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SU(3) IMPLICATIONS OF A MEASUREMENT OF THE RELATIVE SIGNS OF COUPLING  
CONSTANTS FOR RESONANT AMPLITUDES

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SU(3) IMPLICATIONS OF A MEASUREMENT OF THE  
RELATIVE SIGNS OF COUPLING CONSTANTS  
FOR RESONANT AMPLITUDES

Anne Kernan and Wesley M. Smart

July 26, 1966

Su(3) Implications of a Measurement of the  
Relative Signs of Coupling Constants  
for Resonant Amplitudes\*

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July 26, 1966

A measurement of the relative signs of the product of the coupling constants,  $g_{N\bar{K}Y}^* g_{\Lambda\pi Y}^*$ , for  $Y_1^*(1660)$ ,  $Y_1^*(1765)$ ,  $Y_1^*(1915)$ , and  $Y_1^*(2030)$  is used to assign these resonances to SU(3) multiplets and to place limits on the value of  $\alpha$  (the D-F mixing parameter) for the resonances assigned to octets.

In a partial-wave analysis of the reaction  $K^- + n \rightarrow \Lambda + \pi^-$  in the c. m. energy interval 1660 to 1900 MeV, we measured the relative phases of the resonant amplitudes  $Y_1^{*-}(1660)$ ,  $Y_1^{*-}(1765)$ ,  $Y_1^{*-}(1915)$  and  $Y_1^{*-}(2030)$ .<sup>1</sup> In the elastic channel  $K^- + n \rightarrow K^- + n$  the relative phase of two resonant amplitudes, taken at the resonant energy  $E_R$ , is always zero because the resonant amplitude  $T_R$  is proportional to  $g_{NK\bar{Y}}^2 / (E_R - E - i\Gamma/2)$ . In the inelastic channel  $\Lambda\pi$  the amplitude varies as  $g_{NK\bar{Y}}^* g_{\Lambda\pi Y}^* / (E_R - E - i\Gamma/2)$ , and the relative phase of two resonant amplitudes may be 0 or 180 deg depending on the relative sign of  $g_{NK\bar{Y}}^* g_{\Lambda\pi Y}^*$  for the two  $Y^*$  states.

The analysis in reference 1 showed that  $Y_1^{*}(1765)$  and  $Y_1^{*}(1915)$  were in phase at energy  $E_R$  and 180 deg out of phase with  $Y_1^{*}(1660)$  and  $Y_1^{*}(2030)$ . This implies that  $g_{NK\bar{Y}}^* g_{\Lambda\pi Y}^*$  is of one sign ( $\pm$ ) for  $Y_1^{*}(1765)$  and  $Y_1^{*}(1915)$  and of opposite sign ( $\mp$ ) for  $Y_1^{*}(1660)$  and  $Y_1^{*}(2030)$ . The ambiguity in sign arises because the experiment does not measure the phase relationship of the resonant amplitude in the  $\Lambda\pi$  and  $N\bar{K}$  channels.

In this paper we show that knowledge of the relative sign of the coupling constants is a powerful aid in assigning particles to SU(3) multiplets.

$Y_1^*$  resonant states have hypercharge  $Y = 0$  and isotopic spin  $I = 1$ . Table I shows the Clebsch-Gordan coefficients of SU(3) for the decay of a  $Y_1^{*-}$  into a baryon B and a meson M, both members of octets.<sup>2</sup> We consider the case that the baryon is a member of the  $J^P = 1/2^+$  ( $N\Lambda\Sigma\Xi$ ) octet, and the meson is a member of the pseudoscalar ( $K\eta\pi K$ ) octet.

