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Intentional Youth Development Activities and Peer Effects in a Gang Prevention Program

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***Abstract*—We analyze the impact of group activities targeting Social-Emotional Learning (SEL) and peer effects on the risk and protective factors associated with gang involvement among youth participating in the Los Angeles Mayor's Office of Gang Reduction and Youth Development (GRYD) Prevention program. We compare the impact of targeted and non-targeted activities in decreasing Internal Risk, External Risk, and Family Norms Risk, as measured by a standardized questionnaire. We show that targeted activities are effective in decreasing Internal Risk and that activities focusing on Emotional Management are the most beneficial. Since targeted activities involve group interactions, we investigate the impact of peer network effects on outcomes using both a linear-in-means model and dynamic mode decomposition with control (DMDc). Our analysis suggests that peer network effects contribute to beneficial changes in risk and protective factors above and beyond the skill-building content of the activities.**

***Index Terms*—difference in differences, lasso regression, linear in means, dynamic mode decomposition with control**

I. Introduction

The City of Los Angeles Mayor's Office of Gang Reduction and Youth Development (GRYD) was established in 2008 with a goal of putting into place a comprehensive strategy for preventing and reducing youth gang involvement and interrupting gang violence [12], [17]. Within GRYD's comprehensive strategy, Prevention Services are focused on identifying at-risk youth aged 10-15 and providing a program of activities and services effective in building resilience against the allure of gangs. Youth referred to the program by community members including faith leaders, teachers, parents, or police, may agree to complete the Youth Services Eligibility Tool (YSET) questionnaire. The YSET is designed to measure risk and protective factors for gang involvement. It includes more than fifty attitudinal questions, scored on a five-point Likert scale, in areas related to the risk of gang involvement.[1] Risk scores for individual youth are evaluated using thresholds established in prior validation studies [6], [7]. Youth who exhibit risk score above threshold are referred to the Secondary Prevention program. Youth who score below, but close to the risk threshold may be referred to the Primary Prevention program, which includes only a subset of the services offered by Secondary Prevention. Secondary Prevention services include family meetings, individual meetings, and group activities. After six months of attendance, a YSET retest is administered to assess progress in the program. If the risk measured at retest falls below the established thresholds, the youth successfully "graduates" from the program. If a youth scores above the threshold upon retest, they may choose to continue in the program for another cycle of services.

The overall effectiveness of GRYD Prevention program has already been investigated in [5], [13], [19]. In these studies the authors used a range of statistical and mathematical models to establish an association between programming and decreases in risk scores as measured by the YSET. However, it is not clear whether the apparent effectiveness of the program comes from building important life skills or is the result of the prosocial opportunities provided by activities [11]. In other words, does risk appear to decline among program youth because of the content of the activities or is it simply the social interaction that is important? To better understand how activities help the youth, the Activating Intentional Youth Development Approach (AIYDA) was first launched in 2018 [11]. AIYDA activities include content that is explicitly designed to activate different social-emotional learning (SEL) domains intended to foster youth critical life-optimizing skills. In this work, we investigate how AIYDA activities impact risk scores as measured in the YSET and whether the peer networks created by AIYDA group activities amplify or mitigate this impact. Correspondingly, this work is divided into two parts.

In Section II we use difference-in-differences (DID) models to assess whether AIYDA activities are associated with greater decreases in YSET scores compared with non-AIYDA activities. To do this, we group YSET questions into five categories: Internal Resilience, External Resilience, Strength of Family Norms, Gang Social Activity, and Peer Gang Involvement. This grouping was introduced by the GRYD research and evaluation team in [4]. However, to be consistent with previous

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[1] In general, specific risk factors such as impulsive risk taking are the compliment of protective factors such as self-control. We focus on risk scores to be consistent with existing literature, but note risk measures can be easily converted to resilience measures without changing our results.

work [5], [13], we use a reversed measurement scale compared to [4]. Therefore, we talk about Internal Risk, External Risk, Family Norms Risk, Gang Activity Risk, and Peer Gang Involvement Risk. We find that AIDYA is more effective in reducing Internal Risk compared with the other categories. This observation aligns with expectations about what AIYDA can reasonably influence: Internal Risk is the only category that is endogenous to the individual, while the other categories are exogenous, related to family and friends. Furthermore, we study which of the specific SEL domains have more impact in reducing Internal Risk by performing a lasso regression, and find Emotional Management to be the most influential.

In Section III we investigate peer effects. AIYDA services are group activities, in which individuals interact with each other. We model each individual as a node in a social network, where two youths are connected if they interacted in the same AIYDA activity. It is thus natural to raise the question of whether the change in an individual's measured risk is affected by the change in risk of their peers. There are various ways to identify peer effects from a social network [2]. In this report, we study peer effects using (*i*) a linear-in-means model [3], and (*ii*) Dynamic Mode Decomposition (DMD) [18]. Using the linear-in-means model [3] we show that the percentage change in an individual's risk score is positively correlated with the mean of the percentage change in the risk score of her neighbors. The YSET score before and after receiving Secondary Prevention services can also be seen as a dynamical system, which we approximate using Dynamic Mode Decomposition [5], [13]. Using the social network structure to control DMD, we observe that peer effects appear to influence the dynamics of the system.

