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ABSTRACT

The assumption of vanishing neutrino mass leads to a group of transformations of the neutrino field which leave commutation relations and free-field Hamiltonian invariant but change the interaction into an equivalent one giving the same physical results. This concept, which is attributable to Pursey and Pauli, is here applied to μ-e decay under the assumption of local nonderivative coupling with no restrictions as to conservation of parity or lepton charge. The physically relevant invariant combinations of coupling constants are derived, and relations between them are discussed. Use is made of a recent paper by Kinoshita and Sirlin to express all experimental information on μ-e decay in terms of these invariants.

INVARIANTS IN MU DECAY*

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A. Construction of the Invariants

It has been shown recently by Pauli[1] that the assumption of vanishing neutrino mass leads to a group of linear transformations of the neutrino field operators which leave both commutation relations and free Hamiltonian invariant. The group is generated by the following two commuting subgroups,

$$\psi_\nu' = a\,\psi_\nu + b\gamma_5\, C^{-1}\bar{\psi}_\nu\,, \quad \bar{\psi}_\nu' = a^*\bar{\psi}_\nu + b^*\psi_\nu C\gamma_5 \tag{I}$$

with

$$|a|^2 + |b|^2 = 1\,, \tag{I}$$

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†On leave of absence from Istituto di Fisica dell' Universita' di Roma, Italy.

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[1a] W. Pauli, Nuovo cimento 6, 204 (1957). A similar discussion on the basis of Transformation (II) only was given independently by

[1b] D. L. Pursey (to be published). A somewhat more complete analysis of this concept in beta decay can be found in

[1c] G. Lüders, On the Pursey-Pauli Invariants in the Theory of Beta Decay UCRL-3903, Aug. 1957; Nuovo cimento (to be published). A discussion for K-meson decay can be found in R. Gatto, Invariants in K_{μ_3} and K_{e_3} Decays, UCRL-3949, September 1957; Progress of Theoretical Physics (to be published).

and

$$\psi'_\nu = e^{i\alpha\gamma_5}\,\psi_\nu\,, \qquad \bar{\psi}'_\nu = \bar{\psi}_\nu\, e^{i\alpha\gamma_5}\,, \tag{II}$$

with α real. The symbol C in Transformation (I) denotes the 4-by-4 charge-conjugation matrix. An interaction Hamiltonian containing neutrinos does not stay invariant under this transformation. It is transformed, however, into a Hamiltonian that is equivalent to the original one in the sense that it leads to the same observable effects.

In this paper the concept of equivalent Hamiltonians is studied for the μ-e decay. The interaction is assumed to be invariant under the proper Lorentz group and local, and without derivative couplings but with no restrictions as to conservation of parity or of lepton charge. The most general expression for the interaction then is[2]

$$H_{int} = \sum_i (\bar{\psi}_e\,\Gamma_i\psi_\mu)\ \{(\bar{\psi}_\nu\,\Gamma_i\,(g_i + g_i'\gamma_5)\,\psi_\nu) + (\bar{\psi}^C_\nu\,\Gamma_i\,(f_i + f_i'\gamma_5)\,\psi_\nu) + (\bar{\psi}_\nu\Gamma_i\,(h_i + h_i'\,\gamma_5)\,\psi^C_\nu) + \text{Hermitian conj.}\} \tag{2}$$

with

$$\psi^C_\nu = C^{-1}\,\bar{\psi}_\nu, \qquad \bar{\psi}^C_\nu = -\psi_\nu\,C \tag{3}$$

The sum in Eq. (2) goes over all five covariant combinations of matrices corresponding to scalar (S), vector (V), tensor (T), axial vector (A), and pseudoscalar (P) coupling. It is well known that other orderings of the field operators in Eq. (2) do not lead to expressions that are not already contained in this equation.[3]

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[2]For the notation we follow a paper by T. Kinoshita and A. Sirlin (Phys. Rev., to be published). We also use their definition of the Γ matrices.

[3]L. Michel, Proc. Phys. Soc. A 63, 514 (1950);
M. Fierz, Z. Physik 104, 553 (1937).

