

UC San Diego

UC San Diego Electronic Theses and Dissertations

Title

An Optimal Grip on Mathematics: An Embodied Critical Inquiry into Problem-Solving

Permalink

<https://escholarship.org/uc/item/1fw2d061>

ISBN

9798190928358

Author

Hernandez, Gabriela

Publication Date

2026-09-11

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA SAN DIEGO

SAN DIEGO STATE UNIVERISTY

An Optimal Grip on Mathematics: An Embodied Critical Inquiry into Problem-Solving

A Dissertation submitted in partial satisfaction of the requirements

for the degree Doctor of Philosophy

in

Math and Science Education

by

Gabriela Maria Hernandez

Committee in charge:

University of California San Diego

Sherice Clarke, Co-Chair

Stanley Lo

Dor Abrahamson

San Diego State University

Mary Pilgrim, Co-Chair

Sasha Chizhik

Toni Saia

2026

Copyright

Gabriela Maria Hernandez, 2026

All rights reserved.

The Dissertation of Gabriela Maria Hernandez is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Co-Chair

Co-Chair

University of California San Diego
San Diego State University

2026

Dedication

"I came to theory because I was hurting-the pain within me was so intense that I could not go on living. I came to theory desperate, wanting to comprehend-to grasp what was happening around and within me. Most importantly, I wanted to make the hurt go away. I saw in theory then a location for healing...Living in childhood without a sense of home, I found a place of sanctuary in 'theorizing'" in making sense out of what was happening. I found a place where I could imagine possible futures, a place where life could be lived differently. This lived experience of thinking, of reflection and analysis, became a place where I worked at explaining the hurt and making it go away." - bell hooks, *Theory as Liberatory Practice*

This dissertation is for little Gabby, who never had a safe place to call home, who found sanctuary in theorizing her way through the world. She just wanted to make the hurt make sense. This work is for her. It's proof that the theorizing mattered, that the little girl who couldn't stop thinking her way toward safety became someone who may help others find their way through constraint.



Table of Contents

Dissertation Approval Page	iii
Dedication	iv
List of Figures	xiv
Acknowledgments	xv
Vita	xviii
Abstract of Dissertation.....	xviii
Chapter 1: Introduction	1
Purpose and Research Questions	3
Chapter 2: Literature Review	6
Ableism	7
Problem-Solving in Mathematics Education	9
Why Problem-Solving Matters	9
What is Problem-Solving	10
Affect and Belief in Problem-Solving.....	11
Metacognition in Problem-Solving	12
Platonic Roots of Math	14
The Three Pillars	16
Enactivism	17
Empirical Evidence of Embodied Cognition.....	18
Enactivism: Thinking is Movement	20
Math from an Enactivist Perspective	21
Ecological Psychology.....	22
Affordances	23
Affordances in Math	24
Affordances in Dance and Problem-Solving	25
Movement as Critical Inquiry	28
Constraints.....	31
Constraints in Ecological Dynamics	32
All the Ecological Psychology Analytics Tools Together	33
Ecological-Enactive Model of Disability	34
Disabled is not a Bad Word.....	34
Identity-First Language	35
The Medical Model of Disability.....	36
The Social Model of Disability.....	36

Ecological-Enactive Model of Disability	37
Disability and Activist Affordances.....	39
Ecological Shrinkage: A Bridge.....	41
Shrinkage in Math Education Cultural Ecology	42
Theoretical Framework	44
Chapter 3: Methods.....	48
Positionality.....	48
Research Design.....	50
Emic-Etic Positioning and Intervention.....	52
Context and Setting.....	54
The Climbing Gym.....	54
Puzzles	55
Recruitment.....	56
Validation Check	57
Member Checking	57
Participants	57
Route-Setters	58
Taylor.....	58
Christopher.....	58
Chad.....	59
Kenneth	59
Sofia	59
Ashley.....	60
Data Sources	60
Phase 1: Route-Setter Focus Group	60
Phase 2: Rock Climbing	60
Semi-Structured Initial Interview.....	61
Rock Climbing Session	61
Rock Climbing Stimulated Recall Interview	62
Phase 3: Mathematics	62
Task-Based Clinical Math Interview.....	62
Mathematics Stimulated Recall Interview.....	64
Data Collection	66
Phase 1: Route-Setter Focus Group	66
Phase 2: Rock Climbing	66

Semi-Structured Initial Interview	67
Rock Climbing Session	67
Rock Climbing Stimulated Recall Interview	67
Phase 3: Mathematics	67
Task-Based Clinical Math Interview	68
Mathematics Stimulated Recall Interview	68
Data Analysis	68
Analysis for RQ1	68
Analysis for RQ2	71
Analysis for RQ3	72
Phase 2 Analysis: Climbing Context	72
Phase 3 Analysis: Mathematical Context	76
Chapter 4: Findings	78
Phase 1: Environmental Considerations	78
Affordances	78
Different Solution Paths	79
Affordances as Relational, Not Universal	81
Constraints	81
“Killing” Opportunities to Induce Discomfort	81
Space not for Disability	82
Conceptualizations of Problem-Solving: RQ2	83
Taylor	84
Coherences	84
Divergences	85
Christopher	86
Coherences	86
Divergences	87
Chad	87
Coherences	87
Divergences	88
Kenneth	90
Coherences	90
Divergences	91
Sofia	91
Convergences	92

Divergences	93
Ashley	94
Coherences	94
Divergences	95
Themes Across Participants: Math Problem-Solving as Decontextualized	96
Phase 2: Rock Climbing	99
Emergent Codes	99
Problem-Solving Strategies	99
Disposition.....	101
Affect	103
Expertise Building	104
Environmental Factors	106
Efficiency	107
Rock Climbing Case Studies	108
Taylor Rock Climbing Problem-Solving	109
Christopher Rock Climbing Problem-Solving.....	114
Kenneth Rock Climbing Problem-Solving	118
Chad Rock Climbing Problem-Solving	124
Sofia Rock Climbing Problem-Solving	129
Ashley Rock Climbing Problem-Solving	135
Connections Across Participants.....	140
Efficiency Across Participants	140
Disposition Across Participants	140
Environment Across Participants	141
Affect Across Participants	142
Expertise Across Participants.....	142
Strategies Across Participants	143
Phase 3: Mathematics	144
Applying the A Priori Framework from Phase 2.....	144
Taylor Math Problem-Solving.....	146
Approach to SOMA	146
Approach to Towers of Hanoi.....	146
Approach to Tangram.....	147
Problem-Solving Attributes	147
Christopher Math Problem-Solving.....	152

Approach to SOMA	152
Approach to Towers of Hanoi.....	153
Approach to Tangram.....	154
Problem-Solving Attributes	155
Kenneth Math Problem-Solving	160
Approach to SOMA	160
Approach to Towers of Hanoi.....	161
Approach to Tangram.....	162
Problem-Solving Attributes	163
Chad Math Problem-Solving.....	167
Approach to SOMA	167
Approach to Towers of Hanoi.....	168
Approach to Tangram.....	168
Problem-Solving Attributes	169
Ashley Math Problem-Solving	173
Approach to SOMA Cube	173
Approach to Towers of Hanoi.....	174
Approach to Tangram.....	175
Problem-Solving Attributes	176
Phase 2 and 3 Comparison: RQ3	180
Taylor Similarities Across Domains	181
Taylor Differences Across Domains.....	182
Christopher Similarities Across Domains	185
Christopher Differences Across Domains.....	186
Kenneth Similarities Across Domains	188
Kenneth Differences Across Domains	190
Chad Similarities Across Domains	191
Chad Differences Across Domains.....	193
Ashley Similarities Across Domains.....	195
Ashley Differences Across Domains	197
RQ3 Themes Across Participants.....	197
Disposition is Foundational	198
Affect is Constituent to Problem-Solving	198
Efficiency Is Almost Always A Consideration.....	199
Environmental Attunement is Key	199

Expertise as Flow	200
Constraints and Affordances Across the Board	200
Chapter 5: Discussion.....	202
Connecting Back to Research Questions	202
Research Question 1: Constraints and Affordances	202
Research Question 2: Pre-Intervention Assessment.....	203
Research Question 3: Domain Bridging Between Math and Climbing	205
Conceptual Bridges	206
The Role of Language in Articulating about Unfamiliar Domains	206
Comparing Mathematical Puzzles to Different Styles of Climbing	207
Towers of Hanoi and “Progressive” Thinking.....	208
Embodied Problem-Solving Cycle	209
The Four-Phase Cycle	210
Phase 1: Beta (Orientation and Assessment)	210
Phase 2: Sequence (Execution).....	210
Phase 3: High Point (Maximum Progress).....	211
Phase 4: Feedback (Evaluation and Adjustment)	211
Relating to Carlson and Bloom and Beyond: The High Point.....	212
Disability Unveils the High Point	213
Chapter 6: Implications	216
Epistemic Climb-it	216
Supporting Disabled Learners in the Mathematics Classroom	217
Sociomathematical Norms and Classroom Practices	218
Problem-Solving is not a Monolithic Capacity	220
The Periodic Table Of Embodied Activities	221
Applying the Humanistic Problem-Solving Framework	222
Applying the Embodied Problem-Solving Cycle	222
Burning Questions	223
Chapter 7: Limitations	225
Chapter 8: Conclusion.....	227
References	230
Appendix A: Interview Protocols	241
Route-Setter Focus Group Protocol	241
Semi-Structured Climber Protocol	243
Stimulated Recall Climber Protocol	245

Math Task-Based Clinical Math Interview Protocol	247
Math Stimulated Recall Interview Protocol	249
Contact Summary Form	250
Appendix B: Mathematical Tasks	251
Appendix C: Climbing Glossary	254
Appendix D: Problem-Solving Strategy Codebook	256
Appendix E: Disposition Codebook	261
Appendix F: Affect Codebook	265
Appendix G: Expertise Building Codebook.....	267
Appendix H: Environmental Factors Codebook.....	269
Appendix J: Pictures of Different Types of Climbing	271

List of Figures

Figure 1: Theoretical Framework of the Study.....	45
Figure 2: Taylor Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	113
Figure 3: Christopher Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	117
Figure 4: Kenneth's Personal Conceptualization of Problem-Solving.	120
Figure 5: Kenneth's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	123
Figure 6: Chad's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	128
Figure 7: Sofia's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	134
Figure 8: Ashley's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	139
Figure 9: Taylor Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	151
Figure 10: Christopher Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	159
Figure 11: Kenneth Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	166
Figure 12: Chad Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	172
Figure 13: Ashley Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart	179
Figure 14: Taylor Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange). 181	
Figure 15: Taylor Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right).	181
Figure 16: Christopher Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange)	185
Figure 17: Christopher's Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)	185
Figure 18: Kenneth Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange)	188

Figure 19: Kenneth's Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)	188
Figure 20: Kenneth's Phase 1 & 2 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)	191
Figure 21: Chad's Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)	191
Figure 22: Ashley Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange)	195
Figure 23: Ashley's Phase 1 & 2 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)	195
Figure 24: Four-Phase Embodied Problem-Solving Cycle.....	212
Figure 25: Manifestations of the four-phase cycle across rock climbing and math.	215

List of Tables

Table 1: Overview of Study Data Collection and Analysis Timeline.	52
Table 2: Mathematical Task Design Rationale.....	65
Table 3: Example of Affordances Content Analysis	70
Table 4: Example of coherences and divergences in categorization	72
Table 5: Example of Axial codes in relation to some Open codes	75
Table 6: Example of Emergent Axial codes in Phase 3	77
Table 7: Summary of convergences and divergences in conceptualizations of problem-solving	97

Acknowledgments

I would not be here if it were not for my community. I mean this wholeheartedly. Thank you so much to my participants and dear friends for being a part of this study. I am so grateful.

Thank you, Momma, for supporting every wild decision I've made—the rock climbing, the years in Asia, this PhD journey. I know having a daughter who can't sit still has required patience and trust, but you've given me both without hesitation. Your belief in me has been the biggest constant in my life, and I'm so grateful you've supported and celebrated my unconventional path.

Mr. Harden, it has been an honor having you in my life for so long. You saw potential in me long before I saw it in myself. I truly value the relationship we have cultivated and maintained over the last fifteen years.

To my incredible co-chairs: Mary, you've been with me since CSU, through every doubt, every pivot. Thank you so much for your unwavering support. Sherice, your advice that I am "the route-setter of my own study" empowered me to approach this research with agency and creativity. You both are exactly the hype women I needed to do this work while still holding me accountable and always pushing me to grow. Thank you Toni for your crucial and extremely valuable perspective on disability throughout this process, and Sasha for pushing my thinking, especially the mathematics!

