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LARGE ANGLE SCATTERING OF NEGATIVE PIONS
IN ALUMINUM, COPPER, AND LEAD

H. H. Heckman

(thesis)

December 23, 1952

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ABSTRACT

A new technique is used to measure the cross sections for large angle scattering of negative pions as they traverse a semi-infinite scatterer. A stripped emulsion is embedded in the scatterer, exposed to an incident beam of 50^{+15}_{-5} Mev π^- mesons, and scanned. Most of the mesons stop at the expected distance from the absorber edge as determined from the range-energy relation. A few mesons are found at smaller depths of penetration and are attributed to large angle scattering. Star forming mesons that enter the emulsion traveling opposite (90° - 180°) to the direction of the incident beam are attributed to nuclear backscattering and are used in the cross section calculation. This method affords an effective solid angle of 2π steradians for observing nuclear backscattering. The cross section is proportional to the ratio of backward flux to incident flux, both of which are observed in the same strip of emulsion. The conclusions of this investigation are: a) The scattering of negative pions from complex nuclei is consistent with energy independence in the region of 32 ± 10 Mev. b) The S- and P-waves contribute to backscattering. c) The cross section for backscattering is proportional to the mass number, A, which indicates that pion - nucleon collisions are observed. The calculations of the cross sections include corrections for inelastic collisions. The total nuclear backscattering cross sections (elastic and inelastic) for negative pions in aluminum, copper, and lead are 59.6 ± 11 mb., 192 ± 27 mb., and 577 ± 80 mb., respectively.

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I INTRODUCTION

In 1935, Yukawa¹ proposed his celebrated meson theory of nuclear forces. The theory, developed along the formalism of electromagnetic theory, predicted that the "quantum", or meson, associated with the short range of nuclear field would have a rest mass of about 137 times the mass of the electron. Two years later, in 1937, Neddermeyer and Anderson² detected mesons in their cosmic ray studies.

It was apparent that if mesons were to be related to nuclear forces, there should be a strong interaction between mesons and nuclei. The question then arose as to whether or not the cosmic ray mesons observed at sea level could be identified with the particle proposed by Yukawa. The experiment of Conversi, Pancini, and Piccioni³ definitely excluded this supposition. The conclusion of their experiment was that the interaction of these mesons with the nucleus was the order of 10^{10} times too small to account for nuclear forces⁴. This dilemma could be resolved by the hypothesis that there was more than one kind of meson.

In 1947, Lattes, Occhialini, and Powell⁵ produced evidence that there was indeed a type of meson that interacted strongly with nuclei. Notwithstanding the fundamental importance of cosmic ray studies to meson physics, it was not until mesons were produced in the laboratory that the decay schemes and physical properties of mesons could be extensively analysed. A meson of $276.1 \pm 2.3 m_e$ ⁶ was found to have the fundamental property of strong interaction with nuclei, as shown through its production by nuclear collision, star formation upon absorption into the nucleus, and nuclear scattering. These strongly interacting mesons have zero or unit electric charge, both positive and negative, and have zero spin^{7,8}. This species of meson is designated as the π meson, or simply, the pion. In free space, the charged pion will decay spontaneously into a neutrino and a charged μ meson, or muon, with a mean lifetime

of $(2.54 \pm 0.11) \times 10^{-8}$ sec.^{9,10} The muon is found to have very little interaction with matter, and it is this type of meson that must be identified with the weakly interacting type observed in cosmic radiation at sea level. The muon has a mass of $209.6 \pm 2.4 m_e$ and spin of $1/2 \hbar$. It decays with a mean lifetime of 2.15×10^{-6} sec.¹¹ into two neutrinos and an electron or positron. The neutral pion decays with a mean lifetime of about 1×10^{-14} sec.¹², converting its rest mass of $264 m_e$ into two high energy gamma rays of about 70 Mev each.

One of the most direct ways to gather information on the nature of the pion - nucleon interaction is the study of the magnitude and angular distribution of pion scattering by nuclei. The scattering of pions from hydrogen lends itself to theoretical interpretation much more readily than the analysis of pion scattering from the more complex nuclei. Initial evidence on the interaction of negative pions of 55 to 85 Mev energy with protons indicated a total cross section much smaller than expected^{13,14}. Later, however, H. L. Anderson, et al^{15,16}, continued these transmission experiments to higher pion energies. The results of these investigations show that the total cross section rises rather rapidly above 80 Mev, reaching an apparent maximum value of $66 \pm 6 \times 10^{-27} \text{ cm}^2$ at 150 Mev.

Regardless of the difficulty of exact theoretical interpretation, the interaction of negative pions with heavier nuclei is certainly of physical interest. And indeed, quite general features of the pion - nucleon coupling may be deduced by their study. The total nuclear cross section of negative pions in various attenuating materials has been shown to be independent of energy between 85 Mev¹³ and 137 Mev¹⁷. The only possible exception to this for the elements that were studied might be carbon, which showed a slight decrease in the cross section with decreasing energy. The cross sections are in all cases close to the geometrical value of $\pi A^{2/3} (\frac{\hbar}{\mu c})^2$. This energy independence is in contrast to the strong energy dependence of the total cross section exhibited by pions on protons.

Some of the earliest information about the negative pion - nuclei interaction was obtained by the use of nuclear track emulsion. In this work, primarily carried out by Bradner and Rankin^{18,19} and Bernardini, Booth, Lederman, and Tinlot²⁰⁻²³, the negative pion energies studied

were in the regions of 30-35 Mev and 30-110 Mev respectively. In the energy region where these experiments can be compared, the total nuclear interaction cross sections were given as the nuclear area for the elements of the emulsion. Bradner and Rankin observed a cross section for elastic scatters that was equal to the combined cross sections for stars, disappearances, and inelastic scatters.

The use of cloud chambers has been particularly successful in the study of the negative pion interaction with pure materials used as scatters, usually carbon or aluminum²⁴⁻²⁷. Up to the present time, the most detailed investigation of the scattering and absorption of negative pions by complex nuclei, i. e. carbon, has been done by Lederman et al²⁷. 62 Mev pions were used in this experiment. The most interesting feature of the data was the observation of a Coulomb-nuclear interference in the elastic scattering. Also significant was the large amount of large angle scattering greater than 90° . The angular distribution for elastic back scatters indicated a minimum around 90 degrees and rising to a maximum at 180 degrees.

The differential cross section for inelastic scatters larger than 30 degrees appeared to be approximately one-third as large as for elastic scattering and it was consistent with spherical symmetry.

The intent of this paper is to study the large angle scattering of negative pions of about 30 Mev in pure substances. The method does not differentiate between elastic and inelastic scattering, but does exclude star formation and charge exchange in the calculation of the scattering cross section, which attenuation experiments do not. The method is adaptable most easily to elements that have a large stopping power, which precludes the use of the very light elements. For this reason, we have chosen for our study the elements aluminum, copper, and lead.