In the limit of unitary symmetry the coupling of a member of a multiplet  $\{\mu\}$  to  $\{8\} \otimes \{8\}$  is described by a single invariant coupling constant  $g_\mu$ . Then the coupling constant  $g_{\text{BMY}}^*$  is  $g_\mu$  times the Clebsch-Gordan coefficient for the transition  $Y^* \rightarrow B + M$  for  $Y^*$  a member of a  $\{27\}$ ,  $\{10\}$ , or  $\{10^*\}$ . The situation is more complicated when  $Y^*$  is a member of an octet. Because  $\{8\} \otimes \{8\}$  contains the  $\{8_1\}$  and  $\{8_2\}$  representation, there are two coupling constants for this process,  $g_1$  and  $g_2$ . The coupling of an octet to  $\{8\} \otimes \{8\}$  is described in the notation of de Swart<sup>3</sup> by two parameters  $g_8$  and  $\alpha$ , where  $g_8 = (\sqrt{30}/40) g_1 + (\sqrt{6}/24) g_2$  and  $\alpha = (\sqrt{6}/24) g_2/g_8$ . The coefficient, which when multiplied into  $g_8$  gives the coupling constant  $g_{\text{BMY}}^*$  for  $\{8\} \otimes \{8\}$  is shown in the last column of Table I. The quantity  $g_{\text{NKY}}^* g_{\text{A}\pi\text{Y}}^*$  is simply the product of the first and fifth row in Table I times  $g_\mu$ , and is shown in Table II.

Using Table II we make some observations on SU(3) assignments for  $Y_1^*(1660)$ ,  $Y_1^*(1765)$ ,  $Y_1^*(1915)$ , and  $Y_1^*(2030)$ .

The  $Y_1^*(2030)$  has  $J^P = 7/2^+$ , and it has been suggested that this particle along with  $N_{3/2}^*(1920)$  belongs to a  $7/2^+$   $\{10\}$  multiplet, which is the Regge recurrence of the  $3/2^+$   $\delta$  decuplet.<sup>4</sup>

In order to make an SU(3) assignment for  $Y_1^*(1765)$ , we assume that  $Y_1^*(2030)$  is a member of a  $\{10\}$  representation. Table II shows that  $g_{\text{NKY}}^* g_{\text{A}\pi\text{Y}}^*$  is positive for a  $\{10\}$ . Since  $g_{\text{NKY}}^* g_{\text{A}\pi\text{Y}}^*$  has opposite sign for  $Y_1^*(2030)$  and  $Y_1^*(1765)$  by reference 1, the  $\{27\}$  and  $\{10\}$  assignments are ruled out for  $Y_1^*(1765)$ , as is  $\{8\}$  with  $1/2 < \alpha < 1$ . The only possible assignment for  $Y_1^*(1765)$  is  $\{10^*\}$  or  $\{8\}$  with  $\alpha < 1/2$  or  $\alpha > 1$ , if  $Y_1^*(2030)$  is a member of a  $\{10\}$ . A measurement of the relative phase of  $Y_1^*(1765)$  and

$Y_1^*(2030)$  in the  $\Sigma\eta$  channel would resolve this ambiguity. Unfortunately the branching ratio for  $Y_1^*(1765)$  decaying to  $\Sigma\eta$  is probably very small because of limited phase space, since the  $Q$  value for the decay is only 30 MeV. A measurement of the relative sign of  $g_{NKY^*}$   $g_{\Sigma\pi Y^*}$  for  $Y_1^*(1765)$  and  $Y_1^*(2030)$  could restrict further the value of  $\alpha$ , and possibly rule out  $\{10^*\}$ . A  $\{10^*\}$  multiplet would contain a  $Y = 2, I = 0$  resonance; evidence for this state has recently been reported.<sup>5</sup> A recent study of the branching ratios of  $Y_1^*(1765)$  favors the octet assignment with  $\alpha = -1.5^{+0.7}_{-1.1}$  or  $-0.5^{+0.2}_{-0.3}$ , depending on the energy-dependent form assumed for the resonant width  $\Gamma$ .<sup>6</sup> This is consistent with our limits on  $\alpha$ .

