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Essays on the Joint Dynamics of Family Housing and Childbearing Decisions

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Chia-Lo Chen

June 2021

Dissertation Committee:

Dr. Richard James Arnott, Chairperson
Dr. Michael Bates
Dr. Yang Xie

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The Dissertation of Chia-Lo Chen is approved:

Committee Chairperson

University of California, Riverside

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Some of the data used in this dissertation are derived from Sensitive Data Files of the Panel Study of Income Dynamics, obtained under special contractual arrangements designed to protect the anonymity of respondents. These data are not available from the authors. Persons interested in obtaining PSID Sensitive Data Files should contact through the Internet at `PSIDHelp@isr.umich.edu`.

ABSTRACT OF THE DISSERTATION

Essays on the Joint Dynamics of Family Housing and Childbearing Decisions

by

Chia-Lo Chen

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2021
Dr. Richard James Arnott, Chairperson

Rising housing prices and falling fertility rates are two features of cities in industrialized countries. The impact of housing market shocks on family decisions concerning home purchase and childbearing is the center of the dissertation. The first two essays explore the relationships between the housing market conditions and the interdependence of family home purchase and childbearing decisions. The third essay provides a theoretical analysis of how an increase in home price uncertainty affects family saving and home-buying decisions.

The first essay empirically investigates how the local house price level and its recent growth affect the frequency and timing of family home-buying and childbearing events. Taking the American urban families in the Panel Study of Income Dynamics database as the sample, it finds that the timing of a childbearing event relative to the date of the family's entering homeownership is more responsive to house prices and their changes than is the frequency of fertility event. Recent house price appreciation has a considerably smaller impact on the home buying and childbearing decisions than the current house price level,

while it disproportionately deters poor families from a home purchase.

The second essay extends the basic lifecycle-consumption model to treat homeownership and childbearing decisions by taking each event as a phase transition. Operating via an intertemporal budget constraint and a mortgage downpayment requirement, the analysis demonstrates the interdependence of the two decisions in a family's life course. It explores some comparative dynamic properties of the model.

The third essay explores how an increase in future house price uncertainty affects the probability of a home-renting individual buying a home in the next period. Applying the portfolio optimization theory, the model shows that how the form of the uncertainty growth and the home buyer's tastes towards risk affect its saving strategy and the probability of entering homeownership. It particularly identifies that the role of prudence, the third-order derivative of the utility function, is central to the effect of increased uncertainty on savings.

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Introduction

Most individuals cherish the ideal of raising a family in a comfortable and affordable home. In most major metropolitan areas around the world, this ideal has become increasingly difficult to realize because of high and rapidly rising housing prices. Over the same time span, industrialized countries have been experiencing a "missing babies" problem—low and declining fertility rates. The average age of the first birth has advanced and the number of children in a family has fallen substantially. Thus, increasing costs of homeownership and declining fertility rates are correlated. Are there causal mechanisms underlying this correlation? If so, what are they?

Declining fertility rates are commonly regarded as a part of the great shift of family values in modern society. But the ideological shift does not fully explain this demographic change in the industrialized countries, since women's realized number of children are often lower than their desired number.[1] The change in family values from the obsolete "be fruitful and multiply'' toward appreciating small family sizes is only one force contributing to the shrinking young generation. Another reason for the enduring phenomenon is the growing economic cost of children. The transition in social structure over the past century has

[1] In contrast, most industrializing countries have an inverse gap due to lack of access to contraceptive services.

substantially reshaped the family economy, too. With much better access to the labor market, women are no longer bound to household labor, so that the opportunity cost of parenting is much higher than before. Meanwhile, the social transition from the extended to the nuclear family, as well as urbanization, bar many young couples from the free child-caring services formerly provided by relatives or local communities. The same transition has also raised social norms concerning private expenditure on infant care, daycare, early education, and children's entertainment. As a result, the high costs of children have forced couples to put more weight on the economic aspect in their family formation plan. The decisions on whether and when to have children have become more responsive to family finance.

Compared to the rich literature on the relationship between female labor force participation and childbearing, the relationship between housing and childbearing is much less studied. The dynamics of home purchase and childbearing are interrelated through both the family preferences and family finances. Perhaps high and rapidly growing house prices cause households to delay home purchase but have little effect on childbearing; perhaps they have little effect on the home purchase because households delay having children to absorb the price shock; perhaps households plan to have their first child and to buy their first home at the same time, with high and growing house prices causing both to be postponed.

This dissertation contains three essays that look at different aspects of the joint dynamics of family housing and childbearing decisions. The first essay empirically investigates the effects of house prices and their growth on household childbearing and homeownership. It is the first empirical research to examine their effects simultaneously. The essay employs a multinomial-logit model to estimate how households adjust their housing and childbearing decisions to the local house price level and recent price history, focusing on the relative timing of the events. The dependent discrete choice set captures the probability of homeownership and childbearing state transitions between surveys. The sample used in this research is drawn from the Panel Study of Income Dynamics (PSID). It comprises American families in the PSID database living in US metropolitan areas who are not currently homeowners and did not migrate to other cities recently. The study finds that, while high house prices suppress the probability of home buying, the timing of household fertility events relative to the home purchase event is more responsive to the level of house prices than childbearing itself. Higher house prices have a weak, positive effect on childbearing suggesting at least a temporary substitution away from owning a home to having children. House price appreciation over the previous two and four years does not on average affect either the probability of homeownership or the probability of childbearing but does discourage their joint occurrence and disproportionately deters low-income households from entering homeownership.

To provide a theoretical foundation for the empirical work on the relationship between homeownership and childbearing, the second essay constructs a stylized life-cycle consumption model of the individual household. In the model, the household chooses not only a savings-consumption plan but also when to switch from renting to homeownership and when to have a single child, with a downpayment requirement constraining the household from a home purchase without sufficient savings. This model provides a simple way of thinking dynamically about the interdependence between homeownership and childbearing decisions of a household, especially their relative timings, and the effects of exogenous shock on these decisions. The essay analyzes a permanent income shock as a sample comparative dynamic exercise. It shows that the household may respond to a positive income shock by choosing to delay home purchase shortly in order to have a child earlier.

The deterministic setting of the joint-dynamic model makes the theoretical analysis simpler but at the same time drops considerations of risk and uncertainty as determinants. If market house prices are stochastic, a household's fear of insufficient funds for a planned home mortgage downpayment will alter the shadow value of its assets. Inspired by the observation, the third essay looks at a single source of uncertainty and a single decision—how an increase in uncertainty about future house prices affects the homeownership decision of a household that rents during the first period in the context of a simple two-period model with a mortgage downpayment constraint. Using portfolio optimization theory, the model shows the critical role played by the risk preferences of the household. Risk aversion and prudence, the third-order derivative of the utility function, determine the household demand for precautionary savings, which have an option value in home purchase.

Chapter 1

The Joint Dynamics of Family Housing and Childbearing Decisions: An Empirical Application

abstract

Housing price growth is now and then blamed for causing fertility decline in cities. As the cost of housing rises over the years, it is likely the increasing financial burden not only bars new home buyers from entering homeownership but also has an impact on their family plan to raise children. The net impacts on fertility and on sequencing of home buying and childbearing are unclear however. By formulating the family behavior of housing and

childbearing as a joint decision-making process, we investigate the effect of local housing price variation on both behaviors simultaneously for non-homeowning women in the United States. We estimate a multinomial logit model of the interaction of the two binary choices, entering homeownership and childbearing, using family data from the Panel Study of Income Dynamics between 1985 and 2015 and the corresponding metropolitan statistical area level house price data imputed from the Federal Housing Financial Agency and the Census. The results show that, high house price level strongly discourages the probability of entering homeownership, while it has a very mild net positive relation with the likelihood of childbearing for non-homeowning women. In areas with high house price, families are more likely to have a new baby before buying a home, mostly because of the substantial drop the probability of entering homeownership and childbearing in tandem in one or two years. Though the net effect on childbearing is small, high house price would nonetheless raise the chance of parenting without homeownership. On the other hand, the effect of house price change, regardless of the price level, is hardly found.

1.1 Introduction

The decision to buy a home and to rear children are two major and interrelated decisions a family faces. On the one hand, home and children are somewhat complementary to achieve a paradigm family in most cultures. Especially where a nuclear family is the prevailing family ideal, owning a home in general gives a family more and usually better living space to raise children (Haurin et al., 2002; Kulu and Vikat, 2007), and it even serves as a qualification of formal family formation in some countries (Wei et al., 2012). Moreover, the desire to have a bigger family usually calls for a higher demand for housing space (Mulder, 2006b; Kulu and Steele, 2013). On the other hand, both homeownership and parenting are constrained by a single family budget, and home price variations in the market can theoretically affect the fertility choices of families who seek them both owing to economic reasons. For potential home buyers (non-homeowners), an increase in housing costs would crowd out the expense of child-rearing and force them to postpone or even give up one of the two goals. For homeowners, although the added wealth from property appreciation would ease the family financial burden and allow them to raise more children (Clark and Ferrer, 2019; Cloyne et al., 2019), their ability to move to a larger home could be limited. This interplay of homeownership, childbearing, and the housing market implies a complicated relationship between house price and fertility. The impact of growing housing costs is unlikely to be confined to the tightened budget for consumption. The welfare loss of delayed family formation can also be a consequence, especially for non-homeowners.

Now and again, the issue of climbing house prices enters the center of public policy debates such as on public housing and inequality, city gentrification and redevelopment, home mortgage market regulations, and the policy response to low fertility rates. The association between housing and fertility decisions has attracted extensive attention across multiple fields of the social sciences in the past decade. It is however difficult to disentangle the interaction between homeownership and childbearing in analysis because of their interweaving influences on each other's decision. Recent empirical literature has been investigating the effect of house price on fertility decision mostly by restricting the research sample either to the homeowners or the renters, to explore the one-to-one causality between house price and childbearing choice to prevent the confounding factor of homeownership. Notwithstanding their findings, the underlying mechanism of the relationship between house price and childbearing decisions is yet to be explored. House price variation has a more direct impact on the home buying decision. As homeownership and childbearing are correlated, the home buying decision can be involved in the childbearing decision, and *vice versa*. Focusing on the influences on childbearing choice alone tends to overlook the variation in homeownership transition and its subsequent effect on childbearing for either group. This omission may downplay the negative effect of growing house prices on the well-being of non-homeowners.

We are interested in how the local housing market affects the dynamics of the family decisions of homeownership transition and childbearing of potential home buyers. Not only do we revisit the question of the impact of high local house price level on the childbearing decision, we also investigate how the variation in price impacts the family life course plan of both housing and parenting. Does high house price shift the order of the transitions? Does the booming housing market have the same effect? These are the questions we contemplate. To answer them, we estimate a multinomial logit (MNL) model of the crossing of the two binary choices, becoming a homeowner and childbearing, on local median house price level and changes for women living in the U.S. between the years 1985 and 2015. The data mainly comes from the Panel Study of Income Dynamics (PSID) and the corresponding metropolitan statistical area (MSA)-level house price data imputed from the Federal Housing Financial Agency (FHFA) and the Census.

The results draw a pattern of the impact of the housing market on the behavior of non-homeowning women in the U.S. High house price level strongly discourages the probability of entering homeownership, while it has a very mild net positive relationship with the likelihood of childbearing for non-homeowning women. In areas with high house prices, families are more likely to have a new baby before buying a home mostly because of the substantial drop in the probability of entering homeownership and childbearing alongside each other in one or two years (hereafter referred to as "doing both"). The effect of house price change, regardless of the price level, is hardly found. The estimations only observe a negative effect of four-year price growth on doing both. Overall, house price level, instead of short-term variation in price, has a more prominent impact on the family choice on homeownership and childbearing. An analysis on the interaction effect also shows the effect scales differ by family income level, partnership, and race. Though the net effect on childbearing is small, high house prices would nonetheless raise the chance of parenting without homeownership.

Unlike Beckerian static theoretical models, both of the two decisions are dynamic over time. If a family is heading for both homeownership and a new child while the credit constraint is binding, it must postpone its plan, save money and wait for a better chance, and now and again adjust to the varying costs. Facing a growing housing price, a renting family would either delay having a child to accumulate the mortgage down payment first or have a child right away and postpone the plan to enter homeownership. The relative costs and preferences on homeownership and children are critical in making the decision. The recent price trend can also shape the expected price for the future and alter the decision if the family has a forward-looking perspective.

Knowledge of the association of housing and fertility, especially to the behavior of newly formed families, is important to the assessment of relevant housing policies. Its efficacy is closely tied with family responses to the change in their financial budget. Our results suggest a small side effect on childbearing could accompany policies targeting the affordability of homeownership. A better understanding of the dynamics of family decisions will help policymakers to predict both the direct and indirect outcomes of those policies on either homeownership rate or birth rate more accurately, and the evaluation of relevant policies will be more conclusive and precise.

1.2 Background

Financial consideration is one major material restriction for families to have more and better care for children in industrialized countries. Since housing is typically the largest part of living expense and often the largest store of wealth for families, it is natural to wonder how local house prices could affect family size, especially as many urban areas have experienced property appreciation recently. Growing empirical literature in recent years investigates this relationship around the world thanks to the increasing data accessibility of housing markets and demography, but the findings diverge with different methodologies and data source. Some time series (e.g. Yi and Zhang (2010) on Hong Kong) and cross-sectional regression (e.g. Simon and Tamura (2009) on the U.S.) studies conclude that high house price has a negative impact on local fertility rate. Others cast doubts on such a conclusion and instead argue for a positive relation through the wealth channel. For instance, Feyrer et al. (2008) argue that the U.S. data shows a positive correlation, though weak, between housing prices and fertility rates.

A few studies isolate the wealth effect of property appreciation by controlling homeownership. Dettling and Kearney (2014) examine the relationships of the lagged house price index and homeownership rate on the MSA-level fertility rates and find that the positive home price effect is greater in areas with higher homeownership rate, and the prediction implies that the overall relation between home price and fertility is slightly positive in the United States. Lovenheim and Mumford (2013) employ a panel dataset of U.S. households between 1985 and 2005 from the PSID data to show a positive effect on homeowners but find no significant repercussion for renters. Clark and Ferrer (2019) find a similar result for homeowners and non-homeowners using a Canadian longitudinal data, and lately Daysal et al. (2021) report an empirical finding that the observed effect for homeowners in Denmark has almost the same scale with that observed in the U.S. By and large, the leading evidence suggests that house price appreciation generates a dominating positive wealth effect on childbearing for homeowners but only a weak negative-to-none price effect for non-homeowners (renters) on the likelihood of childbearing for families in general.

The association of housing and childbearing is yet to be fully explained. Though these empirical works directly estimate the impacts of house prices on fertility decisions, preventing the confounding factor of homeownership, they fail to consider the accompanying movement in homeownership change. Most studies on the demographic impacts of housing price fluctuation focus on a single variable and control for the other, implicitly assuming the choice of housing independent from childbearing.[1] As emphasized earlier, childbearing is not extraneous to housing status and *vice versa*. Major changes in human life course such as family formation, career building, and childbearing are all likely to raise the possibility of entering homeownership and settling down for the subsequent needs of mental, material, and spatial accommodation. The demand for children can reinforce the demand for more and better housing service and even the desire of moving up the housing ladder in the long term (Clark and Onaka, 1983; Banks et al., 2004), not to mention that private-owned homes are deemed as a status good to signal a qualified man for marriage in many cultures. There is also the inverse influence. Stable homeownership not only allows households to allocate more resources to other activities but also provides a stable environment for child raising (Aaronson, 2000; Dietz and Haurin, 2003). This could increase the incentive to have a larger family. Mulder (2006a,b) summarizes well this complexity between housing, family formation, and fertility rate conceptually.[2] In particular, the requirement on home location, space, quality, and ownership can be seen as part of the demand for the quality of family life

[1] This does not mean housing is totally silent from the studies of fertility decision or the other way around. A common argument presumes housing space and children are strong complements, and one of the choices is usually set as given in analysis.

[2] We limit the scope of this paper from the impact of transformation in social norms in recent years, which could dampen housing demand prompted by other family status transitions. For more on this issue, see the discussions in Mulder (2006a) and Drew (2015). Additionally, the influence of fertility rate on housing prices is assumed away because such a channel would hardly be captured in micro-level analysis.

and of the children's upbringing, to which Kulu and Vikat (2007), Clark (2012), and Clark and Lisowski (2018) partly call to attention.[3] Accordingly, Enström Öst (2012) argues for the growing simultaneity of family housing and fertility decisions over time, showing that the positive correlation of the two actions has become more prominent in the younger cohort in Sweden as the young generations are facing an increasingly insecure occupational and financial environment.[4] In the U.S., Clark and Withers (2009) report a slightly higher probability of moving right before and after a new birth.[5] Clearly, homeownership transition should not be removed from the analysis of fertility if we want to gain a full picture of the impact of housing prices.

Non-homeowning families are vulnerable to price shocks. The narrowing affordability of homeownership due to credit constraints and increasing cost would impinge upon the decision or the timing of other important life status transitions such as family formation and workforce participation (Mulder, 2006b; Clark, 2012). This yoke is heavy on young couples in particular: they often do not possess equity or a stable income; at the same time, they are right at the junction of their life course to choose whether or not to have a child (Courgeau and Lelièvre, 1992; Sobotka et al., 2011). Without equity, newly-formed families in a booming housing market face more difficulty to pay for a mortgage down payment should their income remain unchanged, a notable occurrence in the era of a tighter hous-

[3] In a very different setting, Sato and Yamamoto (2005) discuss an equilibrium of fertility rate, population density, and urbanization in Japan, which alludes to the importance of the relationship between residence and childbearing.

[4] The caveat of this positive relation is that homeownership reflects the fulfillment of the desired housing space. If homeownership *per se* instead of its incurred housing service becomes the goal, the relationship may not hold.

[5] The effect of the status of having a child and the event of childbirth are different. It is the event that would raise the likelihood of moving. The status of having children decreases the likelihood empirically perhaps because children fortify family and reinforce the desire of sedentariness (Clark, 2013).

ing mortgage market after the Financial Crisis (Lennartz et al., 2016). If the social norm strongly regards homeownership as a requirement of formal family formation, the rising financial burden from the growing housing cost would drain a family's savings and further crowd out the resources that could be used elsewhere, such as parenting, as observed in some parts of Europe and East Asia.[6] High homeownership rates under this circumstance would not reflect the pervasive positive wealth effect caused by a housing price increase but instead stress the heavy burden of housing. In Southern Europe, where such preference for homeownership is strong, insufficient housing rental market and inferior access to housing mortgages accompanied with a high homeownership rate are likely attributable to the extreme-low fertility rates (Mulder and Billari, 2010). Lo (2012) also observed a negative relationship between homeownership and fertility in a cross-county study on Taiwan. Indirectly, yet untested, Turner and Seo (2007) suggest this possible substitution observed from the U.S. data.

In a microeconomic analysis, if we regard children as an economic good, the decision on home buying and childbearing should not be considered in a static framework but as if they are durable.[7] Families desiring homeownership and a child contemplate not only their current but future satisfaction from the housing service and children's development, and the expectation of future prices determines their willingness to take action today. Both

[6]Though the homeownership rate is falling among the younger cohorts in the U.S., it is not decisive evidence for the preference change of American young families. See Myers et al. (2020) for the discussion.

[7]The Beckerian setting is to assume the utility of parenting comes from not only the number of children but their "quality." The discussion of the effect of home price on fertility in this paper intentionally circumvents the question of the demand for children's quality to prevent digression. (For a discussion, see Becker and Lewis (1973).) Different from the effect of average permanent income, the variation of home prices does not involve a potential change of the opportunity cost of mother's time nor the expected return of the child's education investment (Becker, 1960, 1965). Moreover, this inverse effect on fertility through quality demand change may not yet be dominating in individual-level analysis on American data, as Black et al. (2013) discovered a positive relationship of husband income and fertility using the 1990 U.S. census data.

the current price level and the expected price are in the family's consideration. A large literature about housing and fertility choice has discussed these respective dynamics (e.g. Hotz et al. (1997); Ortalo-Magne and Rady (2006)). In addition, buying a home is not an action to which perfect capital market can be unconditionally assumed (Daysal et al., 2021). A potential buyer must accumulate enough savings in practice to pay at least part of the value of home.[8] To achieve that, families may alter the preferred time of childbearing to achieve their financial goal. Such lagged effect of home prices is the same type of regular tempo effect caused by rising female education and average income (Lutz and Skirbekk, 2005), except it is purely due to economic reasons. A high house price implies a tougher barrier of credit constraint, which bars families from their unconstrained optimal timing of homeownership transition and childbearing. The discretion in family finance and the possibility of intertemporal choice on home buying and childbearing reflect the importance of a dynamic framework for the co-movement to understand the interplay of the two family choices.

1.3 Research Strategy

1.3.1 Conceptual framework

Homeownership and childbearing are not mutually independent decisions for a family, and house price has an impact on both simultaneously. Due to credit constraint and the expectation of future prices, the optimal timing of the choices could be determined by the current price level and its trend. To examine this co-movement, we extend the

[8] In the U.S., a conforming home mortgage with down payment ratio smaller than 20% requires the borrower to buy a mortgage insurance.

common single-causality framework to a dynamic two-dimensional choice question: a non-homeowning family considers home buying and childbearing together in each given time period. Without equity holding nor extraordinary wealth, the family needs to face the credit constraint on a home purchase and adjust its optimal decision based on the prices available. The current price level determines the current affordability, and the recent price variation affects the expectation of future house prices. An increase in local house price would render house buying more difficult and shape the family's belief. A short-term variation may be a transitory deviation. A middle-term change may reflect the trend.

The family does not take the decision once for all of its life course. Before becoming a homeowner or realizing its desired family size, it keeps updating the relevant factors and acts optimally following the current condition. At each time point, the choice is made between the intersection of the two yes or no questions: To buy a home now? To have a baby now? If the desired family status is yet to be satisfied, the question will be reconsidered later once the information is updated. In our exercise we track the behavior of non-homeowners, so the statistics would reflect the state transitions and their association with home price changes.

The extension does not mean we can leap to comprehensive knowledge of the family's behavior. For simplification, this framework limits the choice set in each dimension to be a simple binary one, and the quality and quantity of housing services and children are precluded. Thus, the framework is just an abstract of a general decision between homeownership and parenthood. Many other determinants are also ignored. Unobservable and excluded endogenous factors, such as credit score, abortion access, and migration, all play a part in the determination of home buying and childbearing. We acknowledge the extent of this framework's interpretation power is limited. In any case, as long as one does not overgeneralize the interpretation of the model, it provides some insights on family behavior regarding homeownership and childbearing across time for the investigated group.

1.3.2 Empirical approach

The main purpose of the empirical exercise is to explore the proposed effects of house price level and changes on the joint behavior of childbearing and entering homeownership, deemed as joint dynamics in a family's life course at an individual level. To this end, we examine the relationships between the observed local house price levels and variations and the probabilities of women giving birth and entering homeownership in a single model. Given the distinctive nature of the two family behaviors, as will be discussed soon, and the assumption on adaptive expectation of the family budget, we apply a multinomial logit (MNL) model for the estimations.[9]

We model homeownership and parenthood of a new child as two binary choices. The intersection of the two binary choices creates four choice alternatives with no natural ordering at each time point. In line with the conceptual framework, we consider that a non-homeowning woman f, representing her family, decides what to do among the four alternatives in each time period t. In the next time period, the information of the local housing market is updated, and she makes a new choice. Each choice is considered a decision-making case n such that $n \in \{f \times t\}$. In each case, the utility the woman would

[9]The basic settings of the logit model for multinomial discrete choices are well documented in the econometric literature, including Ben-Akiva and Lerman (1985),Imbens and Wooldridge (2007), Train (2009), and Walker and Ben-Akiva (2011). This paper mostly follows the terms that are used in Imbens and Wooldridge (2007) and Greene (2012).

obtain from the alternatives follows the standard random utility model (RUM), and she makes her choice by choosing the alternative that would give her the greatest utility. The RUM formulates the utility of case n from alternative j as $U_{nj} = V_{nj} + \varepsilon_{nj}$, where V_{nj} is called the representative utility, which is a function of the observable factors, and ε_{nj}, the disturbance component, captures the utility that is influenced by the other unobservable factors. Because the alternative i will be chosen if and only if $U_{nj} > V_{nj}$ or $\varepsilon_{nj} < \varepsilon_{ni} + V_{ni} - V_{nj}$ for all $j \neq i$, the probability of the choice is the product of all the cumulative distribution of ε_{nj} under $\varepsilon_{ni} + V_{ni} - V_{nj}$ for all $j \neq i$ given ε_{ni}. We assume the all the disturbances are independently and identically distributed (*i.i.d.*) and following Gumbel type-1 distribution, and we can derive the closed-form expression of the probability of the choice in the following equation, which stands for the logit choice probability.

$$\begin{aligned} P_{ni} &= Pr(\varepsilon_{nj} < \varepsilon_{ni} + V_{ni} - V_{nj} \ \forall j \neq i) \\ &= \int \left(\prod_{j \neq i} F(\varepsilon_{ni} + V_{ni} - V_{nj}) \right) f(\varepsilon_{ni}) d\varepsilon_{ni} \\ &= \int \left(\prod_{j \neq i} e^{-e^{-(\varepsilon_{ni} + V_{ni} - V_{nj})}} \right) e^{-e^{-\varepsilon_{ni}}} d\varepsilon_{ni} \\ &= \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}}. \end{aligned} \tag{1.1}$$

For each case, we observe a vector of factors, x_n, which consists of the woman's information at the time, including her demographic characteristics and financial conditions, the local economic indices, and the local house price index. Because the two binary choices are of different nature (homeownership and parenthood do not have accurate measures in common save the pecuniary costs), there is no common variable that varies among the alternatives.[10] Thus, the model assumes no alternative-specific variable and all the observable variables are individual- or case-specific.

For the purpose of empirical implementation, we assume V_{nj} as a linear function of x_n with a vector of choice-specific parameters, β_j. The convenient specification is defined as

$$V_{nj} \equiv x_n'\beta_j = \beta_{j0} + \beta_{j1}p_{st} + z_n'b_{1j} + \varphi_{st}b_{2j} + v_{sj} + w_{tj}, \tag{1.2}$$

where in our benchmark model p_{st} denotes the real-term median local market house price level in MSA s in year t, z_n denotes the vector of case-specific variables of case n for woman f in year t, and $varphi_{st}$ denotes the vector of local economic factors collected.[11] The vector z_n includes the woman's age, race, education level, partnership (marriage or cohabitation) status and change, employment status, and the current total family income. The vector φ_{st} includes the state-level unemployment rate and MSA-level personal income per capita and an index for nationwide recession. The potential geographic fixed effects and the year effects are captured by variables v and w, which represent the census division invariant effect and the five-year group invariant effect.[12] Given that the independent variable is at the MSA level, we implement clustering of the standard errors at MSA-level throughout the estimations, following the suggestion of Abadie et al. (2017). Since we are also interested in the effect of house price variation, we replace the price level with Δp_{st} and $\%\Delta p_{st}$ in the

[10] As the theoretical framework links the two behaviors through the family budget, the consumption of a composite of all other goods, or inversely the cost of the actions, is a legitimate alternative-specific variable. However, we cannot observe the price of homeownership a woman faced nor the accurate cost of childbearing from the data.

[11] Empirically, the estimations with the time-lagged house price information do not differ from the ones with the current price information. In Appendix 1.A, we give a brief discussion about why we choose to use the current price information instead of the time-lagged one.

[12] This setting of the fixed effect groups, which differs from the common state or city fixed effect and year fixed effect, is designed due to the lower limit of choice sample size for estimation.

alternative reduced-form model, which represents the two- or four-year price change and growth rate in real term in MSA s of year t.

Regrading with the parameters, the constant term, β_{j0}, and the parameter of our main interest, β_{j1}, are scalar, and b_{1j} and b_{2j} are two sub-vectors in β_j. The alternative $j = 0$, denoting the choice that the woman would not enter homeownership nor have a child, is set to be the base outcome and β_0 is normalized to a vector of 0. The interpretation of the estimates is however not very straightforward. By Equation (1.1), the value of a parameter β_{ik} directly expresses the marginal effect of the k-th variable x_k on the natural logarithm of the relative probability for alternative i, RP_{i0}, which is defined as the proportion of the probability of i to the probability of the base outcome ("doing nothing"). The following equation states this relation for case n:

$$\ln RP_{ni0} \equiv \ln \left(\frac{P_{ni}}{P_{n0}} \right) = x_n' \beta_i \tag{1.3}$$

$$\beta_{ik} = \frac{\partial \ln RP_{ni0}}{\partial x_{nk}}. \tag{1.4}$$

Though a positive β_{ik} indicates a positive marginal effect on the log-relative probability, it does not necessarily imply a positive marginal effect on the probability (Greene, 2012). In a MNL model, the marginal effect of x_k on the probability of choosing alternative i is a function of the probability of choosing i and all the estimated parameters. Equation (1.5) shows the calculation. Later, we mainly report the estimated marginal effects on the probability to give a more intuitive interpretation of our findings.

$$
\begin{aligned}
\frac{\partial P_{ni}}{\partial x_{nk}} &= \frac{\partial \left(e^{x_n' \beta_i} / \sum_j^J e^{x_n' \beta_j} \right)}{\partial x_{nk}} \\
&= \frac{e^{x_n' \beta_i}}{\sum_j e^{x_n' \beta_j}} \frac{\partial \left(x_n' \beta_i \right)}{\partial x_{nk}} - \frac{e^{x_n' \beta_i}}{\left(\sum_j e^{x_n' \beta_j} \right)^2} \left(\sum_j e^{x_n' \beta_j} \frac{\partial \left(x_n' \beta_j \right)}{\partial x_{nk}} \right). \qquad (1.5) \\
&= P_{ni} \left(\beta_{ik} - \sum_j P_{nj} \beta_{jk} \right) \equiv P_{ni} \left(\beta_{ik} - \bar{\beta}_{ik} \right)
\end{aligned}
$$

The MNL model is estimated by using the maximum likelihood method. Back to (1.1), this model naturally requires the outcomes to be exclusive, exhaustive, and finite and satisfies the property of independence from irrelevant alternatives (IIA). It is a consequence of the assumption of *i.i.d.* disturbance. We argue it is reasonable to assume the outcomes more or less meet the requirements.[13] As they are the intersection of two binary choices, the first three properties are automatically satisfied. The satisfaction of the IIA property is more debatable since, had we excluded the choice to buying a home only or giving birth only, the predicted odds of the remaining alternatives may not remain identical, especially when a large set of variables is considered. Nevertheless, we show that the estimation of the main parameters of interest passes the Hausman tests for IIA property for all combinations of alternatives with the base outcome. Also, applying to panel data, we implicitly assume the disturbances, as well as the unobserved factors, are independent over time. It is, again, a simplistic assumption in compromise for the convenience of the model estimations.[14] One related underlying assumption is that women would adapt their optimal path in each time

[13] As for the choice of a logit model, according to Amemiya (1981), the estimations under the logit assumption do not visibly differ from those under the assumption of normal distribution (a probit model). Small et al. (2007) argue that the advantage of the MNL in its simplicity outweighs the cost of the assumptions.

