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RESEARCH ARTICLE

Rational exaggeration and counter-exaggeration in information aggregation games

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Abstract We study an information aggregation game in which each of a finite collection of "senders" receives a private signal and submits a report to the center, who then makes a decision based on the average of these reports. The integration of three features distinguishes our framework from the related literature: players' reports are aggregated by a mechanistic averaging rule, their strategy sets are intervals rather than binary choices, and they are ex ante heterogeneous. In this setting, players engage in a "tug-of-war," as they exaggerate and counter-exaggerate in order to manipulate the center's decision. While incentives to exaggerate have been studied extensively, the phenomenon of counter-exaggeration is less well understood. Our main results are as follows. First, the cycle of counter-exaggeration can be broken only by the imposition of exogenous bounds on the space of admissible sender reports. Second, in the unique pure-strategy equilibrium, all but at most one player is constrained with positive prob-

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ability by one of the report bounds. Our third and fourth results hold for a class of "anchored" games. We show that if the report space is strictly contained in the signal space, then welfare is increasing in the size of the report space, but if the containment relation is reversed, welfare is independent of the size of the space. Finally, the equilibrium performance of our heterogeneous players can be unambiguously ranked: a player's equilibrium payoff is inversely related to the probability that her exaggeration will be thwarted by the report bounds.

Keywords Information aggregation · Majority rule · LIBOR · Baltic Dry Index · Yelp · Online reviews · Exaggeration · Counter-exaggeration · Mean versus median mechanism · Strategic communication · Incomplete-information games · Strategic information transmission

JEL Classification F71 · D72 · D82

1 Introduction

Information aggregation is ubiquitous in modern economies, and information averaging is perhaps the most commonly used aggregation tool. Several key financial benchmarks, for example, are set by having market participants submit price quotes to a central agency, which then averages them in some way to determine a market rate. The best known example is LIBOR, which is the trimmed mean of estimates submitted by money-center banks of the rates at which they could borrow for a given maturity and denomination. Similarly, the Baltic Dry Index (BDI) is set by averaging responses from shipping brokers to daily questions about the cost of booking various cargoes of raw materials on various routes. In these examples, a data contributor typically has a vested interest in the rate that emerges from the process and thus an incentive to manipulate it. Indeed, a flurry of recent criminal and civil manipulation charges against LIBOR panel members has culminated in huge settlements. Significantly, in spite of the obvious opportunities for manipulation of simple averaging rules, reform proposals currently on the table do *not* include procedures that would enable the aggregator to reverse-engineer submitted quotes and recover agents' actual information.

As use of the Internet has grown, online information aggregation/averaging sites have enabled consumers to submit opinions relating to a host of products and services, ranging from plumbers to restaurants to university courses. Services such as AngiesList.com, Yelp.com, and Ratemyprofessors.com aggregate these opinions and report averages that influence consumer choices significantly.[1] As the importance of these services and their summary statistics grows, parties with vested interests in the reports have greater incentives to manipulate them. Not surprisingly, a cottage industry has evolved around submitting positive restaurant reviews to Yelp.[2] Despite these problems, however, simple averaging procedures remain the norm.

[1] See Anderson and Magruder (2012) for a discussion of the literature on online reviews and for measurements of the impact on restaurant profits of a 4-star Yelp rating. See also Ye et al. (2009).

[2] It has been claimed that as many as 40 % of Yelp reviews are biased in some way Guynn and Chang (2012).

Information averaging is also commonplace in small-group settings, such as faculty committees and boards of directors or trustees. A typical function of such groups is to gather together several parties, each with idiosyncratic expertise, and apply their collective wisdom to a single decision problem, such as designing a compensation package for a new hire or allocating funds to an investment project. In these settings, participants typically have heterogeneous preferences regarding the decision being made; consequently, each has an incentive to package the information she contributes in a way that will steer the collective outcome in her preferred direction.

An extensive literature has emerged to study information aggregation (see Sect. 2). Most contributions adopt a mechanism-design approach, exploring whether there are aggregator responses that would induce full information revelation. As we have emphasized, however, straightforward report-averaging has proved extremely resilient as an aggregating instrument in spite of its deficiencies. One reason is that the ground rules within which many real-world aggregators operate require them to provide their patrons and/or clients with a service which is passive rather than strategic. Accordingly, it seem worthwhile to investigate the strategic incentives for manipulation that are embedded into real-world mechanistic aggregation institutions.

In this paper, we define a class of games that we call "aggregation games." There is a finite collection of agents. Each agent is characterized by two parameters. The first is a privately observed signal, and the second is an observable characteristic, representing the player's bias w.r.t. the game's outcome. After simultaneously observing their respective signals, agents submit reports to a central authority, who averages them in order to determine some payoff-relevant variable. Our framework focuses on identifying agents' incentives to misreport their information. For example, if an agent has a preference for outcomes exceeding those that would otherwise result from the aggregation process, she has an incentive to bias her reports upwardly, that is, she can be expected to "rationally exaggerate" her private information. Of course, in many aggregation situations—e.g., online reviews, teaching evaluations—many (perhaps most) senders forgo the opportunity to act strategically. Accordingly, in Sects. 5.3 and 6.3, we introduce the possibility that some players will report their signals truthfully, ignoring their incentives to do otherwise.

The problem of *exaggeration* in the context of multisender information transmission has been widely studied (see Sect. 2 below). In our framework, there is a distinct phenomenon that we call *counter-exaggeration*, which, to our knowledge, has not previously been rigorously modeled.[3] When sender r's message is only one of several messages that contribute to the average, she obviously has an incentive to exaggerate her message, to induce the center to assign more weight to her contribution. Moreover, if r knows that sender i has a bias in the reverse direction, and thus will exaggerate in a direction contrary to r's interests, r will have a further incentive to exaggerate.[4]

[3] The phenomenon is certainly alluded to by many of the authors we discuss [e.g., Kawamura (2011), Krishna and Morgan (2001)] but as we argue below, a specific set of model properties is required in order to explore its implications.

[4] Counter-exaggeration is an acknowledged necessity within the online reviewing community. For example, as reputation.com (n.d.) observes: ". . .In these situations, there is realistically only one way to effectively handle false reviews: having an overwhelming majority of positive reviews will discredit & nullify any false & misleading reviews."

In turn, i will be motivated to "counter-counter exaggerate" in order to offset r's counter-exaggeration. Such a process could potentially result in a endless spiral of ever-increasing exaggerations. Because agents are ex ante homogeneous in the literature most relevant to our analysis (see pp. 8–10), this secondary incentive does not arise, so that its implications have until now been neglected.

Some other key features of our setup are as follows: aggregate information is multi-dimensional, but each agent privately observes the true value of exactly one component of the vector-valued state of nature. However, the outcome of the game is a scalar, over which all agents have (univariate) preferences. There is ex ante common knowledge of bias heterogeneity. In the reporting process, talk is "cheap" in the sense that misreporting is not penalized. Finally, prespecified bounds on the message space constrain the reports that agents can make.

To guide the reader's intuition, we offer two illustrative vignettes, constructed to highlight various key features of our model. Our first vignette concerns a faculty committee, convened to decide how aggressively to pursue to a candidate whom the department hopes to hire. Each committee member is assigned the task of obtaining information about the candidate along a particular dimension (research, teaching, outreach, service) and to rate her performance from that perspective according to a common scale. The committee chair (who is required by institutional procedures to comply with prespecified rules) averages the reported ratings, and the average is mapped to a recruitment effort level. Although all faculty members agree that this effort should reflect the profession's collective assessment of the candidate's merit, it is common knowledge that each member is biased either in favor of or against her. A familiar scenario would be: those in her field advocate for a particularly aggressive recruitment campaign, while others with less sympathy for her research interests are inclined to adopt a more relaxed approach. In our second vignette, a corporate board is tasked with determining the size or level of investment in a new project. The suitability of the project must be evaluated from many perspectives (e.g., revenue, cost, risk, and strategic complementarities). Accordingly, the board commissions a member with recognized expertise in each one of the relevant perspectives to investigate the project from that particular perspective and to report its evaluation using a common numerical rating scale. [Following Crawford and Sobel (1982), we assume that each expert is able to assign without error an objective "score" to the project.] The board then (mechanically) constructs an overall assessment of the project's potential by averaging the reported specialized ratings.[5] This assessment is then mapped to a (scalar) level of commitment to the project. Although all experts agree that the firm's commitment should reflect market realities, it is common knowledge that each one is biased either in favor of or against the project. Some are less risk-averse than others; some approve the direction in which the project would take the company; others do not. We could expect that experts' reports will be slanted in directions consistent with their biases. It would be natural to expect that the project's supporters (detractors) would inflate (deflate) the scores they report, relative to their privately observed information. Moreover, knowing

[5] Mechanical averaging seems plausible in this context: a board with its own heterogeneous preferences would likely be too unwieldy to implement a sophisticated scheme designed to reverse-engineers experts' biases.

that others will behave in exactly the same way, the inflaters will need to inflate more than they otherwise would, in order to counterbalance the impact of the deflaters.

Relative to the literature, our framework has a unique combination of features, most notably a continuum message space, a naive aggregation rule, and senders with ex ante heterogeneous preferences. This combination yields a unique set of results relating to a number of questions about the ubiquitous aggregation mechanism of simple averaging. We discuss three of them here. First, how effectively does it aggregate information? Consistent with the literature, we find that some information loss is a necessary social cost of obtaining an equilibrium when agents are heterogeneous. Specifically, the map from signal vectors to sender reports is not 1–1. This is because in the unique pure-strategy equilibrium of an aggregation game, all but at most one player is constrained with positive probability by one of the bounds on the report space.[6] This result highlights the pivotal role that these boundaries play in our model. Without them, players would engage in an endlessly escalating tug-of-war: those with positive biases would distort their messages farther and farther to the right, in order to offset the increasingly magnified leftward distortions of players with negative biases. But the constraints also have welfare consequences: they result in "message-bunching," and hence information loss, at one or both ends of the signal spectrum; this is more prevalent, the greater is both the number of players and the degree of player heterogeneity.[7] Second, can information losses be mitigated by restricting the set of admissible reports? While the literature [e.g., Kawamura (2011), Rosar (2010)] suggests an affirmative answer, our answer is no: while it is possible to restrict the space *too much*, thus increasing information losses beyond the minimal level necessary for equilibrium existence, further expansions of the message space beyond a certain point have no real impact, i.e., the information content of equilibrium and hence payoffs and aggregate welfare are independent of the size of the space. Third, among our heterogeneous senders, which ones are more negatively impacted by the aggregation process? Our framework yields a sharp and intuitive answer to this question: a player will do better in equilibrium, the less likely it is that her exaggeration will be thwarted by the bounds on admissible reports. In a symmetric game, whose bounds are equidistant from the origin, players with more extreme biases are more likely to want to send extreme messages and hence are more likely to be constrained. On the other hand, when the bounds are one-sided (e.g., any nonnegative report is admissible), the player with the most extreme positive (negative) bias does best (worst).

The paper is organized as follows. The symbol † appended to a proposition title indicates that its proof is in the "Appendix." When a proposition follows immediately from arguments in the text, its formal proof is omitted. Section 2 relates our model to the literature. In Sect. 3, we introduce our model. Section 4 presents some general results on the equilibrium properties of aggregation games. Aggregation games are particularly tractable when there is exactly one player who is either unconstrained, or

6 The role of a compact message space in limiting information transmission has been noted in the literature, [e.g., Rosar (2010)] but usually in contexts that differ from ours. See, for example, Ottaviani and Squintani (2006).

7 Rosar (2010) also observes bunching which he restricts the space of admissible reports, but in his context, bunching is not a *necessary* cost of obtaining an equilibrium.

symmetrically constrained, by the bounds on reports. We call this player the "anchor," identify a class of games called anchored games, and conclude Sect. 4 with some general results on anchored games. Sections 5 and 6 focus on two special classes of anchored games. In Sect. 5, we study games that are symmetric in a strong sense: there is a right-wing and a mirror-image left-wing faction. Because other model parameters are also symmetric, neither faction has an advantage. Symmetric games have striking properties, of which several have already been mentioned. Another is the impact of player heterogeneity. Intuitively, payoffs decline as heterogeneity increases. Surprisingly, however, a faction-mean-preserving spread of each faction's biases may actually increase aggregate welfare. Our final results focus on the size of the game. We introduce the possibility that some players will report truthfully rather than strategically, and we consider the effect of increasing the number of either truthful or strategic players. Section 6 studies a quite different class of anchored games, in which the upper bound on the space of admissible reports is so high that it never binds in equilibrium. Games of this kind are anchored by the player with the largest positive bias. In spite of significant structural differences, most properties of this class of games are surprisingly similar to those of symmetric games. Section 7 concludes.

2 Related literature

In this section, we modify the notation in the papers we discuss in order to match our own notation. Our paper contributes to the extensive literature initiated by Crawford and Sobel (1982) [CS] on information transmission and aggregation.[8] In CS, a single sender privately observes a signal θ identifying the state of nature and then transmits a message to a receiver, who makes a decision τ that affects the utility of both. The receiver's objective[9] is to maximize $-(\tau-\theta)^2$; the sender's utility is $-(\tau-(\theta+k))^2$, where $k\geq 0$. When $k>0$, the sender has an incentive to announce a message that exceeds her observed signal; the receiver, recognizing this incentive, "de-biases" the sender's message, in order to learn all that can be learned about the true value of θ. CS's main result is that perfect information communication is attainable only if $k=0$. In equilibrium, the sender partitions her information space into intervals and reports only the interval to which her signal belongs.

An early extension of the CS framework, by Melumad and Shibano (1991), demonstrates that when the sender's and the receiver's preferences are disparate, information transmission may do more harm than good. Blume et al. (2007) introduce noise into the senders' signals and find that noise can improve social welfare. Ottaviani and Squintani (2006) and Kartik et al. (2007) extend CS by introducing the possibility that the receiver is *naive*, i.e., takes the sender's report at face value. Both papers point out that when there is only one sender, a necessary condition for exaggeration—they call it "language inflation"—is that the receiver is naive with probability less than one.

[8] A portion of this literature has been published in this journal. See, for example, Dickhaut et al. (1995), Plott et al. (2003), Gunay (2008), Yang (2010), and Chen and Gordon (2014).

[9] CS specify a more general class of utility functions. The quadratic loss function we use, which in CS is only an example, has been widely adopted in the subsequent literature.

Clearly, when there are multiple senders, this condition is no longer necessary. Chen (2010) extends the preceding specification by allowing that with positive probability, the sender truthfully reports her signal. Within this setting, Chen can explain sender exaggeration, receiver skepticism, and the clustering of messages at the top end of the message space. While our mechanistic receiver does not exhibit skepticism, our model also exhibits clustering and, of course, exaggeration.

A number of papers extend CS's model to a multisender setting. In several, the receiver (henceforth *the center*) decides according to majority rule. We discuss two of these papers, Austen-Smith and Banks (1996) [AB] and Feddersen and Pesendorfer (1997) [FP]. The starting point for both is Condorcet (1785). His Jury Theorem established conditions under which, when voters with identical preferences select non-strategically (or *sincerely*) between two alternatives based on their private information, and the majority prevails, then, as the number of voters increases without bound, information is in the limit perfectly aggregated, in the sense that the majority's choice coincides with the choice that would be taken if all private information were publicly available. (FP later call this property "full information equivalence.") AB study the relationship between sincerity and rationality. Under majority rule, rationality dictates that one should decide how to vote conditional on the assumption that one's vote is decisive (or *pivotal*). Based on this assumption, one can make inferences about the distribution of other senders' realized signals and thus about the true state of the world. AB show that under three specifications, voting sincerely is, except in very special circumstances, incompatible with voting *informatively*, i.e., in a way that depends non-trivially on one's private signal. While AB focus on small games, FP explores the implications of pivotality in large ones. FP's specification of senders' preferences is quite similar to ours, except that their biases are private information. They consider a sequence of games in which n increases without bound; when senders condition on pivotality, their limit game exhibits full information equivalence. An important difference between both these papers and ours is that their senders' reports are binary; hence, they can *lie* but they cannot *exaggerate*.