II. AIYDA vs. Non-AIYDA

GRYD's Activating Intentional Youth Development Approach (AIYDA) seeks to tie specific, actionable behaviors to more abstract goals in social-emotional learning (SEL) domains. For example, it may be difficult to envision how to engage "emotional management", a key SEL domain, as part of an activity. However, "self-control", which is essential to emotional management, is more easily explained and put into practice. Here, self-control is a so-called "activator" tied to emotional management. Each unique AIYDA activity identifies a single SEL domain and either one or two unique activators tied to that domain. An AIYDA activity is intentionally designed to generate situations where activators are put into practice to make progress on SEL goals. AIYDA activities may be contrasted with non-AIYDA activities, which do not explicitly seek to generate SEL learning opportunities. The expectation is that AIYDA activities lead to greater reductions in risk compared to non-AIYDA activities.

A. DID Analysis

In this subsection, we performed a difference-in-differences analysis AIYDA activities compared to non-AIYDA activities. We assume that the change in risk among youth engaged in AIYDA activities *would have been* the same as youth in non-AIYDA activities had their received this alternative treatment. As long as the parallel trend assumption holds, it is possible to estimate the unique effect of this AIYDA on risk scores [8], [20]. Our setting consists of two groups and two time periods. Participants in Secondary Prevention who participated in both AIYDA and non-AIYDA activities are considered part of the treatment group. Participants who receive only non-AIYDA activities are considered to be the comparison group. Many youth participated in very few AIYDA activities. We therefore only considered youth with more than ten percent of AIYDA activities in their combined treatment. After filtering, the AIYDA group consisted of 940 youth while the non-AIYDA comparison group consisted of 772 youth. For the AIYDA group, 50% of the components are males, 49.6% are females, and 0.4% did not specify their gender. The ethnicity of the first group are 91% Latino, 8% Black and 1% other. For the non-AIYDA group, 60% of the components are males and 40% females. The ethnicity are 75% Latino, 23% Black and 2% other. For both groups, we consider the intake score (pre-treatment) and first retest score (post-treatment) as the two time periods. Mathematically, the regression formula of DID can be expressed as

$$\mathbf{Y} = \beta_0 + \beta_1 \mathbf{G} + \beta_2 \mathbf{T} + \beta_3 \mathbf{D} + \boldsymbol{\epsilon}, \tag{1}$$

where $\mathbf{Y}$ is the sum of the answers on each section of the YSET questionnaire, $\mathbf{G}$ is a group indicator with $\mathbf{G} = 1$ representing the AIYDA treatment group and $\mathbf{G} = 0$ the non-AIYDA comparison group, $\mathbf{T}$ is a time indicator denoting pre-treatment ($\mathbf{T} = 0$) and the post-treatment measures ($\mathbf{T} = 1$), and $\mathbf{D}$ is defined to be $\mathbf{D} = \mathbf{G} \times \mathbf{T}$, an indicator of treatment effect. Further, $\beta_0, \beta_1, \beta_2, \beta_3$ are coefficients and $\boldsymbol{\epsilon}$ corresponds to the error term. More specifically, β_3 represents the estimated effect of treatment. A value of β_3 that is negative and statistically significant suggests that participation in AIYDA activities is associated with a reduction in risk.

We explore different groupings of the scores according to different risk factors targeted by the GRYD program [4]. We only consider three subgroups: Internal Risk, External Risk, and Family Norms Risk. Table I shows the estimated coefficients and p-values for each of three domains considered. The results suggest that AIYDA activities lead to a decrease of Internal Risk factors, but not to a decrease of the other risk factors. Figure 1-3 provide a visualization of the results. The results are sensible given that individuals presumably have some control over their own attitudes and behaviors, which comprise the elements of Internal Risk, but much less control over the attitudes and behaviors of friends and family, which comprise the elements of External Risk and Family Norms Risk.

B. Variable Selection

We apply Lasso regression to study which of the six SEL domains targeted by AIYDA activities has more influence on the reduction of risk scores as measured by the YSET.

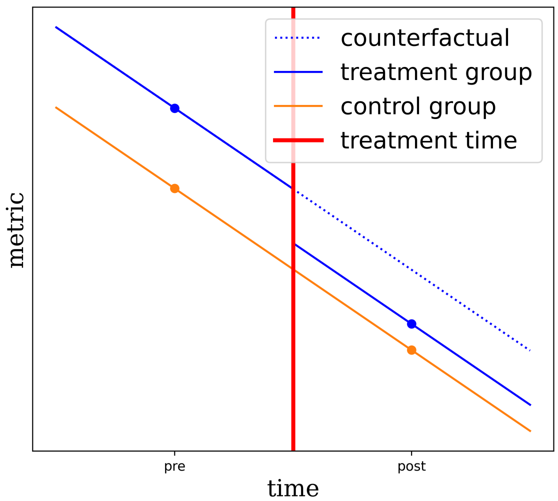

Figure 1: The plot shows a significant decrease of the Internal Risk measure in the AIYDA group. The blue line represents the Internal Risk measure in the AIYDA group pre and post treatment (red line). The orange line is the Internal Risk measure in the non-AIYDA group pre and post treatment.

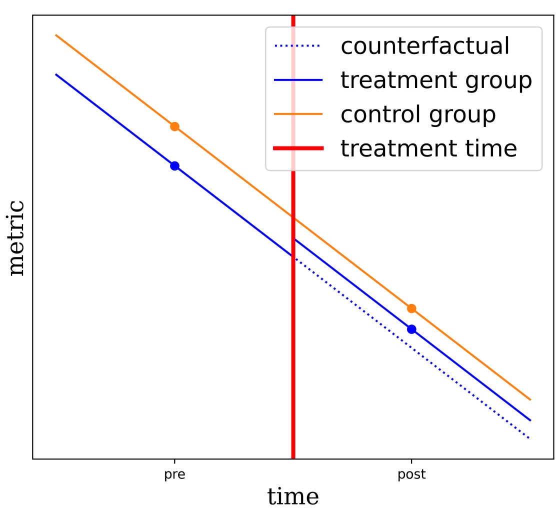

Figure 2: The plot shows a non significant increase of the External Risk measure in the AIYDA group. The blue line represents the External Risk measure in the AIYDA group pre and post treatment (red line). The orange line is the External Risk measure in the non-AIYDA group pre and post treatment.