[14] For example, family wealth and health condition of the family member are likely to be autocorrelated but difficult to be observed due to the data limitation.

period given the new state variables. The idea *per se* is very similar to discrete-choice dynamic programming except that it contains irreversible state transitions and the only state variable connecting period is the amount of private asset (Keane et al., 2011).

A few more elements are taken into consideration in this model. First, Mulder (2006b) presents the possibility of inter-correlation between housing prices and local fertility rate. Though an individual decision can hardly affect the whole MSA's housing price level, it is nonetheless a threat to identification. Local housing prices correlate with local macroeconomic conditions. Therefore, the state-level unemployment rate and personal real income are introduced to control for the macroeconomic variation of the region. Second, we limit our samples to the women who did not move across MSAs during the time of the tracked house price change (two or four years). This is to prevent the endogeneity problem of the movers who choose the place where the housing market is preferred. Lastly, the estimated standard errors are calculated using the sandwich estimator in order to be robust against the unspecified heteroskedasticity.

1.4 Data

We construct an individual-level panel dataset of women in non-homeowning and independent families with the local house price and other economic indices in order to investigate the effect of the house price variations on family behavior.[15] Our main data sources are the restricted-used Panel Survey of Income Dynamics (PSID) and the Cross-National Equivalent File (CNEF). The local house price data is built from the MSA-level

[15]Though a great proportion of our sample are presumably first-time home buyers, we refrain from using the term because some women in the sample are reportedly living in an owner-occupied unit initially and then move out.

Housing Price Indices (HPI) from the Federal Housing Financial Agency (FHFA) and the Longitudinal Tract Database (LTDB). Other supplementary data for the local and national economic performances come from multiple resources including the Bureau of Economic Analysis (BEA), the Local Area Unemployment Statistics (LAUS), and Federal Reserve Economic Data (FRED). All the monetary measures are in real term, inflated to 2011 dollars using the CPI for All Urban Consumers (CPI-U).

The PSID is a public longitudinal survey on the financial conditions of U.S. families conducted by the University of Michigan since 1968. It drew a group of families in the first survey and then follows those families and their descendants and records their financial and demographic information including moving, homeownership, and childbearing every one or two years.[16] Its restricted-used version provides the geographic information of the observations. This allows us to pinpoint the respondent's residence and link to the local economy. This advantage makes tracking family status transition and its relation to the local housing price level possible. We take the sample from the surveys from 1985 to 2015, in a total of 22 waves.[17] Our sampling strategy imitates the work of Lovenheim and Mumford (2013) to a certain degree. Women aged 20 to 44 in a financially independent family who are either the family head or the partner (spouse or cohabitator) of the head are selected. This choice is based on the common childbearing period (age below 45) and the likelihood that the respondent (or her partner) is financially independent. There are in total 77,792 such observed cases in PSID. As we are interested in the behaviors of families who are facing

[16]The PSID was initially an annual survey with detailed financial variables (especially regarding family wealth) collected every four years. Since 1997, the PSID survey became biennial. A group of Latino and immigrant families were later added into the survey. In our analysis we exclude the Latino families added in 1990 and 1992 because the PSID does not assign proper weights to them.

[17]Some key variables were added into the survey only since the wave of 1985, and the CNEF has not covered the 2017 data by the date of writing.

the dual decisions, we further limit our sample to be the group who live in the area where the local HPI are available, who are not a homeowner, and did not move to other MSAs during the time window of the house price change considered (2 or 4 years).

The original PSID data structure is mostly family based. To construct an individual level panel, we borrow the data framework of the CNEF-PSID. The CNEF is a research project organized by the Ohio State University; it aims to construct uniform international social and economic data sets. Its PSID branch publishes a processed individual-level panel data set of the PSID up to the year 2015, with a limited number of variables. Although it only contains limited information, it serves as the backbone of comprehensive individual-level data for our purpose. With the help of the WZB-PSID tools developed by Ulrich Kohler at the Berlin Social Science Center (Wissenschaftszentrum Berlin für Sozialforschung (WZB)), we merge the PSID data with the CNEF-PSID data set and build our main data set.[18] In the analysis, we apply the standard cross-sectional PSID weight constructed by the CNEF-PSID. The standard weight provided by the PSID accounts for the original family's national representativeness and attrition over time.[19] In estimation, the case weight is the individual's PSID weight divided by the number of cases of the individual in the sample to prevent the over-representation issue. One feature of the PSID weight is that it excludes women who appeared in the sample by marrying or cohabitating with the core PSID members. This setting avoids data attrition due to divorce or cohabitation break-up, but also causes the loss of a considerable number of observations. In the section of alternative spec-

[18]We thank David Brady and Ulrich Kohler for their help in reconstructing the PSID data for the analysis using the WZB-PSID tools and relevant commands they developed.

[19]The broad idea of the PSID weight is that the members and the descendants of the original surveyed families (the "core members") are assigned a weight that reflects the possibility of the family being selected in 1968. Then, accounting for the conditional probability of attrition, the weights of remaining respondents grow slightly in every survey.

ifications, we construct a supplementary weight (the “extended weight”) to include these women in the estimation by assigning them the same weight from their partner. As will be discussed later, adding these samples would affect the results only mildly.

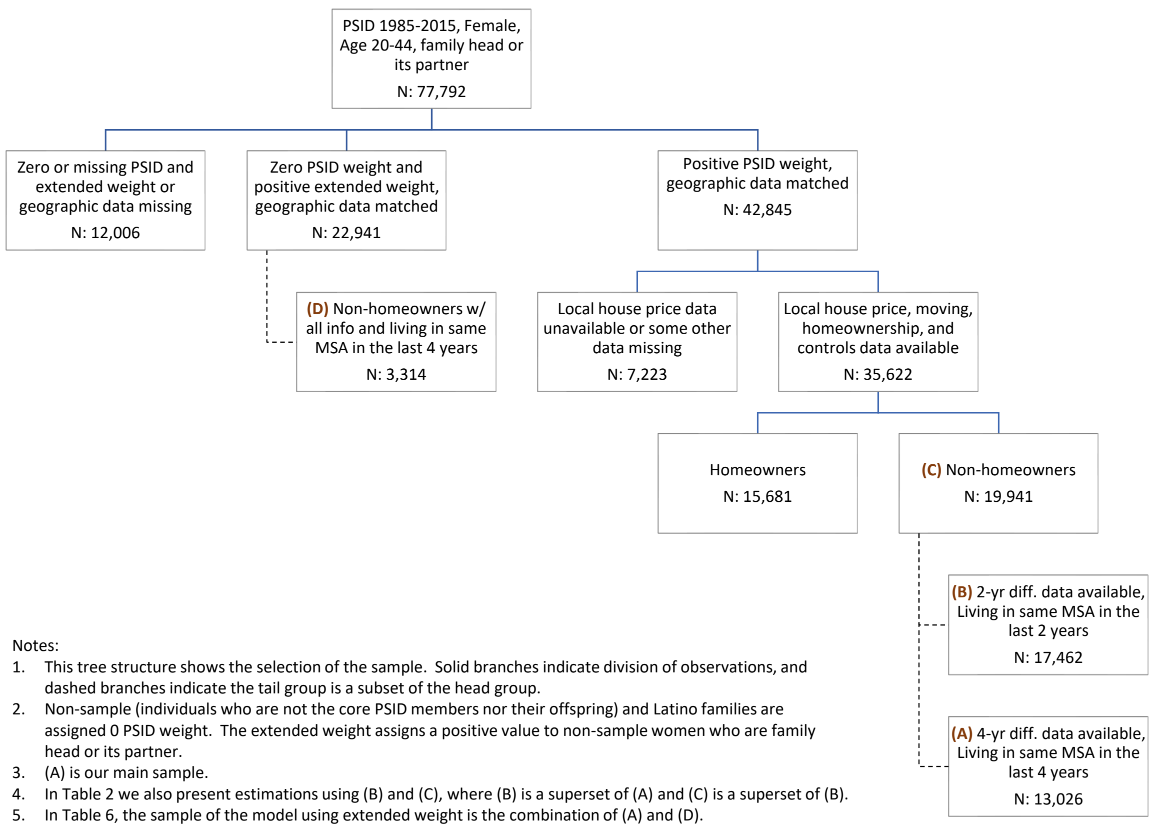

Figure 1.1: The tree structure of PSID sample selection.

After censoring PSID weight and (cross-sectional) data completeness, 35,622 cases are left. Among them, 19,941 non-homeowner cases are available for analysis. To concentrate on the effect of housing and childbearing, only women who did not move inter-MSA in the past four years are kept, which yields a sample size of 13,026 cases. This is our main sample. Figure 1.1 shows the method of sample selection, and the main sample is presented as box (A). In this sample, the unconditional probability of entering homeownership only is 9.2%, of childbearing only 6.5%, and of doing both in the same time period 0.7%. Later, we also estimate models using sample box (B) and (C) to examine the representativeness of our main sample. Moreover, box (D) is added to our main sample when we use the extended weight.

Table 1.1 presents the summary statistics of the sample with positive PSID weight and no missing information. Except for the first three rows (the dependent variable) and the last three rows (the independent variable of interest), all the other variables are introduced into the model as controls. Column (1) summarizes the whole available sample, regardless of whether the women are existing homeowners or have moved from other MSA recently. This sample is represented by the right-most box of the third tile in Figure 1.1. Column (2) restricts the sample to only non-homeowners, which is indicated as box (C) in the figure. Column (3) summarizes the main sample, namely box (A) in the figure. There is a clear demographic difference between the groups.

Comparing column (1) and (2), the non-homeowners are in general younger, with a lower education level, much less likely to have a partner, and with a higher rate being black, while the unconditional likelihood of childbearing is almost the same with the whole sample. Not surprisingly, they also tend to have a lower family income, on average $42,572 versus $65,323 annually.[20] They are more vulnerable to the house price growth not only because of the lower income but also because they do not possess any equity hedge. The demographic difference between columns (2) and (3) is much smaller, except the average

[20]The distribution of total family income is right skewed. The medians are both lower.

Table 1.1: Summary statistics of the PSID sample.

Category	Variable	(1) All available cases with full info		(2) Non-homeowners		(3) Non-homeowners and non-migrants	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Decision	New homeownership only	0.092	(0.289)	0.099	(0.299)	0.092	(0.288)
(in prob.)	New birth only	0.073	(0.260)	0.073	(0.260)	0.065	(0.247)
	New ownership and new birth	0.009	(0.097)	0.009	(0.095)	0.007	(0.084)
Demographic	Age	33.766	(5.983)	32.125	(5.945)	33.603	(5.470)
	Partnership	0.586	(0.493)	0.391	(0.488)	0.359	(0.480)
	White	0.528	(0.499)	0.397	(0.489)	0.330	(0.470)
	Black	0.415	(0.493)	0.545	(0.498)	0.618	(0.486)
	Other race	0.057	(0.232)	0.058	(0.233)	0.051	(0.220)
Education	High school diploma	0.442	(0.497)	0.489	(0.500)	0.533	(0.499)
	Some college	0.289	(0.453)	0.309	(0.462)	0.307	(0.461)
	College graduate	0.270	(0.444)	0.202	(0.401)	0.160	(0.367)
Family size	No child	0.293	(0.455)	0.342	(0.474)	0.277	(0.447)
	One child	0.243	(0.429)	0.245	(0.430)	0.246	(0.431)
	Two children	0.277	(0.448)	0.228	(0.420)	0.255	(0.436)
	More than two	0.187	(0.390)	0.184	(0.388)	0.222	(0.416)
Financial	Employment status	0.812	(0.391)	0.798	(0.401)	0.783	(0.412)
	Real family income in $1,000	65.323	(64.829)	42.572	(41.348)	40.730	(41.675)
Housing	2-year price change in $1,000					2.941	(29.836)
market	4-year price change in $1,000					5.073	(49.219)
	Median price in $1,000	176.984	(89.604)	181.891	(95.517)	181.085	(96.106)
Observations		35622		19941		13026	

Note: The sample are women who are either family head or its partner, age between 20 and 44, with positive weight and all information available from PSID 1985-2015. Non-homeowners are equivalent to the Sample box (C) in Figure 1, and non-homeowners and non-migrants are equivalent to Sample box (A) in Figure 1, where non-migrants means people who stayed in the same MSA in the last four years. All monetary means are inflated using the CPI-U in real 2011 dollars.

age of non-migrant women is similar with the whole sample. Notably, women who stayed in the same metropolitan area in the last four years have a lower probability of entering homeownership and childbearing, probably because their family income is on average lower and they are more likely to live alone. This suggests a relation between migration and housing and fertility. Though this is not the focus of this paper, it is a fact that deserves more attention.

Our outcome variable for the regression model is defined as four mutually exclusive alternatives of actions in a time window. It is set as the time period between the last pair of surveys, and we track the survey dates by month. During each time window, a female respondent chooses either to do nothing, buy a home, have a child, or both. There is however a doubt for the setting. Since the PSID shifted from annual to biennial in 1997, the time window for respondents after year 1997 became one year longer than for those who were surveyed before. To examine whether this would become a confounding factor, we set another group of estimations that fix the time window of the outcome variable to the period between the current survey and the survey taken two years ago, and regress with the respondents from surveys of the odd years only. The outcomes are presented in section 5.3. We show that the results from the two groups are qualitatively the same, while the estimations from the second group inevitably suffer from higher standard errors. For convenience, we call the outcome variable in the standard group the "flexible window output" and in the alternative group the "fixed window output" in the following sections.

The house price data of the MSAs are imputed from two sources. On the one hand, the FHFA publishes the quarterly Housing Price Indices (HPI) of 403 MSAs.[21] The earliest

[21]The list of the MSAs changes over time due to the demographic change. We update all the geographic

recording dates from 1976, though most of the series start from the 1980s. These indices estimate the longitudinal trend of the price level of local single-family houses using both repeat-sales prices and appraisal data (not seasonally adjusted). Except for eight MSAs, all indices set 1995 as the base year. On the other hand, the Longitudinal Tract Database (LTDB) from Brown University has the data of the cross-sectional tract-level median home values, which it calculated from the decennial Census. Since the HPIs do not represent cross-sectional price differences between cities, we take the cross-sectional home value data of year 2000 from the LTDB and calculate the average median home value of each MSA, then together with the FHFA HPI we construct a panel of imputed yearly average house prices. Although the LTDB also has the median rent data, it is unfortunate we cannot find a reliable local longitudinal information for rents.[22]

The last three columns in Table 1.1 show a summary of the house price changes in real term.[23] On average, the house prices experienced a net growth in the past 30 years, despite a huge slide between 2007 and 2011. For our main sample, the net average two-year real house price change is $2,941 and the four-year change is $5,073. The high standard errors partly reflect the fluctuations in time series and partly reflect the huge diversity of house price growth between cities. Even at the census division level, this spatial difference can be easily spotted by comparing the distributions of house price change. As Figure 1.2 indicates, the local house variations in the coast areas are much greater than the cities in the Midwest and the South for households in PSID.

information in accordance to the September 2018 Delineation of the United States Office of Management and Budget (OMB).

[22] The FHFA recommends the CPI-U of all items less shelter for estimating the inflation of HPI. However, a pilot estimation shows it does not produce a notable change on the results.

[23] The statistics of house price changes for columns (1) and (2) are suppressed because, without dropping

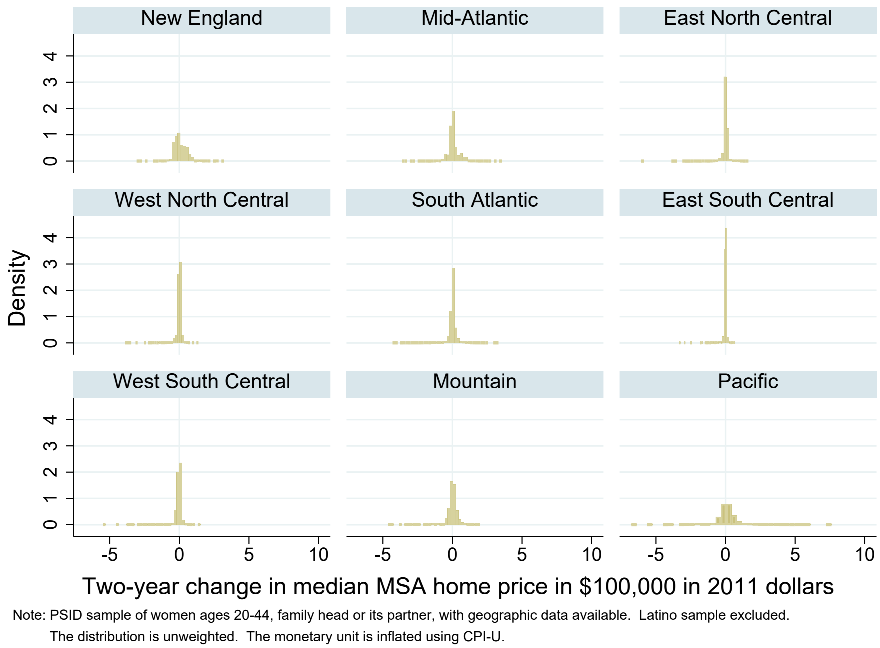

Figure 1.2: The distribution of 2-year home price change between 1985 and 2015, by census division.

Other supplementary data sources provide aggregate level data to control for the regional or national macroeconomic conditions. We take the MSA-level average personal income from the BEA, the state-level unemployment rate from the LAUS, and create a yearly national recession index by taking the annual average of the quarterly recession index from the FRED. These variables reflect the state economic performance and the broad national health of the economy about which families are likely to be concerned when they formulate their expectation for the future market condition.

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the sample who migrated, migration made house price change endogenous.

1.5 Results

Our benchmark statistical model presented below takes the flexible window output and the PSID standard weight. Without further specification, the sample is women who did not move inter-MSA in the past four years, namely the sample box (A) in Figure 1.1. Most tables in this section report the estimated marginal effect to give an intuitive and comparable interpretation. It should be remembered that the predicted effects are quantitatively meaningful only at the margin of the change.

1.5.1 The marginal effect of variables of interest

Table 1.2 reports the main results of the estimated average marginal effects of the house price value and variations from the MNL model described earlier.[24] Each column presents the estimated marginal effects and their standard error of the specific independent variable on the three alternatives, with no action as the base alternative with all coefficients normalized to 0. The upper panel reports the results from the benchmark model. The sample is limited to women whose residential data in the past four years is available and records no change in residential MSA during that time, namely the sample (A) in Figure 1.1. All estimations include the full set of controls listed in Table 1.1, geographic fixed effects at the census division level, and time fixed effects at the 5-year level. The standard errors are estimated by the sandwich estimator clustered at the MSA level. Column (1) shows the marginal effects of the real house price level, columns (2) and (4) show the effect

[24]The MNL model passes the IIA tests for all the parameters of the main variables of interest. Table 1.7 shows the test results. Moreover, because the baseline probability of the alternatives varies in a wide range, elasticity does not provide intuitive interpretation. Because of this reason, we report the marginal effects only. Table 1.8 reports the elasticity table for the models in the upper panel of Table 2.

of the two-year and four-year real price change, and columns (3) and (5) show the effect of the two-year and four-year real price growth rate. The bottom panel reports the estimation results using more general sample selection rules for comparison. The model for the left three columns slackens the rule of selection to two-year data availability and no residential MSA change. This increases the sample size by more than four thousand. The model for the right-most column includes all women regardless of whether they moved from another MSA in the past, further adding two thousand observations. They are represented by box (B) and (C) in Figure 1.1.

Table 1.2: The MNL model estimates of marginal effects of house price level and variations.

Dependent Choice	(1)	(2)	(3)	(4)	(5)
	Independent Variable				
No inter-MSA move in the past four years					
	Median House Price ($100,000)	2-Year Price Change ($100,000)	2-Year Price Growth Rate	4-Year Price Change ($100,000)	4-Year Price Growth Rate
Homeownership only	-0.064***	-0.026	-0.076	-0.020	-0.064*
	(0.0169)	(0.0230)	(0.0718)	(0.0140)	(0.0388)
Birth only	0.018***	-0.006	0.013	0.005	0.021
	(0.0058)	(0.0104)	(0.0338)	(0.0092)	(0.0237)
Both	-0.011***	-0.009	-0.034	-0.015***	-0.036**
	(0.0050)	(0.0058)	(0.0232)	(0.0046)	(0.0164)
N	13026	13026	13026	13026	13026
No inter-MSA move in the past two years				Regardless of moving	
	Median House Price ($100,000)	2-Year Price Change ($100,000)	2-Year Price Growth Rate		Median House Price ($100,000)
Homeownership only	-0.058***	0.000	-0.001		-0.053***
	(0.0146)	(0.0147)	(0.0601)		(0.0144)
Birth only	0.013**	-0.015*	-0.021		0.011**
	(0.0039)	(0.0086)	(0.0241)		(0.0041)
Both	-0.005	-0.003	-0.021		-0.007
	(0.0043)	(0.0081)	(0.0234)		(0.0046)
N	17462	17462	17462		19941

Note: Models differ only in the independent variable of interest. All estimates include controls for partnership, race, number of children, total family income, and employment status, age group dummies, educational attainment dummies, state-by-year unemployment rate, MSA-by-year real income per capita, national recession index, and geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at the MSA level. Significant at *** $p < 10\%$, ** $p < 5\%$, * $p < 1\%$.
Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, with positive weight and all information available from PSID 1985-2015.

The estimation on the effect of the real local house price level shows that families living in an expensive area generally have a lower chance to enter homeownership but a higher chance to have a new child at the renting stage. A $100,000 difference in the local house price unsurprisingly leads to a 5.3 to 6.4 percentage points decrease of the probability of entering homeownership, which is in line with earlier work using PSID (Henretta, 1987). Oppositely, it contributes to 1.1 to 1.8 percentage points of the probability of women giving birth. With regard to the doing both alternative, it seems that the discouragement to homeownership slightly outweighs the encouragement to childbearing, leading to a 0.5 to 1.1 percentage point decrease in probability. Yet the standard errors are relatively high as a result of the small size of observations on this choice. The results of the first two alternatives are robust, as dropping the last alternative does not create a substantial drift of the estimated values (results not shown here). To make sense of the number, the difference between the median sales prices of houses between 2011Q1 and 2015Q1 is $50,163 in 2011 USD, according to the Federal Reserve. If we take the estimates from the upper panel for granted, this price level difference by itself would generate a 3.2 percentage points decline, or about a 34.8% decrease, in the likelihood of homeownership transition only; a 0.9 percentage points increase, or 12.4% growth, in the likelihood of childbearing only; a roughly 0.6 percentage point decline, or 61.1% decrease, in the likelihood of doing both, if other things remain equal.

On the other hand, we find only weak evidence in support of the marginal effects of house price change. The results posted in Table 1.2 suggest that the experience of house price growth can deter women from entering homeownership, as the estimates of the effect on both entering homeownership only and doing both are negative. Consistent estimates notwithstanding, the suggestive effect is only statistically meaningful on doing both, for the four-year price change. The estimates from column (4) indicate a $100,000 price increase would result in a 1.5 percentage points decrease in the probability of the choice, and the estimates from column (5) allude that a 42% increase in house price would lead to an effect of the same magnitude had the effect been linear to price growth rate, *ceteris paribus*. The estimates of marginal effects on childbearing are weak and mixed. For two-year price change and growth, the estimates are small and inconsistent in direction with large standard errors. For four-year price change and growth, the estimates are consistently positive but very weak and lack statistical power. However, combined with the estimated effect of doing both, these results signify a net decline in childbearing likelihood.

These results imply an interesting interaction between house price variation and the dynamics of homeownership and childbearing, and at the same time go along with the literature. Lovenheim and Mumford (2013) argue that the increasing trend in fertility in the early 2000s is likely contributed by the wealth effect of homeowners due to the home equity appreciation, while neither house price level nor change surge are statistically associated with the childbearing likelihood of renters.[25] Our estimations indicate instead that high price leads to a tradeoff between homeownership and childbearing for those families, or at least they are compelled to postpone homeownership and switch the order of homeowning and parenting in their family's life course. To link this to Lovenheim and Mumford's results, we estimate the effects of house price level and change on childbearing likelihood only, using

[25]The focus of Lovenheim and Mumford (2013) is the fertility behavior of homeowners. In their analysis, they present the regression results of renters as the side finding to compare with their main results. Here we compare our results with their side finding.

a simple linear regression model with the same set of controls plus year and state fixed effects and report the results in the upper panel of Table 1.3. In the first three columns, the model used regresses the probability of a woman giving birth between the last two surveys, regardless of whether she entered homeownership during the same time period. State fixed effects and year fixed effects are controlled. This is in line with the spirit of Lovenheim and Mumford's main model. Though the detailed settings and variable definitions are different, we obtain the result of rejecting the relationship between house price and net childbearing likelihood, which is consistent with Lovenheim and Mumford's finding on renters.

The model for columns (4) to (6) in the upper panel regresses the probability of woman giving birth only between the last two surveys, excluding incidents of entering homeownership and giving birth during the same period, and in the bottom panel the model regresses the other two alternatives on the same set of independent variables. These regressions show that local house price has a prominent positive relationship with the probability of childbearing only and a negative relationship with the probability of entering homeownership as well as doing both in the same time period. In combination, the evidence suggests a consequential delay of families entering homeownership caused by high local house prices, whereas the course of parenting is likely to take place at any rate, and even with a minor increase in the net probability. A \$100,000 increase in local house price relates to a 1.6 percentage points increase in the probability of childbearing only and a 1.1 percentage points decrease in doing both in the same period, resulting in a 0.5 percentage point net increase in the probability of childbearing, which reflects a 7.7% growth.

Financial constraint is the most plausible mechanism to explain this outcome.

Table 1.3: The linear probability model estimates of marginal effects of house price level and variations.

Dependent Choice	(1)	(2)	(3)	(4) Independent Variable	(5)	(6)
	Median House Price ($100,000)	2-Year Price Change ($100,000)	4-Year Price Change ($100,000)	Median House Price ($100,000)	2-Year Price Change ($100,000)	4-Year Price Change ($100,000)
Birth	0.005 (0.0079)	0.002 (0.0133)	0.001 (0.0106)			
Birth only				0.016*** (0.0070)	-0.001 (0.0114)	0.008 (0.0097)
N	13026	13026	13026	13026	13026	13026
Homeownership only	-0.051** (0.0177)	-0.049* (0.0255)	-0.027* (0.0143)			
Doing both				-0.011*** (0.0049)	0.004 (0.077)	-0.007 (0.0044)
N	13026	13026	13026	13026	13026	13026

Note: Models differ only in the independent variable of interest. All estimates include controls for partnership, race, number of children, total family income, and employment status, age group dummies, educational attainment dummies, state-by-year unemployment rate, MSA-by-year real income per capita, national recession index, and geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at the MSA level. Significant at *** p <10%, ** p <5%, * p <1%.
Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, with positive weight and all information available from PSID 1985-2015.

Because of the larger amount of required mortgage down payment and a higher expected mortgage payment thereafter, prospective home buyers are apt to prefer to wait longer before entering the market. Meanwhile, the childbearing decision seems merely affected by the price shock, and, if anything, is probably due to the temporarily loosened family budget since the expense on the mortgage is postponed. This suggests more children will be born into non-homeowning families, though it does not necessarily imply a smaller housing space for children as these families can still expand their housing space by moving to a bigger rental unit. The underlying limitation of the estimates here is that it only covers women who did not move across MSAs through the survey period.

The results in Table 1.2 and 1.3 both suggest no strong relationship in general between short-term house price change or growth and the two family decisions, hinting that such a short shock itself is not really an influential factor. One possible interpretation is that short-term price change may not be enough for families to give up or delay homeownership, as house price variations can be transitory. If the growth persists longer, an expectation of a continuing growth trend may be formed in the public. This starts to have an impact on the costliest choice: doing both at the same time. Thus, we could observe the suggestive effect in columns (4) and (5) of the upper panel of Table 1.2, though it may still be marginal because the linear probability model does not support the statistical significance. Another possible interpretation is that childbearing and sometimes the realization of homeownership transition arrive months after the decision is made. Two-year price change partly takes place after the family decision time point and thus has null explanation power. Nonetheless, a series of estimations on the marginal effect of lagged two-year price change all return insignificant results, suggesting short-term price variation is no an influence on general family behavior.