The behavior of the Dirac matrices Γ under charge conjugation and their commutativity or anticommutativity with γ_5 determine whether a particular type of coupling characterized by the subscript i really needs six coupling constants, as in the general expression (2), or less. One finds that only the following coupling constants are multiplied by nonvanishing expressions:

$$g_T,\ g_T';\ g_V,\ g_V';\ f_V';\ h_V';\ g_A,\ g_A';\ f_A;\ h_A;\ \text{all constants for S and P.} \quad (4)$$

Since equivalent Hamiltonians lead to the same observable effects, all physical results can be characterized by combinations of the coupling constants that are invariant under the group of transformations of the neutrino field. In the Appendix the transformation properties of the coupling constants are discussed in detail; there it is also explained how the most general invariant combinations of them can be constructed. For practical purposes, however, only those invariants are needed which can occur in a first-order perturbation treatment, i.e., which are bilinear expressions formed by products of coupling constants and complex conjugate coupling constants. Before such invariants are given we want to discuss which simplifications arise from the fact that, for all practical purposes, the mass of the electron, can be neglected in calculations of the μ-e decay. Putting the mass of the electron formally equal to zero one sees that then also the electron field admits a group of transformations of the type (II):

$$\psi_e' = e^{i\epsilon\gamma_5}\psi_e, \qquad \bar{\psi}_e' = \bar{\psi}_e e^{i\epsilon\gamma_5} \quad (\epsilon \text{ real}). \quad \text{(III)}$$

Because, for the electron the particle and antiparticle are distinguished by their charges, there is, however, no analogue to Transformation (I). Under Transformation (III), electron states with spin parallel or opposite to the direction of motion are only multiplied by a phase factor. Results of experiments in which only the intensity of the decay electrons and their longitudinal polarization is observed can therefore be expressed in terms of combinations of coupling constants that are invariant under this wider group (Class A in the following list). Observations of the transversal polarization of an electron beam do not admit the group of transformations of the electron field; they have to be expressed in terms of the Class B invariants. The situation is not quite so simple if the electron mass is not neglected.[4]

The list of the invariants is as follows;

Class A

$$K^{(2)} = |g_T|^2 + |g_T'|^2 , \quad L^{(2)} = 2\,\mathcal{R}\,(g_T g_T'^{*})$$

$$K^{(1)} = |g_V|^2 + |g_A'|^2 + 2\,(|f_V'|^2 + |f_A|^2 + |h_V'|^2 + |h_A|^2)$$

$$K_2^{(1)} = |g_V'|^2 + |g_A|^2 \tag{5}$$

$$L_1^{(1)} = 2\,\mathcal{R}\,[g_V g_A'^{*} + 2\,(f_V' f_A^{*} + h_V' h_A^{*})]$$

$$L_2^{(1)} = 2\,\mathcal{R}\,(g_V' g_A^{*})$$

$$K_1^{(0)} = |g_S|^2 + |g_S'|^2 + |g_P|^2 + |g_P'|^2 + 2(|f_S|^2 + |f_S'|^2 + |f_P|^2 + |f_P'|^2 + $$
$$+ |h_S|^2 + |h_S'|^2 + |h_P|^2 + |h_P'|^2)$$

$$K_2^{(0)} = 2\,\mathcal{R}\,[g_S g_P^{*} + g_S' g_P'^{*} + 2(f_S f_P^{*} + f_S' f_P'^{*} + h_S h_P^{*} + h_S' h_P'^{*})]$$

$$L_1^{(0)} = 2\,\mathcal{R}\,[g_S g_S'^{*} + g_P g_P'^{*} + 2\,(f_S f_S'^{*} + f_P f_P'^{*} + h_S h_S'^{*} + h_P h_P'^{*})]$$

$$L_2^{(0)} = 2\,\mathcal{R}\,[g_S g_P'^{*} + g_S' g_P^{*} + 2\,(f_S f_P'^{*} + f_S' f_P^{*} + h_S h_P'^{*} + h_S' h_P^{*})] ,$$

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[4] Even for nonvanishing rest mass the electron field admits multiplication by γ_5 connected with the formal substitution $m_e \rightarrow -m_e$. The invariants of Class A stay invariant under such a transformation, whereas those of Class B take up a minus sign. Therefore in expressions for intensity and longitudinal polarization, terms with an even power of the electron mass appear multiplied by invariants of Class A, and those with an odd power of the electron mass appear multiplied by invariants of Class B. Similarly, in expressions for transversal polarization, terms with an even (odd) power of the electron mass appear multiplied by invariants of Class B (A). Compare the similar considerations for beta decay with neutrinos of nonvanishing mass in Lüders, 1c footnote 5.