Wolinsky (2002) adopts a mechanism-design approach. Each of several experts receives a binary signal about whether a policy should be implemented; as in our model, each expert's signal is interpreted as the observation of a single dimension of a multidimensional state of nature. Wolinsky's players can, without detection, under- but not overreport. His center is more inclined than any sender to implement the policy. If the center cannot commit to a decision rule, then in equilibrium the experts reveal no meaningful information and the policy is implemented with probability zero.

Another strand of the literature addresses issues closely related to the ones that we confront. Battaglini (2002) extends CS by introducing multidimensional signals and policies. His primary contribution is to show that when there are two senders who perfectly observe the multidimensional state of nature, full revelation of information is generically possible, regardless of the degree of heterogeneity between senders. This result contrasts starkly with CS's result for unidimensional signals. Battaglini (2004) extends his earlier contribution to the case in which senders' private information is noisy. Ambrus and Takahashi (2008) point out that Battaglini's full revelation result depends critically on the assumption that the state and policy spaces are unbounded. Levy and Razin (2007) consider a model with a multidimensional state space in which the degree of sender heterogeneity differs across dimensions. They focus on informational spillovers. If information along the first dimension reveals information along the second, sender heterogeneity w.r.t. the second may inhibit information revelation about the first. Chakraborty and Harbaugh (2007) study a game in which a single sender has multidimensional comparative information; for example, a professor will know the quality of multiple students. They show that the sender's tendency to exaggerate can be mitigated by restricting the message space to a rank ordering. In spite of the links between these papers and ours, their approaches are tangential, for three reasons. First, while the state space in our model is also multidimensional, each sender observes only one dimension of the state. Second, in our model, the outcome space on which senders' preferences are defined is unidimensional. Third, while much of the literature just discussed focuses on constructing mechanisms to induce full revelation, our concern is with the welfare properties of the passive method of aggregating information (averaging) that prevails in a wide variety of institutional contexts, so that the key question that these papers address, how revealing is the most revelatory mechanism that can be designed, does not arise.

To conclude this section, we discuss the four papers most closely related to ours, which consider averaging as (possibly one of several) decision rule(s)[10]: Morgan and Stocken (2008) [MS], Gruner and Kiel (2004) [GK], Rosar (2010) [RR], and Kawamura (2011) [KA]. In these papers, the sender privately observes a scalar iid random variable, which is binary in MS and continuous in the others. In MS, as in our paper, private information can be interpreted as a signal about a state of nature that is of common concern; in KA, each player is concerned only about her own signal; in GK and RR, the interpretation of signals is less transparent. In each case, senders simultaneously submit reports to a central receiver, who aggregates them with the goal either of neutrally reflecting aggregate private information (MS), or of maximizing aggregate welfare (the others). Thus, MS's paper is furthest from ours in its modeling of private information, but closest in other respects.

Each of MS' senders receives a binary signal and sends a binary report. The receiver aggregates reports and chooses the policy which would be optimal for an unbiased decision maker. When the number of senders is small, a sender might be deterred from misreporting by the possibility that she might overshoot. But this possibility decreases as the number of senders increases, so that more and more constituents will vote according to their biases rather than their information. Since MS's senders' reports are binary, once again lying is possible, but exaggeration is not.

To highlight the difference between our model and those of GK/RR/KA, we return to our faculty recruitment vignette (pp. 3–4). To sharpen the comparison, we consider an extreme version of GK/RR's model (in which their parameter $\alpha = 0$). In their models, as in ours, a sender's private information reveals the candidate's quality along the single dimension that he investigates. In their models, the sender's *evaluation* of

[10] Krishna and Morgan (2001) [KM] is less relevant to us, since their experts submit reports sequentially not simultaneously. On the other hand, KM explicitly address the issue of counter-exaggeration: "For instance, hawks may choose more extreme positions on an issue if they know that doves are also being consulted, and vice versa." (p. 748).

the candidate would depend *exclusively* on her quality along this dimension.[11] In our model, by contrast, there is an information-pooling component that is independent of the degree of sender heterogeneity. All committee members in our model evaluate the candidate's quality according to the same criterion, i.e., the average of her qualities along *all* dimensions. Thus, a sender in our model may be biased toward a candidate *because* she works in his field, but the intensity of his bias is independent of her *quality* in that particular field.

A related difference between these three models and ours is that their senders are ex ante homogeneous, that is, since senders' private signals are iid, it follows that before any private information has been observed, their ex ante expected utilities—computed as functions only of the center's action and integrating w.r.t. the joint distribution over signals—are identical. This property has striking implications for equilibrium behavior. In particular, in each of GK's equilibria under the averaging rule (regardless of the value of their α parameter) and for *any* sender, the interim expected action by the center—i.e., the center's action conditional on the signal that the sender has observed—coincides with the sender's ideal action, that is, regardless of the signal she receives, each sender can ensure that the center delivers, in expectation, her ideal outcome. As a consequence, while there is a tension between GK's center and each individual sender, there is no tension whatsoever *between* GK's senders. In their equilibria, as in ours, each sender exaggerates the magnitude of her signal in order to influence the center. However, as we noted on pp. 2–3, there is no impetus in their models for competitive *counter*-exaggeration, i.e., exaggeration motivated by the need to offset the exaggerations of other senders.[12]

In our model, by contrast, the degree of heterogeneity between senders is independent of the state of nature, that is, for *every* state of nature, any two senders will disagree about what is the best action for the center to take. By implication, if the ex ante value of the center's decision is ideal from one sender's perspective, it *cannot* be ideal from the other's. A consequence of this irreducible heterogeneity is that our model has a pure-strategy equilibrium only if the space of admissible reports is compact.[13] Without this restriction, senders would be locked into an endless "tug-of-war" of exaggeration and counter-exaggeration, each trying to offset the "adverse" influences of all the others on the center's decision (cf. pp. 2–3). This cycle can be broken only by imposing bounds on the space of reports and thereby limiting the extent to which a sender can exaggerate.

This difference has spillover implications on the welfare impact of heterogeneity on information aggregation. In GK's model, the social cost of incomplete information is that the variance of the center's decision is higher than it would be under truthful revelation. We later refer to this consequence as a *second-moment effect*. In our model, for all senders except at most one, there is a positive measure of types for whom one of the bounds on reports is binding. For any type in this set, the second-moment effect is

[11] When GK/RR's α parameter is positive, other senders' dimensions matter also.

[12] When in RR's model the report space is restricted, incentives for counter-exaggeration may arise.

[13] RR considers compact report spaces, but they are not *necessary* for equilibrium.

augmented by a *first-moment effect*: the expected value of the center's decision differs from that type's most preferred outcome.

3 The model

We study an incomplete-information, simultaneous-move game with $n > 1$ players, indexed by $r = 1, \ldots, n$. For any variable $\mathbf{x} \in \mathbb{R}^n$, the symbol $\mu(\mathbf{x})$ will denote the average of $\mathbf{x}$'s components. The state of nature is represented by an n-vector $\boldsymbol{\theta} \in \mathbb{R}^n$. Player r receives a private, perfectly informative signal[14] about the r'th component of $\boldsymbol{\theta}$ and submits a report s_r about this signal to the center. The center then takes a decision based on the average $\mu(\mathbf{s}(\boldsymbol{\theta}))$ of these reports. Each player's preferences depend on $\mu(\boldsymbol{\theta})$, as well as on a publicly known bias factor, and thus has an incentive to distort the average signal received by the center. We reiterate for emphasis that while the social *information* space is multidimensional, the *decision* space, over which players have preferences, is unidimensional.

Player characteristics Each player has an *observable characteristic* and a *type*. Player r's *type* is $\theta_r \in \mathbb{R}$, which is her private information about a unique dimension of the state of nature $\boldsymbol{\theta}$ (e.g., the riskiness of the project being evaluated[15]). Ex ante, before players observe their private signals, the θ_r's are identically, independently, and continuously distributed on the compact interval $\Theta \equiv [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$, with $\overline{\theta} > \underline{\theta}$. Let $h(\cdot)$ denote the density, and $H(\cdot)$ the c.d.f., of players' types. We assume that $h(\cdot)$ is bounded. Let $\boldsymbol{\Theta} = \Theta^n$ denote the space of *type profiles*, with generic element $\boldsymbol{\theta}$. Similarly, let $\boldsymbol{\Theta}_{-r} = \Theta^{n-1}$ be the space of types for players other than r, with generic element $\boldsymbol{\theta}_{-r}$. For $\boldsymbol{\theta}_{-r} \in \boldsymbol{\Theta}_{-r}$, let $\mathbf{h}_{-r}(\boldsymbol{\theta}_{-r}) = \prod_{i \neq r} h(\theta_i)$. When we integrate w.r.t. either player r's type or all other players' types, we will use, respectively, the variants ϑ_r and $\boldsymbol{\vartheta}_{-r}$ of θ_r and $\boldsymbol{\theta}_{-r}$ to distinguish dummy variables of integration.

Player r's observable characteristic is denoted by $k_r \in \mathbb{R}$ and is interpreted as r's bias w.r.t. revealed information: a player whose characteristic is positive prefers that the center overestimate the aggregate information, i.e., the mean of players' types. For example, if board member r is particularly opposed to the project being evaluated, then $k_r < 0$. We refer to the vector $\mathbf{k} = (k_r)_{r=1}^n$ as the *observable characteristic profile*. To avoid special cases and/or additional notation, Assumption A1 imposes two restrictions on observable characteristics: players' biases cancel each other out in the aggregate and they are distinct.

Assumption A1 (i) $\sum_i k_i = 0$; (ii) $i \neq r \implies k_i \neq k_r$.

Restriction (i) yields a clean expression for welfare, while (ii) ensures uniqueness.

The utility function The *utility function* is a mapping $u : T \times \boldsymbol{\Theta} \times \mathbb{R} \to \mathbb{R}$, where $T \subset \mathbb{R}$ is compact. The scalar first argument of u is the decision taken by the center in response to information provided by the players: $u(\tau, \boldsymbol{\theta}, k)$ is the utility to a player with

[14] In this sense, our model can be viewed as an n-dimensional extension of Crawford and Sobel (1982) and its many successors, in which there is a single sender who is perfectly informed about a scalar state of nature.

[15] Here and below, references to "the project" and its "net return" relate to the second of the vignettes outlined on pp. 3–4, in which a corporate board is tasked with determining the size of an investment project.

observable characteristic k, when the central authority's decision is τ and the vector of unobservable characteristics is $\boldsymbol{\theta}$. The essence of an aggregation game is that players' types affect their utility only through their effect on the average of all players' types, i.e., $u(\tau, \boldsymbol{\theta}, k) = u(\tau, \boldsymbol{\theta}', k)$ if $\mu(\boldsymbol{\theta}) = \mu\left(\boldsymbol{\theta}'\right)$. In our vignette, the project's net return depends on *aggregate* information, while each board member has private information about some component of the project. When convenient, we write the second argument of u either as the vector $\boldsymbol{\theta}$ or the scalar $\mu(\boldsymbol{\theta})$. We further restrict utility to be a quadratic loss function:[16]

Assumption A2

$$u(\tau, \mu(\boldsymbol{\theta}), k) = -\left(\tau - (k + \mu(\boldsymbol{\theta}))\right)^2 . \tag{1}$$

In (1), $\mu(\boldsymbol{\theta}) + k$ would be a player's ideal outcome—it depends on the aggregate information $\mu(\boldsymbol{\theta})$ and her own bias k—while τ, the center's decision, depends on the aggregate information reported by the players. The quadratic function captures information losses that arise because players act strategically and misreport their information.

Pure strategies We assume that the center rejects reports that lie outside a compact interval denoted by $A = [\underline{a}, \bar{a}]$. We refer below to $\underline{a}$ and $\bar{a}$ as the *announcement bounds*. Given the structure of our model, a player whose unconstrained optimal report exceeds $\bar{a}$ necessarily weakly prefers to have a report of $\bar{a}$ accepted than to have her report rejected. Accordingly, to streamline the exposition, we impose as a restriction that each player must choose a report in A. A *pure strategy* for player r is a function $s_r\colon \Theta \rightarrow A$, where $s_r(\theta_r)$ denotes r's announcement when her type is θ_r. (Henceforth, the symbol s_r will denote a *function* from types to A, while a_r will denote a particular value of $s_r(\theta_r)$.) The vector $\mathbf{s} = (s_1, \ldots, s_n)$, called a *pure-strategy profile*, is thus a mapping from $\boldsymbol{\Theta}$ to $\mathbf{A} = A^n$. A pure-strategy $s_r(\cdot)$ is said to be *monotone* if it is non-decreasing and strictly increasing except when $s_r(\cdot)$ is at the boundary of A. Since the space A is bounded both above and below, if s_r is monotone, there exists a *low threshold type* $\underset{\sim}{\theta}_r \in [\underline{\theta}, \overline{\theta}]$ and a *high threshold type* $\tilde{\theta}_r \in [\underline{\theta}, \overline{\theta}]$ such that s_r equals $\underline{a}$ on $[\underline{\theta}, \underset{\sim}{\theta}_r)$, is strictly increasing on $(\underset{\sim}{\theta}_r, \tilde{\theta}_r)$ and equals $\bar{a}$ on $(\tilde{\theta}_r, \overline{\theta}]$.[17] Formally,

$$\underset{\sim}{\theta}_r(s_r) = \begin{cases} \underline{\theta} & \text{if } s_r(\underline{\theta}) > \underline{a} \\ \sup\{\theta \in \Theta : s_r(\theta) = \underline{a}\} & \text{if } s_r(\underline{\theta}) = \underline{a}, \end{cases} \tag{2a}$$

$$\tilde{\theta}_r(s_r) = \begin{cases} \overline{\theta} & \text{if } s_r(\overline{\theta}) < \bar{a} \\ \inf\{\theta \in \Theta : s_r(\theta) = \bar{a}\} & \text{if } s_r(\overline{\theta}) = \underline{a} \end{cases}. \tag{2b}$$

The outcome function The *outcome function*, $t\colon \mathbf{A} \times \mathbb{R}^n \rightarrow \mathbb{R}_+$, maps players' announcements and the vector of observable characteristics to actions by the cen-

[16] This is the classical specification for problems of the kind we are analyzing. See, e.g., Crawford and Sobel (1982), Gilligan and Krehbiel (1989), Krishna and Morgan (2001), and Morgan and Stocken (2008), to mention just a few.