1) Lasso Regression: For data $(\mathbf{x}_i, y_i)$, $i = 1, 2, \ldots, N$, and coefficients $\beta_j, j = 0, 1, 2, \ldots, p$, lasso regression is a modified linear regression model with an l_1 penalization term that minimizes the following optimization problem

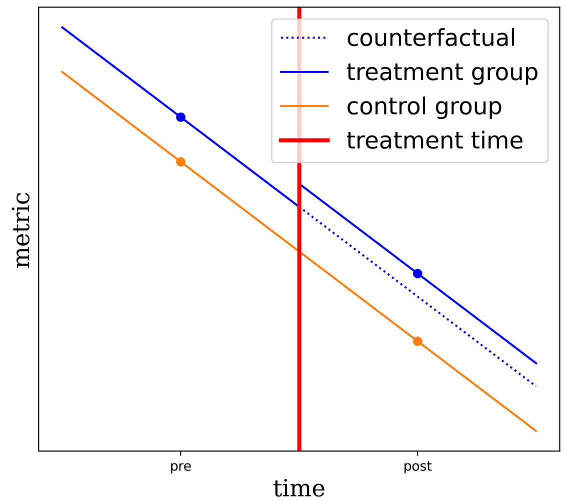

Figure 3: The plot shows a non significant increase of the Family Norms Risk measure in the AIYDA group. The blue line represents the Family Norm Risk measure in the AIYDA group pre and post treatment (red line). The orange line is the Family Norms Risk measure in the non-AIYDA group pre and post treatment.

$$\arg\min_{\beta} \left\{ \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j \right)^2 + \alpha \sum_{j=1}^{p} |\beta_j| \right\}. \quad (2)$$

The added l_1 regularization term increases model interpretation by performing variable selection and shrinkages at the same time [16], and thus allows us to select the most influential features.

In our model, we regress the counts of individual participation in activities targeting different SEL domains x_{ij} against the percent change in Internal Risk between intake and retest y_i. By varying the degree of regularization α, variable importance can be decided by how fast they shrink to zero for increasing α (Figure 4). The Problem Solving SEL domain is the fastest to go to zero, followed by Empathy and Responsibility, which thus have the least influence on the percent change in YSET scores. Emotion Management, Teamwork, and Initiative are slower the go to zero, and therefore have greater influence over the percent change in YSET scores.

III. Peer Effects and Network Analysis

Since AIYDA activities involves groups it is natural to represent the interactions among individuals in these groups as a social network [4]. Here we examine how the characteristic of these networks impact changes in individual risk using a linear-in-means model and dynamic model decomposition (DMD).

Table I: Result of DID on Different Cumulative Scores

Internal Risk	Estimate	p value
$\hat{\beta}_0$: Intercept	10.1425	<0.001
$\hat{\beta}_1$: Group	0.226	<0.001
$\hat{\beta}_2$: Time	-2.5130	<0.001
$\hat{\beta}_3$: Treatment	-0.8424	0.009

External Risk	Estimate	p value
$\hat{\beta}_0$: Intercept	7.9171	<0.001
$\hat{\beta}_1$: Group	-0.3086	0.070
$\hat{\beta}_2$: Time	-1.4275	<0.001
$\hat{\beta}_3$: Treatment	0.1466	0.543

Family Norms Risk	Estimate	p value
$\hat{\beta}_0$: Intercept	39.0207	<0.001
$\hat{\beta}_1$: Group	1.2367	0.046
$\hat{\beta}_2$: Time	-4.9741	<0.001
$\hat{\beta}_3$: Treatment	0.6401	0.465

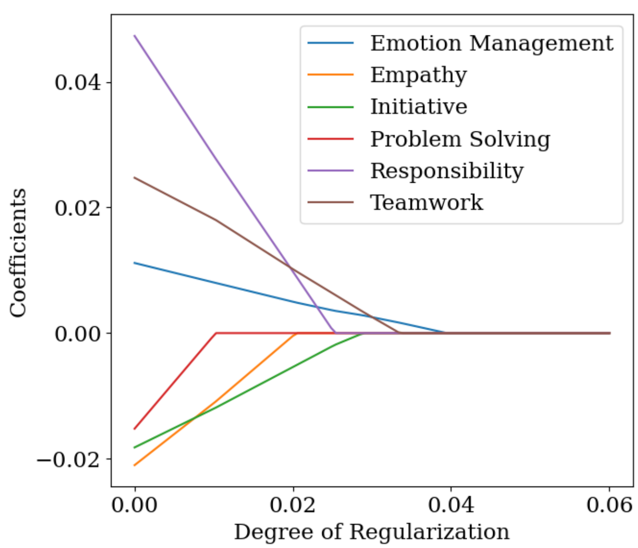

Figure 4: The plot shows how the estimated coefficients for the different SEL domains decrease to zero as the degree of Lasso penalization is increased. Emotion Management is the last variable to shrink to zero, suggesting a higher importance in the linear model.

