String/M-branes for Relativists

Donald Marolf

February 6, 2008

Department of Physics, Syracuse University, Syracuse, NY 13244, USA,

and

Institute for Theoretical Physics,
University of California, Santa Barbara, California, 93106, USA.

Abstract

These notes present an introduction to branes in ten and eleven dimensional supergravity and string/M-theory which is geared to an audience of traditional relativists, especially graduate students and others with little background in supergravity. They are designed as a tutorial and not as a thorough review of the subject; as a result, many topics of current interest are not addressed. However, a guide to further reading is included. The presentation begins with eleven dimensional supergravity, stressing its relation to 3+1 Einstein-Maxwell theory. The notion of Kaluza-Klein compactification is then introduced, and is used to relate the eleven dimensional discussion to supergravity in 9+1 dimensions and to string theory. The focus is on type IIA supergravity, but the type IIB theory is also addressed, as is the T-duality symmetry that relates them. Branes in both 10+1 and 9+1 dimensions are included. Finally, although the details are not discussed, a few comments are provided on the relation between supergravity and string perturbation theory and on black hole entropy. The goal is to provide traditional relativists with a kernel of knowledge from which to grow their understanding of branes and strings.

TABLE OF CONTENTS

1 Introduction

This contribution to the Proceedings of the 3rd Mexican School on Gravitation and Mathematical Physics (Mazatlán, Mexico) is intended to be an introduction to the branes which have contributed so much to studies of string/M-theory in recent years. The goal of this work is to open string/M-theory to traditional relativists and students of general relativity by describing a few features in an accessible language. Our aim here is not in any way to be complete. In fact, we will purposely leave out many of the points which practicing string theorists deem to be most relevant, such as the detailed setup of D-brane perturbation theory and its relation to super Yang-Mills theory, S-duality, and the Maldacena conjecture. Some general commentary on the conceptual framework within which D-brane perturbation theory is to be viewed will, however, be presented in section 6. The hope is that this will provide a chunk of material that a member of the relativity community can latch on to and use to develop a perspective on both branes in particular, and string/M-theory in general. From here, one can move on to more advanced topics. A guide to further reading is given in appendix A.

We will focus here on aspects of branes which can be described in terms of supergravity. To provide a context for this discussion, the first part of the text will introduce various aspects of supergravity. We draw heavily on Polchinski's treatment [1], though the style is (hopefully) more adapted to the current audience. We begin in section 2 with a discussion of supergravity in eleven dimensions, which is both a particularly relevant and a particularly simple case. We then introduce the associated branes in section 3.

The case of ten dimensional supergravity is also central to our mission. We choose to approach this subject via Kaluza-Klein reduction of eleven dimensional supergravity. Section 4 thus begins with some introductory remarks on Kaluza-Klein compactifications and then constructs a supergravity theory (the type IIA theory) in 9+1 dimensions. Section 5 introduces the branes of type IIA theory and then comments briefly on type IIB supergravity and the so-called T-duality symmetry that relates the two theories. We close with some remarks on D-brane perturbation theory and a few words about black hole entropy via D-branes in section 6.

In broad outline, our discussion will follow the lectures originally given in Mazatlán. For the sake of both the author and the reader, no attempt will be made to be substantially more comprehensive or to include many relevant points which are not truly central to our discussion. However, the details of the presentation will be rather different and, in particular, the connections with eleven dimensional supergravity will be emphasized far more heavily than in the original lectures. This will in fact provide a more comfortable perspective for relativists, as the various classical brane solutions are somewhat less singular in eleven dimensional supergravity. As a result, the weight of the discussion has been shifted relative to the original lectures away from type IIB supergravity and toward the IIA case which is more directly related to the eleven dimensional theory.

The literature on branes and string theory is rather vast. Our goal here is to provide a tutorial and not a complete review. As a result, our referencing of the original works will at times be rather spotty. For more complete reference lists, see the reviews mentioned in appendix A, especially [2].

2 Supergravity in Eleven Dimensions

Before diving into the details, a few words of orientation are in order. We will shortly see that supergravity in eleven (10+1) dimensions is really not much more complicated than the 3+1 Einstein-Maxwell theory of Einstein-Hilbert gravity coupled to Maxwell electrodynamics. The same is not as true of supergravity in lower dimensions. In ten (9+1) dimensions and below, many interesting supergravity theories contain a so-called dilaton field which couples non-minimally to the Maxwell-like gauge fields. As a result, the equivalence principle does not hold in such theories and different fields couple to metrics that differ by a conformal factor. However, in eleven dimensions, properties of the supersymmetry algebra guarantee that any supergravity theory containing no fields with spin higher than two[1], has no dilaton. In fact, there is a a unique supergravity theory in eleven dimensions and it contains only three fields: the metric, a U(1) (i.e., abelian, Maxwell-like) gauge field, and a spin 3/2 gravitino.

2.1 On n-form gauge fields

We first address just the bosonic part of eleven-dimensional supergravity, setting the fermionic fields to zero. The differences between this truncated theory and 3+1 Einstein-Maxwell theory amount to just differing numbers of dimensions. This happens in two ways: The first is the obvious fact that the theory lives in a 10+1 spacetime instead of a 3+1 spacetime. The second is that the gauge field is slightly 'larger' than that of Maxwell theory. Instead of having a *vector* (or, equivalently, a one-form) potential, the potential is a 3-form: $A_3 = \frac{1}{3!} A_{3,\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma$. See appendix B for a discussion of the conventions used here for n-forms.

We will be seeing a lot of n-form potentials below. Although they may at first seem unfamiliar, they are in fact a very natural (and very slight) generalization of Maxwell fields. An n-form gauge potential A_n is associated with an $(n+1)$-form field strength of the form $F_{n+1} = dA_n$, where d is the exterior derivative. As a result, the field strength satisfies a Bianchi identity $dF_{n+1} = 0$. As with the familiar Maxwell field, there is an associated set of gauge transformations

$$A_n \to A_n + d\Lambda_{n-1} \tag{1}$$

where Λ_{n-1} is an arbitrary $(n-1)$ form. Such gauge transformations leave the field strength F_{n+1} invariant.

[1]Except for anti-symmetric tensor fields, which propagate on curved manifolds without the constraints associated with other higher spin fields.

In D spacetime dimensions (generally taken to be 11 in this section), the equation of motion for such a gauge field is typically of the form

$$d \star F_{D-(n+1)} = J_{D-n}, \tag{2}$$

where, in a slight abuse of notation, $\star F_{D-(n+1)}$ denotes the $D-(n+1)$ form that is the hodge-dual of $F_{(n+1)}$ and J_{D-n} is a $(D-n)$-form current that serves as a source for the field strength. Thus, the natural coupling between an n-form gauge field and its current is of the form $\int_M A_n \wedge J_{D-n}$, where M denotes the spacetime manifold.

As usual, gauge symmetry implies that the current is conserved. However, current conservation for a $(D-n)$-form current with $D-n>1$ is, in a certain sense, a much stronger statement than conservation of the current in 3+1 Maxwell theory. Note that the analogue of Gauss' law in the present context is to define the charge Q_B contained in a $(D-n)$-ball B by the integral $Q_{D-n} = \int_{\partial B} \star F_{D-(n+1)}$ over the boundary ∂B of that ball. Now, suppose that the current J_{D-n} in fact vanishes in a neighborhood of the surface ∂B. Then by stokes theorem and equation (2) we can deform the surface ∂B in any way we like and, as long as the surface does not encounter any current, the total charge Q_{D-n} does not change.

Now, in familiar 3+1 Maxwell theory, charge is measured by integrals over 2-surfaces. This is associated with the fact that an electrically charged particle sweeps out a worldline in spacetime. Note that any sphere which can be collapsed to a point without encountering the worldline of the particle must enclose zero net charge. The important fact is that, in four dimensions, there are two-spheres which 'link' with any curve and which cannot in fact be shrunk to a point without encountering the particle's worldline. In contrast, circles do not link with worldlines in 3+1 dimensions. For this reason, particles in 3+1 dimensions cannot be charged under any gauge field whose field strength is, for example, a 3-form. In considering a one-form field strength, note that 3-spheres generically *intersect* with worldlines and so do not *enclose* charge as 2-spheres do. This illustrates a general relation between a gauge field and the associated charges: unless the world-volume of an object can link with surfaces of dimension $D-(n+1)$, it cannot be electrically charged under an n-form gauge potential A_n.

While we are here, we may as well work out this counting. Let us suppose that we have an n-form gauge field A_n in D spacetime dimensions. Then, we must integrate $\star F_{D-(n+1)}$ over a $D-(n+1)$ surface in order to calculate the charge. Now, in D dimensions, surfaces of dimensions k and m can link if $k+m+1=D$ (i.e., curves and curves in three dimensions, 2-surfaces and worldlines in four dimensions, etc.). Thus, non-zero electric charge of A_n is associated with n dimensional worldvolumes. Such objects are generically known as 'p-branes' (as higher dimensional generalizations of the term membrane). Here, p is a the number of spatial dimensions of the object; i.e., the electric charge of an n-form gauge potential is carried by $(n-1)$-branes, whose world volume has $n-1$ spatial dimensions and time. This is how strings, membranes, and other branes will arise in our discussion of supergravity.

Note that, although the p-branes are extended objects, the concept of a charge *density* on the branes is not appropriate. Recall that the charge is measured by any $D-(p+2)$ surface surrounding the brane and that, by the above charge conservation argument, the charge detected by any such surface must necessarily be the same. Thus, the equations of motion tell us that moving the $D-(p+2)$ surface *along* the brane cannot *ever* change the flux through the surface; see Fig. 1 below. Thus, 'non-uniform' p-branes cannot exist! The proper concept here is to assign to such a p-brane only one number, the total charge. It simply happens that the particular type of charge being measured is somewhat less local than the familiar electric charge; it is fundamentally associated with $p+1$ dimensional hypersurfaces in the spacetime.

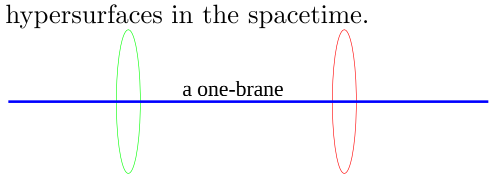

Fig. 1. By charge conservation, both circles necessarily capture the same flux.

As a small complication, we will be interested not only in electric charges, but also in magnetic charges. Indeed, in supersymmetric string theory, both electric and magnetic charges appear to be on an equal footing. A useful point of contact for the present discussion is to realize that, in a certain sense, both electrically and magnetically charged 'objects' occur in pure Einstein-Maxwell theory without any matter fields. These are just the electrically and magnetically charged eternal black hole solutions. Although the Maxwell field satisfies both $dF=0$ and $d\star F=0$ at every non-singular point of such spacetimes, the black holes can still be said to 'carry charge' due to topological effects: the electric or magnetic flux starts in one asymptotic region, funnels through the Einstein-Rosen bridge at the 'throat' of the black hole, and out into the other asymptotic region[2]. Note that black holes (i.e., point-like or zero-brane objects) may carry both electric *and* magnetic charge for the Maxwell field in 3+1 dimensions.

The counting of dimensions for magnetic charges proceeds much like the counting for electric charges. To define what we mean by a magnetic charge, we recall that hodge duality $F\to\star F$ in Maxwell theory interchanges electric and magnetic charge. Thus, since electric charge is associated with integrals of $\star F_{D-(n+1)}$, magnetic charge is defined by integrating the field strength F_{n+1} itself over an $n+1$ surface. In d dimensions, an $n+1$ surface can link with $D-n-2$ worldvolumes, or $(D-n-3)$-branes. As a check, for 3+1 Maxwell theory, we have magnetic $4-1-3=0$ branes.

Let's take a look at the eleven-dimensional context. Without knowing anything more about supergravity than we already do, we can expect two types of 'objects' to be of particular interest from the point of view of the 3-form

[2]As a result, the electric charge of the black hole measured in one asymptotic region is the opposite of the charge measured in the other asymptotic region. However, this need not trouble us so long as we understand that we must first orient ourselves by picking an asymptotic region in order to discuss the notion of charge.

gauge field A_3. There may be 2+1 electrically charged objects (2-branes) and $(D-n-3)+1 = (11-3-3)+1 = 5+1$ dimensional magnetically charged objects (5-branes). Since there are no explicit charges in the theory, these 'objects' (if they exist) must be black-hole like 'solitonic' solutions. We will see below that black two-brane and black five-brane solutions carrying the proper charges do indeed exist in eleven-dimensional supergravity. What is more, and what is different from lower dimensional supergravity, is that the horizons of these black branes remain smooth in the extremal limit of maximal electric or magnetic charge. The extremal versions of these brane-solutions are what are usually referred to as 'the M-theory two-brane' or 'M2-brane,' and 'the M-theory five-brane' or 'M5-brane.' We will discuss these in more detail in section 3.2.

2.2 Dynamics

I hope the discussion of section 2.1 has provided some orientation to supergravity in eleven dimensions. Now, however, it is time to fill in a few details. For example, it is appropriate to write down the full dynamics of the system. This is conveniently summarized by the action [3]

$$\begin{aligned} S &= \frac{1}{2\kappa_{11}^2}\int d^{11}x\, e\Big[\big(R - \frac{1}{2}|F_4|^2\big) \\ &- \frac{1}{2^3\cdot 4!}(\overline{\psi}_\alpha \Gamma^{\alpha\beta\gamma\delta\sigma\lambda}\psi_\lambda + 12\overline{\psi}^\beta\Gamma^{\gamma\delta}\psi^\sigma)(F+\hat{F})_{\beta\gamma\delta\sigma} \\ &- \overline{\psi}_\alpha\Gamma^{\alpha\beta\gamma}D_\beta\big(\frac{1}{2}(\omega+\hat{\omega})\big)\psi_\gamma\Big] - \frac{1}{12\kappa_{11}^2}\int A_3\wedge F_4\wedge F_4. \end{aligned} \tag{3}$$

Here R is the Ricci scalar of the metric $g_{\alpha\beta}$, e_α^a is the vielbein which squares to $g_{\alpha\beta}$ (and e is its determinant), A_3 is the three-form field discussed in the previous section, and ψ is the spin 3/2 gravitino. Here we use the notation

$$\begin{aligned} \hat{\omega}_{\alpha ab} &= \omega_{\alpha ab} + \frac{1}{8}\overline{\psi}^\beta\Gamma_{\beta\alpha ab\gamma}\psi^\gamma, \\ \hat{F}_{\alpha\beta\gamma\delta} &= F_{\alpha\beta\gamma\delta} - 3\overline{\psi}_{[\alpha}\Gamma_{\beta\gamma}\psi_{\alpha]}. \end{aligned} \tag{4}$$

In the above, Greek letters $(\alpha, \beta, ...)$ denote spacetime indices and Latin letters $(a, b, ...)$ denote internal indices. The square brackets [...] indicates a completely antisymmetric sum over permutations of the indices, divided by the number of terms. Our conventions for spinors and Γ-matrices are those of [4]. We will not state them explicitly here as spinors only make appearances in this section and section 2.3 and, in both cases, the details can be safely glossed over.

This looks a little complicated, but let's take a minute to sort through the various terms. We'll begin with the least familiar part: the gravitino. Since our attention here will be focused on classical solutions, we will be able to largely ignore the gravitino. The point here is that the gravitino is a fermion and, due to the Pauli exclusion principle, fermion fields do not have semi-classical states of the same sort that bosonic fields do. It is helpful here to think about the electron field as an example. There are, of course, states with a large number of electrons that are well described by a classical charged fluid. However, because of the exclusion principle, there are no semi-classical coherent states of the electron field itself; i.e., no states for which the dynamics is well described by a *classical spinor field*. In the same way, we might expect that there are states of the gravitino field that are well described by some sort of classical fluid, but we should only expect the classical action (3) to be a good description of the dynamics when the gravitino field vanishes. Thus, we will set $\psi = 0$ throughout most of our discussion. This is self-consistent as setting $\psi = 0$ in the initial data is enough to guarantee $\psi = 0$ for all time.

A study of (3) shows that the dynamics of the solutions for which $\psi = 0$ can be obtained by simply setting ψ to zero in the action. This simplifies the situation sufficiently that it is worth rewriting the action as:

$$S_{\text{bosonic}} = \frac{1}{2\kappa_{11}^2} \int d^{11}x(-g)^{1/2}\big(R - \frac{1}{2}|F_4|^2\big) - \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4. \qquad (5)$$

This sort of presentation, giving only the bosonic terms, is quite common in the literature and is sufficient for most solutions of interest[3]. Now that the gravitino has been set to zero, we see that our action contains only three terms: the Einstein (scalar curvature) term R, the Maxwell-like term F_4^2, and the remaining so-called 'Chern-Simons term.'