[17] Either one of the half-open intervals can be empty. For example, if $s_r(\cdot) > \underline{a}$ on Θ, then $[\underline{\theta}, \underset{\sim}{\theta}_r(s_r)) = \emptyset$.

tral authority. (Our notation distinguishes the outcome *function* t from the realized *outcome* τ.) Our center aggregates information mechanically rather than strategically: it takes each player's report at its face value.[18] Indeed, we restrict outcomes to be *complete information socially efficient (CISE)*, meaning that if players were to truthfully reveal their types on average, the outcome would maximize social welfare, defined as the average of players' utilities. Defining the *social welfare function* as

$$w(\tau, \boldsymbol{\theta}, \mathbf{k}) = \sum_i u(\tau, \boldsymbol{\theta}, k_i)/n, \tag{3}$$

the CISE outcome function is $t(\boldsymbol{\theta}, \mathbf{k}) = \text{argmax } w(\cdot, \boldsymbol{\theta}, \mathbf{k})$. An outcome implemented by a CISE outcome function is called a *CISE outcome*. From (1) and Assumption A1(i), the CISE outcome is

$$t(\mathbf{a}, \mathbf{k}) = \mu(\mathbf{a}). \tag{4}$$

Player's expected payoff functions Player r's expected payoff function, U_r, maps her own announcement and type into her utility, given other players' strategies. Our expression for U_r suppresses r's observable characteristic and the outcome function. Formally, given a profile $\mathbf{s}_{-r}$ of strategies for players other than r, player r's expected payoff function $U_r\colon \mathbf{A} \times \Theta \to \mathbb{R}_+$ is

$$U_r(a, \theta; \mathbf{s}_{-r}) = \int_{\boldsymbol{\Theta}_{-r}} u\left(t\left((a, \mathbf{s}_{-r}(\boldsymbol{\vartheta}_{-r})), \mathbf{k}\right), (\theta, \boldsymbol{\vartheta}_{-r}), k_r\right) d\mathbf{h}_{-r}(\boldsymbol{\vartheta}_{-r}). \tag{5}$$

Henceforth, when confusion can be avoided, we will abbreviate the derivative $\frac{\partial U_r}{\partial a}$ to U_r'.

Equilibrium A *monotone pure-strategy Nash equilibrium* (MPE) for an aggregation game is a monotone strategy profile $\mathbf{s}$ such that for all r, $\theta \in \Theta$, and $a \in A$, $U_r(s_r(\theta), \theta; \mathbf{s}_{-r}) \geq U_r(a, \theta; \mathbf{s}_{-r})$. From (1), (4), and (5), we know that

$$\text{for all } r, \text{ all } a, \text{ all } \theta, \text{ and all } \mathbf{s}_{-r}, \quad \frac{\partial^2 U_r(a, \theta; \mathbf{s}_{-r})}{\partial a \partial \theta} > 0. \tag{6}$$

Inequality (6) states that U_r satisfies Milgrom–Shannon's "single crossing property of incremental returns (SCP-IR)" in $(a; \theta)$ Milgrom and Shannon (1994). In our context, this property implies Athey's sufficiency condition, SCC, for existence of a pure-strategy equilibrium, i.e., "the single crossing condition for games of incomplete information" (Athey 2001, Definition 3).[19]

Proposition 1 (Existence of an MPE)† *Every aggregation game has an MPE,* $\mathbf{s}$, *with the property that for each* r, s_r *is continuously differentiable on* $(\underset{\sim}{\theta}_r(\mathbf{s}), \tilde{\theta}_r(\mathbf{s}))$.

[18] In our framework, we envisage the center not as an autonomous player—for example, a committee chair with discretionary powers—but rather as a set of bureaucratic rules or procedures.

[19] Athey's condition requires that U_r satisfy SCP-IR only if other players play non-increasing strategies. Our U_r's satisfy SCP-IR regardless of other players' choices.

While Athey's machinery ensures the existence of a monotone equilibrium, it follows immediately from (6) that monotonicity is a *necessary* condition for existence. Prop. 2 makes this precise.

Proposition 2 (Every equilibrium is an MPE)† *If* $\mathbf{s}$ *is a pure-strategy equilibrium for an aggregation game, then for each* r, s_r *is strictly increasing on* $(\underset{\sim}{\theta}_r(\mathbf{s}), \tilde{\theta}_r(\mathbf{s}))$.

4 Equilibrium properties of aggregation games

4.1 CUA strategies

We say that r's strategy is *constrained unit affine (CUA)* if it is unit affine except at the announcement bounds, i.e., if for some $\lambda_r \in \mathbb{R}$, $s_r(\cdot) = \min\{\bar{a}, \max\{\underline{a}, \iota(\cdot) + \lambda_r\}\}$. The defining property of a CUA strategy is that the extent of r's misrepresentation of her type is independent of this type, except when r is constrained by the boundaries of A. The parameter λ_r indicates the extent of r's misrepresentation. The quadratic specification in (1) ensures that equilibrium strategies will be CUA. Indeed, if r were not required to respect the bounds $\underline{a}$ and/or $\bar{a}$ on λ_r, her optimal response to $\mathbf{s}_{-r}$ would be the affine strategy $\theta_r + \lambda_r(k_r, \mathbf{s}_{-r})$, where

$$\lambda_r(k_r, \mathbf{s}_{-r}) = \underbrace{nk_r}_{\text{exaggeration}} + \underbrace{\sum_{i \neq r} \mathbb{E}_{\vartheta_i}(\vartheta_i - s_i(\vartheta_i))}_{\text{counter-exaggeration}}. \tag{7}$$

In general, $\theta_r + \lambda_r(k_r, \mathbf{s}_{-r})$ will not belong to A for extreme realizations of θ_r. Accordingly, r's *constrained* optimal response will be the CUA strategy:

$$s_r(\theta_r) = \min\{\bar{a}, \max\{\theta_r + \lambda_r(k_r, \mathbf{s}_{-r}), \underline{a}\}\}. \tag{8}$$

Equation (7) highlights the strategic misreporting incentives in an aggregation game. A player *exaggerates* her information both to shift the center's decision in the direction of her bias k_r [the first term on the rhs of (7)] and *counter-exaggerates* to offset the aggregate exaggerations of other players (the second term). As players who prefer higher outcomes attempt to influence the center by increasing their announcements, other players who prefer lower outcomes will counter by decreasing theirs. As noted earlier (p. 4), if there were no bounds on announcements, this tug-of-war would escalate endlessly. Thus, a necessary condition for the existence of an MPE is that the announcement space A is bounded at least on one end. The announcement bounds essentially limit the extent to which players can misreport their types. We will observe below that players with different observable characteristics are restricted by the bounds to different degrees.

To clarify concepts, we introduce some definitions. We will say that player r's strategy $s_r(\cdot)$ is

(1) *non-degenerate (resp. degenerate)* if the interval $(\underset{\sim}{\theta}_r(s_r), \tilde{\theta}_r(s_r))$ is non-empty (resp. empty);

(2) *up-constrained* if $\underset{\sim}{\theta}_r(s_r) = \underline{\theta}$ and $\tilde{\theta}_r(s_r) < \overline{\theta}$;

(3) *down-constrained* if $\underset{\sim}{\theta}_r(s_r) > \underline{\theta}$ and $\tilde{\theta}_r(s_r) = \overline{\theta}$;

(4) *single-constrained* if it is either up-constrained or down-constrained;
(5) *bi-constrained* if it is both up-constrained and down-constrained; and
(6) *almost-never-constrained* if it is neither up-constrained nor down-constrained.

Degenerate / *Almost-never-constrained* strategies pick boundary / interior points of A with probability one.[20] An MPE in which each player's strategy is non-degenerate is called an NMPE.

All members of the set $\{s_r(\cdot) = \min\{\bar{a}, \max\{\underline{a}, \iota(\cdot) + \lambda_r\}\}: \lambda_r \leq \underline{a} - \overline{\theta}\}$ of down-constrained degenerate strategies are functionally equivalent: in each case, $s_r(\cdot) = \underline{a}$. Similarly, all up-constrained strategies with $\lambda_r \geq \bar{a} - \underline{\theta}$ are equivalent. Hence, we can impose without loss of generality (w.l.o.g.):

$$s_r(\cdot) = \min\{\bar{a}, \max\{\underline{a}, \iota(\cdot) + \lambda\}\} \text{ is an } \textit{admissible CUA strategy} \text{ iff } \lambda_r \in \Lambda \equiv [\underline{a} - \overline{\theta}, \bar{a} - \underline{\theta}]. \tag{9}$$

Since $\bar{a} > \underline{a}$ and $\overline{\theta} > \underline{\theta}$, $\Lambda \neq \emptyset$. From (2a) and (2b), if s_r is an admissible CUA, then

$$\underset{\sim}{\theta}_r(s_r) = \max\{\underline{\theta}, \underline{a} - \lambda_r\} < \min\{\overline{\theta}, \bar{a} - \lambda_r\} = \tilde{\theta}_r(s_r). \tag{10}$$

If $\Theta \subseteq [\underline{a}, \bar{a}\,]$, we say that the announcement interval is *inclusive*. It follows from (10) that

$$\text{if } \Theta \text{ is inclusive, then no CUA strategy is bi-constrained.} \tag{11}$$

To see this, note that if Θ is inclusive and $\lambda_r \geq 0$, then $s_r(\underline{\theta}) = \underline{\theta} + \lambda_r \geq \underline{a} + \lambda_r \geq \underline{a}$. Similarly, if $\lambda_r \leq 0$, then $s_r(\overline{\theta}) \leq \bar{a}$. Since we focus exclusively on CUA strategies in the remainder of the paper, we will sometimes use the symbol λ_r as a shorthand for the CUA strategy with parameter λ_r. To identify an NMPE, we compute the $\boldsymbol{\lambda}$ vector that solves the set (7) of n equations subject to the constraint (8). As a first step, we let $\xi_r(\cdot)$ denote player r's *deviation from affine*, defined as the difference between the CUA strategy $s_r(\cdot)$ and the affine strategy $\iota(\cdot) + \lambda_r$, that is, from (8)

$$\xi_r(\cdot) = \min\{\bar{a}, \max\{\theta_r + \lambda_r, \underline{a}\}\} - (\theta_r + \lambda_r) = \begin{cases} \bar{a} - (\theta_r + \lambda_r) & \text{if } \theta_r > \tilde{\theta}_r(\lambda_r) \\ (\theta_r + \lambda_r) - \underline{a} & \text{if } \theta_r < \underset{\sim}{\theta}_r(\lambda_r) \\ 0 & \text{otherwise} \end{cases}. \tag{12}$$

[20] These distinctions relate to the concept of *informative* voting, which recurs throughout the information-transmission literature. (It appears to have been introduced in Austen-Smith and Banks (1996).) Almost-never-constrained strategies are informative, and degenerate ones are uninformative; the remaining types are somewhere in between.

Given λ_r, let $\mathbb{E}\xi_r$ denote r's *expected deviation from affine* (henceforth EDFA):

$$\mathbb{E}\xi_r \equiv \mathbb{E}_{\vartheta_r}(s_r(\vartheta_r) - (\vartheta_r + \lambda_r)) \tag{13}$$

$$= \int_{\underline{\theta}}^{\underset{\sim}{\theta}_r} (\underline{a} - (\vartheta_r + \lambda_r))\mathrm{d}H(\vartheta_r) + \int_{\tilde{\theta}_r}^{\overline{\theta}} (\bar{a} - (\vartheta_r + \lambda_r))\mathrm{d}H(\vartheta_r). \tag{14}$$

Thus, $\mathbb{E}\xi_r$ is a measure of the impact of bounds $\bar{a}$ and $\underline{a}$ on r's expected announcement. Note that

$$\text{if } r \text{ is single-constrained and } \mathbb{E}\xi_r \neq 0, \;\; \text{then } \lambda_r \mathbb{E}\xi_r < 0. \tag{15}$$

Deviations from affine play a critical role in our model. First, they limit the extent of exaggeration and thus ensure that an MPE exists. Second, as Prop. 6 below shows, the equilibrium expected payoff vector can be expressed exclusively in terms of the first and second moments of the ξ_r's.

Substituting $\theta_i - s_i(\theta_i) = -(\lambda_i + \xi_i)$ into (7) and rearranging, we obtain that if $\boldsymbol{\lambda}^*$ is an MPE, then

$$nk_r = \sum_i \lambda_i^* + \sum_{i \neq r} \mathbb{E}\xi_i(\lambda_i^*), \qquad \text{for all } r \text{ with } \lambda_r^* \in \text{int}(\Lambda). \tag{7$'$}$$

If $\boldsymbol{\lambda}^*$ is an MPE, then for all i, j with $\lambda_i^*, \lambda_j^* \in \text{int}(\Lambda)$, (7′) immediately implies that

$$n(k_i - k_j) = \mathbb{E}\xi_j(\lambda_j^*) - \mathbb{E}\xi_i(\lambda_i^*). \tag{16}$$

Figure 1 provides some intuition for the determination of equilibrium strategies for a game with two players i and j, with $0 < k_i = -k_j$. The figure is a diagonal cross section of the three-dimensional graph from $\Theta \times \Theta$ to outcomes, that is, the graph depicts the event that i and j observe signals with the same value. Player i is up-constrained; player j is down-constrained. The thick piecewise-linear line represents the outcome as a function of type realizations, given the two players' strategies. The important property highlighted by the heavy solid line is that the outcome accurately reflects the aggregate signal only when $\theta_r \in [\underset{\sim}{\theta}_r, \tilde{\theta}_r]$, for $r = i, j$; on the other hand, when $\genfrac{}{}{0pt}{}{\theta_i > \tilde{\theta}_i}{\theta_j < \underset{\sim}{\theta}_j}$ (and $\theta_j \in [\underset{\sim}{\theta}_j, \tilde{\theta}_j]$), the realized outcome is an $\genfrac{}{}{0pt}{}{\text{under-}}{\text{over}}$estimate of the realized type. Now, consider the outcome from player i's perspective. For concreteness, suppose $\theta_i = 0$ and the horizontal axis represents j's type. Player i's ex post ideal outcome, as a function of j's type, is represented by the dashed line above the diagonal: for every value of j's type, i's ex post ideal outcome exceeds this type by k_i. When j is unconstrained, her underreport exactly counteracts i's overreport, resulting in an outcome that is suboptimal from i's perspective. However, at low values of θ_j, the constraint $\underline{a}$ binds j's underreporting, resulting in an outcome *exceeding* i's ideal. Equation (7′) indicates how the over- and underestimates are balanced in equilibrium: in our two-player example, in which $\lambda_i + \lambda_j = 0$, Eq. (7′) reduces to $2k_i = \mathbb{E}\xi_j(\lambda_j^*)$.

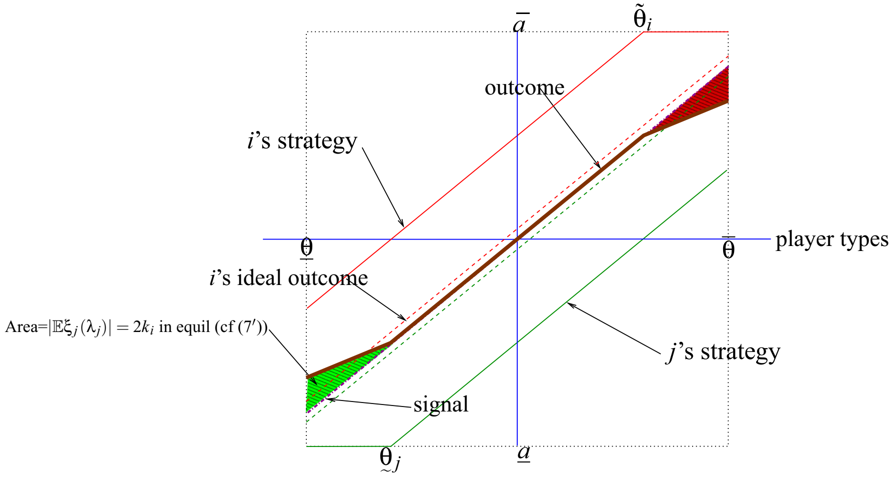

Fig. 1 Intuition for display (7′)

The right-hand term—the absolute magnitude of j's deviation from affine—is the area of the bottom left shaded triangle.

Since $\mathbb{E}\xi_r(\lambda_r^*) = 0$ if r's equilibrium strategy is almost never constrained, (16) and A1(ii) imply

Proposition 3 (Necessity of constraints) *In any MPE, at most one player's strategy is almost never constrained.*

An important property of an MPE is that, regardless of the width of the announcement interval A, players misreport to the extent that the announcements of all but one player are constrained, with positive probability, by one of the boundaries. The following corollary of Prop. 3 is immediate.