Dor, you changed everything with one question. In our first Zoom meeting, when you said, "Why don't you just do your study on climbing? It seems so obvious," you gave me permission to bring my whole self to this research. Thank you for being a steady anchor throughout this journey and for paving the way for this work to exist.

Christian, thank you for supporting me and always grounding me in my most turbulent moments. It's been so wonderful writing papers with you, exploring so many ideas with you, and having you as a dear friend. Charles, it has been such a delight working with you for the last five

years. You are such a wise and kind soul. Beth and Angel, y'all are the real MVPs in MSED. Thank you for keeping me sane(-ish). Thank you CJ, for helping me make edits, generating my visuals, and being an amazing and absolutely brilliant and wonderful human.

Glascoe, thank you for helping me see my worth when we met sixteen years ago and for encouraging me to go to community college. I've absolutely loved the evolution of our relationship. Thank you, Andrew, for being a pillar in my life since I began this program and for spending countless hours exploring theoretical concepts with me, helping me bring this study to life. I am infinitely grateful to have you as my ride or die.

To my incredible community of badass women--Meg, Christine, Madisyn, Dani, Rach, Natasha, Jung-In, Bri, Carolyn, Rawan, Samantha, Sara--and several others, y'all have been my rock over the last few years. Thank you for embodying strength and grace and for being there for me in my darkest moments.

Oliver, I can't even begin to express what our friendship means to me--I feel SO privileged to have you in my life and to share so many amazing adventures together.

Chaz, Avery, Mason, thank you for always being down for adventures and all the time shared together. You guys remind me to hold life lightly.

Finally, my sweet Brian, I am so happy I found you. Thank you for feeding me and always reminding me to slow down. You ground me and make me feel safe in ways I never knew possible. I can't wait for our life with our future child (aka blue heeler) Horchata (Horchy!!) together. I love you so much.

All in all, thank you to my community for being my family. Thank you for giving me grace as I stumble and unlearn. Thank you for holding me accountable and helping me to become the woman I am today.

Vita

2014 Bachelor of Science in Mathematics, Georgia State University

2017 Master of Science in Mathematics with an Interdisciplinary Specialization in Education,
Colorado State University

2026 Doctor of Philosophy in Mathematics and Science Education, University of California San
Diego/San Diego State University

PUBLICATIONS

Pilgrim, M. E., Wilkes, C. E., II, Hernandez, G., & Stringer, B. P. (2025). We're all human, so why does equity matter? *Journal of the California Mathematics Project*, 13.
<https://journals.calstate.edu/jcmp/article/view/6860>

Hernandez, G. M. & Kronsted, C. (2025). Integrating rock climbing into the mathematics curriculum: An enactive affordance-based approach. *Philosophy of Mathematics Education Journal*.

Stringer, B. P., Hernandez, G. M. & Moschetti, M. G. (2022). Honoring identity and building community in the mathematics classroom. *Teaching for Excellence and Equity In Mathematics*.

Abstract of Dissertation

An Optimal Grip on Mathematics: An Embodied Critical Inquiry into Problem-Solving

by

Gabriela Hernandez

Doctor of Philosophy in Math and Science Education

University of California San Diego, 2026
San Diego State University, 2026

Professor Mary Pilgrim, Co-Chair
Professor Sherice Clarke, Co-Chair

This dissertation challenges the dominant view of mathematical problem-solving as disembodied and abstract by examining how disabled rock climbers problem-solve across the domains of rock climbing and mathematics. Drawing on enactivism, ecological psychology, and critical disability scholarship, this qualitative study centers six disabled climbers' experiences in constraint navigation as an epistemic resource for reimagining mathematics education. Through focus groups, semi-structured individual interviews, rock climbing observations, and task-based mathematical interviews, this study addresses three research questions: 1) What are the affordances and constraints designed into the context of the problem space for rock climbing?; 2) What are individuals' perceived similarities and differences concerning problem-solving across

the domains of (a) rock climbing; (b) mathematics; and (c) their daily lives?; and 3) What are the observed similarities and differences with regard to the problem-solving strategies amongst the domains of rock climbing and mathematics?

Findings show that while participants initially characterized mathematics as decontextualized and disembodied, their enacted problem-solving in mathematics demonstrated cross-domain coherences when climbing expertise was explicitly positioned as relevant. Through grounded theory analysis, I created a holistic problem-solving framework with six interconnected dimensions -- Disposition, Affect, Efficiency, Environment, Expertise, and Strategies—revealing that attributes transfer in varied patterns while also flexibly responding to domain affordances and constraints. Exploring how disabled individuals navigate constrained environments can provide a magnifying glass into how learning can emerge from constraints. Ultimately, mathematics education would benefit all students by honoring the embodied intelligence they bring from rich lived experience.

Chapter 1: Introduction

In rock climbing, finding an *optimal grip* means achieving the perfect balance between not holding on too tightly or too loosely while maintaining the ideal grip, enabling efficient movement and clear perception of what is to come ahead. This tenuous balance requires a strategic navigation of constraints: the type of hold, individual body positioning and strength limits, and how much energy is left. These constraints shape what optimal means in each moment to the climber. Optimal grip is leveraged from ecological psychology's notion of maintaining optimal grip on one's environment, defined as the tendency to reduce uncertainty and improve one's hold on relevant possibilities for action (Bruineberg & Rietveld, 2014). It captures a fundamental principle of problem-solving explored in this dissertation: constraints are crucial to how we make sense of problems and discover pathways forward (Araújo et al., 2006; Newell, 1986).

Consider a prototypical high school algebra task: "solve for x." Constraints manifest as expectations for a single correct answer and normative solution procedures. Yet multiple representations (e.g., graphical, symbolic, numeric) and corresponding solution paths exist, each offering different affordances for engagement with the problem. In fact, actual problem-solving is not merely applying memorized procedures; instead, it is actively identifying and working within constraints to unveil affordances, finding an optimal grip on the problem. However, the constraints within mathematics extend beyond these prototypical tasks to the culture itself, what I frame as residue of ableism. Mathematical thinking has been shaped through narrow, ableist conceptions derived from studies in mathematics education and developmental psychology that privilege particular bodies and ways of knowing while marginalizing others (Healy et al., 2024). Mathematics education's emphasis on speed, visual-spatial reasoning, abstract symbolic manipulation, and individual silent work systematically

disadvantages non-“normative” learners while presenting these constraints as inherent to mathematics itself (Boaler, 2022; Lakoff & Núñez, 2000; Tan et al., 2019).

What if that framing flipped? If mathematics education has been designed around particular bodies and styles of learning, then perhaps disabled individuals navigating these spaces are demonstrating a different type of expertise that often remains invisible. Indeed, they are navigating environments designed without them in mind. This reframing positions the “problem” not in disabled bodies but in a mismatched coupling between the individual and the environment, and suggests that disabled learners bring sophisticated constraint-navigation skills that mathematics education has traditionally failed to recognize or value.

Disabled learners are exceptionally skilled at navigating constraints because they are accustomed to navigating a world not designed for their bodies, thus developing sophisticated problem-solving strategies in response to embodied and ecological constraints (Hamraie, 2017; Wong, 2020). Rather than viewing disability as limitation, critical disability studies scholars position disability as *generative* (Siebers, 2008). I argue that disabled climbers' adaptive strategies for achieving optimal grip, both literally on climbing walls and figuratively in mathematics, offer valuable insights into mathematical cognition that challenge deficit-oriented frameworks. This dissertation examines how disabled rock climbers identify, navigate, and leverage constraints in their problem-solving processes across climbing and mathematical domains, exploring how their problem-solving in rock climbing transfers to seemingly abstract problem-solving of mathematics.

Despite decades of evidence suggesting that movement-based learning leads to better educational outcomes, much of the United States's high school and college education remains disembodied (Becker, 2023). As some researchers in education have recently argued, it is essential to integrate embodiment into the K-12 and college classrooms (Becker, 2023), particularly in the math classroom (Abrahamson & Bakker, 2016; Gandell, 2024). There has been a growing body of research on embodied cognition that seeks to challenge cognition as a

purely mental process, highlighting the role of the environment as well as the sensorimotor processes in thinking (Abrahamson, 2021; Alibali, 2005; Lakoff & Nunez, 2000; Varela et al., 1992). While scholarship on embodied cognition is expanding the scope of learning, there has been limited attention to disability. Drawing on critical disabilities studies, in this dissertation, I develop a humanizing problem-solving framework in order to investigate disabled learners' problem-solving processes.

This dissertation is a study of the problem-solving strategies of physically disabled climbers across problem-solving domains. Studying embodied problem-solving across contexts can yield significant insights into the nature of cognition, challenging traditional views and highlighting the role of the body, environment, and interaction in shaping thinking and learning. This research uncovers how problem-solving strategies, influenced by constraints and affordances, transfer across domains, suggesting common underlying cognitive (embodied) mechanisms. The overarching goal of this dissertation is to illuminate the rich problem-solving strategies employed by learners (particularly disabled learners). This research has important implications for challenging deficit-oriented perspectives by showcasing unique assets that *all* learners bring to the process of problem-solving.

Purpose and Research Questions

My dissertation aligns with a current trend in mathematics education research, which aims to evaluate the relevance of the embodiment paradigm drawn from cognitive science (Abrahamson & Bakker, 2016; Abrahamson 2021; Gandell, 2024). Enactivism (Varela et al., 1992; Gallagher, 2017; Di Paolo et al., 2017), ecological psychology (Gibson, 1979) and ecological-enactive model of disability (particularly, activist affordances (Dokumaci 2023)), serves as the pillars of theoretical framework, seeking to redefine problem-solving in mathematics by drawing insights from embodied activities like rock climbing.

By exploring how disabled rock climbers navigate a landscape of affordances designed for non-disabled individuals, I gain valuable insights into their rich problem-solving strategies grounded in their lived experiences. This is particularly relevant to mathematics, where many students, especially those from marginalized groups, face a shrinkage of their field of affordances due to ableist assumptions and practices (Lambert, 2018). I aimed to unveil and leverage the lived experiences of disabled climbers in my dissertation in response to Dokumaci's (2023) call to "name, recognize, and map out" their activist affordances (p. 113). Conceptualizing activist affordances as problem-solving strategies, I analyzed disabled individuals' problem-solving strategies in rock climbing, where constraints are amplified and visible. The ecological and embodied constraints disabled climbers faced on the wall led to inspectable strategies that I delineated to develop a humanizing problem-solving framework. This problem-solving framework was then applied to the selection of and analysis of participant engagement with mathematics tasks, aiming to explore transfer among problem-solving orientations and problem-solving strategies between rock climbing and mathematics. In this dissertation, I illuminated that individuals bring to bear rich and effective strategies from their lived and embodied experiences, particularly in domains outside of mathematics. Once identified and articulated into a coherent framework, it can then inform how we design for learning and teaching.

The following research questions (RQs) guide this dissertation:

RQ1) What are the affordances and constraints designed into the context of the problem space for rock climbing?

RQ2) What are individuals' perceived similarities and differences concerning problem-solving across the domains of (a) rock climbing; (b) mathematics; and (c) their daily lives?

RQ3) What are the observed similarities and differences with regard to the problem-solving strategies amongst the domains of rock climbing and mathematics?

Chapter 2: Literature Review

This chapter begins by establishing a comprehensive understanding of ableism as a pervasive system of discrimination that shapes societal structures, attitudes, and practices, privileging non-disabled people. With this lens in place, the exploration shifts to the crucial role of problem-solving in mathematics education, highlighting the limitations of traditional, platonic approaches that focus on abstract, disembodied thinking. Building upon this disembodied critique, the chapter dives into the embodied nature of problem-solving, drawing on enactivism and ecological psychology. It emphasizes the importance of movement and sensorimotor experience in shaping cognition and problem-solving abilities.

Ecological psychology — particularly the concept of affordances — with its focus on the dynamic interaction between individuals and their environments, provides a crucial bridge for the review of literature. It reveals how the built environment, often designed with ableist assumptions in mind, can create barriers and limitations for disabled individuals. This understanding allows us to critically examine the ways in which our world is structured and how it can either constrain or afford opportunities for individuals from marginalized groups.