The Chern-Simons term has the same form ($A \wedge F \wedge F$) as the action for 2+1 Chern-Simons theory, but in the current eleven dimensional context it is not a topological invariant. Its variation is not a total divergence and it contributes to the equations of motion. Suppose that we couple a source to (5) and vary the resulting action with respect to A_3. The equation of motion is of the form

$$d(\star F_4 + (const) F_4 \wedge F_4) = J. \qquad (6)$$

Thus, the charge conservation arguments of section 2.1 continue to hold, but with the charge computed from integrals of $\star F_4 + F_4 \wedge F_4$ and not just $\star F$. This is quite important (see e.g. [6]) for certain features of brane physics, such as some types of brane intersections and is essential for the BPS bound to hold in eleven dimensions [7]. Nevertheless, we will be able to ignore the Chern-Simons term in our discussion below since we will consider only relatively simple intersections. Specifically we note that since $dF = 0$, the variation of the Chern-Simons term gives a contribution to the equations of motion proportional to $F_4 \wedge F_4$. Thus, whenever there are 4 or more linearly independent vectors k at each point such that $k^\alpha F_{\alpha\beta\gamma\delta} = 0$, we have $F_4 \wedge F_4 = 0$ and the Chern-Simons term does not affect the equations of motion. For the cases we consider below, this property is satisfied as all of the non-vanishing components of F will lie in a subspace of dimension seven or less.

[3] Typically, an interesting bosonic solution in fact corresponds to one of the quantum states in a supersymmetry multiplet. The supersymmetry algebra can often be used to construct from the original solution the related spacetimes in which the fermions are excited. See for example [5].

2.3 Supersymmetry and BPS States

Some comments are now in order on the subject of supersymmetry, so that we may introduce (and then use!) the concept of BPS states. Again, I would like to begin with a few heuristics to provide a rough perspective for the traditional relativist.

We will see that BPS solutions are closely related to extremal solutions; in particular, to extremally charged solutions. As a result, most of our intuition from extreme Reissner-Nördstrom solutions carries over to the general BPS case. A particularly useful property is that any BPS solution has a Killing vector which is either timelike or null. If, furthermore, the solution is asymptotically flat in the usual strict sense[4] then in fact this Killing vector field must be timelike [8].

The setting for any discussion of BPS solutions is the class of supergravity solutions with a certain amount of asymptotic flatness, though we will not go into the details of the boundary conditions and fall-off rates here. For the remainder of this work, we will follow the usual terminology of string theory and use a somewhat less restrictive notion of asymptotic flatness than is common in traditional relativity. The main difference is that we will *not* require the topology of the asymptotic region to be of the form R^n minus a compact set. Instead, we will allow it to be of the form $(R^n - \Sigma) \times Y$, for any compact set $\Sigma \subset R^n$, Y any homogeneous manifold, and[5] $n \geq 4$. We will use the term 'strict asymptotic flatness' to indicate the special case where the homogeneous manifold Y is a single point.

In the setting of pure gravity, one would expect that (under appropriate fall-off conditions at infinity) such spacetimes would exhibit asymptotic symmetries that correspond to the Poincaré group in the appropriate number of spacetime dimensions together with the symmetries of Y. Some particular solutions in this class will even have Killing vectors which make some subgroup of the Poincaré group into an *exact* symmetry of the spacetime; e.g. the rotation subgroup in spherically symmetric cases. Poincaré group. These are just the metric and matter fields that are invariant under some non-trivial continuous subgroup of the diffeomorphism group.

Now, supersymmetry is best thought of as an (anti-commuting) extension of the diffeomorphism group. Indeed, diffeomorphisms form a subgroup of the supersymmetry gauge transformations and, in the asymptotically flat setting just described, the asymptotic Poincaré transformations will be a subgroup of the asymptotic supersymmetry transformations. Solutions that are invariant under a subgroup of the supersymmetry transformations containing non-trivial anti-commuting elements are said to have a 'Killing Spinor' and are known as BPS (Bogomuln'yi-Prasad-Sommerfeld) solutions. We will see that they are closely related to extremal solutions.

[4]If the asymptotically flat region has the topology of $\mathbf{R^n}$ minus a compact set.

[5]We impose this last condition as a substitute for spelling out the fall-off conditions. As is well known (see e.g. [9]), the asymptotic fall-off conditions for 2+1 (and smaller) dimensional spacetimes are qualitatively different from those in higher dimensions.

Having oriented ourselves with this intuitive introduction, it is now time to examine the details of the eleven-dimensional supersymmetry transformations and their algebra. The infinitesimal supersymmetry transformations are in one-to-one correspondence with Grassmann valued (Majorana[6]) spinor fields $\eta(x)$. The transformation associated with η is given by

$$\begin{aligned}
\delta e^a_\alpha &= \frac{1}{2}\overline{\eta}\Gamma^a\psi_\alpha, \\
\delta A_{\alpha\beta\gamma} &= -\frac{3}{2}\overline{\eta}\Gamma_{[\alpha\beta}\psi_{\gamma]}, \\
\delta\psi_\alpha &= D_\alpha(\hat{\omega})\eta + \frac{\sqrt{2}}{(4!)^2}\big(\Gamma^{PQRS}_{M} - 8\delta^P_M\Gamma^{QRS}\big)\eta\hat{F}_{PQRS} \equiv \hat{D}_\alpha\eta,
\end{aligned} \tag{7}$$

where the last line defines the supercovariant derivative $\hat{D}_\alpha$ acting on the spinor η.

The details of the supersymmetry transformations are not particularly important for our purposes. What *is* important is the general structure. Note that the variation of the vielbein e involves the gravitino ψ, but then the variation of the gravitino involves the connection $\hat{\omega}$ which contains derivatives of the vielbein. Similarly taking two variations of the gauge field A_3, we find terms involving derivatives of the gauge field. It turns out that, in fact, the proper second variations give just diffeomorphisms of the spacetime.

Recalling that the variation of A_3 contains ψ, we also note that the first variation of the gravitino field involves a derivative of the spinor η. Thus, the second variation of A_3 is something that involves the derivative of some gauge parameter. With the proper choice of spinors η, one can construct a second supersymmetry variation that gives just the usual gauge transformation $A_3 \to A_3 + d\Lambda_2$ on the gauge field. Thus, both diffeomorphisms and gauge transformations are in fact contained in the spacetime supersymmetry algebra. The supersymmetry algebra can be thought of as a sort of 'square root' of the diffeomorphism and gauge algebras. The fact that diffeomorphisms and gauge transformations are expressed as squares leads to extremely useful positivity properties.

We will not need the details of the local supersymmetry algebra below. However, it is useful to display the algebra of the asymptotic supercharges. Just as for the diffeomorphisms and gauge transformations, the asymptotic supersymmetries lead, in the asymptotically flat context, to conserved 'supercharges.' In fact, for the eleven dimensional case, there are several relevant notions of the asymptotic algebra. This is because there are interesting p-branes with several values of p. Thus, there are several interesting classes of 'asymptotically flat' structures associated with different choices of the homogeneous manifold $Y = \mathbf{R}^{11-p}$ in our generalization of strict asymptotic flatness.

However, all of these algebras are rather similar. If Q is the generator of supersymmetry transformations, so that the asymptotic versions of the trans-

[6] i.e., satisfying the reality condition $\eta^* = B\eta$ where $B = \Gamma^3\Gamma^5...\Gamma^9$ and $*$ denotes complex conjugation.

formations above are generated by taking super Poisson brackets with $Q\overline{\eta}$, then the algebra associated with the p-brane case has the general form

$$\{Q_a^A, \overline{Q}^{Bb}\}_+ = -2P_\mu \Gamma_a^{\mu b} \delta^{AB} - 2iZ^{AB}\delta_a^b, \tag{8}$$

where we have used a, b for the internal spinor indices. Here, P_μ are the momenta per unit p-volume and Z^{AB} is an antisymmetric real matrix associated with the asymptotic gauge transformations. In particular, the eigenvalues of Z are of the form $\pm iq$ where q is the appropriately normalized charge carried by the p-brane. Our notation reflects that fact that it is natural to split the SUSY generator Q, which is an eleven dimensional Majorana fermion, into a set of $(11-p)$ dimensional fermions Q^A. Thus, the indices a, b take values appropriate to spinors in $(11-d)$ dimensions.

The most important property of this algebra is that it implies the so-called BPS bound on masses and charges. To get an idea of how this arises, recall that while $\overline{Q}Q$ is a Lorentz invariant, it is $Q^\dagger Q$ that is a positive definite operator. Thus, a positivity condition should follow by writing the algebra in terms of $Q^\dagger$ and Q. For simplicity, let us also choose an asymptotic Lorentz frame such that energy-momentum of the spacetime is aligned with the time direction: $P_\mu = T\delta_{\mu 0}$, where T is the brane tensions, or mass per unit p-volume. The algebra then takes the form

$$\{Q_a^A, Q^{\dagger Bb}\}_+ \quad = \quad 2T\delta^{AB}\delta_a^b + 2iZ^{AB}\Gamma_a^{0b} \tag{9}$$

It is useful to adopt the notation of quantum mechanics, even though we are considering classical spacetimes. Thus, we describe a spacetime by a state $|\psi\rangle$ and we let the generators Q act on that state as $Q|\psi\rangle$. Contracting the above relation (9) with η_{bB} and $\eta_A^{\dagger a}$ for a set of spinor fields η_A, taking the expectation value in any state, and using the positivity of the inner product and the fact that the eigenvalues of Γ_a^{0b} are ± 1 yields the relation

$$T \geq |q|. \tag{10}$$

See [7] for a full classical supergravity gravity derivation in the context of magnetic charge in eleven dimensions and [8] for a complete derivation in classical $N = 2$ supergravity in four dimensions. See also [10] for details of the above argument in the four dimensional context.

This is the BPS bound. A spacetime in which this bound is saturated is called a BPS spacetime and the corresponding quantum states are known as BPS states. Note that, from our above argument, a state is BPS only if it is annihilated by one of the supersymmetry generators; that is, if the spacetime is invariant under the transformation (7) for some spinor η. The converse is also true; any asymptotically flat spacetime which is invariant under some nontrivial supersymmetry transformation is BPS. Given a solution s and a spinor η for which the transformation (7) vanishes on s, one says that η is a *Killing spinor* of s. Since the gravitino ψ vanishes for a bosonic solution, in this context we see from (7) that η is a Killing spinor whenever it is supercovariantly constant; i.e when it satisfies $\hat{D}_\alpha \eta = 0$.

Consideration of the full local spacetime supersymmetry algebra in a BPS spacetime with Killing spinor η shows [8] that not only is the energy determined by the charge, but that the spacetime in fact has a non-spacelike Killing field $\overline{\eta}\Gamma^\mu \eta$. When the spacetime is asymptotically flat in all directions (i.e., the fields decay in an asymptotic region diffeomorphic to $\mathbf{R}^n$ minus some compact set), this Killing field is in fact timelike. The structure of the argument is very much like the Witten proof of the positive energy theorem [11].

The bound (10) is reminiscent of the extremality bound for Reissner-Nördstrom black holes. It turns out that the relationship is a strong one. Given the similarity of eleven dimensional supergravity to Einstein-Maxwell theory, it will come as no surprise that there is a supergravity theory in 3+1 dimensions that contains Einstein-Maxwell theory, together with a few extra fields. When the extra fields vanish on an initial slice, they remain zero for all time. Thus, Einstein-Maxwell theory is a 'consistent truncation' of the supergravity. In this context, the BPS bound and the extremality bound for charge coincide when there is no angular momentum. Thus, any asymptotically flat solution of Einstein-Maxwell theory with extremal charge and vanishing angular momentum defines a BPS solution of the supergravity.

In general, any BPS black hole solution will be extremal, though the converse is not always true. An important example occurs in four dimensions where all BPS states must have zero angular momentum. Thus, the 3+1 extreme Kerr solution is not BPS.

Now that we have come to terms with supersymmetry, we can proceed to ignore fermions completely in the sections below.

3 M-branes: The BPS Solutions

Although we wish to focus on the eleven dimensional case, as indicated above supersymmetry and supergravity can also be considered in less than eleven dimensions; for example, in 3+1 dimensions. In that case, any asymptotically flat solution of Einstein-Maxwell theory with extremal charge and zero angular momentum is a BPS solutions of 3+1 supergravity. But this is just the class of Majumdar-Papapetrou solutions, which consist of some number of extremal Reissner-Nördstrom black holes in static equilibrium. Thus, the Majumdar-Papapetrou solutions are a more familiar analogue of the eleven dimensional M-brane solutions which we wish to discuss. We therefore present a brief review of the Majumdar-Papapetrou solutions in section 3.1 below as an introduction to the world of M-branes. We will examine the M-branes themselves in section 3.2 and find that they strongly resemble the Majumdar-Papapetrou solutions.

3.1 The 3+1 Majumdar-Papapetrou solutions

Recall that the Reissner-Nördstrom solution with mass M and charge Q takes the form

$$ds^2 = -(1 - \frac{2MG}{R} + \frac{GQ^2}{R^2})dt^2 + \frac{1}{1 - \frac{2MG}{R} + \frac{GQ^2}{R^2}}dR^2 + R^2 d\Omega_2^2, \tag{11}$$

with R the usual Schwarzschild radial coordinate, t the Killing time, and $d\Omega_2^2$ the metric on the unit two-sphere. Here, Q and M are the charge and mass of the black hole, with Q measured in units of $\sqrt{(mass)(length)}$ as is natural in classical mechanics with $c = 1$ but $G \neq 1$. The factors of Newton's constant G have been left explicit for consistency with the rest of this exposition. The extremal situation is $Q = M$ and, in this case, the solution is controlled by a single length scale $r_0 = GM = GQ$. Since r_0 is defined by the charge, we will refer to it as the "charge radius" of the black hole. The metric simplifies to take the form

$$ds^2 = -(1 - r_0/R)^2 dt^2 + (1 - r_0/R)^{-2} dR^2 + R^2 d\Omega_2^2. \tag{12}$$

We now change to so-called isotropic coordinates in which the spatial part of the metric is conformally flat. Let $r = R - r_0$, so that the horizon lies at $r = 0$. Introducing the Cartesian coordinates x^i as usual on $\mathbf{R}^3$, we have

$$ds^2 = -f^{-2}dt^2 + f^2 \sum_{i=1}^{3} dx^i dx^i, \tag{13}$$

where $f = 1 + r_0/r$. Similarly, the electro-magnetic potential is given by $A_t = -f^{-1}$ with the spatial components of A vanishing. As the function f satisfies Poisson's equation with a delta function source,

$$\partial_x^2 f := \sum_{i=1}^{3} \partial_i \partial_i f = -4\pi \delta^{(3)}(x), \tag{14}$$

the solution for the extreme black hole takes a form similar to that seen in electrodynamics (except that the Poisson equation is for the *inverse* of the electrostatic potential). Note that the relevant differential operator is the Laplacian on a *flat* three-space and not the one directly defined by the metric. Such differential operators will often appear below, and we will use the convention that ∂_x^2 will always denote the flat-space Laplacian associated with the coordinates x. Similarly, we will write $dx^2 := \sum_i dx^i dx^i$.

The analogy with electrostatics is quite strong. The above metric (13) and the associated electric field in fact define the class of Majumdar-Papapetrou solutions [12]. These are, in general, solutions of the Einstein-Maxwell system coupled to extremal dust. Recall that extremal dust has the property that when two grains of dust are at rest, their electrostatic repulsion is exactly sufficient to balance their gravitational attraction and they remain at rest. Modulo the conditions below, any choice of the function f in (13) yields a static solution of the field equations corresponding to some distribution of this dust. For an asymptotically flat solution, we should take f to be of the form $1 + Q/r$ near infinity. The one restriction on f is that $\rho = -\frac{1}{4\pi}\nabla^2 f$ must be everywhere positive. In particular, we will take it to be of the form $\rho_0 + \sum_{k=1}^N r_k \delta(x - x_k)$ where ρ_0 is continuous. The density (defined with respect to the Cartesian coordinate system x_i) of extremal dust is given by ρ_0 and each delta function will result in the presence of an extremal black hole with charge radius r_k. In particular, near $x = x_k$, the metric takes the same form as for an isolated extremal black hole.

Extremality is quite important for the simple form of this class of solutions. It is only in the extremal limit that the repulsion induced by the electric charge can 'cancel' the gravitational attraction so that the solution can remain static. If one adds any additional energy to the solution, the non-linearities of gravity become more directly manifest.

Note that the source in (14) lies at the origin of the x-coordinates; i.e., at the horizon of the black hole. However, since the horizon of the black hole is in fact not just a single point in space, $x = 0$ is clearly a coordinate singularity. This means that although the support of the delta function lies at $x = 0$, this should not be interpreted as the location of the black hole charge. Rather, the role of this delta function is to enforce a boundary condition on the electric flux emerging from the black hole so that the hole does indeed carry the proper charge.