Corollary 4 (Announcement bounds are necessary for equilibrium) *If each player is free to choose any report in* $\mathbb{R}$*, then no pure-strategy equilibrium can exist.*

Two further properties of MPEs are as follows:

Proposition 5 (Uniqueness and monotonicity of MPE)[†] *Every aggregation game has a unique MPE. In any MPE if* $k_i > k_j$*, then* $\lambda_i^* > \lambda_j^*$.

4.2 MPE outcomes and payoffs

We next analyze two aspects of player r's equilibrium performance: the extent to which the expected outcome of the game deviates from r's ideal, and the link between r's equilibrium payoff and k_r. We define as a benchmark the *first-best outcome* for player r: this outcome would maximize r's payoff if she had complete information about the average type. We denote this "ideal" outcome from r's perspective by $\hat{t}(\boldsymbol{\theta}, k_r)$. From (1), it is

$$\hat{t}(\boldsymbol{\theta}, k_r) = \mu(\boldsymbol{\theta}) + k_r. \tag{17}$$

Recalling that $\mu(\mathbf{s}^*)$ is the equilibrium outcome of the game, the difference $\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu(\mathbf{s}^*(\boldsymbol{\vartheta})) - \hat{t}(\boldsymbol{\vartheta}, k_r)\right)$, which we label as r's *expected deviation from first-best*, is a measure of the degree to which the equilibrium outcome differs in expectation from player r's first-best. If $\mathbf{s}^* = \boldsymbol{\theta} + \boldsymbol{\lambda}^*$ is an MPE profile and $\lambda_r^* \in \text{int}(\Lambda)$, then from (13) and (7′), r's expected deviation from first-best is a scalar multiple of r's EDFA, that is,

$$\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\left(\mathbf{s}^*(\boldsymbol{\vartheta})\right) - \hat{t}(\boldsymbol{\vartheta}, k_r)\right) = \mathbb{E}\xi_r/n. \tag{18}$$

Property (18) establishes that in an NMPE, r's expected deviation from first-best is determined entirely by the probability with which the announcement bounds restrict r's announcement: the higher this probability is, the greater is the expected deviation, which we shall call the *first-moment effect* on r's expected payoff. If $\boldsymbol{\theta}$ is inclusive, then r's expected deviation from first-best is $\substack{\text{positive}\\\text{negative}}$ if r is $\substack{\text{down-constrained}\\\text{up-constrained}}$. Even when the first-moment effect is zero, r's payoff will be negatively impacted by a *second-moment effect*, arising from the randomness in all players' deviations from affine. From (1), r's expected payoff from a strategy profile $\boldsymbol{\lambda}$ is $-\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu(\boldsymbol{\vartheta}) + k_r - \mu(\mathbf{s}(\boldsymbol{\theta}))\right)^2$, i.e., the expectation of the squared difference between r's first-best outcome and the realized outcome. Let $V\xi_r(\lambda_r)$ denote the ex ante variance of r's deviation from affine, i.e.,

$$V\xi_r(\lambda_r) = \text{Var}_{\vartheta}\left(s_r(\vartheta_r) - (\vartheta_r + \lambda_r)\right) \leq \text{Var}(\theta). \tag{19}$$

To verify the inequality in (19), note that when λ_r equals the upper bound $\bar{a} - \underline{\theta}$ on Λ, so that $s_r(\cdot) = \bar{a}$, $V\xi_r(\lambda_r)$ reduces to $\text{Var}_{\vartheta}\left(\underline{\theta} + \vartheta_r\right) = \text{Var}(\theta)$. The inequality now follows from the fact that $\frac{\partial V\xi_r}{\partial \lambda_r} > 0$ (expression (43) in the "Appendix").

Proposition 6 (Equilibrium payoffs)[†] *Let* $\mathbf{s}^* = \boldsymbol{\theta} + \boldsymbol{\lambda}^*$ *be an MPE profile of an aggregation game. For each player r with* $\lambda_r^* \in$ *int*(Λ), *r's expected equilibrium payoff is*

$$\begin{aligned}\mathbb{E}_{\boldsymbol{\vartheta}} u\left(\mu\left(\mathbf{s}^*\right), \mu(\boldsymbol{\vartheta}), k_r\right) &= -\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu(\boldsymbol{\vartheta}) + k_r - \mu\left(\mathbf{s}^*\right)\right)^2 \\ &= -\left((\mathbb{E}\xi_r(\lambda_r^*)/n)^2 + \mu\left(\mathbf{V}\,\boldsymbol{\xi}(\boldsymbol{\lambda}^*)\right)/n\right).\end{aligned} \tag{20}$$

Proposition 6 shows that r's payoff is the sum of two negative effects: the first is the square of the first-moment effect [see (18)], which differs for each player; the second is the second-moment effect—i.e., the average, deflated by n, of the variances of players' deviations from affine. The latter effect is common to all players. Thus, the difference between two players' expected equilibrium payoffs is proportional to the difference between the squares of their expected deviations from affine.

Proposition 6 highlights an important property of our model: welfare losses arise not from misreporting per se, but only from the indirect effects of misreporting via players' deviations from affine. The reason for this is that in equilibrium, players can reverse-engineer and hence offset the information distortions resulting from the unconstrained misreports by others. However, players cannot offset the information losses that result from deviations from affine. Specifically, since players know each others' λ-values, player r can infer the average state of nature $\mu(\boldsymbol{\theta})$ from a profile of announcements $\mathbf{s}_{-r}(\boldsymbol{\theta}_{-r})$ iff for each $i \neq r$, player i's announcement is unconstrained by the announcement bounds, that is, i's announcement does not deviate from i's affine function $\theta_i + \lambda_i$.

We now consider social welfare. The standard benchmark is utilitarian.[21] We refer to this as *average private welfare* (APW). A natural alternative in our context is to view social welfare from an *unbiased* perspective, i.e., from the perspective of a player whose observable characteristic is zero, reflecting a preference for truthful revelation. We define this measure as *unbiased social welfare* (USW). We will place more emphasis on USW, since it has cleaner comparative statics properties.

$$\text{USW} = \mathbb{E}_{\boldsymbol{\vartheta}} u(\mu(\mathbf{s}), \mu(\boldsymbol{\vartheta}), 0) = -\mathbb{E}_{\boldsymbol{\vartheta}} \left(\mu(\boldsymbol{\vartheta}) - \mu(\mathbf{s}(\boldsymbol{\theta}))\right)^2. \tag{21a}$$

Under Assumption A1, the two measures differ by a scalar that is independent of strategies.[22]

$$\text{APW} = \text{USW} - \sum_i k_i^2/n. \tag{21b}$$

From (34) in the proof of Prop. 6, the following result is immediate.

Proposition 7 (Unbiased social welfare) *If $\boldsymbol{\lambda}^*$ is an MPE profile of an aggregation game, then*

$$USW = -\left\{\left(\mu\left(\mathbb{E}\boldsymbol{\xi}(\boldsymbol{\lambda}^*)\right) + \mu\left(\boldsymbol{\lambda}^*\right)\right)^2 + \mu\left(V\boldsymbol{\xi}(\boldsymbol{\lambda}^*)\right)/n\right\}. \tag{22}$$

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[21] See, e.g., Morgan and Stocken (2008), Gruner and Kiel (2004), Rosar (2010), Kawamura (2011).

[22]

$$\begin{aligned}
\text{APW} &= \frac{1}{n}\mathbb{E}_{\boldsymbol{\vartheta}}\left(\sum_{i=1}^{n} u(\mu(\mathbf{s}(\boldsymbol{\theta})), \mu(\boldsymbol{\vartheta}), k_i)\right)\\
&= -\frac{1}{n}\mathbb{E}_{\boldsymbol{\vartheta}}\sum_{i\in I}(\mu(\boldsymbol{\vartheta}) + k_i - \mu(\mathbf{s}(\boldsymbol{\theta})))^2\\
&= -\frac{1}{n}\left\{\sum_i k_i^2 + 2\sum_i k_i \mathbb{E}_{\boldsymbol{\vartheta}}(\mu(\boldsymbol{\vartheta}) - \mu(\mathbf{s}(\boldsymbol{\theta}))) + n\mathbb{E}_{\boldsymbol{\vartheta}}(\mu(\boldsymbol{\vartheta}) - \mu(\mathbf{s}(\boldsymbol{\theta})))^2\right\}\\
&= \text{USW} - \sum_i k_i^2/n.
\end{aligned}$$

4.3 Anchored games

Our discussion so far has highlighted the central role played by the EDFA in determining equilibrium payoffs. The EDFA is an especially useful concept for games in which some player j's equilibrium EDFA is zero. This property arises when j's equilibrium strategy is either never constrained or else the constraints on j imposed by the two announcement bounds cancel each other out in expectation. We shall refer to such games as *anchored games* and to player j as *the anchor*. Anchored games exhibit strong properties and are particularly easy to analyze, since there are closed-form expressions relating the endogenous variables to primitives of the game, such as the vector $\mathbf{k}$, the announcement bounds, and the size of the game. In Sects. 5–6 below, we study games in which a unique MPE with an anchor necessarily exists. In Sect. 5, she is the "middle" player in a symmetric game. In Sect. 6, she is the player with the largest observable characteristic in a game in which $\bar{a}$ never binds.

To streamline our analysis of these games, we impose an additional assumption:

Assumption A3 (i) The announcement space is inclusive (cf. p. 16); (ii) the type distribution is uniform with density parameter $h = 1/(\overline{\theta} - \underline{\theta})$; (iii) $||\mathbf{k}||_\infty < (\overline{\theta} - \underline{\theta})/4n$.

Parts (i) and (ii) simplify our analysis. Moreover,

Remark 1 Every anchored game satisfying A3(iii) has an NMPE

Part (iii) ensures that every player's equilibrium strategy is non-degenerate: without this property, the $\mathbf{k}$ vector could be sufficiently dispersed that some players, in their efforts to shift the average announcement in their favor, would choose strategies constrained with probability one by one of the announcement bounds.

Proposition 8 (Properties of anchored games) *Let* $\boldsymbol{\lambda}^*$ *be an MPE profile of an anchored aggregation game, and let j be the anchor. For each player r with* $\lambda^*_r \in int(\Lambda)$*,*

(1) r's EDFA, $\mathbb{E}\xi_r = n(k_j - k_r)$*, and*
(2) r's expected deviation from first-best is $(k_j - k_r)$*.*

Part i) is obtained by combining (16) with the defining property of an anchored game, i.e., $\mathbb{E}\xi_j(\lambda^*_j) = 0$. Part ii) then follows from (18). Strikingly, r's expected deviation from first-best depends exclusively on the gap between j's observable characteristic and r's, while r's EDFA depends both on this gap and n. To see why the latter is proportional to n, recall that r's objective is to shift the mean announcement by a magnitude k_r that is independent of n; the greater n is, the smaller is r's contribution to the mean, and hence, the more r must misreport. Note that the more r misreports, the more likely it is that she will be constrained by the announcement bounds. Proposition 8(ii) shows that the farther away a player's observable characteristic is from the anchor's, the larger in absolute value is her expected deviation from first-best, and from Prop. 6, the lower is her equilibrium payoff relative to other players.

5 Symmetric games

In this section, we study games that are symmetric in a strong sense. We say that the announcement space is symmetric if the announcement bounds are symmetric about zero (i.e., if $\underline{a} = -\bar{a}$), and that the type distribution is symmetric if $\underline{\theta} = -\overline{\theta}$. (Assumption A3(ii) then implies that θ is symmetrically distributed around its mean of zero.) We say that the observable characteristic vector $\mathbf{k}$ is symmetric if

$$\text{for every player } \bar{r} \text{ with } k_{\bar{r}} > 0, \text{ there exists a } \textit{matched player } \underline{r} \text{ with } k_{\underline{r}} = -k_{\bar{r}}. \tag{23}$$

We will refer to players whose observable characteristics are $\substack{\text{positive} \\ \text{negative}}$ as the $\substack{\textit{right-wing} \\ \textit{left-wing}}$ faction. There is in addition a *middle* player m with $k_m = 0$. We will refer to a game satisfying A1–A3 and the above symmetry conditions as a *symmetric aggregation game* (SAG).

Proposition 9 (NMPE of symmetric games)[†] *Every SAG has a unique NMPE satisfying*

$$\mathbb{E}\xi_r(\lambda_r^*) = -nk_r, \quad \textit{for all } r \textit{ with } \lambda_r \in int(\Lambda). \tag{24}$$

Moreover,

(1) $\lambda_m^* = 0$;
(2) *for each player* $\bar{r}$ *and matched player* $\underline{r}$, $\lambda_{\underline{r}}^* = -\lambda_{\bar{r}}^*$.

Only the middle player m announces truthfully in equilibrium. Assumption A3(i) implies that this player is never constrained; hence, every SAG is anchored (see Sect. 4.3). All players other than m misreport; their EDFA's are determined entirely by their observable characteristics and n.

It is immediate from Props. 6 and 9 that r's equilibrium expected payoff in a SAG is entirely determined by k_r and the average of the second moments of all players' deviations is from affine.

$$\mathbb{E}_{\boldsymbol{\vartheta}} u\left(\mu(\mathbf{s}^*), \mu(\boldsymbol{\vartheta}), k_r\right) = -\left(\mu(V\boldsymbol{\xi})/n + k_r^2\right). \tag{20$'$}$$

Moreover, Prop. 9 implies that the squared term in expression (22) is zero, so that USW depends *only* on the average second-moment effect, that is, the potentially negative first-moment effect of exaggeration by player $\bar{r}$ is fully offset by the equal and opposite first-moment effect of her matched player, $\underline{r}$. Specifically, in a SAG,

$$\text{USW} = -\,\mu\big(V\boldsymbol{\xi}(\boldsymbol{\lambda}^*)\big)/n. \tag{22$'$}$$

Prop. 9 is useful for analyzing the comparative statics of aggregation games. The parameters we study below are (1) the magnitude of the bounds on the announcement space (Sect. 5.1); (2) the heterogeneity of players' observable characteristics (Sect. 5.2); and (3) the number and composition of players (Sect. 5.3). Our analysis sheds light on the "design" of information aggregation mechanisms. In the language of our illustrative vignette—a board making an investment decision—we will investigate the effect on (a) the strategic misreporting of board members; (b) the truthfulness of aggregate reporting; and (c) overall welfare as (1) the board relaxes the restrictions on its allowable messages; (2) the heterogeneity of board members' biases increases; and (3) the board becomes larger.

Throughout this section, whenever we make a statement relating to either $\overline{\theta}$, $\bar{a}$, or $k_{\bar{r}}$, we will be implicitly also making the matching statement about $\underline{\theta}$, $\underline{a}$, or $k_{\underline{r}}$. Further, when we study the effect of increasing $\bar{a}$, we will be simultaneously, but implicitly, reducing $\underline{a}$ to preserve symmetry.

5.1 Effects of changing the announcement bounds

Since players strategically misreport their private information, one might expect that welfare would be enhanced by tightening the announcement bounds to eliminate extreme messages. But this intuition is incorrect. From Prop. 9, player r's EDFA, $\mathbb{E}\xi_r(\cdot)$, is independent of $\bar{a}$. If $k_r \neq 0$, then expression (14) implies that as $\bar{a}$ changes, λ_r must adjust so that $\mathbb{E}\xi_r(\cdot)$ remains equal to $-nk_r$. Specifically,

Proposition 10 (Effect of changing $\bar{a}$)[†] *In the unique NMPE of an SAG:*

$$\frac{\mathrm{d}\lambda_r}{\mathrm{d}\bar{a}} = \begin{cases} 0 & \text{if } r \text{ is the middle player} \\ 1 & \text{if } r \text{ is up-constrained} \\ -1 & \text{if } r \text{ is down-constrained} \\ \frac{(1-H(\tilde{\theta}_r))-H(\underset{\sim}{\theta}_r)}{H(\underset{\sim}{\theta}_r)+(1-H(\tilde{\theta}_r))} & \text{if } r \text{ is bi-constrained} \end{cases} \tag{25}$$

Proposition 10 implies that if r is single-constrained, she increases the degree of her misreporting at exactly the rate that the bounds are relaxed; she responds more slowly if she is bi-constrained.