1.5.2 The other controls

The estimation results shown above suggest a local house price growth would cause a net negative effect on home buying but net zero effect on childbearing among households who chose not to move to another MSA. Of course, other controls also play an important role in the family decision. Table 1.4 reports the full table of marginal effects of controls less the fixed effects of two selected estimations, grouped in three column groups, which correspond to the model of columns (1) and (4) of the upper panel of Table 1.2. Each column group consists of three columns, in tandem reporting the marginal effects on entering homeownership only, childbearing only, and doing both.

The estimates of the controls show the consistency between the two models.[26] Partnership (either by marriage or cohabitation) always has positive effects on both homeownership and childbearing. Whether it is newly formed matters only for homeownership. Total family income is positive related to home buying, but has a negative effect on childbearing on a smaller scale, presumably due to the greater opportunity cost of work time, which leads to a net substitution for children. This substitution effect of work time is also weakly reflected by the negative effect of the employment status of the woman. However, it is not statistically significant.

Demographically, parenthood is a positive indicator for an additional child, but the ability to enter homeownership may be deferred when the number of children is greater than two. This is probably because a large family size erodes the financial affordability of homeownership. Non-homeowning black women have a relatively higher likelihood of giving birth, and they are also less likely to enter homeownership compared to white women.[27] It should be noted that this does not imply a disparity of fertility rates between races but a higher likelihood of parenthood without homeownership for black families. This difference may not only be attributable to social and economic inequality but also to the divergence in social norms of expected life course. Women's education level shows a positive effect on both homeownership and childbearing. Again, we should be cautious of the interpretation, as the baseline is a woman with low education but already independent from her parents. The positive effect can reflect delayed fertility due to prolonged education time, so it does not necessarily reflect fertility difference by education. Regarding age, it seems the woman's

[26] Not showing in the table, the estimates are also consistent between the model for two-year price change and four-year price change.

[27] Table 4 does not show the latter directly because the reference group is the other race.

age does not have a privilege or penalty on entering homeownership, but inevitably the childbearing likelihood declines steadily with age. All these observations are in line with the demographic regularities.

The MSA-level personal income per capita is the most important regional macroeconomic factor in the family's decision. Higher personal income implies higher house price level in the local area. The unconditional correlation coefficient between the two variables in the sample is 0.71. Naturally as a result, a high personal income level exerts a similar effect as the high house price level, and the effect is suppressed when both variables are included, as in column (1-1). However, this is not true for the change in personal income. By estimations not shown here, a short-term change of local personal income does not intervene in family decisions on homeownership and childbearing directly. Recession has a negative impact on becoming a homeowner in that year, but in general has no substantial effect on the probability of childbearing.

So far, the estimates reflect the observed average marginal effect of house price level and changes. As Table 1.4 indicates that other factors are also associated with the probabilities of home buying and childbearing, it is reasonable to argue that the effects of house price level and changes are different for families under different financial and demographic conditions. An earlier study suggests non-linear and interaction effects are common in the decision of family homeownership (Li, 1977). Here, we briefly examine this potential heterogeneity of the effect by estimating an interaction term of the main variables of interest with the three most outstanding controls, the real term family income, partnership, race, and parenthood. Without diving too deeply into this issue, as each of the

Table 1.4: The MNL model estimates of the marginal effects of other controls.

Controls	(1-1)	(1-2)	(1-3)	(2-1)	(2-2)	(2-3)
	Dependent Choice					
	Ownership only	Birth only	Both	Ownership only	Birth only	Both
Median House Price ($100,000)	-0.064***	0.018***	-0.011**			
	(0.0169)	(0.0058)	(0.0050)			
4-Year Price Change ($100,000)				-0.020	0.005	-0.015***
				(0.0140)	(0.0092)	(0.0046)
Partnership	0.102***	0.027***	0.040***	0.102***	0.028***	0.040***
	(0.0180)	(0.0076)	(0.0105)	(0.0180)	(0.0077)	(0.0105)
Enter Partnership	0.023	0.004	-0.007	0.024	0.003	-0.007
	(0.0219)	(0.0109)	(0.0089)	(0.0223)	(0.0107)	(0.0090)
Total Family Income ($100,000)	0.128***	-0.029**	0.003	0.125***	-0.029**	0.003
	(0.0368)	(0.0115)	(0.0060)	(0.0396)	(0.0116)	(0.0062)
Employment	-0.030	-0.008	0.000	-0.028	-0.008	0.000
	(0.0190)	(0.0069)	(0.0067)	(0.0189)	(0.0070)	(0.0068)
Number of Children (base = 0)						
One	-0.002	0.048***	0.025***	-0.001	0.048***	0.026**
	(0.0196)	(0.0069)	(0.0068)	(0.0200)	(0.0068)	(0.0069)
Two	-0.009	0.087***	0.018***	-0.008	0.088***	0.018**
	(0.0217)	(0.0120)	(0.0078)	(0.0223)	(0.0123)	(0.0079)
More than 2	-0.076***	0.129***	0.015*	-0.076***	0.128***	0.015**
	(0.0236)	(0.0145)	(0.0077)	(0.0239)	(0.0147)	(0.0079)
Race (base = other)						
White	0.029	0.021***	-0.000	0.032	0.020**	-0.001
	(0.0380)	(0.0079)	(0.0088)	(0.0311)	(0.0080)	(0.0089)
Black	-0.025	0.027**	-0.005	-0.023	0.028**	-0.006
	(0.0322)	(0.0101)	(0.0098)	(0.0323)	(0.0102)	(0.0098)
Education (base = no high school)						
High school diploma	0.061***	-0.001	0.005	0.062***	0.000	0.005
	(0.0222)	(0.0073)	(0.0034)	(0.0223)	(0.0073)	(0.0034)
Some college	0.038	0.016*	0.018***	0.041	0.016**	0.018***
	(0.0251)	(0.0073)	(0.0062)	(0.0253)	(0.0073)	(0.0064)
College graduate	0.109***	0.019	0.028***	0.108***	0.020	0.027***
	(0.0292)	(0.0118)	(0.0083)	(0.0286)	(0.0121)	(0.0082)
Age group (base = 20-24)						
25-29	-0.018	-0.039	-0.001	-0.021	-0.037	-0.002
	(0.0383)	(0.0272)	(0.0228)	(0.0388)	(0.0271)	(0.0234)
30-34	0.006	-0.070***	-0.018	0.003	-0.069***	-0.018
	(0.0411)	(0.0262)	(0.0230)	(0.0419)	(0.0262)	(0.0236)
35-39	0.012	-0.101***	-0.028	0.008	-0.100***	-0.028
	(0.0410)	(0.0266)	(0.0225)	(0.0417)	(0.0264)	(0.0229)
40-44	-0.008	-0.121***	-0.034	-0.014	-0.119***	-0.035
	(0.0454)	(0.0259)	(0.0225)	(0.0457)	(0.0258)	(0.0230)
State umemployment rate	-0.009*	-0.001	-0.001	-0.011**	-0.001	-0.003
	(0.0046)	(0.0022)	(0.0020)	(0.0049)	(0.0025)	(0.0022)
MSA personal income per capita	-0.002	-0.046	0.115**	-0.548***	0.119**	0.040
	(0.1977)	(0.0787)	(0.0492)	(0.1085)	(0.0480)	(0.0353)
Average recession indicator	-0.035	0.006	-0.011	-0.028	0.005	-0.013*
	(0.0306)	(0.0118)	(0.0075)	(0.0305)	(0.0119)	(0.0078)
N	13026	13026	13026	13026	13026	13026

Note: Models differ only in the independent variable of interest. All estimates include geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at individual level. Significant at *** p <10%, ** p <5%, * p <1%.
Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, stayed in the same MSA in the last four years, with positive weight and all information available from PSID 1985-2015.

interaction effects is potentially a topic pending for further research, we look into interaction effects by showing the estimated marginal effects and the exponentiated coefficient of the interaction terms.

The results of the interaction effects with total family income are presented in Table 1.5. Panel A reports the ratio of relative-risk ratio (RRR) of the multiplicative term.[28] For the alternatives of birth only and doing both, none of the estimates stray significantly from 1, indicating that family income level does not affect the relative volume of the effect. The estimates for the effect on entering homeownership are different. For house price level, the ratio of RRR is 0.78, meaning that the RRR on average is about 0.78 times that for women with \$100,000 higher in total family income. Because the effect of house price on ownership is negative, the RRR of the effect is less than 1. As shown in Panel B, the RRR of house price level on entering homeownership is 0.76, which means the odds of entering homeownership would drop one quarter given a \$100,000 increase in house price level. The 0.78 ratio of RRR times 0.76 is about 0.59, indicating that, given a \$100,000 increase in total family income, the RRR of house price level is lower. In other words, the impact of house price on home buying is relatively larger for the higher income group. Differently, the ratio of RRR of the four-year price change is greater than 1, indicating its impact is smaller for women with higher family income, while the average marginal effect is insignificant. In sum, this result suggests that families with higher income are more responsive to high price level and more resilient to price change in home buying, but they have no significant difference in the childbearing decision. These families have more financial capability against

Table 1.5: The estimated ratio of RRR of the interaction effect of total family income and the marginal effects (ME) of house price level and variations, by income level.

		(1)	(2)	(3)	(4)	(5)
		Interaction with total family income				
A. Ratio of RRR	Dependent Choice	Median House Price ($100,000)	2-Year Price Change ($100,000)	2-Year Price Growth Rate	4-Year Price Change ($100,000)	4-Year Price Growth Rate
	Homeownership only	0.786***	1.641	3.986	1.545**	2.977**
		(0.0598)	(0.4943)	(3.9519)	(0.2712)	(1.5592)
	Birth only	1.101	1.339	1.164	1.320	2.143
		(0.2317)	(0.9311)	(2.3898)	(0.5783)	(2.6486)
	Both	0.943	0.826	0.359	1.175	1.176
		(0.1566)	(0.2817)	(0.4870)	(0.3923)	(1.8359)
	N	13026	13026	13026	13026	13026
B. RRR						
	Homeownership only	0.757**	0.525*	0.174*	0.583***	0.245**
		(0.0941)	(0.1818)	(0.1748)	(0.1193)	(0.1348)
	Birth only	1.244	0.674	0.893	0.870	0.817
		(0.2231)	(0.3208)	(1.3502)	(0.2828)	(0.7724)
	Both	0.465**	0.630	0.257	0.350**	0.090
		(0.1535)	(0.2768)	(0.4113)	(0.1623)	(0.1654)
C. ME on ownership only						
Real Family Income at	$10,000	-0.032**	-0.062*	-0.175*	-0.051**	-0.136**
		(0.0139)	(0.0346)	(0.1021)	(0.0207)	(0.0565)
	$30,000	-0.042**	-0.057*	-0.160*	-0.047**	-0.126**
		(0.0147)	(0.0322)	(0.0939)	(0.0194)	(0.0526)
	$50,000	-0.054***	-0.050*	-0.138	-0.040**	-0.110**
		(0.0155)	(0.0296)	(0.0861)	(0.0181)	(0.0490)
	$70,000	-0.068***	-0.040	-0.108	-0.031*	-0.089*
		(0.0168)	(0.0275)	(0.0822)	(0.0173)	(0.0474)
	$90,000	-0.082***	-0.028	-0.070	-0.020	-0.061
		(0.0186)	(0.0271)	(0.0869)	(0.0175)	(0.0498)

Note: Models differ only in the independent variable of interest. All estimates include controls for partnership, race, number of children, total family income, and employment status, age group dummies, educational attainment dummies, state-by-year unemployment rate, MSA-by-year real income per capita, national recession index, and geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at the MSA level. For the panel of the ratio of RRR, the value in parentheses reports the robust standard errors times ratio of RRR. Significant at *** p <10%, ** p <5%, * p <1%. Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, stayed in the same MSA in the last four years, with positive weight and all information available from PSID 1985-2015.

price appreciation.

It should be noted that this measure compares the RRRs, which is itself a ratio. The scale of RRR is determined by the marginal effect as well as the baseline odds. Because the baseline probability of home buying varies largely by income group, the ratio of RRR does not necessarily provide insights on the comparison of marginal effect in different income groups. To affirm the conclusion above, we show the marginal effects at different family income levels in Panel C. The marginal effects are consistent with the ratios of RRR. For house price level, the marginal effect is increasing with family income and for house price change it is decreasing.

Ratio of RRR is not an intuitive measure. It is nevertheless a convenient tool to show the presence of an interaction effect. Regarding partnership, race, and parenthood, the analysis reveals several notable points. We can see the results in Table 1.9. Women in partnership behave differently from those who are not in partnership for home buying and doing both, though the greater value of the latter results from the extremely small odds of doing both for women who are not in a relationship. There is also an interaction effect of a two-year house price change and partnership on the childbearing decision. Race difference creates a large divergence in the effect of a two-year house price change on the childbearing decision, indicating different norms on childbearing between Blacks and Whites. Parenthood does not present a strong interaction effect except for doing both, which is again due to the extremely small likelihood of doing both at the first birth. At any rate, this exercise shed some light on the more intricate mechanisms of the family's decision. The more solid argument requires far deeper investigations than the simple interaction term analysis.

[28] Appendix 1.B provides a short introduction to the ratio of RRR.

1.5.3 Other specifications

The estimated marginal effect might not reflect the true underlying mechanism if the statistics are unique to certain specifications. Sample selection, weighting, clustering, and other specifications on the variable could all affect the estimated results. Here we test the robustness of our findings by estimating models with different specifications. Table 1.6 presents the test results, with each panel reporting the estimates from a model one specification from the benchmark.

Panel A reports the results from the model that defines the alternatives as the actions taken place in the fixed two-year time window. As discussed in the empirical design, this setting is to prevent the uneven behavior accounting time after year 1997. The sample for this model only includes women from the odd year surveys, so that the sample sizes are noticeably smaller than in the benchmark model. The results are qualitatively similar to the upper panel of Table 1.2, but cannot reject the null hypothesis of the four-year marginal effect on doing both. This is not surprising. For the samples before year 1997, the new definition means a double length of the behavior time window. If a woman became a homeowner and had a child in two consecutive years before 1997, she is considered to have taken the two actions separately in each year in the flexible time window scheme, but in the fixed time window scheme her behavior is classified as doing both during the two-year period.[29] This could reduce the sensitivity of the suggestive impact of house price change.

The model for panel B uses the extended sample weight that includes women who join the survey because they enter the families of core survey members. For this reason,

[29]Because of this, the unconditional probability of doing both under the flexible time window is 0.6% and under the fixed time window is 1.1% for the same sample.

the newly added women have a much higher rate of being in a partnership and having a higher average family income. This change accounts for an additional thirty-four hundred observations in the sample. At any rate, the estimates of the modified model are still consistent with the results in Table 1.2. Though the suggestive evidence of the marginal effect of a four-year house price change is still marginal, expanding the sample size does not really upset our main finding.

All the reported standard errors so far are accounted for by clustering MSAs. The geographical dimension of standard error correlation is justified by the fact that our variables of interest and the controls for the local economy are at the same geographical level. However, time-series correlation for the same individual is also possible in a panel case. Individual time-invariant traits are common, and they can lead to a correlation between the unobservable components of each individual. Thus, we re-estimate the standard errors by clustering samples by individuals and present the results in panel C. Clearly, this modification does not change the main results. The standard errors are floating around the same level. Moreover, though not reported here, combining these specification changes does not generate notable difference in the results.

1.6 Discussion

How does house price affect the decision of non-homeowning families on home buying and childbearing? Our analysis presented in this paper endeavors to sketch a big picture about the impact of house price level and variation for American urban families over the past thirty years. If it is not too arbitrary to assume these families have some-

Table 1.6: The MNL model estimates of marginal effects of house price level and variations with other specifications.

	(1)	(2)	(3)	(4)	(5)
Dependent Choice			Independent Variable		
A. Alternative dependent variable: two-year fixed time window					
	Median House Price ($100,000)	2-Year Price Change ($100,000)	2-Year Price Growth Rate	4-Year Price Change ($100,000)	4-Year Price Growth Rate
Homeownership only	-0.053***	-0.014	-0.048	-0.005	-0.019
	(0.0166)	(0.0238)	(0.0722)	(0.0136)	(0.0403)
Birth only	0.021***	-0.007	0.006	0.007	0.030
	(0.0067)	(0.0102)	(0.0371)	(0.0085)	(0.0231)
Both	-0.016***	-0.005	-0.007	-0.008	-0.019
	(0.0051)	(0.0051)	(0.0344)	(0.0058)	(0.0185)
N	9074	9074	9074	9074	9074
B. Alternative weight: extended weight					
Homeownership only	-0.080***	-0.014	-0.036	-0.027**	-0.085**
	(0.0166)	(0.0211)	(0.0642)	(0.0120)	(0.0355)
Birth only	0.023***	-0.007	0.018	0.004	0.023
	(0.0054)	(0.0105)	(0.0286)	(0.0090)	(0.0221)
Both	-0.011**	-0.009	-0.030	-0.010*	-0.024
	(0.0054)	(0.0070)	(0.0248)	(0.0053)	(0.0152)
N	16340	16340	16340	16340	16340
C. Alternative clustering: clustering by individual					
Homeownership only	-0.064***	-0.026	-0.076	-0.020	-0.064
	(0.0158)	(0.0265)	(0.0781)	(0.0175)	(0.0465)
Birth only	0.018***	-0.006	0.013	0.005	0.021
	(0.0059)	(0.0122)	(0.0371)	(0.0082)	(0.0226)
Both	-0.011**	-0.009	-0.034	-0.015***	-0.036**
	(0.0043)	(0.0067)	(0.0240)	(0.0052)	(0.0173)
N	13026	13026	13026	13026	13026

Note: Models differ only in the independent variable of interest. All estimates include controls for partnership, race, number of children, total family income, and employment status, age group dummies, educational attainment dummies, state-by-year unemployment rate, MSA-by-year real income per capita, national recession index, and geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at the MSA level except Panel C. Significant at *** p <10%, ** p <5%, * p <1%.

Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, stayed in the same MSA in the last four years, with positive weight and all information available from PSID 1985-2015. Panel A excludes all sample from the odd-year surveys. Panel B adds women with positive extended weight.

what homogenous preferences on housing and children and are statistically representative, the results reveal a few key insights. First, a higher median house price would lower the probability of a family entering homeownership and raise the probability of childbearing slightly, given other conditions unchanged. According to the statistics, a $100,000 increase in local median house price relates to a 5.7 percentage points net decline in the probability of becoming a homeowner and a 0.6 percentage points net increase in the probability of childbearing, by and large. Unequivocally, home buying is sensitive to house price level for the obvious economic reason. Childbearing, however, is not affected by the high price to the same degree. A marginal substitution for homeownership can only be inferred, not directly observed, from the estimations. More interesting are the dynamics of the two behaviors. Women are more likely to have a child without entering homeownership around the same time interval. In other words, more families decide to have a new child before becoming homeowners in areas with an expensive median house price.

Second, the experience of a two-year price change does not have an observable effect on the decisions of home buying and childbearing; and, the evidence reports only a weak negative effect of a four-year price change on doing both during the same time window. This suggests the temporal change in house price does not have a strong impact on the family's behavior, neither in absolute value nor ratio. There are two possible explanations. One is that recent local housing market variation simply does not alter family behavior on home buying and childbearing nor even their expectation on the future trend of equity value and child-rearing costs. Households care about only the current total cost of homeownership. The other explanation is that such effects do exist, but the negative impact of lower relative income due to the increased price is offset by the positive expectation on future equity appreciation. We cannot directly tell which explanation is closer to the reality, but the negative marginal effect of a four-year price change on concurrent home buying and childbearing hints that a family's willingness to take the costliest move is eroded by house price growth, implying that the negative impact might surpass upon a high cost condition. Therefore, the argument of the co-existing offsetting effects is more plausible.

Third, no matter whether the average marginal effect is significant from zero, the results do not imply a linear effect across the whole sample. Families with higher income are hit more by a high house price level to enter homeownership probably due to their higher unconditional likelihood of home buying. The effect of price appreciation behaves oppositely. Women with lower income are affected more by a high house price level, likely because they face a tighter credit constraint and have a lesser chance to acquire benefit from equity appreciation. The analysis also shows significant interaction effects of house price with partnership and race, while a woman's childbearing choice seems less critical. This finding signifies the complexity of a family's decision, as the influence of a single factor is multi-dimensional, entangled with numerous other considerations. Greater economic inequality and a declining marriage rate (but compensated by a growing cohabitation rate) are both likely to play a role at the aggregated level (Lutz et al., 2006; Lesthaeghe, 2010). A more detailed mechanism may hide beneath the surface, though it is out of the scope of this paper.

Considering the housing market only, house price may not directly affect current regional birth rate according to our results, but it not at all unimportant to family fertility.

An expensive housing market would alter the family's life course plan, push homeownership behind parenthood, an allow more children to be raised in rental units during their infancy. Although in this paper only homeownership is considered as a family housing choice, it encapsulates the common differences between rental and owner-occupying housing units, including floor space, maintenance quality, tenure stability, surrounding amenities and facility, all of which could lead to a profound legacy for children, as Haurin et al. (2002) and other authors argue. On the other hand, the recent lowering fertility rate nationwide seems not to be attributable to the rising house prices. At least for non-homeowners staying in the same city, increasing house prices may only generate a temporary discouragement on childbearing for women with partners. Other economic and demographic transformations within society should have a greater and perpetual influence on aggregate fertility rate.

Our findings are in line with the literature and contribute to a deeper understanding of the association between the housing market and family homeownership and childbearing. The dynamics between the two family behaviors are shown to be sensitive to the market variation. Nevertheless, this analysis has clear limitations. In order to prevent house price endogeneity, people who migrated to other metropolitan areas are excluded from our research. But migration is a crucial dimension in family life course. It allows a family to actively choose the house price it would encounter and closely relates to family income and the living conditions of environment. Though it is a relatively small group, empirically women from the migration group have a higher probability of childbearing, suggesting the importance of migration on fertility. Inversely, local house price variations or even spatially relative house price disparity can also alter the migration decision and in tandem affect the home buying and childbearing decisions. In addition, the interaction of housing and childbearing is also influenced by other major life course transitions such as partnership and employment. These transitions are treated as exogenous in our analysis for the purpose of our research. In reality, they are not. Regarding family life course, they are as substantial as housing and childbearing. Investigations on multi-dimensional choice model in a dynamic framework could reveal more insights into individual decisions, and this research is just a start. As we show, the dynamics of major family transitions are sensitive to house price, so it may well happen to partnership and career paths also. We look forward to more detailed studies to disentangle the underlying secret of the economic-demographic interplay.

Another challenge to the analysis of the impact of house price is the difficulty of accurately measuring the real cost families are facing. Besides the fact that house prices may vary to a remarkable degree in a big city and families have divergent housing demand, other factors, including the loan-to-value ratio, mortgage interest rate, and current rental cost, are also accountable for estimating the financial cost of homeownership. The credit constraint of home mortgage is specifically the major obstacle to homeownership, and its volume depends on the proportion of the property value that banks are willing to loan out. The mortgage interest rate also plays an important role as it determines the overall property cost. Unfortunately, we do not have the complete information about what kind of mortgage offer respondents can obtain. In this paper we instead assume the financial burden is exclusively proportional to the local median house price. We expect more questions about the joint family behavior of homeownership and childbearing could be answered with the help of more detailed data on the real cost of homeownership in the future.

Appendix 1.A The Time-lagged Price Effects

Since a birth event is a consequence of the fertility decision (if not an accident) of around 40 weeks ago, the current local house price level and change may not truly reflect the housing market condition at the decision time.[30] The one-year lagged price as well as other time-variant controls may better serve as the factors for the determination of childbearing, as Dettling and Kearney (2014) and Lo (2012) did. In this research, we stick to using the current price and price change because we are interested in the joint dynamics of childbearing and home buying. Besides, the date of conception can be broadly inferred by assuming all pregnancies are full-term, but there is no credible way to find the date of property transactions. Due to the data limitation, we only know the time interval of homeownership transition, gridded by the survey dates.[31] Lacking the precise timing information, we choose the current estimated current price level and price change as the indicators of the housing market conditions.

Using the current price is in line with relevant research in the literature. In a recent paper, Clark et al. (2020) argue that the current price is a valid approximation of the lagged price due to its high correlation and persistence in trend. It is also true for our main research sample. The correlation coefficient of the current and one-year lagged price is 0.97, and that of the price changes are between 0.81 and 0.87. In results not reported here, the

[30] According to CDC, the rate of preterm birth is about 10% in the US, and the early preterm birth (less than 34 weeks) rate is 2.75%. National Vital Statistical Reports, November 27, 2019. https://www.cdc.gov/nchs/data/nvsr/nvsr68/nvsr68_13-508.pdf.

[31] Most surveys recorded the date (to month) of moving, but the transaction and the moving-in do not necessarily happen in the same time interval. The surveys are usually taken in April and around.

regression results using the one-year lagged price information are not significantly different from the results in Table 1.2, except for a slight increase in the scale of the estimated marginal effect of the two-year price change on doing both. We also estimate the marginal effects with both the current and lagged price variables. Because of the high collinearity of the current and lagged prices, the results are highly blurred for all alternatives.[32] This result is different from the estimations with current price level and price change in the same regression, which return both effects as estimated separately.

Appendix 1.B Ratio of RRR

Because our model is non-linear, the interaction effect cannot be simply identified by the coefficient on the interaction terms alone. Instead, the exponentiated coefficient of the multiplicative term between two explanatory variables can imply the presence of interaction effects (Buis, 2010). For a MNL model, the exponentiation of coefficient β_{ik} is called the RRR for alternative i of an independent variable x_k. It is defined as the ratio of the relative probability of i for a one unit increase in x_k. If the value is greater than one, it means that the relative probability of i is greater given an increase in x_k. This interpretation is derived from Equation (1.3).[33]

$$RRR(\beta_{ik}) \equiv e^{\beta_{ik}} = \frac{e^{x'\beta_i+\beta_{ik}}}{e^{x'\beta_i}} = \frac{P_i(x_k+1)/P_0(x_k+1)}{P_i(x_k)/P_0(x_k)}. \tag{1.6}$$

[32] Regarding a longer-time-lagged effect, Lovenheim and Mumford (2013) estimate the effect of the four-year-lagged four-year price change and find that it has minimal influences on the childbearing decision.

[33] Mathematically, RRR is also the proportion of the risk ratio of alternative i for a unit increase in x_k to the risk ratio of the base outcome for a unit increase in x_k. However, this form does not provide an intuitive interpretation.

The exponentiation of a multiplicative term is the ratio of RRR for the two explanatory variables (Norton et al., 2004). It tells the relative volume of effect, in terms of the RRR of one variable, for a one unit increase in the other variable. If we add an interaction term of x_k and x_l to the RUM model and let β_{ikl} be its coefficient for alternative i, we have

$$e^{\beta_{ikl}} = \frac{e^{x'\beta_i+\beta_{ik}+\beta_{il}+\beta_{ikl}}/e^{x'\beta_i+\beta_{il}}}{e^{x'\beta_i+\beta_{ik}}/e^{x'\beta_i}} = \frac{RRR(\beta_{ik} \mid x_l+1)}{RRR(\beta_{ik} \mid x_l)}. \tag{1.7}$$

In our case, we set the first variable as the variable of interest, and the second one is the interacted control variable. The ratio of RRR shows how many times the RRR of the variable of interest would change given a unit increase of the control. If RRR is greater than 1, a greater ratio of RRR indicates the effect is intensified by the interaction. If it is less than 1, a greater ratio indicates the effect is diminished by interaction.

Table 1.7: The IIA property test for the MNL model.

Independent Variable	Alternative Dropped	Null Hypothesis					
		$\hat{y}_1$ (full) = $\hat{y}_1$		$\hat{y}_2$ (full) = $\hat{y}_2$		$\hat{y}_3$ (full) = $\hat{y}_3$	
		χ^2	p-value	χ^2	p-value	χ^2	p-value
Median House Price ($100,000)	Ownership only			0.661	0.416	0.201	0.654
	Birth only	1.356	0.244			0.226	0.634
	Both	1.046	0.347	0.653	0.419		
2-Year Price Change ($100,000)	Ownership only			0.179	0.672	0.076	0.783
	Birth only	1.028	0.311			0.013	0.911
	Both	2.191	0.139	0.111	0.740		
2-Year Price Growth Rate	Ownership only			0.768	0.381	0.570	0.450
	Birth only	0.907	0.341			0.040	0.842
	Both	3.456	0.063	0.387	0.534		
4-Year Price Change ($100,000)	Ownership only			1.022	0.312	0.106	0.745
	Birth only	0.059	0.809			0.066	0.797
	Both	0.700	0.403	0.336	0.562		
4-Year Price Growth Rate	Ownership only			2.292	0.130	0.106	0.747
	Birth only	0.002	0.966			0.001	0.978
	Both	0.218	0.641	1.214	0.271		

Note: The null hypothesis for all tests is that the estimated odds of the alternative from the benchmark model (full alternatives) is the same as the estimated odds of the alternative from the model with one other alternative dropped. $\hat{y}_1$ denotes the odds of "ownership only," $\hat{y}_2$ the odds of "birth only," and $\hat{y}_3$ the odds of "doing both."

Table 1.8: The MNL model estimates of elasticities of house price level and variations.