Since  $g_{NKY^*}$   $g_{\Lambda\pi Y^*}$  has the same sign for  $Y_1^*(1765)$  and  $Y_1^*(1915)$ , the same possible assignments are indicated for  $Y_1^*(1915)$ .

It has been suggested that  $Y_1^*(1915)$  belongs to a  $5/2^+$  octet along with  $N_{1/2}^*(1688)$ ,  $Y_0^*(1815)$ , and  $\Xi^*(1933)$ ,<sup>7</sup> and this is consistent with the conclusions drawn above. If  $Y_1^*(2030)$  is a member of a  $\{10\}$ , then  $\alpha$  is less than  $1/2$  or greater than  $1$  for the  $5/2^+$  baryon octet. The  $1/2^+$  baryon octet has  $\alpha \approx 1/4$ ,<sup>8,9</sup> so the results are consistent with  $\alpha$  being the same for the  $1/2^+$  and  $5/2^+$  baryon octets.

$Y_1^*(1660)$  is usually assigned the  $3/2^-$   $\gamma$  octet of baryons. Then from Table II the  $3/2^-$   $\gamma$  octet has  $1/2 < \alpha < 1$  if  $Y_1^*(2030)$  is assigned to  $\{10\}$ .

Regardless of the  $Y_1^*(2030)$  assignment, one can still state that  $\alpha$  is different for the  $3/2^-$   $\gamma$  baryon octet and the proposed  $5/2^+$  baryon octet: For one octet  $\alpha$  lies in the range  $1/2 < \alpha < 1$ , and for the other  $\alpha < 1/2$  or  $> 1$ . Cutkosky has discussed the conditions under which  $\alpha$



might be the same for different baryon octets.<sup>9</sup>

The following reservations must be made:

(i) Most of the above conclusions are based on the assumption that  $Y_1^*$  (2030) belongs to a  $\{10\}$  representation. Ideally one should measure the  $Y_1^*$  phases relative to  $Y_1^*$  (1385), which is firmly established as a member of the  $3/2^-$  baryon  $\delta$   $\{10\}$ .

(ii) The experimental measurement of the phase of the  $Y_1^*$  (1660) and  $Y_1^*$  (1915) amplitudes is not conclusive, because these amplitudes are relatively weak in the  $\Lambda\pi$  channel.<sup>1</sup>

We have used the experimental data from reference 1 primarily to illustrate that a measurement of the relative phase of resonant amplitudes in a two-body inelastic reaction can be used to make SU(3) assignments. This method is applicable to the higher spin resonances formed in  $\pi$ -N and K-N scattering, and may prove to be more reliable than assignments made on the basis of measured partial decay widths. The SU(3) predictions of the relative signs of coupling constants involve only one parameter,  $\alpha$ , and this only for octets. On the other hand, SU(3) calculations of partial widths depend upon  $g_\mu$  and kinematical factors, as well as on  $\alpha$ . Inexactness of SU(3) symmetry may cause a splitting in  $g_\mu$ , giving rise to discrepancies between calculated and experimental partial decay rates. For example, a calculation of  $\Gamma_{\Xi\pi}$  for  $\Xi_{1/2}^*$  (1530) (a member of the  $\delta$  decuplet), using as input the current values of  $\Gamma_{\Lambda\pi}$  and  $\Gamma_{\Sigma\pi}$  for  $Y_1^*$  (1385) and  $\Gamma_{N\pi}$  for  $N_{3/2}^*$  (1236),<sup>10</sup> predicted  $\Gamma_{\Xi\pi} = 16$  MeV, compared to the measured value of  $7.5 \pm 1.7$  MeV.

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FOOTNOTES AND REFERENCES

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Table I. SU(3) Clebsch-Gordan coefficients for the decomposition of  $|\mu, Y, I, I_3\rangle = |\mu, 0, 1, -1\rangle$  into  $|8, y, i, i_3\rangle \otimes |8, y', i', i_3'\rangle$ .