The final section delves into a nuanced understanding of disability, moving beyond medical and social models toward the ecological-enactive model, emphasizing the importance of lived experiences, environmental factors, and the political context of disability. All together, enactivism, ecological psychology, and ecological-enactive model of disability work as the theoretical framework of this study.

This study highlights the potential of disabled individuals as experts in navigating constraints, offering valuable insights into rich problem-solving strategies that can benefit all learners. By recognizing and valuing the expertise of the disabled community, I posit that we can

learn to create more inclusive and engaging math learning experiences that expand possibilities for all students.

Ableism

In simple terms, *ableism* is the deeply ingrained systemic discrimination of disabled individuals (Campbell, 2009). Ableism fosters a narrative where disabled people must achieve extraordinary feats to be valued, rather than being accepted and respected for who they are (Shapiro 1994; Hehir 2002). The narrative not only devalues the everyday lives and experiences of disabled individuals, but it also serves to reinforce the very systems of oppression that marginalize them. This devaluation is interwoven into the fabric of society, perpetuating a hierarchy based on perceived normalcy and productivity, as defined by dominant groups, where dominance grows out of socially established superiority and inferiority as driving social constructs. Ableism permeates society, impacting not only those with visible disabilities but also those with invisible disabilities, chronic illnesses, and anyone arbitrarily deemed non-normative (Wolbring, 2008).

Ableism intersects with other forms of discrimination, such as racism, sexism, and ageism, marginalizing and devaluing individuals based on multiple aspects of their identity (Lewis, 2022). For instance, Hispanic disabled students may face both racist assumptions about intellectual inferiority and assumptions about mathematical capacity due to their disability, experiencing aggregated marginalization (Annamma et al., 2013). The interaction with these other identities exacerbates the systemic oppression that disabled individuals with intersectional identities face (Sojitra 2023). Indeed, all individuals, including disabled individuals, can be unconsciously influenced by societal biases, limiting our potential and the potential of others.

Though there is a substantial amount of research that examines the role of disability in math education (e.g., Lambert and Tan 2020; Tan et al. 2019), few researchers explicitly center the lens of ableism. Centering ableism makes visible how mathematics education privileges certain bodies and minds while systematically excluding others. For example, timed tests

privilege those who process quickly while disadvantaging students with processing differences; visual-heavy instruction excluding blind students; and classroom norms favoring individual silent work exclude students whose learning depends on verbal processing or collaborative sense-making (Lambert, 2020; Tan et al., 2019). Borgioli (2008) applies a critical pedagogy perspective with the lens of ableism, asserting that ableism in mathematics education goes beyond merely enabling students to become better mathematicians. Instead, it's a discriminatory system where those in power have narrow beliefs about what is considered acceptable proficiency in mathematics. Therefore, students are stratified by the creation of an inequitable environment where some students are labeled as "able" or "capable" while others are not, perpetuating social and political inequalities within the field.

Through empirical study of the expertise and assets of disabled learners engaged in problem-solving, this dissertation seeks to desettle ableist assumptions that are all too ubiquitous in the math education practice and research. Therefore, it is crucial to center the lens of ableism moving forward. Problem-solving, embodied cognition, and affordances coalesce as the musculature of my study and ableism is the backbone. Without considering ableism and its impact on all the pieces of the study as I move forward, as well as my very own biases and my participants' biases informed by internalized ableism, the resulting framework could unintentionally perpetuate the very inequities and limitations it seeks to address in math education (D'souza 2017). By explicitly considering ableism throughout the study, from data collection to analysis and interpretation, I aim to ensure that my work contributes to a more inclusive and equitable understanding of mathematical learning, challenging the dominant narratives that have historically marginalized and excluded learners.

Because problem-solving is at the very core of mathematical thinking (Halmos, 1980), understanding disabled climbers' problem-solving strategies requires first understanding what constitutes problem-solving itself. This chapter therefore continues with a review of problem-solving literature.

Problem-Solving in Mathematics Education

This section of this literature review explores the multifaceted nature of problem-solving, beginning with an overview of its fundamental role in mathematics education and everyday life. This section highlights the necessity of problem-solving skills for success in mathematics and introduces several different conceptions of problem-solving. The discussion then delves into the nebulous nature of problem-solving, acknowledging the challenges in defining a precise and all-encompassing conceptualization of the term. Researchers have proposed a range of definitions, emphasizing different aspects of the problem-solving process. I also discuss the role of belief, affect, and metacognition in problem-solving. I then add a unique dimension to the existing literature on mathematical problem-solving by focusing on Mason and Spence's (1999) concept of knowing-to, emphasizing awareness and real-time decision-making in problem-solving. While the knowing-to perspective marks a pivotal step towards a more embodied approach to problem-solving, it still retains the idea of problem-solving as a cerebral activity. This reinforces a rigid and potentially dehumanizing view of mathematics education. My argument focuses on the need to move beyond these limitations by embracing embodied cognition, particularly through the lens of enactivism.

Why Problem-Solving Matters

Our daily lives are a constant test of our problem-solving abilities. From the simple task of fixing a leaky faucet to the intricacies of navigating complex interpersonal relationships, these skills are the tools we use to overcome challenges every day (Cornelius & Caspy, 1987). When faced with an obstacle, effective problem-solving enables us to make sense of the situation, analyze root causes, and consider a range of potential strategies towards a solution (Blanchard-Fields, 2000). This critical thinking allows us to not only develop creative fixes but also adapt those strategies on the fly as circumstances change. This adaptability is crucial because real-world problems rarely present in a neat and tidy package (Lave et al., 1988). Problem-solving goes

beyond technical solutions; it hones our critical thinking by encouraging us to analyze information, identify patterns, and make sound judgments (National Research Council, 2012). Most importantly, it fosters our ability to adapt by teaching us to be flexible and think on our feet. All these attributes contribute to real-world problem-solving, and engaging with mathematics provides a powerful platform for learning and practicing the problem-solving process.

What is Problem-Solving

Halmos (1980) asserts:

I do believe that problems are the heart of mathematics, and I hope that as teachers, in the classroom, in seminars, and in the books and articles we write, we will emphasize them more and more and that we will train our students to be better problem posers and problem solvers than we are (p. 524).

Building on this, Schoenfeld (1985, 1992) contributed to the transformation of mathematics education research by positioning problem-solving not as an isolated skill, but as the central activity through which mathematical understanding develops. For the last three decades, the National Council of Teachers of Mathematics (1989) has advocated for students to have "opportunities to formulate problems from given situations and to create new problems by modifying the conditions of given problems" as the "essence of studying mathematics is itself an exercise in exploring, conjecturing, examining and testing, all aspects of problem-solving." (p. 95).

More recently, NCTM in 2020 continue to emphasize problem-solving but broaden the scope. They advocate for developing students' mathematical mindsets alongside procedural fluency. This includes fostering perseverance, strategic thinking, and the ability to make connections across mathematical concepts. Indeed, over the years, NCTM has conceptualized problem-solving as a core component of learning mathematics focused on applying knowledge creatively and adaptively in new situations. This aligns with the Common Core State Standards (CCSS) for Mathematical Practice, which call for students to "make sense of problems

and persevere in solving them" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6).

But what exactly constitutes "problem-solving"? While the definition of problem-solving has been historically nebulous (Bokhove and Jones, 2018; Borasi, 1986), reaching a clearer consensus of a definition within the field holds the potential to significantly benefit both research and curriculum development (Chamberlin, 2008). In his seminal work in mathematics education, *How to Solve It*, Pólya (1945) introduced a heuristic approach involving understanding the problem, devising a plan, executing the plan, and reflecting on the process. Pólya describes problem-solving as navigating through challenges and barriers to discover an unknown solution to a given problem. Sowder (1985) defined problem-solving as "what it is you do when you don't know what to do" (p. 141). Schoenfeld (1987; 2016) viewed problem-solving as grappling with new tasks in the absence of fully mastered solution methods, and in the spirit of Polya, considered it an art form integral to the very essence of mathematics. Further, the complexity of problem-solving has also been highlighted, necessitating the management and coordination of various cognitive operations (Garofalo & Lester, 1985). Bodner (2012) conceptualizes problem-solving as an anarchistic pursuit that is "...cyclic, reflective, and might appear irrational to anyone watching us use it" (p. 27). Some researchers are careful to make a clear distinction between exercises—which are linear—and problems because the two are often conflated (Bodner, 2012; Schoenfeld, 1992). Beyond cognitive strategies and heuristics, researchers increasingly recognize that problem-solving is fundamentally shaped by affect and belief.

Affect and Belief in Problem-Solving

What determines whether students successfully engage in problem-solving? Research reveals that cognitive strategies alone are insufficient, increasingly recognizing that problem-solving is fundamentally shaped by affect and belief (Schoenfeld, 2016; Lester, 1994; Carlson and Bloom, 2005; Garofalo & Lester, 1985). A student's belief system about math significantly

impacts their approach to problem-solving (Schoenfeld, 2016). Students who believe math is difficult and unyielding are more likely to give up easily; students who view math as a challenge to be overcome are more likely to persevere and develop effective strategies (Boaler, 2002; Dweck, 2006). Moreover, emotions (affect) play a powerful role in learning. Anxiety towards math can cloud a student's thinking and hinder their ability to solve problems (Ashcraft & Kirk, 2001). Positive emotions like curiosity and excitement can fuel motivation and lead to deeper engagement with the content (Tobias, 1993). Yet even when students possess relevant knowledge, productive beliefs, and positive affect, they may still struggle if they lack awareness of and control over their own thinking processes, what researchers term metacognition.

Metacognition in Problem-Solving

Some researchers, drawing upon Flavell (1976), describe metacognition as the knowledge concerning one's cognitive processes and products or anything related to them, including the active monitoring, regulation, and orchestration of these processes about cognitive objects, typically in pursuit of specific goals (Garofalo & Lester, 1985; Stillman & Galbraith, 1998). In essence, cognition involves thinking and learning processes, while metacognition involves analyzing and regulating how one thinks and learns. Garofalo and Lester (1985) developed the *Cognitive-Metacognitive Framework for Studying Mathematical Performance*, comprising four main stages: Orientation, Organization, Execution, and Verification, with shifts between phases often driven by metacognitive decisions. They argue that historical emphasis on heuristics like Polya's has overshadowed the importance of metacognition. Lester (1994) finds metacognition to be the driving force in problem-solving, embedded within each phase. Schoenfeld (1992) emphasizes metacognition and self-regulation as critical components of effective problem-solving, defining metacognition as awareness and understanding of one's cognitive processes, particularly in problem-solving tasks, involving active monitoring and regulation.

Mason and Spence (1999) add a unique dimension to the existing literature on mathematical problem-solving by focusing on the concept of *knowing-to*, which they define as a state of awareness and preparedness that enables real-time decision-making and creativity in problem-solving. They discuss the other types of knowing: *knowing-that*, *knowing-how*, and *knowing-why*. *Knowing-that* refers to factual knowledge, the what of a situation. *Knowing-how* involves procedural knowledge, the ability to perform a skill or complete a task. Finally, *knowing-why* delves deeper, encompassing the justification or explanation behind the what and how. What must first happen to initiate the cascade of knowings is the “bolt of lightning” (Mason & Spence, p. 146)—the *knowing-to*—that is understood as a sudden shift in attention, or “sudden shift in the structure of attention, and such shifts are instantaneous” (p. 146). In other words, in the problem-solving process, *knowing-to* acts as the spark, igniting a moment of creative insight. However, this spark relies on a foundation of existing knowledge (*knowing-that*) to build upon. *Knowing-why* then provides direction and justification, helping us navigate any difficulties that may arise. Finally, *knowing-how* translates these ideas into action, transforming the situation. Importantly, all these elements operate within a specific context. It is our awareness of this context, combined with past experiences that resonate with the situation that ultimately triggers the crucial *knowing-to act* in the moment.

While many researchers discuss the cognitive and metacognitive aspects of problem-solving, Mason and Spence (1999) delve into the epistemological underpinnings, emphasizing that problem-solving is not just about applying learned strategies but also about being aware and adaptable in the moment. They claim that in problem-solving, “....context is novel and resolution non-routine or multi-layered” (p. 136) and that students must be able to act creatively in the moment rather than respond to prescribed stimuli. Their work challenges the traditional educational paradigm that treats knowledge as an object to be transferred from external authority to the learner, alongside many influential and groundbreaking researchers (Freire 1978; Dewey 1986). Mason and Spence also believe that knowledge has been institutionalized in

education as objective rather than a state of awareness. Instead, they advocate for a shift towards internal authority in learning, rooted in *knowing-to act*. This form of knowledge is not just an accumulation of facts but an active state of presence that allows for creative and adaptive problem-solving.