A. Network Construction

To analyze peer effects, the first task is to construct a network where nodes are participants and edges represent peer social interactions. We focus on interactions that happen during each individual's first cycle of AIYDA activities between their intake YSET and first retest. Youth i and j may share a directed edge if they participated in the same activity. We use directed edges to define peer relations to capture the interactions between participation in activities the length of time youth have been in the GRYD Prevention program. Specifically, because youth may remain in GRYD Prevention for multiple six-month cycles, there is a chance that someone in their first cycle, say youth i, interacts with someone a second or third cycle, say youth j. From this point of view, youth j may have a peer effect on youth i's first cycle outcome, but the reverse is not possible because youth j already completed their first cycle before ever encountering youth i. The edge $j \to i$ (i.e. edge pointing from j to i) marks a possible casual effect. The absence of an edge from i to j indicates that no such causal effect is possible. Thus, the direction of the arrow is from a subject's peers to the subject conditioned on whether or not a causal effect is possible. The peers of node are defined by its in-degree.

B. Linear-in-Means

One method to explore peer effects among GRYD participants is a regression model called linear-in-means (see, for instance, [3]). We assume that the risk the score of a participant may be affected by the mean of the risk scores and mean of other features of her peers. Assuming $\mathbb{E}[\epsilon_i \,|\, \boldsymbol{x}] = 0$ (i.e., the $\boldsymbol{x}$ are strictly exogenous), the linear-in-means model for each participant i is formally written as

$$y_i = \alpha + \boldsymbol{\beta}^\top \boldsymbol{x}_i + \boldsymbol{\gamma}^\top \frac{1}{d_i} \sum_{j \in N_i} \boldsymbol{x}_j + \delta \frac{1}{d_i} \sum_{j \in N_i} y_j + \epsilon_i, \quad (3)$$

where

$$y_i = \frac{\text{retest risk score - intake risk score}}{\text{intake risk score}},$$

is the percent change in risk score, $\boldsymbol{x}_i$ is the vector of exogenous features including the sources of referrals, and total number of successfully completed activities and general demographic information.

The number d_i is the in-degree of node i, N_i is the collection of peer nodes influencing i (i.e., the set of nodes having an outgoing edges to i), $\sum_{j \in N_i} \frac{1}{d_i} y_j$ is the mean of percent change in risk among peers, $\sum_{j \in N_i} \frac{1}{d_i} x_j$ is the mean of certain features among peer nodes, and the error term ϵ_i represents unobserved characteristics associated with i. The vector $\boldsymbol{\beta}^\top$ represents the coefficients of exogenous features of node i and $\boldsymbol{\gamma}^\top$ the coefficients of exogenous features of peers of i, while δ represents the coefficients of endogenous features among the peers of i.

Let n be the number of nodes in the graph and m be the number of features, and assume $\mathbb{E}[\boldsymbol{\epsilon} \,|\, \boldsymbol{x}] = 0$. Define the adjacency matrix of the graph such that the entry in row i, column j is 1 if there is at least one edge pointing from i to j, and 0 otherwise. We can write the matrix form of the model as

$$\boldsymbol{y} = \alpha \mathbf{1} + \boldsymbol{\beta}^\top X + \boldsymbol{\gamma}^\top G X + \delta G \boldsymbol{y} + \boldsymbol{\epsilon}, \quad (4)$$

where $\mathbf{1}$ is an $n \times 1$ vector, X is an $n \times m$ feature matrix, G is an $n \times n$ matrix obtained by first transposing the adjacency matrix of the graph and then dividing each row by the common in-degree, $\boldsymbol{y}$ is an $n \times 1$ vector representing the percentage change in risk scores. In this case, with the normal Ordinary Least Square (OLS) method, coefficient estimates of all variables are biased because $\frac{1}{d_i}\sum_{j \in N_i} y_j$ is endogenous [9], so we adopt a Two-Stage Least Squares (2SLS) method to resolve this issue. We observe that I, G, G^2, with I being the identity matrix, are linearly independent, and by Proposition 1 in [3], we conclude that X, GX, G^2X can be used as valid instrument variables to estimate δ. Therefore, α, $\boldsymbol{\beta}^\top$ and $\boldsymbol{\gamma}^\top$ can all be estimated without bias. The instrument variables have realistic interpretation in this specific application. For instance, G^2X represents the weighted average characteristics of second-order neighbors (i.e., neighbors' neighbors) in the social network. The output of 2SLS is shown in Table II.

The coefficient δ represents peer effects. It is estimated to be a positive number, 0.4855, and a $p-$value of 0.001. The standard error is relatively large compared with a coefficient of 0.4855. This is expected because less precision for estimation of coefficients in the instrumental model results from the loss of information. However, the slope remains above zero within the confidence interval, suggesting a nontrivial peer effect has been successfully identified by the linear-in-means model. Moreover, the also reveals variation in the percent change in risk with respect to each individuals' own characteristics and provides estimation for the exogenous social effects. In particular, the result shows the percent change in risk tends to be higher with high initial risk score and being referred to GRYD Prevention by law enforcement. In terms of exogenous social effects, the percent change in risk tends to have a higher value given neighbors with higher initial risk scores.

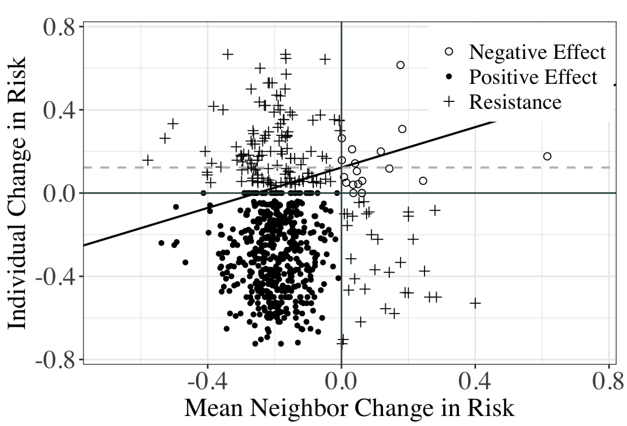

Figure 5: The plot shows how the individual change in risk is associated to the mean neighbor change in risk. A negative value for the change in risk means that the treatment was effective. In the plot, we identify three different kind of individuals: (i) top-right quadrant, negative peer effect; (ii) bottom left quadrant, positive peer effect; and (iii) other points, resistance.