Dependent Choice	(1)	(2)	(3) Independent Variable	(4)	(5)
No inter-MSA move in the past four years					
	Median House Price ($100,000)	2-Year Price Change ($100,000)	2-Year Price Growth Rate	4-Year Price Change ($100,000)	4-Year Price Growth Rate
Homeownership only	-0.471***	-0.227	-0.630	-0.179*	-0.535*
	(0.1219)	(0.1702)	(0.5112)	(0.1053)	(0.2798)
Birth only	0.269**	-0.206	0.057	0.028	0.259
	(0.1295)	(0.2465)	(0.7711)	(0.2144)	(0.5427)
Both	-0.786**	-0.666*	-2.225	-0.943***	-2.346**
	(0.3074)	(0.3504)	(1.3703)	(0.2710)	(0.9780)
N	13026	13026	13026	13026	13026

Note: Models differ only in the independent variable of interest. All estimates include geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at the MSA level. Significant at *** p <10%, ** p <5%, * p <1%.
Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, stayed in the same MSA in the last four years, with positive weight and all information available from PSID 1985-2015.

Table 1.9: The estimated ratio of RRR of the interaction effect of partnership, race, and parenthood.

Interacted with:	Choice	(1)	(2)	(3) Independent Variable	(4)	(5)
B. Partnership		Median House Price ($100,000)	2-Year Price Change ($100,000)	2-Year Price Growth Rate	4-Year Price Change ($100,000)	4-Year Price Growth Rate
	Ownership	0.908	0.803	0.560	1.004	0.890
		(0.1053)	(0.2632)	(0.5113)	(0.1969)	(0.4510)
	Give a birth	0.970	0.349**	0.054**	0.883	0.535
		(0.1350)	(0.1431)	(0.0787)	(0.2020)	(0.4156)
	Both	14.046***	0.860	0.567	0.938	1.546
		(14.4356)	(0.2809)	(0.7676)	(0.2721)	(1.5134)
	N	13026	13026	13026	13026	13026
C. Black						
	Ownership	1.229	0.706	0.451	0.917	0.826
		(0.1564)	(0.2145)	(0.4253)	(0.2710)	(0.4762)
	Give a birth	1.111	2.522**	13.428	1.282	2.619
		(0.1301)	(1.1189)	(24.6520)	(0.3608)	(2.6480)
	Both	0.539	2.747	59.614	1.518	5.602
		(0.2660)	(1.9277)	(161.0310)	(0.7242)	(8.3308)
	N	12359	12359	12359	12359	12359
D. Parenthood						
	Ownership	1.319**	1.047	1.529	1.163	1.840
		(0.1800)	(0.2954)	(1.2084)	(0.2013)	(0.8211)
	Give a birth	1.039	0.680	0.483	0.587*	0.259*
		(0.1328)	(0.3892)	(0.8031)	(0.1854)	(0.2038)
	Both	1.196	3.401**	10170.613***	2.204	460.829**
		(0.4278)	(2.0199)	(32159.4459)	(1.1060)	(1202.1338)
	N	13026	13026	13026	13026	13026

Note: Models differ only in the independent variable of interest. All estimates include controls for partnership, race, number of children, total family income, and employment status, age group dummies, educational attainment dummies, state-by-year unemployment rate, MSA-by-year real income per capita, national recession index, and geographic (census division) and time (five-year period) fixed effects. Robust standard errors in parentheses account for clustering at the MSA level. For the panel of ratio of RRR, the value in parentheses reports the robust standard errors times ratio of RRR. Significant at *** $p < 10\%$, ** $p < 5\%$, * $p < 1\%$.
Source: Non-homeowning women who are either family head or its partner, age between 20 and 44, stayed in the same MSA in the last four years, with positive weight and all information available from PSID 1985-2015.

Chapter 2

The Joint Dynamics of Family Housing and Childbearing Decisions: A Theory

abstract

In the life course of a nuclear family, homeownership and parenthood are two critical stages, and a young couple's decisions to buy a home and bear a child are connected. While owning a property and having children are typically desired, both of them are costly. Without external financial support nor unlimited credit, a young couple may have to choose whether to achieve the stages and determine the time in their life span to do that. We present a theoretical model to capture the decision-making rule of the interdependent dual-stage transitions. The unitary model follows the neoclassical utilitarian view under a dynamic

and deterministic framework. It demonstrates the determination of the household's relative preferences on homeownership and children and time preferences on its optimal behavior. We then use the model to show the comparative statics of the household choice on the timing of homeownership and parenthood transitions.

2.1 Introduction

Homeownership interlinks with the decision of family formation. The utility of homeownership ranges across social, psychological, and economic domains. An owner-occupying housing unit provides a more stable and secure housing environment, usually accompanied by higher housing and neighborhood satisfaction (Elsinga and Hoekstra, 2005). In some cultures, it serves as a signal of a mature household (Wei et al., 2012; Mulder and Billari, 2010). It also plays a means of wealth storage, a financial hedging tool, and a long-term investment in some circumstances (Ortalo-Magne and Rady, 2006; Sinai and Souleles, 2005; Han, 2008). Socially and psychologically, homeownership gives a household incentive to raise children, as the available living space and its stability and quality influence a household's desire for children. As a social qualification, homeownership signifies the readiness of family formation and justifies such a decision. In the Beckerian altruistic view, providing children a better growing environment returns more pleasure to the parents. This relationship makes homeownership and children complementary to some degree. The demand for one reinforces the demand for the other (Kulu and Vikat, 2007).[1] However, both homeownership and children are costly. A borrowing constraint stands before the transition into homeownership, and a substantial and long-lasting expense is subject to both the home mortgage and childrearing. A young couple with limited resources may have to compromise on the transitions in housing tenure and parenthood. Even if they plan to achieve both in the end, a trade-off between the transition timings may be necessary.

The relationship between family housing and childbearing has attracted attention

[1] As we are interested in the optimal–or planned–path of the life course, we do not consider unintended pregnancies, which inversely cause a family union, in this paper.

across sociology, demography, and economics. A growing literature studies the correlations and interrelations between family childbearing and moving (Clark and Onaka, 1983; Clark and Withers, 2009; Clark, 2013), living space (Kulu and Vikat, 2007; Kulu and Steele, 2013), house prices (Lovenheim and Mumford, 2013; Dettling and Kearney, 2014), and homeownership (Mulder, 2006a,b). The last relationship is particularly intricating for its correlation ambiguity, which is attributable to the tension between the economic side and the social/psychological side. For instance, the comparative analysis between European countries by Mulder and Billari (2010) indicates that the housing market conditions may influence the associations between homeownership rate and the fertility rate.

With the support of rich micro-level longitudinal data, researchers further examined the relationship between the timing of housing status transition and childbearing in the past two decades to explore the relationship between housing and women fertility. In addition to Clark's pioneering work on the relationship between moving and childbearing, Mulder and Wagner (2001) shows the positive connection of homeownership and family formation using West Germany's data. More directly, Enström Öst (2012) presents a growing correlation of family housing and first birth decisions in the younger cohort of Swedish households.

Notwithstanding a growing empirical literature on the interdependent decision-making process in recent years, the theoretical base of the issue has not kept up. Whereas more and more works reveal the connections between the two family events, acknowledging their dynamic nature, the commonly employed empirical methods such as reduced-formed regressions and estimations essentially presume a static optimization problem for households or a mechanistic behavioral response between home buying and childbearing. The relative timing and sequencing of the events are discounted from the decision framework, and dynamic models on the two decisions are only discussed separately.

A modern model on the multiple family life-course decisions should treat the decisions as dynamic. Household preferences and costs on homeownership and children could be interrelated and evolving. Early homeownership can encourage the decision to have more children and the same for the other way around. At the very least, the incurred costs from the home buying and childbearing event are not one-shot but continuous. Thus, the marginal effects of these events are persistent to the household, at least on the economic side. Timing and ordering of the events are, in this sense, one critical decision for household welfare. Moreover, such a model should also take uncertainties and the corresponding adaptive expectation into account. Factors such as family income, costs of housing and childrearing, and even the pregnancy rate evolve over time according to some stochastic process. A structural model incorporating a dynamic and stochastic feature could better help decipher the interdependence of these major family events.

The aim of this paper is a modest one. It constructs a dynamic and deterministic model of the joint decisions of a household on childbearing and home buying, in the presence of a home mortgage downpayment constraint and derives its comparative static properties. The purpose of the exercise is to build a stepping-stone for the further construction of stochastic models that match the sophistication of real family life-course decisions, which can provide a structural foundation for the modern micro-level empirical investigations on the transitions of family homeownership and parenthood simultaneously. The stepping-stone shall also help decipher the intertwining relationship in the multiple domains in general, not confined in life-course decisions but any intertemporal joint decisions.

We formulate a unitary family model to capture the decision-making rule of two interdependent shifts of life course stage using an optimal control framework. The household has to decide whether to buy a property, whether to bear a child, and, more importantly, when to do them, under a Utilitarian design. By limiting the financial feature of the property by assuming a deterministic frame, this study focuses on how a household determines the timing of home buying and childbearing to realize its desired life course. The comparative statics demonstrates the determination of the household's relative preferences on homeownership and children and time preferences on its optimal behavior. Because of the different expense schemes of real estate transactions and childrearing, the time preferences relative to the market interest rate are critical to the household's choices.

2.2 Model

In a deterministic world, consider that a young household (HH hereafter) ponders when to bear a new child and when to become a homeowner in its life course: a continuous and finite interval of time. In the beginning, it has no initial endowment but has a fixed income. Raising a child requires a continued expense. Buying a home requires a mortgage, imposing a credit constraint for the downpayment and subsequent interest-only periodical payments. HH's objective is to maximize the overall utility during the considered time, regardless of other social ties or norms that would potentially affect its preferences on homeownership and children.

2.2.1 Model Settings

The basic framework of the utilitarian model with continuous time consists of the following features:

Housing decision The housing state of HH at time t is a dummy, $H_t \in \{0, 1\}$. We confine HH's action on housing to buying one house for its use. Let p equals the price of a preferred house and α the downpayment ratio of the mortgage. By the time HH buys the house (time t which H_t first changes to 1, denoted as $\tilde{t}$,) a downpayment with the amount $d = \alpha p$ is required. After the mortgage is originated, there is a mortgage payable in each period, and the net added cost m accounts for the mortgage payable minus the previous rental expense. The net added cost is increasing in p but not necessarily a positive number. HH has to pay the cost for the rest of the time in the program. An additional assumption we impose in the model is that HH cannot sell the house once it becomes the homeowner but can borrow against the procured equity.

Child raising decision Similarly to the housing decision, the child raising state at time t is represented by $N_t \in \{0, 1\}$. We consider only the decision of the first child. Once the child enters the family, HH cannot dump the child. We assume a fixed cost of child raising s in each period.[2]

Asset accumulation HH can borrow money and then pay it back at the terminal time,

[2]Decisions on the number of children do not fall within the bounds of the study. Multiple children question is doable but in this stage does not add valuable implications to the analysis on the decision and timing of the first child. The setting of a exogenous childrearing cost circumvents the classical Beckerian quantity-quantity trade-off problem (Becker, 1960) and the altruistic consideration as parents (Becker and Barro, 1986).

as the model does not restrict borrowing for consumption. The only exception is the home mortgage. Homebuyers need to prepare a positive amount of net assets to satisfy the downpayment requirement. HH is not allowed to buy a house without holding enough net assets.

Utility function Let c_t be a continuous variable that indicates HH's consumption of a composite of all goods other than the expenditure on housing and childrearing in time t. Therefore, HH's utility is a function of homeownership, child, and other consumption, $u(c_t, H_t, N_t)$. It increases with all the variables, one continuous and two discrete. To simplify the analysis without losing generality, we assume that the utility brought by homeownership and parenting is additively separable from that by other consumption. We further assume that the utility of consumption is concave in a logarithm form, a special case of a CRRA utility function. It is just a cardinal representation and can be transformed into other forms through any monotonic function without losing generality.

$$u(c_t, H_t, N_t) = \ln c_t + v(H_t, N_t) \tag{2.1}$$

For simplicity in the following discussions, hereafter we let $v(0,0) = 0$ and define $\Delta v_n^c \equiv v(n,1) - v(n,0)$ and $\Delta v_n^h \equiv v(1,n) - v(0,n)$. Δv_n^c represents the utility gain by having a child with n house where $n \in \{0,1\}$, and, likewise, Δv_n^h represents the utility gain by purchasing a house with n child. The complementary feature between housing state and children implies that $\Delta v_1^c > \Delta v_0^c$ and $\Delta v_1^h > \Delta v_0^h$.

Budget constraint HH chooses the share of income for consumption and the necessary expenditure and saves the rest in each period. We use a_t to denote the net wealth of HH at time t, which consists of its financial wealth and equity. The growth in a_t in each period comes from the interest payment on the existing assets plus a fixed period income w (assumed being constant) minus consumption c_t, the net mortgage payment m if HH owns a house, and the cost of child raising s if there is a child.

$$\dot{a}_t = ra_t + w - c_t - mH_t - sN_t \tag{2.2}$$

At the onset of the program, HH has no endowment, namely, $a_0 = 0$. HH must rent a house and pay rent every period before owning a house. The rental nevertheless is omitted in the mathematical representation as we normalize m to the mortgage-rental difference. At the time of entering homeownership, HH must possess $a_t \geq d$ in order to pay for the downpayment. We assume that the downpayment does not decrease HH's wealth; it simply transforms from the financial asset to home equity. Note that, income does not necessarily need to be greater than the sum of child raising cost and mortgage payable. HH can save enough money before buying a house ($a_t > d$) to pay for the mortgage in the rest of its life course.

Time preference HH has a constant and non-negative rate of time preference $\rho \geq 0$. It values early enjoyment more than later. Like other individual models, the difference between ρ and r has is one critical determinant for the decision maker in an intertemporal model.

Program length A finite, continuous time problem is assumed. HH only considers its utility in a continuous time support $t \in [0, T]$. It does not care about anything after the termination of the program. HH is not altruistic to its descendants as in the Becker-Barrow family decision models.

2.2.2 The Objective Function

Given this information, we write down the objective function of the decision-maker. HH has to decide its consumption path and whether to have a child and buy a home and, if so, the timing of childbearing and buying a home to maximize its utility.

$$\begin{aligned}
&\max_{\{c_t, H_t, N_t\}} \int_0^T u(c_t, H_t, N_t) e^{-\rho t} dt \\
&\quad\quad \text{s.t. } a_t = a_0 + \int_0^{\tilde{t}} (ra_t + w - c_t - mH_t - sN_t)\, dt \\
&\quad\quad\quad H_0 = N_0 = 0,\ H_T = N_T = 1, \\
&\quad\quad\quad a_{\tilde{t}} \geq d,\ a_T \geq 0, \\
&\quad\quad\quad a_0 = 0 \text{ is given.}
\end{aligned} \tag{2.3}$$

Equation (2.3) embodies HH's maximization problem. HH decides how much income to spend and saves the rest. It also decides the timings of having a child and purchasing a house to maximize its utility given all the information available. It is free to choose the order of the actions and even whether or not to do them. The only confinement comes from the budget (and the "no dumping rule" described earlier.) It is impossible for HH to buy a house at the onset of the program because $a_0 = 0$. The following sections show how to determine HH's optimal strategy by solving the maximization problem of the program.

2.3 Household Optimization

Depending on the decision of home buying and childbearing and their order, the feasible choice set can be classified into six strategy groups, marked as six sequences. Three strategies will end up with a child and an owner-occupying house. First, having a child first and then buying a house ($0 \leq \hat{t} < \tilde{t} \leq T$). Second, buying a house first and then having a child ($0 < \tilde{t} < \hat{t} \leq T$) Third, buying a house and having a child at the same time ($0 \leq \hat{t} = \tilde{t} \leq T$). Other than these, HH may choose to end up with a child but never owning a house, owning a house but childless, or neither become a homeowner nor parents. The existence of the later three sequences fully depends on the extreme exogenous setting of utility or cost setting, and their analysis is trivial. As this paper intends to highlight the decision of event timings, we focus on the former three possibilities, especially the first sequence, with only a brief discussion about the others at the end. Moreover, since the third sequence is a special case of the first two, we discuss them together in the following subsection.

2.3.1 Sequence 1: Having a Child First ($0 \leq \hat{t} \leq \tilde{t} \leq T$)

Sequence 1 includes the feasible life course strategies that HH has a child first and later becomes a homeowner. With the presence of the homeownership and parenthood transition, the objective function is essentially a dynamic optimization problem with switches in state equations (Kamien and Schwartz, 2012). We divide the program into three stages:

before having a child ($0 \le t < \hat{t}$, Stage 1), having a child while not a homeowner ($\hat{t} \le t < \tilde{t}$, Stage 2), and having a child and an owner-occupying unit ($\tilde{t} \le t \le T$, Stage 3).[3] Given the sequence, HH looks for the optimal strategy to maximize its utility subject to the budget constraints and the downpayment constraint. It needs to accumulate wealth to pay the downpayment of a house at $\tilde{t}$. After that, HH can borrow money against the home equity. We can rewrite the objective function as a summation of three separate value functions. The choice variables are the sequence of consumption and the timing of childbearing and home buying.

$$\max_{\{c_t\},\hat{t},\tilde{t}} \int_0^{\hat{t}} u(c_t,0,0)e^{-\rho t}dt + \int_{\hat{t}}^{\tilde{t}} u(c_t,0,1)e^{-\rho t}dt + \int_{\tilde{t}}^{T} u(c_t,1,1)e^{-\rho t}dt$$

$$\text{s.t. } \dot{a}_t = \begin{cases} ra_t + w - c & \text{for Stage 1} \\ ra_t + w - c - s & \text{for Stage 2} \\ ra_t + w - c - s - m & \text{for Stage 3} \end{cases} \tag{2.4}$$

$$a_0 = 0 \text{ is given}, a(\tilde{t}) \ge d, a(T) \ge 0.$$

Let $V_1 \equiv \int_0^{\hat{t}} u(c_t,0,0)e^{-\rho t}dt$, $V_2 \equiv \int_{\hat{t}}^{\tilde{t}} u(c_t,0,1)e^{-\rho t}dt$, and $V_3 \equiv \int_{\tilde{t}}^{T} u(c_t,1,1)e^{-\rho t}dt$. Respectively, we construct three current value Hamiltonians with current value multipliers λ_1, λ_2, and λ_3 for each stage.

[3] In this paper, we refer "state" to the homeownership and parenthood status of the decision-maker and "stage" to the steps in its life course. Each stage corresponds to a unique budget condition in the objective function. A state transition causes a stage switch, but a stage switch only changes one of the two states, except in Sequence 3.

$$\mathcal{H}_1(t) = u(c_t, 0, 0) + \lambda_1(ra_t + w - c_t) \quad (2.5)$$

$$\mathcal{H}_2(t) = u(c_t, 0, 1) + \lambda_2(ra_t + w - c_t - s) \quad (2.6)$$

$$\mathcal{H}_3(t) = u(c_t, 1, 1) + \lambda_3(ra_t + w - c_t - s - m). \quad (2.7)$$

Since there is no change in the homeownership and child status in each stage, the first-order necessary conditions only relate to consumption and the multipliers.

$$\frac{\partial \mathcal{H}_i}{\partial c} = 0 \;\Rightarrow\; u_c = \frac{1}{c} = \lambda_i \quad (2.8)$$

$$\frac{\partial \mathcal{H}_i}{\partial a} = \rho\lambda_i - \dot{\lambda}_i \;\Rightarrow\; \frac{\dot{\lambda}_i}{\lambda_i} = \rho - r \quad \text{for } i = 1, 2, 3. \quad (2.9)$$

Moreover, because the timing of the state transitions are endogenous, the two stage switches in the programs give two more necessary conditions of the optimization:

$$\mathcal{H}_1(\hat{t}^-) = \mathcal{H}_2(\hat{t}^+) \;\text{ if } 0 < \hat{t} < \tilde{t} \quad (2.10)$$

$$\mathcal{H}_2(\tilde{t}^-) = \mathcal{H}_3(\tilde{t}^+) \;\text{ if } \hat{t} < \tilde{t} < T. \quad (2.11)$$

Equations (2.10) and (2.11) indicate that the total current value has to be continuous at stage switches. They represent the equality of marginal benefit and marginal cost at the optimal timing, namely, $\frac{\partial V_1}{\partial \hat{t}} = \frac{\partial V_2}{\partial \hat{t}}$ and $\frac{\partial V_2}{\partial \tilde{t}} = \frac{\partial V_3}{\partial \tilde{t}}$. In particular, since the downpayment constraint must be satisfied before the second stage switch, a discontinuity of consumption sequence at the stage switch is possible. Therefore, $\lambda_2^-(\hat{t}^-) = \lambda_3^+(\hat{t}^+)$ may not hold.

The Euler equation regulates the consumption sequence throughout the program. Because of the model setting, it is the same for all stages and can easily be derived by combining (2.8) and (2.9):

$$\frac{\dot{c}_i}{c_i} = -\frac{\dot{\lambda}_i}{\lambda_i} = r - \rho. \tag{2.12}$$

The Euler equation shows that the growth path of consumption depends only on the difference between the interest rate and the time preference rate, which are both constant and exogenous. If HH is patient enough, $r - \rho > 0$ and the consumption grows as time goes on because the interest of savings is valuable. In contrast, if HH is impatient enough, $r - \rho < 0$ and HH consumes more in the early periods and gradually reduces consumption for savings. Instead, if there is no effective discounting or premium of savings to the HH, or $r = \rho$, consumption would be constant over time and could only change at stage switches. Since the growth rate is exogenous, the consumption path is determined by the initial consumption level and the timing of stage switches.

The Benchmark Case: $r = \rho$

The simplest scenario is a program without effective discounting ($r = \rho$). HH evenly smooths consumption in each stage and minimizes the consumption gap among stages as far as the utility function is concave. The marginal present value of enjoyment at any time point in the program is identical because the interest payment of assets perfectly compensates the distate for waiting.

Canonically, the problem is solved backward stage by stage. In the first step, we solve the optimization problem of the last stage, treating the initial conditions of the stage, which were determined in the last stage, as given. Then we move to the earlier stage and do the same thing all over again until we include all the stages. Following this procedure, we start by optimizing Stage 3. Let $\bar{d}$ refer to the realized amount of savings at the end of Stage 2. At the beginning of Stage 3, $\tilde{t}$ and $\bar{d}$ are given. The optimization problem becomes

$$\begin{aligned} &\max_{\{c_t\}} V_3(\tilde{t}, \bar{d}) = \int_{\tilde{t}}^{T} u(c_t, 1, 1)e^{-\rho t}dt \\ &\text{s.t. } \dot{a}_t = ra_t + w - c - s - m \\ &a(\tilde{t}) = \bar{d} \geq d, a(T) \geq 0. \end{aligned} \tag{2.13}$$

The Hamiltonian, first order conditions, and Euler equation of the problem are the same as before. The consumption, denoted c_3, is a fixed value because the growth rate is zero. Also, because asset has no value to HH after time T, it would not keep any asset at T, resulting in $a(T) = 0$. Integrating the transition equation ($\dot{a}_t e^{-rt} - ra_t e^{-rt} = (w - c - s - m)e^{-rt}$) gives the consumption function of $\tilde{t}$ and $\bar{d}$.

$$c_3(\tilde{t}, \bar{d}) = w - m - s + \bar{d}\frac{re^{-r\tilde{t}}}{e^{-r\tilde{t}} - e^{-rT}}. \tag{2.14}$$

And the maximized value function of Stage 3 can be written as

$$V_3^*(\tilde{t}, \bar{d}) = \int_{\tilde{t}}^{T} u(c_3, 1, 1)e^{-\rho t}dt = u(c_3, 1, 1)\frac{e^{-\rho\tilde{t}} - e^{-\rho T}}{\rho} \equiv \phi_3(\tilde{t}, \bar{d}). \tag{2.15}$$

In the second step, we add the value function of Stage 2 into the objective function and replace V_3 by (2.15). The downpayment constraint requires that the amount of assets at the homeownership transition is more or equal to d. We use the notation K to represent this constraint.

$$\begin{aligned}
&\max_{\{c_t\},\tilde{t},\bar{d}} V_2(\hat{t},\tilde{t}) + V_3^*(\tilde{t},\bar{d}) = \int_{\hat{t}}^{\tilde{t}} u(c_t,0,1)e^{-\rho t}dt + \phi_3(\tilde{t},\bar{d}) \\
&\quad \text{s.t. } \dot{a}_t = ra_t + w - c - s \\
&\qquad a(\tilde{t}) = \bar{d} \geq d \text{ or } K(a(\tilde{t})) = a(\tilde{t}) - d \geq 0 \\
&\qquad a(\hat{t}) \text{ given; } \tilde{t} \text{ is free.}
\end{aligned} \tag{2.16}$$

Because the endpoint $\tilde{t}$ is free and the function K regulates the state variable, two additional transversality conditions apply here:[4]

$$K \geq 0 \Rightarrow \;\lambda_2(\tilde{t}) = \frac{\partial \phi_3}{\partial a} + p\frac{\partial K}{\partial a},\; pK = 0,\; p \geq 0 \tag{2.17}$$

$$\tilde{t} \text{ is free} \Rightarrow \; e^{-\rho t}u(c_t,0,1) + e^{-\rho t}\lambda_2(ra + w - c_t - s) + \frac{\partial \phi_3}{\partial \tilde{t}} + p\frac{\partial K}{\partial \tilde{t}} = 0 \text{ at } \tilde{t} \tag{2.18}$$

Equation (2.17) summarizes the necessary conditions of the optimal solution by which the constraint K is satisfied. The marginal value of assets at the terminal of Stage 2 ($\lambda(\tilde{t})$) equals the assets' contribution to the salvage term plus the assets' marginal cost of obeying the downpayment constraint. This cost is a virtual loss to HH caused by this credit restriction because it confines HH's feasible choice sets. HH would prefer a different plan had the constraint not exist. In the optimization problem, if the house price is zero and the homeownership transition is free, the downpayment constraint is slack. In this special case, the optimal solution is not affected by the downpayment requirement. Most likely, the

[4]For the derivation and proof, see Kamien and Schwartz (2012), Part II, Section 7.

constraint binds (with an exception explained below), and $K = 0$ and $\lambda(\tilde{t}) \leq \frac{\partial \phi_3}{\partial a}$ $(p > 0)$.

The equation also implies the critical value of consumption at $\tilde{t}$. By combining (2.14) and (2.15), we have $\frac{\partial \phi_3}{\partial a} = (e^{-\rho\tilde{t}} - e^{-\rho T})/\rho \frac{u(c_3,1,1)}{\partial a} = \frac{e^{-r\tilde{t}}}{c_3}$. Also, $\frac{\partial K}{\partial a} = 1$ by (2.16) and $\lambda(\tilde{t}) = \frac{1}{c_2}$ by (2.8). Thus, (2.17) indicates that the optimal solution must satisfy either $c_3 \geq e^{-r\tilde{t}}c_2$ and $a(\tilde{t}) = d$ or, in a rare case, $c_3 = e^{-r\tilde{t}}c_2$ and $a(\tilde{t}) > d$. The latter case implies that the downpayment constraint is possibly slack if the optimal c_3 is smaller than c_2. The precondition of this case, however, is singular. HH chooses a smaller c_3 only if both m and the marginal utility of housing are large enough while the required downpayment is sufficiently small. Intuitively, HH's only wish in its life span is to own a luxury mansion, and the bank somehow offers a mortgage with exorbitant interest-only mortgage payment but a incredibly low downpayment. The new equity the allows HH to borrow against its value only helps to pay the mortgage, and the consumption level after the homeownership transition reduces, yet HH does not care the lower life standard other than housing in the rest of the program. Though possible, it is definitely abnormal.[5] For this reason, it is safe to assume that the constraint always binds, i.e. $a(\tilde{t}) = \bar{d} = d$. This implication holds for all non-negative r and ρ. The consumption level in Stage 3 hence is a function of $\tilde{t}$ only, and we replace all $\bar{d}$ by d thereafter.

Using the same technical tool in deriving c_3, we can get the consumption in Stage

[5]It is unrealistic also because of the unique setting of the model, which sees the house purchase is purely an equity transformation without any loss in liquidity or potential value loss. Had we consider the program in an economy with limited equity loan ration or a precarious housing market, in which equity loan is costly, HH would find that the purchased equity could largely devalue, and thus spend less before the housing event.