Now, consider the welfare effects of relaxing the announcement bounds. First, suppose that the announcement space is inclusive, so that no player is bi-constrained. In this case, for the middle player m, $\xi_m(\cdot) = 0$ is identically zero, and thus, $\mathbb{E}\xi_m$ and $V\xi_m$ are independent of $\bar{a}$. For $r \neq m$, r is either up- or down-constrained: from (12) and (25), $\xi_r(\cdot)$ is invariant w.r.t. symmetric changes in $\bar{a}$ and $\underline{a}$, that is, players with $k \neq 0$ adjust their announcements to fully compensate for the change in the announcement bounds. Since, from Prop. 6, r's expected payoff is determined by the first moment of r's own deviation from affine, and the second moments of all players' deviations, changes in $\bar{a}$ and $\underline{a}$, have no impact on r's payoff, provided that the announcement space is inclusive.

This independence property no longer holds when at least one player's equilibrium strategy is bi-constrained. For some intuition for this difference, Fig. 2 considers the impact of relaxing the announcement bounds when only the middle player m is bi-constrained. Whenever m's type lies outside the interval $[\underline{a}, \bar{a}]$, obliging her to misreport her type, all players are negatively affected. The areas of the large triangles at either end of the type spectrum indicate the magnitude of the distortion of m's information. When the bounds are relaxed to $[\underline{a}', \bar{a}']$, the sizes of both triangles shrink, reflecting a decline in the variance of m's deviation from affine. Ex ante, this change

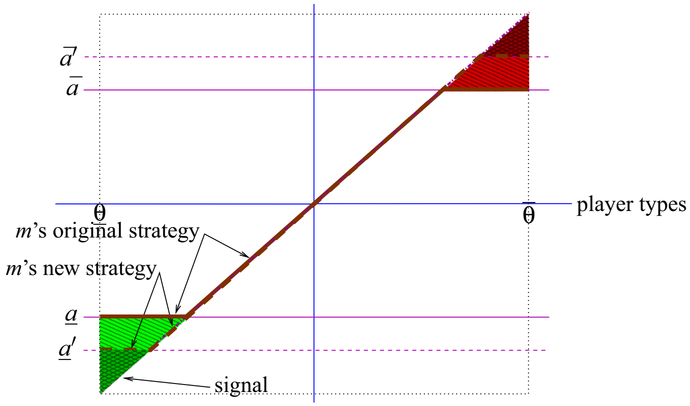

Fig. 2 Intuition for Prop. 11

benefits all players equally, since, from Prop. 6, each player's payoff is decreasing in the variance of *all* players' deviations from affine. Equation (42) in the proof of Prop. 11 provides an expression for the rate at which a bi-constrained player's variance declines as the bounds are relaxed. The more players who are initially bi-constrained, the greater is the collective benefit of a relaxation.

Proposition 11 (Effect of relaxing announcement bounds)† *In the unique NMPE of an SAG, as the announcement space expands:*

(1) *if initially the announcement space is inclusive, then the equilibrium expected payoff of every player remains constant;*

(2) *if initially some player is bi-constrained, then each player's equilibrium expected payoff is equally positively affected, as is the unbiased social welfare.*

Prop. 11 complements our discussion of Prop. 6 on p. 20. While misreporting unambiguously increases as the announcement bounds are relaxed, welfare is either unaffected or increases. This result highlights that it is deviations from affine rather than misreporting per se that reduce welfare. The proposition also delivers a strong normative message: at least from the perspective of information aggregation, the announcement space should be sufficiently large that it contains the type space. For real-world information-transmission problems in which the center is non-strategic, our results thus provide a justification in terms of aggregation for "free speech."

5.2 Effects of increasing player heterogeneity

We next study how player heterogeneity affects information aggregation and welfare. In particular, we study the impacts of increases in both inter- and intra-faction heterogeneities. In SAGs, Part iii) of Prop. 8 implies $\mathbb{E}\xi_r = -nk_r$. Substituting this identity into (14) and totally differentiating w.r.t. k_r and λ_r, we obtain

$$\frac{\mathrm{d}\lambda_r}{\mathrm{d}k_r} = \frac{n}{H(\underset{\sim}{\theta}_r) + (1 - H(\tilde{\theta}_r))} > n \tag{26}$$

where the denominator is the probability with which player r is constrained by the announcement bounds. For $k_r > 0$, as k_r increases and players become more heterogeneous, λ_r also increases and at a rate faster than k_r in order to maintain the equilibrium property that $\mathbb{E}\xi_r = -nk_r$. It now follows, by differentiating (14) and (19) w.r.t. λ_r, that as k_r increases, both $|\mathbb{E}\xi_r|$ and $V\xi_r$ also increase, reducing the payoff not only of r but of all other players as well.

Proposition 12 (Effect of dispersing players' observable characteristics)† *In the unique NMPE of an SAG, an increase in $|k_r|$ reduces each player's expected equilibrium payoff.*

This result is hardly surprising: as players become more heterogeneous, the extent of their misreporting increases and the announcement bounds become more restrictive, thereby reducing welfare. The impact of an increase in *intra-faction* heterogeneity is less intuitive. Recall that in an SAG there is a $\substack{\text{right-wing} \\ \text{left-wing}}$ faction with $\substack{k>0 \\ k<0}$ and that players are heterogeneous within each faction. We say that intra-faction heterogeneity increases when, within each faction, the extremists become more extreme and the moderates become more moderate. To explore the welfare impacts of this shift, we reduce notation by assuming, for the remainder of Sect. 5.2:

Assumption A4 (i) $[\underline{\theta}, \overline{\theta}] = [-1, 1]$, so that $h(\cdot) = 1/2$; (ii) $(n-1)$ is divisible by 4.

Let $\bar{\mathbf{k}}^+ \in (0, 1)^{(n-1)/2}$ be a strictly increasing vector, denoting the observable characteristics of the right-wing faction. Choose vectors $\boldsymbol{\alpha} \in \mathbb{R}_{++}^{(n-1)/4}$ and $\boldsymbol{dk} = (-\boldsymbol{\alpha}, \boldsymbol{\alpha}) \in \mathbb{R}^{(n-1)/2}$. We will consider a family of right-wing profiles of the form $\{\bar{\mathbf{k}}^+ + \gamma \boldsymbol{dk} : \gamma \gtrapprox 0\}$. The observable characteristics of the left-wing faction are implied by symmetry. An increase in γ represents a faction-mean-preserving spread of each faction's profile of observable characteristics. As γ increases in a neighborhood of zero,[23] the moderate members of the faction become more moderate—the dk's are negative for the first $(n-1)/4$ faction members, all of whom have k's below the faction's median—while the extreme members become more extreme. Prop. 13 below establishes the following effects of such a spread. If players' characteristics are initially quite homogeneous—specifically, contained in the interval $(-1/4(n-1), 1/4(n-1))$—the spread will reduce both USW and APW. On the other hand, if the factions are initially quite polarized—specifically, no player's characteristic belongs to $[-1/4(n-1), 1/4(n-1)]$—the spread will increase USW (though not necessarily APW).

Proposition 13 (Effect of a faction-mean-preserving spread of observable characteristics)† *Let USW(γ) and APW(γ) denote, respectively, unbiased social welfare and aggregate private welfare for the unique NMPE of the n-player SAG satisfying Assumption A4, whose right-wing faction has the profile of observable characteristics $\bar{\mathbf{k}}^+ + \gamma \boldsymbol{dk}$.*

[23] If γ is sufficiently small and $\mathbf{k}$ is strictly monotone, the perturbed vector $\bar{\mathbf{k}}^+ + \gamma \boldsymbol{dk}$ will be also strictly monotone.

(1) *if* $\max(\bar{\mathbf{k}}^+) < 1/4(n-1)$, *then* $\left.\frac{\mathrm{d}USW(\gamma)}{\mathrm{d}\gamma}\right|_{\gamma=0} < 0$ *and* $\left.\frac{\mathrm{d}APW(\gamma)}{\mathrm{d}\gamma}\right|_{\gamma=0} < 0$

(2) *if* $\min(\bar{\mathbf{k}}^+) > 1/4(n-1)$, *then* $\left.\frac{\mathrm{d}USW(\gamma)}{\mathrm{d}\gamma}\right|_{\gamma=0} > 0$

To obtain intuition for this surprising result, we return to Fig. 1 and set $j = \underline{r}$, with $k_{\underline{r}} < 0$. Intuitively, the magnitude of $V\xi_{\underline{r}}$ increases with the area of the triangle at the bottom left of the figure. This area increases with the square of $\underline{r}$'s low threshhold type, $\underset{\sim}{\theta}_r$, i.e., $V\xi_{\underline{r}}$ is convex in $\underline{r}$'s threshhold type. On the other hand, in a symmetric game with a uniform distribution over types, $\underline{r}$'s threshhold type is a concave function of $\underline{r}$'s EDFA. The curvature of the convolution relating $V\xi_{\underline{r}}$ to $k_{\underline{r}}$ depends on the balance between these two effects.

5.3 Effects of changing the size of the game

Until now, we have considered games in which the dimensionality of the state of nature $\boldsymbol{\theta}$ has been fixed. We now allow this dimensionality to vary and consider the possibility that information is collected about only some of the dimensions. Specifically, we assume $\boldsymbol{\theta} \in \mathbb{R}^N$ and let $\{\mathcal{N}_1, \mathcal{N}_2\}$ be a partition of $\{1,\ldots, N\}$, with $n = \#\mathcal{N}_1$. We assume that $\mathcal{N}_1$ is symmetric in the sense of (23) and replace n with N in the statement of A3(iii), so that $||\mathbf{k}||_\infty < \overline{\theta}/2N$. For each component θ_i of $\boldsymbol{\theta}$ such that $i \in \mathcal{N}_2$, there is no corresponding player. To motivate this structure, consider again our vignette of a board of directors analyzing an investment project. Uncertainty relating to the project's success has N dimensions. Each of the n board members is assigned exactly one dimension to investigate; since $n \leq N$, there may be dimensions of uncertainty that are not "covered" by the board. We refer to such games as *partially covered* games and the dimensions in $\mathcal{N}_2$ as *uncovered dimensions*. In this section, we study the impacts on equilibrium strategies and welfare in partially covered games of varying n. For example, our original board could be augmented (increasing n) in order to obtain information about more dimensions of the project. This would enable the center to make a more informed decision and increase welfare. However, as n increases, more players will engage in the tug-of-war of exaggeration and counter-exaggeration. This would increase the first and second moments of players' deviations from affine and thus lower welfare. Since the true level of information is $\mu(\boldsymbol{\theta}) = \sum_{i=1}^{2}\sum_{j\in\mathcal{N}_i}\theta_j/N$, the CISE outcome in this context [cf. (4)] is given by

$$t(\mathbf{a}) = \frac{\sum_{i\in\mathcal{N}_1} a_i + \sum_{k\in\mathcal{N}_2}\theta_k^e}{N} = \mu_1(\mathbf{a}) + \mu_2(\boldsymbol{\theta}^e) \tag{4$'$}$$

where $\theta_k^e = \mathbb{E}\theta_k$, $\mu_1(\mathbf{a}) = \frac{\sum_{i\in\mathcal{N}_1} a_i}{N}$, and $\mu_2(\boldsymbol{\theta}^e) = \frac{\sum_{k\in\mathcal{N}_2}\theta_k^e}{N}$. In equilibrium, each player still adopts a CUA strategy; for $r \in \mathcal{N}_1$, the level of misreporting is given by

$$\lambda_r = Nk_r + \sum_{i\in\mathcal{N}_1\backslash\{r\}} \mathbb{E}_{\vartheta_i}\left(\vartheta_i - s_i(\vartheta_i)\right). \tag{7$''$}$$

Since the center averages all N dimensions, r's level of exaggeration increases from nk_r to Nk_r [cf. (7)], but her level of counter-exaggeration depends only on n. The analog of Prop. 9 [cf. (24)] is now

$$Nk_r = -\mathbb{E}\xi_r(\lambda_r^*) \quad \text{for all } r. \tag{24$'$}$$

From the equilibrium strategies in (7$''$), we obtain the USW measure: [cf. (22$'$)]:

$$\text{USW} = -\left\{\mu_1(V\boldsymbol{\xi}(\boldsymbol{\lambda}^*))/N + \text{Var}(\theta)\,(1 - {}^{n}/_{N})\right\} \tag{22$''$}$$

The first and second terms are, respectively, the welfare losses that result from misreporting by strategic players and those from "uncovered" dimensions due to the lack of information. Prop. 14 further exploits symmetry to obtain a closed-form expression for (22$''$):[24]

Proposition 14 (Welfare in Partially Covered Games)† *For partially covered SAGs, USW is given by*

$$\begin{aligned} USW &= -\sum_{i\in\mathcal{N}_1} k_i^2\left(\sqrt{\frac{8}{9Nh|k_i|}} - 1\right) - \frac{\overline{\theta}^2}{3}\left(1 - \frac{n}{N}\right) \\ &\le -\sum_{i\in\mathcal{N}_1} k_i^2\left(\frac{4\sqrt{2}}{3} - 1\right) - \frac{\overline{\theta}^2}{3}\left(1 - \frac{n}{N}\right) < 0 \end{aligned} \tag{27}$$

Consider an increase in n while holding N fixed, so that more uncovered dimensions are covered by strategic players. From (27), USW rises at the rate of $\overline{\theta}^2/3N$, as information is obtained about more dimensions, but decreases at the rate of $k_i^2\left(\sqrt{\frac{8}{9Nh|k_i|}} - 1\right)$ as more players misreport their information and the variances of players' deviations from affine increase. Strikingly, as Prop. 15 proves, the welfare gains unambiguously outweigh the welfare losses:

Proposition 15 (Effect of adding players)† *Given assumptions A1–A3, $k_i^2\left(\sqrt{\frac{8}{9Nh|k_i|}} - 1\right) < \overline{\theta}^2/3N$, so that in partially covered SAGs with $N > n$, USW rises as n increases holding N fixed.*

Thus, increasing n while holding N fixed always improves welfare from an information perspective, even when the additional players act strategically. The explanation for this is apparent from an examination of (22$''$). Since the new players' strategies are non-degenerate (Remark 1), each new player provides *some* information about the dimension that she covers, which is an improvement on the zero information that is obtained if the dimension remains uncovered. In other words, the second-moment effect contributed by even a strategic player is necessarily of smaller magnitude than the variance of θ_i.

[24] Since the distribution of θ is uniform, and $\overline{\theta} + \underline{\theta} = 0$ in a SAG, $\text{Var}(\theta) = \overline{\theta}^2/3$.

6 Single-bounded games

In many applications, the announcement space is naturally bounded on one side but not the other. Most obviously, this occurs when announcements are restricted to be nonnegative but there is no natural upper bound. (For example, our board members might be reporting prices, interest rates, or the variances of some privately observed statistic.) We refer to games satisfying this condition as *single-bounded aggregation games*. In this context, it would be natural to model the upper bound on actions in a single-bounded game as infinite. However, to maintain consistency with the framework in Sect. 3, we impose an artificial upper bound that will never bind. Since, from (7), no player's equilibrium announcement will exceed $n\left(\max(\mathbf{k}) + \overline{\theta} - \underline{a}\right) + \underline{a}$, we impose w.l.o.g. in Sect. 6

Assumption A5 $\underline{a} = \underline{\theta} = 0$, and $\bar{a} = n\left(\max(\mathbf{k}) + \overline{\theta}\right)$.