In the learning process, Mason and Spence (1999) believe that understanding a student's attention is critical. What aspects of the problem or task are they focusing on? Is their focus narrow and singular, or are they considering multiple ideas at once? Is their attention clear and precise, or is it scattered and unfocused? Additionally, is their attention rigid and inflexible, or can they easily shift gears when needed? Perhaps most importantly, are they aware of their own attention? Can they recognize when they are stuck or relying on unhelpful habits? By helping students become more aware of their attention patterns, educators can help students build more robust and flexible knowledge structures that can be applied in novel situations.

In conclusion, while the nature of problem-solving, particularly in mathematics, remains multifaceted, research highlights the interplay between cognition, metacognition, and affective factors like belief (Schoenfeld, 2016; Lester, 1994; Carlson & Bloom, 2005; Garofalo & Lester, 1985). Beyond established frameworks and heuristics, Mason and Spence (1999) introduce the concept of *knowing-to*, emphasizing the importance of real-time decision-making and awareness in navigating problem-solving situations. This broadened perspective suggests that fostering such awareness alongside cognitive and metacognitive strategies holds promise for advancing problem-solving education. However, it is crucial to acknowledge that even this expanded view remains focused on the mind and does not consider the role of embodiment in the problem-solving process.

Platonic Roots of Math

Seminal math education researchers in problem-solving (e.g., Polya, Schoenfeld, Lester) focus heavily on internal cognitive processes, aligning with an in-the-head ontology of problem-

solving. Indeed, the emphasis is on cognition (thinking and learning processes) and metacognition (analyzing and regulating those processes). Even Mason and Spence's (1999) knowing-to concept, while introducing situational awareness, remains internal-- they argue it is about the learner's state of awareness of themselves, not necessarily the external environment, social interaction, or the learner's body.

Platonic beliefs position math as existing independently, with symbols simply representing eternal, abstract truths that exist in some non-physical realm. Despite advancements in the classroom, many students still grapple with this romanticized view of mathematics (Lakoff & Nuñez 2000; Nuñez. 2008). This perspective portrays math as an objective, abstract, and formula-driven subject, disconnected from the real world, and is pervasive in the classroom to this day (Battey & Marshall, 2023). This myth perpetuates the concept that mathematics is an objective reality, independent of humans, with mathematics merely a glimpse into this grand structure (Nuñez, 2008). This romance extends to the myth of innate mathematical ability: some individuals are seen as naturally gifted in accessing this "true" reality, while others are deemed incapable.

Problem-solving is crucial, and at the heart of mathematics (Halmos 1980), yet it remains clouded by platonic ideals. The platonic myth promotes a view of problem-solving as disembodied, often framed as a novice-to-expert progression and relies heavily on information processing and metacognition (Lester, 1994; Smith, 2012). While metacognition is a crucial component for problem-solving, current notions of it traditionally exclude embodiment. The platonic and disembodied view of math hinders efforts to integrate real-world applications into math education because it separates the world of perfect mathematical forms from our messy, physical reality. Indeed, though word problems are often included in math curriculum, they are often superficial and not meaningful to students (Wagner & Borden, 2020).

It is time to move beyond the myth and humanize mathematics (Gutiérrez, 2018). To examine problem-solving through a more human-centered lens, I draw on theoretical frameworks that challenge platonic assumptions. I position cognition as embodied action rather

than abstract symbolic manipulation (enactivism), see environments as offering opportunities rather than serving as neutral backdrops (ecological psychology), and conceptualize disability as revealing human-environment relationships rather than individual deficits (ecological-enactive model of disability).

The Three Pillars

This dissertation is grounded in three complementary frameworks: *enactivism* (Varela et al., 1992; Gallagher, 2017; Di Paolo et al., 2017) as the overarching approach to cognition and learning; *ecological psychology* (Gibson, 1979), where I leverage the analytical concept of *affordances*; and the *ecological-enactive model of disability* (Toro et al., 2020; Dokumaci, 2023) which synthesizes these perspectives to conceptualize disability as environmental shrinkage rather than individual deficit.

This section first offers an overview of the epistemological framework of embodied cognition, an alternative framework to the in-the-head traditional learning perspectives of problem-solving, specifically drawing on a more radical subset of embodied cognition, enactivism. The section begins by introducing the embodied cognition perspective. I then will introduce the concept of affordances from ecological psychology, relied on by some enactivists to explain action-centered cognition. The discussion of affordances leads to a connection to movement and its role in critical thinking. The concepts of embodied cognition, enactivism, affordances, and movement set the stage for a discussion of constraints in ecological dynamics, which enables me to frame the math education cultural ecology as constrained. Ultimately, the embodied cognition perspective offers a new way of understanding mathematics and problem-solving that is more in line with lived experience. However, in my dissertation I go one step further: I will make contributions to embodied cognition, specifically enactivist philosophy, by desettling the hegemonic able-bodied scholarship of enactivism and enriching the field with diverse perspectives and experiences of disabled individuals. All of these

ideas coalesce to create a rich and complex tapestry of problem-solving that is ultimately more humanizing.

Enactivism

Embodied cognition is an alternative epistemological framework to representationalist epistemologies that conceptualize cognition as being bound by the brain, relying on mental representations and computational manipulation. Embodiment theory conceptualizes the mind to extend beyond the head, into the body, and the environment, rejecting the fundamental tenets of Cartesian dualism (Lakoff & Johnson, 1999). From the Cartesian perspective, the body is simply an input/output machine (Anderson & Hinton, 2014; Fodor, 1983). That is, the brain is perceived to be the command center that instructs the body on physical actions. In Cartesian dualism, the main role of perceptual faculties, especially vision, is to monitor and guide these actions while providing feedback to the brain about outcomes (Abrahamson & Bakker, 2016). Embodied cognition represents the merging of various perspectives, each bringing a unique angle to the central idea that our bodies are essential to how we think, feel, and interact with the world. Embodied cognition draws from a gradient of perspectives, all contributing to the understanding of how our bodies shape our thinking. The degree to which our body plays a role in our cognition can be best conceptualized through the collective term of *4E (embodied, embedded, extended and enacted cognition)*.

Though embodied cognition is rather loosely defined, what unites the different branches is the core notion that the body acts as an integral part and parcel of the mind, rather than being subordinate to it. To put it in other words, embodied cognition is the notion that cognitive processes are grounded in, constituted, and shaped by our bodies, perception, action planning, and emotional response (Macrine & Fugate, 2022). The interconnected aspects of *embodied, embedded, extended and enacted cognition* (4E cognition) provide a comprehensive framework for understanding how our bodies influence our thinking (Newen et al., 2018). Here,

cognition is: a) not just embodied; b) but also embedded within the specific context in which it arises (Shapiro, 2019); c) extended beyond the individual, incorporating the tools and practices of our culture (Hutchins, 1996); and d) enacted as part of a dynamic system, where the body itself is constantly adapting and influencing its environment (Varela et al., 1992). Moving forward, I will assume an enactivist epistemology, but first I will provide some evidence that supports cognitive development is shaped by embodiment.

Empirical Evidence of Embodied Cognition

The concept of embodied cognition emphasizes the interconnection between our bodies, minds, and the environment. This theory suggests that our physical experiences and interactions with the world shape our cognitive processes. In mathematics specifically, embodied cognition manifests in multiple ways. Goldin-Meadow et al., 2001 shows that gesturing while explaining mathematical concepts reduces cognitive load, suggesting embodied action supports abstract reasoning. Similarly, Alibali and Nathan (2012) demonstrate that students' spontaneous gestures reveal embodied understanding of mathematical concepts like equations and graphs often expressing insights not yet articulated verbally.

At the most basic developmental level, finger counting creates persistent embodied numerical representations. Fischer & Brugger (2011) found that spatial-numerical associations formed through finger counting persist into adulthood, shaping how we mentally represent numbers. Sixtus et al. (2023) demonstrate how early bodily engagement shapes numerical understanding, and Neveu et al. (2023) found that awareness of individual fingers and fine motor skills directly contribute to early numerical and arithmetic abilities, with 3D motion analysis even revealing specific connections between digital dexterity and mathematical competence. Willoughby et al. (2021) extended Neveu et al.'s findings by showing that improvements in motor competence skills are associated with improvements in both executive function and math problem-solving skills in early childhood. This evidence from cognitive development reinforces

that mathematical ability is not separate from bodily capability but emerges through and depends upon sensorimotor development.

Calvo-Merino and colleagues (2005) investigate the link between action observation and motor skills. Using brain imaging, they found that expert dancers showed increased activity in motor areas when observing dance movements, compared to control subjects. This suggests that observing actions activates the same brain regions used to perform those actions, highlighting the role of our own motor repertoire in perception. Jasmin and Casasanto (2012) explore how the physical layout of our cultural mediation tools can influence our understanding of language. Their study demonstrates that people rated words with more letters on the right side of the QWERTY keyboard as more positive and words with more letters on the left side as negative. This effect, even observed in newly invented words, suggests that our physical interaction with keyboards shapes how we perceive and evaluate language.

Landy and Goldstone (2007) investigate how visual layout affects mathematical reasoning. They found that participants struggled to solve equations when the background on the screen that the equation was presented on moved in a way that contradicted the manipulation required to solve the problem. In other words, when participants were asked to solve algebraic equations on computer screens, the spatial arrangement of mathematical terms shifted dynamically. They found that participants struggled when visual spacing contradicted the algebraic operations needed. For instance, when terms requiring combination were visually separated, or independent terms were grouped together. This demonstrates that even seemingly abstract concepts in math are influenced by our embodied experiences and understanding of spatial relationships, and symbolic notations are perceived as objects. These studies collectively highlight the core tenets of embodied cognition: our ability to perceive, understand, and reason is not limited to abstract processes in the brain, but is fundamentally shaped by our physical interactions with the world. Next, we take embodied cognition one step further to enactivism.

Enactivism: Thinking is Movement

Enactivism, as proposed by Varela et al. (1991), offers a powerful perspective on embodied cognition. It emphasizes the role of action in shaping our understanding of the world. They argue that cognition, even "off line," is not simply a matter of processing information but rather an *enaction*, a process of actively engaging with the environment. This engagement happens through "a network consisting of multiple levels of interconnected, sensorimotor networks" (p. 17). They state the following:

In a nutshell, the enactive approach consists of two points: (1) perception consists in perceptually guided action; and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided (p. 172).

From this perspective, action, perception, and cognition are fundamentally intertwined. While we can intellectually separate them, in practice, they function as a dynamic and continuous loop. Our actions shape our perceptions, simultaneously bringing forth and shaping our understanding of the world. This interaction between organism and environment is driven by a fundamental desire for equilibrium. Each cognitive cycle involves a constant and recursive exchange between the two.

The enactivist perspective challenges the traditional view of thinking as a purely cerebral activity. It proposes that even complex processes like problem-solving are fundamentally tied to bodily movement. In simpler terms, thinking is movement. Our movements aren't simply the result of mental calculations; the movement itself is the processing. As Di Paolo et al. (2017) explain, sensorimotor interactions are what constitute cognition from an enactivist viewpoint. Perception and cognition are not separate but deeply intertwined, happening together as we engage with the world. The formation of sensorimotor schemes is a crucial aspect of embodied cognition. These schemes are patterns of interaction with the world that emerge through repeated cycles of sensing and moving. As we engage with our environment, successful sensorimotor patterns become reinforced as durable cognitive structures, leading to habits or refined skills (i.e. our

perception can better guide our actions). This process reflects the constant drive for equilibrium—organisms need to maintain internal consistency despite a dynamic environment. When challenges disrupt this equilibrium, organisms explore new sensorimotor patterns that would enable perception to guide action. This leads to either creating new schemes or reshaping existing ones, ultimately driving adaptation to ensure continued viability (Di Paolo, 2023). This enactivist framework dismantles the notion of math as a purely abstract, disembodied subject and, from this perspective, even mathematical would-be abstractions can be grounded in our physical interactions with the environment. Thus, as I will argue, enactivism acts as an epistemological thread between two otherwise disparate cultural practices, rock climbing and math, demonstrating how both simply rely on problem-solving rooted in our physical interactions with the world.