2 as a function of $\tilde{t}$, $\hat{t}$, and $a(\hat{t})$.

$$c_2(\tilde{t}, \hat{t}, a(\hat{t})) = w - s - \left(de^{-r\tilde{t}} - a(\hat{t})e^{-r\hat{t}}\right) \frac{r}{e^{-r\hat{t}} - e^{-r\tilde{t}}} \tag{2.19}$$

Since we let $a(\tilde{t}) = \bar{d} = d$, by (2.18) we get a non-linear function with single unknown $\tilde{t}$ given $\hat{t}$ and $a(\hat{t})$.

$$e^{-\rho\tilde{t}}u(c_2, 0, 1) + e^{-\rho\tilde{t}}u_{2c}(rd + w - c_2 - s) + u_{3c}\frac{e^{-r(\tilde{t}+T)}}{e^{-r\tilde{t}} - e^{-rT}}rd - u(c_3, 1, 1)e^{-\rho\tilde{t}} = 0 \tag{2.20}$$

Equation (2.20) is the necessary condition of the optimization of Stage 3 given $\hat{t}$. When $\tilde{t}$ approaches to T, c_2 and c_3 both increase according to Equations (2.14) and (2.19). At the same time, the marginal utility of consumption decreases by the assumption of concave utility function. The necessary condition shows that, as far as the decrease in marginal utility is moderate enough, a meaningful solution of the optimal timing of the second stage switch exists in the form of a function of $\hat{t}$ and other exogenous parameters.

$$V_2^* + V_3^* = u(c_2, 0, 1)\frac{e^{-\rho\hat{t}} - e^{-\rho\tilde{t}}}{\rho} + u(c_3, 1, 1)\frac{e^{-\rho\tilde{t}} - e^{-\rho T}}{\rho} \equiv \phi_2(w, s, d, m, r, \rho; \hat{t}, a(\hat{t})) \tag{2.21}$$

The last step is to add Stage 1 into the objective function, just like what we did in the second step, to complete the objective function.[6] Like in Stage 2, the free endpoint

[6]

$$\max_{\{c_t\}, \hat{t}, a(\hat{t})} \int_0^{\hat{t}} u(c_t, 0, 0)e^{-\rho t}dt + \phi_2(\hat{t}) \quad \text{s.t. } \dot{a}_t = ra_t + w - c;\ a(0) = 0, \hat{t} \text{ is free}$$

gives an additional necessary condition to the problem, and the consumption level at Stage 1 can be derived by integrating the transition equation.

$$\hat{t} \text{ is free} \Rightarrow \; e^{-\rho t}u(c_t, 0, 0) + e^{-\rho t}\lambda_1(ra + w - c_t) + \frac{\partial \phi_2}{\partial \hat{t}} = 0 \text{ at } \hat{t} \tag{2.22}$$

$$c_1(\hat{t}, a(\hat{t})) = w - ra(\hat{t})\frac{e^{-r\hat{t}}}{1 - e^{-r\hat{t}}} \tag{2.23}$$

Theoretically, one can solve for the optimal timing of the first stage switch and the optimal savings at the switch by combining Equations (2.22) and (2.23). What remains unanswered is whether there is an analytical solution and the economic implication behind it.

An Alternative Method

The stage value functions for the benchmark model is simple because consumption is constant in each stage, Π. Since the object of the program is to maximize HH's overall value of the program, it is possible to solve the problem in an alternative way. We rewrite the objection function as a maximization problem for the sum of the value function of all stages:

$$\max_{\hat{t}, a(\hat{t}), \tilde{t}} \Pi = \sum_{i=1}^{3} V_i = u(c_1, 0, 0)\frac{1 - e^{-\rho\hat{t}}}{\rho} + u(c_2, 0, 1)\frac{e^{-\rho\hat{t}} - e^{-\rho\tilde{t}}}{\rho} + u(c_3, 1, 1)\frac{e^{-\rho\tilde{t}} - e^{-\rho T}}{\rho} \tag{2.24}$$

subject to (2.14), (2.19), (2.23), the consumption functions.[7] Written in this form, with the additional necessary conditions embedded in the consumption function, we do not need to be listed them out explicitly. The consumption path is determined by the timing of state switches and the savings at the switches. Only three choice variables left; HH simply solves the maximization problem at once by choosing the optimal $\hat{t}$, $\tilde{t}$, and $a(\hat{t})$. Therefore, the solution must satisfy the first-order conditions of the program with respect to $\hat{t}$, $\tilde{t}$, and $a(\hat{t})$, with proofs of the existence of local maximum. We can immediately find that $\frac{\partial \Pi^*}{\partial \tilde{t}} = 0$ is exactly Equation (2.18). This connection indicates the essence of the necessary condition: the choice is optimal when the marginal benefit of prolonging Stage 2 equals its marginal cost. A marginal postponement of $\tilde{t}$ 2 leads to a utility loss due to the extended wait to enter homeownership and a utility gain from the greater c_2 and c_3. The consumption levels in both stages increase because there is more time to save for the downpayment in Stage 2, and HH *has* to spend the same amount of asset in a shorter length of Stage 3. Rearranging the components, we can get that the marginal gain of time in Stage 2 and its consumption growth ($\frac{\partial V_2}{\partial \tilde{t}}$) equals the marginal loss of time in Stage 3 minus the marginal gain from the increase in Stage 3 consumption ($\frac{\partial V_3}{\partial \tilde{t}}$). The optimal $\tilde{t}$ occurs at $\frac{\partial V_2}{\partial \tilde{t}} = \frac{\partial V_3}{\partial \tilde{t}}$. Likewise, we can see the condition $\frac{\partial \Pi^*}{\partial \hat{t}} = 0$ is equivalent to Equation (2.22), or $\frac{\partial V_1}{\partial \hat{t}} = \frac{\partial V_2}{\partial \hat{t}}$. It indicates that the gain of time in Stage 1 and its consumption growth by postponing childbearing ($\frac{\partial V_1}{\partial \hat{t}}$) equals the marginal loss of time in Stage 2 minus the marginal gain from the increase in Stage 2 consumption ($\frac{\partial V_2}{\partial \hat{t}}$).

Before diving into details of the equations, it is better to check the optimal con-

[7]The alternative method is available only if we assume the downpayment condition is binding. In a more general framework, we have to first judge whether $a(\tilde{t}) = d$ should always hold, as discussed earlier regarding Equation (2.17).

dition of the amount of asset at the end of Stage 1, $a(\hat{t})$. The first-order condition with respect to $a(\hat{t})$ gives

$$\frac{\partial \Pi}{\partial a(\hat{t})} = u_{1c}\frac{\partial c_1}{\partial a(\hat{t})}\frac{1-e^{-\rho\hat{t}}}{\rho} + u_{2c}\frac{\partial c_2}{\partial a(\hat{t})}\frac{e^{-\rho\hat{t}}-e^{-\rho\tilde{t}}}{\rho} = -u_{1c}e^{-\rho\tilde{t}} + u_{2c}e^{-\rho\tilde{t}}.$$

As far as $\frac{\partial \Pi^*}{\partial a(\hat{t})} = 0$, it is obvious that $u_{1c} = u_{2c}$, and equivalently $c_1 = c_2$ since we assume a separable utility function. Also, it is easy to show that this point is the local maximum by checking its second-order differentiation. Two conclusions immediately come after the result. First, there should be no discrete jump at the first stage switch at the optimal path. HH consumes the same amount of its income in Stage 1 and 2, and, mathematically, the asset value throughout the two stages are the same ($\lambda_1 = \lambda_2$.) Second, by equating c_1 and c_2, the optimal $a(\hat{t})$ is determined given $\hat{t}$ and $\tilde{t}$, that is

$$a(\hat{t})^* = \left[\left(e^{-r\hat{t}} - e^{-r\tilde{t}}\right)s + rde^{-r\tilde{t}}\right]\frac{1-e^{-r\hat{t}}}{re^{-r\hat{t}}\left(1-e^{-r\tilde{t}}\right)} \tag{2.25}$$

where $a(\hat{t})$ increases with $\hat{t}$. At one end, if HH decides to have child at the beginning of the program, we have $a(\hat{t})^* = a(0) = 0$ because the program start from Stage 2. At the other, if $\hat{t} = \tilde{t}$, or HH decides to have child at the same time it purchases house, we have $a(\hat{t})^* = a(\tilde{t}) = d$.

Let us return to the first-order condition with respect to $\hat{t}$, $\frac{\partial \Pi^*}{\partial \hat{t}} = 0$. The continuity of consumption ($c_1 = c_2$) and Equation (2.25) largely simplify Equation (2.22):

$$\begin{aligned}\frac{\partial \Pi^*}{\partial \hat{t}} &= e^{-\rho t} u(c_t, 0, 0) + e^{-\rho t} \lambda_1 (ra + w - c_t) + \frac{\partial \phi_2}{\partial \hat{t}} \\ &= e^{-\rho t} u(c_1, 0, 0) + u_{1c} \frac{\partial c_1}{\partial \hat{t}} \frac{1 - e^{-\rho \hat{t}}}{\rho} + u_{2c} \frac{\partial c_2}{\partial \hat{t}} \frac{e^{-\rho \hat{t}} - e^{-\rho \tilde{t}}}{\rho} - e^{-\rho t} u(c_2, 0, 1) \\ &= \left(u_{1c} s - \Delta v_0^c\right) e^{-\rho \hat{t}}.\end{aligned}$$

At the optimum, $\frac{\partial \Pi^*}{\partial \hat{t}} = 0$. Thus, at $\hat{t}^*$, we have

$$u_{1c} = u_{2c} = \lambda_1 = \lambda_2 = \frac{\Delta v_0^c}{s}. \tag{2.26}$$

Equation (2.26) determines the threshold of the first stage switch. HH chooses to have a child once the consumption reaches $s/(\Delta v_0^c)$, from which the marginal utility of having a child exceeds the marginal utility of consumption. The same result can be derived from Equation (2.10) if we have $\lambda_1(\hat{t}^-) = \lambda_2(\hat{t}^+)$, which is a reasonable presumption because no other constraint regulates this stage switch. In this case, the Hamiltonians in Equation (2.10) is unfolded us below combining Equations (2.5), (2.6), and (2.8), and the same conclusion can be achieved.

$$\ln c + v(0,0) + \lambda_1(ra + w - c) = \ln c + v(0,1) + \lambda_2(ra + w - c - s), \quad \lambda_1 = \lambda_2 = \frac{1}{c}.$$

Since consumption is constant in each period in the benchmark example, this condition concludes that the optimal solution is always a corner one given $r = \rho$: $\hat{t}^*$ is either 0 or $\tilde{t}$. If HH adds a child at the onset of the program, the objective function in

Equation (2.24) is simplified to $u(c_2,0,1)\frac{1-e^{-\rho\tilde{t}}}{\rho}+u(c_3,1,1)\frac{e^{-\rho\tilde{t}}-e^{-\rho T}}{\rho}$. If, instead, $\hat{t}^*=\tilde{t}$, we have Sequence 3, and Equation (2.24) is reduced to $u(c_1,0,0)\frac{1-e^{-\rho\tilde{t}}}{\rho}+u(c_3,1,1)\frac{e^{-\rho\tilde{t}}-e^{-\rho T}}{\rho}$. In either case, as $\frac{\partial\Pi}{\partial\tilde{t}}>0$ when $\tilde{t}=0$ and $\frac{\partial\Pi}{\partial\tilde{t}}<0$ when $\tilde{t}=T$ by Equation (2.18), the optimal condition of $\tilde{t}$ contains at least one interior solution by the intermediate value theorem.

In short, HH in the benchmark example chooses between having a child at the beginning of the program or at the time of house purchase by comparing the optimal overall value in each possible sequence. The decision depends on the parameters which determine the time needed to accumulate enough assets to pay for the downpayment. Though this result looks trivial, it sheds light on the analysis for more general cases.

Phase Diagram

In light of the work by Arnott et al. (1983) and Powell (1993), this part explores the phase diagram of the optimization program. A phase diagram analysis helps us look for HH's optimal behavior visually and geometrically and gives more intuition on household behavior. It shows the dynamics of the state variable, a, and its shadow value, λ_i by identifying the boundaries of directional momentum. Based on that, we can depict the trajectory of the optimal strategy.

The loci of $\dot{a}=0$ and $\dot{\lambda}=0$ correspond to the equations derived from the budget constraints and the Euler equation (Equation (2.12)):

$$\dot{a} = 0 \;\Rightarrow \begin{cases} ra + w - \frac{1}{\lambda_1} = 0 & \Leftrightarrow \quad \lambda_1 = \frac{1}{ra+w} \quad \text{in Stage 1} \\ ra + w - \frac{1}{\lambda_2} - s = 0 & \Leftrightarrow \quad \lambda_2 = \frac{1}{ra+w-s} \quad \text{in Stage 2} \\ ra + w - \frac{1}{\lambda_3} - s - m = 0 & \Leftrightarrow \quad \lambda_3 = \frac{1}{ra+w-s-m} \quad \text{in Stage 3} \end{cases} \tag{2.27}$$

$$\dot{\lambda}_i = 0 \;\Rightarrow\; \lambda_i = 0 \quad \text{for } i = 1, 2, 3.$$

A $\dot{a} = 0$ locus is a rectangular hyperbolic curve that changes over stages, together with the changes in the transition equation. It shifts up as HH bears a higher expense in each period by starting raising a child or paying the mortgage. In Stage 1, the locus intersects with $a = 0$ line at $\lambda = 1/w$, in Stage 2 at $\lambda = 1/(w - s)$, and in Stage 3 at $\lambda = 1/(w - s - m)$. Any point right to it has a momentum to move rightward, so $\dot{a} > 0$; in contrast, any point left to the locus has a momentum to move leftward and $\dot{a} < 0$.

At the other dimension, two conditions lead to $\dot{\lambda}_i = (\rho - r)\lambda = 0$: either $\lambda_i = 0$ or $\rho - r = 0$ according to the Euler equation. In the benchmark case, the equality between the interest rate and the time preference rate implies that any combination of λ and a satisfies $\dot{\lambda}_i = 0$, which echoes that the optimal consumption remains fixed within stages. In other cases, the locus goes along the horizontal axis ($\lambda_i = 0$). For instance, if the interest rate is greater than the time preference rate, every point above the locus has a momentum $\dot{\lambda}_i < 0$, as will be shown later in Figure 2.2.

A life-course strategy is represented as a trajectory of household assets and the synchronous shadow value in the phase diagram. It starts from $a = 0$ and ends up back to it. To achieve the topmost happiness in life, HH starts raising a child whenever the shadow value of childrearing cost falls below the marginal utility of the child. Moreover, once the savings reach the required downpayment, HH will immediately buy a house.

Figure 2.1 shows the dynamics of the benchmark case. The optimal path consists of two parts: a rightward bound trajectory above the $\dot{a} = 0$ locus and a leftward bound one below. In the first part, HH consumes at a fixed level till it accumulates enough assets to invest in the house purchase. The optimal timing of having a child is either at the onset of the program or by the time entering homeownership. In other words, it is always a corner solution, depending on whether the path starts above or below the $\lambda = \frac{\Delta v_0^c}{s}$ line, by Equation (2.26). So the first part of the path belongs to either Stage 1 or 2. Since $\lambda = \frac{1}{c}$, we can see that in either stage the value of assets is constant and always above its respective the $\dot{a} = 0$ locus by Equations (2.23) or (2.19).

$$\lambda_1 = u_c = \frac{1}{c_1} = \frac{1}{w - a(\hat{t})e^{-r\hat{t}}\frac{1}{1-e^{-r\hat{t}}}} > \frac{1}{w}$$

This result remains when the program starts from Stage 2.

Once entered homeownership, HH does not need to hold such a high amount of assets, and it will start to spend them by borrowing against the equity of the house. The consumption level jumps up and stays high until the end. The second part of the path in the diagram reflects this behavior. Equation (2.14) implies that

$$\lambda_3 = u_c = \frac{1}{c_3} = \frac{1}{w - m - s + \bar{d}\frac{re^{-r\hat{t}}}{e^{-r\hat{t}} - e^{-rT}}} < \frac{1}{rd + w - s - m}.$$

The shadow value of assets equals the marginal utility of consumption when the expenditure

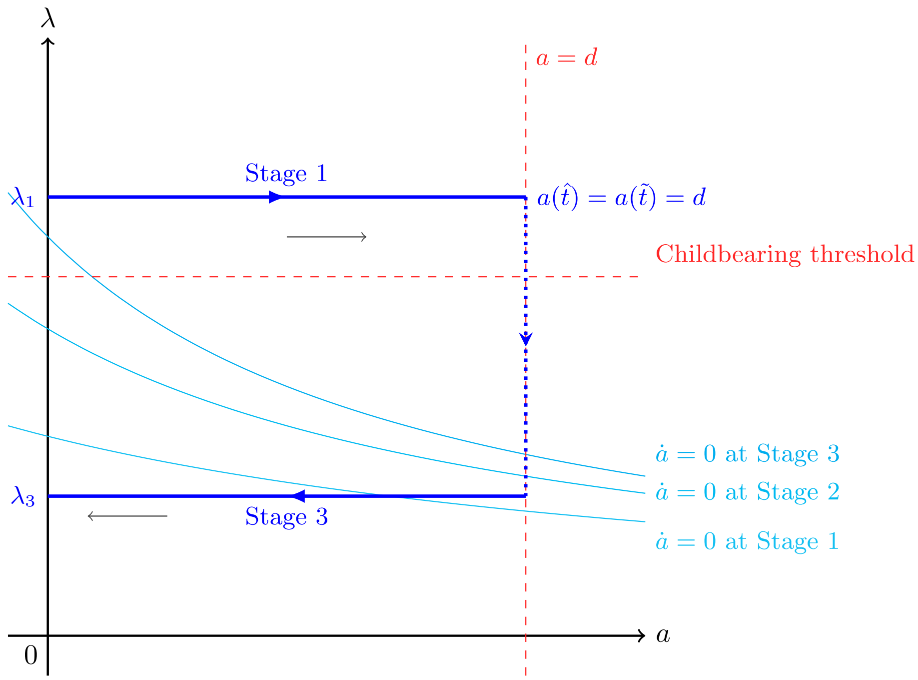

Figure 2.1: A phase diagram of the benchmark example ($r = \rho$) with corner solution $\hat{t} = \tilde{t}$.

exceeds income, so the optimal consumption must lead to asset reduction. This condition satisfies the regulation of zero net assets at the end of the program; the HH has no altruism to its child and cannot play a Ponzi game.

It is worth noting that the sequence is chosen only if the marginal utility at c_3 is less than $\frac{\Delta v_1^h}{m}$, the marginal utility of buying a house after having a child. Otherwise, HH would decide not to become a homeowner in the life course and simply enjoying spending money on other hobbies. Nevertheless, this condition is actually very strict. HH would choose not to buy a house as far as the maximum of the overall value function is less than the maximum of the overall value function without Stage 3. At any rate, the condition is easy to check once the optimal strategy is visualized on a phase diagram.

The General Case: $r \neq \rho$

In many circumstances, the interest rate does not equal the time preference rate, especially when there is individual heterogeneity. A more general case should lie on $r \neq \rho$. Different from the benchmark, in this case, consumption evolves within stage as time goes on. Notwithstanding the divergence, the analysis follows the same procedure, and readers would find that it is nothing but a slight extension of the benchmark case.

The canonical stage-by-stage backward analysis is skipped here. After all, it provides no more intuition than what has been discussed in the previous sections. We start from the alternative method. First, we need to check its availability, the legitimacy of assuming a binding downpayment constraint. If $r \neq \rho$, consumption grows changes positively or negatively with a constant rate $r - \rho$, as we see from the Euler equation.

$$\begin{aligned} c_3(t) &= c(\tilde{t})e^{(r-\rho)(t-\tilde{t})} \\ &= \left[(w-m-s)\frac{\rho}{r}\frac{e^{-r\tilde{t}} - e^{-rT}}{e^{-\rho\tilde{t}} - e^{-\rho T}} + \frac{\rho a(\tilde{t})e^{-r\tilde{t}}}{e^{-\rho\tilde{t}} - e^{-\rho T}}\right] e^{(r-\rho)\tilde{t}} e^{(r-\rho)(t-\tilde{t})} \quad t \in \left(\tilde{t}, T\right] \end{aligned} \tag{2.28}$$

$$\begin{aligned} c_2(t) &= c(\hat{t})e^{(r-\rho)(t-\hat{t})} \\ &= \left[(w-s)\frac{\rho}{r}\frac{e^{-r\hat{t}} - e^{-r\tilde{t}}}{e^{-\rho\hat{t}} - e^{-\rho\tilde{t}}} - \frac{\rho\left(a(\tilde{t})e^{-r\tilde{t}} - a(\hat{t})e^{-r\hat{t}}\right)}{e^{-\rho\hat{t}} - e^{-\rho\tilde{t}}}\right] e^{(r-\rho)\hat{t}} e^{(r-\rho)(t-\hat{t})} \quad t \in \left(\hat{t}, \tilde{t}\right] \end{aligned} \tag{2.29}$$

$$\begin{aligned} c_1(t) &= c(0)e^{(r-\rho)t} \\ &= \left[w\frac{\rho}{r}\frac{1 - e^{-r\hat{t}}}{1 - e^{-\rho\hat{t}}} - \frac{\rho e^{-r\hat{t}}a(\hat{t})}{1 - e^{-\rho\hat{t}}}\right] e^{(r-\rho)t} \quad t \in \left[0, \hat{t}\right] \end{aligned} \tag{2.30}$$

To check the availability we go back to the Equation (2.17) again. In this case, we have $\phi_3 = \int_{\tilde{t}}^{T} u(c_3, 1, 1)e^{-\rho t}dt$ as consumption is growing over time, and $\frac{\partial \phi_3}{\partial a} = \frac{e^{-\rho \tilde{t}}}{c(\tilde{t})} = \frac{e^{-\rho \tilde{t}}}{c_3(\tilde{t})}$. Thus, the optimal solution needs to satisfy $c_3(\tilde{t}^+) \geq e^{-\rho \tilde{t}} c_2(\tilde{t}^-)$ and $a(\tilde{t}) = d$, or, again rarely, $c_3(\tilde{t}^+) = e^{-\rho \tilde{t}} c_2(\tilde{t}^-)$ and $a(\tilde{t}) > d$. The precondition has the same feature as the benchmark example. A slack downpayment constraint occurs only if $c_3(\tilde{t}^+)$ is smaller than $c_2(\tilde{t}^-)$, which is an abnormal occasion. This allows us to safely assume the constraint is always binding as well as the benchmark case.

The maximization problem can be addressed with a small twist on Equations (2.14), (2.19), (2.23), and (2.24):

$$\max_{\hat{t}, a(\hat{t}), \tilde{t}} \Pi = \int_0^{\hat{t}} u(c_1(t), 0, 0)e^{-\rho t}dt + \int_{\hat{t}}^{\tilde{t}} u(c_2(t), 0, 1)e^{-\rho t}dt + \int_{\tilde{t}}^{T} u(c_3(t), 1, 1)e^{-\rho t}dt \quad (2.31)$$

subject to the consumption functions.

Like before, the two transversality conditions of free endpoints can be acquired by optimizing the equation with respect to the both timings, namely by solving $\frac{\partial \Pi^*}{\partial \hat{t}} = 0$ and $\frac{\partial \Pi^*}{\partial \tilde{t}} = 0$, applying the Leibniz integral rule. Both equations lead to the same conditions in the benchmark case. The first stage switch still occurs when the marginal utility of child-bearing exceeds the marginal utility of consumption, and an interior solution of the second stage switch exists, given proper parameters. The feature of this case is that consumption varies over time, thereby making an interior solution of the first stage switch possible. The continuity of the program value also allows the analytical form of the optimal $a(\hat{t})$ by taking

$\frac{\partial \Pi^*}{\partial a(\hat{t})} = -\frac{e^{-r\hat{t}}}{c(0)} + \frac{e^{-\rho\hat{t}}}{c(\hat{t})} = 0$. Thus,we get

$$a^*(\hat{t}) = \frac{1-e^{-\rho\hat{t}}}{1-e^{-\rho\tilde{t}}}\frac{e^{-\rho\tilde{t}}}{e^{-\rho\hat{t}}}d + \frac{1-e^{-r\hat{t}}}{1-e^{-\rho\tilde{t}}}\frac{e^{-\rho\hat{t}}-e^{-\rho\tilde{t}}}{e^{-r\hat{t}}}\frac{w}{r} - \frac{1-e^{-\rho\hat{t}}}{1-e^{-\rho\tilde{t}}}\frac{e^{-r\hat{t}}-e^{-r\tilde{t}}}{e^{-r\hat{t}}}\frac{w-s}{r}. \qquad (2.32)$$

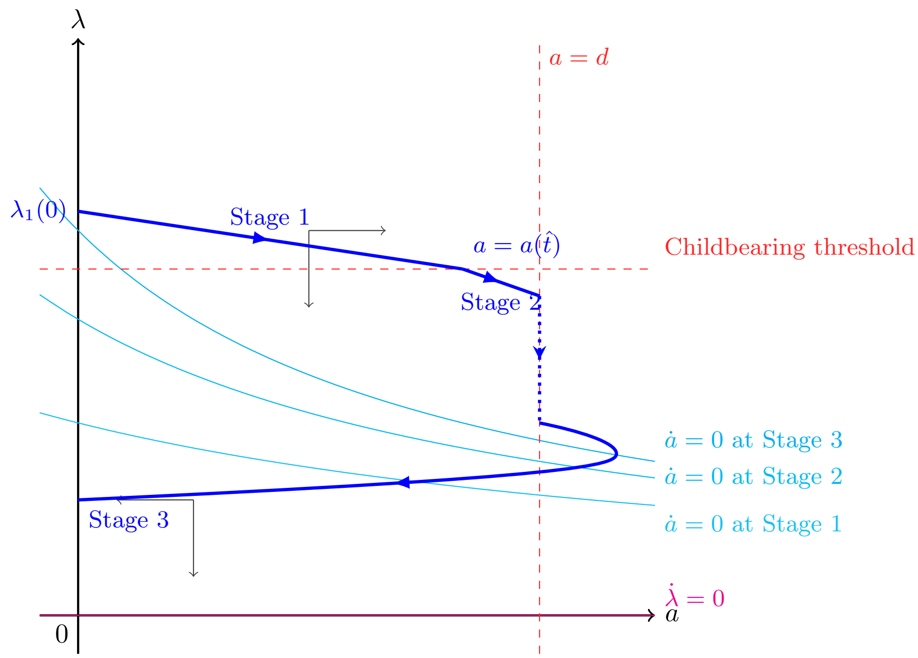

Figure 2.2: A phase diagram of the program when $r > \rho$, with interior $\hat{t}$.

Figure 2.2 presents the phase diagram of a possible $r > \rho$ case.[8] In this case, $\dot{\lambda}$ is negative anywhere in the first quadrant, reflecting that the shadow value of assets is decreasing and therefore consumption goes up over time. The time path starts at $\lambda_1 = \frac{1}{c(0)}$. If it is above the childbearing threshold regulated by Equation (2.26), the path starts from Stage 1 and switches to Stage 2 at the time λ falls to the childbearing threshold. If $\lambda_1(0)$ is below the threshold, we have a corner solution that $\hat{t} = 0$ and the path starts from Stage 2. If the whole upper part of the path is above the threshold, there would be no Stage 2 but

[8]The time chart of the same asset accumulation path is displayed by the blue line later in Figure 2.4.

$\hat{t} = \tilde{t}$.

If the timing of childbearing is not at the corner, we can see that the momentum of asset growth in Stage 2 is slower because HH bears the child raising costs in the stage. The downward kink of the trajectory at the stage switch point in the figure shows this change. When the path reaches $a = d$, HH switches to Stage 3, and a leap up in consumption occurs because the accumulated assets are now free to use; the HH can now borrow against the home equity. In Stage 3, HH may keep saving for some time for the sake of interest return and later spend every penny before the end of the program. This happens when the trajectory segment of Stage 3 starts above $\dot{a} = 0$. HH continues increasing its consumption level over time, and the amount of assets eventually reduces to zero at the end of the program.

Figure 2.3 presents the opposite case, $r < \rho$. It has the same $\dot{a} = 0$ loci, but the sign of $\dot{\lambda}$ is positive everywhere in the quadrant. There is no interior solution for the childbearing timing because the consumption level is decreasing over time, except the sudden jump at the time of house purchase. It is signified in the figure by the growing shadow value of assets. If the marginal utility of consumption is always below the childbearing threshold, we have a corner solution that HH decides to have a child at the onset of the program. If the initial marginal utility of consumption is higher than the threshold, we have another corner solution with $\hat{t} = \tilde{t}$. These two scenarios are the same as in $r > \rho$ case. However, because it is infeasible to dump a child, HH determines whether to have a child at the beginning or at the time buying a house by comparing the total value of each application of the programs if the trajectory intersects with the childbearing threshold. In Figure 2.3, HH chooses to have a child at the beginning of the program.

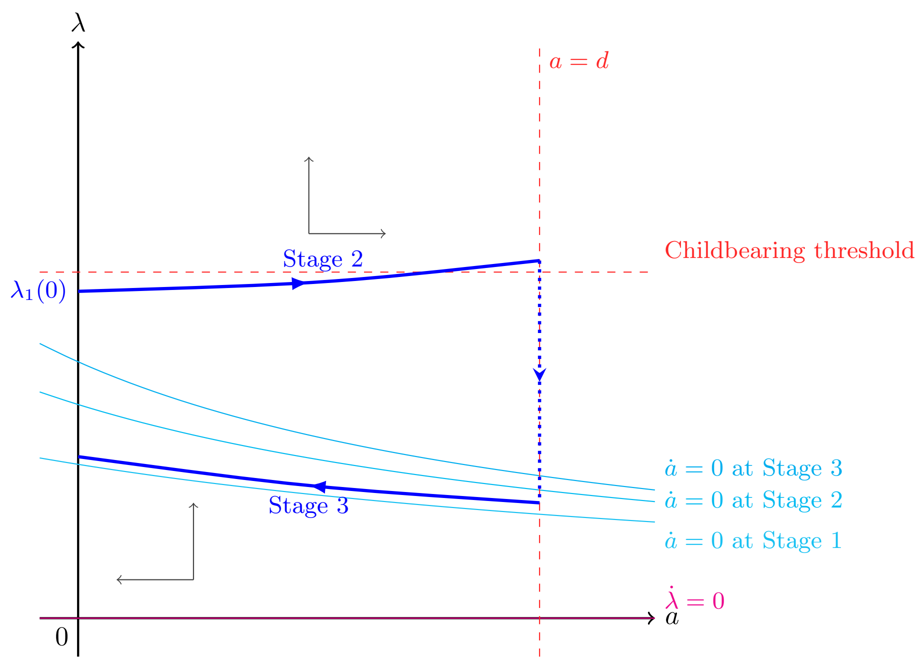

Figure 2.3: A Phase diagram of the program when $r < \rho$, with $\hat{t} = 0$.