	$y \ i \ i_3 ; y' \ i' \ i_3'$		$Y_1^{*-}$						
			$\{\mu\}$	$\{27\}$	$\{10\}$	$\{10^*\}$	$\{8_1\}$	$\{8_2\}$	$\{8\}$
			Y	0	0	0	0	0	0
			I	1	1	1	1	1	1
			$I_3$	-1	-1	-1	-1	-1	-1
$nK^-$	$1 \ \frac{1}{2} \ -\frac{1}{2}$	$-1 \ \frac{1}{2} \ -\frac{1}{2}$		$\sqrt{1/5}$	$-\sqrt{1/6}$	$\sqrt{1/6}$	$-\sqrt{3/10}$	$\sqrt{1/6}$	$-\sqrt{16(1-2a)}$
$\Sigma^0 \pi^-$	$0 \ 1 \ 0$	$0 \ 1 \ -1$		0	$\sqrt{1/12}$	$-\sqrt{1/12}$	0	$\sqrt{1/3}$	$\sqrt{32} a$
$\Sigma^- \pi^0$	$0 \ 1 \ -1$	$0 \ 1 \ 0$		0	$-\sqrt{1/12}$	$\sqrt{1/12}$	0	$-\sqrt{1/3}$	$-\sqrt{32} a$
$\Sigma^- \eta$	$0 \ 1 \ -1$	$0 \ 0 \ 0$		$\sqrt{3/10}$	$\sqrt{1/4}$	$\sqrt{1/4}$	$\sqrt{1/5}$	0	$\sqrt{32/3(1-a)}$
$\Lambda \pi^-$	$0 \ 0 \ 0$	$0 \ 1 \ -1$		$\sqrt{3/10}$	$-\sqrt{1/4}$	$-\sqrt{1/4}$	$\sqrt{1/5}$	0	$\sqrt{32/3(1-a)}$
$\Xi^- K^0$	$-1 \ \frac{1}{2} \ -\frac{1}{2}$	$1 \ \frac{1}{2} \ -\frac{1}{2}$		$\sqrt{1/5}$	$\sqrt{1/6}$	$-\sqrt{1/6}$	$-\sqrt{3/10}$	$-\sqrt{1/6}$	$-\sqrt{16}$

Table II. Quantity  $g_{N\bar{K}Y}^* g_{BMY}^*$  for  $Y_1^*$  a member of a {27}, {10}, {10\*}, or {8} multiplet; B and M denote members of the  $J^P = 1/2^+$  baryon octet and of the pseudoscalar meson octet respectively;

B, M	{27}	{10}	{10*}	{8}
$\Sigma^0 \pi^-$	0	$-g_{10}^2 \sqrt{1/72}$	$-g_{10^*}^2 \sqrt{1/72}$	$-g_8^2 \sqrt{512} a(1-2a)$
$\Sigma^- \pi^0$	0	$g_{10}^2 \sqrt{1/72}$	$g_{10^*}^2 \sqrt{1/72}$	$g_8^2 \sqrt{512} a(1-2a)$
$\Sigma^- \eta$	$g_{27}^2 \sqrt{3/50}$	$-g_{10}^2 \sqrt{1/24}$	$g_{10^*}^2 \sqrt{1/24}$	$-g_8^2 \sqrt{512/3} (1-a)(1-2a)$
$\Lambda \pi^-$	$g_{27}^2 \sqrt{3/50}$	$g_{10}^2 \sqrt{1/24}$	$-g_{10^*}^2 \sqrt{1/24}$	$-g_8^2 \sqrt{512/3} (1-a)(1-2a)$
$\Xi^- K^0$	$g_{27}^2 \sqrt{1/25}$	$-g_{10}^2 \sqrt{1/36}$	$-g_{10^*}^2 \sqrt{1/36}$	$g_8^2 \sqrt{256} (1-2a)$

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