Math from an Enactivist Perspective

Enactivism has the potential to transform math education from a subject focused on abstract concepts to a process grounded in embodied experiences. Some researchers believe that the way that we experience and work with mathematical objects is analogous to how we manipulate physical objects (Abrahamson & Bakker, 2016; Kronsted & Gallagher, 2021). Mathematical objects exist in a semiotic social space, meaning they become part of the shared symbols and signs used in communication and discourse. However, unlike material objects, mathematical objects are often not physically present or literally visible in the learning environment. They need to be constructed collaboratively through discourse, or the exchange of ideas. The uncertain state of being of mathematical objects can make instructional conversations difficult. This is because these conversations often depend on some shared reference point, even if it is somewhat ambiguous or still being defined. Therefore, concretizing mathematical concepts is crucial in building a strong foundation for mathematical understanding (Abrahamson & Bakker, 2016). Enactivist learning activities, designed to engage

students' bodies and senses in exploring mathematical concepts, can play a vital role in this process. By physically manipulating objects, moving through space, engaging in embodied simulations, and thus developing new cognitive structures that enable perception to guided action, students can make sense of mathematical ontologies as lived experiences.

The Embodied Design Research Laboratory (EDRL), as described by Abrahamson (2024) is dedicated to exploring how enactivist principles can guide educational practices. At the EDRL, researchers create learning environments where students engage in problem-solving through movement with activities involving physical actions like manipulating virtual objects, where solution movements are designed to instantiate mathematical concepts. For example, Abrahamson discusses how coordinated hand movements can create opportunities for students to ground what, through appropriate discourse, will become proportional reasoning. As students move and solve these proportion problems, they develop new ways of perceiving and understanding the situation and ultimately use symbols to express their movement strategies, enabling students to articulate their understanding more formally. Abrahamson argues that this way of learning allows students to ground math understanding because the movement forms constitute the embodied–enactive meaning of the concepts.

Ecological Psychology

Ecological psychology, developed by Gibson (1979), provides a theoretical framework for understanding how organisms perceive and act within their environments, which builds upon and complements enactivism as the foundational theoretical framework. Ecological psychology provides several analytical tools for examining organism–environment relations. In this section, I will discuss the different analytic tools leveraged in this study: a) affordances; b) constraints; and c) exploratory activity and movement to understand how affordances are discovered through physical exploration.

Affordances

The constant loop of action, perception, and cognition that we see in enactivism often relies on a crucial concept from ecological psychology: *Affordances*. Coined by Gibson (1977), affordances are essentially the opportunities for action that our environment presents. These opportunities arise from the way our bodies interact with the world around us (sensorimotor coupling or relations). In other words, the environment does not simply provide us with information; it offers possibilities for what we can do. These possibilities are not absolute but rather relational, depending on our individual characteristics, like physical abilities and current state (Chemero, 2013). This can be understood as *ecological information* (Bruineberg et al., 2019)—the patterns and relationships within an environment that inform an individual's ability to perceive and act upon affordances. Individuals experience their environment in relation to what they can do with the environment, which holds also for semiotic representations of symbolic notation (Kronsted & Gallagher, 2021).

Affordances are not inherent properties of objects but, rather, possibilities for action that emerge from the relationship between an individual and their environment (Baggs & Chemero, 2020). Affordances are contextual, shaped by capabilities, dynamic and selective, nested, and socio-cultural. For instance, a cup with a lid only affords drinking to someone with a human body, not a cat. The affordances an individual perceives depend on their physical structure, skills, and current state (Rucińska, 2021). For example, a rock wall offers different grip affordances to a novice climber versus an expert (Rucińska 2021). As skills develop, the environment becomes richer in affordances. A trained mathematician sees a math problem as offering problem-solving actions, unlike someone unfamiliar with math (Cuffari et al. 2015; Schoenfeld, 2013). Our *field of affordances* constantly changes based on various factors like thirst, mood, social norms, ongoing experiences, and available tools (van Dijk & Rietveld, 2016). For instance, the "eatability" of a sandwich is highly salient to a hungry person as opposed to someone who is not hungry.

Affordances can be nested, meaning acting on one affordance sets you up for future actions. For example, during rock climbing, the evolving sensorimotor coupling with the environment leads to an ongoing discovery of affordances, creating a cascade of affordances based on successful actions. Indeed, each move changes the readily available information. New features might become visible or holds previously overlooked might become relevant within the new context. An expert climber sees a climbing wall as a series of connected affordances, each leading to the next move (Hacques et al., 2021). They have learned through experience what information is most relevant to the specific task at hand, allowing them to quickly focus on the affordances that matter, enabling efficient movement. Similarly, in mathematics, students are experiencing a comparable cascading of affordances via sensorimotor exploration. As they move through a math problem, new affordances are unveiled, revealing new information in the problem. New features, relationships, or variables may become visible, and previously overlooked information may become salient within the new context. Affordances are also influenced by culture and social interactions (Rietveld & Kiverstein, 2014). For example, mathematical symbols only become meaningful affordances within specific cultural contexts. Mathematical symbols have no inherent meaning. It is through a shared cultural agreement that they become tools for problem-solving (Lakoff & Nuñez, 2000). Similarly, in rock climbing, advice or watching others tackle a problem can change what affordances an individual discerns during a route.

Affordances in Math

Students engage with mathematical symbols in several ways— pointing, gesturing, moving, etc. These actions are not merely peripheral; rather, they are integral to and constituent of the mathematical process. Nemirovsky (2003) articulates the relationship with mathematical symbols and student engagement through movement with three conjectures about body activity and mathematical understanding. His first conjecture posits that mathematical

abstraction emerges mostly out of embodied activities. His second conjecture asserts that understanding and thinking are perceptuomotor activities distributed across different areas of perception and motor action, shaped by how we have learned the subject. For instance, understanding a trigonometric function as circular motion versus as an infinite sum engages entirely different perceptuomotor resources that students call upon. His third conjecture extends this further: he argues that what we think emerges from and within these activities themselves. So mental calculation, rather than processing abstract numerical properties, combines elements of writing numbers, uttering names, watching shapes, grouping objects, and tracing lines.

Equations, like physical objects, have affordances and there are mechanisms that can be leveraged to illuminate affordances (like the activities in the EDRL). Therefore, doing math involves detecting and engaging these affordances. Anderson (2021) asserts that:

A natural extension of the view that equations have spatial affordances is that they invite us to act on and *with them*, manipulating and moving their parts around as part and parcel of the problem-solving process. In this view, to learn algebra is to acquire a sensorimotor skill, and acting in accord with the rules of algebra is a matter of learning to see and act in accordance with the transformations that the equations afford (p. 235).

In other words, learning algebra, or more broadly math, can be understood as the acquisition of an embodied skill, where following the rules involves seeing and acting on the transformations these manipulations afford (acting on affordances). Symbolic math is possible because we have bodies that can interact with the world through perception and action. Even symbolic systems can be understood through this embodied lens.

Affordances in Dance and Problem-Solving

Philosophers have used dance as a metaphor to explore how affordances shape critical thinking. In their article, Kronsted and Gallagher (2021) propose an enactivist and ecological affordance-based approach to understanding the connection between conceptual problem-solving and dance. They assert that dance training enhances conceptual problem-solving by improving our ability to explore affordances through movement and shifts in attention. They

argue math itself can be seen as "an affordance-based sensorimotor activity" (p. 45). Building on Anderson (2013), they suggest mathematical thinking is not just internal but involves physically manipulating symbols (like numbers on a board). Interestingly, neuroscience research shows similar regions activate during both dance and symbol manipulation, suggesting shared cognitive mechanisms (Calvo-Merino et al., 2005). This implies that dance training strengthens neural processes used in math and symbol manipulation (increased neuroplasticity). In essence, both dance and math involve embodied skills within an affordance–action cycle, where we constantly adapt to new possibilities through action. By enhancing our ability to engage affordances through dance, we can improve our problem-solving skills in symbolic tasks. In short, as we enhance our affordance-manipulation through dance, we can enhance our affordance manipulation in the problem-solving process of symbolic manipulation.

Kronsted and Gallagher (2021) argue that dance training enables students to practice shifting attention from conventional object features to novel affordances, aiding problem-solving. In dance improvisation, heightened awareness of the body's interaction with the environment fosters perceiving objects as novel, encouraging experimentation with their affordances. This approach treats objects as if encountered for the first time, creatively exploring their possibilities. For example, dancers might "dance with" objects like boxes, discovering novel affordances. Dance improvisation practices attention-shifting for exploration, uncovering hidden affordances. Comparable to trial and error in problem-solving in math, when students are stuck on a problem, problem-solving looks like shifting your attention to new features within the structure and then exploring new ways of engagement from that newly found starting point (Mason, 1989; Tancredi et al., 2022). Trial and error can be seen as identifying new affordances and then "pursuing a line of engagement from those affordances" (Kronsted & Gallagher, 2021, p. 49). In math, the solution may more readily flow when we start the problem in a new way. Hence, the connection between dance improvisation and attention shifting becomes evident. The process of redirecting attention to explore new possibilities is consistent whether

one is dancing, rock climbing, or solving mathematical problems. The skill of shifting attention to previously overlooked object affordances learned in many types of movement can be applied to conceptual problem-solving. It trains the individual to navigate new affordance paths within various constraints. Indeed, shifting attention through embodied engagement with object affordances is a universal skill.

Barbour (2011) argues that dancing is a critical process of discovery. Through the conceptualization of movement as a tool for interrogation, she sees dance beyond the expression of emotions or creating aesthetics. It can be a powerful tool for questioning assumptions, social norms, and power structures. By embodying these challenges physically, dancers can reveal hidden meanings and complexities. She also highlights the importance of narrative in dance research and that through sharing the stories behind their movements, dancers offer deeper insights into their creative process and the knowledge they have gained through dance. Dunagan et al. (2019) compare dance to a complex puzzle, highlighting the active decision-making required in the movement. They see this dynamic, embodied problem-solving as critical thinking at its core.

Studies on dance suggest that this type of physical engagement cultivates critical thinking skills. Therefore, investigating the relationship of rock climbing and mathematical problem-solving could offer a valuable and novel contribution to the field of mathematics education. While dance has been extensively theorized and researched, the field of embodied cognition could benefit from exploring other domains like rock climbing. While dance is often linked to critical thinking and movement, I argue that rock climbing offers a potentially a more explicit connection between movement and critical thinking. Math and rock climbing share a core principle: navigating a structured environment with a defined goal while adhering to rules and procedures, simultaneously exploring affordances within those constraints. Similar to a dancer interpreting choreography within a specific style, a climber navigates a route with

established holds, yet must discover movements to find a solution that works for their unique bodies (Hernandez & Kronsted, 2025).

Movement as Critical Inquiry

Welch (2022) proposes a relationship between movement and critical inquiry, where bodily exploration of affordance informs abstract thinking, and vice versa. Drawing on Sheets-Johnstone (2012) and Varela et al. (1992), Welch holds that our bodies shape how we perceive the world through affordances - potential actions revealed by our sensorimotor abilities. So basically, movement is essentially our bodies seeking relevant affordances. Welch suggests that activities like dance, which heighten this awareness, can improve problem-solving. The sharpened sensorimotor skills translate into a better grasp of spatial concepts like math. Conversely, abstract thinking in math can enhance critical thinking in dancers or athletes. By reconceptualizing unexplored spaces, they discover new movement possibilities (affordances) that were previously unseen. In Welch's view, movement is a form of curiosity.

As we move, we are constantly questioning and exploring our environment, seeking new possibilities. As evidenced by research, dance and other expressive movements such as rock climbing enhance our kinesthetic awareness and ability to explore, exposing us to diverse affordances (Rucińska, 2021) . This exploration, prompting critical inquiry from different perspectives, deepens our understanding of ourselves and our place in the world. This resonates with experiences in math classrooms where students excitedly manipulate equations and proofs (Abrahamson & Sánchez-García, 2016), playing around to grasp underlying structures (similar to movement as curiosity). Both scenarios involve actively exploring an environment (dancing, rock climbing or mathematics) to discover new possibilities and insights, ultimately enriching problem-solving abilities and expanding our worldview. Movement as curiosity is the foundation for *critical inquiry* (not just critical thinking):

Critical inquiry is one kind of critical thinking that is the creative and innovative approach to asking questions. All critical thinking is done in response to one kind of question or another. Critical inquiry is the critical thinking activity of pursuing and constructing the questions that the critical thinking activity of answering follows (Welch, 2022, p. 81).