Class B

$$M^{(2)} = |g_T|^2 - |g'_T|^2 \qquad N^{(2)} = 2\,\mathfrak{J}\,(g_T g'^*_T)\,;$$

$$M_1^{(1)} = |g_V|^2 - |g'_A|^2 + 2(|f'_V|^2 - |f_A|^2 + |h'_V|^2 - |h_A|^2)$$

$$M_2^{(1)} = |g'_V|^2 - |g_A|^2 \tag{6}$$

$$N_1^{(1)} = 2\,\mathfrak{J}\,[g_V g'^*_A + 2\,(f'_V f_A^* + h'_V h_A^*)\,]$$

$$N_2^{(1)} = 2\,\mathfrak{J}\,(g'_V g_A^*)$$

$$M_1^{(0)} = |g_S|^2 + |g'_S|^2 - |g_P|^2 - |g'_P|^2 + 2(|f_S|^2 + |f'_S|^2 - |f_P|^2 - |f'_P|^2 + |h_S|^2 + |h'_S|^2 - |h_P|^2 - |h'_P|^2)$$

$$N_1^{(0)} = 2\,\mathfrak{J}\,[g_S g_P^* + g'_S g'^*_P + 2\,(f_S f_P^* + f'_S f'^*_P + h'_S h'^*_P + h'_S h'^*_P)]$$

$$N_2^{(0)} = 2\,\mathfrak{R}\,[g_S g'^*_S - g_P g'^*_P + 2\,(f_S f'^*_S - f_P f'^*_P + h_S h'^*_S = h_P h'^*_P\,)\,]$$

$$N_3^{(0)} = 2\,\mathfrak{J}\,[g_S g'^*_P + g'_S g_P^* + 2\,(f_S f'^*_P + f'_S f_P^* + h_S h'^*_P + h'_S h_P^*\,)\,]$$

One notices that the invariants fall into three distinct groups (SP, AV, T) so that there are no interference terms between the three groups.[5]

The particular invariant combination of S and P or of A and V in Class A comes about by the action of Transformation (III) of the electron field, which just mixes the coupling constants in these pairs of types of interactions among themselves. The list contains 20 invariants, but not all of them appear independently in lowest-order expressions for experimental

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[5]Pursey[1c] gives only two such groups (STP and AV); there is, however, no interference between T on the one hand and SP on the other. This follows for Pursey's interaction Hamiltonian ($f_i = f'_i = h_i = h'_i = 0$) from the observation that both Transformation (II) and charge conjugation of the neutrino field transform the Hamiltonian into an equivalent one.

distributions. Rather

$$K_2^{(0)},\ L_1^{(0)},\ M^{(2)},\ N^{(2)},\ N_1^{(0)},\ N_2^{(0)}$$

do not appear at all, whereas the VA invariants occur only in the combinations

$$K_1^{(1)} + K_2^{(1)},\quad L_1^{(1)} + L_2^{(1)},\quad M_1^{(1)} + M_2^{(1)},\quad N_1^{(1)} + N_2^{(1)} \ .$$

The reason for this very limited experimental information as compared with beta decay is twofold; there is only one charged--and therefore observable--decay product, and we observe only the decay of the free μ meson (in contrast to the beta decay of nucleons bound in nuclei).

Radiative corrections[6] have to be expressable in terms of the same invariants, since Transformations (I) and (II) of the neutrino field do not affect the interaction between the charged particles involved in the decay, and the electromagnetic field. Furthermore, even radiative corrections in experiments in which only the intensity of longitudinal polarization of the electrons is observed can depend only upon the invariants of Class A, since the electron transformation (III) does not affect the electron current,

$$j_\mu = e\,\overline{\psi}_e \gamma_\mu \psi_e' = e\overline{\psi}_e e^{i\epsilon\gamma_5}\gamma_\mu e^{i\epsilon\gamma_5}\psi\ , \qquad (7)$$

whose interaction with the Maxwell field gives rise to these corrections.

The invariants (5) and (6), in contrast to the invariants given for beta decay,[1] do not form a complete set. Therefore their equality for two different interactions is a necessary but not a sufficient condition for the equivalence of these two interactions in the sense that they can be transformed into each other by a combination of Transformations (I) and (II) of the neutrino field.

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[6]Such corrections were calculated for a particular case (two-component theory in the Salam-Lee-Yang-Landau form) by T. Kinoshita and A. Sirlin (Phys. Rev. 107, 593 (1957)).