IV2SLS Regression Results			
Dep. Variable:	ratio		
Model:	IV2SLS		
Method:	Two Stage Least Squares		
No. Observations:	679		
Df Residuals:	643		
Df Model:	35		
R-squared:	0.369		
Adj. R-squared:	0.334		
F-statistic:	8.506		
Prob (F-statistic):	2.02e-34		
Variables	**coef**	**std err**	**P> |t|**
const	0.1227	0.341	0.719
δ : neighbor_percent_change	0.4855	0.141	0.001
score_intake	-0.0286	0.002	0.000
Ethnicity_Black	-0.0444	0.093	0.632
Ethnicity_Latino	-0.0387	0.097	0.692
Referral_Other	0.0225	0.060	0.709
Referral_OtherSchool	-0.0182	0.066	0.782
Referral_Parent	0.0301	0.050	0.546
Referral_Retest	-0.0326	0.057	0.566
Referral_SchoolCounselor	0.0822	0.057	0.147
Referral_YouthWalkin	0.0455	0.044	0.304
Gender_male	0.0263	0.052	0.610
Gender_female	-0.0021	0.051	0.968
Ethnicity_Others	0.1462	0.115	0.203
Gender_Others	-0.1250	0.128	0.331
Referral_Law	0.3504	0.104	0.001
Referral_Others	-0.0373	0.082	0.649
Duration_Normalized	-0.1460	0.091	0.107
activity_count_normalized	0.0997	0.113	0.377
G_score_intake	0.0240	0.006	0.000
G_Ethnicity_Black	-0.1821	0.295	0.538
G_Ethnicity_Latino	-0.2763	0.307	0.368
G_Referral_Other	0.1293	0.133	0.331
G_Referral_OtherSchool	-0.0329	0.152	0.828
G_Referral_Parent	0.1027	0.123	0.403
G_Referral_Retest	0.0144	0.123	0.907
G_Referral_SchoolCounselor	-0.0907	0.135	0.503
G_Referral_YouthWalkin	0.2201	0.108	0.041
G_Gender_male	0.0414	0.137	0.762
G_Gender_female	0.0019	0.136	0.989
G_Ethnicity_Others	-0.3238	0.296	0.274
G_Gender_Others	0.1674	0.218	0.443
G_Referral_Law	-0.1302	0.200	0.515
G_Referral_Others	0.0387	0.161	0.810
G_Duration_Normalized	-0.0632	0.135	0.640
G_activity_count_normalized	0.1839	0.161	0.253

Table II: IV2SLS Regression summary statistics. The neighbors' characteristics includes sources of referral, gender, race-ethnicity, total number of completed activities and total duration in the program. The variables with names starting with G represent the exogenous effects, i.e. the mean of neighbors' characteristics.

We visualize the strength of peer effects in Figure 5. The vertical axis is the percent change in risk for each individual on the YSET between intake and retest, while the horizontal axis is the mean of the percent change in risk among the peers of each individual. Accordingly, the slope of linear model is the peer effect. It is important to acknowledge that the intercept describes a baseline when for all features, so it does not necessarily provide informative insights. A positive percentage change in risk score (points above zero on the vertical axis) signifies increased risk, while a negative percent change signifies decreased risk (points below zero on the vertical axis).

The first quadrant, on the top right represents negative peer effects, in which both variables are positive. This implies that when peers see increasing risk scores, each individual in this quadrant also sees an increasing risk score. The second quadrant, on the top left, represents negative resistance. Here peers see decreasing risk scores, but each individual in this quadrant sees movement in the opposite direction. The third quadrant, on the bottom left, represents a positive peer effect; when peers see decreasing risk scores so does each individual in this quadrant. The final quadrant, on the bottom right, represents positive resistance where peers show an increase in risk while individuals see a decrease. Notably the third quadrant is the most populous suggesting that there is a strong positive peer effect overall.

IV. DMD and DMDc for Nework Effect

In this section, we first use Dynamic Mode Decomposition (DMD) to examine whether GRYD Prevention Services have an effect on reducing the participants' risk score as measured by the YSET. We then employ Dynamic Mode Decomposition with control (DMDc) to study peer effects from AIYDA activities by treating the social network structure as a set of controls. Before presenting our experimental results, we recall the theory of DMD [18] and DMDc [14].

A. DMD

We start with the Koopman operator [10], which is an infinite-dimensional linear operator describing how a nonlinear dynamical system evolve. With the Koopman operator providing a reasonable representation to the dynamic system, we then use DMD as a finite-dimensional approximation of the infinite-dimensional Koopman operator [15].

1) Koopman Analysis: Consider a discrete-time dynamical system

$$z_{k+1} = F(z_k), \tag{5}$$

where z_k is a state variable at time k defined on a smooth manifold $\mathcal{M}$, and $F : \mathcal{M} \to \mathcal{M}$ is a function representing the dynamics.

Assume we have observables $x_k = g(z_k)$ where $g : \mathcal{M} \to \mathbb{C}$ is a function mapping the state of the dynamics at time k to a scalar value, which in our case is an observation from the YSET questionnaire.