Though not showing here, it is possible that HH runs in debt for some time after becoming a homeowner without extra credit constraints when we have $r < \rho$. When it happens, the asset path goes to the second quadrant ($a < 0$) and then turns back and perches on $a = 0$ at the end of the program. The occasion results from HH's impatient nature, for which HH cares less about its happiness in the future than that in the present days.

2.3.2 Other Possible Sequences

Sequence 2: Owning a House First $(0 \leq \tilde{t} \leq \hat{t} \leq T)$

HH likely prefers this sequence if the marginal utility of homeownership without child

$v(1,0)$ is sufficiently large or the marginal utility of childbearing without house $v(0,1)$ is sufficiently small. Intuitive, the sequence is preferred when a life with homeownership is more important than family formation. Not only an individual's taste but social norms may play a factor in the preferences. An extreme example is that a society has a stringent social or religious rule such that the prerequisite of family formation is to own a home. In this case, $v(0,1)$ can even be negative if such behavior incurs penalty. The objective function of Sequence 2 is

$$\int_0^{\tilde{t}} u(c(t),0,0)e^{-\rho t}dt + \int_{\tilde{t}}^{\hat{t}} u(c(t),1,0)e^{-\rho t}dt + \int_{\hat{t}}^{T} u(c(t),1,1)e^{-\rho t}dt.$$

HH decides by assessing which strategy is the best as before. The necessary condition for the binding downpayment constraint remains the same, while binding becomes even more likely. It is because HH can manage consumption by adjusting the childbearing event time after becoming a homeowner. If children are preferable, Sequence 3 is very likely better than Sequence 2. Like Sequence 1, it only occurs when $r > \rho$. The main factor to cause a household chooses Sequence 2 in this model is a high parenting cost, no matter financially or psychologically.

Sequence 3: Having a Child and Buying a House together ($0 \leq \tilde{t} = \hat{t} \leq T$)

This sequence is treated as a special case of Sequences 1 and 2. The earlier discussion has fully covered this possibility.

Sequence 4: Never Owning a House

If the maximum of value function of Sequences 1 and 2 are both less than the maximum of a value function without Stage 3, $\int_0^{\hat{t}} u(c(t),0,0)e^{-\rho t}dt + \int_{\hat{t}}^{T} u(c(t),0,1)e^{-\rho t}dt$, HH will not buy a house during the program. On a phase diagram, one can judge whether HH buy house or not by checking whether the behavior path in Sequences 1 and 2 after house purchase are always above $\lambda = \frac{\Delta v_1^h}{m}$ line. If it is true, HH prefers not to buy a house. If the trajectory intersects the line, the decision is indeterminate and depends on numerical analysis.

Sequence 5: Never Having a Child

Sequence 5 is similar to Sequence 4. If Δv_0^c and Δv_1^c are both sufficiently low, which implies a low value of having a child to HH, an optimal path without having a child is possible. As discussed earlier, the low marginal utility of having a child leads to a high threshold of consumption level to reach in the phase diagram. In Figure 2.2, for instance, a very low Δv_0^c results in no intersection of the curve and the behavior path, implying HH would not want to have a child throughout the program since the joy from high consumption can substitute the happiness of having a child. Yet, it is considered an atypical situation in general.

Sequence 6: Never Having a Child nor Owning a House

The last sequence is an extreme case which satisfies both the requirements for having Sequences 4 and 5. In this case, Δv_0^c, Δv_0^h, and $v(1,1)-v(0,0)$ are all sufficiently small. Alternatively, HH's income maybe so low that the marginal utility of consumption is way higher than that of homeownership and children. In either case, HH has no desire to own a house or have a child in the life course.

Which sequence is best for HH? The precise answer to the question requires a numerical application. Nevertheless, the phase diagrams analysis shed light on a qualitative examination of the optimal solution. For any program under this model, we can start from the phase diagram of Sequence 6. If the time path is everywhere below the marginal benefit of child raising or house purchase, the optimal solution should be either Sequence 4 or 5; if it is below both thresholds, HH would prefer one of the first three sequences. Which is the best is determined by the relative marginal benefits and the required volume of downpayment.

2.4 Comparative Analyses

After showing the characteristics of the optimal strategy of the model, we now use the phase diagram analysis to investigate the impacts of exogenous parameter shocks on the household's optimal solutions. We focus on a permanent shift of household income at the beginning of the program. Any shock in the middle of the program leads to a behavior readjustment, essentially a new optimization question with a shorter time span. If a shock occurs after the one stage transition, it reduces to a classical two-state dynamic program. We select Sequence 1 with $r > \rho$ as the model optimal path for the comparative analysis. The comparative static on an optimum with interior solutions can draw more insights on household decision of state transition timings.

A positive shock in permanent income w may in the first glimpse bring both the optimal action times earlier. However, we shall see that the optimal movement does not always follow the intuition, according to the model. Figure 2.4 shows the optimal asset trajectory over time for Sequence 1 with $r > \rho$, reflecting the optimal path in Figure 2.2.

When w increases permanently, the path shifts up with a positive amount of terminal assets, given the initial optimal strategy. This is shown by the dashed path in the figure. Without adjustment, the asset path violates the transversality condition. The strategy cannot be optimal because assets are not fully utilized. The most direct reaction to the shock is an increase in non-housing consumption. It decreases the current value of assets at every moment and, by Equation (2.26), reduces $\hat{t}$ because parenting is now relatively cheaper. As childbearing takes place in advance, HH starts bearing its costs earlier and, depending on the shift of $\hat{t}$ and the change in w, HH may require more time to save enough money for the house downpayment. The house purchase is postponed, the time interval of Stage 3 becomes shorter, and Stage 3 consumption rises to exhaust the same amount of equity in a shorter time span. This induces a counter marginal effect to the timing of the stage switch. However, it cannot offset the raised incentive of having a baby earlier. Therefore, under the framework, HH would postpone the house purchase for a short time in exchange for having a child earlier.

Though lack of analytical solution, the mathematical examination supports the result of the diagrammatical analysis. The total derivative of Equation (2.26) respect to income gives $\frac{d}{dw}c_1(\hat{t}) = 0$, and, replacing the consumption function by Equations (2.23) and (2.32), it gives the same negative relationship between w and $\hat{t}$.

$$\frac{d\hat{t}}{dw} = -\frac{\rho}{r}\frac{1-e^{-r\tilde{t}}}{1-e^{-\rho\tilde{t}}}e^{-(r-\rho)\hat{t}}\left(\frac{\partial c_1}{\partial \hat{t}}\right)^{-1} < 0.$$

The other condition of free endpoints does not provide a determinate sign of the

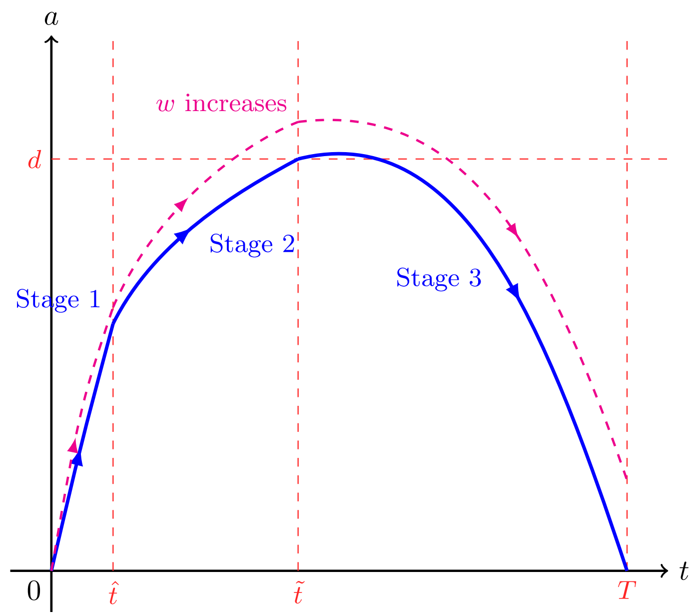

Figure 2.4: Dynamics of household asset when there is a permanent income shock.

change in home-buying timing. Though, Equation (2.18) however gives some clues on how the dynamic would be. The total derivative of the condition respect to income gives the function of $\frac{d\tilde{t}}{dw}$ such that

$$\frac{d\tilde{t}}{dw} = \frac{u_{2c} - u_{3c} + \left(u_{2cc}\dot{a}(\tilde{t}^-)\frac{\partial c(\tilde{t}^-)}{\partial w} - u_{3cc}\dot{a}(\tilde{t}^+)\frac{\partial c(\tilde{t}^+)}{\partial w}\right) + u_{2cc}\dot{a}(\tilde{t}^-)\frac{\partial c_2}{\partial \hat{t}}\frac{d\hat{t}}{dw}}{-u_{2cc}\dot{a}(\tilde{t}^-)\frac{\partial c_2}{\partial \tilde{t}} + u_{3cc}\dot{a}(\tilde{t}^+)\frac{\partial c_3}{\partial \tilde{t}}}. \tag{2.33}$$

Because the consumption level is always larger in Stage 3 than in Stage 2, the rate of asset accumulation higher, and $\frac{\partial c_2}{\partial \tilde{t}} > \frac{\partial c_3}{\partial \tilde{t}} > 0$, the denominator of Equation (2.33) is positive. The sign of the numerator is more intricate. The first component, $u_{2c} - u_{3c}$, is always positive, and the component in the parenthesis is negative for the same determination of the denominator. The last component is non-negative. If the inital optimal timing of childbearing is a corner solution, this component is zero because $\frac{d\hat{t}}{dw} = 0$ given $\hat{t} = 0$ or $\tilde{t}$. However, an interior solution for $\hat{t}$, causing $\frac{d\hat{t}}{dw} < 0$, leads a positive value of the component and thus raise the possibility of a positive $\frac{d\tilde{t}}{dw}$. This gives a theoretical base of our argument.

2.5 Discussion

In this paper, we construct a dynamic household choice model to capture the decision rules of a household's optimal course of life in a joint decision scheme. It displays the interactions between the decisions on the order and the timings of household childbearing and home-buying events, particularly through the channel of the intertemporal budget constraint and the unique credit constraint for home buying. Although the building blocks of the model are simple, its mathematical analysis is surprisingly complex, and no closed-form solution can be derived, even with the simple logarithm utility function. We can only draw out the general rules of the decision-making process without giving a one-shot conclusion. Still, the examinations explore the methods to analyze and interpret a joint-dynamic model, and the phase diagram analysis helps us decipher the mechanism of household intertemporal resource allocation. We see the finding as an intermediate achievement toward a more general behavior model incorporating uncertainty and time-varying parameters for multi-dimensional individual decision making.

Besides income shocks, the precarious nature of the market almost certainly grants the uncertainties in expected house prices and rents, mortgage interest rates, and the down-payment requirement. A prudent household has to assess these possibilities and probably save extra money to avoid downside risk. The fluctuation of house prices also implies an opportunity to gain from holding home equity. This character of real estate as an investment is silent in our model, but it is always a substantial determinant for individual home-buying decisions. A residential property can provide its owner with not only a safe housing service but also a means to hedge probable housing market booms and even to profit.[9] On the fertility side, childbearing is not an event that can be totally controlled. Unintended pregnancy may happen to couples, and infertility is also not uncommon. Moreover, the probability of infertility and having a baby with chromosomal disorders increases with age, giving pressure to women not to delay their childbearing effort. The next step of the model development is to include stochastic processes into the theoretical model to approximate these uncertainties.

Even at this halfway stage, there are still many things we can do. While this research only investigates one type of shock, the procedure is capable of extracting the comparative statics for any parameter shocks in the model. It will be interesting to see how a change in house prices and the downpayment constraint would affect the optimal timing and ordering of the childbearing and home-buying events. The comparative statics will provide more insights into the economic impact on family life courses. They can also contribute to a theoretical base of empirical works on these socio-economic issues. With the help of micro-level longitudinal data on household wealth, we can examine the validity of the model implications.

The framework can be further generalized to any two-dimensional dynamic decision problems. In the household choices domain, the model has the potential to investigate the interactions between housing, childbearing, marriage, migration, and individual career choice. Because only home buying normally entails a credit constraint, it can be expected

[9] In the reduced-form utility function, we can see this benefit is embedded in the marginal utility of homeownership with a fixed value.

that the relationships of entering homeownership with other household life state transitions should share the same set of features.

Chapter 3

Increases in House-Price Uncertainty and the Discouragement of First-Time Homeownership

abstract

Does an increase in housing-market volatility discourage first-time homeownership? We develop a simple two-period model of housing tenure choice in which the prospective first-time home-buying household enters homeownership only if it can afford the mortgage downpayment. The probability of homeownership then increases with first-period savings. We show that an increase in volatility, modeled as a mean-preserving spread in the distribution of

the second-period house price, has an ambiguous effect on homeownership probability. The model serves as a theoretical application of the portfolio optimization theory on home-buying decisions. If the upside and downside risk were symmetric, a prudent household would respond to the increased risk by increasing precautionary saving, which would increase homeownership probability. But the outside option of renting limits the possible loss, causing the upside and downside risks to be asymmetric and much like an option-type right. The overall effect depends on the prospective home buyer's preferences toward risk (in the form of risk aversion and prudence) and the particular form of the change in risk. The generalized model can also be applied to investigate the effects of other types of uncertainty and to explore the policy-related parameters of homeownership.

3.1 Introduction

Most societies highly value homeownership. Households view owning a home as conducive to freedom and family well-being, and governments view a higher homeownership rate as conducive to economic prosperity and social stability. This paper addresses the question: Does an increase in housing-market volatility discourage first-time homeownership? With a simple two-period model of housing tenure choice, we explore the saving behavior of identical prospective home buyers in a credit-constrained housing market against increasing risks in the form of mean-preserving spreads. The model essentially applies the portfolio optimization theory to precautionary savings on household home-buying decisions. In the model, savings plays a dual role; it adjusts and smooths intertemporal consumption and affords the mortgage downpayment required for homeownership. We show, in line with theory, that although risk aversion is generally a determining characteristic, prudence is also a crucial determinant of saving behavior toward risk changes.[1] Increased house-price uncertainty can, under certain conditions, raise or reduce home buyers' desired savings for a downpayment. Neither characteristic has decisive power on the direction of the effect of an increase in price uncertainty. While prudence compels precautionary savings against a rising risk, an asymmetry between the upside and downside risks exists because of the outside option of home buyers. That is, the worst that can happen is having to rent in both periods. This option limits the possible loss caused by a very high house price. Depending on home buyers' preferences and the form of the house-price distribution, the general effect of house-price volatility on homeownership is indeterminate.

[1]See Appendix 3.B for an introduction on prudence and its relationship with risk aversion.

The market for housing is typically an incomplete credit market. Most home mortgages require a downpayment at the time of origination. This borrowing constraint regulates housing choices and consumption (Bernanke and Gertler, 1995). Even though the downpayment eventually feeds into the homeowner's equity, the requirement compels potential home buyers to save before purchase. Savings in this context can be interpreted as a real call option for homeownership with a price upper-limit and an option price that is simply the opportunity cost of the savings. If the realized price is lower than expected, the buyer can enter homeownership at the planned time. Otherwise, if the realized price is higher than expected and the savings are insufficient for the downpayment, the investment is just a waste. The cost of this investment is a period of relative frugality before becoming a homeowner. To understand how risk affects homeownership, we explore the relationship between future house-price volatility and household saving behavior.

The impact of housing-market volatility on housing demand has garnered attention in the recent empirical literature. Evidence suggests that the transition to homeownership is sensitive to house-price uncertainty; though, the observed impacts do not have a uniform direction. On the one hand, Turner (2003) and Turner and Seo (2007) show a negative effect of house-price uncertainty on home buying using cross-sectional and panel data analyses. On the other, Banks et al. (2016) and Han (2010) conclude that households are more likely to enter homeownership earlier due to the desire for insurance or hedging. In a different context, Sinai and Souleles (2005) observe that the likelihood of homeownership increases with rental price risk. They interpret their findings to mean that housing is both consumption and investment. Overall, the theoretical examination on the effect of house price volatility remains scarce. The exceptions are Li and Yao (2007) and Banks et al. (2016). They apply multi-period life-cycle models to simulate the consequence of higher uncertainty in house prices, assuming a constant-relative-risk-aversion within-period utility function.

We construct a two-period homeownership choice model with an incomplete credit market and uncertainty in house prices. The choice of housing service is binary: owning or renting. No direct cost is imposed for the transition from one status to another. This simple design allows us to bypass the effect of housing capital gain and other endogenous choices and isolate the influence of house-price variation.[2] The increases in risk from the framework of Rothschild and Stiglitz (1970) and Diamond and Stiglitz (1974) are applied here to capture the comparative statics under uncertainty. Our main result demonstrates that prudence is the critical trait in increasing savings for a mortgage downpayment. If prudence is a common trait among households, this result suggests they are prone to save more to enter homeownership earlier against house-price volatility. This harmonizes the existing analytical findings of (Banks et al., 2016) even without considering owner-occupying housing as a financial hedging tool.

In Section 3.2, we introduce the model framework, and the following two sections demonstrate the comparative statics in the discrete and continuous uncertainty scenarios. In Section 3.5, we conclude the findings and discuss the extension capacity of the model to other uncertainty schemes.

[2]The saving behavior of potential home buyers also depends on their long-term housing plans and the expected capital return from holding properties as an investment. Price uncertainty is crucial to both effects (Henderson and Ioannides, 1983; Han, 2008). Fu (1995) and Li and Yao (2007) argue that the impact of price uncertainty on the homeowner's decision regarding further housing investment is indeterminate.

3.2 The Model

Consider a two-period life-cycle model ($t \in \{1, 2\}$) for a risk-averse individual whose current utility is induced from housing services and the consumption of a composite of all other goods. Housing service is a necessity for the individual, and the utility it brings is additively separable from other consumption. Two types of housing services are possible: renting or owner-occupying, $h \in \{0, 1\}$. We assume that the housing market has a uniform unit house price V and unit rental price R, and there is no depreciation in the house quality over time in the model. The individual's goal is to maximize their life-cycle utility by finding the optimal housing tenure strategy and inter-temporal consumption allocation. In each period, depending on the housing choice, the individual has to pay the housing rent (R) or the cost of owner-occupation ($P = rV$), which is the interest-only mortgage payment with a mortgage interest rate that is the same as the safe interest rate, r.[3]

We denote the disposable wealth of the individual at the beginning of Period t as a_t, where a_1 represents the wealth endowment. Income of Period t is denoted by Y_t. From the outset of each period, before income is earned, the individual receives new information about the housing market and chooses the type of housing service for the current period and the level of savings for future spending (or current consumption) based on the information in hand. (The timing of information update matters when we introduce the uncertainty in section 3.3). The credit market is assumed complete except for the home mortgage. By the time of entering homeownership ($\tilde{t}$), the individual must possess sufficient wealth for the downpayment ($a_t \geq \tilde{D}V$), where $\tilde{D}$ is the market downpayment rate. We assume there

[3] See Appendix 3.A for a full notation list.

is no transaction fee, and the downpayment is fully transformed to home equity against which the owner can borrow in the complete credit market. In this model, the only cost of tenure transition is the opportunity cost of savings.[4] Similarly, when a homeowner decides to give up homeownership or is in the final period of the program, the home equity is fully transformed to cash.[5]

To focus on the specific point of interest, we rule out other options that exist in real estate transactions. The utility generated from either type of housing service is assumed identical, so we can simply drop housing services from the utility function as it becomes a constant term.[6] To the individual, the only difference between renting and owning is the financial cost incurred. This limits the relationship between housing and other consumption to a single channel. Finally, unless specified, we generally assume the individual's time discount is fully compensated by the interest payment, equivalent to $\beta(1+r) = 1$; consumption smoothing is preferred for any concave utility function.

Without uncertainty in the model, the individual has perfect foresight at the out-

[4]The result would not change if we loosened these assumptions. In the two-period model, adding transaction fees and an equity borrowing constraint is equivalent to an increase in owner-occupation costs.

[5]These settings can be easily generalized without disturbing the main result. For instance, we can set a positive real estate transaction fee. The fee will generally discourage homeownership but, from a marginalist perspective, it does not overturn the effect of a risk increase.

[6]This setting is in parallel to the assumption that housing quality and space demanded are independent of the homeownership decision. Loosening the assumption would create a non-zero marginal utility of homeownership, but it can be mathematically transformed to being part of the cost gap between owning and renting the house.

set. Algebraically, one can write the optimization problem as the following:

$$
\begin{aligned}
&\max_{\{c_t,h_t\}} \sum_{t=1}^{2} \beta^{t-1} u(c_t) \\
&\text{s.t. } a_2 = (1+r)\left[a_1 + Y_1 - c_1 - (1-h_1)R_1 - h_1 P_1\right] \\
&\quad c_2 \leq a_2 + Y_2 - (1-h_2)R_2 - h_2 P_2 \\
&\quad a_{\tilde{t}} \geq DP_{\tilde{t}} \text{ (the downpayment constraint for buying a home at } \tilde{t}) \\
&\quad a_1 \text{ is exogenous}
\end{aligned}
\tag{3.1}
$$

where $D = \tilde{D}/r$ is the effective downpayment rate, so $DP_t = \tilde{D}V_t$.

The downpayment requirement is activated only if the individual chooses to enter homeownership. The requirement leads to two potential credit constraints in the program, regulating the wealth requirement for tenure transition in either period. If the individual decides to buy a home in Period 1, the wealth endowment at the beginning of that period must be at least as much as the downpayment requirement. If the individual rents in Period 1 ($h_1 = 0$) and buys a home in Period 2, the wealth at the beginning of Period 2 has to be greater or equal to the downpayment requirement in the period, or $a_2 = (1+r)(a_1 + Y_1 - c_1 - R_1) \geq DP_2$. This constraint forces potential buyers to save more than they would have otherwise whenever the downpayment payable exceeds the savings for the smoothed consumption. The downpayment constraint further renders entering homeownership in Period 2 impossible once the present price of the downpayment is higher than $a_1 + Y_1 - R_1$. In the remainder of this paper we assume $a_1 = 0$ (homeownership unavailable in Period 1), $R_2 > E[P_2]$ (homeownership in Period 2 is preferred), and $R_1 < Y_1 - \frac{DP_2}{1+r}$ (entering homeownership in Period 2 is possible), to avoid trivial choices.

Figure 3.1 presents the comparison between renting and owner-occupying in Period 2 in the case that the latter is the better strategy. The four-quadrant diagram lines up the relationships of tenure choice, consumption, and total utility resulting from different strategies. Quadrant 1 sets out the c_1-c_2 plot with the constraints. Given the assumption that $\beta = \frac{1}{1+r}$, the unconstrained optimal inter-temporal consumption bundle lies on the 45° line. When owning a home is cheaper, owner-occupation has a higher budget constraint, but the feasible consumption bundles on the budget line are truncated by the downpayment constraint (DPC); the DPC imposes a cap on the first-period consumption in order to afford a mortgage downpayment in Period 2. The individual would prefer to buy a home if the owner-occupation cost is sufficiently low and the DPC is not too strict. IC_r and IC_b represent the indifference curves at the optimal utility level for renting and owning a home in Period 2. Quadrants 2 and 4 are the transformation quadrants that map c_2 to $u(c_2)/(1+r)$ and c_1 to $u(c_1)$. The utility function is assumed to be concave here. Quadrant 3 is thus the inter-temporal utility plot in which the indifference curves in Quadrant 1 are transformed to straight lines orthogonal to the 45° line. From Quadrants 1 to 3, the graph is a straightforward mapping of the indifference curves for comparative statics. Indeed we can simply draw the indifference curves in Quadrant 1 in cases without uncertainty; mapping is necessary once uncertainty is involved and multiple states are attainable.

In brief, savings (or loans if these are preferred) play a dual role in the program. These help to balance inter-temporal consumption and to fulfill the credit constraint for homeownership. Without sufficient savings, the individual would not be able to become a homeowner in either period.

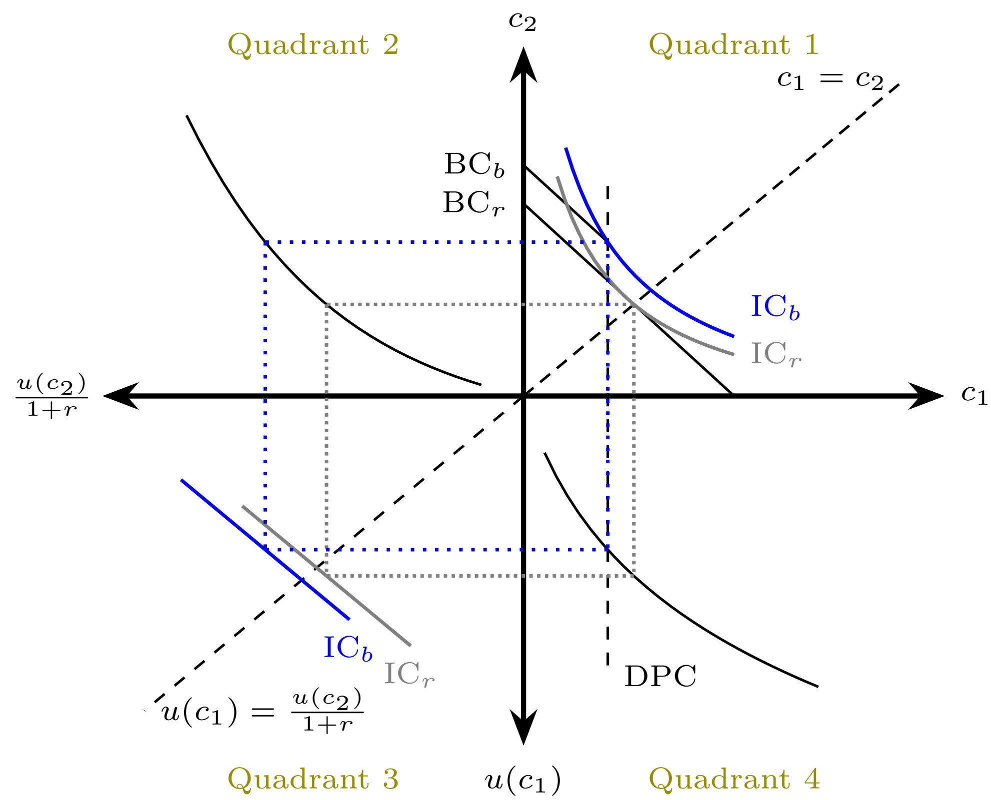

Figure 3.1: The four-quadrant diagram comparing continuing to rent (denoted with subscript r) and buying a home (b) in Period 2 (without uncertainty).

The following discussion concentrates on the effect of house-price uncertainty. Without losing generality, uncertainty is discussed in terms of the distribution of owner-occupation costs in Period 2, the expectation of which is identical to the cost in Period 1, or $P_1 \equiv P = E[P_2]$. The individual holds rational expectations and knowledge of the distribution of future prices while all other things remain equal. We let both rent and income be fixed over time, so $R_1 = R_2 \equiv R$ and $Y_1 = Y_2 \equiv Y$. As a potential home buyer, the individual makes a planned saving in Period 1 ($s_1 = \frac{a_2}{1+r}$). At the outset of Period 2, the new price is realized, and the individual decides whether homeownership is feasible given this new information.[7] The amount of savings determines whether the individual can afford the downpayment given the new price. Frugality in Period 1 would provide the individual the advantage of keeping their tenure options open under uncertainty.[8]

3.3 Uncertainties with Discrete Distributions

We start from a simple distribution consisting of only two possible states, the high price state $P_2^h = P + \Delta$ and the low price state $P_2^l = P - \Delta$, where Δ is a small positive value relative to the owner-occupation cost. The probability of each state is 0.5, so the expected owner-occupation price is unchanged. This distribution is regarded as a simple mean-preserving spread (MPS) from a constant lottery, as proposed by Rothschild and Stiglitz (1970). In the next section, we extend the discussion to the case with continuous distributions.

Had the DPC not been binding, a risk-averse individual would naturally save more in Period 1 to compensate for the distaste for uncertainty in Period 2. This corollary is drawn from Jensen's inequality under rational expectations.[9] When the credit constraint is binding, namely, the wealth for the mortgage downpayment is needed, the response of risk-

[7] This setting is in line with temporally uncertain prospects, as discussed in Dreze and Modigliani (1975).

[8] One limitation of the two-period model in this context is that it is incapable of including the expected housing capital gain for buyers in Period 2. It is important to keep this in mind when we deduce any results from the model.