B. Discussion of the Invariants

(a) The invariants (5) and (6) are not all independent, but are restricted by a number of relations between them. First of all, all K invariants with the exception of $K_2^{(0)}$ are nonnegative; they vanish only if all coupling constants occurring in them vanish separately. Further, one has the following identities and inequalities between the invariants:

$$\begin{aligned}
&K^{(2)2} = L^{(2)2} + M^{(2)2} + N^{(2)2} , \\
&-K_1^{(1)} \leq (L_1^{(1)}, M_1^{(1)}, N_1^{(1)}) \leq K_1^{(1)}, \\
&K_2^{(1)2} = L_2^{(1)2} + M_2^{(1)2} + N_2^{(1)2} , \\
&-K_1^{(0)} \leq (K_2^{(0)}, L_1^{(0)}, L_2^{(0)}, M_1^{(0)}, N_1^{(0)}, N_2^{(0)}, \\
&\qquad N_3^{(0)}) \leq + K_1^{(0)} .
\end{aligned} \tag{8}$$

(b) The necessary condition for conservation of parity is: all L and all N, with the exception of $N_1^{(0)}$ vanish. The necessary condition for invariance with respect to charge conjugation is: all L and $N_1^{(0)}$ as well as $N_2^{(0)}$ vanish. The necessary condition for invariance under time reversal is: all N with the exception of $N_2^{(0)}$ vanish. These conditions are not sufficient, because the list of invariants is not complete; there may be practically unobservable higher-order effects that violate one or several of these invariance properties, and which depend on other invariant combinations of the coupling constants. The conditions are, however, sufficient in the weaker sense that if they are satisfied there are no first-order violation effects.

If one goes over from μ^+ decay to μ^- decay, all observable quantities that are multiplied by invariants that remain invariant under charge conjugation stay unchanged, whereas those which are multiplied by invariants that would have to vanish if charge conjugation were satisfied take up a minus sign.

(c) Necessary conditions to be satisfied by an interaction which conserves lepton charge,[7] in the sense that μ meson and electron with the same electric charge also carry the same lepton charge, are given by:

[7] The concept of conservation of lepton charge appears to have been put forward first by E. J. Konopinski and H. M. Mahmoud, Phys. Rev. 92, 1045 (1953).

No additional condition for

$$K^{(2)},\ L^{(2)},\ M^{(2)},\ N^{(2)} \text{ and } K_2^{(1)},\ L_2^{(1)},\ M_2^{(1)},\ N_2^{(1)};$$

$$K^{(1)^2} = L_1^{(1)^2} + M_1^{(1)^2} + N_1^{(1)^2} \quad ; \tag{9}$$

S, P identities.

Here "S, P identities" refers to complicated relations between the invariants characteristics for scalar and pseudoscalar coupling which are of the type of 5-by-5 determinants (constructed from the invariants) set equal to zero. The same identities, which we do not give explicitly, will appear at a few other places. These relations are derived by observing that there has to exist an equivalent Hamiltonian with $f_i = f_i' = h_i = h_i' = 0$. Because of the incompleteness of the list of invariants the conditions are again only necessary but not sufficient.

(d) Conservation of leptons, when μ meson and electron with equal electric charge have opposite lepton charge, leads to the necessary conditions:

$$K^{(2)} = L^{(2)} = M^{(2)} = N^{(2)} = K_2^{(1)} = L_2^{(1)} = M_2^{(1)} = N_2^{(1)} = 0,$$

$$K_1^{(1)^2} = L_1^{(1)^2} + M_1^{(1)^2} + N_1^{(1)^2}, \tag{10}$$

S, P identities.

(e) A general two-component theory with no restrictions as to conservation of lepton charge[8] leads to the following conditions on the invariants:

$$K^{(2)} = L^{(2)} = M^{(2)} = N^{(2)} = 0,$$

$$K_1^{(1)} = K_2^{(1)},\quad L_1^{(1)} = L_2^{(1)},\quad M_1^{(1)} = M_2^{(1)},\quad N_1^{(1)} = N_2^{(1)},$$

S, P identities.

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[8]This possibility has been considered for μ - e decay by M. H. Friedman, Phys. Rev. 106, 387 (1957). It was used in the discussion of beta decay by Pauli.[1c]

(f) A two-component coupling, with conservation of lepton charge (i.e., a two-component theory in the conventional sense[9]), leads to the following necessary conditions:

(i) equal electric charge of μ meson and electron implies equal lepton charge:

all invariants with superscript (2) and (0) vanish,

$$K_1^{(1)} = K_2^{(1)}, \quad L_1^{(1)} = L_2^{(1)}, \quad M_1^{(1)} = M_2^{(1)}, \quad N_1^{(1)} = N_2^{(1)}; \tag{12}$$

(ii) equal electric charge of μ meson and electron implies opposite lepton charge:[10]

all invariants with superscript (2) and (1) vanish;

$$K_1^{(0)} = \pm L_1^{(0)}, \quad K_2^{(0)} = \pm L_2^{(0)}, \quad M_1^{(0)} = \pm N_2^{(0)}, \quad N_1^{(0)} = \pm N_3^{(0)};$$

$$K_1^{(0)^2} = K_2^{(0)^2} + M_1^{(0)^2} + N_1^{(0)^2}. \tag{13}$$

The sign (+ or -) in the second line has to be the same in all four cases.