The Koopman operator $\mathcal{K}$ for discrete-time dynamical systems is a linear operator that acts on all scalar measurement functions g so that

$$\mathcal{K}g(z_k) = g(F(z_k)) = g(z_{k+1}). \tag{6}$$

In this way, we transform a state-space representation to a Koopman representation, trading nonlinear, finite-dimensional dynamics for linear, infinite-dimensional dynamics. Different from local linearization, the Koopman operator preserves a system's global nonlinear features as mentioned in [1].

Consider the spectral theory for infinite-dimensional vector spaces, let $\varphi_j(z)$ be a Koopman eigenfunction with corresponding eigenvalue λ_j, we have

$$\mathcal{K}\varphi_j(z) = \lambda_j \varphi_j(z), \tag{7}$$

where $\varphi_j(z)$ define a set of intrinsic measurement coordinates, which enable us to approximate them with a linear dynamical system. Assume the eigenvalues are numbered so that their magnitude decreases as the index increase. A set of observables (risk scores in our case) g can be written in terms of Koopman eigenfunctions

$$g(z) = \sum_{j=1}^{\infty} \varphi_j(z) v_j, \tag{8}$$

where v_j is the coefficients in the expansion. The spectral representation of the continuous dynamic system follows from (7) and (8) that

$$\mathcal{K}g(z) = \mathcal{K}\sum_{j=1}^{\infty} \varphi_j(z) v_j = \sum_{j=1}^{\infty} \lambda_j \varphi_j(z) v_j. \tag{9}$$

Combining with result from (6), future nonlinear observables x_{k+1} can be approximated from time k by

$$x_{k+1} = \mathcal{K}g(z_k) = \sum_{j=1}^{\infty} \mathcal{K}\varphi_j(z_k) v_j = \sum_{j=1}^{\infty} \lambda_j \varphi_j(z_k) v_j. \tag{10}$$

Equation (10) allows us to describe the asymptotic behavior of the system as time goes to infinity. In particular, when the leading eigenvalue has norm less than one, we have an exponential decay in time.

Repeating this process until reaching the initial status, we can then write observables for any time k

$$x_k = \sum_{j=1}^{\infty} \mathcal{K}^k \varphi_j(x_0) v_j = \sum_{j=1}^{\infty} \lambda_j^k \varphi_j(x_0) v_j. \tag{11}$$

In our problem setting, at each time slot k, we arrange n individuals and their scores from the YSET into a $p \times n$ matrix X_k

$$X_k = [\mathbf{x}_k^{(1)}, \cdots, \mathbf{x}_k^{(n)}] \in \mathbb{R}^{p\times n}, , \tag{12}$$

where $\mathbf{x}_k^{(i)}$ is a $p \times 1$ vector indicating the p section scores of the i-th participant at time k. For us, $k = 0$ represents the intake scores and $k = 1$ represents the retest score.

2) DMD Algorithm and its connection to the Koopman Operator: In this section we introduce Dynamic Mode Decomposition (DMD) as a way to approximate the infinite dimensional Koopman operator using a finite attainable data set.

Suppose at time k there are a total of n participants, indexed by $j : \{\mathbf{x}_k^{(j)}\}_{k=0}^{m}$. We arrange the entire ensemble of participants and their risk scores by defining matrices $X, X' \in \mathbf{R}^{p\times n}$ as

$$X = \begin{bmatrix} | & | & | & | \\ X_0 & X_1 & ... & X_{m-1} \\ | & | & | & | \end{bmatrix}$$
$$= \begin{bmatrix} | & | & | & | & | & | & | \\ \mathbf{x}_0^{(1)} & ... & \mathbf{x}_0^{(n)} & ... & \mathbf{x}_{m-1}^{(1)} & ... & \mathbf{x}_{m-1}^{(n)} \\ | & | & | & | & | & | & | \end{bmatrix}, \tag{13}$$

$$X' = \begin{bmatrix} | & | & | & | \\ X_1 & X_2 & ... & X_m \\ | & | & | & | \end{bmatrix}$$
$$= \begin{bmatrix} | & | & | & | & | & | & | \\ \mathbf{x}_1^{(1)} & ... & \mathbf{x}_1^{(n)} & ... & \mathbf{x}_m^{(1)} & ... & \mathbf{x}_m^{(n)} \\ | & | & | & | & | & | & | \end{bmatrix}. \tag{14}$$

DMD approximates the dynamical system $X \mapsto X'$ by finding a matrix A such that $X' \approx AX$. More precisely, A is defined as the minimizer of $\|X' - AX\|_F^2$ on the set of $p \times n$, matrices. Here $\| \cdot \|_F$ denotes the Frobenius norm of a matrix. DMD modes are eigenvectors the matrix A, which can be expressed as

$$A = X'X^\dagger \tag{15}$$

where † denotes the Moore–Penrose pseudoinverse.

In our problem setting, we have only time $k = 0$ indicating the intake test and $k = 1$ representing the retake test. Thus, our X and X' become

$$X = \begin{bmatrix} | & | & | \\ \mathbf{x}_0^{(1)} & ... & \mathbf{x}_0^{(n)} \\ | & | & | \end{bmatrix}, \quad X' = \begin{bmatrix} | & | & | \\ \mathbf{x}_1^{(1)} & ... & \mathbf{x}_1^{(n)} \\ | & | & | \end{bmatrix}, \tag{16}$$

both of which are $p \times n$ real matrices.