Critical inquiry involves the creative process of crafting the questions that will ultimately enrich any problem-solving activity. Further, using critical inquiry as a tool when solving a problem has the potential to lead to change rather than perpetuating the status quo. This emphasizes asking the right questions, challenging assumptions, and finding new ways to frame a problem.

Traditional critical thinking, on the other hand, focuses on providing answers, analyzing, and constructing sound arguments once a question is established. Welch emphasizes that all critical thinking relies on questions, but it is the process of critical inquiry that determines how innovative and insightful our approach will be. By carefully examining the questions themselves, we open up new possibilities for understanding and transformative solutions. Welch encourages us to question the question itself, leading to a more comprehensive understanding of the problem. This has implications for embodied practices like rock climbing, where climbers can physically explore and refine their questions through movement. By training in domains like rock climbing, we can potentially cultivate similar skills in other areas like mathematics. In math education, this means encouraging students to question the problem itself, explore multiple paths, and seek creative solutions, rather than focusing solely on finding the one "right" answer.

Whether tackling a rock climbing route or a mathematical problem, in both domains, success depends on active exploration. We uncover possibilities through playful experimentation, strategically testing different approaches within the constraints of the environment. In climbing, this might mean trying a new foothold or a dynamic move. In mathematics, we might manipulate equations or apply a theorem in an unexpected way. This initial curiosity evolves into critical inquiry as we evaluate our results, refine our understanding of the problem space, and adapt our strategies accordingly. This process of seeking out affordances, critically assessing them, and ultimately leveraging them for success applies

equally to physical movement on a rock wall and the manipulations of mathematical problem-solving. Welch argues that a necessary feature of critical inquiry is *creativity*, claiming that creativity can be seen in the unique way individuals tackle problems. It is not just about answering the questions to the problems, but rather figuring out the right questions to ask and how to ask them. This process often leads to deconstruction—breaking things down to understand the different elements that once made up a structure.

In sum, Welch's framework highlights a powerful dance between embodiment and abstraction. By training our focus through movement – whether exploring the affordances of a climbing wall or the affordances of mathematical symbols – we actively cultivate critical inquiry. This approach fosters an inquisitive mindset, prompting us to question assumptions, embrace curiosity, and creatively reframe problems. The insights gained through physical exploration enhance our problem-solving in domains like mathematics, and vice versa. Ultimately, this embodied and critical approach breaks down rigid barriers within traditional education. It encourages us to see problem-solving as a dynamic and holistic process, where exploring affordances, interrogating questions, and finding creative solutions become valuable skills.

With this review of the literature, I have established the foundation for constructing a bridge between math and rock climbing. By shifting away from traditional conceptions of mathematics as purely abstract, I argue that both rock climbing and math can be seen as sensorimotor explorations in search of new affordances, and this is my working definition of problem-solving moving forward. This means engagement in both fields is the active interaction with the environment, seeking out possibilities (affordances), and adapting strategies based on the results. This embodied understanding offers a unique way to understand problem-solving across these domains, breaking down the artificial divide between the physical and the mental. I have now set the stage for exploring how embodied experiences in rock climbing might offer valuable insights and strategies to enhance the way we learn and teach mathematics.

Building upon the framework of ecological psychology, I will now delve deeper into the concept of constraints and their significance in continuing to forge a bridge between rock climbing and mathematics. Within ecological dynamics, constraints act as essential guiding forces, shaping our interactions and actions within a specific environment. The addition of constraints offers a powerful lens to further understand and explore these interconnected domains.

Constraints

Newell (1986) conceptualizes constraints as boundaries or features that limit the possibilities for action within some system. Rather than prescribing specific movements, constraints shape and channel behavior by eliminating certain configurations while leaving others available. Newell proposes three categories of constraints that interact to determine optimal patterns of coordination: *organismic* constraints (individual characteristics like body structure, physical capabilities, and developmental stage), *environmental* constraints (external factors like gravity, ambient temperature, and available surfaces), and *task constraints* (goal-related factors such as rules, implements, and specific performance demands). This constraints perspective suggests that the same individual can produce different movement patterns when constraints change, and similarly, that different individuals can produce similar patterns when facing similar constraints. Understanding how constraints shape action provides a framework for examining how individuals, particularly those navigating constrained environments, develop adaptive problem-solving strategies. Ecological dynamics applies this constraints framework to learning environments, using task constraints as pedagogical tools to guide discovery and coordination development (Vilar et al., 2012).

Constraints in Ecological Dynamics

Focused on natural learning environments, *ecological dynamics* (Vilar et al., 2012) is a theoretical framework traditionally used in sports science to understand how skills are acquired. It blends dynamical systems theory and Gibson's concept of affordances to explain the learning of physical skills as the complex self-organizing of subject–environment dynamical systems. In the context of mathematics education, ecological dynamics offers a framework for designing, implementing, and analyzing activities in which students develop fundamental understandings of mathematical concepts via solving and reflecting on motor-control inquiry problems. This approach suggests that mathematical meanings emerge from the guided signification of situated motor-action coordinations, emphasizing the role of constraints in the process of teaching and learning (Abrahamson & Sánchez-García, 2016). Put simply, students essentially do something physical (like moving their hands); teachers then add constraints (rules about how to move), and through this scaffolded instruction, students begin to recognize patterns. So the math doesn't exist separately in some abstract world; it emerges from doing coordinated movements under specific constraints.

According to Abrahamson and Sánchez-García (2016), constraints play a crucial role in ecological dynamics and its application in mathematics education. In the context of ecological dynamics, constraints are seen as influential factors that shape the dynamics of a system. In the context of mathematics education, the application of ecological dynamics involves the use of constraints to guide students in developing fundamental understandings of mathematical concepts through motor-action inquiry problems. These task constraints perturb the existing dynamics of the student–environment system, leading to adaptations in the student's sensorimotor relations and the emergence of new ways of moving, thinking, and acting in relation to mathematical concepts. The role of constraints in ecological dynamics is thus closely tied to the process of learning and the development of mathematical coordination through guided, goal-oriented interactions with manipulable features of learning environments. With this

in mind, the pedagogical goal is to never teach anyone how precisely to do something; rather, it is to create conditions with optimal constraints where students need to figure out how to make that action possible for themselves, always underspecifying actions. Constraints, therefore, serve as a powerful analytical lens for understanding how learning systems function.

All the Ecological Psychology Analytics Tools Together

In sum, I have discussed how ecological psychology provides a powerful set of analytical tools for examining individual problem-solving. *Affordances* serve as a primary lens for identifying action possibilities and understanding those possibilities from the relationship between an individual and their environment—whether that environment is a climbing wall or a mathematical problem. Exploratory activity and movement as critical inquiry illuminate how affordances are discovered through physical exploration, shifting attention, and creative questioning—processes equally relevant whether one is navigating a climbing route or manipulating symbolic representations. Finally, constraints within ecological dynamics serve as an analytical lens for understanding how learning systems function: by attending to how constraints shape available affordances, how individuals adapt their sensorimotor coordinations within constrained environments, and how new patterns of action emerge through exploration, researchers can understand the dynamics of embodied cognition in problem-solving contexts. Together, these analytical tools reveal problem-solving as an embodied, adaptive and exploratory process—one where solutions can emerge through the skilled perception and manipulation of affordances within constrained environments, with plenty of space for exploration.

These tools, appropriated from ecological psychology, allow me to analyze how disabled climbers navigate problem-solving across rock climbing and mathematics. Rock climbing and mathematics both involve sensorimotor exploration in search of new affordances within constrained, rule-bound environments. This parallel then makes it possible to compare

how individuals problem-solve across these domains, revealing what is fundamental to their specific embodied problem-solving and what is contingent on the context that they are problems-solving in.

Ecological-Enactive Model of Disability

In this final section, I begin with an introduction to the concept of disability, discussing the importance of language and identity when talking about disability. Next, I provide a brief historical overview of disability, discussing the medical model (Retief & Letšosa, 2018) and the social model (D'Alessio, 2011). I then argue for the enactive-ecological model (Toro et al., 2020) and the concept of activist affordances (Dokumaci, 2023) to build upon the social and medical model. I emphasize the active role of disabled individuals in shaping their interactions with the environment, highlighting the importance of adaptability, agency, and potential in these interactions. I will argue for a more nuanced understanding of disability, one that acknowledges the role of both individual factors and societal factors that contribute to a more nuanced understanding that considers disability as a fluid continuum rather than a rigid binary. By centering the voices of disabled individuals and highlighting their problem-solving strategies in constrained environments, I will contribute to the desettling of traditional ableist scholarship of enactivism, ecological psychology, and math education more broadly.

Disabled is not a Bad Word

Vasu Sojitra, disability advocate and professional skier, states:

I say the word 'Disabled' loud and proud. 'Disabled' is not a bad word. 'Ableism' is. I use the word Disabled because it acknowledges the social impacts on Disabled people. I believe that saying the name brings awareness to the ableist standards that make spaces inaccessible – from health care, to education, to caregivers, to transportation, to job opportunities, to intimate relationships. It acknowledges the issues that language like 'differently-abled' erases. While differently-abled individualizes the disability, it does not acknowledge the issues our resilient community continues to face daily. Now, once you add other identities to the mix – like race, ethnicity, gender, sexuality, socio-economic status, zip code – these issues are exacerbated tenfold. This is the meaning of intersectionality. It is okay to

have a disability, we just have to make sure the barriers that are put on us by systems of ableism are being dismantled and people are actively working to help support folks with disabilities (2023).

In this dissertation, I will use the word *disabled*— not differently abled or dis/abled— to describe my participants. My decision aligns with the language preferred by many disability rights advocates and is congruent with the stance taken by the National Disability Rights Network (2020). Through this very explicit and intentional choice of language, I aim to center the lived experiences of my participants and amplify their voices while highlighting the injustices they face.

Identity-First Language

Disability, like other social identities, is a construct shaped by societal forces and carries significant political and economic weight (Oliver, 1990). Therefore, to align with this understanding, I will use identity-first language (e.g., disabled person) rather than person-first language (e.g., person with a disability). This is also the preferred language of my participants. Identity-first language allows disabled individuals to reclaim the term, expressing solidarity and pride in their identity (Brown, 2011). Identity-first language also challenges the harmful connotations historically associated with disability terminology. While person-first language may be seen as less offensive, it perpetuates the idea that disability is an individual problem rather than a reflection of broader societal issues (Saia, 2019; Dunn & Burcaw 2013; Swain et al., 2003).

Next, I will provide a brief historical context of disability in the US, beginning with the medical model, followed by a discussion of the social model. I will discuss the limitations of each model, namely, the ways in which both of these models create a disabled/non-disabled binary. I will then present the enactive-ecological model, ultimately providing a more nuanced understanding of disability that this dissertation leverages.

The Medical Model of Disability

The medical model started to gain traction in the mid-1800s with advancements in medical science (Retief & Letšosa, 2018). This conception of disability focuses on the pathological and abnormal embodiment of the disabled individual. For instance, people who use wheelchairs may be viewed as having "broken" bodies because they require medical tools to achieve mobility, rather than as people encountering inaccessible built environments. In this way, disability is often conflated with a tragedy due to its intrinsic negative perception. Medical professionals who subscribe to the medical model often do not consider the complex lived experiences of disabled individuals; instead, they are seen as a problem to be solved (Woods & Thomas, 2003). Disabled individuals are blamed in this view, suggesting they need to be "fixed" rather than considering that ableism is interwoven into the fabric of society. The medical model is still pervasive in society even today, where medical professionals euthanize, institutionalize, and even sterilize disabled people (Saia, 2019).

A particularly salient example is the medical model's approach to Deaf individuals. Rather than recognizing Deaf culture, with its own language (American Sign Language), history, community, and identity, the medical model frames deafness as a defect that requires correction through cochlear implants, speech therapy, and oralism (Baynton, 1996). This approach to deafness treats Deaf people as "broken hearing people" rather than as members of a rich linguistic and cultural community.

The Social Model of Disability

Inspired by the activism of the British disability movement in the 1960s and the 1970s, the social model of disability developed in reaction to the limitations of the medical model of disability (D'Alessio, 2011). The social model of disability locates disability within societal structures rather than individual deficits. The social model offers a clear framework for addressing disability through social change, advocating for removing physical, social, and attitudinal barriers. For

example, under the social model, people who use wheelchairs are not disabled by their inability to walk but instead by buildings designed with stairs and no ramps. This approach empowers disabled people and fosters a sense of community by shifting blame from the individual to societal structures. However, the social model has its limitations.