II EXPERIMENTAL METHOD

Arrangement

The technique used in this experiment was developed to use advantageously the high density of π^- meson flux that is produced by bombarding an internal target with the 340 Mev circulating proton beam of the synchro-cyclotron. These π^- mesons are then separated from the positively charged particles by the magnetic field of the cyclotron. Specialized methods have been devised to use the mesons that remain near the median plane. The usual method is to design the experiment so that the entire set-up can be placed on a cart, which can then be rolled through a port into the cyclotron vacuum tank. The use of nuclear track emulsions has been an integral part of the technique as their continuous sensitivity, compactness, and versatility make them ideally suited for such experiments.

The principal limitation when nuclear emulsions are exposed internally is the amount of low energy heavy particle background that accumulates in the emulsion during the bombardment. This background is generally isotropic in nature and is due primarily to knock-on protons produced by the neutron "sea" present in the cyclotron during a bombardment. The directional background coming from the target can be greatly reduced by taking proper precautions for shielding. To a trained observer the π^- meson is easily distinguished from the heavy particle background through its characteristic scattering, rapid change of grain density, and star production. When this background becomes extensive, however, detection becomes exceedingly difficult if not impossible.

The design of a meson scattering experiment must necessarily then take into account the problem of reducing the heavy particle background and the subsequent limitation of exposure time. These limiting factors can be partially alleviated by designing a scattering and absorbing system that reduces the geometrical loss of scattered mesons by using large solid angles. This was accomplished by having the scattering element function as an absorber as well as a scatterer. Figure 1 shows a diagram of the experimental arrangement that embodies the general features outlined above. The cart assembly was placed in the

cyclotron so that the $1/4 \times 1/4$ inch beryllium target was located at a radius of 79 inches. The light element target was chosen primarily to reduce the directional background that originates from the target. At this radius, the beam energy is 330 Mev. The beam current is approximately 5×10^{-7} amperes; the effective current, however, is somewhat higher than this value since an average of three traversals of the proton beam is expected for a 0.25 inch beryllium target²⁸.

Since the experimental arrangement was confined to the cart, the maximum distance the scatterer could be placed from the target was 25 to 30 inches. This distance from the target established an energy spectrum for the negative pions that had a minimum cut-off at about 42 Mev. Figure 1 shows typical meson orbits of 42, 45 and 50 Mev which have left the target at 0 to 40 degrees to the beam direction. By leaving the target at other angles, mesons of higher energies could arrive at the scatterer. Suitable arrangement of the lead shielding (Figure 1) allowed no energy greater than 70 Mev to reach the scatterer regardless of the angle of emission from the target. In the energy region of 50 to 70 Mev, one can expect a π^- meson flux of approximately 400 mesons/sec/cm². Except for limiting the maximum energy, no collimation was used since it was desired to eliminate as much as possible mesons that could lose energy by scattering from the walls of the collimator and still be able to reach the scatterer.

Figure 2 is a photograph of the experimental set-up with the aluminum scatterer-absorber in position on the cart as it is placed in the cyclotron for exposure to the proton beam. With this experimental arrangement, exposures of ten minutes were possible without undue background radiation.

Details of Scatterer-Absorber

Figure 3 is an isometric sketch of the aluminum absorber which shows schematically how the incident flux and large angle scatters are detected. Most of the incident π^- mesons, upon entering normally to the surface of the scatterer, lose energy by ionization and come to rest after traversing a range appropriate to their energy. A small fraction of the incident mesons will suffer a nuclear interaction, such as a large angle

nuclear scatter, and will come to rest at a smaller depth of penetration. To sample the distribution of π^- endings, a 250 micron NTB stripped nuclear emulsion was embedded in the scatterer. The figure gives an example of an incident particle that has suffered a large angle scatter and has stopped in the emulsion. The position of the π^- ending, as well as the angle of entrance, can be recorded for each event observed. Also shown are two examples of non-scattered particles coming to rest in the emulsion at their appropriate ranges. To insure that the mesons traverse the absorber, rather than the emulsion, before entering and stopping in the emulsion, the plane of the emulsion was tilted $7\frac{1}{2}$ degrees from the perpendicular to the scatterer face.

Analysis of the spectrum of π^- meson endings is simplified when the scattering medium is considered to be semi-infinite. This condition can be fulfilled experimentally by embedding the stripped emulsion in the scatterer-absorber so that the medium extends from the leading edge of the emulsion for a distance greater than the maximum range of the particles allowed to strike the surface. The maximum energy was chosen to be 70 Mev for all cases. The ranges in aluminum, copper, and lead for this maximum energy are 7.3 cms., 2.6 cms., and 2.75 cms. respectively. The dimensions a), b), and c) of Fig. 3 were for aluminum, 21.6 cms., 8.75 cms., and 9.75 cms., and for copper and lead, 10 cms., 4.5 cms., and 5.0 cms. One could then consider that an area in the center of the stripped emulsion with a minimum width of 4.5 cms. was effectively placed in a semi-infinite absorber. When the plates were scanned, only a band 2.55 cms. wide was studied.

To facilitate the embedding of the stripped emulsion, the absorbers were designed to consist of two separate pieces of equal size. Figure 4 shows an exploded view of the copper absorber illustrating the method used to "sandwich" the stripped emulsion between the absorber halves. The 2 x 4 inch sheets of 250 micron NTB stripped emulsion were trimmed so that, when centrally placed on the interface of the absorbers, they would cover an area 2 inches wide from the front to rear edges. Photographic tape was used to bind the "sandwich" together, making a compact, light tight unit.

Unlike copper and lead, the aluminum absorber was large enough so that one could anticipate some variations in the meson flux incident

upon the absorber. For the orbits that leave the target at 0 degrees, the gradient of the energy at the surface of the absorber is approximately $0.75 \frac{\text{Mev}}{\text{cm}}$. An energy interval of the order of 10 Mev was therefore expected in the mean energy of the meson flux at this surface. Of particular importance is the determination of any deviation in the flux density of the incident meson beam from that recorded in the primary emulsion at the center of the absorber. This deviation could arise from an energy dependent production cross section of negative pions from the beryllium target and also from the fact that equal areas at the absorber did not subtend equal solid angles from the target. In this special case of aluminum, two additional pieces of stripped emulsion 1 inch wide were located equidistant and parallel to the center-line of the primary nuclear emulsion. The center-line to center-line distance of the 1 inch strips was 16 cms. These secondary emulsion detectors thus served to augment the information on the structure of the incident meson flux.