A complete discussion of all observable effects in μ-e decay (without radiative corrections) has been given recently by Kinoshita and Sirlin.[11] They use an interaction in which they assume

$$f_i = f'_i = h_i = h'_i = 0, \tag{14}$$

i.e., they assume conservation of lepton charge and electric charge equal to lepton charge for μ meson and electron. The concept of invariants permits us to generalize their results immediately to the interaction Hamiltonian Eq. (2): the coupling constants f_i, f'_i and h_i, h'_i can enter into expressions for observable results only in such a way that the invariants (5) and (6) are obtained. Kinoshita and Sirlin express all observable quantities in terms

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[9] A. Salam, Nuovo cimento 5, 299 (1957);
T. D. Lee and C.N. Yang, Phys. Rev. 105, (1957);
L. Landau, Nuclear Phys. 3, 127 (1957).

[10] This case is definitely excluded experimentally by the observed spectrum.

[11] T. Kinoshita and A. Sirlin, Phys. Rev. (to be published).

of ten combinations of coupling constants. Their results hold also for the more general interactions (2) if one puts

$$\begin{aligned} a &= K_1^{(0)}, \quad b = K_1^{(1)} + K_2^{(1)}, \quad c = K^{(2)}, \\ a' &= L_2^{(0)}, \quad b' = L_1^{(1)} + L_2^{(1)}, \quad c' = L^{(2)}, \\ \alpha &= M_1^{(1)} \qquad \beta = M_1^{(1)} + M_2^{(1)}, \\ \alpha' &= -N_3^{(0)} \qquad \beta' = N_1^{(1)} + N_2^{(1)}. \end{aligned} \tag{15}$$

One recognizes the result stated in Section A that some invariants do not occur at all and others in particular combinations.

Especially for the so-called ρ value,[12] they find

$$\rho = \frac{3b + 6c}{a + 4b + 6c} . \tag{16}$$

Since a measurement of the spectrum does not show violation effects and since one has, neglecting the mass of the electron, invariance with respect to Transformation (III) of the electron field, ρ can depend only upon K invariants, as is indeed the case. The inequality for ρ,

$$0 \leq \rho \leq 1, \tag{17}$$

is always fulfilled. The same inequality holds if conservation of lepton charge (μ mesons and electrons of equal electric charge have equal lepton charge) is postulated.[12] For conservation of lepton charge with μ meson and electrons of equal electric charge having opposite lepton charge, one finds[12]

$$0 \leq \rho \leq \frac{3}{4} . \tag{18}$$

[12] L. Michel, Proc. Phys. Soc. A63, 514 (1950);
C. Bouichat and L. Michel, Phys. Rev. 106, 170 (1957).

This is also true in a general two-component theory with no restriction as to the conservation of lepton charge.[13, 14]

In two-component theory with conservation of lepton charge, with μ meson and electron of equal electric charge having equal lepton charge, one has

$$\rho = \frac{3}{4} \tag{19}$$

and with μ mesons and electrons of equal electric charge having opposite lepton charge.[9]

$$\rho = 0. \tag{20}$$

All these relations are found if one applies the restrictions on the K invariants in the particular cases as summarized in Section B.

Kinoshita and Sirlin [1c] write the general electron spectrum for the decay of (negatively charged) μ mesons in the form

$$\begin{aligned} dN = C_0 + C_1 (\vec{P}_e \cdot \vec{\sigma}_e)(\vec{P}_e \cdot \vec{\sigma}_\mu) + C_2 (\vec{P}_e \times \vec{\sigma}_e)(\vec{P}_e \times \vec{\sigma}_\mu) + \\ + C_3 \vec{P}_e (\vec{\sigma}_e \times \vec{\sigma}_\mu) + C_4 (\vec{P}_e \cdot \vec{\sigma}_e) + C_5 (\vec{P}_e \cdot \vec{\sigma}_\mu). \end{aligned} \tag{21}$$

The relevance of the various terms for violation effects is seen if it is recognized that under reflections in space, one has[15]

$$\vec{P} \to -\vec{P}, \quad \vec{\sigma} \to \vec{\sigma}, \tag{22}$$

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[13] M. H. Friedman, Phys. Rev. 106, 387 (1957).