Of course, in 3+1 dimensions, we can also have magnetically charged black holes. In fact, we can have dyons, carrying both electric and magnetic charge. The corresponding extremal solutions are given directly by electro-magnetic duality rotations of the above solution.

For future reference we note that there is a similar set of solutions in 4+1 dimensions, though black holes in five dimensions can carry only electric charge. They take the form

$$ds^2 = -f^{-2}dt^2 + f\sum_{i=1}^{4} dx^i dx^i = -f^{-2}dt^2 + f dx^2, \tag{15}$$

where $\partial_x^2 f = -2\omega_3(\rho_0 + \sum_{k=1}^N r_k^2 \delta(x - x_k))$, ω_3 is the volume of the unit three-sphere, ρ_0 is the charge per unit d^4x cell, and r_k is the charge radius of the k-th extremal black hole. The fact that r_k^2, as opposed to r_k, appears as the source reflects the fact that the fundamental solution of Poisson's equation in four dimensions is of the form r^{-2}.

As we have already commented, there is a coordinate singularity at the black hole horizon. Thus, the isotropic form of the metric does not allow us to see to what extent the black hole, or even the horizon, is non-singular. However, if the black hole is to have a smooth horizon, then a necessary condition is that the horizon have non-zero (and finite) area. That this is true of the above metrics is easy to read off from (13) and (15) by realizing that the divergence of f^2 or f cancels the r^2 factor that arises in writing $dx^2 = dr^2 + r^2 d\Omega^2$ in spherical coordinates. While this is certainly not a sufficient condition for smoothness of the horizon, it will serve as a useful guide below.

Finally, for completeness, we display the conformal diagrams for these solutions. They are, in fact, identical except for the dimension of the (suppressed) spheres of symmetry.

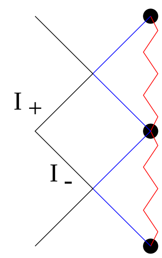

Fig. 2. Conformal diagram for extreme Einstein-Maxwell Black Holes.

Here I_+, I_- denote the past and future null infinities of a particular asymptotic region and the wavy line on the right denotes the (timelike) singularity. The black circles mark the "internal infinities." These points lie at an infinite affine parameter along any geodesic (spacelike, timelike, or null) from the interior. This is often referred to as the "deep throat" of the black hole. The reason for this should be clear from the sketch below showing the embedding of a spacelike slice at constant Killing time into flat space.

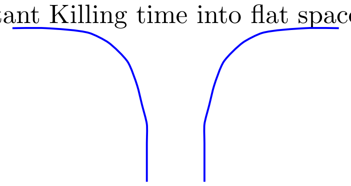

Fig. 3. Embedding diagram for a Killing slice of the extreme Einstein-Maxwell Hole.

3.2 Brane solutions in eleven dimensions

There are four (basic) solutions of eleven dimensional supergravity that are of particular importance in string/M-theory. These are known as the M-theory wave (basically the eleven-dimensional version of the Aichelburg-Sexl metric [13]), the M2-brane [14] (electrically charged under A_3), the M5-brane (magnetically charged under A_3), and the eleven dimensional version of the Kaluza-Klein monopole [15, 16].

Below, we discuss only the extremal versions of the solutions. The non-extremal forms of the M-branes may be found in, e.g., [7]. In familiar 3+1 Einstein-Maxwell gravity, we are used to thinking of extreme black holes as being some sort of marginal and perhaps unphysical case. Indeed, it is an important part of black hole thermodynamics that one cannot by any finite (classical) process transform a non-extreme black hole into an extreme black hole. Moreover, a real astrophysical black hole will quickly loose its charge due to interactions with the interstellar medium. Even in a pure vacuum, quantum field theory effects in the real world cause black holes loose their charge and to evolve toward neutral black holes. to loose mass. However, this last statement is a consequence of the large charge to mass ratio of the electron. In a supersymmetric theory of the type we discuss here, the charge of any particle is bounded by its mass through a BPS bound so that objects like the familiar electron do not exist[7]. As a result, in a supergravity theory BPS black holes in a vacuum do not discharge. In fact, non-extreme black holes decay through the Hawking process *toward* extremality and, due to quantum effects, one expects that any non-extreme black hole will decay to an extreme black hole in a very large but finite time. Thus, extreme black holes are of central importance in understanding supersymmetric theories as they represent stable 'ground states' for black holes.

The extremal forms of the basic solutions are BPS, and in particular they each have 16 Killing spinors, preserving half of the supersymmetry. It is often said that an arbitrary BPS solution can be built from these basic solutions. To understand the sense in which this is true, recall that a BPS solution is extremal, and so carries charge. BPS solutions are classified in terms of the charges they carry, and the above 'basic' solutions are in one-to-one correspondence with the types of charge present in eleven dimensional supergravity. Since the charges are additive, one is tempted to say that any solution with arbitrary amounts of the various charges can be built up by 'combining' these basic solutions. We will even see that certain simple solutions carrying multiple charge are in fact built from the basic solutions in a simple way. However, there is as yet no known method for writing down a general BPS solution at all, much less in terms of the basic solutions.

Nevertheless, it is these four basic solutions (and those which are built from them simply) which we will study here, leaving the more complicated cases for the literature (see in particular [17, 18] for what is known about the supergravity solutions corresponding to more complicated cases). Although it may not be obvious from the names, all four of the basic solutions are associated with branes in string/M-theory.

Let us begin with the most obviously brane-like of the cases, the BPS M2- and M5-branes. These are straightforward supergravity analogues of the extreme Reissner-Nördstrom black holes of Einstein-Maxwell theory[8]. There is a corresponding notion of isotropic coordinates in which the multi black hole

[7]One may wish to ask what relevance such supersymmetric theories can have to the real world. The hope is that supersymmetry represents a 'broken symmetry' of physics, so that studies of supersymmetric theories can be relevant at some fundamental level even though the connection with everyday physics is rather complicated.

[8]Interestingly, the global structure of the *non-extreme* M2- and M5-brane solutions is much like that of the Schwarzschild black hole, as opposed to that of non-extreme Reissner-Nördstrom. In particular, there is no inner horizon and the singularity is spacelike as opposed to timelike.

solutions are given by solving a flat space Poisson equation with delta-function sources. The solutions of this Poisson equation are typically denoted H_2 for the M2-brane and H_5 for the M5-brane and are referred to as 'Harmonic' functions. The details are different for the two branes, but both should seem quite familiar from our review of the Majumdar-Papapetrou solutions.

For the M2-brane, we introduce a set of three coordinates $x_{\|}$ which should be thought of as labeling the directions along the brane, and a set of eight coordinates $x_{\perp}$ which should be thought of as labeling the directions orthogonal to the brane. As one of the $x_{\|}$ directions is the time direction, we define $dx_{\|}^2 = -(dx_{\|}^0)^2 + (dx_{\|}^1)^2 + (dx_{\|}^2)^2$. The solution takes the form:

$$\begin{aligned} A_3 &= -H_2^{-1} dt \wedge dx_{\|,1} \wedge dx_{\|,2} \\ ds^2 &= -H_2^{-2/3} dx_{\|}^2 + H_2^{1/3} dx_{\perp}^2 \end{aligned} \tag{16}$$

with $\partial_{\perp}^2 H_2$ equal to a sum of delta-functions. Note that, near the delta function source, H_2 will diverge like r^{-6}, where r is the $x_{\perp}$ coordinate distance from the source. As a result, $H_2^{1/3}$ diverges like r^{-2}, and the sphere at the horizon will have non-zero (finite) area. This suggests that the horizon of the BPS M2-brane is smooth, and a careful investigation [7] does indeed show that this is the case. This is rather interesting, as the extremal limits of black branes in lower dimensional supergravity theories tend, because of the dilaton, to have singular horizons. The global structure of the M2-brane is in fact much like that of the extreme Reissner-Nördstrom black holes discussed above. The conformal diagram is just that of Fig. 1, except that each point on the diagram now represents a surface with both the topology and metric of $\mathbf{R}^2 \times S^7$ instead of just a sphere.

For the M5-brane, we introduce a set of six coordinates $x_{\|}$ along the brane, and a set of five coordinates $x_{\perp}$ orthogonal to the brane. Again, the $x_{\|}$ directions include the time t. The solution takes the form:

$$\begin{aligned} dA = F &= -\frac{1}{4!} \partial_{x_{\perp}^i} H_5 \epsilon^{ijklm} dx^j \wedge dx^k \wedge dx^l \wedge dx^m \\ ds^2 &= -H_5^{-1/3} dx_{\|}^2 + H_5^{2/3} dx_{\perp}^2, \end{aligned} \tag{17}$$

with $\partial_{\perp}^2 H_5$ equal to a sum of delta-functions. The different form of the gauge field as compared with (16) is associated with the fact that this solution carries a magnetic charge instead of an electric charge. Now the field H_5 diverges at a delta-function source as r^{-3}, so that $H_5^{2/3}$ diverges like r^{-2} and again the area of the spheres is finite at the horizon. Once again, a detailed study shows that the horizon is completely smooth. In fact, it turns out [7] that this solution is smooth *everywhere*, even inside the horizon! Its conformal diagram is rather different from those we have encountered so far and is shown below. The regions marked A and B below ('in front of' and 'behind') the horizon are exactly the same. In familiar cases, the singularity theorems guarantee that something of this kind does not occur: compact trapped surfaces imply a singularity in their future [19]. However, the fact that we deal with a black brane, and not a black hole, means that the trapped surfaces are not in fact compact. The point here is that the horizon is extended in the $x_{\parallel}$ directions. What happens when the solution is toroidally compactified by making identifications in the $x_{\parallel}$ coordinates is an interesting story that will be discussed below.

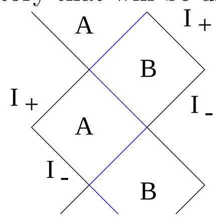

Fig. 4. Conformal diagram for the extreme M5-brane.

The remaining two solutions have the interesting property of being BPS despite the fact that the gauge field A_3 is identically zero. This is not really a contradiction to the condition of extremality when one notes (see section 4.2) that under Kaluza-Klein reduction a momentum can act like a charge. Another useful perspective results from recalling that two parallel beams of light (or two parallel gravitational waves) do not interact gravitationally. The same is true for any null particles. Thus, one may say that the spatial components of the momentum provide a gravitational repulsion and that the case of null momentum is like the case of extremal charge, where this repulsion just exactly balances the gravitational attraction due to the energy of the particles.

The M-theory wave carries just such a null momentum. This solution was originally constructed [13] (in 3+1 dimensions) by boosting a Schwarzschild solution while rescaling its mass parameter M in order to keep the total energy E finite in some asymptotic frame. This explains the null momentum of the resulting solution. It too can be described in terms of a 'harmonic function' H_W. The M-theory wave may be thought of as the gravitational field of a null *particle*, such as a graviton or a quantum of the A_3 field in the short wavelength (WKB) approximation. We introduce a time coordinate t, a coordinate z in the direction of motion of the particle, and a set of nine additional coordinates $x_{\perp}$. In isotropic coordinates, the solution takes the form

$$ds^2 = -dt^2 + dx_{\perp}^2 + dz^2 + (H_W - 1)(dt - dz)^2 \tag{18}$$

where $H_W(x_{\perp})$ is a solution of $\partial_{\perp}^2 H_W = -7\omega_9 \rho$, where ρ is again a source and ω_9 is the volume of the unit 9-sphere. When ρ is a delta-function, this solution is in fact singular at the source.

Let us now turn to the Kaluza-Klein monopole. This solution was originally constructed [15, 16] by using the fact that the metric product of any two Ricci flat spaces is Ricci flat. Thus, one can make a static solution of 4+1 Einstein gravity out of any solution to four-dimensional, Euclidean gravity. Such a solution is Ricci flat, so the metric product with a line is also Ricci flat. The metric product of Euclidean Taub-NUT space [20] with a line gives the 4+1 Kaluza-Klein monopole. The eleven dimensional solution of interest here is simply the metric product of Euclidean Taub-NUT space with a 6+1 Minkowski space. Recall that the Taub-NUT solution is not asymptotically flat. Rather, it is asymptotically flat in three directions (to which we assign coordinates $x_\perp$) and the fourth direction (which we will call θ) is an angular coordinate for which the associated S^1 twists around the two-sphere to make a non-trivial asymptotic structure. Introducing coordinates $x_\parallel$ on the 6+1 Minkowski space, the solution takes the form:

$$ds^2 = dx_\parallel^2 + H_{KK} dx_\perp^2 + H_{KK}^{-1}(d\theta + A_i dx_\perp^i)^2 \tag{19}$$

with, of course, $A_3 = 0$. Again, H_{KK} satisfies an equation of the form $\partial_\perp^2 H_{KK} = -4\pi\rho$ and a_k is determined from H_{KK} via $\partial_{x_\perp^i} H_{KK} = \epsilon_{ijk}\partial_{x_\perp^j} a_k$. As usual, we find a coordinate singularity at the location of the delta-function sources.

The story of this singularly is just that of Taub-NUT space. Suppose that θ is periodic with period L. Then the spacetime is in fact smooth in the neighborhood of a 'source' of the form $\rho = \frac{L}{4\pi}\delta^{(3)}(x_\perp)$; in this case, (19) actually represents a smooth geodesically complete solution to the *source-free* 10+1 Einstein equations. A source of this sort is referred to as a monopole of unit charge. A multi-center solution[9] is smooth whenever each separate center has this charge. Now, if we take a limit of a multi-center solution in which several of the centers coalesce into a single center with charge greater than one, the resulting spacetime has a timelike singularity at the source. However, this singularity (with an integer n number of units of the above fundamental charge) has a particularly simple form. It is a quotient of flat space, and in this sense it is a higher dimensional dimensional version of a conical singularity.

A favorite topic to include in discussions of black holes is that of black hole entropy. It is therefore natural to ask about the entropy of the branes that we have discussed above. Similar thermodynamic arguments hold for black branes as for black holes, suggesting that one should associate an entropy of $A/4G_{11}$ with such objects where A is the area (volume) of the horizon and $1/16\pi G_{11} = 1/2\kappa_{11}^{-2}$ is the coupling constant that stands in front of the supergravity action. However, the Kaluza-Klein monopole and M-theory wave have no Killing horizons and so presumably carry no such entropy. The M2- and M5-branes are a bit more subtle. On the one hand, their horizons are homogeneous surfaces that are non-compact. As such, one might be tempted to assign them infinite entropy. Some further insight into the issue is gained by using the fact that the solutions are invariant under translations in the spatial $x_\parallel$ coordinates to make toroidal identifications and compactify the horizons. We can then calculate the horizon area and, because the norms of the Killing fields $\partial_{x_\parallel}$ vanish on the horizon, the result is zero. Thus, at least when compactified in this way, the M2- and M5-branes also carry no entropy. This is another sense in which such solutions are 'basic.'

One might guess that there is something singular about the zero-area horizons of the compactified M2- and M5-branes. However, since those solutions were constructed by making discrete identifications of spacetime with smooth

[9]One with several delta-function sources.

horizons, the curvature and field strength cannot diverge at the zero-area horizon. It turns out that the situation is essentially the same as that which arises [21] when AdS_3 is identified to make the $M = 0$ BTZ black hole. The initially spacelike Killing fields $\partial_{x_\parallel}$ become null on the horizon but also have fixed points. Thus, the horizon of the compactified solution has both closed null curves and a 'Lorentzian conical singularity.'

3.3 Brane Engineering

Before leaving eleven dimensions, a few words are in order on two of the basic techniques in 'Brane Engineering,' constructing new brane solutions from old. The particular techniques to be discussed are known as smearing and combining charges.

Smearing is particularly straightforward. It is based on the observation that each type of 'basic' solution above is related to the solution of a linear differential equation. Using a delta-function source gives a solution which preserves some set of translation symmetries (in the parallel directions) and breaks another set (in the $x_\perp$ directions). However, a solution can be obtained that preserves more translational symmetries by using a more symmetric source, e.g., one supported on a line, plane, or a higher dimensional surface. Constructing such a solution can be thought of as 'smearing out' the charge of a less symmetric solution. Smearing out a given brane solution often results in a spacetime with a singular horizon. However, this need not be especially worrying if one regards the smeared solution as merely an effective description analogous to describing a collection of discrete atoms as a continuous fluid. One imagines an array of branes in which a large number of unsmeared basic branes are placed in the spacetime with a small spacing between the branes. We will soon see that smearing is an important step in the construction of BPS black brane solutions with finite entropy.