After leaving the target, mesons could possibly scatter from parts of the cyclotron, cart, or shielding, and arrive at the scatterer with largely reduced energies. To exclude this expected background, only mesons that were observed to enter the emulsion at angles greater than 90 degrees to the direction of the incident meson beam were used to evaluate the scattering cross sections. Since the muon exhibits a very small nuclear interaction, it can be expected that all mesons that undergo large angle scatters can be identified as pions. The incident flux that entered the scatterer was certain, however, to contain an unknown amount of μ^- meson contamination that originated from negative pions which decayed in flight. The mesons comprising the spectrum of the main flux could not then be considered as consisting of all pions. For this reason, only mesons ending in the forward and backward flux that resulted in nuclear stars were classified as negative pions. It was upon these star forming mesons that the data were analysed.

After the scattering assembly was placed in the meson beam, the stripped emulsion was removed, developed, and mounted on glass plates. The procedure used for developing and mounting the emulsion is described in Appendix I.

III THEORETICAL ANALYSIS OF PION ENDINGS IN A SEMI-INFINITE ABSORBER

The theoretical spatial and directional distribution of π^- endings in the absorber clearly depends upon the model chosen to describe the process of the nuclear scattering of negative pions by complex nuclei. An exact analysis of this problem would require knowledge of 1) the angular distribution of the scattering in the laboratory frame, 2) the energy dependence of the scattering process, and 3) the distribution of energy loss suffered by the pion in scattering from the nucleus. Of these, the experimental information on the angular distribution of scattering is the most complete. Data from nuclear emulsion plates have contributed preliminary results pertaining to the problem of inelastic scattering of pions by complex nuclei. In Section I, the experimental evidence was reviewed with these three points in mind.

General Formulation

A particle of total range R enters normally to the absorber at 0, and scatters, elastically or inelastically, at x_1 in an interval dx_1 between θ and $\theta + d\theta$. The ending is observed between x and $x + dx$. The problem is to find the number of mesons that can be expected to stop between x and $x + dx$, i.e. $N(x)dx$. Multiple small angle scattering is neglected in the theoretical analysis, although its effect upon the experimental data will be considered in Section V. Let:

$N(R)dR$ = The number of incident pions with range between R and $R + dR$.

$\sigma(\theta)$ = The differential cross section for scattering (barns per steradian). It will be assumed $\sigma(\theta)$ is energy independent over the energy interval considered.

f = The fraction of the residual range ($R - x_1$) retained after scattering.

$\pi(f)df$ = The probability that f lies between f and $f + df$.

n = The density of scattering centers.

The number of particles with initial range between R and $R + dR$ that are scattered between x_1 and $x_1 + dx_1$, θ and $\theta + d\theta$ and retain a fraction f to $f + df$ of its residual range after scattering is:

$$N(x_1, \theta, R, F) dx_1 d\theta dR df = 2\pi \sigma(\theta) \sin \theta d\theta N(R) dR n dx_1 \pi(f) df \quad (1)$$

From Fig. 5, the relation between x and x_1 is found to be

$$\cos \theta = \frac{x - x_1}{f(R - x_1)}$$

or

$$x_1 = \frac{x - fR \cos \theta}{1 - f \cos \theta}$$

The differential relationship is then

$$dx_1 = dx / (1 - f \cos \theta) \quad (2)$$

Substituting Equation (2) into Equation (1), we obtain

$$N(x, \theta, R, f) dx d\theta dR df = 2\pi \frac{\sigma(\theta) \sin \theta d\theta}{1 - f \cos \theta} N(R) dR n dx \pi(f) df \quad (3)$$

which is the number of particles that are observed between x and $x + dx$, that had an original range between R and $R + dR$, and that were scattered between θ and $\theta + d\theta$ retaining f to $f + df$ of its residual range $(R - x_1)$. The distribution $N(x)dx$ of the endings for particles scattered less than 90° is obtained by integrating the variables f , R , and θ over the proper limits:

$$N(x)dx = 2\pi n dx \int_R \int_f \int_{\theta=\cos^{-1} \frac{x}{fR}}^{\pi/2} N(R) \pi(f) \frac{\sigma(\theta) \sin \theta}{1 - f \cos \theta} d\theta df dR \quad (4)$$

Similarly, the distribution $N(x)dx$ of backscattered particles is given by:

$$N(x)dx = 2\pi n dx \int_{R_{\min}}^{R_{\max}} N(R) dR \int_{f=0}^1 \pi(f) df \int_{90^\circ}^{180^\circ} \frac{\sigma(\theta) \sin \theta}{1 - f \cos \theta} d\theta$$

Since $\int_{R_{\min}}^{R_{\max}} N(R) dR = N$, the total number of incident particles, we have for $R_{\min}$ the backscatters

$$N(x)dx = 2\pi nN dx \int_{f=0}^1 \pi(f) df \int_{90^\circ}^{180^\circ} \frac{\sigma(\theta) \sin\theta}{1 - f \cos\theta} d\theta \quad (5)$$

Equations 4 and 5 constitute the formal solution of the problem of finding the distribution of pion endings in the absorber.

The distribution of backward scattered particles is constant for depths of penetration, x , less than $R_{\min}$. This unique result is a direct consequence of the two integrals in Eq. 5 being definite. The spatial distribution of backscatters will be flat, regardless of the angular distribution and the functional form of $\pi(f)$, which is related to the distribution of energy loss upon scattering. Also, it is independent of the structure of the initial flux, depending only on the total number of incident particles. Figure 6 illustrates graphically why equal populations of stopped backscatters can exist in equal volume elements dV and dV' that are at different depths of penetration. Particles a and b, to reach dV and dV' respectively, must scatter in the parabolic shells A and B. Since these shells subtend equal solid angles to the elemental volumes, the density of backscatters will be the same in each.

The initial assumption in this derivation was the energy independence of the differential cross section, $\sigma(\theta)$. An experimental determination of $N(x)dx$ would give, in addition to the evaluation of the total cross section for backscattering, a test of the assumption of energy independent scattering. A constant distribution would confirm the hypothesis, whereas a variation of the distribution from flatness would indicate that the scattering process is energy dependent.

Equations (4) and (5) will now be applied to several specific examples of angular distributions, and assumed functional forms of $\pi(f)$.

Specific Applications

1. Isotropic Elastic Scattering

The simplest examples of $N(x)dx$ (0° to 90°) and $N(x)dx$ (90° to 180°), Equations (4) and (5), are for spherically symmetric, elastic scattering. In this case we have

$$\sigma(\theta) = \frac{\sigma(0^\circ - 90^\circ)}{2\pi} \quad \text{for forward going flux}$$

$$\sigma(\theta) = \frac{\sigma}{2\pi} \quad \text{for backward going flux}$$

$\pi(f) = \delta(f - 1)$ where $\delta(f - 1)$ is the delta function for $f = 1$.