[9] For buyers in Period 2, if there is no uncertainty and the income of the individual is sufficiently high such that the DPC is slack ($a_2(c_1 = c_2) > DP$), the net saving in Period 1 will be $\underline{s}_1 = -\frac{R-P}{2+r} < 0$. With uncertainty, the expected lifetime utility in (3.1) after substituting the budget constraints becomes

$$E[U] = u(Y - R - s_1) + \beta \left[\frac{1}{2} u(Y - P - \Delta + (1+r)s_1) + \frac{1}{2} u(Y - P + \Delta + (1+r)s_1) \right]. \quad (3.2)$$

The first part of the right-hand side represents $u(c_1)$. The second part represents the expected utility in Period 2, which consists of the high price state, $u(c_2^h)$, and the low price state, $u(c_2^l)$. By Jensen's inequality, $\frac{\partial U}{\partial \dot{a}}\Big|_{s_1 = \underline{s}_1} > u'(Y - R - \underline{s}_1) - u'(Y - P + (1+r)\underline{s}_1) = 0$ so the net saving in the first period is positive. Here we omit the possibility of $P + \Delta > R$.

averse individuals to the introduction of uncertainty may not be uniform in the increase or decrease of savings. From here on, we refer to this optimal approach in the scenario without uncertainty as the "benchmark approach." On this approach, the individual presumes $P_2 = P_1 = P$ and prepares for a downpayment which is just DP in Period 2. Once the uncertainty is introduced, the individual has a 50% chance of failing to enter homeownership if no adjustment is made. It is unlikely to be the optimal approach because the opportunity cost of savings has one-to-one odds of receiving no returns. Two options lie before the hopeful buyer in response to the uncertainty. The individual may, at the cost of highly unbalanced consumption, practice austerity to save more in Period 1 and ensure the affordability of the downpayment in either state (the "austerity approach"). Alternatively, the individual may give up a 50% chance to enter homeownership and practice indulgence to save less, to the point that the wealth in the second period is just enough to pay for the downpayment in the low price state and smooth consumption as much as possible (the "indulgence").

First, consider that $P - \Delta < R < P$ or $P < R < P + \Delta$. The individual would buy a home at $t = 2$ in the low price state but keep renting in the high price state. This would be the case no matter which approach is taken because the owner-occupation price is higher than the rental price. We can easily check which approach is better.

Between the benchmark approach and that of practicing indulgence (saving less), we set $\delta \in [0, 1]$ to depict the home buyer's consumption behavior. We have $\delta = 0$ if the individual takes the benchmark approach and behaves as if there is no uncertainty, and $\delta = 1$ if the indulgence approach is taken. This setting allows any consumption behavior between the two approaches for the purpose of marginal analysis. We can rewrite (3.2) as a function of δ:

$$E[U] = u(Y - R - \frac{D(P-\Delta\delta)}{1+r}) + \frac{\beta}{2}\left[u\left(Y + DP - R - D\Delta\delta\right) + u\left(Y + DP - P + \Delta - D\Delta\delta\right)\right].$$

The derivative of the value function with respect to δ at $\delta = 0$ shows that the benchmark approach is not optimal.

$$\begin{aligned} \frac{\partial E[U]}{\partial \delta}\Big|_{\delta=0} &= \frac{D\Delta}{1+r}\left[u'(c_{1\ \delta=0}) - \frac{1}{2}u'(c^h_{2\ \delta=0}) - \frac{1}{2}u'(c^l_{2\ \delta=0})\right] \\ &> \frac{D\Delta}{1+r}\left[u'(c_{1\ \delta=0}) - u'(c^h_{2\ \delta=0})\right] \\ &\geq 0. \end{aligned} \tag{3.3}$$

The first inequality comes from the fact that c_2 is always higher in the low price state than in the high price state. The second inequality holds because c^h_2 is always greater or equal to c_1. Similarly, this inequality holds at $\delta = 1$. From (3.3), it is clear that the indulgence approach is always preferred to the benchmark approach. The same analysis can be easily applied to show that the benchmark approach is always preferred to the austerity approach. Thus, we can conclude that the indulgence approach is optimal if $P - \Delta < R < P$ or $P < R < P + \Delta$.

However, this analysis does not work when the highest possible owner-occupation cost is less than the rent.[10] We can see that the indulgence approach is still always preferred to the benchmark, as in the previous case. However, whether the austerity or indulgence approach is better is indeterminate unless we have more specific information. In this case, homeownership is also preferred in the high price state, and this gives an extra return on

[10] We do not consider $R < P - \Delta < P$ here as the individual has no incentive to enter homeownership.

the austerity approach.

An example is set out in Figure 3.2. Due to the uncertainty of the house price in Period 2, the original homeownership budget line (the short gray line in Quadrant 1) splits into two lines, representing the homeownership budget in the low price state (BC(P^l)) and high price state (BC(P^h)). Every approach leads to two possible outcomes of the consumption bundle, labeled in Quadrant 1, one for each of the high price and low price states. Following the benchmark or the indulgence approach, the individual would not be able to afford the mortgage downpayment in the high price state. The high price homeownership budget is only available on the austerity approach. The corresponding inter-temporal utilities are mapped in Quadrant 3, and the expected utility is simply the mid-point of those. Though the indifference curves show the indulgence approach to be optimal in this example, it is not necessarily the case. Had the return of the austerity approach becomes higher or the concavity of the utility function become greater, the austerity approach might be optimal. Intuitively, this is likely for individuals who are less risk-averse (or even risk-loving) and highly value homeownership (so their effective owner-occupation cost is much lower than the effective rent).

3.4 Uncertainties with Continuous Distributions

The analysis in the previous section gives us an approximate idea of how an individual in our theoretical setting would react to house-price uncertainty. Discrete distribution is, however, a restrictive assumption. The following analysis extends the discussion to continuous uncertainty and draws on the financial portfolio management literature, following

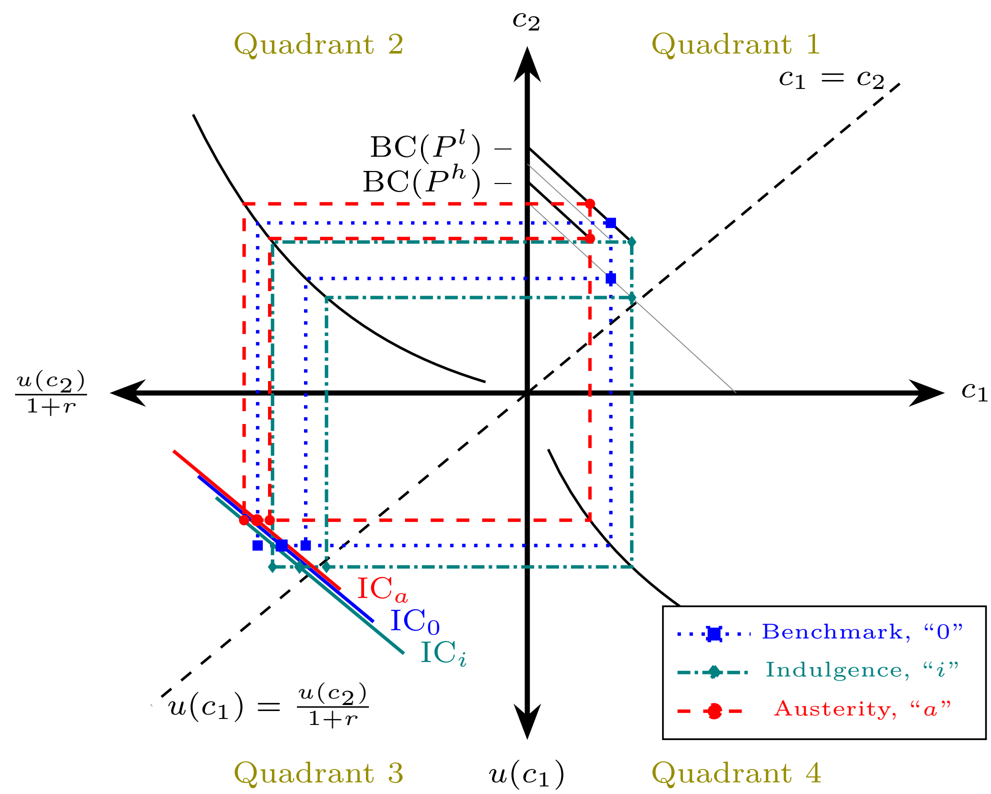

Figure 3.2: The four-quadrant diagram of the austerity, benchmark, and indulgence approaches in the housing market with price uncertainty.

the work of Diamond and Stiglitz (1974) on risk and risk aversion.

Consider a home owner-occupation-cost price distribution function $F(P, \alpha)$ where P is the random variable defined over $[0, \infty)$ and α is an exogenous index of riskiness of the distribution, which will be used later. The individual faces an optimization problem in that the consumption in the first period determines the upper limit of the affordable house price in the next period ($\hat{P}$). The probability of homeownership affordability $\int_0^{\hat{P}} dF(P, \alpha)$ is endogenous and is determined simultaneously with the level of current consumption. Rewriting the optimization problem described in (3.1), we have the new objective function to maximize the expected utility:

$$
\begin{aligned}
&\max_{\{c_t\},\hat{P}} E[U] \equiv u(c_1) + \beta \left[\int_0^{\hat{P}} u(c_2^b) f(P,\alpha) dP + \int_{\hat{P}}^{\infty} u(c_2^r) f(P,\alpha) dP \right] \\
&\text{s.t. } c_2^b - [Y - P + (1+r)(Y - R - c_1)] \leq 0 \\
&\qquad c_2^r - [Y - R + (1+r)(Y - R - c_1)] \leq 0 \\
&\qquad D\hat{P} - (1+r)(Y - R - c_1) \leq 0
\end{aligned} \tag{3.4}
$$

where if $P \leq \hat{P}$, the individual buys a home and the second-period consumption level is c_2^b, otherwise the individual continues renting with the consumption level c_2^r. The maximization problem technically allows negative saving, but this would not be optimal because, unlike the discrete case, the probability of being able to afford the downpayment continuously and proportionally increases with savings. The setting also inherently regulates $\hat{P} \leq R$ because people whose holding cost is higher than their rent have no incentive to buy a home.

Since all constraints must bind, c_2^r and $\hat{P}$ are actually functions of c_1 and c_2^b is a function of c_1 and P. We can simplify (3.4) and rewrite it as a maximization problem with the single control variable c_1. The first-order condition is then easier to obtain. To keep the derivation as succinct as possible, we denote $u_1 = u(c_1)$, $u_2^b = u(c_2^b)$, $u_2^r = u(c_2^r)$, and $\hat{u}_2^b = u(c_2^b(\hat{P}))$ where $c_2^b(\hat{P}) = c_2^b(P = \hat{P}(c_1), c_1)$.[11] Assuming the optimal $\hat{P}$ is interior, we get the first-order condition by Leibniz's rule:

[11] The maximization problem therefore becomes

$$
\max_{c_1} u_1 + \beta \left[\int_0^{\hat{P}} u_2^b f(P,\alpha) dP + \int_{\hat{P}}^{\infty} u_2^r f(P,\alpha) dP \right]
$$

where $c_2^b = Y - P + (1+r)(Y - R - c_1)$, $c_2^r = Y - R + (1+r)(Y - R - c_1)$, and $\hat{P} = \frac{1}{D}[Y - (1+r)(Y - R - c_1)]$. And the derivatives of the variables with respect to c_1 are $\frac{d\hat{P}}{dc_1} = -\frac{1+r}{D}$, $\frac{dc_2^b}{dc_1} = \frac{dc_2^r}{dc_1} = -(1+r)$, and $\frac{dc_2^b(\hat{P})}{dc_1} = -(1+r)\frac{D-1}{D}$.

$$u_1' + \beta \hat{u}_2^b f(\hat{P},\alpha)\left(-\tfrac{1+r}{D}\right) - \int_0^{\hat{P}} \beta u_2^{b\prime}(1+r) f(P,\alpha) dP - \beta u_2^r f(\hat{P},\alpha)\left(-\tfrac{1+r}{D}\right) - \int_{\hat{P}}^{\infty} \beta u_2^{r\prime}(1+r) f(P,\alpha) dP = 0.$$

Rearranging the equation above we obtain the inter-temporal Euler equation for this problem.

$$u_1' = \frac{f(\hat{P},\alpha)}{D}\left(\hat{u}_2^b - u_2^r\right) + \int_0^{\hat{P}} u_2^{b\prime} f(P,\alpha) dP + \int_{\hat{P}}^{\infty} u_2^{r\prime} f(P,\alpha) dP. \qquad (3.5)$$

Equation (3.5) has a very straightforward interpretation. The first component of the right-hand side represents the marginal expected benefit of savings through increasing downpayment affordability.[12] The marginal increase in savings raises the probability of $P \leq \hat{P}$ by $\frac{f(\hat{P},\alpha)}{D}$, and the utility difference between renting and owner-occupying at that price is $\hat{u}_2^b - u_2^r$. The second component is the expected marginal benefit of savings on Period 2 consumption if $P \leq \hat{P}$ (the individual enters homeownership), and the final component is the expected marginal benefit of savings if $P > \hat{P}$ (the individual continues renting). Together these components represent the expected marginal return of savings through increasing c_2. The Euler equation demonstrates that, with the optimal choice, the sum of the three marginal benefits of savings should be equal to the marginal benefit of c_1.

3.4.1 Increase in Risks

Let us now consider how this optimal level of saving would change with an increase in future house-price uncertainty. We define $F(P,\alpha)$ to be a sequence of distribution functions indexed by α, the shift parameter of the distribution, as defined by Diamond and Stiglitz (1974). For any adjacent distribution pairs $F(P,\alpha_i)$ and $F(P,\alpha_j)$ such that

[12] As D becomes smaller, $\hat{P}$ approaches to R and $\hat{u}_2^b - u_2^r$ to zero. When there is no mortgage constraint ($D = 0$), and thus $\hat{P}$ is no longer regulated by the third constraint in (3.5), the first component is dropped.

$\alpha_j > \alpha_i$, the latter differs from the former in the shifting of a portion of its distribution weight to the tails, indicating greater volatility. In formal terms, $F(P, \alpha_j)$ is a simple MPS of $F(P, \alpha_i)$ and is therefore riskier (Rothschild and Stiglitz, 1970). $F(P, \alpha)$ follows two sufficient conditions of MPS:

$$\int_0^\infty F_\alpha(P, \alpha) dP = 0 \tag{3.6}$$

and

$$T(y, \alpha) = \int_0^y F_\alpha(P, \alpha) dP \geq 0 \qquad y \in [0, \infty) \tag{3.7}$$

where F_α represents the derivative of F with respect to α, the marginal shift of distribution in the sequence. The first condition regulates the mean, ensuring it does not change in the function family. The second condition, where $T(y, \alpha)$ represents the cumulative distributional difference of adjacent pairs at α assures that every distributional change in the sequence of the functions is a simple MPS that satisfies the single-crossing property.

Undoubtedly, the expected utility for any risk-averse individual is decreasing as α increases, *ceteris paribus* (Hirshleifer and Riley, 1992). In response, they would adjust their optimal consumption level c_1^* to satisfy Equation (3.5). Assuming all other things remain unchanged, the optimal consumption is a function of the riskiness of the house price, namely the shift parameter, or $c_1^* = c_1^*(\alpha)$. Taking the derivative of (3.5) with respect to α, we have

$$u_1'' \frac{dc_1^*}{d\alpha} = \frac{\hat{u}_2^b - u_2^r}{D} \left[f_P(\hat{P}, \alpha) \frac{d\hat{P}}{dc_1} \frac{dc_1^*}{d\alpha} + f_\alpha(\hat{P}, \alpha) \right] +$$
$$\frac{f(\hat{P}, \alpha)}{D} \left[\hat{u}_2^{b\prime} \frac{dc_2^b(\hat{P})}{dc_1} \frac{dc_1^*}{d\alpha} - u_2^{r\prime} \frac{dc_2^r}{dc_1} \frac{dc_1^*}{d\alpha} \right] +$$
$$\hat{u}_2^{b\prime} f(\hat{P}, \alpha) \frac{d\hat{P}}{dc_1} \frac{dc_1^*}{d\alpha} - 0 + \int_0^{\hat{P}} \left[u_2^{b\prime\prime} \frac{dc_2^b}{dc_1} \frac{dc_1^*}{d\alpha} f(P, \alpha) + u_2^{b\prime} f_\alpha(P, \alpha) \right] dP +$$
$$0 - u_2^{r\prime} f(\hat{P}, \alpha) \frac{d\hat{P}}{dc_1} \frac{dc_1^*}{d\alpha} + \int_{\hat{P}}^\infty \left[u_2^{r\prime\prime} \frac{dc_2^b r}{dc_1} \frac{dc_1^*}{d\alpha} f(P, \alpha) + u_2^{r\prime} f_\alpha(P, \alpha) \right] dP.$$

Rearranging this equation and we obtain the function of the marginal change for optimal consumption in Period 1 with respect to the shift parameter:

$$\frac{dc_1^*}{d\alpha} = \frac{\frac{f_\alpha(\hat{P},\alpha)}{D} \left(\hat{u}_2^b - u_2^r \right) + \int_0^{\hat{P}} u_2^{b\prime} f_\alpha(P, \alpha) dP + \int_{\hat{P}}^\infty u_2^{r\prime} f_\alpha(P, \alpha) dP}{\frac{1+r}{D^2} f_P(\hat{P}, \alpha) \left(\hat{u}_2^b - u_2^r \right) + \frac{1+r}{D^2} f(\hat{P}, \alpha) \left[(2D-1) \hat{u}_2^{b\prime} - 2D u_2^{r\prime} \right] + (1+r) \left[\int_0^{\hat{P}} u_2^{b\prime\prime} f(P, \alpha) dP + \int_{\hat{P}}^\infty u_2^{r\prime\prime} f(P, \alpha) dP \right] + u_1''.} \tag{3.8}$$

Using the implicit function theorem, the denominator of (3.8) is the second-order condition of the value function U with respect to c_1, which must be negative for a maximization problem ($U_{cc} < 0$). The numerator is the partial derivative of (3.5) with respect to α. The first component indicates the "marginal risk effect," the effect of the probability change at $\hat{P}$ on the expected marginal return of savings for the mortgage downpayment. The sign of the effect depends solely on $f_\alpha(\hat{P}, \alpha)$ and the effect is null when $f_\alpha(\hat{P}, \alpha) = 0$ or the mortgage downpayment is not needed. The remaining components indicate the "cumulative risk effect," which captures the effect of the entire perturbation on the expected utility of marginal consumption in Period 2. By applying integration by parts twice, the numerator of (3.8) is expanded to

$$\frac{f_\alpha(\hat{P},\alpha)}{D}\left(\hat{u}_2^b - u_2^r\right) + F_\alpha(\hat{P},\alpha)\left(\hat{u}_2^{b\prime} - u_2^{r\prime}\right) + T(\hat{P},\alpha)\left(\hat{u}_2^{b\prime\prime} - u_2^{r\prime\prime}\right) + \int_0^{\hat{P}} u_2^{b\prime\prime\prime} T(P,\alpha)dP + \int_{\hat{P}}^{\infty} u_2^{r\prime\prime\prime} T(P,\alpha)dP. \tag{3.9}$$

Expression (3.9) is positive when $f_\alpha(\hat{P},\alpha) \geq 0$, $F_\alpha(\hat{P},\alpha)\left(\hat{u}_2^{b\prime} - u_2^{r\prime}\right) \leq 0$, and the marginal utility function is a strictly convex function or, equivalently, the individual is prudent ($u''' > 0$) in accordance with the definition of Kimball (1990). The first term determines the sign of the marginal risk effect. The second term is positive if either $F_\alpha(\hat{P},\alpha) \leq 0$ for a risk-averse individual or $F_\alpha(\hat{P},\alpha) \geq 0$ for a risk-loving individual. This is because the two types of decision makers have opposite risk preferences. However, neither being risk-averse nor risk-loving by itself determines saving behavior if the perturbation of the distribution is unknown. The third term, prudence, has clear implications. A more prudent individual would save more to prevent the downside risk of not being able to afford the downpayment and having to pay rent, and the consumption in Period 2 is therefore eroded.

The overall interpretation of (3.9) is that, with either of the two presuppositions that follow, a prudent individual will save more ($\frac{dc_1^*}{d\alpha} < 0$) when future house-price uncertainty is higher. One presupposition is that such change in the housing market does not alter the probability density nor the cumulative distribution at the optimal home purchase upper-limit, so $f_\alpha(\hat{P},\alpha) = F_\alpha(\hat{P},\alpha) = 0$. In other words, the shift of distributions takes place below or above the critical $\hat{P}$. Alternatively, the individual has to be risk-averse (loving), and $\hat{P}$ is located where its probability becomes bigger, and the cumulative probability of lower prices becomes smaller (bigger). Figure 3.3 gives an example of a simple MPS from

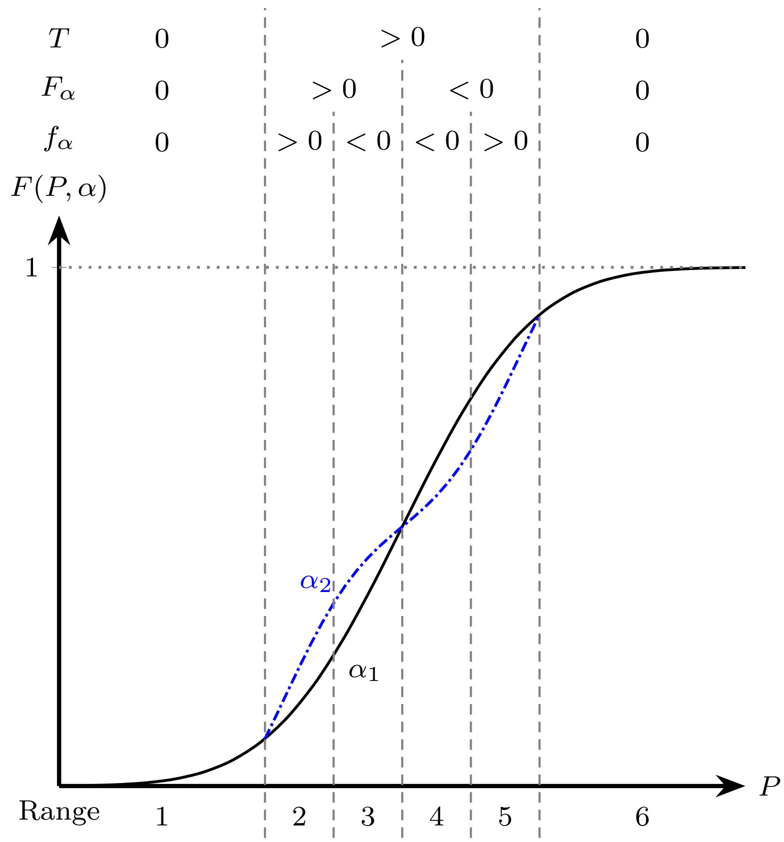

Figure 3.3: A simple MPS of a normal distribution.

the index α_1 to α_2. In the graph, the first circumstance corresponds to Ranges 1 and 6 and the crossing point of the MPS, and the second corresponds to Range 5 for risk-averse individuals and Range 2 for risk-loving individuals. The behavior of the individual in other ranges remains undetermined since we do not know the specific form of the utility function and the distribution.

We may also determine that $\frac{dc_1^*}{d\alpha} > 0$ if we have $T(\hat{P}, \alpha) = 0$, $f_\alpha(\hat{P}, \alpha) \leq 0$, $F_\alpha(\hat{P}, \alpha) \geq 0$ (for risk-averse individuals), and $f_\alpha(\hat{P}, \alpha) F_\alpha(\hat{P}, \alpha) \neq 0$. But no range of the distribution has this property, as Figure 3.3 shows. $T(\hat{P}, \alpha) = 0$ means that the perturbation does not alter the distribution at $\hat{P}$, so it is contradictory to any non-zero change in $f_\alpha(\hat{P}, \alpha)$

and $F_\alpha(\hat{P},\alpha)$. Therefore, if $\hat{P}$ falls outside the supports listed above, the individual's reaction to the perturbation is indeterminate.

The comparative statics for other control variables are straightforward from (3.8) as they are all linear functions of c_1. With respect to the planned downpayment upper-limit, as well as the Period 2 consumption in the renting state and the expected consumption in the buying state, we have

$$\frac{d\hat{P}^*}{d\alpha} = \frac{d\hat{P}}{dc_1}\frac{dc_1^*}{d\alpha} = -\frac{1+r}{D}\frac{dc_1^*}{d\alpha} \tag{3.10}$$

$$\frac{dc_2^{r*}}{d\alpha} = \frac{dE\left[c_2^{b*}\right]}{d\alpha} = -(1+r)\frac{dc_1^*}{d\alpha}. \tag{3.11}$$

If rather, we had a risk-neutral individual ($u'' = 0$), the direction of the reaction would be determined solely by the marginal change of the probability of the critical house price. The individual would not make a change if $f_\alpha(\hat{P},\alpha) = 0$. This indeterminacy echoes the literature on temporal uncertainty.[13]

The analysis here is a reduced version of a more general analysis using the system of implicit functions. Appendix 3.C addresses the procedure for the generalized process. It carries a slightly greater calculating burden but can extract more economic intuitions from the model. It also serves as the general way to find the comparative statics of scenarios with uncertainties on any exogenous variable.

[13] See, for example, Eeckhoudt et al. (1995).

3.5 Discussions

Below we discuss the interpretations of the results in the previous sections and possible extensions of those. In short, the model combines the effects of the borrowing constraint and the general implications of precautionary saving and provides a compatible framework that potentially allows for the inclusion of additional choices and dimensions of uncertainty.

3.5.1 The Role of Saving

Our results show a homogeneous reaction to a rise in the mean-preserving risk if the optimal affordable upper-limit of owner-occupation $\hat{P}$ is above or below the range of the MPS or exactly at the crossing point of the MPS. This is the case whether or not risk aversion is assumed. The individual, believing that the variance of the future price will increase, would choose to save more to be better prepared for the mortgage downpayment. This results from the particularity of the setting of uncertain future consumption. In this model, uncertainty affects only consumption in the owner-occupying state. Because the individual is prudent, when the distribution of the future house price becomes more variable, the expected marginal utility for future consumption in the owner-occupying state will increase. In response, the individual will increase saving for higher consumption in Period 2, which would raise the marginal return of c_1 and lower the marginal return of $E\left[c_2^b\right]$ and c_2^r such that the Euler equation continues to hold.

The second role of savings in this model is to reflect the indeterminacy of the individual's reaction in some conditions. In addition to being a means of inter-temporal wealth transfer, savings help to fulfill the DPC for home buyers. As such, savings can be seen as an investment in the option for future homeownership. The more one saves, the higher the chance of exercising the option, the value of which at $\hat{P}$ is $\frac{f(\hat{P},\alpha)}{D}\left(\hat{u}_2^b - u_2^r\right)$. When the perturbation does not vary the frequency density and cumulative distribution at $\hat{P}$, the value of the option remains constant, and the sign of $\frac{dc_1^*}{d\alpha}$ is determined by prudence. Otherwise, the change in the marginal gain from affordability must be considered as shown in (3.8) and (3.9).

If $f_\alpha(\hat{P},\alpha)$ is negative, the marginal risk effect on savings is decreasing with growing risk because the probability of the extra investment in the option being useful is dropping; the expected utility gain from the additional affordability is not as large as before. That $F_\alpha(\hat{P},\alpha)$ is positive implies a higher probability of $P \leq \hat{P}$. A risk-averse individual would be less worried about the current investment being useless and, consequently, will face a reduced incentive to save more wealth. These two factors would have an opposite effect in relation to prudence-led precautionary savings. There is thus the possibility that the individual might prefer to save less under particular circumstances.

3.5.2 Comparative Statics of the Effective Downpayment Rate

Fixing the uncertainty on owner-occupation costs, we can, from this model, identify some implications of the comparative statics of other variables in an uncertain housing market. Taking the effective downpayment as an example and applying the same process as above or the procedure set out in Appendix 3.C, we can obtain the derivative of optimal consumption in Period 1 with respect to D starting from (3.5).

$$\frac{dc_1^*}{dD} = \frac{\frac{\hat{P}f_P(\hat{P},\alpha)+f(\hat{P},\alpha)}{D^2}\left(\hat{u}_2^b - u_2^r\right) - \frac{\hat{P}f(\hat{P},\alpha)}{D^2}\hat{u}_2^{b\prime} + \frac{\hat{P}f(\hat{P},\alpha)}{D}\left(\hat{u}_2^{b\prime} - u_2^{r\prime}\right)}{U_{cc}}. \tag{3.12}$$

The function shows that the sign of $\frac{dc_1^*}{dD}$ is again indeterminate. The denominator is the same as in (3.8) and is always negative. For the numerator, the first two components indicate the effect on the marginal value of the homeownership option at $\hat{P}$. This effect on savings is more likely to be positive if the marginal probability density is positive. The last component, which is always negative for a risk-averse individual, indicates the marginal utility loss due to additional savings. Whether or not the net effect is positive depends on the curvature of the risk preference and the price distribution. More risk-averse individuals would be less likely to save more when the downpayment rate goes up. They are more concerned about the possible waste in savings incurred by the shrinking probability of entering homeownership. Similarly, lowering the downpayment rate does not necessarily remove the incentive to save if the individual finds that the expected option value outweighs the marginal cost. The total expected utility would fall no matter which behavior is adopted. Analysis of the change in income also points to an implication of indeterminacy.

3.5.3 Other Uncertainties

One way to extend the model to make it somewhat more realistic is to consider the relationship between the owner-occupation cost and rent. So far, rent is assumed to be constant. In the short run, this might be acceptable but not if we want to use a life-cycle framework to interpret the model. Income is the other variable that may experience uncertainty. The logic of analysis is not different from the case of rent uncertainty. These are highly connected since, in the model, only consumption is accounted for in the utility function, and an increase in future income is equivalent to a decrease in future rent.

There are several conceivable arrangements for the modification of the model. The one that appears simplest is to assume a one-to-one monotonic relationship between two prices in each period: we can write $R_2 = R(P)$ while P follows the same distribution $f(P, \alpha)$. In this case, rent is always less (or higher) than the owner-occupation cost, and there is no implicit upper-limit for $\hat{P}$. The randomness of future price now affects consumption not only in the owner-occupation state but also in the renting state, in the same direction. It is not difficult to follow the steps used previously to solve for the new equilibrium. For a risk-averse individual, the optimal $\hat{P}$ would be higher than before, in so far as rent is proportional to owner-occupation cost. One advantage to this method is that we should be able to obtain this relationship locally from the empirical evidence. A more general (and more theoretical) arrangement is to set P and R to follow a bivariate distribution. The two variables are persistent if their covariance is high, but their association is not likely to be mechanical (Wang et al., 2018). In the real world, divergence in trends occurs due to speculation movements (Shiller, 2007) or exogenous shocks outside the housing market. For instance, a rent control policy or higher property tax may undo the proportional relationship between rent and owner-occupation cost.