The next technique to discuss is that of combining the basic types of charge. As mentioned above, this is in general rather difficult. If, however, two solutions preserve some of the same supersymmetries *and* they have been engineered to have the same translation symmetries (for example, by smearing), then they tend to be rather easy to combine. Making a simple guess as to the way in which the relevant harmonic functions (H_2, H_5, H_W, H_M) should enter the metric and gauge fields tends to lead to a solution to the supergravity equations which preserves the common supersymmetries.

So far as I know, there are no general theorems available on this subject. We will thus content ourselves with a few simple examples. We have already discussed the solution (16) corresponding to a set of parallel M2-branes. This solution preserves half of the original 32 supersymmetries of 10+1 supergravity. The particular supersymmetries that are broken are related to the plane in space along which the M2-branes are oriented. Let us call the spatial coordinates along these branes $x^1_\parallel$ and $x^2_\parallel$. We could also consider another set of M2-branes oriented along another plane associated with two other coordinates $y^1_\parallel$ and $y^2_\parallel$, which are to be orthogonal to the $x_\parallel$ coordinates. A set of solution containing both types of branes and preserving the 8 supersymmetries common to both sets of M2-branes separately is given by:

$$
\begin{aligned}
A &= -H_x^{-1} dt \wedge dx_{\|,1} \wedge dx_{\|,2} - H_y^{-1} dt \wedge dy_{\|,1} \wedge dy_{\|,2} \\
ds^2 &= -H_x^{-2/3} H_y^{-2/3} dt^2 + H_x^{-2/3} H_y^{1/3} dx_{\|}^2 + H_x^{1/3} H_y^{-2/3} dy_{\|}^2 \\
&+ H_x^{1/3} H_y^{1/3} dx_{\perp}^2
\end{aligned}
\tag{20}
$$

where H_x, H_y are functions only of the six spatial coordinates $x_\perp$ that are transverse to both sets of branes. The functions H_1 and H_y are, as usual, related to source distributions through $\partial^2_{x_\perp} H_x = -7\omega_8 \rho_x$ and $\partial^2_{x_\perp} H_y = -7\omega_8 \rho_y$ and the distributions ρ_x and ρ_y may be arbitrary functions of $x_\perp$.

Note that the form (20) is just like that of (16) except that we include two Harmonic functions. A given term in the metric (20) is multiplied by a power of each harmonic function determined by whether the term refers to distances along or transverse to the corresponding brane. These powers are identical to the ones in (16).

In the solution (20), we have taken the two sets of branes to be completely orthogonal to each other. However, other choices of the relative angle still preserve the same amount of supersymmetry. If one thinks about the coordinates $x_\|$, $y_\|$ as two holomorphic coordinates on $\mathbf{C}^2$, then the requirement for a supersymmetric solution is that the $x_\|$ and $y_\|$ planes are related by a $U(2)$ transformation [22] as opposed to a more general $O(4)$ transformation. The metric in this case takes a similar form, with the part of the metric on the four-space spanned by $x_\|$, $y_\|$ taking a certain Hermitian form.

Combining the two sets of branes without first smearing them to generate four translation symmetries is, however, rather more difficult. It turns out that, when one or both of the sets of branes is 'localized' (i.e., not completely spread out along the other set of branes) then the supergravity equations no longer cleanly divide into pieces describing each set of branes separately. The case where only one set is localized (and two translational symmetries remain) is still tractable, however. The solution still takes the same basic form (20) and construction of the solution still splits into two parts. One can first solve a standard flat space Poisson equation for the Harmonic function H_x associated with the delocalized set of branes. One then has a linear differential equation to solve for the localized brane Harmonic function H_y, where H_x appears in the particular differential operator to be inverted. When the separation (in the $x_\perp$ directions) of the two sets of branes vanishes, this 'interaction' between the two sets of branes generates some interesting effects related to black hole no-hair theorems [23, 24, 18], see [25] for a discussion geared to an audience of relativists. Localizing both branes requires the solution of a non-linear partial differential equation (see [17] for discussion of the related M5-brane case). Although their solution is not yet understood, it appears [18] that it will be rather far from the simple structure associated with the basic branes.

Let us now return to the smeared solution (20) and consider the case in which $\rho_x = \rho_y = r_0^4 \delta^{(6)}(x_\perp)$. We then find that H_x and H_y diverge at $x_\perp = 0$ like

$|x_\perp|^{-4}$. As a result, the 5-spheres at $x_\perp = 0$ are infinite in volume. Thus, this solution is somewhat singular. However, adding a third M2-brane in another completely orthogonal $(z_\parallel^1, z_\parallel^2)$ plane yields a non-singular solution. The metric and gauge field

$$\begin{aligned}
A &= -H_x^{-1} dt \wedge dx_{\parallel,1} \wedge dx_{\parallel,2} - H_y^{-1} dt \wedge dy_{\parallel,1} \wedge dy_{\parallel,2} - H_z^{-1} dt \wedge dz_{\parallel,1} \wedge dz_{\parallel,2} \\
ds^2 &= -H_x^{-2/3} H_y^{-2/3} H_z^{-2/3} dt^2 + H_x^{-2/3} H_y^{1/3} H_z^{1/3} dx_\parallel^2 + H_x^{1/3} H_y^{-2/3} H_z^{1/3} dy_\parallel^2 \\
&+ H_x^{1/3} H_y^{1/3} H_z^{-2/3} dz_\parallel^2 + H_x^{1/3} H_y^{1/3} H_z^{1/3} dx_\perp^2 \qquad (21)
\end{aligned}$$

for $\partial^2_{x_\perp} H_{x,y,z}(x_\perp) = -2\omega_3 \rho_{x,y,z}(x_\perp)$ yields a BPS solution of the supergravity equations that preserves 1/8 of the supersymmetry (i.e., 4 supercharges) and has a smooth horizon. Moreover, this solution has the property that the translational Killing fields $\partial x_\parallel^i$, $\partial y_\parallel^i$, $\partial z_\parallel^i$ have norms that do not vanish on the horizon.

In contrast, recall that while the solution (16) for a single M2-brane has a smooth horizon, the spatial translational Killing fields have vanishing norm there. As mentioned above, this means that compactifying a single M2-brane by, for example, taking the coordinate $x_\parallel^1$ to live on a circle, yields a solution with a conic singularity at the horizon and vanishing entropy. On the other hand, because the norms of the spatial translations do not vanish for the solution (21), it compactifies nicely into a black object with finite horizon area. This is the simplest BPS black brane solution with a finite entropy and, as a result, it is the simplest solution for which a microscopic accounting of the entropy has been given in string theory. A straightforward calculation shows the the horizon area is

$$A = \omega_3 r_x r_y r_z L_{1x} L_{2x} L_{1y} L_{2y} L_{1z} L_{2z}, \qquad (22)$$

where ω_3 is the volume of the unit three-sphere and the L's are the lengths of the various circles on which the solution has been compactified.

Now, charges are quantized in string/M- theory and it is useful to express the entropy in terms of the number of charge quanta Q_x, Q_y, Q_z carried by the various branes. The tension of a single M2-brane is $(2\pi)^3 l_p^{-3}$, where l_p is the eleven-dimensional Plank length, defined by $16\pi G_{11} = 2\kappa_{11}^2 = (2\pi)^8 l_p^9$. Note that r_x^2 is a measure of the charge *density* of the x-type branes per unit cell of the y, z four-space. As such, r_x^2 is proportional to $Q_x / L_{1y} L_{2y} L_{1z} L_{2z}$. Inspection of the area formula (22) thus shows that rewriting the area in terms of the integer charges will remove the factors of L. Putting in the proper normalization coefficients, the result turns out to be

$$A/4G_{11} = 2\pi \sqrt{Q_x Q_y Q_z}. \qquad (23)$$

We will comment briefly on the corresponding microscopic counting of states in section 6.2.

4 Kaluza-Klein and Dimensional Reduction

So far, we have dealt almost exclusively with branes in eleven dimensional supergravity. We focused on this case for two reasons. The first reason is that eleven is the maximal number of dimensions in which there is a supergravity theory with fields of spin less than or equal to two. As such, it seems to play a role of fundamental importance in string/M-theory. Most other lower dimensional supergravity theories can be obtained through the Kaluza-Klein mechanism in which some subset of the dimensions are taken to be compact and small. This mechanism is discussed in sections 4.1 and 4.2 below.

The second reason is that supergravity is in fact simpler in eleven dimensions than in lower dimensions. The eleven dimensional theory contains only two bosonic fields, the metric and the 3-form, and one fermionic field. In particular, there is no dilaton and the 3-form is minimally coupled. In contrast, supersymmetry in lower dimensions forces the theory to contain a plethora of different bosonic fields, including a dilaton. It is the dilaton and its cousins, the other "moduli," which make classical supergravity in ten dimensions or less so much different from familiar Einstein-Maxwell theory. This is because the dilaton couples non-minimally to the various fields, with the result that different fields couple to what are effectively different metrics. In other words, the equivalence principle is violated. The various metrics share the same conformal structure and are related by different factors of e^{ϕ}, where ϕ is the dilaton. Thus, by focusing first on the eleven dimensional case, we have been able to make maximal use of our 3+1 Einstein-Maxwell intuition.

However, it turns out that supergravity in 9+1 dimensions (just one dimension down from the maximum) has an important property that does not follow just by thinking of it as the compactification of a 10+1 theory: The 9+1 theories admit a self-consistent perturbative quantization in terms of strings[10]. This means that the powerful technology of perturbative quantum field theory can be brought to bear on questions concerning their quantum dynamics. This perturbative technology can in particular be applied to certain branes in 9+1 supergravity. It is through this fusion of supergravity and perturbative field theory that string/M-theory has been revolutionized in recent years via studies of duality, black hole entropy, and more recently the Maldacena conjecture or AdS/CFT correspondence.

This article is not the place to enter into a detailed discussion of string perturbation theory, hough we will comment briefly on the subject in section 6. The reader interested in learning that subject should consult the standard references [1, 4]. Our purpose here is to provide a clear picture of the supergravity side of BPS brane physics, and in particular to discuss their relationship with the eleven dimensional theories. Thus, we begin with a discussion of Kaluza-Klein compactification in non-gravitational theories. We then discuss in detail the Kaluza-Klein reduction of eleven-dimensional supergravity in the presence of a

[10]It should be mentioned that string theory is not a quantization of pure 9+1 supergravity; string theory modifies the physics even at the classical level. See section 6.1 for (a few) more details.

small S^1, which yields so-called type IIA supergravity in 9+1 dimensions. This sets the stage for our discussion of 9+1 branes in section 5.

4.1 Some remarks on Kaluza-Klein reduction

The idea of the Kaluza-Klein mechanism is that, at low energies, a *quantum* field theory on an $n+d$-dimensional spacetime in which d of the dimensions are compact behaves essentially like a quantum field theory on an n-dimensional spacetime. To see why, consider a free scalar field on $M^n \times S^1$ where M^n is n-dimensional Minkowski space. Let us consider the mode spectrum of the scalar field. Modes are labeled by an n-vector momentum p and an integer k corresponding to momentum around the S^1. Suppose that the length of the S^1 is L, so that the dispersion relation associated with one-particle excitations is $E^2 = p^2 + (k/L)^2$. If we now consider the theory at energy scales less than $1/L$, the only states with such a low energy are states with $k = 0$; i.e., states that are translationally invariant around the S^1.

In this way, our scalar field reduces at such energy scales to a quantum field on n-dimensional Minkowski space. Note that this is an intrinsically quantum mechanical effect. Note also that it is associated with the discrete spectrum of the Laplacian on a circle. Since the Laplacian has a discrete spectrum on any compact space, the same mechanism operates with any choice of compact manifold. The simplest cases to analyze are those in which the spacetime is a direct product of a non-compact spacetime M with a compact manifold K, and in which K is a homogeneous space. In that case, the lower dimensional (reduced) theory is obtained from the higher dimensional one simply by taking the fields to be invariant under the symmetry group that acts transitively on the compact manifold. On a general manifold of the form $M \times K$, the reduced theory is given by considering the zero-modes of the Laplacian (or other appropriate differential operator) on K. Similar, but less clean, mechanisms may apply even when the spacetime is not a direct product of a compact and a non-compact spacetime.

The effect of compactification on interacting fields is similar. At the perturbative level the story is exactly the same, and non-perturbative effects seldom change the picture significantly.

4.2 Kaluza-Klein in (super)gravity

Let us now consider this mechanism in a theory with gravity. Since the spacetime metric is dynamical, this case is perhaps not as clean cut as the scalar field example just discussed. However, at the perturbative level, one may treat gravity just as any other field. To this extent then, the same conclusions apply. Also, our general experience with quantum mechanics and the uncertainty principle makes it reasonable on more general grounds to expect that excitations associated with the small compact space will be expensive in terms of energy. Thus, at least at first glance, we expect that gravity on a manifold of the form $M \times K$ reduces at low energies to a theory on the non-compact manifold M.

The case in which we are most interested is the Kaluza-Klein reduction of eleven-dimensional supergravity to 9+1 dimensions. We expect the reduced theory to be obtained by considering the class of eleven-dimensional field configurations that are translationally invariant around the S^1. Let us therefore assume that our eleven dimensional spacetime $\mathcal{M}$ has a spacelike Killing vector field λ^μ whose orbits have the topology S^1. It is convenient to normalize this Killing field to have norm $+1$ at infinity and to denote the length of the Killing orbits there by L. The Killing field is not necessarily hypersurface orthogonal.

Since the translation group generated by the Killing field acts nicely (technically, 'properly discontinuously' [19] on our eleven dimensional spacetime $\mathcal{M}$, we may consider the quotient of the smooth topological space $\mathcal{M}$ by the action of this group. The result is a new topological space M, which is a ten dimensional smooth manifold. This is the manifold on which our $10 = 9 + 1$ dimensional reduced theory will live.

By using the metric, we define a set of projection operations on the various 10+1 fields with each projection providing a different field in the 9+1 dimensional spacetime. Recall that a field is an object which transforms in a certain way under local Lorentz transformations (i.e., diffeomorphisms) of the manifold. The diffeomorphisms of the 9+1 manifold will be that subgroup of the eleven-dimensional diffeomorphisms that leaves the killing field λ^μ invariant. Thus, the transformations that become the diffeomorphisms of the $9 + 1$ manifold are a proper subgroup of the 10+1 diffeomorphisms and a single 10+1 field can contain several 9+1 fields.

To see how the 9+1 fields are constructed, consider any coordinate patch U (with coordinates x^a) on the 9+1 manifold M. If $V \subset \mathcal{M}$ is the preimage of U under the above quotient construction, then each x^a defines a function on V. Since no linear combination of the gradients of the x^a functions can be proportional to the Killing field λ^μ, we can complete this set of functions to a coordinate patch[11] on V by adding a coordinate θ which is proportional to the Killing parameter along any orbit of λ^μ; i.e., satisfying $\theta_{,\mu}\lambda^\mu = \lambda^\mu\lambda_\mu$.

This coordinate system gives an explicit realization of the natural decomposition of the 10+1 fields into a set of 9+1 fields. The set of gradients $x^a_{,\mu}$ of the 9+1 coordinates define a projection operation on any contravariant (upper) index, as does the gradient $\theta_{,\mu}$ of the coordinate θ. Thus, from the 10+1 contravariant metric $g^{\mu\nu}$, we can define the 9+1 metric $g^{ab} = x^a_{,\mu} x^b_{,\nu} g^{\mu\nu}$, a 9+1 abelian vector field $A^a_1 = -x^a_{,\mu}\theta_{,\nu} g^{\mu\nu}$, and a 9+1 scalar field ϕ through $Le^{4\phi/3} = \lambda^\mu g_{\mu\nu}\lambda^\nu$. The particular coefficient of ϕ is chosen so that it is canonically normalized[12]. This ϕ is the famous dilaton of string theory, and it is this field which is responsible for many of the differences between supergravity in less than eleven dimensions and familiar Einstein-Maxwell theory. It is clear that all of these fields transform in an appropriate way under 9+1 diffeomorphisms.

There are several important observations to make about these definitions.

[11] Technically, θ is not quite a valid coordinate because it is periodic instead of single valued. The reader can easily fill in the appropriate details if desired.

[12] When $2\kappa^2_{10}$ is set to one, see below.

The first is that nondegeneracy of the 10+1 metric implies non-degeneracy of the 9+1 metric. Thus, g^{ab} has an inverse which gives the covariant metric g_{ab}.