Equation (4) becomes

$$\begin{aligned} N(x)dx (0^\circ \text{ to } 90^\circ) &= ndx \int_R N(R) dR \int_{\theta=\cos^{-1} \frac{x}{R}}^{\pi/2} \frac{\sigma \sin \theta d\theta}{1 - \cos \theta} \\ &= n \sigma dx \int_R N(R) \ln \frac{R}{R-x} dR \end{aligned}$$

If we have a monoenergetic flux, R is a constant and $N(x)dx (0^\circ \text{ to } 90^\circ)$ is then expressed as

$$N(x)dx (0^\circ \text{ to } 90^\circ) = \sigma(0^\circ - 90^\circ) n N dx \left[\ln \frac{R}{R-x} \right] \quad (6)$$

where N is the total number of particles incident, and $\sigma(0^\circ - 90^\circ) = \int_{0^\circ}^{90^\circ} \sigma(\theta) d\Omega$. For the backward flux

$$\begin{aligned} N(x)dx &= \sigma n N dx \int_{90^\circ}^{180^\circ} \frac{\sin \theta}{1 - \cos \theta} d\theta \\ &= \sigma n N dx [\ln 2] \end{aligned} \quad (7)$$

where σ is the cross section for backscattering and N is the total number of incident particles. Equations (6) and (7) are plotted in Fig. 7 to show the distribution of particle endings in the absorber for a monoenergetic flux of range R . The dark area of constant magnitude is the backscatter spectrum, Eq. 7. The ordinate is a measure of the number of endings per unit cell length (0.1 range).

2. Non-Spherically Symmetric Elastic Backscattering

As pointed out in Section I, there is strong experimental evidence that the angular distribution for backscattering of negative pions from complex nuclei cannot be described completely by S-wave scattering^{27,29}.

The results of this experiment also indicate an angular distribution that cannot be reconciled with isotropic backscattering. If P-wave scattering were present, the peak in the observed angular distribution could be

interpreted. We assume the angular distribution to be of the type $a + b \cos^2 \theta$. Since a reasonable fit is obtained with the two parameters, a and b , no higher terms are used to fit the distribution empirically.

The elastic backscattering spectrum, using this angular distribution and letting $\pi(f) = \delta(f - 1)$, from Eq. 5 is:

$$\begin{aligned} N(x)dx &= \sigma n N dx \int_{90^\circ}^{180^\circ} \frac{a + b \cos^2 \theta}{1 - \cos \theta} \sin \theta d\theta \\ &= \sigma n N dx \left[a \ln 2 + b (\ln 2 - 1/2) \right] \end{aligned} \quad (8)$$

where

$$\sigma(\theta) = \frac{\sigma}{2\pi} (a + b \cos^2 \theta).$$

σ is the total cross section for elastic backscattering. a and b are the probabilities for S-wave and P-wave scattering respectively. a and b are normalized by the condition

$$a + b/3 = 1 \quad (9)$$

3. Isotropic Inelastic Scattering

The process through which the pion loses a large fraction of its initial energy upon scattering from a nucleus is not well understood. If inelastic scattering is the result of an elastic collision between a pion and a single nucleon within the nucleus, the angular distribution very probably would not be isotropic. If the inelastic collision could be described as taking place through a type of compound nucleus, the distribution would, a priori, be isotropic. The data of Lederman, et al²⁷ indicates that the inelastic scattering of 60 Mev π^- mesons may be isotropic. This result forms the basis for the otherwise arbitrary assumption that the angular distribution of inelastic pion collisions with nuclei is spherically symmetric.

There is no experimental information at the present time that gives a clue as to the distribution of energy loss that results from the inelastic collisions between nuclei and negative pions of about 30 Mev.

By studying the nuclear collisions of 60 to 90 Mev negative pions in photographic emulsions, Bernardini et al²³ were able to measure the approximate initial and final energies of the inelastically scattered pions. An inelastic collision could be identified if the energy loss was larger than 15 percent. Approximately 11 inelastic scattering events were observed, the residual energies ranging from 5 to 60 Mev. The number of events was too small to show any definite structure in the energy loss distribution, but it was consistent with a constant energy loss distribution, i.e. $P(E)dE = \text{constant} \cdot dE$. It is shown later that the functional form of $P(E)dE$, and therefore $\pi(f)df$, that is chosen to approximate the distribution of energy loss does not greatly affect the total number of backscatters that can stop in the absorber.

To transform $P(E)dE$ to $\pi(f)df$, we use the empirical range-energy relation $E = c(R - x_1)^m$, where m is taken as 0.60 for Al, Cu, and Pb, and $(R - x_1)$ is the residual range. For $P(E)$ a constant we have

$$P(E)dE = \text{constant} \cdot m(R - x_1)^{m-1} d(R - x_1) = \pi(R - x_1) d(R - x_1)$$

In terms of f , where the true range $= f(R - x_1)$, and the particle is inelastically scattered:

$$\pi(f)df = mf^{m-1} df \quad (10)$$

Substituting Eq. 10 and $\sigma_{in}(\theta) = a_{in} \frac{\sigma(\text{total})}{2\pi}$ into Eq. 5

$$\begin{aligned} N(x)dx &= a_{in} \sigma_{in} N dx \int_0^1 m f^{m-1} df \int_{90^\circ}^{180^\circ} \frac{\sin \theta}{1 - f \cos \theta} d\theta \\ &= a_{in} \sigma_{in} N dx \quad [0.863] \quad (\text{for } P(E) = \text{constant}) \end{aligned} \quad (11)$$

where $N(x)dx$ is the number of inelastically backscattered pions that stop between x and $x + dx$ in the absorber, σ is the total cross section for backscattering and a_{in} is the probability for inelastic scattering.

Johnson³⁰ has set forth an argument for a distribution favoring small energy losses in inelastic collisions in a theoretical treatment of inelastic meson scattering by nuclei. The theoretical model is that

inelastic scattering results from elastic collisions between mesons and free nucleons. The Fermi gas model of the nucleus is used to obtain the internal momentum distribution of the nucleons. $\pi(f) = \text{const.}$ is a function that can be used since it has the characteristic of favoring small energy loss. In terms of energy, $P(E)$ varies as $E^{0.6}$. The functions $P(E) = \text{const.}$ and $P(E) \sim E^{0.6}$ are plotted in Fig. 8. Taking $P(E) \sim E^{0.6}$ as the energy loss distribution, the expression for $N(x)dx$ becomes:

$$N(x)dx = a_{\text{in}} \sigma_n N dx [0.822] \quad (12)$$

A comparison of Eq. 11 with Eq. 12 shows that the number of backscattered pions calculated to stop in the absorber differ by only 5 percent. This illustrates that $N(x)dx$ is insensitive to the assumed distribution of $P(E)$. Experimenters^{18,19,20} have shown that $a_{\text{in}} \sigma$ (total) is much less than the total scattering cross section σ (elastic + inelastic) in the energy region of this experiment. Thus, the relative contribution of inelastic backscattering to the total number of backscattering pions that remain in the absorber can be expected to be less than the statistical error of the experimental data.

