects and possibly to a somewhat larger increase in $\sigma_T(s)$. No attempt has been made here to consider the results from higher order iterations but this possibility should be kept in mind.

We now mention some other mechanisms, consistent with the present formalism, which may lead to an increasing total cross section. One candidate is the production of low-mass fireballs internally by repeated pomeron exchange. In our description this is represented by the coupling v which we have neglected. It is believed, however, that the contribution of such processes to σ_T is small [4], [16]. A local increase may also result from nonleading triple-Regge couplings such as PPR. In spite of the fact that $R (= P', \rho, \dots)$ is non-leading it may be significant since the energy scale is e^Δ rather than 1 GeV. If this is indeed the explanation, then σ_T may begin to flatten out at accessible energies. Another interesting possibility is the association of a rising σ_T with the observed sudden and substantial increase in anti-baryon production at high energies [21].

Another popular approach, somewhat similar to ours, is that of the absorbed multiperipheral model [13]. These authors also employ a perturbative approach using small corrections to a bare pomeron pole with, however, unit intercept. Absorption is then necessary to

preserve the Froissart bound. The absorptive corrections necessarily eventually destroy factorization, which experiment approximately respects at present.*

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APPENDIX A

We shall discuss here some features of the singularities belonging to D_2 . It has been shown that apart from the cut all the singularities of the model are generated from zeros of $D(J)$ which appears in eqs. (12) and (13). The function $D(J)$ has been split into $D_1(J)$ and $D_2(J)$ (eqs. (15) - (17)) where the mathematical structure of the singularities stemming from $D_1(J)$ has been already discussed in ref. [11]. Let us again write the expression for $D_2(J)$;

$$D_2(J) = J - \alpha_0 + \sqrt{\epsilon} e^{-\Delta(J-\alpha_0)}. \quad (\text{A.1})$$

The contribution of a pole at α_2 , such that $D_2(\alpha_2) = 0$, to the scattering amplitude is given in eq. (21). However (21) is meaningless if the denominator vanishes, namely if

$$\alpha_2 = \alpha_d = \alpha_0 - \frac{1}{\Delta}, \quad (\text{A.2})$$

or, in other words, when $D'(\alpha_2) = 0$. This is the case of a dipole, which cannot occur in $D_1(J)$ because there (eq. (16)) the coupling $\sqrt{\epsilon}$ appears with a different sign than in $D_2(J)$. The contribution of the dipole singularity to σ_T will be evaluated below.

Since in the dipole limit we have the two conditions, $D_2(\alpha_d) = 0$ and $D_2'(\alpha_d) = 0$ one obtains also a relation for ϵ , namely

$$\epsilon = \epsilon_c = 1/e^2 \Delta^2 \quad (\text{A.3})$$

which for $\Delta = 3$ is $\epsilon_c = 0.015$.

In contrast to the family D_1 , where in addition to the pomeron there are only complex singularities for every value of ϵ , here in D_2 if $\epsilon < \epsilon_c$ the leading singularities are two real poles and if

* Since our approach really uses an effective pomeron pole in the input, some absorptive corrections may already be included.

$\epsilon > \epsilon_c$ they become a complex pair. The dipole case, $\epsilon = \epsilon_c$, is a transition point.

Mathematically the point $J = \alpha_d = \alpha_0 - \frac{1}{\Delta}$ may be viewed as a critical point with a critical coupling $\epsilon = \epsilon_c$. For ϵ near ϵ_c one may expand about the "critical point" and easily obtain that $D_2(J)$ leads to the following pair of poles:

$$\alpha_{\pm} = \alpha_0 - \frac{1}{\Delta} \left(\frac{\epsilon_c}{\epsilon} \right)^{1/2} \pm \frac{1}{\Delta} \left(\frac{\epsilon_c}{\epsilon} - 1 \right)^{1/2}. \quad (A.4)$$

Indeed, these poles are real if $\epsilon < \epsilon_c$ and become complex for $\epsilon > \epsilon_c$. Also since the "critical point" is a dipole it is obvious that for $\epsilon < \epsilon_c$ one of the poles must have a negative residue.