The second is that the scalar has been defined by the norm of the Killing field and not the norm of $\theta_{,\mu}$ as one might expect. The point is that these two objects are related. To see this, let us first note that the coordinates x^a are constant along the orbits of the Killing field. Thus, the Lie derivative of x^a along λ^μ vanishes, and we have $x^a_{,\nu} g^{\nu\mu}\lambda_\mu = 0$. This means that the gradients $x^a_{,\mu}$ span the space orthogonal to λ_μ at each point. But, by definition, $\lambda^\mu \theta_{,\mu} = \lambda^\mu \lambda_\mu$. Thus, we find that $\theta_{,\mu} - \lambda_\mu$ is of the form $c_a x^a_{,\mu}$ where c_a is some function on the 9+1 spacetime. This fact, together with the definition of A_1, can be used to derive the relation:

$$c_a = -g_{ab} A_1^b. \tag{24}$$

Thus, we have

$$\theta_{,\mu}\theta_{,}{}^{\mu} = \lambda^\mu \lambda_\mu + A_{1a} A_1^a. \tag{25}$$

We see that the definition of ϕ differs from the seemingly more natural one only by a function of the vector field A_1. Choosing to write ϕ directly in terms of the Killing field λ^μ removes a mixing between the vector field and scalar that would otherwise obscure the physics. Note that we have related the scalar field ϕ to the logarithm of the norm of the Killing field, and that this norm is positive by assumption.

Finally, let us consider the vector field A_1. Although we have $x^a_{,\nu} g^{\nu\mu}\lambda_\mu = 0$, the vector field A_1 need not vanish. It represents the twist of the gauge field; that is, the failure of the gauge field to be hypersurface orthogonal. Note that there is a freedom to redefine the zero of θ at each value of the x^a. This amounts to the transformation $\theta \to \theta - \Lambda(x)$. Under this operation, we see that the $9+1$ metric g^{ab} is not affected, and neither is the scalar (since it depends only on the norm of the Killing field) while the vector field transforms as $A_1^a \to A_1^a + \Lambda_{,b} g^{ab}$; i.e., $A_{1a} \to A_{1a} + \Lambda_{,a}$. Thus, we see that A_1 is in fact an abelian gauge field.

It is interesting to ask about the charge to which this gauge field couples, as the field itself arose directly from the reduction of the gravitational field in eleven dimensions. Let us therefore consider the transformation generated by the electric charge, a global $U(1)$ gauge rotation. As we have just seen, in terms of the higher dimensional spacetime this is a translation along the Killing field λ^μ. It is therefore generated by a momentum. Thus we see that the charge to which the gauge field A_1 couples is nothing other than momentum around the internal S^1 when viewed from the higher dimensional perspective. A timelike energy-momentum vector in the eleven dimensional spacetime translates in the ten-dimensional context into a charge and a ten-dimensional energy-momentum tensor satisfying a BPS bound.

In performing calculations, it is often useful to express the above decomposition in terms of the eleven dimensional *covariant* metric ds^2_{11}. The reader may check that we have

$$ds^2_{11} = g_{ab} dx^a dx^b + e^{4\phi/3}[d\theta + A_{1a} dx^a]^2 \tag{26}$$

One might think it is natural to decompose, the antisymmetric 3-form into a 9+1 3-form $\hat{A}_3^{abc} = A_3^{\mu\nu\rho} x^a_{,\mu} x^b_{,\nu} x^c_{,\rho}$ and and a 2-form $A_2^{ab} = A_3^{\mu\nu\rho} x^a_{,\mu} x^b_{,\nu} \lambda_\rho$ in order that both be invariant under the gauge transformations $A_1 \to A_1 + d\Lambda_0$. However, it turns out that the 3-form $\hat{A}_3$ transforms nontrivially under the gauge transformations associated with the 2-form potential A_2. The various gauge transformations cannot be completely disentangled and in fact the standard choice is to use

$$A_3 = \frac{1}{3!} \tilde{A}_{3abc} dx^a \wedge dx^b \wedge dx^c + \frac{1}{2!} A_{2ab} dx^a \wedge dx^b \wedge d\theta. \tag{27}$$

The decomposition of the fermionic fields is similar, but we will not go into this in detail.

The gauge symmetry of the eleven dimensional A_3 implies that there are independent 9+1 gauge symmetries $\tilde{A}_3 \to \tilde{A}_3 + d\Lambda_2$ and $A_2 \to A_2 + d\Lambda_1$ where Λ_n are arbitrary n-forms. The two form is invariant under $A_1 \to d\Lambda_0$, but we have $\tilde{A}_3 \to \tilde{A}_3 + A_2 \wedge d\Lambda_0$. From here on, we drop the tilde ($\tilde{}$) on $\tilde{A}_3$. This mixing of gauge transformations leads to interesting phenomena involving non-conservation of charge in the reduced theory, but this will not arise in the simple spacetimes discussed below. As a result, we will not go into the details here.

4.3 On 9+1 Dynamics: Here comes the dilaton

The dynamics for the 9+1 theory follows from that of eleven dimensions by inserting the relations between the 9+1 fields and the 10+1 fields into the action. The result is an action principle for the 9+1 theory which takes the form

$$\begin{aligned} S_{9+1,bosonic} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \Big[\sqrt{-g} \left(e^{2\phi/3} R - \frac{1}{2} e^{2\phi} F_2^2 \right) \\ &- \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(e^{-2\phi/3} F_3^2 + e^{2\phi/3} \tilde{F}_4^2 \right) \\ &- \frac{1}{4\kappa_{10}^2} \int A_3 \wedge F_3 \wedge F_4 \Big]. \end{aligned} \tag{28}$$

where all quantities refer to the 9+1 dimensional fields, $F_n = dA_{n-1}$, and $\tilde{F}_4 = dA_3 - A_1 \wedge F_3$. As opposed to F_4 itself, the new field strength $\tilde{F}_4$ is in fact invariant under the gauge transformations of the A_1 potential. We have also defined $\kappa_{10}^2 = \kappa_{11}^2 / L$.

One important feature of (28) is that the field ϕ appears all over the place, with different factors of e^ϕ appearing in different terms. The upshot of this is that the various gauge fields do not couple minimally to the metric g, but instead the action includes derivative couplings between ϕ and the gauge fields. Now, we do in fact have the freedom to mix the metric with ϕ by rescaling the metric by some power of e^ϕ. This can be used to make any one of the gauge fields couple minimally to the new metric, or to remove the factors of e^ϕ in front of the scalar curvature term, and put the action in a form more like that of familiar Einstein-Hilbert gravity. However, because of the way that different factors of e^{ϕ} appear in the different terms, this cannot be done for all fields at once. Thus, we may think of each different gauge field as coupling to a different metric.

A short calculation shows that the gauge fields F_2 and F_4 couple minimally to $e^{2\phi/3}g$ while the gauge field F_3 couples minimally to $e^{-\phi/3}g$. In doing this calculation, it is important to realize that terms like F_2^2 contain implicit factors of the metric g which has been used to contract the indices (see appendix). On the other hand, it is for the 'Einstein metric' $e^{\phi/6}g$ that the gravitational part of the action takes the standard Einstein-Hilbert form (the integral of the scalar curvature density) without any extra factors of e^{ϕ}.

The choice of a particular metric in the class $e^{\alpha\phi}g$ is known as the choice of conformal frame. One can make a choice of frame that simplifies a given calculation, if one desires. It is interesting to note that, in the conformal frame which follows from the Kaluza-Klein reduction, the field ϕ has no explicit kinetic term so that its variation leads to a constraint. It turns out that this is just a combination of the usual constraints that one would expect in a gravitating theory, and that a term of the form $\partial_a \partial^a \phi$ does appear in the equations of motion obtained by varying the metric in that frame.

The two most useful choices of conformal frame are the Einstein frame discussed above, and the so-called string frame. The action in the Einstein frame is a handy thing to have on hand, so we will write it down here. If we now let g_E denote the metric in the Einstein frame and let R_E be the associated curvature, the action is

$$\begin{aligned} S_{bosonic} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_E}\left(R_E - \frac{1}{2}\partial_a\phi\partial^a\phi\right) \\ &- \frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g_E}\left(e^{3\phi/2}|F_2|^2 + e^{-\phi}|F^2|_3 + e^{\phi/2}|\tilde{F}_4|^2\right) \\ &- \frac{1}{4\kappa_{10}^2}\int A_2 \wedge F_4 \wedge F_4. \end{aligned} \tag{29}$$

Note that, in Einstein frame, the gauge fields are all sources for the dilaton but the metric is not. Also, since the kinetic term for the dilaton now take the standard form, we can see that the dilaton would be canonically normalized if we set $2\kappa_{10}^2$ to one. Finally, since it is in this frame that the gravitational dynamics take the familiar Einstein-Hilbert form, this is the frame in which the standard ADM formulas for energy and momentum may be applied.

The string frame is defined by taking the metric to be $e^{2\phi/3}g$, where g is the original metric from dimensional reduction. That is, the string metric g_S and the Einstein metric g_E are related by $ds_E = e^{-\phi/2}ds_S$. It should not be a surprise that the string metric is useful as two of the gauge fields (F^2 and F^4) couple minimally to this metric. These two gauge fields are known as *Ramond-Ramond* (R-R) gauge fields while F_3 is known as the *Neveu-Schwarz Neveu-Schwarz* (NS-NS) gauge field. For an explanation of how this terminology arose in string perturbation theory, see [1]. The potential A_2 for this field is commonly written B_{ab} and when string theorists discuss "the B-field," it is this potential to which they are referring.

However, what makes the string metric especially useful is that it turns out to be the metric to which fundamental strings (which we have not yet discussed) couple. For this reason, it is in the string frame that one can make the most direct contact with string perturbation theory. This, however, is a discussion for another place and time. Here, we wish only to record the metric in the string frame for the reader's future use.

$$\begin{aligned} S_{bosonic} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_S}e^{-2\phi}\left(R_S + 4\partial_a\phi\partial^a\phi - \frac{1}{2}|F_3|^2\right) \\ &- \frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g_S}\left(|F_2|^2 + |\tilde{F}_4^2|\right) \\ &- \frac{1}{4\kappa_{10}^2}\int A_2\wedge F_4\wedge F_4. \end{aligned} \tag{30}$$

After setting $c = \hbar = 1$, the parameter κ_{10}^2 has units of $(length)^8$. It is useful to write $\kappa_{10}^2 = (2\pi)^6 g_s^2 l_s^8$ where l_s is the "string length" and g_s is the "string coupling." For more on the separate role of g_2 and l_2, see section 6.

In the above, we have discussed only the compactification of eleven dimensional supergravity on a circle. One can, of course, consider further compactifications to smaller dimensional manifolds. The story in that case is much the same except that the number of (lower-dimensional) fields generated increases rapidly. In particular, further compactification generates large numbers of massless scalars that couple non-minimally to the various gauge fields. These cousins of the dilaton are generally referred to as *moduli*.

All of these moduli have a tendency to diverge at the horizon of an extreme black hole, making the solution singular. One may think of the issue as follows: the moduli, like the dilaton, couple to the gauge fields so that the squared field strengths F^2 act as sources. This can be seen from the action (29) in the Einstein frame. Non-singular extremal black hole solutions typically have an infinite throat, as in the four and five dimensional Einstein-Maxwell examples discussed earlier. This means that a smooth such solution would have an infinite volume of space near the horizon in which the gauge field strengths are approximately constant. Thus, unless these gauge fields are tuned to have $F^2 = 0$ or the various gauge fields are somehow played off against one another, there is an infinite source for the moduli. As a result, it requires some care to construct an extremal black hole solution with a smooth horizon and such solutions necessarily carry more than one charge. For a brane solution, the norm of each Killing field acts like a modulus whose sources must be properly tuned.

This is essentially the issue encountered at the end of section 3.3 in which it was found that three charges (in the case, three different types of M2-branes) were required to obtain a brane solution in which the norms of the Killing fields did not vanish on the horizon. Recall that a Killing field with positive norm is required to Kaluza-Klein reduce the spacetime to a solution of lower-dimensional supergravity. Because the three charge solution (21) has six Killing vector fields whose norms do not vanish on the horizon, it may be reduced all the way down to a solution of 4+1 gravity. In this context, it represents an extreme black *hole*. In fact, it reduces to just the standard 5+1 extremal black hole (15) of Einstein-Maxwell theory.

By the way, the theory discussed above is far from the only supergravity theory in ten dimensions. It is a particular kind called 'type IIA.' The 'II' refers to the fact that there are two independent gravitino fields. In type IIA theory, these gravitinos have opposite chirality. This is turn allows type IIA theory to be defined even on non-orientable manifolds. There is also a type IIB theory which has two gravitinos, but of the same chirality. Thus, type IIB theory can only be defined on manifolds with a global notion of chirality and, in particular, only on orientable spacetimes. We will discuss type IIB theory further in section 5.3 below. Two other supergravity theories with less supersymmetry are known as the type I and heterotic theories. Each of these types of supergravity in ten dimensions is associated with its own version of string theory. We will not discuss type I or heterotic supergravity here, but a discussion of these theories and how they are related to the type II theories can be found in [1].

5 Branes in 9+1 type II Supergravity

We now wish to discuss the basic brane solutions of type II supergravity in 9+1 dimensions. Since any solution of type IIA theory is really a solution of eleven-dimensional supergravity (which just happens to have a Killing field) in disguise, any brane solution of type IIA theory immediately defines a brane solution of eleven dimensional supergravity. Thus, we should be able to construct the basic brane solutions of type IIA theory by working with the basic brane solutions of section 3.2. For this reason we address the type IIA solutions first in section 5.1. Next follows a short aside on brane singularities in section 5.2. We then briefly discuss type IIB supergravity, and its relation through so-called T-duality with the type IIA theory, in section 5.3. When this is done, we will be in a position to provide a few short comments in section 6 on perturbative string theory, in order to give the traditional relativist a useful first intuitive picture of the subject. Below, we will discuss only BPS branes, although there has recently been significant interest in non-BPS D-branes[13] [26, 27].

5.1 The type IIA branes

It is useful here to recall our decomposition of the eleven-dimensional metric and gauge field into the various fields of ten dimensional supergravity. We proceeded by projecting the fields along various directions associated with the Killing field. In order to get brane solutions of type IIA theory that are charged under all of the type IIA gauge fields, a similar operation will need to be performed on

[13]The term 'non-BPS D-branes' refers to a type of D-brane that have no BPS version, as opposed to just the non-extremal branes obtained by adding energy to the BPS D-branes.

the brane solutions. For any given brane in eleven dimensions, we will need to reduce both a basic brane solution in which the Killing field acts along the brane (i.e., is a symmetry of the brane), and one in which it acts transverse to the basic brane.

One may at first wonder what it means for the brane to be transverse to the Killing field since translations along a Killing field must leave the solution invariant, and therefore must preserve the brane. The answer to this puzzle is the smearing mentioned in section 3.3. One can take a basic brane solution, pick a direction transverse to the brane, and smear the brane in that direction. The result is an eleven dimensional brane solution with a Killing field 'transverse to the *basic* brane.' The Killing field means that the smeared brane is then easily compactified and reduced to 9+1 dimensions. All of the basic branes of type IIA supergravity are generated by this procedure.

Performing these reductions amounts to no more than using the relations between the 9+1 fields and the 10+1 fields given in section 4 to write down the 9+1 solutions from the branes given in section 3.2. We leave the details of the calculations to the reader, but we provide a list here of the various 9+1 brane solutions. Below, we group together those branes charged under the Ramond-Ramond gauge fields and those charged under the NS-NS gauge fields. This grouping is natural from the point of view of the type IIA theory (and of string perturbation theory), though we will see that it is somewhat less natural from the eleven dimensional point of view.

Let us begin with the Ramond-Ramond branes. It turns out that type IIA theory has p-brane solutions with Ramond-Ramond charge for every even p. What is very nice is that, in terms of the string metric, all of these solutions take much the same simple form. In order to treat all of the branes at once, it is useful to introduce a uniform notation for both the electrically charged branes and the magnetically charged branes. For each gauge field A_n, we can introduce (at least locally) a magnetic dual gauge field A_{9-n} through $dA_{8-n} = \star F_{n+1}$. A brane which couples magnetically to A_n then couples electrically to A_{9-n} and vice versa. In type IIA theory, this notation should introduce no confusion as the standard gauge fields have $n = 1, 2, 3$ while these new (dual) gauge fields have $n = 5, 6, 7$.