In general, it is clear from Eqs. 8, 11, and 12 that the distribution $N(x)dx$ can be written as:

$$N(x)dx = k N n \sigma dx \quad (13)$$

where σ is the total cross section and k is a function of θ and f . If the differential cross section $\sigma(\theta)$ is written as $\frac{\sigma}{2\pi} K(\theta)$ where $K(\theta)$ is a function of θ only and $\int_{4\pi} K(\theta) d\Omega = 1$, then from Eq. 5 k is defined by:

$$k = \int_f \pi(f) df \int_{90^\circ}^{180^\circ} \frac{K(\theta) \sin\theta}{1 - f \cos\theta} d\theta \quad (14)$$

By considering the geometry employed in this experiment for detecting backscattered pions, one can arrive at a simple physical interpretation of k . When a pion enters the absorber and backscatters, there is a probability that it will be able to escape the absorber via the front surface and not be detected. For a backscatter to be observed, the total path

length must be within the absorber. k is then interpreted as the fraction of backscatters that remain in the absorber. In other words, k is the efficiency of the nuclear emulsion for detecting backscattered pions when it is embedded in the absorber. Table I gives numerical values of k for several distributions of $K(\theta)$ and $\pi(f)$.

TABLE I

$K(\theta)$	$\pi(f)$	Description	Detection Efficiency k^*
any	$\delta(f - 0)$	Completely inelastic, $f = 0$	1.000
1	1	Partially inelastic isotropic scattering	0.822
$\sin^2 \theta$	$\delta(f - 1)$	Elastic scattering, $\sin^2 \theta$ distribution	0.750
1	$\delta(f - 1)$	Elastic isotropic scattering	0.693
$\cos^2 \theta$	$\delta(f - 1)$	Elastic scattering, $\cos^2 \theta$ distribution	0.580
*Fraction of backscatters that remain in absorber			

In the order listed in the table, the combinations of the distributions chosen for $K(\theta)$ and $\pi(f)$ become more favorable for backscattering out of the absorber. Consequently, the fraction of backscatters that remain in the absorber decreases. This behavior is characterized by the values of k . When no particles escape the absorber, as would be expected for highly inelastic collisions, the value of k is one, the upper limit. The lower limit is 0.50. This would occur if all scatters were elastic and, at $\theta = 180^\circ$, since all particles of range R that were backscattered at a depth of penetration less than $R/2$ would be able to escape the absorber.

IV ANALYSIS AND RESULTS OF EXPERIMENTAL DATA

Distribution of Negative Pion Endings

To sample the spatial and directional distribution of pion endings, the emulsions which were embedded in the aluminum, copper, and lead absorbers were scanned under oil immersion 90-X objectives and 5- or 10-X eyepieces. The method of scanning was to start 1.5 to 2.0 mm from the leading edge of the emulsion and progress along the direction of the incident beam, i.e. the x-coordinate, while keeping the y-coordinate constant. Five to ten scans through the complete range spectrum were sufficient to determine the incident flux. The scanning effort was then turned to the collection of backscattered pions that stopped in the emulsion at the smaller depths of penetration. The data taken for each meson ending observed was a) depth of penetration, b) projected angle of entrance into the emulsion, c) approximate range in emulsion, d) type of ending, σ (star forming) or ρ (non-star forming), and e) the number of prongs in a σ -type meson.

Figures 9, 10, and 11 are histograms showing the distribution of the pions ending in aluminum, copper, and lead. The dark areas represent backscattered pions. Figures 12a), b), and c), show the distribution of backscatters ending in the range intervals 2 to 26 mm in aluminum, 2 to 11 mm in copper, and 1.5 to 10.5 mm in lead. Figure 12d is the combined data for all backscatters. Figures 13 and 14 show the theoretical energy distributions of the scattered pions that can contribute to the detected backward flux. The scattering energy in aluminum is 30.7 ± 7.0 Mev; in copper and lead it is 33.2 ± 8.6 Mev. The limits defined are standard deviations determined by

$$\sigma_E^2 = \frac{\int P(E) (E - \bar{E})^2 dE}{\int P(E) dE}$$

The derivation of the energy distribution of the backscattered pions is given in Appendix II. The spatial distributions of the endings of backscattered pions, Figs. 13a, 13b, and 13c, can be fitted within the expected statistical limits, to a straight line of zero slope. Figure 13d gives a fit to a straight line much better than is expected. This experimental

result indicates that in the energy region 32 ± 10 Mev the nuclear scattering process is energy independent. The validity of the energy independent assumption in the theoretical analysis (Section III) is therefore verified.

Angular Distribution of Backscattered Negative Pions

Figure 15 shows the projected entrance angles of star forming pions that entered the emulsion 90° or greater to the direction of the incident flux. Figure 15d is the combination of the angular distributions observed for aluminum, copper, and lead. Since the number of events for each element is small, the angular distributions were combined to obtain as large a sampling of the distribution as possible. The coefficients a and b which give the best fit of $a + b \cos^2 \theta$ to this combined angular distribution will be considered to describe the angular dependence for pion backscattering by each element.

The angular distribution expected for the backscattered pions that stop in the embedded emulsion is, from Eq. 14

$$k(\theta) d\theta = \frac{K(\theta) \sin\theta d\theta}{1 - \cos\theta} = \frac{(a + b \cos^2 \theta) \sin\theta}{1 - \cos\theta} d\theta \quad (15)$$

It is assumed that the effect of inelastic scatters is small, and the true angular distribution $k(\theta)$ is closely approximated by considering elastic ($f = 1$) scattering only. To compare this angular distribution with the experimental data, $k(\theta)d\theta$ is expressed as $k(\delta)d\delta$, where δ is the horizontal projection of θ and $k(\delta)$ is the projected angular distribution. When the experimental angular resolution is folded into $k(\delta)$, the theoretical angular distribution can then be directly applied to the experimental data to determine the coefficients a and b .*

The horizontal projected angular distribution, $k(\delta)d\delta$, is

$$k(\delta)d\delta = k_0(\delta)d\delta + k_1(\delta)d\delta \quad (16)$$

*The fold $f(\delta)$ of the function $k(\delta)$ and $g(\delta)$ is defined by the formula

$$f(\delta) = \int_{-\infty}^{\infty} k(\delta)g(t - \delta)dt.$$

where

$$k_0(\delta) d\delta = \frac{a}{2\pi} \left[\frac{\pi - \delta}{\sin \delta \cos \delta} - \frac{\pi}{2 \cos \delta} \right] d\delta$$

$$k_1(\delta) d\delta = \frac{b}{2\pi} \left[\frac{\pi - \delta}{\sin \delta \cos \delta} - \frac{\pi}{2 \cos \delta} - \frac{\pi}{4} \cos \delta - 1 \right] d\delta$$

for $\pi/2 < \delta < 3\pi/2$

$k_0(\delta) d\delta$ is the projected angular distribution that corresponds to an isotropic distribution in θ and $k_1(\delta) d\delta$ corresponds to a $\cos^2 \theta$ angular distribution. $k_0(\delta)$ and $k_1(\delta)$ are plotted in Fig. 16a.