Let r be the rank of the matrix A, we can write the solution to this system in terms of the eigenvalues μ_k eigenvectors ϕ_k and coefficients of initial condition b_j

$$\mathbf{x}_{k+1} = \sum_{j=1}^{r} \phi_j \mu_j^k b_j. \tag{17}$$

Matrix A approximates the Koopman operator, and in particular we have $\mu_j \approx \lambda_j$, $\phi_j \approx \varphi_j(x_0)$, and $b_j \approx \mathbf{v}_j$. We can thus rewrite

$$x_k \approx \sum_{j=1}^{r} \lambda_j^k \varphi_j(x_0) \mathbf{v}_j. \tag{18}$$

In practice, when the state dimension or number of participants is large, it is hard for the matrix A to be analyzed directly. The DMD algorithm, shown in Algorithm 1, provides a manageable way to avoid the eigendecomposition of A.

Algorithm 1: Dynamic Mode Decomposition (DMD)

1 Input matrices $X, X' \in \mathbb{R}^{p \times n}$, and an integer $r > 0$
2 Compute the rank-reduced SVD of $X = U_r \Sigma_r V_r^*$
3 Define $\tilde{A} = U_r^* X' V_r \Sigma_r^{-1}$
4 Compute the eigendecomposition of $\tilde{A}$, i.e., $\tilde{A}W = W\Lambda$, the eigenvalues of A are given by the diagonal entries of Λ
5 Reconstruct the eigendecomposition of A from W and Λ, the eigenvectors of A (DMD modes) are given by columns of Φ: $\Phi = X' V_r \Sigma_r^{-1} W$

B. DMDc for Network Effect

In this section, we use Dynamic Mode Decomposition with control (DMDc) [14] to analyze peer network effects on changes in risk. Consider discrete-time dynamic system sampled every Δt

$$\mathbf{x}_{k+1} = \tilde{A}\mathbf{x}_k + \tilde{B}\mathbf{u}_k, \tag{19}$$

where $\tilde{A}$ describes the dynamic of the unforced system. The operator $\tilde{B}$ characterizes the impact of the input u_k on state x_{k+1}. Similar to our construction in the previous section, we change the state snapshot matrix into a matrix that has each columns showing the risk score for each participant. We then have a mapping from intake scores to retest scores given the underlying dynamics and peer network structure

$$X' = \tilde{A}X + \tilde{B}\Gamma, \tag{20}$$

where X and X' are as in the previous section and Γ is defined as follows

$$\Gamma = \begin{bmatrix} | & | & | & | \\ U_0 & U_1 & ... & U_{m-1} \\ | & | & | & | \end{bmatrix}$$
$$= \begin{bmatrix} | & | & | & | & | & | & | \\ \mathbf{u}_0^{(1)} & ... & \mathbf{u}_0^{(n)} & ... & \mathbf{u}_{m-1}^{(1)} & ... & \mathbf{u}_{m-1}^{(n)} \\ | & | & | & | & | & | & | \end{bmatrix}. \tag{21}$$

In our problem setting, the matrix Γ encodes the control imposed by the peer network structure. Specifically, Γ is the adjacency matrix of the n participants as our network structure.

Based on the setup, we feed in X and X' and solve for $\tilde{A}$ and $\tilde{B}$. In standard DMD, described in the previous section, the resulting solution for A in system $X' = AX$ does not identify a separate peer network effect. The leading eigenvalue of A captures features of the whole dynamical system. In the present case, $\tilde{A}$ captures the dynamics of the system controlling for peer network effects and $\tilde{B}$ captures the network effects controlling for other underlying dynamics. By comparing the magnitude of the eigenvalue of A and $\tilde{A}$, we are able to examine the impact of peer network interactions on the effectiveness of the program.

C. Experiment Result

We compare the eigenvalues from DMD and DMDc algorithm applied to the data from each GRYD Zone separately. GRYD Zones are geographic regions within the overall GRYD