A5 implies that the announcement space of a single-bounded game is inclusive. Further, since the upper bound $\bar{a}$ is never binding, $\xi_r(\cdot) \geq 0$ (from 12). Hence, A5, together with (15), implies

$$\lambda_r \geq 0 \Longrightarrow \mathbb{E}\xi_r(\lambda_r) = 0. \tag{28}$$

In a single-bounded game, a key role is played by the player h, whose observable characteristic exceeds that of any other player. Since $\sum_i k_i = 0$, k_h is necessarily positive.

Proposition 16 (Single-Bounded Games)† *Every single-bounded aggregation game satisfying A1, A2, and A5 has a unique MPE $\boldsymbol{\lambda}^*$ in which $\lambda_h^* \geq 0$ and $\mathbb{E}\xi_h(\lambda_h^*) = 0$. Moreover, for all $r \neq h$, $\lambda_r^* \in int(\Lambda)$ implies $\mathbb{E}\xi_r(\lambda_r^*) = n(k_h - k_r) > 0$.*

Since $\mathbb{E}\xi_h(\lambda_h^*) = 0$, every single-bounded game is anchored, with h as the anchor. While aggregation games satisfying Assumption A5 look and feel quite different from the SAGs studied in Sect. 5, the comparative statics properties we obtain in Sect. 5 and in this section are remarkably similar, at least for games in which the spread of $\mathbf{k}$ is small enough relative to n that an NMPE exists.[25] The similarity of the properties they exhibit reflects the dominant role played by the anchor. We begin by characterizing the equilibrium of an arbitrary single-bounded game and then discuss its comparative statics properties. To avoid repetition, no formal results will be presented; we merely relate these properties to the corresponding ones in Sect. 5.

Since $\mathbb{E}\xi_h = 0$, (18) implies that the equilibrium outcome implements h's first-best outcome in expectation. Since $\underline{a} = \underline{\theta}$, $\mathbb{E}\xi_r > 0$ for $r \neq h$ implies $\lambda_r < \underline{a} - \underline{\theta} = 0$, that is, every other player, even including ones whose observable characteristicsare very close to h's, will underreport to counteract h's extreme overreporting. Indeed, from (7′)

[25] It is straightforward to identify conditions analogous to Assumption A3(iii) that guarantee the existence of an NMPE. To save space, we leave this as an exercise for the reader.

and Prop. 8,[26] $\sum_i \lambda_i = n(1-n)k_h < 0$, i.e., when $n > 2$, h's overreporting is more than compensated by the sum of all other players' (unconstrained) underreporting. Since player $r \neq h$ is constrained by the lower bound $\underline{a}$, her expected first-best outcome differs from the expected equilibrium outcome. From Prop. 16, r's EDFA, $\mathbb{E}\xi_r = n(k_h - k_r)$, is greater, the more different is r's characteristic from h's.

To compute USW in a single-bounded game, we first note from (7′) that

$$\lambda_h^* = nk_h - \sum_{i \neq h}(\mathbb{E}\xi_i(\lambda_i^*) + \lambda_i^*). \tag{7'''}$$

Substituting this expression for λ_h^* into (22), we obtain

$$\text{USW} = -\left(k_h^2 + \mu\big(V\boldsymbol{\xi}(\boldsymbol{\lambda}^*)\big)\right)/n. \tag{22'''}$$

There is a striking similarity between (22‴) and the corresponding expression (22′) for SAGs. Except for the k_h term, the first-moment effects in single-bounded games cancel each other out; the only remaining source of welfare loss is the n-deflated average of the second-moment effects. Note, further, that since $k_j = 0$ for the anchor $j = m$ in an SAG, we can write, for either class of anchored game, $\text{USW} = -\left(k_j^2 + \mu(V\boldsymbol{\xi}(\boldsymbol{\lambda}^*))\right)/n$. The difference between the two classes is that in an SAG, each player's first-moment effect is offset by her matched player's, while in a single-bounded game, the anchor offsets unilaterally *all* the other players first-moment effects.

6.1 Effects of changing the announcement bound

Suppose the lower announcement bound, $\underline{a}$, decreases, holding $\underline{\theta}$ constant at zero, ensuring that the announcement space remains inclusive. The effects of this change are identical to those discussed in Sect. 5.1: each player's strategy adjusts to hold constant the first and second moments of her deviation from affine. The equilibrium outcome remains unchanged, as do all the other players' expected payoffs.

6.2 Effects of increasing player heterogeneity

Once again, the effects here are qualitatively similar to the effects described in Sect. 5.2. In the present context, we interpret an increase in heterogeneity as an increase in all components of the gap vector $\boldsymbol{\Delta k} = (k_h - k_i)_{i \neq h}$. Such a change unambiguously lowers all players' expected payoffs and USW. The proof closely parallels the proof of Prop. 12. Again, it is more interesting to consider the impact of an increase in

[26] Using (7′), then Prop. 8, and finally, Assumption A1(i), we obtain

$$nk_h = \sum_i \lambda_i + \sum_{i \neq h} \mathbb{E}\xi_i(\lambda_i) = \sum_i \lambda_i + n\sum_{i \neq h}(k_h - k_i) = \sum_i \lambda_i + n^2 k_h.$$

intra-faction heterogeneity—for example, the effect of a mean-preserving spread of $\mathbf{\Delta k}$. If we impose Assumption A3 and let $[\underline{\theta}, \overline{\theta}] = [0, 1]$, the result we obtain is very similar to Prop. 13. If the largest element of $\mathbf{\Delta k}$ is less than $1/4(n-1)$, USW declines with a mean-preserving spread of $\mathbf{\Delta k}$. On the other hand, if the smallest element is greater than $1/4(n-1)$, USW increases.

6.3 Effects of changing the size of the game

Finally, we consider the effect in a single-bounded, partially covered game of varying the numbers n_1 and n_2 of strategic and non-strategic players as well as the dimensionality N of the state of nature. For consistency, we assume that the identity of the anchor player h remains constant throughout. The effects of these variations are similar in most respects to the effects analyzed in Sect. 5.3. As in an SAG, r's EDFA is proportional to N. In this case, if $\boldsymbol{\lambda}^*$ is an equilibrium profile, then $\mathbb{E}\xi_r(\lambda_r^*) = N(k_h - k_r) > 0$. The expression for r's expected payoff is identical to the right-hand side of (20′), except that the k_rs are replaced by $(k_h - k_r)$s. The comparative statics of USW and expected payoffs w.r.t. n_1 and N are comparable to those summarized in Prop. 15. The one striking difference between symmetric and single-bounded games concerns the strategic role played by the anchor player. An SAG is anchored by the middle player m, whose role is entirely passive: regardless of who else is playing the game, $\lambda_m^* = 0$. A single-bounded game is anchored by player h, whose strategy λ_h^* plays a pivotal equilibrating role. For $r \neq h$, r's expected deviation from first-best is positive, and independent of n, even though as n increases, each new player contributes an additional downward bias to the mean report (i.e., $r \neq h \implies \lambda_r^* < 0$). This independence property holds because, as noted above (p. 31), the anchor h single-handedly balances the sum of all other players' negative biases. More precisely, we replace n by N in (7‴) to obtain $\lambda_h = Nk_h - \sum_{i \neq h} (\mathbb{E}\xi_i(\lambda_i) + \lambda_i)$; since the number of terms in the summation increases, and each one is negative, λ_h increases super-proportionally as N increases.

7 Summary

This paper contributes to the literature on information aggregation. When private information is aggregated, or multiple expert opinions are solicited, incentives arise for agents to exaggerate or to counter-exaggerate, in order to offset the exaggeration by other players. While the former concept has been addressed in the literature, the latter has not. We analyze the incentives for both tendencies in a model with multiple heterogeneous information providers and a single recipient. Three features distinguish our framework from others in the literature: players' reports are aggregated by a mechanistic averaging rule, their strategy sets are intervals rather than binary choices, and they are ex ante heterogeneous. Our model can be applied to a wide range of institutional settings, including committees, media exchanges, judges' panels, online review sites, and many others.

In our model, the "center" is restricted by governance structures to be non-strategic, as often happens in real-world settings such as faculty committees, corporate boards, and governmental working groups. This restriction renders it critically important to select "appropriate" committee or board members whenever the center can do so.[27] While our model assumes a passive center that takes both the reports and the membership of the players as given, our results on player heterogeneity suggest guidelines for how committee members might be selected. For instance, suppose $m < n$ members of a committee with known biases are already on board, and the center needs to select the remaining $n-m$ members. Sections 5.2 and 6.2 imply that it is not necessarily true that the center should select members with the lowest degrees of biases. Instead, the $n - m$ members should be chosen to minimize the overall heterogeneity of the entire committee. While this translates into selecting (symmetric) member pairs with the lowest biases for symmetric games, for single-bounded games, the remaining members should be chosen, so that their biases are as close as possible to the most biased of the m existing members, i.e., the one with the highest bias k.

8 Appendix: Proofs

Proof of Proposition 1 To prove the proposition, we apply Theorems 1 and 2 of Athey (2001). The first of these theorems is used to establish existence for finite-action aggregation games. The second implies existence for general aggregation games. To apply Athey's first theorem, we define a *finite-action aggregation game* to be one in which players are restricted to choose actions from a finite subset of A. In all other respects, finite-action aggregation games are identical to (infinite action) aggregation games. We now check that u satisfies Athey's Assumption A1. Clearly, our types have joint density w.r.t. Lebesgue measure, which is bounded and atomless. Moreover, the integrability condition in Athey's A1 is trivially satisfied, since u is bounded. Moreover, inequality (6) implies that the SCC holds. Therefore, every finite-action aggregation game has an MPE in which player r's equilibrium strategy s_r is non-decreasing. By Athey's Theorem 2, the restricted game has an MPE, call it $\mathbf{s}^*$. To show that $\mathbf{s}^*$ will also be an equilibrium for the original, unrestricted game, it suffices to show that for all r, all θ and all $a > \bar{a}$, $\frac{\partial U_r(a,\theta;\mathbf{s}^*_{-r})}{\partial a} < 0$. To establish this, note that $\mathbf{s}^*_{-r} \geq \underline{\mathbf{s}}_{-r} \geq 0$, so that since t is strictly increasing, $a > \bar{a}$ implies

$$U'_r(a, \theta; \mathbf{s}^*_{-r}) < U'_r(\bar{a}, \theta; \mathbf{s}^*_{-r}) \leq U'_r(\bar{a}, \theta; \underline{\mathbf{s}}_{-r}) \leq U'_r(\bar{a}, \theta; 0) \leq 0$$

Finally, to establish that s_r is strictly increasing and continuously differentiable on $(\underset{\sim}{\theta}_r(\mathbf{s}), \overline{\theta}]$, note that $U'_r(s_r(\cdot), \cdot; \mathbf{s}_{-r}) = 0$ on $(\underset{\sim}{\theta}_r(\mathbf{s}), \overline{\theta}]$. From (6), Assumption A2 and the implicit function theorem, we have, for all $\theta \in (\underset{\sim}{\theta}_r(\mathbf{s}), \overline{\theta}]$, $\frac{\mathrm{d}s_r(\theta)}{\mathrm{d}\theta} = -\frac{\partial^2 U_r(s_r(\theta),\theta;\mathbf{s}_{-r})}{\partial a \partial \theta} \Big/ \frac{\partial^2 U_r(s_r(\theta),\theta;\mathbf{s}_{-r})}{\partial a^2} > 0.$ □

Proof of Proposition 2 Suppose that $\mathbf{s}$ is a non-monotone pure-strategy profile, that is, for some r, and some θ_r, as well as $\delta > 0$ with $\underset{\sim}{\theta}_r(\mathbf{s}) < \theta_r$ and $\theta_r + \delta < \tilde{\theta}_r(\mathbf{s})$, we have $da = s_r(\theta + \delta) - s_r(\theta) \leq 0$. Since, clearly, $U_r(\cdot, \cdot; \mathbf{s}_{-r})$ is twice continuously

[27] We thank an anonymous referee for this insightful observation.

differentiable, it follows from the Taylor–Lagrange theorem that there exists $\zeta \in [0, 1]$ such that

$$U_r'\big(s_r(\theta+\delta), \theta+\delta; \mathbf{s}_{-r}\big) - U_r'\big(s_r(\theta), \theta; \mathbf{s}_{-r}\big)$$
$$= \nabla U_r'\big(s_r(\theta)+\zeta\, da, \theta+\zeta\,\delta; \mathbf{s}_{-r}\big)\cdot(da, \delta) \geq \frac{\partial^2 U_r\big(s_r(\theta)+\zeta\, da, \theta+\zeta\,\delta; \mathbf{s}_{-r}\big)}{\partial a \partial\theta}\,\delta \geq 0$$

The weak inequality holds because $da \leq 0$, and from Assumption A2 and (4), $U_r(\cdot, \cdot; \mathbf{s}_{-r})$ is strictly concave in r's action. The strict inequality follows from (6), given that $\delta > 0$. But optimality requires that $U_r'\big(s_r(\cdot), \cdot; \mathbf{s}_{-r}\big) \equiv 0$ on $(\underset{\sim}{\theta}_r(\mathbf{s}), \tilde{\theta}_r(\mathbf{s}))$. Hence, $\mathbf{s}$ cannot be an equilibrium profile. □

Proof of Proposition 5 We will prove uniqueness only for non-degenerate equilibrium profiles. Uniqueness for other profiles is ensured by restriction (9), but we omit the details. Let $\boldsymbol{\lambda}^*$ be an NMPE for the aggregation game, and let $\boldsymbol{\lambda}$ be any other profile of strategies such that for some j, $\lambda_j \neq \lambda_j^*$. We will show that if $\boldsymbol{\lambda}$ satisfies the necessary condition (16), then it fails the other necessary condition (7′).

We first establish a property of EDFA that will be useful. Differentiating (14) w.r.t. λ_r and inferring from (10) that $H(\underset{\sim}{\theta}_r) < H(\tilde{\theta}_r)$, we obtain

$$\frac{\mathrm{d}\mathbb{E}\xi_r}{\mathrm{d}\lambda_r} = -\left(H(\underset{\sim}{\theta}_r) + 1 - H(\tilde{\theta}_r)\right) \subset (-1, 0] \tag{29}$$
$$\text{and} \quad \frac{\mathrm{d}\mathbb{E}\xi_r}{\mathrm{d}\lambda_r} = 0 \qquad \text{iff } r \text{ is almost never constrained}$$

Suppose w.l.o.g. that $\lambda_j > \lambda_j^*$. From (29), $\mathbb{E}\xi_j(\lambda_j) < \mathbb{E}\xi_j(\lambda_j^*)$. For all $r \neq j$, (16) implies that $\mathbb{E}\xi_r(\lambda_r) < \mathbb{E}\xi_r(\lambda_r^*)$, and (29), in turn, implies that $\lambda_r > \lambda_r^*$. To establish that $\boldsymbol{\lambda}$ cannot satisfy (7′), it suffices to show that

$$\left(\sum_i \lambda_i + \sum_{i\neq j}\mathbb{E}\xi_i(\lambda_i)\right) > \left(\sum_i \lambda_i^* + \sum_{i\neq j}\mathbb{E}\xi_i(\lambda_i^*)\right) = nk_j$$

or, equivalently,

$$\lambda_j - \lambda_j^* + \sum_{i\neq j}\left(\lambda_i - \lambda_i^*\right) > \sum_{i\neq j}\left(\mathbb{E}\xi_i(\lambda_i^*) - \mathbb{E}\xi_i(\lambda_i)\right)$$

This last inequality is indeed satisfied, since by assumption, $\lambda_j > \lambda_j^*$, while (29) implies that for all $i \neq j$, $\lambda_i - \lambda_i^* > \mathbb{E}\xi_i(\lambda_i^*) - \mathbb{E}\xi_i(\lambda_i)$.