The angular resolution for the backscatters, Fig. 16b was determined by measuring the projected entrance angles of the pions that made up the incident flux. The average energy of the pions as they entered the emulsion surface was about 4 Mev. The angular resolution for the backscatters is closely approximated by the angular deviations of the incident flux since multiple scattering takes place predominately at low energies. To illustrate, the differences between the r.m.s. angles for pions with initial energies of 50 and 30 Mev which have final energies of 4 Mev is only about 10 percent. The standard deviation of the projected angular distribution was for aluminum $\pm 16^\circ$, copper $\pm 23.5^\circ$, and for lead $\pm 33^\circ$. The root mean angle of the weighted distribution, Fig. 16b, is 27.6° . The distribution is closely approximated by a gaussian with $\sqrt{\sigma_E^2}$ equal to 27.6° . Figure 17a is the fold of the theoretical distributions $k_0(\delta)$ and $k_1(\delta)$ and the experimental angular resolution. A least squares analysis of the experimental data gives the best fit when $b = 3.5 a$. The folded horizontal projection of $k(\theta) = (1 + 3.5 \cos^2 \theta)$ is one of the theoretical distributions superimposed upon the experimental data in Fig. 17b.

Total Cross Sections for Backscattering of Negative Pions

1. Effective Detection Efficiency

The total scattering cross section σ (elastic + inelastic) is given by Eq. 13

$$\sigma = \frac{1}{k_{\text{eff}} n} \frac{N(x)}{N} \quad (17)$$

- where k_{eff} is the effective detecting efficiency and takes into account both elastic and inelastic backscatters
- n is the density of scattering centers, (number per cm^3)
- $N(x)$ is the number of backscatters per cm observed (per scan through the spectrum of backscatters)
- N is the total number of particles observed in the incident flux (per scan through the whole spectrum)

The evaluation of the effective detection efficiency k_{eff} (inelastic and elastic) requires the ratio of inelastic to elastic collisions between negative pions of about 30 Mev energy and complex nuclei. From the work of Bradner and Rankin^{18,19}, and Bernardini, et al^{20,21,22,23}, an estimate for this ratio, to one place accuracy, is 0.2. The ratio of the isotropic to $\cos^2\theta$ distribution by least squares is 3.5. It has been assumed inelastic collisions are isotropic (Section IV B.3) and elastic collisions are either S-or P-wave. The fractions of the observed backscattered pions, $N(x)$, that must be attributed to the following modes of scattering are then:

- (1) $0.165 = a_{\text{in}}$ for isotropic inelastic scattering (Eqs. 11 and 12)
- (2) $0.295 = a$ for isotropic elastic scattering (Eq. 8)
- (3) $0.54 = \frac{b}{3}$ for $\cos^2\theta$ elastic scattering (Eq. 8)

The equation for the effective k follows, using Eqs. 8 and 11

$$k_{\text{eff}} = a [0.693] + b [0.193] + a_{\text{in}} [0.863]$$

$$= 0.659 \text{ (for energy loss probability } P(E) = \text{const}) \quad (18)$$

If Eqs. 8 and 12 are used, then:

$$k_{\text{eff}} = 0.652 \text{ (for energy loss probability } P(E) \sim E^{0.6}) \quad (19)$$

The difference of 1 percent between these values of k_{eff} typifies the insensitivity of k_{eff} , and therefore the calculated cross section, upon the function assumed for $P(E)$.

2. Scanning Efficiencies

Table II tabulates the ratio of $N(x)/N$ obtained by the two observers. The weighted mean is used to calculate the total backscattering cross sections, Eq. 17. If the scanning efficiencies of the observers are the same for $N(x)$ and N , the ratio $N(x)/N$ obtained is the true ratio. The

TABLE II

	Aluminum	Copper	Lead
Observer A	$2.26 \pm 0.22 \times 10^{-3}$	$12.13 \pm 3.04 \times 10^{-3}$	$15.07 \pm 4.3 \times 10^{-3}$
Observer B	2.55 ± 0.83	10.20 ± 1.72	12.64 ± 2.1
Mean	2.33 ± 0.44	10.73 ± 1.51	13.15 ± 1.83

relative efficiencies of the observers for finding π^+ stars was 75 ± 5 percent. A direct check on the relative efficiencies for finding backscatters was not possible due to the fact that these events were extremely rare. In the areas scanned for backscatters, however, there were enough low energy background particles, principally μ -mesons, from which an estimate of the relative efficiencies for the detection of backscatters could be made. Within statistical limits, it was concluded that efficiencies of the observers for finding $N(x)$ and N were the same. The percentage of star forming backscatters was observed to be 72.8 ± 6.0 percent. This agrees well with the accepted value of 72.2 ± 2.0 ³¹. This agreement suggests that all backscatters observed were pions, and indicates also that the scanning technique was not biased towards many pronged stars.

In lead, Fig. 11, 2.3 percent of the pions comprising the incident flux of the spectrum is made up of pions which were identified as backscatters. This percentage of backscatters in the main flux is consistent with the number expected from multiple small angle scattering with energy loss from 50 to 4 Mev. With a root mean square angle of about 42° , which compares favorable to the observed projected root mean angle at 33° , one can account for this amount of Coulomb contamination in the backscatters. The greatest contribution to multiple scattering occurs

at low energies. Thus, approximately 2 percent of the low energy pions that stop in the region scanned for backscatters can contribute to $N(x)$. The correction to be applied to $N(x)/N$ for this Coulomb contamination is -4.5 percent. The corrections for this effect in copper and aluminum are negligible. The correction to $N(x)/N$ for aluminum due to flux deviations at the surface of the absorber (Section III) amounts to +1.6 percent. Out of 3.3×10^3 events, one $\pi - \mu$ decay was observed. As it entered the emulsion in the direction expected for negative particles, it was identified as a negative pion which decayed near the end of its range.