Introducing the usual set of $p + 1$ coordinates $x_\parallel$ along the brane and $9 - p$ coordinates $x_\perp$ transverse to the brane we have, for all even p,

$$\begin{aligned} ds^2_{string} &= H_p^{-1/2} dx_\parallel^2 + H_p^{1/2} dx_\perp^2 \\ A_{p+1} &= -H_p^{-1} dx_\parallel^0 \wedge ... \wedge dx_\parallel^p \\ e^{2\phi} &= H_p^{(3-p)/2} \end{aligned} \tag{31}$$

where H_p is a function only of the $x_\perp$ coordinates and satisfies

$$\partial_\perp^2 H_p = -(7-p)\omega_{8-p} r_0^{7-p} \delta^{(9-p)}(x_\perp) \tag{32}$$

for the basic brane solution. Here ω_{8-p} is the volume of the unit $(8-p)$-sphere and r_0 is the charge radius of the brane. These are the solutions known as extreme R-R p-branes or, in a slight abuse of language, as (extreme) Dp-branes. See section 6.2 for an explanation of this terminology. As usual, we also obtain a solution by considering any source term on the RHS of eq. (32). For odd p, there are no gauge fields A_{p+1} in type IIA supergravity so (31) does not yield a solution to this theory for such cases[14]. Solutions for the non-extremal branes may be found in, e.g. [2]. As for the M-branes, their global structure is like that of the Schwarzschild solution as opposed to that of non-extremal Reissner-Nördstrom.

Although all of these R-R branes take the same simple form (31), they proceed by quite different routes from the eleven dimensional branes. A short list follows: The D0-brane solution follows by reducing the smeared M-theory wave along the smearing direction. The D2-brane follows by reducing the smeared M2-brane along the smearing direction. The D4-brane is the reduction of the unsmeared M5-brane in a direction along the brane. Finally, the D6-brane is the reduction of the unsmeared Kaluza-Klein monopole along the S^1 fibers.

Next, there are the Neveu-Schwarz branes. Since the only Neveu-Schwarz gauge field is A_2, we expect to find two types of Neveu-Schwarz branes. The gauge field A_2 should couple electrically to a 1-brane (a string) and it should couple magnetically to a 5-brane. The 1-brane follows by reducing the M2-brane in a direction along the brane. The resulting solution

$$\begin{aligned} ds^2_{string} &= -H_F^{-1} dx_\parallel^2 + dx_\perp^2 \\ A_2 &= H_F^{-1} dx_\parallel^0 \wedge dx_\parallel^1 \\ e^{2\phi} &= H_F^{-1} \end{aligned} \tag{33}$$

is known as the *fundamental string*. The reason for this is that this solution represents the classical limit of a long, straight version of the *same* string that appears in string perturbation theory[15].

The Neveu-Schwarz 5-brane (NS5-brane) is constructed by smearing the M5-brane in a transverse direction and then reducing along the smearing direction. The result is

$$\begin{aligned} ds^2_{string} &= -dx_\parallel^2 + H_5 dx_\perp^2 \\ F_3 &= -\frac{1}{3!}\partial_{x^i_\perp} H_5 \epsilon_{ijkl} dx^j_\perp \wedge dx^k_\perp \wedge dx^l_\perp \\ e^{2\phi} &= H_5. \end{aligned} \tag{34}$$

An interesting property of the NS5-brane is that, in the string metric, the timelike Killing field has no horizon; its norm is constant across the spacetime.

[14] One might also ask about the case $p = 8$, since we have not discussed a 9-form gauge potential. It turns out that there is in fact a Ramond-Ramond 8-brane in type IIA theory and that its existence is tied to the Chern-Simons term in the type IIA action. In this work, we follow a policy of considering only the asymptotically flat brane solutions, we will not discuss the 8-brane explicitly.

[15] The name 'fundamental string' is, however, a bit of a historical artifact as such strings are no longer regarded as significantly more fundamental than the other branes. They are, however, quite useful due to the existence of string perturbation theory.

The would-be horizon at $x_\perp = 0$ has receded to infinite proper distance in all directions, not just along a Killing slice as for the extreme Reissner-Nördstrom black hole. As a result, the coordinate patch above actually covers a manifold that, in the string frame, is geodesically complete[16].

Finally, there are the purely gravitational 'branes' given by the 9+1 versions of the Aichelburg-Sexl metric and of the Kaluza-Klein monopole with all gauge fields set to zero and constant dilaton. These may be either written down directly by analogy with (18) and (19) or constructed by first smearing the 10+1 solutions along some $x_\perp$ direction and reducing the result to 9+1 dimensions.

This exhausts the possible ways to make extremal 9+1 branes by reducing (and perhaps smearing once) the basic eleven-dimensional branes. Below, we provide a few words on their global structure and singularities.

5.2 On brane singularities

We have constructed the D-brane spacetimes from what, in many cases, are nice eleven-dimensional solutions which are smooth except perhaps at a singularity hidden inside a horizon. However, the construction of the reduced solutions introduces singularities. Of the 9+1 branes, only the NS 5-brane does not have a naked singularity[17]. For the D4- and D6-branes and the fundamental string, this happens because the Killing field used in the reduction has fixed points. In the case of the D4 brane, we can see from (17) that the norm of the Killing field vanishes on the horizon of the M5-brane. To some extent, this simply means that the Killing field becomes null on the horizon, but a careful analysis verifies that this Killing field does indeed have a fixed point on the horizon. Thus, when we identify points in the 10+1 spacetime under discrete Killing translations to get a spacetime with compact Killing orbits, we create a "Lorentzian conical singularity" on the horizon. As stated above, this is very much like the singularity that arises on the horizon of the $M = 0$ BTZ black hole when it is constructed by identifying AdS_3 under translations along a Killing field.

A conical singularity is perhaps not so bad from the eleven dimensional perspective but, because the norm of λ^μ vanishes at the fixed point, e^ϕ vanishes there as well. The result is that, when expressed in terms of the string metric, the D4-brane has a (null) curvature singularity on its 'horizon.' The resulting conformal diagram takes the form below. The story of the fundamental string is much the same.

[16]However, it not geodesically complete either in the Einstein frame or as viewed from the eleven-dimensional perspective. In each of these cases, there is a null singularity at the horizon.

[17]This statement refers to the metric in the string frame. In the Einstein frame there is a naked singularity on the horizon. Its story is much like that of the D2-brane discussed below.

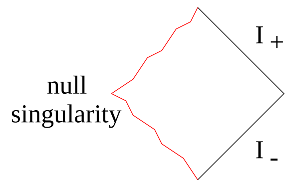

Fig 5. Conformal Diagram for the extreme D4-brane or fundamental string.

In the case of the D6-brane, the Killing field used in the reduction of the Kaluza-Klein monopole clearly has a (6+1)-plane of fixed points at $x_\perp = 0$ even for the singly charged (smooth) case. From the geometry of the eleven-dimensional solution, it is clear that this results in a 6+1 dimensional timelike singularity in the 9+1 dimensional D6-brane solution. A short calculation shows that this timelike singularity in fact resides at $x_\perp = 0$ in (31). One would be tempted to call this location the 'horizon' of the D6-brane as the norm of the Killing field (as measured in the string metric) vanishes there, and in fact this is the standard nomenclature. However, one should remember that for the D6-brane[18], this horizon is a timelike singularity. The proper conformal diagram for the D6-brane is therefore the one given in Fig. 6 below.

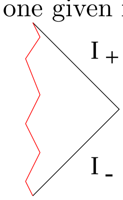

Fig. 6. Conformal diagram for the extreme D6-brane.

Let us now consider the D2-brane solution, which is the reduction of a smeared M2-brane. Although smearing the M2-brane in a transverse direction makes the horizon of the eleven dimensional solution singular, one may take the perspective that the smeared solution represents the approximate solution for an array of M2-branes for which the associated charge radius is much larger than the spacing between the branes. In this case, one interprets the D2-brane horizon as being non-singular[19] from the eleven dimensional point of view. However, from the 9+1 perspective, there is a null curvature on the horizon as in the case of the D4-brane. Unlike the D4-brane, however, the curvature scalar R of the string metric does not diverge at the horizon and one must study more carefully the null components of the curvature in order to see the singularity explicitly. The simplest way to detect the singularity from the metric (31) is to note that the spheres around the brane shrink to zero size at the horizon so that, if the solution were smooth, the horizon could have only a single null generator, which is impossible. Thus the conformal diagram of the 9+1 D2-brane solution is again given by Fig. 5.

With an eleven dimensional perspective in mind, the singularities of the D2-, D4-, and D6-brane solutions might not be considered especially troubling. Nev-

[18] And for the larger D-branes which are not asymptotically flat and which we do not discuss here.

[19] Actually, as consisting of many separate non-singular horizons.

ertheless, from the 9+1 perspective the singularities are quite real and represent places where the 9+1 equations of motion break down. Let us recall that, even in eleven dimensions, the D0-brane or M-wave solution *is* singular and should be thought of as describing the approximate field produced by some 'source.' For the M-wave, one may think of this source as being a short wavelength graviton, with the solution (18) itself representing just the Coulomb part of the field. Similarly, looking at the way that the D-brane singularities interact with the equations of motion through (32), it is natural to think of the singularities as representing bits of matter, like a braney form of extremal dust, which are coupled to the supergravity. One would then consider the explicit solutions given above in (31) to be degenerate cases in which the density of this dust of branes has been taken to be distributional instead of smooth. With a smooth source, the solutions (31) yield smooth solutions with a Killing field ∂_t that is everywhere timelike, so that there are no horizons.

It is worth mentioning that, when the dust distribution becomes too close to the delta-function of (32), the curvature becomes very high near the center of this distribution. In the region where it is larger than the string (or Planck) scale, one no longer expects the classical supergravity metric to accurately describe the physics. However, when the dust configuration is close to the delta-function case, the two supergravity solutions (smooth and singular) will differ significantly only near the source and, in particular, in the region where the curvature is beyond the string scale. Thus, from a practical point of view, there is no difference between considering the singular metrics (31) and smooth metrics given by smooth sources which are close approximations to the delta-function. The surprising thing from the perspective of familiar 3+1 Einstein-Maxwell theory is that a distributional dust configuration makes sense and that it leads to 'point-like' objects instead of to black holes with a non-zero horizon area. This difference can be traced to the existence of the dilaton field, which also diverges at the horizon.

By the way, this dual perspective of thinking of branes either as solitonic objects intrinsic to some basic version of the theory (like supergravity or string theory) or as external objects or sources coupled to such a theory is pervasive in current work on string/M-theory. It is arguable that there is no 'right' choice to make and that in fact some sort of self-consistent combination of the two perspectives is more appropriate. For the purposes of our exposition here, however, it is useful to keep the perspective that the branes are solitonic objects in eleven dimensional supergravity[20] but that, to properly represent them in ten dimensional Kaluza-Klein reduced theories it is useful to consider them to be extra bits of extremal 'dust' that we couple to the 9+1 supergravity theory.

5.3 T-duality and the type IIB theory

The other type of maximally supersymmetric gravity theory in 9+1 dimensions is called type IIB theory. It is not given by the dimensional reduction of a 10+1

[20] Except perhaps for the M-wave, as discussed above.

theory, though it has many of the same properties as the type IIA theory. For example, the subtheories of IIA and IIB supergravity obtained by setting the Ramond-Ramond fields to zero are identical. One difference, however, is that whereas type IIA supergravity has Ramond-Ramond gauge fields of odd rank, the type IIB theory has Ramond-Ramond gauge fields of even rank.

The full IIA and IIB string theories are related by a symmetry called T-duality. At the level of supergravity, this is a symmetry that maps solutions of type IIA theory with a Killing field into solutions of type IIB theory with a Killing field. It appears that T-duality is an exact symmetry of the underlying string/M-theory whether or not there is a Killing field but that, in the case where there is no Killing field, a nice, nearly classical spacetime is mapped by T-duality into a complicated highly quantum mechanical state. It is only in the presence of a Killing symmetry that this duality maps classical spacetimes to classical spacetimes. T-duality is a true duality in the sense that the transformation squares to the identity.

It is useful to first write down the explicit action of T-duality on the metric and Neveu-Schwarz fields. Let us introduce a coordinate z such that translations in z are a Killing symmetry. Let x^α be any other collection of coordinates which makes (z, x^α) a coordinate patch. Here we write the anti-symmetric Neveu-Schwarz field as B instead of A_2. If the original solution is (g, B), then the transformed solution $(\tilde{g}, \tilde{B})$ is given [28] by

$$\begin{aligned} \tilde{g}_{zz} &= 1/g_{zz}, \quad \tilde{g}_{z\alpha} = B_{z\alpha}/g_{zz}, \\ \tilde{g}_{\alpha\beta} &= g_{\alpha\beta} - (g_{z\alpha}g_{z\beta} - B_{z\alpha}B_{z\alpha})/g_{zz}, \quad \tilde{B}_{z\alpha} = g_{z\alpha}/g_{zz}, \\ \tilde{B}_{\alpha\beta} &= B_{\alpha\beta} - (g_{z\alpha}B_{\beta z} - g_{z\beta}B_{\alpha z})/g_{zz}, \quad \tilde{\phi} = \phi + \log g_{xx}. \end{aligned} \tag{35}$$

Note in particular that T-duality essentially interchanges the $g_{z\alpha}$ part of the metric with the $B_{z\alpha}$ part of the gauge field. Now, in the asymptotically flat context, the g_{z0} component of the metric is associated with momentum in the z-direction while the B_{z0} component of the gauge field is associated with electrically charged strings that extend in the z-direction. Thus, one finds [29] that T-duality interchanges momentum, and the associated Aichelburg-Sexl type solutions, with fundamental strings (which carry the electric charge to which B couples).

Strictly speaking, the T-duality of string theory requires a Killing field with compact (S^1) orbits, through (35) maps solutions to solutions in any case. The original spacetime should be asymptotically flat and, if the original z coordinate is identified such that the length of the S^1 at infinity is L, then the z coordinate of the transformed spacetime should be identified such that the length of the S^1 at infinity is $\frac{4\pi^2 l_s^2}{L}$. The point is that, if the orbits of the Killing field are compact, then quantum mechanics implies that the momentum component around the compact direction is quantized. The proper normalization guarantees that T-duality takes a solution with one quantum of momentum to a solution containing a single fundamental string.

Since the Neveu-Schwarz gauge fields are the same in both the IIA and IIB theories, the branes carrying only Neveu-Schwarz charges and the purely gravitational branes also have identical forms. On the other hand, the Ramond-Ramond fields differ somewhat. Since the IIB theory has Ramond-Ramond gauge fields of even rank, it has R-R p-branes for odd p; i.e. 1-branes, 3-branes, 5-branes, 7-branes, and 9-branes. It turns out that these solutions again take the form (31), but now with odd p. The 3-brane is notable as it has constant dilaton and a non-singular horizon. In fact, the conformal diagram for the 3-brane looks essentially like Fig. 4, the conformal diagram for the M5-brane. For this reason, the D3-brane is closely associated with 5+1 anti-De Sitter space.

The effect of T-duality on the Ramond-Ramond fields is as follows:

$$\begin{aligned}\tilde{F}_{n,\alpha_1\ldots\alpha_n} &= (const)F_{n+1,z\alpha_1\ldots\alpha_n},\\ \tilde{F}_{n,z\alpha_1\ldots\alpha_{n-1}} &= (const)F_{n-1,\alpha_1\ldots\alpha_{n-1}}.\end{aligned} \tag{36}$$

Thus, if one takes a Dp-brane and T-dualizes in some direction along the brane, one obtains a D$(p-1)$-brane solution which is smeared along the T-duality direction. In string theory, D-brane charge is quantized. The normalization constants in (36) are chosen so that, and if the original solution had one unit of Dp-brane charge, then the transformed solution has one unit of D$(p-1)$-brane charge. Similarly, if one smears a unit charge Dp-brane solution in a transverse direction keeping the total charge equal to one quantum, the T-dual solution is a unit charge D$(p+1)$-brane. See [1] for a discussion of D-brane tensions, charge quanta, etc.

6 Some Remarks on D-brane Perturbation Theory

In the preceding sections we have discussed the supergravity aspects of the various branes, including the D-branes. However, the real power of D-branes, and thus their importance, stems from the fact that there is a renormalizable (in fact, order by order finite) quantum perturbation theory to complement the classical supergravity description. This perturbation theory describes both the internal dynamics of D-branes and their interactions with the supergravity fields. While not yet a full quantum theory, it does allow one to perform certain interesting calculations. In particular, this is the key to the famous counting of states of BPS and near-BPS black holes.

Thus, although we will not discuss the details, it is worthwhile to say a few words here about this perturbation theory. Below we will attempt to give, in language appropriate to an audience of relativists, an intuitive understanding of how string perturbation theory is supposed to be viewed. We hope this gives a useful complement to standard presentations which concentrate more on the perturbation theory details.

6.1 Background field expansions and perturbative string theory

The proper framework from which to view string perturbation theory is that of the background field expansion (see e.g. [30]). Let us first review this idea in the context of standard quantum field theory. For definiteness, the reader may choose to focus on a familiar low dimensional interacting scalar field theory or even quantum mechanics. We will use ϕ to denote the scalar field or, more generally, as a schematic notation for the collection of all relevant fields.