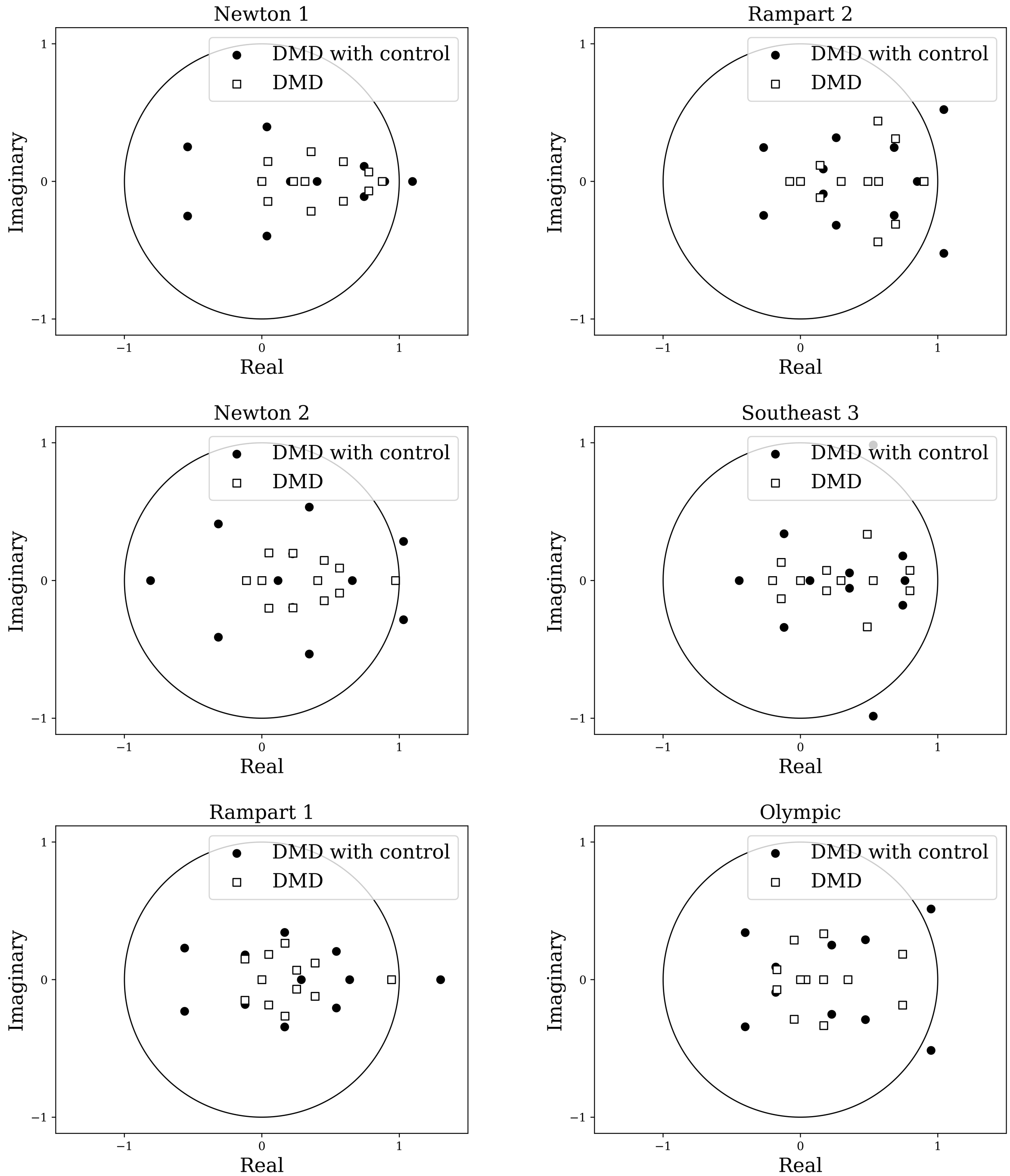

Figure 6: Each of the six plots represents the DMD analysis in a particular GRYD Zone. For each plot, the white squares are the eigenvalues of the matrix A obtained as the DMD approximation of the treatment dynamics in the given zone, while the dark dots are the eigenvalues of the matrix $\tilde{A}$ obtained as the DMDc approximation of the same dynamics. For each plot, we see that the magnitude of the eigenvalues of $\tilde{A}$ tends to be higher than the ones of A, suggesting therefore a positive network effect.

program that are serviced by a single contract provider. We restrict analyses to Zones that served more than 45 participants, namely Newton 1, Newton 2, Rampart 1, Rampart 2, Southeast 3, and Olympic. In Newton 1 there were 85 participants, 50% male, 45% female, and 5% did not specify their gender. In Newton 2 there were 97 participants, 53% male, 39% female, and 8% did not specify their gender. In Rampart 1 there were 67 participants, 54% male, 39% female, and 7% did not specify their gender. In Rampart 2 there were 52 participants, 54% male, 40% female, and 6% did not specify their gender.

In Southeast 3 there were 45 participants, 49% male, 49% female, and 2% did not specify their gender. In Olympic there were 43 participants, 40% male, 58% female, and 2% did not specify their gender. We use $\mathbf{x}_k$ to denote each person where $(\mathbf{x}_k)_i$ indicates the i-th section score of the k-th person. As the number of sections is small, we apply full DMD instead of the reduced version as it is computationally tractable, which means we directly computed the eigenvalues instead of using the reduced rank SVD algorithm presented in Algorithm 1. For the control matrix, we use the adjacency matrix of the peer network.

Table III: Leading Eigenvalues of DMD for each Zone

Zones	DMDc	DMD
Newton 1	1.30	0.94
Netwon 2	1.10	0.88
Rampart 1	1.17	0.90
Rampart 2	1.07	0.97
Southeast 3	1.12	0.80
Olympic	1.08	0.77

Table III shows the norms of leading eigenvalue of $\tilde{A}$ from DMDc and A from DMD. Notice that all of the norms of the leading eigenvalues from the full DMD are less than one, which indicates that GRYD Prevention drives decreases in participant risk scores. By contrast, the norms of the leading eigenvalue from DMDc are consistently larger than those from full original DMD. One interpretation is that the leading eigenvalue of $\tilde{A}$ captures the effect of activities if there were no peers. Since the leading eigenvalue of $\tilde{A}$ is greater than the leading eigenvalue of A, this suggests that peer network effects contribute substantially to the dynamical decrease in risk scores. In other words, in the absence of peer network effects risk scores might actually grow, as indicated by a leading eigenvalue > 1 (Figure 6).

V. Conclusions

We investigated the effectiveness of intentional youth development (AIYDA) activities offered to clients by the Los Angeles Mayor's Office of Gang Reudction and Youth Development (GRYD) Prevention program. Using data from a services eligibility questionnaire (YSET), a measure of the risk of youth gang involvement, we studied the relationships between participation in AIYDA activities and improvements in YSET scores. Leveraging a difference-in-differences model (DID), we demonstrated statistically that engaging in AIYDA activities plays a significant role in improving the youth's performance in the Internal Risk section compared to non-AIYDA activities. We further studied the Social Emotional Learning domains targeted by AIYDA activities, and illustrated that emotional management tends to be the the most influential in improving risk scores. In addition, we examined the peer networks effects arising from participants' engagement in group activities. We utilized a linear-in-means model and showed that positive peer effects exist. Youth tend to improve their risk scores in concert with their peers. Finally, we also performed a dynamic mode decomposition analysis. We demonstrated that the norm of the leading eigenvalue from DMDc, with the peer adjacency matrix as control, is consistently larger than the leading eigenvalue from regular DMD, suggesting that there is a positive peer network effect on the dynamics of the system.

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