[14] In this paper we assume local interaction (cf. Eq. (2)). Possible nonlocal effects are, however, not unlikely to show up in μ-e decay. Lee and Yang (Phys. Rev., to be published) show that the assumption of a reasonable nonlocality permits one to reconcile the observed electron spectrum with the two-component theory, without violation of the conservation of lepton charge.

[15] This rule appears to be more simple than other rules in which a distinction between vectors and pseudovectors as well as between scalars and pseudoscalars is to be made.

and under time reversal, T, one has[16]

$$\vec{P} \to -\vec{P}, \qquad \vec{\sigma} \to -\vec{\sigma}. \qquad (23)$$

If validity of the TCP theorem is assumed, then violation of charge conjugation C, can be recognized from terms which change sign under the transformation

$$\vec{P} \to \vec{P}, \qquad \vec{\sigma} \to -\vec{\sigma}. \qquad (24)$$

From Eqs. (21) through (23) one sees immediately which terms are characteristic for the various violation effects (Table I). This table, apart from the last column, can also be used for other decay processes if the

Table I

Behaviour of the various terms of Eq. (21) under T, C, P.

Number of momenta	Number of spins	Violation	Coefficients
even	even	None	C_0, C_1, C_2
odd	any	Parity	C_3, C_4, C_5
any	odd	charge conjugation	C_4, C_5
even	odd[a]	time reversal	C_3
odd	even		

[a]A more concise statement would be: number of factors odd.

[16]The operation of time reversal maps a process into another one which develops with time in the opposite sense; therefore a decay situation is mapped into a built-up situation. The usual tests of time reversal (cf. the above transformation formulae) are possible only if this second process can again be related to the original one, i. e., if the transition matrix is not only unitary but also Hermitian. For this to be true the process need not be strictly of first order. It is, however, important that the contribution from intermediate states that fulfill energy-momentum conservation, and therefore could be real states, be negligible. In μ-e decay there are no such intermediate states, since they would also appear as another (and more frequent) decay channel.

assumption on test of time reversal[16] is satisfied and if the TCP theorem is assumed to hold. Coefficients characteristic for violation of charge conjugation (i.e., C_4 and C_5) change sign if one goes over to the decay of μ mesons of the opposite charge; the others stay unchanged. Kinoshita and Sirlin make the interesting remark that this behavior of signs could be used for a model-indpendent test of the TCP theorem.

From the table and the discussion in Section B(ii) it is clear which invariants can occur in the various coefficients. If the mass of the electron is neglected, C_0, C_1, and C_4 (describing experiments in which intensity and (or) longitudinal polarization of the electrons is measured) must be expressible in terms of invariants of Class A only; it must be possible to express the other coefficients in terms of invariants of Class B. From the explicit expressions for the coefficients as given by Kinoshita and Sirlin it can be seen that the following invariants or combinations of them appear in the above spectrum:

$K_1^{(0)}$, $K_1^{(1)} + K_2^{(1)}$, and $K^{(2)}$ in C_0 and C_1;

$M^{(0)}$ and $M_1^{(1)} + M_2^{(1)}$ in C_2; $N_3^{(0)}$ and $N_1^{(1)} + N_2^{(1)}$ in C_3;

$L_2^{(0)}$, $L_1^{(1)} + L_2^{(1)}$, and $L^{(2)}$ in C_4 and C_5.

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The authors wish to thank Professor Pauli for his interest in the work and for stimulating correspondence. They also want to acknowledge discussions with Dr. Tsuneyuki Kotani on various aspects of the theory of μ-e decay.

Appendix

Transformation Properties of Coupling Constants

The group generated by Transformations (I) and (II) is isomorphic with the unitary group in two dimensions. (Gerhart Lüders,[1c] Sec. 3). The interaction Hamiltonian of beta decay contains only one neutrino operator; the coupling constants therefore transform like vectors in the two-dimensional space. The interaction Hamiltonian of μ-e decay contains in each term two neutrino operators. It is therefore to be expected that the coupling constants will transform like tensors of second rank.[17] For the systematic construction of irreducible representations of the group occurring as transformations of these coupling constants the application of tensor calculus is useful.