Let us begin by supposing that there is some complete quantum theory of this field, consisting of a set of field operators $\hat{\phi}(x)$ and an associated set of composite operators acting on a Hilbert space. Exact calculations for interacting quantum field theories are seldom possible, and one must resort to various approximation schemes and expansions in small parameters in order to obtain results. For situations where the field is nearly in its vacuum state, standard perturbation theory (see e.g. [31]) can be a useful technique.

However, this is not the only case of interest. For example, it may be that a laboratory device (or a star, black hole, or astrophysical event) produces a large, essentially classical, disturbance in the field ϕ and that one wishes to study small quantum effects in the resulting behavior. It is in such a regime that background field methods are useful. One first considers the solution ϕ_0 to the classical field equations that would describe the situation if $\hbar$ were set to zero. One then rewrites the theory in terms of the field $\widehat{\delta_0\phi}(x) = \hat{\phi}(x) - \phi_0(x)$. Assuming that there is in fact a set of semi-classical states in which the expectation value of $\hat{\phi}(x)$ is close to $\phi_0(x)$ and in which the fluctuations are 'small,' it makes sense to attempt a perturbative treatment in terms of the field $\widehat{\delta_0\phi}$.

This is the basic idea behind the background field expansion. However, there is one additional subtlety. Although one expects any differences to vanish as $\hbar \to 0$, there need not be any state in which the expectation value of $\hat{\phi}(x)$ is exactly $\phi_0(x)$. In a perturbative framework, one assumes that the difference between the actual expectation value and the classical solution ϕ_0 can be expanded in powers of $\hbar$ and one proceeds to solve for it at each order of perturbation theory. It is useful to take the expectation value calculated at order n, which we write as $\overline{\phi}_n$, to be an effective 'classical field' and to work at order n with the perturbation $\widehat{\delta_n\phi} = \phi(x) - \overline{\phi}_n$.

Within the range of validity of this perturbation theory, one can (see [30]) expand about a general classical solution ϕ_0 and obtain, at order n in perturbation theory, an 'effective action' for the 'effective classical background field' $\overline{\phi}$. The variations of the effective action with respect to the effective field yield the classical equations of motion for ϕ corrected by terms of up to order n in $\hbar$ such that the solutions of these equations yield the expectation value of $\hat{\phi}(x)$ (to order n) in a semi-classical state.

One could also attempt to follow the same general framework but to expand around some arbitrary field ϕ_0 which is not a solution to the classical equations of motion. In this case, the field $\widehat{\delta_0\phi}(x)$ is not small and the perturbation theory will not contain anything like a stable vacuum. Because the variation of the action does not vanish at the chosen background, the action contains a term linear in $\widehat{\delta_0\phi}(x)$ which acts as a source. This typically leads to various infrared divergences in the perturbation theory since, when integrated over all time, this source will produce an infinite number of particles. Thus if, for some reason, someone had handed us not the full classical dynamics of the field but only the equations of the perturbation theory around an arbitrary background, the classical solutions of the theory would still be recognizable.

String perturbation theory is in fact a version of background field theory in which the 'strings' correspond to excitations of the field $\widehat{\delta_0\phi}(x)$. However, the logical order of the background field framework is reversed or, perhaps more accurately, turned inside out. Instead of starting with a classical theory, quantizing, and performing the background field expansion, one instead *postulates* the perturbative expansion[21] about any background field and then reconstructs the 'classical' dynamics of the background field in the manner discussed above from the condition that the perturbation theory is well-defined.

This seemingly odd logical structure makes more sense when one recalls that string theory is not, at present, a complete theory based on any particular set of fundamental principles or axioms. Rather, it is really an accidentally discovered set of mathematical phenomena which seem to hang together and which appear to have something to do with quantum gravity, the unification of forces, and so on. The way that string perturbation theory arose historically was through interest in QCD and possible 'strings' of gauge field flux that would connect quarks in hadrons. While studying such strings, it was discovered that they defined a perturbation theory which was finite order by order and which contained a spin two particle which could be interpreted as a graviton. Since finding a perturbative treatment of quantum gravity, or even constructing a new theory of gravity which could be treated perturbatively, had been a question of interest for some time, string theory presented a solution to this technical problem: Simply take this accidentally discovered perturbation theory and use it to construct an associated theory of quantum (and classical) gravity. In the case of string theory, the postulated perturbation theory was used to construct not only the classical dynamics of the various fields, but also to deduce the classical field content itself. The rest, as they say, is history.

Our story of supergravity discussed in the previous sections is relevant here because the dynamics of string theory reduces in a certain classical limit to classical supergravity. A few fine points are worth mentioning briefly. The first is that, when viewed as a background field theory of the sort discussed above, classical string theory actually contains not just the fields of classical supergravity, but an infinite tower of massive fields as well. The masses of these *classical* fields are, however, on the order of the string scale (and therefore considered to be large). Thus, one expects there to be a large regime in which these fields are not independently excited. Instead, the heavy (massive) fields are 'locked' to the values of the massless fields. At the extreme end of this

[21] More accurately, the S-matrix corresponding to such an expansion.

regime, the massive fields are completely irrelevant. However, as one pushes toward the boundaries of this regime, the massive fields may still have some effect on the dynamics. If one solves the classical equations of motion for the heavy fields, one finds that they are disturbed slightly by the massless fields and then in turn provide small sources for the massless fields. This analysis, known in the lingo of path integrals as 'integrating out' the massive fields, leads to additional effective interactions between the massless fields. Such interactions are non-local on a scale set by the masses of the heavy fields; i.e., on the scale of the string length. When expanded in a power series, they lead to a series of higher derivative terms in the action suppressed by powers of the string scale. These are the so-called α'-corrections. The parameter α' is equal to l_s^2 where l_s is the string length.

In this way the string scale explicitly appears in the dynamics of classical string theory. Now, it is true that in the 'real world' the string length is likely to be within a few orders of magnitude of the Planck scale. In principle, however, the two scales are completely independent and should not be confused. The string scale controls the corrections to classical supergravity caused by the tower of massive fields and the (9+1) Planck scale is the true quantum scale. Their ratio defines the string coupling g_s. The regime in which string perturbation theory is useful is $g_s \ll 1$, in which the string length is much greater than the 9+1 Planck length.

6.2 Strings and D-branes

In order to describe how D-branes fit into this picture, we should say just a few more words about the relation of strings to supergravity. As mentioned above, strings provide rules for constructing the perturbation theory about a given 9+1 supergravity background. Roughly speaking, one replaces the Feynman diagrams (related to particles) of familiar perturbation theory with a new sort of diagram related to strings. For details, the reader should consult [1] or [4]. For most of our purposes below, it will suffice to think about the strings as classical objects.

One can conceive of two basic types of strings. The first are the so-called closed strings, which at any moment of time have the topology S^1. Thus, they resemble a classical rubber band. It turns out that the closed strings define a consistent perturbation theory in and of themselves, and that it is this case that leads to the type II supergravities on which we have focused. Another version of the closed string leads to heterotic supergravity, which has half as much supersymmetry as the type II theories.

One might also consider so-called open strings which, at any instant of time, have the topology of an interval. In order for the dynamics of such strings to be well-defined, one must specify boundary conditions at the ends. A natural choice is to impose Neumann boundary conditions to describe free ends. Such strings are quite similar to classical rubber bands that have been cut open. It turns out that this type of string does not yield a consistent perturbation theory by itself, as two open strings can join together to produce a closed string. When open and closed strings are taken together, a consistent perturbation theory does result. This theory is associated with type I supergravity, having half as much supersymmetry as the type II theories.

The other type of boundary condition that one can impose at the end of a string is the Dirichlet boundary condition, requiring the end of the string to remain fixed at some point in space. One can also consider a mixture of Dirichlet and Neumann boundary conditions, insisting that the end of the string remain attached to some submanifold of spacetime, but otherwise leaving it free to roam around the surface. Surfaces associated with such Dirichlet boundary conditions are known as Dirichlet submanifolds; i.e. D-branes. Again, for a consistent perturbation theory, one must consider closed strings in addition to these open strings. Since we have singled out this submanifold as a special place in the spacetime, this perturbation theory should not describe an expansion about empty space. However, there remains the possibility that it can describe an expansion about a background in which certain sub-manifolds are picked out as special; i.e., near a background which includes certain brane-like features. Recall that, as a background field expansion, this perturbation theory should tell us about all of the dynamics of the background, including any dynamics of the branes.

To make a long story short, it turns out that the Dirichlet submanifolds are sources of the Ramond-Ramond gauge fields and of the gravitational field. That is, they carry both stress-energy and Ramond-Ramond charge. Thus, one might expect that they have something to do with the branes discussed earlier that carry Ramond-Ramond charge. In fact, this D-brane perturbation theory is supposed to give the expansion about a background that includes such a charged gravitating R-R brane in the asymptotic regime of small string coupling g_s, which controls the strength of all interactions. The perturbation theory describes both the dynamics of the bulk fields (roughly speaking, through the closed strings) and of the brane itself (roughly speaking, through the open strings). The two parts are coupled and interact.

6.3 On branes and perturbative expansions

There is more to be said about the relation between perturbative expansions and spacetime brane solutions, and this subsection contains a quick overview of the relevant features. It is an important aspect of the full story, and certainly a favorite part of D-brane lore. However, it is arguably tangential and perhaps not of central interest to a relativist. As a result, on a first reading one might wish to skip directly to section 6.4 for a few comments on the accounting of black hole microstates.

For those readers who wish to more fully appreciate the perturbative D-brane construction, we now go back a bit and clarify an issue in closed string perturbation theory. This theory is supposed to describe the expansion around some supergravity background. However, it turns out that string perturbation theory is only understood around backgrounds in which the Ramond-Ramond gauge fields vanish. Since one can see the Ramond-Ramond fields in the string perturbation theory about backgrounds in which they happen to vanish, and since their classical dynamics is in fact determined from the other fields by supersymmetry, the viewpoint is that the lack of string perturbation theory about backgrounds with nontrivial Ramond-Ramond fields is purely a technical one. In fact, in the last year, some steps toward solving this problem have been taken [32, 33, 34, 35, 36, 37]. Nevertheless, it means that one cannot use standard closed string techniques to do perturbative calculations about backgrounds containing branes with Ramond-Ramond charge. D-brane techniques thus provide the only tools to access this case.

In contrast, one can, at least in principle, use standard closed string perturbation theory to study the behavior of excitations about branes carrying only Neveu-Schwarz charge. For electric NS charge (the fundamental string) this works quite well while for magnetic NS charge (the NS 5-brane) it turns out that such techniques cannot access the most interesting physics.

There are several relevant points here. One is that the effective coupling constant in string theory is not really just the constant g_s, but is in fact $g_s e^{\phi}$; i.e., it can vary over the classical solution. Now, a perturbation theory is useful only at weak coupling, so we imagine taking g_s to be very small (with the asymptotic value of the dilaton field set to one). Perturbation theory about the background can only be useful if the dilaton is such that the effective coupling remains small over the entire spacetime. The other relevant point has to do with charge quantization in string/M- theory, as we will discuss below.

Perturbation theory about the fundamental string solution works quite well since, for small g_s, the effective coupling is small everywhere in the spacetime. In fact, the dilaton is such that the effective coupling goes to zero near the singularity. This will be true for any value of the string charge radius.

Nevertheless, it is an interesting question to ask whether the inherent strength of the fundamental string solution should change as we take g_s to zero. For the fundamental string solution discussed in section 5, the charge radius is set by the strength of the delta-function in the source ρ. As ρ represents the quantity that stands on the right-hand side of the (super) Einstein equations, the coefficient r_0^6 of the delta-function may be thought of as being of the form GT, where G is the ten-dimensional gravitational constant and T is the tension (mass per unit length) of the fundamental string.

In string theory, as in any theory with both electric and magnetic charges, the charge is quantized in integer multiples of some fundamental charge. The BPS bound relates the charge to the tension, so the tension of a BPS object is quantized as well. For the fundamental string with n units of charge, this tension is proportional to n and does not depend on the string coupling. Recall that G is proportional to g_s^2. Thus, if we consider a string with a fixed number n of charge units, the parameter r_0 that controls the departure of the supergravity fields from flat empty space vanishes in the limit of weak string coupling. As a result, weak coupling perturbation theory in the fundamental string background reduces, at least away from the singularity, to just perturbation theory about flat space. As mentioned earlier, the singularity of the fundamental string solution is supposed to represent a source due to the presence of fundamental strings.

Thus, perturbation theory about the fundamental string background is naturally associated with string perturbation theory about flat space, in a sector where the strings are arranged to have n units of electric Neveu-Schwarz charge.

Now, the story for the NS 5-brane is quite different. This time, the charge quantization specifies that, with n units of charge, the tension of the brane is proportional to n/g_s^2. Thus, $GT \sim n$ and the supergravity fields remain unchanged as we take $g_s \to 0$. Reading off the dilaton behavior from the 5-brane solution (34), we see that the effective string coupling becomes of order one at some finite place in the spacetime and then diverges as we look deep into the throat of the 5-brane. As a result, string perturbation theory will not be a useful tool to study dynamics associated with the throat of the NS 5-brane.

The Ramond-Ramond branes are a nice intermediate case between these two. It turns out that the tension of any R-R brane (with n units of charge) is proportional to n/g_s. Thus, $GT \sim ng_s$ goes to zero at weak coupling. As a result, the supergravity fields go over to flat empty space in this limit. This means that, even if at finite g_s the dilaton forces the effective coupling to diverge at the singularity of the brane, in the $g_s \to 0$ limit the effective coupling is small over the entire spacetime (except at the singularity itself). On the other hand, since the mass per unit volume of the D-brane is diverging, any internal dynamics associated with motion of the D-brane is frozen out in this limit; the actual dynamics at finite g_s may be thought of as perturbations around an infinitely massive and therefore non-dynamical object. The picture that one obtains strongly resembles the D-brane picture described above; it consists of flat empty space with a preferred submanifold in spacetime occupied by a non-dynamical brane. Thus, one might suppose that the D-branes of perturbation theory should be identified in this way with the Ramond-Ramond branes of supergravity. Additional evidence for this picture comes from the great success of D-brane perturbation theory in reproducing the entropy of black holes [38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48], hawking radiation [49, 50, 51], and the so-called grey-body factors [52] associated with the Ramond-Ramond branes.

6.4 A few words on black hole entropy

This is not the place for an in-depth discussion of just how D-brane perturbation theory can be used to reproduce the properties of supergravity solutions. Such treatments can be found in [53] and in [1]. They involve the fact that the open strings associated with D-branes describe, in the low energy limit, a certain non-abelian Yang-Mills theory. The low energy limit of that theory can then be analyzed and used to study the low energy limit of the brane dynamics. Since BPS branes have the minimal possible energy for their charge, this means that BPS and nearly BPS branes can be addressed by such techniques.

We will, however, close by giving some parts of the entropy calculation for a particular case. As has already been mentioned, the solution (21) with three mutually orthogonal sets of M2-branes is the simplest BPS black brane solution with non-zero entropy. Let us compactify a circle along one of the M2-branes (say, the one associated with the $z_\|$ coordinates) and Kaluza-Klein reduce to

9+1 type IIA supergravity. Then, as we have seen, the z-type M2-branes (which are wrapped around this circle) become fundamental strings in the IIA description while the x- and y-type M2-branes (which do not wrap around the compact circle) become D2-branes. It turns out that a simple description of the microscopic perturbative states can be obtained by T-dualizing this solution to the IIB theory along the direction in which the fundamental strings point. This turns the fundamental strings into momentum and the two sets of D2-branes into D3-branes.

Let us now T-dualize twice more in, say, the two $y_{\|}$ directions. This again yields a solution of IIB theory. The momentum remains momentum in the same direction, but one of the sets of D3-branes has become a set of D1-branes and the other has become a set of D5-branes. The D1-branes (D-strings) are stretched in the same direction that the momentum is flowing, and this all happens in one of the directions along the D5-branes. These T-dualities do not change the integer charges Q_x, Q_y, Q_z associated with the various types of branes: Q_x is now the number of D5-branes, Q_y the number of D-strings, and Q_z the number of momentum quanta. One can check that these T-dualities do not change the Bekenstein-Hawking entropy and, as supposed symmetries of the underlying string theory, they cannot change the number of microstates.

The case of a single D5-brane is particularly simple to discuss. It turns out that the low energy dynamics reduces to what is effectively just a collection of D-strings[22] which are stuck to the D5-brane but free to oscillate within it. The momentum in the solution is just the momentum carried by these oscillations, and the energy of the solution is a linear sum of contributions from the D5-brane rest energy, the D-string rest energy, and the momentum. For a supersymmetric solution, all of the oscillations must move in the same direction along the D-string and so are described by, say, right-moving fields on a 1+1 dimensional spacetime. Oscillations of D-strings propagate at the speed of light and so the associated energy-momentum vector is null.