Critics of the social model argue that it oversimplifies the complex experiences of disabled people, failing to account for the diversity within the disability community. For instance, the social model may not fully capture the challenges faced by individuals with chronic pain or chronic fatigue. Indeed, removing architectural barriers doesn't automatically eliminate the bodily experience of exhaustion or pain that limits activity regardless of environmental design (Wendell, 1996). Additionally, the model's emphasis on social barriers can neglect the reality of some impairments that cannot be solely addressed through environmental changes. Finally, the ideal of a completely barrier-free world may not be entirely realistic (Shakespeare, 2013). In sum, the social model has been instrumental in the disability rights movement by highlighting the role of societal factors in disability; however, it is crucial to acknowledge the model's limitations and continue to develop a more nuanced understanding of disability.

Ultimately, those who adopt the medical model see disability as something wrong with the body that needs to be fixed. Whereas, those who adopt the social model focus on how society excludes disabled people which ultimately creates barriers. Both the medical and the social model do not account for the lived experiences and abilities of disabled individuals. However, disability is about how someone interacts with the world around them, not just a set of limitations.

Ecological-Enactive Model of Disability

Given the lack of nuance of the medical and social model, Toro et al. (2020) propose a new perspective on disability through the *ecological-enactive model*. They argue against existing models that define disability solely through a medical lens (pathology) or a social lens

(exclusion). The authors challenge the idea that disability inherently means a pathological embodiment. They argue that both the medical and social models overlook the lived experience of disabled individuals and their capacity to interact with the world. The core of the ecological-enactive model lies in the concept of "normal" embodiment. A normally embodied person can dynamically adapt their behavior to their environment, constantly rebalancing themselves in familiar and novel situations. For example, the medical model would view a blind climber as pathologically embodied due to lack of vision that requires correction. On the other hand, the social model would locate disability solely in inaccessible routes or lack of tactile route information. Yet, neither model captures how a blind climber actively develops tactile exploration strategies, spatial reasoning, and novel movement patterns, demonstrating adaptive and relational embodiment that is neither simply pathological nor simply socially constructed (Toro et al., 2020).

Adaptability goes beyond simply responding to immediate challenges; it involves the ability to establish new behavioral norms in response to ever-changing circumstances. In contrast, a pathologically embodied person may struggle to adapt. They become fixated on a single behavioral norm to avoid negative reactions, essentially limiting themselves to controlled environments. Toro et al. (2020) offer the example of someone with severe agoraphobia (an anxiety disorder where people fear situations or places where they may feel trapped or helpless) who can only function within their home environment. They have developed a behavioral pattern that works in this single controlled space but cannot flexibly adapt to novel situations outside. The thought of leaving triggers panic, so they become stuck in one way of being.

Toro et al. (2020) argue that disability does not necessarily equate to an " ...inability to tend toward an optimal grip on a field of relevant affordances" (p. 12). Instead, the authors conceptualize disability as the constant correction of the "I-cannot" experience. Every time disabled individuals can leverage affordances that overcome instability, they are also overcoming the limiting idea of I-cannot. The authors believe that able-bodied individuals

should not impose perceptions of pre-established normativity on disabled individuals. Indeed, pathological embodiment can be a product of socio-material practices that constrain disabled individuals' exploration in engagement with relevant affordances, including those that come to meaning in interactions with other people. Toro et al. (2020) believe that imposing societal norms of "normalcy" on disabled individuals can be detrimental and in fact, a socio-material environment that restricts their exploration of alternative ways of engaging with the world can actually push them toward a pathological state of embodiment.

Disability and Activist Affordances

Dokumaci's (2023) ethnographic study spans over 12 years and delves into the lives of individuals with various disabilities across the globe. She emphasizes that disability should not be seen as a fixed state but as a continuum that interacts dynamically with the environment. This interaction often leads to the creation of *activist affordances*—innovative and necessary adaptations that disabled individuals make to navigate and reshape their worlds. One of Dokumaci's key arguments is that disability can be seen as a method of creative practice.

Dokuamaci (2023) explores how disabled individuals, through their innovative adaptations and interactions, challenge and redefine the conventional understanding of affordances in ecological psychology. Activist affordances emerge not just from mere curiosity or rebellion but from the urgent need to adapt to an environment that often does not accommodate the unique experiences of disabled individuals. These affordances, according to Dokumaci, are born out of necessity in environments characterized by *shrinkage*, where the choices for action are severely limited. This contrasts with the traditional notion of affordances, which arise from a plethora of possibilities. In these shrunken environments, disabled people become expert problem-solvers because they must constantly identify new possibilities where non-disabled are restricted. Rather than simply selecting from abundant affordances, disabled individuals must generate new affordances through reconfigurations of their bodies, tools, and

environments. This resourcefulness born from necessity offers critical insights into how all problem-solvers might navigate highly constrained systems—including the constrained cultural ecology of mathematics.

The concept of shrinkage extends beyond the disabled community, suggesting that the approaches developed by disabled individuals in facing constrained environments can offer valuable lessons for broader societal issues, particularly in an era of limited resources in our environment. Furthermore, Dokumaci (2023) argues that disability, seen as *ecological shrinkage*, raises critical questions about the sustainability of our environment. The shrinking of affordances not only affects disabled individuals but also has broader implications for all life forms, urging a reevaluation of our relationship with the planet. In essence, *activist affordances* is an exploration of how the creativity and durability of disabled individuals in the face of constraints can offer insights into making our world more habitable and inclusive for everyone. It is a call to recognize, name, and theorize these adaptations as crucial contributions to understanding and improving our collective ecology.

This principle that designing for those most constrained by a system improves the system for all is thoroughly established in disability studies and universal design literature (Burgstahler, 2015; Hamraie, 2017). For example, curb cuts designed for wheelchair users also benefit parents with strollers, travelers with luggage, and others. Similarly, pedagogical approaches developed to expand affordances for disabled learners can enrich mathematical problem-solving opportunities for all students (Lambert & Tan, 2020; Padilla & Tan, 2019). By centering those who experience the most severe environmental shrinkage (disabled individuals), we gain insights into how to transform constraining systems into more expansive, inclusive spaces where diverse problem-solving approaches can flourish through the intentional consideration of expanding affordances for all students.

Ecological Shrinkage: A Bridge

Inspired by Dokumaci (2023), my conceptualization of disability centers on the environment's *shrinking affordance* set for a particular body. This *shrinkage* refers to the reduced opportunities for action an individual can take within their surroundings (Dokumaci). Rather than the landscape of affordances itself changing, I see this definition as indicating a reduced field of affordances relevant to the individual (Kiverstein et al., 2021; Rietveld & Kiverstein 2014). While the broader possibilities within the environment certainly remain, the person's ability to perceive and act upon those possibilities becomes restricted. This perspective positions disability as a spectrum, rather than a fixed category, as proposed by the social model. While the social model offers a valuable shift away from a medicalized or pathological view of disability towards societal factors, I am moving toward a more comprehensive understanding of the individual-environment interaction. My dissertation seeks a nuanced approach to disability that considers individual experiences, environmental interactions, and the political landscape of living with a disability (Dokumaci, 2017; 2023; Toro et al., 2020).

While Dokumaci (2023) develops the concept of shrinkage to understand how built environments contract affordances for disabled bodies, I take the concept further to extend its analytic power. Shrinkage occurs through physical design and through cultural, institutional, and pedagogical structures. Just as a climbing gym designed without considering diverse bodies shrinks the field of affordances for disabled climbers, mathematics education structured around exclusionary norms and practices (e.g., emphasis on speed and good grades, treating mathematics as disembodied abstract reasoning) shrinks the field of affordances for marginalized students. In both cases, shrinkage is a consequence of environmental design. Understanding how disabled climbers navigate and problem-solving amidst physical shrinkage through activist affordances provides insights into how all learners can navigate and transform systems characterized by cultural shrinkage. With this in mind, I now turn to examining the constrained cultural ecology of mathematics education.

Shrinkage in Math Education Cultural Ecology

There is strong evidence that suggests that constraints of math permeate the field at every level (Boaler, 2016; Gutiérrez, 2018; Leyva et al., 2021; Ernest et al., 2019; Papert, 1980). Its philosophical foundation often rests on Platonism, separating the theoretical realm of perfect mathematical forms from our messier lived experiences. This disconnect is exacerbated by a curriculum that emphasizes rigid rules and procedures over real-world connections, stifling creativity and genuine engagement (Schoenfeld, 2016). Traditional teaching methods that are still pervasive to this day are focused on rote memorization and passive learning, further alienating students (Boaler, 2022; Stains et al., 2018). The broader culture around math portrays it as an exclusive domain for a select few, fostering an intimidating atmosphere that discourages many learners. To be clear, I argue that there is a relationship between Platonism and elitism and between elitism and exclusion. This combination of Platonism, alienating curriculum, disembodied teaching, and an elitist culture poses a substantial barrier for marginalized students to engage in a meaningful and effective math education.

The National Council of Teachers of Mathematics' Annual Perspectives in Mathematics Education (2018) highlights the urgency of rehumanizing mathematics for marginalized groups, with particular emphasis on Black, Indigenous, and Latinx students. Gutiérrez (2018) stresses that to meaningfully address this, we must recognize the intersection of students' identities with power dynamics both inside and outside of schools. She advocates for explicitly centering these marginalized groups, building upon their strengths, as a way to improve math education as a whole, arguing "...in fact, mathematics education cannot truly improve until it adequately addresses the very students who the system has most failed" (p. 2). Padilla and Tan (2019) similarly offer a critical perspective on traditional views of disability in math education. They advocate for inclusive practices for disabled students, moving beyond mere access towards active participation. Influenced by Disability Studies in Mathematics Education, their study investigates guidance documents aimed at supporting the implementation of new standards

with an equity focus. The need to rehumanize mathematics, specifically for marginalized groups as highlighted by Gutiérrez (2018) and Padilla and Tan (2019), demonstrates that existing systems impose constraints including power dynamics and inaccessible design that limit equitable participation and success in mathematics education.

Expanding on equity concerns, Leyva et al. (2021) explore the experiences of Black and Latinx students within precalculus and calculus courses at a predominantly white university. Their critical race and feminist framework illuminates how exclusionary cultures are embedded in math classrooms. These cultures are built on unspoken norms, values, and assumptions – referred to as *logics* – that privilege whiteness and patriarchy. Leyva et al. identified two harmful logics: a) instructor authority discouraging participation, and b) calculus acting as a weeding-out course to eliminate those deemed unfit for STEM fields. These dynamics perpetuate marginalization and harmful stereotypes about belonging in STEM. In other words, the ways in which the culture is constrained can be interpreted as shrinkage as the cultural ecology of mathematics.

Leyva (2021), in a later article, digs deeper by looking specifically at the experiences of Black women in math education. He argues for both increasing representation and dismantling racial and gender biases, asserting that achieving a Black feminist revolution in math education requires this dual approach. Ernest et al. (2019) investigate how gender shapes participation in public and private mathematics classroom spaces, finding a troubling persistence of patriarchy even in seemingly progressive inquiry-based classrooms. These studies by Leyva et al. (2021), Leyva (2021), and Ernest et al. (2019) highlight how deeply embedded social identities and biases act as constraints within the cultural ecology of mathematics education.

The exclusionary logics described by Leyva et al. (2021) and Ernest et al. (2019) manifest psychologically as widespread math anxiety and mathphobia. Math anxiety is a global issue that is known to have a negative impact on performance in mathematics (Dowker et al., 2016; Foley et al., 2017). But what cultivates this anxiety in the first place? Papert (1980) argues that our

culture strongly suggests that only a select few individuals can truly appreciate the beauty of mathematics and experience the joy it offers, leading to mathphobia. Mathphobia is not just a hurdle in learning math and science, instead, it shapes how we see ourselves as learners in general). Papert argues that these anxieties create harmful "cultural toxins" (p. 8) that divide us into rigid binaries like "intelligent/dumb" or "artistic/not-artistic." These labels poison our self-perception, transforming the natural curiosity of a child exploring the world into an existence burdened by insecurities and self-doubt. The power of this idea lies in how it exposes mathphobia as the foundation, the very bricks that build the walls of the box we confine ourselves within. In other words, Papert argues that changing society's relationship to math changes society's relationship to learning.