The basic transformation is[18]

$$(T_{\mu\nu}) = e^{-i\alpha}\begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix}. \qquad (A.1)$$

We further introduce the complex conjugate transformation matrix

$$(T_{\dot{\mu}\dot{\nu}}) = e^{i\alpha}\begin{pmatrix} a^* & -b \\ b^* & a \end{pmatrix}. \qquad (A.2)$$

Then a two-dimensional vector A_μ with undotted index is defined by the transformation law

$$A'_\nu = \Sigma_\mu A_\mu T_{\mu\nu} , \qquad (A.3)$$

and a vector $B_{\dot{\mu}}$ with dotted index by[19]

$$B'_{\dot{\nu}} = \Sigma_{\dot{\mu}} B_{\dot{\mu}} T_{\dot{\mu}\dot{\nu}} . \qquad (A.4)$$

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[17] In this connection a remark in a letter by Professor Pauli was of great value to us.

[18] Pauli,[1c] Eq. (10)

[19] Examples of such vectors are given by Pauli's coupling constants F and G. One has $A_1 = F_1$, $A_2 = F_2$, $B_{\dot{1}} = G_2$, $B_{\dot{2}} = -G_1$. In the sense of this terminology Lüders used only vectors with undotted indices ($H_1 = (B_{\dot{1}})^*$, $H_2 = (B_{\dot{2}})^*$).

A general tensor with undotted and dotted indices transforms under (A. 3) with respect to the undotted indices and under Transformation (A. 4) with respect to the dotted indices. The complex conjugate of such a tensor transforms like one for which all undotted indices are replaced by dotted ones and vice versa. Therefore one may define

$$(A_{\mu\nu \ldots \dot{\rho}\dot{\sigma}\ldots})^* = A_{\dot{\mu}\dot{\nu}\ldots\rho\sigma\ldots} \qquad \text{(A. 5)}$$

Because of the unitarity of Transformation (A. 1), contraction of tensor indices is possible by summing over one dotted and one undotted index.

A tensor of second rank may contain two indices, either of the same kind (both undotted or both dotted) or of different kind (one undotted and one dotted). Tensors with indices of the same kind can be either symmetrical or antisymmetrical in these indices, a property that is invariant under tensor transformations. Therefore one gets the following two types of irreducible tensors.

(a) Antisymmetrical tensors,

$$A_{[\mu\nu]} = -A_{[\nu,\mu]} \quad \text{and} \quad B_{[\dot{\mu},\dot{\nu}]} = -B_{[\dot{\nu},\dot{\mu}]} . \qquad \text{(A. 6)}$$

These tensors contain only one nonvanishing component $A_{[1,2]}$ or $B_{[\dot{1},\dot{2}]}$; therefore one obtains representations of first degree.[20]

(b) Symmetrical tensors,

$$C_{(\mu,\nu)} = C_{(\nu,\mu)} \quad \text{and} \quad D_{(\dot{\mu},\dot{\nu})} = D_{(\dot{\nu},\dot{\mu})} . \qquad \text{(A. 7)}$$

These representation are of third degree. On tensors with indices of different kind the operation of contraction can be applied. Therefore one has two types of irreducible representations.

(c) The trace of a tensor,[21]

$$E = \sum_{\mu} E_{\mu,\dot{\mu}} . \qquad \text{(A. 8)}$$

[20] Pauli's relative invariants (Ref. 1c, Eq. (18) - (18b)) transform according to these representations.

[21] The symbol $\sum_{\mu}$ means here and in some of the following equations that the sum of the two terms with $\mu = 1$, $\dot{\mu} = \dot{1}$ and $\mu = 2$, $\dot{\mu} = \dot{2}$ has to be taken.

This gives a representation which actually is the identical representation: the quantity E is an invariant.

(d) Tensor with vanishing trace,

$$F_{\mu\dot{\nu}} \quad \text{with} \quad \sum_{\mu} F_{\mu\dot{\mu}} = 0. \tag{A. 9}$$

This representation is again of third degree. Tensors of Classes (c) and (d) are invariant under Transformation (II), whereas tensors of Classes (a) and (b) take up factors exp(-2iα) (for undotted indices) or exp(2iα) (for dotted indices). The irreducible representations are characterized by the traces of the transformation matrices (characters)

$$\begin{aligned}
&\text{(a)} \quad e^{-2i\alpha} \quad \text{and} \quad e^{2i\alpha}, \\
&\text{(b)} \quad e^{-2i\alpha}\left[(a + a^*)^2 - 1\right] \quad \text{and} \quad e^{2i\alpha}\left[(a + a^*) - 1\right], \\
&\text{(c)} \quad 1, \\
&\text{(d)} \quad (a + a^*)^2 - 1,
\end{aligned} \tag{A. 10}$$

where the two alternatives for Classes (a) and (b) refer to pairs of undotted or of dotted indices.