No matter which uncertainty is introduced to the model, we can use the same method to analyze the comparative statics of increasing risk on the individual's saving and consumption behavior. Multivariate distributions are also allowed in the model, though unsurprisingly, the result is likely to have a higher level of indeterminacy.

3.5.4 More Decisions

Another possible extension of this model is to provide the individual with an additional choice set. One possible option is to endogenize housing quality or size. However, this requires a full-scale reconstruction of the utility function, which is beyond the scope of this paper. Also, as mentioned at the outset, this analysis does not consider housing capital gains. As stated by Banks et al. (2016), an owner-occupied home is also an investment against housing cost volatility in the future life cycle. We omit this due to the nature of the two-period model. To include the influence of tenure, we would need a dynamic model framework with more than two periods and a heterogeneous market. However, it can be expected that the marginal return on savings would be higher if housing capital gain is taken into consideration.

Another possible extension is to endogenize housing-related life decisions whose utility can be assumed as an additive to the current system. Marriage and childbearing are two sound candidates for inclusion in the model. In the case of childbearing, it is reasonable to assume that the individual is also considering having a fixed number of children. There may be differences in the level of satisfaction of raising children in an owner-occupied home versus a rental home. This modification adds a new discrete choice dimension. The advantages in raising children would become part of the marginal benefit of homeownership besides the possible lower cost of that housing choice. Marriage decision can be included in the model in the same way. Such questions require an analysis like that presented here, and it is possible to introduce additional uncertainty for each newly added dimension.

Appendix 3.A The Notations

For all notations with a time subscript, when none is shown, they are assumed to be constant over time.

a_t	individual's wealth at the beginning of t, where a_1 is equivalent to the wealth endowment
$c_t \in \mathbb{R}^+$	the consumption of a composite of all other goods
D	effective downpayment rate of home purchase such that $D = \frac{\tilde{D}}{r}$
$\tilde{D}$	the market downpayment rate
$F(P, \alpha)$	a family of cumulative distribution functions of P that is ordered by the shift parameter α
$h_t \in \{0, 1\}$	the indicator of homeownership in t
P_t	the cost of owner-occupation in t, equivalent to the interest-only mortgage payment, $P = rV$.
R_t	the cost of rental housing in t
r	general interest rate for assets, assumed constant over time
s_t	the amount of saving in t, $s_t = \frac{a_{t+1}}{1+r}$
$t \in \{1, 2\}$	two periods of the life cycle
$\tilde{t}$	the period in which the individual enters homeownership
$u_t = u(c_t)$	individual's utility function
V_t	house market price in t
Y_t	individual income in t

α	the shift parameter of a distribution function, as an index of the probability spread
β	individual time preference
λ	the shadow price of increasing downpayment affordability
ψ	the shadow price of the Period 2 consumption conditioned to the renting state

Appendix 3.B A Short Note on Risk Aversion and Prudence

This technical note is intended to collect for comparison some of the main properties of risk aversion and prudence, to supplement the discussion in the paper. In a nutshell, risk aversion and prudence are closely related but not equivalent. The nuances between the two have critical implications for savings behavior.

Risk aversion is a characteristic of human behavior that describes people's inclination to avoid uncertainty. Intuitively, a person is considered risk-averse if they prefer a sure outcome to gambling with the same expected value. A risk-averse individual is willing to pay a premium to circumvent the possibility of loss: in financial terms, to hedge against the risk. Mathematically, we say a von Neumann–Morgenstern utility function in relation to wealth $u(w)$ indicates risk aversion if $u(w_0) > pu(w_0 - h) + (1-p)u(w_0 + h)$ for any positive h and $p \in (0, 1)$; or, more generally speaking, if it shows the decision maker's aversion to mean-preserving spreads. If the utility function is twice differentiable, it presents risk aversion if and only if it is a concave function ($u'' < 0$), and the concavity of the function determines the level of that risk aversion. Since the pioneering work of Kenneth Arrow and

John Pratt in the 1960s, the concept of risk aversion has been well studied and documented.

The concept of prudence was formally introduced by Miles Kimball in his work in 1990 to explain precautionary saving and has since attracted much attention, especially in the fields of decision theory and finance. Though the notion of risk aversion well explains hedging and insurance against typical risks, some risks cannot be hedged or insured against. An inevitable zero-mean random return or expense is such a risk. Isolating from risk aversion, a prudent individual prefers to hold more wealth (savings) than usual to alleviate the harm of the potential downside risk.[14] Intuitively and in brief, prudence is defined as the aversion to downside risk (Crainich et al., 2013; Menezes et al., 1980). Mathematically, a utility function has the prudence trait if $u'(w_0 + E[\varepsilon]) = u'(w_0) < E[u'(w_0 + \varepsilon)]$ for any zero-mean, non-degenerate, and independent random variable ε. If the utility function is thrice differentiable, this trait is equivalent to the condition of having a convex marginal utility function, or $u''' > 0$.

The instinctive feeling of the equivalence of risk aversion and prudence is flawed. Indeed, most imaginable risk-averse utility functions are prudent. Utility functions that are decreasing or constant-absolute risk-averse (DARA or CARA) are necessarily prudent.[15] However, it is not impossible that an individual with the increasing absolute risk aversion trait does not perform prudently ($u' > 0, u'' < 0, u''' \leq 0$). Likewise, a prudent individual is not necessarily risk-averse. Literature has shown that prudent risk lovers ($u' > 0, u'' > 0, u''' > 0$) are neither peculiar nor against intuition (Crainich et al., 2013;

[14] A simple framework of lottery pairs can clearly demonstrate how risk aversion and prudence respond to different risks. See, Eeckhoudt and Schlesinger (2006); Schlesinger (2015); Baiardi et al. (2019) for details.

[15] The proof is trivial as the functions satisfy $\frac{\partial A(w)}{\partial w} = -\frac{u'u''' - (u'')^2}{(u')^2} \leq 0$ where $A(w)$ denotes the absolute risk aversion measure. For more discussion about the relationship between DARA and prudence, see Kimball (1993).

Deck and Schlesinger, 2014). A strong risk lover, facing an insurable risk of loss and an uninsurable zero-mean risk of return, which will realize simultaneously in the coming future, would choose to apportion the risks separately to cling to the largest possible prize namely the no-loss state with positive returns.

Appendix 3.C Alternative Constrained Optimization under Uncertainty

This section uses the system of implicit functions to addresses the question of how individuals in the optimization problem in Section 3.4 would react to changes in riskiness. In (3.4), c_2^b is not the true control variable for the individual because it is realized only when P is realized. We thus rewrite the problem and replace c_2^b by the first constraint and set ψ and λ to be the two Lagrangian multipliers of the remaining two constraints. Then, we take the first-order conditions of the problem with respect to the three control variables and the two multipliers as a system of equations. To keep the derivation succinct, we abbreviate $f(P,\alpha)$ to f and $f(\hat{P},\alpha)$ to $\hat{f}$. Likewise, $F(P,\alpha)$ to F and $f_P(P,\alpha)$ and $f_\alpha(P,\alpha)$ to f_P and f_α. We have:

$$\begin{aligned}
c_1: &\quad u_1' - \int_0^{\hat{P}} u_2^{b\prime} f dP - (1+r)\psi - (1+r)\lambda = 0 \\
c_2^r: &\quad \beta \int_{\hat{P}}^{\infty} u_2^{r\prime} f dP - \psi = \beta u_2^{r\prime}\left(1-\hat{F}\right) - \psi = 0 \\
\hat{P}: &\quad \beta \hat{u}_2^b \hat{f} - \beta u_2^r \hat{f} - D\lambda = 0 \\
\psi: &\quad Y - R + (1+r)(Y-R-c_1) - c_2^r = 0 \\
\lambda: &\quad (1+r)(Y-R-c_1) - D\hat{P} = 0.
\end{aligned} \tag{3.13}$$

In (3.13), $\psi = \beta u_2^{r\prime}\left(1-\hat{F}\right)$ represents the shadow price of consumption in the renting state. $\lambda = \frac{\beta \hat{f}}{D}(\hat{u}_2^b - u_2^r)$ represents the shadow price of the downpayment reserve $D\hat{P}$. $(1+r)\lambda$ is exactly the first component of the right-hand side in (3.5). The shadow price of c_2^b is a random variable and cannot be directly calculated. Nevertheless, we can obtain the value by subtracting the shadow prices of the presented values of c_2^r and $D\hat{P}$ from the shadow price of c_1.[16]

The linearization of the system considering changes in the shift parameter α is

$$\left(u_1'' + \int_0^{\hat{P}} u_2^{b\prime\prime}(1+r)f dP\right) dc_1 - \hat{u}_2^{b\prime}\hat{f}d\hat{P} - (1+r)d\psi - (1+r)d\lambda - \int_0^{\hat{P}} u_2^{b\prime} f_\alpha dP d\alpha = 0$$

$$\beta u_2^{r\prime\prime}\left(1-\hat{F}\right)dc_2^r - \beta u_2^{r\prime}\hat{f}d\hat{P} - d\psi - \beta u_2^{r\prime}\hat{F}_\alpha d\alpha = 0$$

$$-\hat{u}_2^{b\prime}\hat{f}dc_1 - \beta u_2^{r\prime}\hat{f}dc_2^r + \beta\left(-\hat{u}_2^{b\prime}\hat{f} + \hat{u}_2^b\hat{f}_P - u_2^r\hat{f}_P\right)d\hat{P} - Dd\lambda + \left(\beta\hat{u}_2^b\hat{f}_\alpha - \beta u_2^r\hat{f}_\alpha\right)d\alpha = 0$$

$$-(1+r)dc_1 - dc_2^r = 0$$

$$-(1+r)dc_1 - Dd\hat{P} = 0.$$

[16]One can also obtain the comparative static properties in terms of elasticities by log-differentiating rather than differentiating the Lagrangian.

Rearranging the equation yields

$$
\begin{pmatrix}
a_{11} & 0 & -\hat{u}_2^{b\prime}\hat{f} & -(1+r) & -(1+r) \\
0 & \beta u_2^{r\prime\prime}\left(1-\hat{F}\right) & -\beta u_2^{r\prime}\hat{f} & -1 & 0 \\
-\hat{u}_2^{b\prime}\hat{f} & -\beta u_2^{r\prime}\hat{f} & a_{33} & 0 & -D \\
-(1+r) & -1 & 0 & 0 & 0 \\
-(1+r) & 0 & -D & 0 & 0
\end{pmatrix}
\begin{pmatrix}
dc_1 \\ dc_2^r \\ d\hat{P} \\ d\psi \\ d\lambda
\end{pmatrix}
=
\begin{pmatrix}
\int_0^{\hat{P}} u_2^{b\prime} f_\alpha dP d\alpha \\
\beta u_2^{r\prime}\hat{F}_\alpha d\alpha \\
-\left(\beta\hat{u}_2^b\hat{f}_\alpha - \beta u_2^r\hat{f}_\alpha\right) d\alpha \\
0 \\
0
\end{pmatrix}
\tag{3.14}
$$

where $a_{11} = u_1'' + \int_0^{\hat{P}} u_2^{b\prime\prime}(1+r)f dP$ and $a_{33} = \beta\left(-\hat{u}_2^{b\prime}\hat{f} + \hat{u}_2^b\hat{f}_P - u_2^r\hat{f}_P\right)$.

By Cramer's rule and the implicit function theorem, we can solve for the marginal change of all the control variables against a marginal variation in α. For instance, using (3.14) to solve for dc_1 we get an equation that is the same as (3.8). Similarly, we can solve for $d\hat{P}$ and find (3.10). With this technique, we can derive the comparative statics for any exogenous changes such as in rent and income.

Conclusions

In modern society, childbearing is associated with other major family life events through both the household's preferences and its finance. Homeownership transition is not independent of these connections. The mortgage downpayment requirement, typically accompanying a home purchase, creates a specific credit constraint limiting families from reaching their ideal life-course plan. This dissertation investigates the relationship between the housing market conditions and the family decisions on home purchase and childbearing and theorizes the problem as a dynamic optimization problem. The first chapter identifies the impacts of local house prices and price appreciations on the timing of home buying and childbearing. It finds a weak temporary substitution between homeownership and children for American urban non-homeowning families. The second chapter formulates the relationship in a deterministic life-cycle consumption model and explores the decision rules under a dynamic framework. It presents how a family makes a trade-off between the timings of home purchase and childbearing by balancing their complementary feature in the social and psychological aspect and the substitutional one in the financial aspect. The third, and a rather independent, chapter studies the household home purchase behavior in a housing market with uncertainty. It explains how personal preferences toward risks determine the impact of growing housing market uncertainties on an individual's consumption and home buying decision.

This study contributes to the understanding of the joint dynamics of household home buying and childbearing events. The empirical investigation is the first one that examines the two events simultaneously and finds evidence of the interrelation between house prices, home purchase, and childbearing. The application of the multinomial discrete-choice model on micro-level panel data highlights the importance of the household decision on the sequencing and timing of home purchase and childbearing. Although it is a rather qualitative study, as the choice is reduced to four, it reveals how households respond to the local housing costs by adjusting their homeownership transition and family formation time. Secondly, this study constructs a dynamic model of home buying and childbearing decisions. There is a long literature about the dynamics of housing and childbearing separately, but a study on the dynamics of the two together is new. The theoretical model studies the two decisions as a joint decision in an optimization problem, and the analysis of the model demonstrates why a downpayment requirement in the housing market can affect a household's family plan not only in their housing tenure decision but also in other domains. The credit constraint is attributable to the obstruction of the ideal life-course plan. This is by far the first economic model explicitly decipher the impact of a credit constraint on two major family decisions. The last chapter is a preliminary study on the introduction of stochastic processes into the joint-decision dynamic model. The examination paves the way for constructing a more general model for multiple interdependent decisions with stochastic processes.

Of course, the contribution does not come without compromises. Due to the data limitations and the nature of the econometric method, the interpretation of the empirical results should not be over-extended. The PSID data is rich in the information of family finance and individual demographic history. However, even this most comprehensive database does not have all the detailed history of individual housing-related information. The timing of homeownership transitions, the purchased prices, the acquired downpayment-to-value ratio, and other financial data are not included, probably due to privacy concerns. The scope of the empirical analysis is therefore limited to the effects of the price level and price appreciation only, instead of the direct impacts of credit constraint. Besides, the price of the detailed personal information is the sample size of the data. The small size of the available sample confines the application of fixed-effect controls. City amenities, family welfare programs, mortgage regulations; these non-random city-wise features are part of the disturbance term. They might contribute to a bias if these features correlate with the family decisions. The sample size also limits the available toolbox in sampling and estimation methods, especially about migration decisions.

Concerning the theory side, the incorporation of uncertainty into the dynamic model should be the first thing for the next-step model development. The growing risk of infertility would give tine pressures on families who plan to have more children. The uncertainty in the house prices and the long-term price growth trend has a unique influence on family willingness to save and buy an owner-occupying housing property. Homeownership under market price uncertainty provides insurance to a housing cost soaring and an investment for growing equity. The two types of risks generate opposite forces in determining the optimal housing and fertility strategies. Chapter 3 summarizes the necessary conditions of the utility functions that have the same reactions to risks in an intertemporal consumption model. The conditions indicate what forms of the utility representations can replace the logarithm utility function in the model without changing the key implications on precautionary savings.

Besides, in Chapter 2, the model assumes that the decision-maker does not consider the quality or quantity of the housing service, nor that of children. Adding any of the degrees of freedom would exponentially increase the complexity of the model, but it is worth doing. For example, the behavioral mechanism for determining when to have the only child could be completely different from determining whether and when to have multiple children. Such an analysis could possibly shed light on the house price effect on the complete fertility rate in a joint-decision framework. How to formulate the differential costs and utilities for children in different orders remains an unsolved issue for the extension.

Notwithstanding these difficulties, the pioneering work has laid the foundation for more researches on the interdependence of housing and fertility decisions and the economic impact on family life-cycle plans. The impact of housing costs on family life and fertility rates remains a big issue today. Whether and how growing living costs deteriorate average living standard and worsen the long-term population aging is associated with the benefit of housing-related public policies. More evidence and analysis will help build a stronger argument for (or against) regulating housing costs and relaxing the credit constraint for first-time home buyers. It will also help understand the reasons for the long-term trend for fertility decline in industrialized society. In the next step of the empirical study, if there is no better individual panel data, we should direct our focus to the US census. City-cohort-level estimations are a possible way to examine the house price effects at a real nationwide scale. The problem of lacking timing information can be circumvented by using the average occurrence through pooling data. It may also generate a larger sample size that will allow the application of nested-type discrete-choice models witch consider a more general choice set, including the decision of moving or migration. The development of the theories, in addition to the abovementioned directions, should also extend to available function-form estimations. Though it will be restricted by the available data, a bridge between the empirical and theoretical study is necessary for further understanding of the household choices in the marginalist eyes. Numerical simulation is likely to be the only solution for a more complicated model, and the quantitative results will be needed for the analysis calibration.

Above all these, we should keep in mind that the relationship between home purchase and childbearing decisions is by no means the only interrelation of family events. Marriage, albeit fading in its necessity in recent years, stands a pivotal position in the decision of housing and childbearing in most circumstances. This study implicitly assumes the formation of a family union has been complete and the new household established when they start considering housing and childbearing. But, as we regard the individual rational behavior on the timing of home buying and childbearing, the timing of getting married may be equally important. It is particularly critical in some Asian countries where births out of wedlock are related to social dishonor, and the timing of marriage and childbearing are closed correlated. Marriage is also one reason for moving to a new place and a reason for buying a house. An unmarried couple is well like to put the three events on the same table in their life-course planning. It will be worth looking for more empirical evidence on the interactions of the three family events. However, any theoretical attempt of dynamic analysis on this multiple-state decision will be inherently challenging and probably mathematically perplexing. After all, like all family stories themselves, the truth is rarely pure and never simple.

Bibliography

Aaronson D (2000) A note on the benefits of homeownership. Journal of Urban Economics 47(3):356–369

Abadie A, Athey S, Imbens GW, Wooldridge J (2017) When should you adjust standard errors for clustering? Tech. rep., National Bureau of Economic Research

Amemiya T (1981) Qualitative response models: A survey. Journal of Economic Literature 19(4):1483–1536

Arnott R, Davidson R, Pines D (1983) Housing quality, maintenance and rehabilitation. The Review of Economic Studies 50(3):467–494

Baiardi D, Magnani M, Menegatti M (2019) The theory of precautionary saving: An overview of recent developments. Review of Economics of the Household 18(2):513–542

Banks J, Blundell R, Oldfield Z, Smith JP (2004) House price volatility and housing ownership over the lifecycle. Discussion papers in economics (04-09), Department of Economics, University College London, London, UK

Banks J, Blundell R, Oldfield Z, Smith JP (2016) House price volatility and the housing ladder. In: Insights in the Economics of Aging, University of Chicago Press, pp 87–119

Becker G (1960) An economic analysis of fertility. In: Demographic and Economic Change in Developed Countries, National Bureau of Economic Research, Inc, pp 209–240, URL https://EconPapers.repec.org/RePEc:nbr:nberch:2387

Becker GS (1965) A theory of the allocation of time. The Economic Journal 75(299):493–517

Becker GS, Barro RJ (1986) Altruism and the economic theory of fertility. Population and Development Review 12:69–76

Becker GS, Lewis HG (1973) On the interaction between the quantity and quality of children. Journal of Political Economy 81(2, Part 2):S279–S288

Ben-Akiva ME, Lerman SR (1985) Discrete choice analysis: theory and application to travel demand, vol 9. MIT Press

Bernanke BS, Gertler M (1995) Inside the black box: The credit channel of monetary policy transmission. Journal of Economic Perspectives 9(4):27–48

Black DA, Kolesnikova N, Sanders SG, Taylor LJ (2013) Are children "normal"? The Review of Economics and Statistics 95(1):21–33

Buis ML (2010) Stata tip 87: Interpretation of interactions in nonlinear models. The Stata Journal 10(2):305–308

Clark J, Ferrer A (2019) The effect of house prices on fertility: Evidence from canada. Economics: The Open-Access, Open-Assessment E-Journal 13(2019-38):1–32

Clark WA (2012) Do women delay family formation in expensive housing markets? Demographic Research 27(1):1

Clark WA (2013) Life course events and residential change: Unpacking age effects on the probability of moving. Journal of Population Research 30(4):319–334

Clark WA, Lisowski W (2018) Examining the life course sequence of intending to move and moving. Population, Space and Place 24(3):e2100

Clark WA, Onaka JL (1983) Life cycle and housing adjustment as explanations of residential mobility. Urban Studies 20(1):47–57

Clark WA, Withers SD (2009) Fertility, mobility and labour-force participation: A study of synchronicity. Population, Space and Place 15(4):305–321

Clark WA, Yi D, Zhang X (2020) Do house prices affect fertility behavior in china? an empirical examination. International Regional Science Review 43(5):423–449

Cloyne J, Huber K, Ilzetzki E, Kleven H (2019) The effect of house prices on household borrowing: A new approach. American Economic Review 109(6):2104–36

Courgeau D, Lelièvre E (1992) Interrelations between first home-ownership, constitution of the family, and professional occupation in france. Demographic applications of event history analysis pp 120–140

Crainich D, Eeckhoudt L, Trannoy A (2013) Even (mixed) risk lovers are prudent. American Economic Review 103(4):1529–1535

Daysal NM, Lovenheim M, Siersbæk N, Wasser DN (2021) Home prices, fertility, and early-life health outcomes. Journal of Public Economics 198:104366

Deck C, Schlesinger H (2014) Consistency of higher order risk preferences. Econometrica 82(5):1913–1943

Dettling LJ, Kearney MS (2014) House prices and birth rates: The impact of the real estate market on the decision to have a baby. Journal of Public Economics 110:82–100

Diamond PA, Stiglitz JE (1974) Increases in risk and in risk aversion. Journal of Economic Theory 8(3):337–360

Dietz RD, Haurin DR (2003) The social and private micro-level consequences of homeownership. Journal of Urban Economics 54(3):401–450

Drew RB (2015) Effect of changing demographics on young adult homeownership rates. Joint Center for Housing Studies Harvard University

Dreze JH, Modigliani F (1975) Consumption decisions under uncertainty. In: Stochastic Optimization Models in Finance, Elsevier, pp 459–486

Eeckhoudt L, Schlesinger H (2006) Putting risk in its proper place. American Economic Review 96(1):280–289

Eeckhoudt L, Gollier C, Schlesinger H (1995) The risk-averse (and prudent) newsboy. Management Science 41(5):786–794

Elsinga M, Hoekstra J (2005) Homeownership and housing satisfaction. Journal of Housing and the Built Environment 20(4):401–424

Enström Öst C (2012) Housing and children: Simultaneous decisions?—a cohort study of young adults' housing and family formation decision. Journal of Population Economics 25(1):349–366

Feyrer J, Sacerdote B, Stern AD (2008) Will the stork return to europe and japan? understanding fertility within developed nations. Journal of Economic Perspectives 22(3):3–22

Fu Y (1995) Uncertainty, liquidity, and housing choices. Regional Science and Urban Economics 25(2):223–236

Greene WH (2012) Econometric analysis, 7th edn. Prentice Hall

Han L (2008) Hedging house price risk in the presence of lumpy transaction costs. Journal of Urban Economics 64(2):270–287

Han L (2010) The effects of price risk on housing demand: empirical evidence from us markets. The Review of Financial Studies 23(11):3889–3928

Haurin DR, Parcel TL, Haurin RJ (2002) Does homeownership affect child outcomes? Real Estate Economics 30(4):635–666

Henderson JV, Ioannides YM (1983) A model of housing tenure choice. The American Economic Review 73(1):98–113

Henretta JC (1987) Family transitions, housing market context, and first home purchase by young married households. Social Forces 66(2):520–536

Hirshleifer J, Riley JG (1992) The Analytics of Uncertainty and Information. Cambridge University Press

Hotz VJ, Klerman JA, Willis RJ (1997) The economics of fertility in developed countries. Handbook of population and family economics 1(Part A):275–347

Imbens G, Wooldridge J (2007) Discrete choice models. NBER Lecture Notes 11

Kamien MI, Schwartz NL (2012) Dynamic optimization: the calculus of variations and optimal control in economics and management. Courier Corporation

Keane MP, Todd PE, Wolpin KI (2011) The structural estimation of behavioral models: Discrete choice dynamic programming methods and applications. In: Handbook of labor economics, vol 4, Elsevier, pp 331–461

Kimball MS (1990) Precautionary saving in the small and in the large. Econometrica 58(1):53–73

Kimball MS (1993) Standard risk aversion. Econometrica 61(3):589–611

Kulu H, Steele F (2013) Interrelationships between childbearing and housing transitions in the family life course. Demography 50(5):1687–1714

Kulu H, Vikat A (2007) Fertility differences by housing type: The effect of housing conditions or of selective moves? Demographic Research 17:775–802

Lennartz C, Arundel R, Ronald R (2016) Younger adults and homeownership in europe through the global financial crisis. Population, Space and Place 22(8):823–835

Lesthaeghe R (2010) The unfolding story of the second demographic transition. Population and Development Review 36(2):211–251

Li MM (1977) A logit model of homeownership. Econometrica: Journal of the Econometric Society 45(5):1081–1097

Li W, Yao R (2007) The life-cycle effects of house price changes. Journal of Money, Credit and banking 39(6):1375–1409

Lo KT (2012) The crowding-out effect of homeownership on fertility. Journal of Family and Economic Issues 33(1):108–117

Lovenheim MF, Mumford KJ (2013) Do family wealth shocks affect fertility choices? evidence from the housing market. Review of Economics and Statistics 95(2):464–475

Lutz W, Skirbekk V (2005) Policies addressing the tempo effect in low-fertility countries. Population and Development Review 31(4):699–720

Lutz W, Skirbekk V, Testa MR (2006) The low-fertility trap hypothesis: Forces that may lead to further postponement and fewer births in europe. Vienna Yearbook of Population Research 4:167–192

Menezes C, Geiss C, Tressler J (1980) Increasing downside risk. The American Economic Review 70(5):921–932

Mulder CH (2006a) Home-ownership and family formation. Journal of Housing and the Built Environment 21(3):281–298

Mulder CH (2006b) Population and housing: A two-sided relationship. Demographic Research 15:401–412

Mulder CH, Billari FC (2010) Homeownership regimes and low fertility. Housing Studies 25(4):527–541

Mulder CH, Wagner M (2001) The connections between family formation and first-time home ownership in the context of west germany and the netherlands. European Journal of Population/Revue europeenne de demographie 17(2):137–164

Myers D, Lee H, Simmons PA (2020) Cohort insights into recovery of millennial homeownership after the great recession. Journal of Housing Economics 47:101619

Norton EC, Wang H, Ai C (2004) Computing interaction effects and standard errors in logit and probit models. The Stata Journal 4(2):154–167

Ortalo-Magne F, Rady S (2006) Housing market dynamics: On the contribution of income shocks and credit constraints. The Review of Economic Studies 73(2):459–485

Powell KK (1993) Essays on the borrowing constraints in the presence of positive equity requirements.

Rothschild M, Stiglitz JE (1970) Increasing risk: I. a definition. Journal of Economic Theory 2(3):225–243

Sato Y, Yamamoto K (2005) Population concentration, urbanization, and demographic transition. Journal of Urban Economics 58(1):45–61

Schlesinger H (2015) Lattices and lotteries in apportioning risk. The Geneva Risk and Insurance Review 40(1):1–14

Shiller RJ (2007) Understanding recent trends in house prices and home ownership. NBER Working Paper No.13553, National Bureau of Economic Research, DOI 10.3386/w13553, URL http://www.nber.org/papers/w13553

Simon CJ, Tamura R (2009) Do higher rents discourage fertility? evidence from us cities, 1940–2000. Regional Science and Urban Economics 39(1):33–42

Sinai T, Souleles NS (2005) Owner-occupied housing as a hedge against rent risk. The Quarterly Journal of Economics 120(2):763–789

Small KA, Verhoef ET, Lindsey R (2007) The economics of urban transportation. Routledge

Sobotka T, Skirbekk V, Philipov D (2011) Economic recession and fertility in the developed world. Population and Development Review 37(2):267–306

Train KE (2009) Discrete choice methods with simulation. Cambridge University P ress

Turner TM (2003) Does investment risk affect the housing decisions of families? Economic Inquiry 41(4):675–691

Turner TM, Seo D (2007) Investment risk and the transition into homeownership. Journal of Regional Science 47(2):229–253

Walker JL, Ben-Akiva M (2011) Advances in discrete choice: mixture models. In: de Palma A, Lindsey R, Quinet E, Vickerman R (eds) Handbook in transport economics, vol 160-187, Edward Elgar

Wang H, Yu F, Zhou Y (2018) Property investment and rental rate under housing price uncertainty: A real options approach. Real Estate Economics 48(2):633–665

Wei SJ, Zhang X, Liu Y (2012) Status competition and housing prices. Tech. rep., National Bureau of Economic Research

Yi J, Zhang J (2010) The effect of house price on fertility: Evidence from hong kong. Economic Inquiry 48(3):635–650