3. Cross Sections

Using the detection efficiency, Eq. 18, and including the above corrections, the total cross sections for large angle scattering of negative pions in aluminum, copper, and lead are as follows:

TABLE III

Cross Sections for Backscattering

Element	$\sigma(\text{elastic} + \text{inelastic}) = \int_{90^\circ}^{180^\circ} \sigma(\theta) d\Omega$	Cross section per nucleon, σ/A , in millibarns
Aluminum	$59.6 \pm 11 \times 10^{-27} \text{ cm}^2$	2.22 ± 0.41
Copper	$192 \pm 27 \times 10^{-27} \text{ cm}^2$	3.00 ± 0.42
Lead	$577 \pm 80 \times 10^{-27} \text{ cm}^2$	2.78 ± 0.38

The backscattering cross sections vs. A are plotted in Fig. 18. The values of the parameters, $\frac{\sigma_{\text{in}}}{\sigma_{\text{el}}}$, the ratio of inelastic to elastic scattering, and b/a , the ratio of $\cos^2\theta$ scattering to isotropic scattering, used to evaluate the cross sections were 0.20 and 3.5, respectively. The small dependence of the calculated cross sections upon these parameters is easily illustrated. For example, if b/a is chosen to be 7 ($\cos^2\theta$ dominant) or zero (no $\cos^2\theta$ scattering) while keeping $\frac{\sigma_{\text{in}}}{\sigma_{\text{el}}} = 0.2$, the percentage

difference between these and the quoted cross sections would be + 2.6 percent and -11.1 percent respectively. Also, if we keep $b/a = 3.5$ and allow $\frac{\sigma_{in}}{\sigma_{el}}$ to take on values: zero (no inelastic scattering) or 0.46 (all S-wave scattering inelastic) the percentage difference would be + 4.1 percent and -7.75 percent respectively. For these extreme examples, the largest percentage difference (-11.1 percent) is less than the minimum statistical error of the experimental data.

TABLE IV

Correction factors F for several values of $\frac{\sigma_{in}}{\sigma_{el}}$ and b/a^*

$\frac{\sigma_{in}}{\sigma_{el}} \backslash b/a$	0	3.5	7
0.0		1.041	
0.2	0.889	1.000	1.026
0.46		0.9225	
*Cross section equals F x cross section of Table III.			

V DISCUSSION

The conclusions that can be obtained from the experimental results can be summarized as follows:

- a) The scattering of negative pions from complex nuclei is consistent with energy independence in the region of 32 ± 10 Mev.
- b) The angular distribution for backscattering indicates that both the S- and P-waves contribute to the backscattering.
- c) The cross section for backscattering is proportional to the mass number, A .

The flatness of the combined data, Fig. 12d, supports, in general, the argument for the energy independence of backscattering. If it were assumed that the energy dependence for backscattering is proportional to E^n , i.e. $\sigma(E) \sim E^n$, then the backscattered pions would be distributed as $(R - x)^{0.6n}$, where R is the total range of the particle and x is the depth of penetration. To estimate the limit of energy dependence, a least squares fit to the combined data gives for the exponent n , (-0.2 ± 0.8) . Of the individual elements, aluminum is the only one in which there is an indication that the scattering may have a greater dependency upon energy. The distribution, however, is still consistent with energy independence.

Of particular interest is the indication that the cross section for backscattering may vary as A , and not as $A^{2/3}$. A possible interpretation of this result is outlined in the following argument. Small angle pion - nuclei scattering are collisions which, in general, involve small momentum transfers. These small angle collisions can be thought of as arising from the interactions between pions and whole nuclei since small momentum transfers preclude small impact parameters. With the whole nucleus entering into the scattering process, one would expect the scattering cross section in this case to vary as $A^{2/3}$. Conversely, one must identify with large angle scattering large momentum transfers between pions and nuclei. For this to occur, the pion must approach the nucleus so closely that the interaction may now take place between the pion and a single nucleon. Under these circumstances, one may expect to observe scatters that actually arise from pion-nucleon collisions within the nucleus,

and therefore the cross section to vary as A , the number of nucleons in the nucleus. The angular distributions observed for π^- and π^+ scattering from free protons ($\sigma_p(\pi^+) = \sigma_N(\pi^-)$ by principle of charge symmetry) was approximately isotropic for $\sigma_p(\pi^-)$ and strongly backward for $\sigma_p(\pi^+)$ ¹⁶. In this experiment, the pronounced backscattering at 180° in the observed angular distribution of the backscattered negative pions also lends support the conclusion that pion - nucleon collisions are observed.

The lowest pion energy used to study π^- - proton scattering has been approximately 50 Mev. At this energy the total cross sections are about 4 mb for π^- mesons¹⁴ and 20 mb for π^+ mesons¹⁵. The total cross sections rise rapidly with increasing energy. In this investigation, the average backscattering cross section per nucleon, $\frac{\sigma}{A}$, for 32 ± 10 Mev π^- mesons is 2.66 ± 0.24 mb per nucleon. In view of the strong energy dependence, the small cross section observed cannot be considered as an anomaly.

The interpretation of the apparent proportional relationship between the backscattering cross sections and A as an indication of pion - nucleon collisions, while cogent, is not conclusive. It is observed that the scattering process is essentially energy independent between 20 and 40 Mev. This seems to be inconsistent with the assumption of pion - nucleon collisions, which are highly dependent upon energy. This contradiction, however, arises from the assumption that the property of strong energy dependence of pion - nucleon collisions can be extrapolated to energies below 50 Mev.

The same relation between σ and A would result if the nucleus were partially transparent to low energy pions of about 30 Mev. The observed total cross sections per nucleon for backscattering are indicative of small total interaction cross sections per nucleon for negative pions at this energy. Under these circumstances, light nuclei, such as aluminum, would be highly transparent, enabling the pion to "see" all the nucleons in the nucleus. It is surprising that the results indicate this argument is apparently valid for the heavier elements, copper and lead. In these cases, less transparency might be expected, and the cross sections would conduce to an $A^{2/3}$ relationship.

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APPENDIX I

Procedure Adopted for Developing and Mounting Stripped Emulsion

The procedure used for developing 250 micron NTB stripped emulsions required no revision of the techniques commonly used for developing glass backed nuclear plates. After 30 minutes of immersion in 6:1 D-19 at 68° F., the emulsion was transferred to a short-stop of distilled water for ten minutes and then fixed by agitation in Kodak acid fixer for 12 hours at 68° F. They were washed for 12 hours and finally placed in a 2 percent solution of glycerine for 8 hours.

The stripped emulsions were mounted on single glass plates, adapting for this purpose an adhesive suitable for mounting transparencies on glass³². The composition of the adhesive which gave the best results in this work was:

Water (120° F.)	150.0 cc.
Gelatin (Photographic)	18.0 grms.
Ethyl alcohol	18.0 cc.
Aerosol (1 percent solution)	1.8 cc.
Add warm water to make	200.0 cc.