The construction of invariants is now quite simple. One forms products of tensors with equal numbers of undotted and of dotted indices and contracts over pairs of such indices. In beta decay all invariants were bilinear in the basic quantities (vectors or coupling constants). The situation is different here. There are linear invariants, i.e., Class (c) of the tensors; they appear, however, not to be interesting from a physical point of view. Then there are bilinear invariants, i.e., apart from products of quantities of Class (c), the combinations

$$\sum_{\mu\nu} A_{[\mu\nu]} B_{[\dot{\mu}\dot{\nu}]}, \quad \sum_{\mu\nu} C_{(\mu\nu)} D_{(\dot{\mu}\dot{\nu})}, \quad \sum_{\mu\nu} F_{\mu\dot{\nu}} G_{\dot{\mu}\nu}. \tag{A. 11}$$

For physical applications, only those invariants in this group are of interest in which one tensor consist of coupling constants and the other of complex conjugate coupling constants. There are also invariants, not expressible by the ones already given, which consist of more than two tensors, e.g.,

$$\sum_{\mu\nu\lambda} A_{[\mu\nu]} C_{(\dot{\mu}\dot{\lambda})} F_{\dot{\nu}\lambda}. \tag{A. 12}$$

But such invariants do not appear in description of experiments that are now possible, because all such experiments can be described by first-order perturbation calculation.

All irreducible tensors of second rank given above do really occur in μ-e decay. By performing Transformations (I) and (II) explicitly one finds

$$A_{[12]} = g_T - g_T' ,$$

$$B_{[\dot{1}\dot{2}]} = g_T + g_T' ,$$

$$C^S_{(12)} = g_S - g_S', \quad C^S_{(11)} = 2\,(f_S - f_S'), \quad C^S_{(22)} = 2\,(h_S - h_S'),$$

$$D^S_{(\dot{1}\dot{2})} = g_S + g_S', \quad D^S_{(1\dot{1})} = 2\,(f_S + f_S'), \quad D^S_{(\dot{2}\dot{2})} = 2\,(h_S + h_S'),$$

$$C^P_{(12)} = g_P - g_P', \quad C^P_{(11)} = 2\,(f_P - f_P'), \quad C^P_{(22)} = 2\,(h_P - h_P'), \qquad (A.\ 13)$$

$$D^P_{(\dot{1}\dot{2})} = g_P + g_P', \quad D^P_{(1\dot{1})} = 2\,(f_P + f_P'), \quad D^P_{(\dot{2}\dot{2})} = 2\,(h_P + h_P'),$$

$$E^V = g_V', \qquad E^A = g_A$$

$$F^V_{(1\dot{1})} = g_V, \quad F^V_{2\dot{1}} = 2\,f_V', \quad F^V_{1\dot{2}} = 2\,h_V',$$

$$F^A_{1\dot{1}} = g_A', \quad F^A_{2\dot{1}} = 2\,f_A, \quad F^A_{1\dot{2}} = 2\,h_A .$$

When only intensity or longitudinal polarization of the outgoing electrons is observed and the electron mass is neglected also, the electron group (III) is admissible. The following combinations of coupling constants,

$$A_{[\mu\nu]}, \quad C^S_{(\mu\nu)} - C^P_{(\mu\nu)}, \quad D^S_{(\mu\nu)} + D^P_{(\mu\nu)}, \quad E^V - E^A, \quad F^V_{\mu\dot{\nu}} - F^A_{\mu\dot{\nu}}, \qquad (A.\ 14)$$

take up an exp $(+i\epsilon)$ under this transformation, and

$$B_{[\dot{\mu}\dot{\nu}]}, \quad C^S_{(\mu\nu)} + C^P_{(\mu\nu)}, \quad D^S_{(\mu\nu)} - D^P_{(\mu\nu)}, \quad E^V + E^A, \quad F^V_{\mu\dot{\nu}} + F^A_{\mu\dot{\nu}} \qquad (A.\ 15)$$

take up an exp $(-i\epsilon)$. The invariants of Class A (Eq. (5)) were obtained by use of these tensors, and their combination according to Eq. (A. 11); to have invariance under the electron group only products of tensors and complex conjugate tensors, both from Eq. (A. 14) or both from Eq. (A. 15), are to be formed. Class B (Eq. (6)), which is not invariant under the electron group, is obtained by multiplying a tensor contained in (A. 14) by the complex conjugate of a tensor contained in (A. 15) or vice versa.