Thus, for each string, one has 4 massless $1+1$ rightmoving scalar fields corresponding to the four internal directions of the fivebrane. Supersymmetry implies that there are also four massless $1+1$ rightmoving fermionic fields for each D-string. A fermion acts roughly like half of a boson, so we may think of this as $6Q_y$ massless right-moving scalars on $S^1 \times R$ (the worldvolume of a one-brane). A standard formula tells us that, given n massless rightmoving scalars with Q_z units of momentum, the entropy at large Q_z is $S = 2\pi\sqrt{Q_z n/6}$. Thus we have $S = 2\pi\sqrt{Q_y Q_z}$, in agreement with the Bekenstein-Hawking entropy $S = A/4G_{11}$ (see eq. (23) with $Q_x = 1$) for the associated black hole.

This gives an idea of the way in which D-brane perturbation theory provides a microscopic accounting of the entropy of this BPS black hole. The other BPS and near-BPS cases are similar in many respects. It is quite satisfying to arrive at exactly the Bekenstein-Hawking entropy formula without having to adjust any free parameters. However, one is certainly struck by the quali-

[22]Or, even better, to a single D-string wrapped Q_y times around the direction in which the momentum flows. See, e.g., [53].

tative differences between the regime in which we are used to thinking about black holes and the regime in which the string calculation is done. We usually consider black holes with large smooth horizons. In contrast, the perturbative calculation is done in the asymptotic regime of small g_s, where spacetime is flat and the horizon has degenerated to zero size. The belief is that supersymmetry guarantees the entropy of the quantum system to be independent of g_s, as it does for other non-gravitational systems[23]. In any case, there is much room for speculation and investigation in trying to match these pictures more closely and in understanding just what form these states take in the black hole regime of finite g_s.

Acknowledgements

The author wishes to thank Gary Horowitz, Rob Myers, Amanda Peet, and especially Joe Polchinski for many discussions on string theory over the years. Thanks also to both the participants of the Mazatlán winter school and the ITP workshop on classical and quantum physics of strong gravitational fields (Santa Barbara, 1999) for listening to various versions of this material and for asking numerous questions. Thanks especially to Abhay Ashtekar, Rafael Sorkin, and Robert Wald for their encouragement to write up these notes. This work was supported in part by NSF grant PHY 9407194 to the ITP (Santa Barbara), NSF grant PHY 9722362 to Syracuse University, and by funds from Syracuse University.

A Guide to Further Reading

The following is a brief guide to a small fraction of the available literature on strings, branes, and M-theory. Other useful references will undoubtedly be forthcoming as well. In particular, the 1999 TASI summer school was devoted to "Branes, Strings, and Gravity." The written versions of many of the lectures presented there should appear on hep-th in the fall of 1999 (most likely with TASI in the title) and should make useful reading. I would guess that the talks by Clifford Johnson and Amanda Peet might be most attuned to the interests and background of relativists, though most of the lectures should probably be recommended as further reading when they appear.

General: The best and most recent general reference available on string theory is Joe Polchinski's two-volume work "String Theory" [1]. This book has an intuitive and pleasantly chatty style, but is of course not aimed at an audience of relativists. It contains much discussion on spacetime aspects of string/M-theory, though generally at a rather conceptual level and details of gravitating solutions are not emphasized. The book provides a useful perspective on many

[23] As supporting evidence, recall that the Bekenstein-Hawking entropy of our BPS black hole does not depend on g_s when written in terms of the integer charges (23).

modern topics, but predates the Maldacena conjecture [54] (aka AdS/CFT). A constructive way to read this book for a relativist with a shortage of time might be to begin with volume II and to simply take for granted all statements that arise from quantum field theoretic or string perturbation arguments. The original text on the subject by Green, Schwarz, and Witten [4] is also a useful reference on many subjects.

Branes and spacetime solutions: A rather thorough, but not particularly recent, review of brane solutions in string/M-theory can be found in the 1997 manuscript [2] by Youm. It contains numerous references to the original works.

D-branes, Yang-Mills Theory, and Black Hole Entropy: As far as the mechanics of this subject is concerned, a canonical reference is Juan Maldacena's Ph.D. thesis [53], although this of course does not include the more modern perspective based on the Maldacena conjecture. For a lighter overview, Joe Polchinski's book [1] also contains the essential points.

The Maldacena Conjecture: Although we have not had a chance to explore it here, the Maldacena conjecture (aka the AdS/CFT correspondence) is an important part of current work in string/M-theory. There is now a quite thorough review of the subject [55] by Aharony, Gubser, Maldacena, Ooguri, and Oz. The address both the motivations behind the conjecture and the evidence in support of it (up to the time of writing). Of course, the subject may continue to develop rapidly, but this review will remain an excellent starting point.

B Conventions

Since there are a number of different conventions for dealing with anti-symmetric tensor (i.e., n-form) fields, it is worthwhile to spell out explicitly the ones used here. In the text, A_n denotes an n-form potential, and F_{n+1} denotes the corresponding $(n+1)$-form field strength. We have $F_{n+1} = dA_n$, though some use is made of a modified "field strength" $\tilde{F}_{n+1}$ defined in the text. The components $A_{\alpha_1\alpha_2\alpha_3\ldots}$ of A are related to the n-form A by

$$A_n = \frac{1}{p!} A_{\alpha_1\ldots\alpha_n} dx^{\alpha_1} \wedge \ldots \wedge dx^{\alpha_n}, \tag{37}$$

so that we have

$$\int A_n = \int A_{0123\ldots(n-1)} d^n x. \tag{38}$$

It is also useful to define

$$|F_m|^2 = \frac{1}{m!} g^{\alpha_1\beta_1} \ldots g^{\alpha_1\beta_1} F_{\alpha_1\ldots.\alpha_m} F_{\beta_1\ldots.\beta_m}. \tag{39}$$

The conventions used here are identical to those in Polchinski's book [1].

References

[1] J. Polchinski, *String Theory* (Cambridge U. Press, Cambridge, 1998).

[2] D. Youm, "Black Holes and solitons in string theory," *Phys. Rept.* **316** 1 (1999) hep-th/9710046.

[3] E. Cremmer, B. Julia, and J. Scherk, "Supergravity Theory in Eleven-Dimensions," *Phys. Lett.* **B76** 409 (1978).

[4] M. Green, J. Schwarz, and E. Witten, *Superstring theory*, (Cambridge U. Press, NY, 1987).

[5] P. C. Aichelburg and F. Embacher, "Exact Superpartners of N=2 supergravity solitons," *Phys. Rev. D* **34** 3006 (1986); M. J. Duff, J. T. Liu, and J. Rahmfeld, "Dipole Moments of Black Holes and String States," *Nucl. Phys.* **B494** 161 (1996) hep-th/9612015; M. J. Duff, J. T. Liu, and J. Rahmfeld, "g=1 for Dirichlet 0-branes," *Nucl. Phys.* **B524** 129 (1998) hep-th/9801072; V. Balasubramanian, D. Kastor, and J. Traschen, "The spin of the M2-brane and spin-spin interactions via probe techniques," *Phys. Rev. D* **59** 984007 (1999).

[6] Townsend, P.K. "Brane Surgery," *Nucl. Phys. Proc. Suppl.* **58** 163 (1997).

[7] G. W. Gibbons, G. T. Horowitz, and P. K. Townsend, "Higher-dimensional resolution of dilatonic black-hole singularities," *Class. Quant. Grav.* **12** 297 (1995).

[8] G. W. Gibbons and C. M. Hull, "A Bogomolny Bound for General Relativity and Solitons in N=2 Supergravity," *Phys. Lett.* **109B** 190 (1982).

[9] A. Ashtekar and M. Varadarajan, "A striking property of the Gravitational Hamiltonian," *Phys. Rev. D* **50** 4944 (1994), gr-qc/9406040.

[10] E. Witten and D. Olive, "Supersymmetry algebras that include topological charges," *Phys. Lett.* **78B** (1978) 97; S. Ferrara, C. A. Savoy, and B. Zumino, "General Massive Multiplets in Extended Supersymmetry," *Phys. Lett.* **100B** (1981) 393.

[11] E. Witten, "A simple proof of the positive energy theorem," *Comm. Math. Phys.* **80** 381 (1981).

[12] S. Majumdar, *Phys. Rev.* **72** (1947) 930; A. Papapetrou, *Proc. Roy. Irish Acad.* **A51** (1947) 191.

[13] P. C. Aichelburg, R. U. Sexl, "On the Gravitational Field of a Massless Particle," *Gen. Rel. Grav.* **2** 303 (1971).

[14] E. Bergshoeff, E. Sezgin, and P.K. Townsend, "Supermembranes and eleven-dimensional supergravity," *Phys. Lett.* **B189** 75 (1987).

[15] R. Sorkin, "Kaluza-Klein monopole," *Phys. Rev. Lett.* **51** 87 (1983).

[16] D. Gross and M. Perry, "Magnetic Monopoles in Kaluza-Klein Theories," *Nucl. Phys.* **B226** 29 (1983).

[17] A. Fayyazuddin and D.J. Smith, "Localized Intersections of M5-Branes and Four-Dimensional Superconformal Field Theories," *JHEP* **9904** 030 (1999), hep-th/9902210.

[18] A. Gomberoff, D. Kastor, D. Marolf, and J. Traschen, "Fully Localized Brane Intersections The Plot Thickens," submitted to *Phys. Rev. D*, hep-th/9905094.

[19] S. Hawking and G. F. R. Ellis, *The large-scale structure of space-time* (Cambridge University Press, 1973).

[20] E. T. Newman, L. Tamburino, and T. Unti, *J. Math. Phys.* **4** (1963) 915.

[21] M. Bañados, C. Teitelboim, and J. Zanelli, "The black hole in three-dimensional space-time," *Phys. Rev. Lett.* **69** 1849 (1992), hep-th/9204099; M. Bañados, M. Henneaux, C. Teitelboim, and J. Zanelli, "Geometry of the (2+1) black hole," *Phys. Rev. D* **48** 1506 (1993), gr-qc/9302012.

[22] K. Behrndt and M. Cvetic, "BPS Saturated Bound States of Tilted p-branes in type II string theory," *Phys. Rev. D* **56** (1997) 1188-1193, hep-th/9702205; J.C. Breckenridge, G. Michaud, and R.C. Myers, "New angles on D-branes," *Phys. Rev.* **D56** (1997) 5172-5178, hep-th/9703041; V. Balasubramanian, F. Larsen, and R.G. Leigh, "Branes at angles and black holes," *Phys. Rev.* **D57** (1998) 3509-3528, hep-th/9704143.

[23] S. Surya and D. Marolf, "Localized Branes and Black Holes," Phys.Rev. D58 (1998) 124013, hep-th/9805121.

[24] D. Marolf and A. Peet, "Brane Baldness vs. Superselection Sectors," to appear in *Phys. Rev. D*, hep-th/9903213.

[25] D. Marolf, "Some brane theoretic no-hair results (and their field theory duals)," for the proceedings of the 1999 Canadian Conference on General Relativity and Relativistic Astrophysics.

[26] O. Bergman, M. R. Gaberdiel, "A nonsupersymmetric open string theory and S-duality," *Nucl. Phys.* **B499** 193 (1997), hep-th/9701137; O. Bergman, M. R. Gaberdiel, "Stable non-BPS D-particles," *Phys. Lett.* **B441** 133 (1998), hep-th/9806155; O. Bergman, M. R. Gaberdiel, "Non-BPS states in Heterotic type IIA duality," *JHEP* **9903** 013 (1999).

[27] A. Sen, "Stable Non-BPS states in string theory," *JHEP* **9806** 007 (1998), hep-th/9803194; A. Sen, "Stable Non-BPS bound states of D-branes," *JHEP* **9808** 010 (1998), hep-th/9805019; A. Sen and B. Zweibach, "Stable Non-BPS states in F Theory," hep-th/9907164; M. R. Gaberdiel and A.

Sen, "Nonsupersymmetric D-brane configurations with Bose-Fermi Degenerate open string spectrum," hep-th/9908060.

[28] T. Buscher, "Path Integral Derivation of Quantum Duality in Nonlinear Sigma Models," *Phys. Lett.* **B201** 466 (1988); "A Symmetry of the String Background Field Equations," *Phys. Lett.* **B194** 59 (1987).

[29] J. H. Horne, G. T. Horowitz, A. R. Steif, "An Equivalence Between Momentum and Charge in String Theory," *Phys. Rev. Lett.* **68** (1992) 568-571; hep-th/9110065.

[30] B. DeWitt, *Relativity, Groups, and Topology II, Proceedings of the 1983 Les Houches Summer School*, ed. by B. DeWitt and R. Stora (Elsevier Science Publishers, Amsterdam, 1984).

[31] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980); S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, New York, 1995).

[32] A. Giveon, D.. Kutasov, and N. Seiberg, "Comments on String theory on ADS_3," *Adv. Theor. Math. Phys.* 2 (1998) 733, hep-th/9806194.

[33] J. de Boer, H. Ooguri, H. Robins, and J. Tannenhauser, "String theory on ADS_3," *JHEP* 9812 (1998) 026, hep-th/9812046.

[34] N. Berkovits, C. Vafa, and E. Witten, "Conformal field theory of ADS background with Ramond-Ramond Flux," *JHEP* **9903** 018 (1999), hep-th/9902098.

[35] D. Kutasov and N. Seiberg, "More comments on string theory on ADS_3, " *JHEP* **9904** 008 (1999).

[36] D. Berenstein and R. G. Leigh, "Superstring Perturbation theory and Ramond-Ramond backgrounds," hep-th/9904104.

[37] N. Berkovits, "Quantization of the type II superstring in a curved six-dimensional background," hep-th/9908041.

[38] A. Strominger and C. Vafa, "Microscopic Origin of the Bekenstein-Hawking Entropy," *Phys. Lett. B* **379** (1996) 99-104, hep-th/9601029.

[39] C. Callan and J. Maldacena, "D-brane approach to black hole quantum mechanics," *Nucl Phys* **B472** (1996) 591-610, hep-th/9602043.

[40] G. Horowitz and A. Strominger, "Counting states of near extremal black holes," *Phys. Rev. Lett.* **77** (1996) 2368, hep-th/9602051.

[41] J. Breckenridge, R. Myers, A. Peet and C. Vafa, "D-branes and spinning black holes," *Phys. Lett. B* **391** (1997) 93-98, hep-th/9602065.

[42] J. Maldacena and A. Strominger, "Statistical Entropy of Four-dimensional extremal black holes," *Phys. Rev. Lett.* **77** (1996) 428-429, hep-th/9603060.

[43] C. Johnson, R. Khuri, and R. Myers, "Entropy of 4-D extremal black holes," *Phys. Lett. B* **378** (1996) 78-86, hep-th/9603061. i

[44] J. Breckenridge, D. Lowe, R. Myers, A. Peet, A. Strominger and C. Vafa, "Microscopic entropy of near extremal spinning black holes," *Phys Lett. B* **381** (1996) 423-426, hep-th/9603078.

[45] I.R. Klebanov, A.A. Tseytlin, "Intersecting M-branes as four-dimensional black holes," *Nucl. Phys.* **B475** (1996) 179-192, hep-th/9604166.

[46] J. M. Maldacena, "N=2 Extremal Black Holes and Intersecting Branes," *Phys. Lett.* **B403** (1997) 20-22, hep-th/9611163.

[47] G.T. Horowitz and D. Marolf, "Counting States of Black Strings with Traveling waves," *Phys. Rev. D* **55** (1997) 835-845, hep-th/9605224.

[48] G. T. Horowitz and D. Marolf, "Counting States of Black Strings with Traveling waves II," *Phys. Rev. D* **55** (1997) 846-852, hep-th/9606113.

[49] A. Dhar, G. Mandal, and S. Wadia, "Absorption vs. Decay of Black Holes in String Theory and T symmetry," *Phys. Lett.* **B388** (1996) 51, hep-th/9605234.

[50] S.R. Das and S.D. Mathur, "Comparing Decay Rates for Black Holes and D-branes," *Nucl. Phys.* **B478** (1996) 561-576, hep-th/9606185.

[51] S.R. Das and S.D. Mathur, "Interactions Involving D-branes," *Nucl. Phys.* **B482** (1996) 153-172, hep-th/9607149.

[52] J. Maldacena and A. Strominger, "Black Hole Greybody Factors and D-Brane Spectroscopy," *Phys. Rev.* **D55** (1997) 861-870, hep-th/9609026.

[53] J. Maldacena, "Black Holes in String Theory," Ph.D. thesis, hep-th/9607235.

[54] J. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," *Adv. Theor. Math. Phys.* **2** 231 (1998), hep-th/9711200.

[55] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large N Field Theories, String Theory, and Gravity," hep-th/9905111.