For the monotonicity of MPE, we prove $\lambda_i^* > \lambda_j^*$ by contradiction. Suppose $\lambda_i^* \leq \lambda_j^*$. The fact that $\underset{\sim}{\theta}_r(\lambda_r) = \underline{a} - \lambda_r$ and $\tilde{\theta}_r(\lambda_r) = \bar{a} - \lambda_r$ imply that $\underset{\sim}{\theta}_i(\lambda_i^*) \geq \underset{\sim}{\theta}_j(\lambda_j^*)$ and $\tilde{\theta}_i(\lambda_i^*) \geq \tilde{\theta}_j(\lambda_j^*)$. Then, from (14), we know that $\mathbb{E}\xi_i(\lambda_i^*) \geq \mathbb{E}\xi_j(\lambda_j^*)$. But (16) and $k_i > k_j$ imply that $\mathbb{E}\xi_i(\lambda_i^*) < \mathbb{E}\xi_j(\lambda_j^*)$. □

Proof of Proposition 6 We first establish a property of $\mathbb{E}\boldsymbol{\xi}$. Aggregating the identity in (13) across players and rearranging, we obtain

$$\mathbb{E}_{\boldsymbol{\vartheta}}\big(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)-\mu(\boldsymbol{\vartheta})\big)=\mu\big(\boldsymbol{\lambda}^*\big)+\mu(\mathbb{E}\boldsymbol{\xi}). \tag{30}$$

Let $\xi_r^*=\xi_r(\lambda_r^*)$. Expanding the left-hand side of (20), we obtain

$$\begin{aligned}\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu(\boldsymbol{\vartheta})+k_r-\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)\right)^2 &= \mathbb{E}_{\boldsymbol{\vartheta}}\big(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)-\mu(\boldsymbol{\vartheta})-k_r\big)^2\\ &= \mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)-\mu(\boldsymbol{\vartheta})\right)^2-2k_r\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)\right.\\ &\quad\left.-\mu(\boldsymbol{\vartheta})\right)+k_r^2\\ &= \mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)-\mu(\boldsymbol{\vartheta})\right)^2-2k_r\big(\mu\big(\mathbb{E}\boldsymbol{\xi}^*\big)\\ &\quad+\mu\big(\boldsymbol{\lambda}^*\big)\big)+k_r^2\end{aligned} \tag{31}$$

The last equality follows from (30). Expanding the first term on the right-hand side of (31),

$$\begin{aligned}\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})\big)-\mu(\boldsymbol{\vartheta})\right)^2 &= \mathbb{E}_{\boldsymbol{\vartheta}}\Big(\mu\big(\underbrace{\mathbf{s}^*(\boldsymbol{\vartheta})-(\boldsymbol{\vartheta}+\boldsymbol{\lambda}^*)}_{\boldsymbol{\xi}^*}\big)+\mu\big(\boldsymbol{\lambda}^*\big)\Big)^2\\ &= \mathbb{E}_{\boldsymbol{\vartheta}}\big(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})-(\boldsymbol{\vartheta}+\boldsymbol{\lambda}^*)\big)\big)^2\\ &\quad+2\mu\big(\boldsymbol{\lambda}^*\big)\mu\big(\mathbb{E}\boldsymbol{\xi}^*\big)+\mu\big(\boldsymbol{\lambda}^*\big)^2\end{aligned} \tag{32}$$

The first equality merely adds and subtracts $\mu(\boldsymbol{\lambda}^*)$ and rearranges terms; the second averages both sides of the identity in (13). Now, expand the first term in (32) to obtain

$$\begin{aligned}\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\big(\mathbf{s}^*(\boldsymbol{\vartheta})-(\boldsymbol{\vartheta}+\boldsymbol{\lambda}^*)\big)\right)^2 &= \left(\sum_i\mathbb{E}_{\vartheta_i}\left(s_i^*(\vartheta_i)-\big(\vartheta_i+\lambda_i^*\big)\right)^2\right.\\ &\quad\left.+\sum\sum\nolimits_{i\neq j}\mathbb{E}\xi_i^*\mathbb{E}\xi_j^*\right)/n^2\\ &= \left(\sum_i V\xi_i^*+\sum_i\big(\mathbb{E}\xi_i^*\big)^2+\sum\sum\nolimits_{i\neq j}\mathbb{E}\xi_i^*\mathbb{E}\xi_j^*\right)/n^2\\ &= \left(\sum_i V\xi_i^*+\left[\sum_i\mathbb{E}\xi_i^*\right]^2\right)/n^2\\ &= \mu\big(\mathbf{V}\,\boldsymbol{\xi}^*\big)/n+\big(\mu\big(\mathbb{E}\boldsymbol{\xi}^*\big)\big)^2\end{aligned} \tag{33}$$

The first equality is obtained by expanding $\mu(\boldsymbol{\vartheta}+\boldsymbol{\lambda}^*-\mathbf{s}^*(\boldsymbol{\vartheta}))$; the second is from the relationship $\mathbb{E}(X^2)=\mathrm{Var}(X)+(EX)^2$ for a random variable X. Now, substituting (33) back into (32),

$$\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu\left(\mathbf{s}^*(\boldsymbol{\vartheta})\right)-\mu(\boldsymbol{\vartheta})\right)^2=\mu\big(\mathbf{V}\,\boldsymbol{\xi}^*\big)/n+\left(\mu\big(\mathbb{E}\boldsymbol{\xi}^*\big)+\mu\big(\boldsymbol{\lambda}^*\big)\right)^2 \tag{34}$$

Finally, substitute (34) back into (31) to obtain

$$\begin{aligned}\mathbb{E}_{\boldsymbol{\vartheta}}\left(\mu(\boldsymbol{\vartheta})+k_r-\mu\left(\mathbf{s}^*(\boldsymbol{\vartheta})\right)\right)^2&=\mu\big(\mathbf{V}\,\boldsymbol{\xi}^*\big)/n+\left(\mu\big(\mathbb{E}\boldsymbol{\xi}^*\big)+\mu\big(\boldsymbol{\lambda}^*\big)-k_r\right)^2\\&=\mu\big(\mathbf{V}\,\boldsymbol{\xi}^*\big)/n+(\mathbb{E}\xi_r^*/n)^2\end{aligned}$$

The last equality is obtained by adding $\mathbb{E}\xi_r^*/n$ to both sides of (7′) and substituting for k_r. □

Proof of Remark 1 To verify that A3(iii) guarantees non-degeneracy, it suffices to check that $\mathbb{E}\xi_r(\lambda_r^*)=n(k_j-k_r)$ is consistent with $\lambda_r^*\in\text{int}(\Lambda)$. Assuming w.l.o.g. that $\lambda_r^*>0$, (14) implies

$$\mathbb{E}\xi_r(\lambda_r^*)=\int_{\bar{a}-\lambda_r^*}^{\overline{\theta}}\big(\vartheta_r+\lambda_r^*-\bar{a}\big)\,\mathrm{d}h(\vartheta_r)=\quad 0.5h\Big(\lambda_r^*+\overline{\theta}-\bar{a}\Big)^2$$

$$\text{so that}\quad\lambda_r^*+\overline{\theta}-\bar{a}=\sqrt{\frac{2}{h}\mathbb{E}\xi_r}=\sqrt{\frac{2}{h}n(k_j-k_r)}\quad\text{if }\lambda_r^*\in\text{int}(\Lambda) \tag{35}$$

The last equality follows from Prop. 8. Also, from A3(iii),

$$2n(k_j-k_r)/h\le\frac{4n}{h}||\mathbf{k}||_\infty<\frac{4n}{h}(\overline{\theta}-\underline{\theta})/4n=(\overline{\theta}-\underline{\theta})^2$$

so that $\lambda_r=\sqrt{\frac{2}{h}n(k_j-k_r)}-(\overline{\theta}-\bar{a})<\big((\overline{\theta}-\underline{\theta})-(\overline{\theta}-\bar{a})\big)=\bar{a}-\underline{\theta}$, verifying that $\lambda\in\text{int}(\Lambda)$. □

Proof of Proposition 9 The existence of a unique MPE was established in Prop. 5. Non-degeneracy is implied by Assumption A3(iii) (see p. 22). Consider $\boldsymbol{\lambda}^*$ such that $\mathbb{E}\xi_r(\lambda_r^*)=-nk_r$ for all r with $\lambda_r\in\text{int}(\Lambda)$, and Parts) and) of the Proposition are satisfied. Our symmetry conditions ensure that such a vector exists, i.e., that if $\bar{r}$ and $\underline{r}$ are matched players, if $\lambda_{\underline{r}}^*=-\lambda_{\bar{r}}^*$, and $\mathbb{E}\xi_{\bar{r}}(\lambda_{\bar{r}}^*)=-nk_{\bar{r}}$, it follows from symmetry, (13), and (14) that $\mathbb{E}\xi_{\underline{r}}(\lambda_{\underline{r}}^*)=-nk_{\underline{r}}$. With the restrictions in (8), we only need to verify that (7′) is satisfied by $\boldsymbol{\lambda}^*$. Since $\sum_i k_i=0$ (Assumption A1), we have

$$-nk_r=\sum_{i\neq r}nk_i=-\sum_{i\neq r}\mathbb{E}\xi_i(\lambda_i^*) \tag{36}$$

Moreover, from Parts) and) of the proposition, $\sum_i\lambda_i^*=0$. Substituting this property and (36) into the right-hand side of (7′), we obtain

$$\sum_i\lambda_i^*+\sum_{i\neq r}\mathbb{E}\xi_i(\lambda_i^*)=nk_r,$$

verifying that (7′) is indeed satisfied. □

Proof of Proposition 10 From Prop. 9, we have

$$\mathbb{E}\xi_r \equiv \int_{\underline{\theta}}^{\underset{\sim}{\theta}_r} (-\bar{a} - (\theta_r + \lambda_r)) dH(\theta_r) + \int_{\tilde{\theta}_r}^{\overline{\theta}} (\bar{a} - (\theta_r + \lambda_r)) dH(\theta_r) \equiv -nk_r, \quad (37)$$

where, in the first integration, we substituted in $\underline{a} = -\bar{a}$. Totally differentiating both sides with respect to $\bar{a}$ and λ_r and noting that $\underset{\sim}{\theta}_r = \underline{a} - \lambda_r = -\bar{a} - \lambda_r$ and $\tilde{\theta}_r = \bar{a} - \lambda_r$, we obtain

$$\left[H(\underset{\sim}{\theta}_r) - (1 - H(\tilde{\theta}_r))\right] + \left[(H(\underset{\sim}{\theta}_r) + (1 - H(\tilde{\theta}_r))\right] \frac{\mathrm{d}\lambda_r}{\mathrm{d}\bar{a}} = 0.$$

Hence, $\frac{\mathrm{d}\lambda_r}{\mathrm{d}\bar{a}} = \frac{(1-H(\tilde{\theta}_r)) - H(\underset{\sim}{\theta}_r)}{H(\underset{\sim}{\theta}_r) + (1-H(\tilde{\theta}_r))}$. When r is bi-constrained, $H(\underset{\sim}{\theta}_r)$ and $H(\tilde{\theta}_r)$ are both nonzero, so that $\frac{\mathrm{d}\lambda_r}{\mathrm{d}\bar{a}} \in (0, 1)$. When r is up-constrained (respectively, down-constrained), $H(\underset{\sim}{\theta}_r) = 0$ (respectively, $H(\tilde{\theta}_r) = 1$), so that $\frac{\mathrm{d}\lambda_r}{\mathrm{d}\bar{a}}$ reduces to 1 (respectively, -1). If r is the middle player, $\lambda_r = 0$, and, since everything is symmetric, $H(\underset{\sim}{\theta}_r) = 1 - H(\tilde{\theta}_r)$ so that $\frac{\mathrm{d}\lambda_r}{\mathrm{d}\bar{a}} = 0$. □

Proof of Proposition 11 Since Part i) of the Proposition follows immediately from the discussion below Prop. 10, we need only to prove in detail Part ii). Specifically, letting I^* be the set of players who are bi-constrained in equilibrium, we will show that player r's expected payoff increases by $-\frac{1}{n^2}\sum_{i \in I^*} \frac{\mathrm{d}V\xi_i}{\mathrm{d}\bar{a}}$, where

$$\frac{\mathrm{d}V\xi_i}{\mathrm{d}\bar{a}} = \frac{4}{H(\underset{\sim}{\theta}_i) + (1 - H(\tilde{\theta}_i))} \left\{ (1 - H(\tilde{\theta}_i)) \int_{\underline{\theta}}^{\underset{\sim}{\theta}_i} (\vartheta_i - \underset{\sim}{\theta}_i) dH(\vartheta_i) \right.$$
$$\left. - H(\underset{\sim}{\theta}_i) \int_{\tilde{\theta}_i}^{\overline{\theta}} (\vartheta_i - \tilde{\theta}_i) dH(\vartheta_i) \right\} < 0$$

Suppose there is a player i whose strategy is bi-constrained. (If i is not the middle player, her matched player is also bi-constrained.) We will show that as $\bar{a}$ increases by da, the variance term $V\xi_i$ decreases, which, from (20′), induces the same increase in $-\frac{1}{n^2}\frac{\mathrm{d}V\xi_i}{\mathrm{d}\bar{a}}da$ in the expected payoff of each player. Let the distribution function of player i's deviation from affine, ξ_i, be denoted as $F_i(\cdot)$. Obviously, $F_i(\cdot)$ is derived from the distribution function of θ, $H(\cdot)$, as well as from i's strategy and the announcement bounds. The random variable ξ_i can be considered as a function of θ_i:

$$\xi_i = \begin{cases} \underline{a} - (\theta_i + \lambda_i) = \underset{\sim}{\theta}_i - \theta_i & \text{if } \theta_i \le \underset{\sim}{\theta}_i \\ 0 & \text{if } \underset{\sim}{\theta}_i < \theta_i \le \tilde{\theta}_i \\ \bar{a} - (\theta_i + \lambda_i) = \tilde{\theta}_i - \theta_i & \text{if } \theta_i > \underset{\sim}{\theta}_i \end{cases}. \quad (38)$$

Given that θ_i is distributed according to $H(\cdot)$, the distribution function $F_i(\cdot)$ of ξ_i can be derived by combining $H(\cdot)$ and (38). Specifically, the support of F_i is $[\tilde{\theta}_i - \overline{\theta}, \underset{\sim}{\theta}_i - \underline{\theta}]$; the fact that i is bi-constrained implies that $\tilde{\theta}_i - \overline{\theta} < 0$ and $\underset{\sim}{\theta}_i - \underline{\theta} > 0$. The values of F_i are given by

$$F_i(x) = \begin{cases} \text{Prob}(\tilde{\theta}_i - \theta_i \le x) = 1 - H(\tilde{\theta}_i - x) & x \in [\tilde{\theta}_i - \overline{\theta}, 0) \\ \text{Prob}(\theta_i \ge \tilde{\theta}_i) = 1 - H(\tilde{\theta}_i) & \text{if } x = 0 \\ \text{Prob}(\underset{\sim}{\theta}_i - \theta_i \le x) = 1 - H(\underset{\sim}{\theta}_i - x) & x \in (0, \underset{\sim}{\theta}_i - \underline{\theta}] \end{cases} \tag{39}$$