We shall now study the contribution of the leading poles in D_2 to the total cross section. It is convenient and sufficient for our purposes to study only the dipole limit. One then needs to calculate also $D''(\alpha_d)$ and $D'''(\alpha_d)$ and a straightforward effort leads to (see eqs. (12) - (14))

$$A_{aa}(J) \xrightarrow{J \rightarrow \alpha_d} \frac{e^{\Delta \alpha_0}}{\Delta} \frac{\left(\beta_{Pa}^{\sim} + v_a \frac{e^{-\Delta(J-\alpha_0)}}{J-\alpha_0} \right)^2}{1 - \frac{\Delta}{3}(J-\alpha_d)} \frac{(s/e^{\Delta})^J}{(J-\alpha_d)^2}. \quad (A.5)$$

The corresponding contribution to $\sigma_T(s)$ is

$$\frac{e^{\Delta \alpha_0}}{\Delta} \left(\beta_{Pa}^{\sim} - \frac{v_a}{\sqrt{\epsilon_c}} \right)^2 \left(s/e^{\Delta} \right)^{\alpha_d} \left(\ln s - \frac{2}{3} \Delta \right). \quad (A.6)$$

Hence the dipole generates a rise in an entirely different energy region than the direct perturbative expansion given in (28). A precise

decoupling occurs when a certain relation between the various mechanisms is fulfilled (see eqs. (A.6) and (23)); in reality the magnitudes of σ_M and σ_D are such that the family D_2 indeed couples more weakly than D_1 .

APPENDIX B

A more realistic model than the one presented in this work has been discussed in great detail in I. Our goal here is to verify that in that model complex poles also appear. Assuming that the pomeron-pomeron-particle coupling is negligible, the denominator function can be written as (see I.5.9, with $v = 0$ and $f_p(t)$ suppressed).

$$D(J) = J - \alpha_P - e^{-\Delta(J-\alpha_P)} \frac{1}{16\pi} \int g^2(t) \frac{e^{-\Delta(J-2\alpha_P(t)+1)}}{J - 2\alpha_P(t) + 1} dt. \quad (B.1)$$

We assume the pomeron has unit intercept. Then, in order to preserve the Froissart bound, the coupling $g(t)$ must vanish at $t = 0$; we write

$$g(t) = -ate^{bt}. \quad (B.2)$$

The integral in (B.1) can be expressed in terms of known functions, resulting in

$$D(J) = J - \alpha_P - \frac{a^2}{16\pi} \times \frac{1}{2\alpha'^3} e^{-\Delta(J-\alpha_P)} e^{(b/\alpha')(J-\alpha_c(0))} \times \left\{ - (J - \alpha_c(0)) \frac{e^{-(\Delta+b/\alpha')(J-\alpha_c(0))}}{\Delta + b/\alpha'} + \frac{e^{-(\Delta+b/\alpha')(J-\alpha_c(0))}}{(\Delta + b/\alpha')^2} + (J - \alpha_c(0))^2 E_1 \left[(\Delta + b/\alpha')(J-\alpha_c(0)) \right] \right\} \quad (B.3)$$

where α' is the slope of the pomeron pole, $\alpha_c(0) = 2\alpha_P(0) - 1$, and E_1 is the exponential integral function defined by

$$E_1(z) = \int_z^\infty dt \frac{e^{-t}}{t}. \quad (B.4)$$

One can easily verify from (B.1) that there exists no real pole below $J = 1$ and hence only complex poles may appear. We shall now show the close similarity of (B.3) to the denominator in the simple model, as given in (9), as far as complex poles are concerned. For concreteness take $b = 2 \text{ GeV}^{-2}$, $\alpha' = 0.2 \text{ GeV}^{-2}$ and $\Delta = 3$, where for these parameters both the real and imaginary parts of the argument in the E_1 function may become large compared to 1. In such a case the following asymptotic expansion is extremely useful [22]:

$$E_1(z) \sim \frac{e^{-z}}{z} \left(1 - \frac{1}{z} + \frac{2}{z^2} + \dots \right); \quad \text{Re } z < 0, \quad |\text{Re } z|, \quad |\text{Im } z| \gg 1 \quad (B.5)$$

The use of (B.5), for the E_1 function in (B.3), greatly simplifies the form of $D(J)$ to

$$D(J) \approx J - \alpha_P - \frac{a^2}{16\pi} \times \frac{1}{(b + \alpha'\Delta)^3} e^{-\Delta(J-\alpha_P)} \frac{e^{-\Delta(J-\alpha_c(0))}}{J - \alpha_c(0)} \quad (B.6)$$

which is exactly the same as the denominator function in eq. (9) with

$$\epsilon = \frac{a^2}{16\pi} \times \frac{1}{(b + \alpha'\Delta)^3}. \quad (B.7)$$

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