Dissolve gelatin in water, add ethyl alcohol and aerosol (acts as wetting agent). Dilute until total volume is 200 cc. An even, thin coat is applied to the surface of a clean glass plate and allowed to dry slowly.

When the adhesive dried sufficiently, the following procedure was devised for mounting the emulsions on the plates. From the 2 percent glycerine solution the emulsion was dipped momentarily in a 50 percent ethyl alcohol and water solution. The edge of the emulsion was placed on the glass and then rolled into contact, carrying air bubbles along the wave. The emulsion was then placed face down on a smooth absorbent material under a small pressure and allowed to set for 12 to 18 hours. Afterwards, the plate was again returned to the 2 percent glycerine solution, this time for 24 hours. The plate was then removed and dried at a constant humidity of about 60 percent. Included in Figure 4 is the completed mount. This plate, from which a portion of the data on copper were collected, had been mounted for 14 months at the time the

illustrative photograph was taken, and has shown no tendency for peeling. When unsuccessful mountings occurred, the emulsion was easily released from the glass by soaking in warm water for a few minutes. The procedure was then repeated until a satisfactory mount was obtained.

APPENDIX II

In Section V, it is seen that the value of k_{eff} , (Eq. 18), and therefore $N(x)dx$, is only 5 percent less than the value obtained by assuming isotropic elastic scattering. To find the distribution of the scattering energies, the approximation of isotropic elastic scattering will be used, since the specification of the scattering energies within the accuracy of 5 percent is sufficient.

From Eq. 1, the probability that a particle of range R_i backscatters between x_1 and $x_1 + dx_1$ and stops in the absorber between A and B is:

$$P(x_1)dx_1 \sim -dx_1 \int_{\cos\theta_{\min}}^{\cos\theta_{\max}} d(\cos\theta) N(R_i) \Delta R_i = -dx_1 N(R_i) \Delta R_i \cos\theta \Big|_{\cos\theta_{\min}}^{\cos\theta_{\max}}$$

The limits, $\cos\theta_{\min}$ and $\cos\theta_{\max}$, depend on the scattering point x_1 , as follows:

	$\cos\theta_{\max}$	$\cos\theta_{\min}$
1) For $R_i - x_1 > x_1 - A$ and $B > x_1$	$-(x_1 - A)/(R_i - x_1)$	0
2) For $R_i - x_1 < x_1 - A$ and $B > x_1$	-1	0
3) For $R_i - x_1 < x_1 - A$ and $R_i - x_1 > x_1 - B$	-1	$(x_1 - B)/(R_i - x_1)$

and $P(x_1)dx_1$ becomes:

$$1) P(x_1)dx_1 = dx_1 \frac{(x_1 - A)}{(R_i - x_1)} N(R_i) \Delta R_i$$

$$2) \quad = dx_1 N(R_i) \Delta R_i$$

$$3) \quad = dx_1 \left(1 - \frac{x_1 - B}{R_i - x_1} \right) N(R_i) dR_i$$

$P(x_1)dx_1$ is also equal to the probability that a particle scatters at x_1 and has the residual range $R_i - x_1$, i.e. $P(R_i - x_1)dx_1$. This can be transformed into the probability of a particle of energy between E_i and $E_i + dE_i$ backscattering and stopping between A and B by the relation: $E_i = c(R_i - x_1)^{0.6}$. Summing over the energy intervals one can determine the distribution of scattering energies contributing to the observed backscatter flux.

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Fig. 1

Schematic drawing of experimental arrangement. Several typical π^- meson orbits of 42, 45, and 50 Mev leaving the Be target at 0 to 40 degrees from the proton beam direction are shown.

Fig. 2

Experimental set-up with aluminum absorber in position and ready to be placed in the cyclotron.

MU-4599

Fig. 3

Isometric sketch of aluminum absorber-scatterer showing principle of detection of large angle scattered π^- mesons.

Fig. 4

Exploded view of copper absorber-scatterer illustrating method of embedding 250 micron NTB stripped emulsion. Shown also is the mounted emulsion after exposure and development.

MU-4600

Fig. 5

Geometrical model of scattering used in theoretical analysis.

MU-4601

Fig. 6

Schematic drawing illustrating parabolic surfaces that contribute backscatters to elemental volumes dV and dV' for particles of range R .

Fig. 7

Theoretical distribution of negative pion endings vs. depth of penetration for a monoenergetic incident flux.

MU-4602

Fig. 8

Approximations used for distributions of residual pion energies after scattering.

MU-4603

Fig. 9

Distribution of π^- stars in aluminum.

MU-4604

Fig. 10

Distribution of π^- stars in copper.

MU-4605

Fig. 11
Distribution of π^- stars in lead.

DISTRIBUTION OF BACKSCATTERED NEGATIVE PION ENDINGS

Fig. 12

Distribution of backscattered negative pion endings in aluminum, copper, and lead. Also combined data.

DISTRIBUTION OF SCATTERING ENERGIES
AND RESIDUAL RANGES IN ALUMINUM

$$E = C(R - X_1)^{0.60}$$

MU-4607

Fig. 13

Distribution of scattering energies residual
ranges in aluminum.

MU-4608

Fig. 14

Distribution of scattering energies and residual
ranges in copper and lead.

HORIZONTAL PROJECTION OF
ENTRANCE ANGLES $>90^\circ$ OF NEGATIVE
PIONS INTO EMULSION

(a) ALUMINUM 18 EVENTS

(b) COPPER 45 EVENTS

(c) LEAD 52 EVENTS

(d) COMBINED DATA 115 EVENTS

MU-4609

Fig. 15

Horizontal projection of entrance angles greater than 90° of negative pions into emulsion from aluminum, copper, and lead.

ANGULAR RESOLUTION

HORIZONTAL PROJECTION OF ENTRANCE ANGLES IN MAIN FLUX n_1 OF EACH ELEMENT PROPORTIONAL TO NUMBER CONTRIBUTING TO ANGULAR DISTRIBUTION OF BACKSCATTERS (SEE FIG. 15-d)

MU-4610

Fig. 16

- Horizontal projection of angular distribution $\frac{a + b \cos^2 \theta}{1 - \cos \theta}$
- Angular resolution for backscattered pions.

FOLDED ANGULAR DISTRIBUTIONS $k_0(\delta)$ AND $k_1(\delta)$ INTO GAUSSIAN
ANGULAR RESOLUTION OF $\sqrt{\delta^2} = 27.6^\circ$

ANGULAR DISTRIBUTION OF BACKSCATTERS FROM
ALUMINUM, COPPER AND LEAD LEAST SQUARES FIT:
 $b = 3.5a$

MU-4611

Fig. 17

- Folded angular distributions $k_0(\delta)$ and $k_1(\delta)$
- Least squares fit to experimental angular distribution

MU-4612

Fig. 18

Backscattering cross sections vs. Mass
number A.