Note, in particular, that $F_i(\cdot)$ jumps up at $x = 0$ from $1 - H(\tilde{\theta}_i)$ to $1 - H(\underset{\sim}{\theta}_i)$. To derive the variance $V\xi_i$, note first that since i is bi-constrained, $\lambda_i \in \text{int}(\Lambda)$. We can, therefore, invoke Prop. 9 to obtain

$$-nk_i \equiv \mathbb{E}(\xi_i) = \int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} \xi_i \mathrm{d}F_i(\xi_i) = \underset{\sim}{\theta}_i - \underline{\theta} - \int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} F_i(\xi_i)\mathrm{d}\xi_i$$

where the last equality is obtained after integrating by parts. Thus,

$$\int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} F_i(\xi_i)\mathrm{d}\xi_i = \underset{\sim}{\theta}_i - \underline{\theta} + nk_i. \tag{40}$$

The variance of ξ_i can now be written as

$$\begin{aligned} V\xi_i &= \int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} (\xi_i - \mathbb{E}(\xi_i))^2 \mathrm{d}F_i(\xi_i) = (\underset{\sim}{\theta}_i - \underline{\theta} - \mathbb{E}(\xi_i))^2 - \int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} F_i(\xi_i) 2(\xi_i - \mathbb{E}(\xi_i))\mathrm{d}\xi_i \\ &= (\underset{\sim}{\theta}_i - \underline{\theta} + nk_i)^2 - 2nk_i \int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} F_i(\xi_i)\mathrm{d}\xi_i - 2\int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} F_i(\xi_i)\xi_i\mathrm{d}\xi_i \\ &= (\underset{\sim}{\theta}_i - \underline{\theta})^2 + 2(\underset{\sim}{\theta}_i - \underline{\theta})nk_i + (nk_i)^2 - 2nk_i(\underset{\sim}{\theta}_i - \underline{\theta} + nk_i) - 2\int_{\tilde{\theta}_i - \overline{\theta}}^{\underset{\sim}{\theta}_i - \underline{\theta}} F_i(\xi_i)\xi_i\mathrm{d}\xi_i \\ &= (\underset{\sim}{\theta}_i - \underline{\theta})^2 - (nk_i)^2 - 2\left[\int_{\tilde{\theta}_i - \overline{\theta}}^{0} (1 - H(\tilde{\theta}_i - \xi_i))\xi_i\mathrm{d}\xi_i \right. \\ &\quad \left. + \int_0^{\underset{\sim}{\theta}_i - \underline{\theta}} (1 - H(\underset{\sim}{\theta}_i - \xi_i))\xi_i\mathrm{d}\xi_i \right], \end{aligned} \tag{41}$$

where the second equality follows from integration by parts, the third from $\mathbb{E}(\xi_i) = -nk_i$, the fourth from (40), and the fifth from (39). Now, differentiating (41) with respect to $\bar{a}$ and noting that $\underset{\sim}{\theta}_i = \underline{a} - \lambda_i = -\bar{a} - \lambda_i$ and $\tilde{\theta}_i = \bar{a} - \lambda_i$, we obtain

$$\frac{dV\xi_i}{\mathrm{d}\bar{a}} = 2\left\{\left[1 - \frac{\mathrm{d}\lambda_i}{\mathrm{d}\bar{a}}\right]\int_{\tilde{\theta}_i - \overline{\theta}}^{0} h(\tilde{\theta}_i - \xi_i)\xi_i\mathrm{d}\xi_i - \left[1 + \frac{\mathrm{d}\lambda_i}{\mathrm{d}\bar{a}}\right]\int_0^{\underset{\sim}{\theta}_i - \underline{\theta}} h(\underset{\sim}{\theta}_i - \xi_i)\xi_i\mathrm{d}\xi_i\right\}$$

$$= \frac{4\left\{H(\underset{\sim}{\theta}_i)\int_{\underline{\theta}}^{\tilde{\theta}_i}(\theta_i - \tilde{\theta}_i)\mathrm{d}H(\theta_i) - (1 - H(\tilde{\theta}_i))\int_{\underset{\sim}{\theta}_i}^{\overline{\theta}}(\theta_i - \underset{\sim}{\theta}_i)\mathrm{d}H(\theta_i)\right\}}{H(\underset{\sim}{\theta}_i) + (1 - H(\tilde{\theta}_i))} < 0 \tag{42}$$

The first inequality holds because $H(\underline{\theta}) = 0$ and $H(\overline{\theta}) = 1$, while $\frac{\mathrm{d}\underset{\sim}{\theta}_i}{\mathrm{d}\bar{a}} \equiv \frac{\mathrm{d}(\underline{a} - \lambda_i)}{\mathrm{d}\bar{a}} = -(1 + \frac{\mathrm{d}\lambda}{\mathrm{d}\bar{a}})$ and $\frac{\mathrm{d}\tilde{\theta}_i}{\mathrm{d}\bar{a}} \equiv \frac{\mathrm{d}(\bar{a} - \lambda_i)}{\mathrm{d}\bar{a}} = (1 - \frac{\mathrm{d}\lambda}{\mathrm{d}\bar{a}})$. The second equality is obtained by substituting in the value of $d\lambda_i/d\underline{a}$ using (25), changing the variables of integration from ξ_i to $\theta_i = \tilde{\theta}_i - \xi_i$ and to $\theta_i = \underset{\sim}{\theta}_i - \xi_i$ in the two integrations, respectively. The term in curly brackets is negative because $\underline{\theta} < \underset{\sim}{\theta}$ while $\overline{\theta} > \tilde{\theta}$. □

Proof of Proposition 12 We already established that as k_r increases, λ_r also increases, raising $\mathbb{E}\xi_r$. We will show that $\mathrm{d}V\xi_r/\mathrm{d}|k_r| > 0$. Then, the proposition follows from (20). By symmetry, we can, w.l.o.g., assume that $k_r > 0$. Using the same procedures as we used to derive (42), we differentiate the expression for $V\xi_r$ in (41) w.r.t. λ_r, to obtain

$$\frac{\partial V\xi_r}{\partial \lambda_r} = 2\int_{\underline{\theta}}^{\underset{\sim}{\theta}_r}(\theta_r - \underset{\sim}{\theta}_r)\mathrm{d}H(\theta_r) + 2\int_{\tilde{\theta}_r}^{\overline{\theta}}(\theta_r - \tilde{\theta}_r)\mathrm{d}H(\theta_r) = -2\mathbb{E}\xi_r = 2nk_r, \tag{43}$$

where the last equality follows from Prop. 9. Note that if r is up-constrained, the first term in expression (43) is zero. Since $\frac{\mathrm{d}V\xi_r}{\mathrm{d}k_r} = \frac{\partial V\xi_r}{\partial k_r} + \frac{\partial V\xi_r}{\partial \lambda_r}\frac{\mathrm{d}\lambda_r}{\mathrm{d}k_r}$, we take the derivative of (41) with respect to k_r and combine (26) with (43) to obtain

$$\frac{\mathrm{d}V\xi_r}{\mathrm{d}|k_r|} = 2n^2|k_r|\left(\frac{1}{H(\underset{\sim}{\theta}_r) + (1 - H(\tilde{\theta}_r))} - 1\right) > 0 \tag{44}$$

To see the effect of increasing $k_{\bar{r}}$ on players' expected payoffs, we totally differentiate (20) w.r.t. $k_{\bar{r}}$, noting that to preserve symmetry, $\frac{\mathrm{d}k_{\underline{r}}}{\mathrm{d}k_{\bar{r}}} = -1$, where $\underline{r}$ is $\bar{r}$'s matched player. As $k_{\bar{r}}$ increases, $\bar{r}$'s and $\underline{r}$'s welfare decline by $\left(\frac{2}{n^2}\frac{\mathrm{d}V\xi_{\bar{r}}}{\mathrm{d}k_{\bar{r}}} + 2k_{\bar{r}}\right)$; for other players, the decline is $\frac{2}{n^2}\frac{\mathrm{d}V\xi_{\bar{r}}}{\mathrm{d}k_{\bar{r}}}$. □

Proof of Proposition 13 Let I^+ denote the members of the right-wing faction, and let I^+_- denote the moderate members of this faction. Pick $r \in I^+$. Let $\xi_r(\gamma)$ denote r's deviation from affine in the equilibrium associated with the parameter γ. Since r is up-bounded, we have

$$n\bar{k}_r^+ = -\mathbb{E}\xi_r(0) = \int_{\tilde{\theta}_r}^{\overline{\theta}}(\theta_r - \tilde{\theta}_r)\mathrm{d}H(\theta_r) = 0.5\int_{\tilde{\theta}_r}^{1}(\theta_r - \tilde{\theta}_r)\mathrm{d}\theta = (1 - \tilde{\theta}_r)^2/4$$

The first equality follows from Prop. 9 and the third from Assumption A4(i). Hence, $\tilde{\theta}_r = 1 - 2\sqrt{n\bar{k}_r^+}$. Moreover, $H(\tilde{\theta}_r) = 0.5\int_{-1}^{\tilde{\theta}_r} d\theta = \frac{1+\tilde{\theta}_r}{2}$. Now, from (44),

$$\left.\frac{dV\xi_r}{dk_r}\right|_{\gamma=0} = 2n^2\bar{k}_r^+\left(\frac{H(\tilde{\theta}_r)}{1-H(\tilde{\theta}_r)}\right) = 2n^2\bar{k}_r^+\left(\frac{1+\tilde{\theta}_r}{1-\tilde{\theta}_r}\right) = 2n^2\bar{k}_r^+\left(\frac{1-\sqrt{n\bar{k}_r^+}}{\sqrt{n\bar{k}_r^+}}\right)$$
$$= 2n\left(\sqrt{n\bar{k}_r^+} - n\bar{k}_r^+\right).$$

Hence, $\left.\frac{d^2V\xi_r}{dk_r{}^2}\right|_{\gamma=0} = n\left(\sqrt{n}/\sqrt{\bar{k}_r^+} - 2n\right) \lesseqgtr 0$ as $\bar{k}_r^+ \gtreqless 1/4(n-1)$, that is, for $k' > k$, $\frac{dV\xi_r(k')}{dk} > \frac{dV\xi_r(k)}{dk}$ if $k' < 1/4(n-1)$, and $\frac{dV\xi_r(k')}{dk} < \frac{dV\xi_r(k)}{dk}$ if $k > 1/4(n-1)$. From Prop. 7, Prop. 9, and symmetry, USW $= -2\sum_{i\in I^+} V\xi_i(\gamma)$, so that

$$\left.\frac{d\text{USW}}{d\gamma}\right|_{\gamma=0} = -2\sum_{i\in I^+}\left.\frac{dV\xi_i(\gamma)}{d\gamma}\right|_{\gamma=0}$$
$$= -2\sum_{i\in I_-^+}\alpha_i\left(\left.\frac{dV\xi_{i+(n-1)/4}(\gamma)}{dk_{i+(n-1)/4}}\right|_{\gamma=0} - \left.\frac{dV\xi_i(\gamma)}{dk_i}\right|_{\gamma=0}\right).$$

Since $\bar{k}_{i+n/4}^+ > \bar{k}_i^+$, $\left.\frac{d\text{USW}}{d\gamma}\right|_{\gamma=0} > 0$ if $\min(\bar{\mathbf{k}}^+) > \frac{1}{4(n-1)}$ and $\left.\frac{d\text{USW}}{d\gamma}\right|_{\gamma=0} < 0$ if $\max(\bar{\mathbf{k}}^+) < \frac{1}{4(n-1)}$. □

Proof of Proposition 14 Under A3, the variance of r's deviation from affine is

$$V\xi_r(\lambda_r) = (\mathbb{E}\xi_r)^2\left(\sqrt{\frac{8}{9h|\mathbb{E}\xi_r|}} - 1\right). \tag{45}$$

To see this, assuming w.l.o.g. that $\lambda_r > 0$, and using the fact that $\lambda_r \in \text{int}(\Lambda)$, we have

$$V\xi_r(\lambda_r) = -(\mathbb{E}\xi_r)^2 + \int_{\bar{a}-\lambda_r}^{\bar{\theta}} (\vartheta_r + \lambda_r - \bar{a})^2\, dh(\vartheta_r) = (h/3)\left(\lambda_r + \bar{\theta} - \bar{a}\right)^3 - (\mathbb{E}\xi_r)^2$$

which, from (35),

$$= \frac{h}{3}\left(\frac{2}{h}\mathbb{E}\xi_r\right)^{3/2} - (\mathbb{E}\xi_r)^2 = \sqrt{\frac{8|\mathbb{E}\xi_r|^3}{9h}} - (\mathbb{E}\xi_r)^2,$$

establishing (45). Note from (24) that in SAGs, $\mathbb{E}\xi_r = -Nk_r$. Substituting this into (45) and then into (22″), and using the fact that $\sum_{i\in\mathcal{N}_1}\mathbb{E}\xi_i = \sum_{i\in\mathcal{N}_1}\lambda_i = 0$ in SAGs, we obtain (27). To see that the inequality holds, recall that $|k_i| < \bar{\theta}/2N$. □

Proof of Proposition 15 Together with the assumption that θ is uniformly distributed, Assumption A3(iii) puts an upper bound on k_i: $k_i < \frac{1}{4Nh}$ for all $i \in \mathcal{N}_1$. We first show that $\Omega(k_i) := k_i^2\left(\sqrt{\frac{8}{9Nh|k_i|}} - 1\right)$ is increasing in k_i; we then show that $\text{Var}(\theta)/N > \Omega(1/4Nh)$. We know that

$$\frac{\mathrm{d}\Omega(k_i)}{\mathrm{d}k_i} \propto \sqrt{\frac{2}{Nh|k_i|}} - 2 > \sqrt{\frac{2}{Nh/4Nh}} - 2 = \sqrt{8} - 2 > 0 \tag{46}$$

where the inequality follows from $k_i < \frac{1}{4Nh}$. From (46) and noting from Assumption A3(ii) that $\mathrm{Var}(\theta) = 1/(12h^2)$, we know that

$$\mathrm{Var}(\theta)/N - \Omega(1/4Nh) = \frac{4(N - \sqrt{2}) + 3}{48N^2h^2} > 0,$$

since $N \geq 2 > \sqrt{2}$. □

Proof of Proposition 16 We first establish $\lambda_h^* > 0$, so that, from (28), $\mathbb{E}\xi_h(\lambda_h^*) = 0$, and thus, h is the anchor of the game. Suppose, instead, that $\lambda_h^* \leq 0$ and $\lambda_h^* \in \mathrm{int}(\Lambda)$. (We can easily rule out the situation when $\lambda_h^* = \min(\Lambda) = \underline{a} - \overline{\theta}$; we omit the details.) Since $k_h > k_r\ \forall r \neq h$, (16) implies that $\mathbb{E}\xi_h(\lambda_h^*) < \mathbb{E}\xi_r(\lambda_r^*)$ and thus $\lambda_r^* < \lambda_h^* \leq 0$. Since $\mathbb{E}\xi_r(\lambda_r) = 0$ when $\lambda_r = 0$, (29) and $\lambda_r^* < 0$ imply

$$\lambda_r^* + \mathbb{E}\xi_r(\lambda_r^*) < 0 \tag{47}$$

From (7′) and $\lambda_h^* \in \mathrm{int}(\Lambda)$, $\lambda_h^* = nk_h - \sum_{r\neq h}(\lambda_r^* + \mathbb{E}\xi_r(\lambda_r^*)) > 0$, where the inequality is due to $k_h > 0$ and (47). This contradicts our supposition that $\lambda_h^* \leq 0$. Property (28) now ensures that $\mathbb{E}\xi_r(\lambda_r) = 0$, so that single-bounded aggregation games are anchored with anchor h. The second part of the proposition now follows from Prop. 8. □

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