Understanding math education as a constrained ecology — a cultural ecology that induces anxiety for many people and leads to marginalization for historically oppressed populations— motivates me to explore how students adapt to these limitations and, more importantly, how novel approaches to learning and problem-solving emerge within this constrained system. The constrained cultural ecology of math education shapes the dynamics of the system and impacts the opportunities for action (affordances) within a given system. To delve deeper into individual adaptation strategies within constrained environments, I will focus my research on disabled climbers. These individuals navigate a world inherently filled with constraints, requiring them to become resourceful in identifying and utilizing new possibilities to overcome these barriers. By studying how they problem-solve in both rock climbing and math, I gained valuable insights into how students in general can both adapt and thrive within a constrained math education system.

Theoretical Framework

Figure 1 encapsulates the theoretical framework showing the six layers from this literature review: starting with ableism (foundational critical lens) through problem-solving, embodied

cognition, ecological psychology, and disability models, culminating in analysis of cultural shrinkage in mathematics education. Disabled climbers' navigation of both physical and cultural shrinkage positions them as ideal informants for understanding constraint navigation across domains.

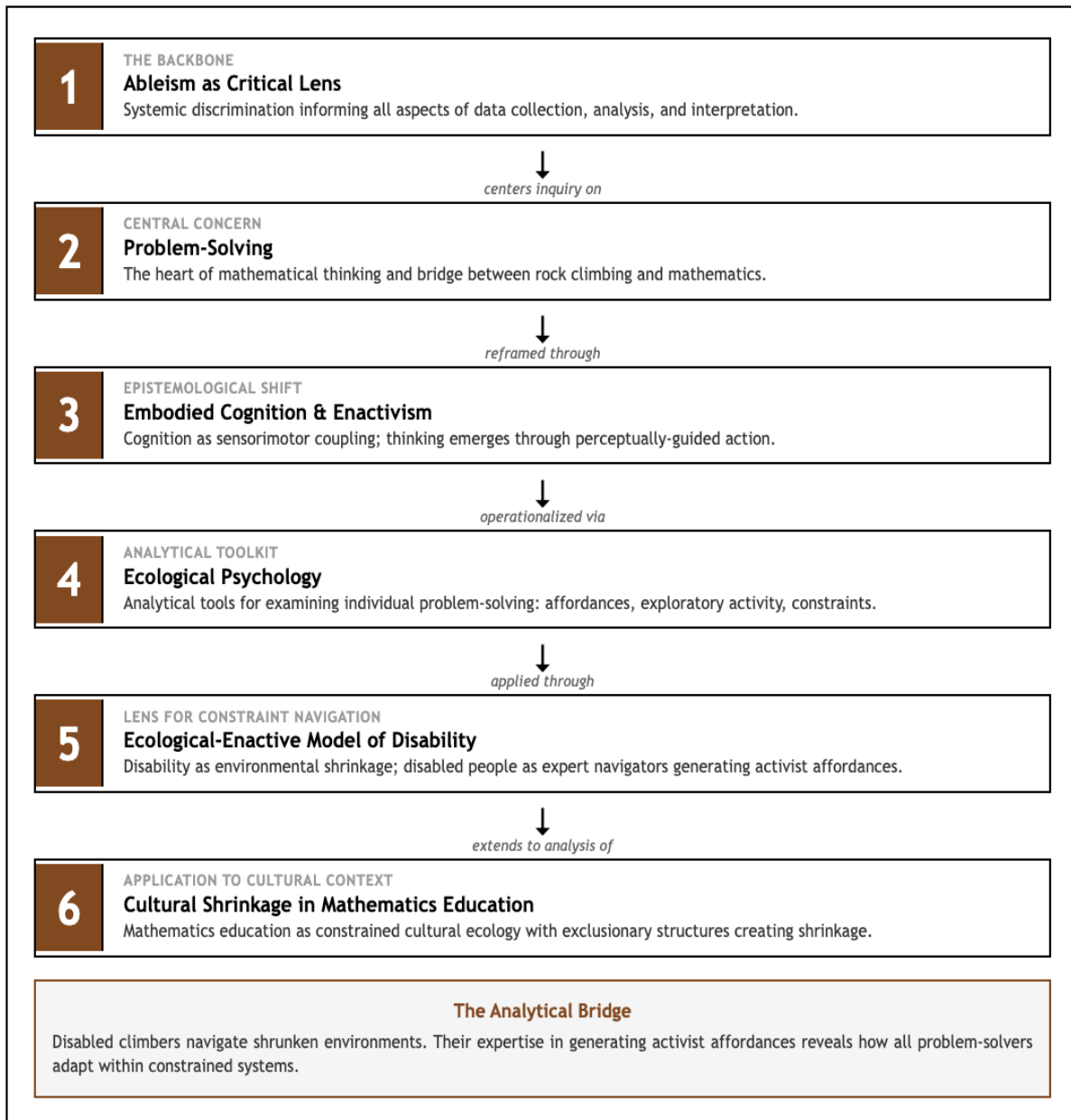


Figure 1: Theoretical Framework of the Study

The research questions guiding this dissertation (RQs) are as follows:

RQ1) What are the affordances and constraints designed into the context of the problem space for rock climbing?

RQ2) What are individuals' perceived similarities and differences concerning problem-solving across the domains of (a) rock climbing; (b) mathematics; and (c) their daily lives?

RQ3) What are the observed similarities and differences with regard to the problem-solving strategies amongst the domains of rock climbing and mathematics?

RQ1 leverages ecological psychology's analytical toolkit (Layer 4 from Figure 1), examining the gym climbing environment through the lens of affordances and constraints. By understanding how route-setters design problem spaces, I establish a foundation for analyzing how disabled climbers problem-solve within these constrained systems. The ecological-enactive model of disability (Layer 5 in Figure 1) is then deployed to reveal how the gym climbing environment is fundamentally designed for non-disabled climbers, creating ecological shrinkage (reduced field of affordances) for disabled climbers. Central to this process of analysis, ableism as critical lens (Layer 1 in Figure 1) ensures I examine whose bodies are centered in design decisions and whose are treated as an afterthought.

For RQ2, embodied cognition and enactivism (Layer 3 in Figure 1) is deployed to analyze participants' pre-intervention conceptualizations and unveil a disparity in the conceptualizations across domains. The enactivist principle would infer a coherence across domains, yet we will soon see that they see math as abstract and disembodied, unlike their perception of climbing. Thus, cultural shrinkage in mathematics education (Layer 6 in Figure 1) provides context for this gap. Again, ableism as critical lens (Layer 1 in Figure 1) is foundational to this analysis because it

reveals how ableist assumptions about proper mathematical thinking have systematically devalued embodied ways of knowing.

RQ3 is a conglomeration of all six layers. Problem-solving (Layer 2 in Figure 1) provides the central phenomenon under investigation: problem-solving as the bridge between rock climbing and mathematics. Embodied cognition and enactivism (Layer 3 in Figure 1) are what enable the bridge to be constructed (i.e., the epistemological thread) between the two domains: problem-solving is the sensorimotor exploration in search of new affordances in both domains. Ecological psychology (Layer 4 in Figure 1) operationalizes this framework through concrete analytical tools (affordances, exploratory activity, constraints). The ecological-enactive model of disability (Layer 5 in Figure 1) is then applied to interpret their problem-solving orientations across domains as activist affordances, born from navigating shrinking environments (Layer 6 in Figure 1). Finally, and most importantly, ableism as a critical lens (Layer 1 in Figure 1) operates throughout RQ3, as it informed all aspects of data collection, analysis, and interpretation.

Chapter 3: Methods

In this study, I investigated problem-solving strategies of disabled rock climbers across climbing and mathematical domains. The study employed a three-phase design. In Phase 1, I interviewed route setters at the gym to understand the context of the study and to understand the constraints and affordances designed into the climbing problem-solving space. In Phase 2, I examined climbing problem-solving through interviews, observations, and stimulated recall, and then I ultimately developed a holistic problem-solving framework. In Phase 3, I investigated mathematical problem-solving and used the framework from Phase 2 to select tasks and analyze the data to eventually examine transfer across domains. In this chapter, I discuss my positionality as a researcher, the setting and context of the study, and the methods for data collection and analysis that I employed to address the research questions.

Positionality

In high school, I was stripped from mainstream schooling and placed into special education. Poked and prodded, test after test, I was numbed by a cocktail of medications as psychiatrists and educators tried to understand why I learned so differently. I found solace in mathematics, which wholeheartedly saved me at the time. I plunged into a world that made sense, finding safety in the logical certainty of math. Many years later in my master's program, that same mathematical culture nearly destroyed me, demanding I fragment myself to succeed in a cold and rigid environment devoid of emotion, embodiment, and wholeness. But then climbing changed everything.

In climbing, I began discovering embodiment—what it meant to solve problems with my body, what it meant to know and listen to my body. Yoga deepened this understanding, teaching me the wisdom that the body holds. I could no longer partake in mathematics as a disembodied pursuit. I knew I wanted to help shift the field toward honoring the brilliance of the

body and of the rich lived experiences we all bring. When I came to San Diego for my PhD to pursue this vision, I was welcomed into the adaptive climbing community. Here, I witnessed disabled climbers alchemizing constraints into the most badass pathways up the wall. I witnessed brilliant, creative problem-solving that academia had trained me not to see. I listened to their stories and knew the world needed to learn from them too. This dissertation is my attempt to honor what they taught me.

Though I am not explicitly disabled in the ways participants in this study experience, I know what it means to navigate constraints. I know what it means to be otherized, to have my ways of learning pathologized. I know what it is like to develop adaptive strategies in spaces not designed for me. I inhabit what Anzaldúa (1987) terms *nepantla*—the space between worlds. I live on multiple bridges: between mathematics' logical precision and the embodied and emotional realm; and within the disability community itself, where my neurodivergence positions me on the margins due to its lack of visibility. As a neurodivergent brown woman who grew up in the Deep South, I intimately understand constraint navigation. Yet, I am often received as non-disabled, affording me cultural capital in a way that my participants seldom receive. This liminal positioning within disability spaces therefore means I benefit from privilege while simultaneously experiencing exclusion. It has been such a reflective journey of understanding when my voice amplifies and when it is speaking over. Indeed, being an ally means not being the loudest voice in the room.

Given my unique positionality, I was attentive throughout this study to the potential for interpretations to be shaped by my biases. Several measures were taken to address this. A second coder reviewed and applied the coding scheme to excerpts from phase 2 data. Additionally, I engaged in member checking with all six participants across both phases. These measures are discussed in further detail later in this chapter.

Research Design

I first conducted a pilot study to investigate the feasibility and validity of the research methods and procedures. This pilot phase involved a small-scale trial run of the study design, including data collection techniques. The pilot study aimed to identify any potential challenges or limitations early on, allowing me to make necessary adjustments before proceeding to the main phases of the research. Given that this study is collecting observational data from rock climbers while they are climbing, it was imperative to conduct a pilot study as a proof of concept. By embarking on this preliminary investigation, I aimed to validate some of the methods and techniques employed, ensuring their feasibility, reliability, and efficacy. The pilot study served as a crucial step in refining the protocols, identifying potential challenges, and making appropriate adaptations to the research design. Practical adjustments were also made to the interview protocols, ensuring that the conversations flowed naturally and captured the data needed. Additionally, I successfully tested a more streamlined technology setup, switching to a phone and earbud combination for recording from my initial plan of using a GoPro and camcorder. This change improved both efficiency and the overall participant experiences. These outcomes validated my chosen methodology for data collection, providing evidence of the feasibility and reliability for the data collection methods used in this dissertation.

In Phase 1, I conducted a focus group (to address RQ1) with the route setters at the main climbing gym in San Diego (where most of the participants climb) of the study to get a lay of the land to understand the environment that I am working within, as well as to understand their methodology in terms of constraints and affordances when it comes to problem creation.

Phase 2 addresses RQ2 and part of RQ3. In RQ2, I aimed to understand (through semi-structured interviews) how participants conceptualize problem-solving across domains and extend this conceptualization to everyday life. After I understood their articulated conceptions of problem-solving, I then turned to the rock climbing phase of the study. This part of the study focused on data collection from disabled climbers through a rock climbing observation session

and a stimulated recall interview. Here, I investigated their experiences, challenges, and problem-solving strategies in navigating climbing routes. The primary goal of this phase was to gain an in-depth understanding of conceptions as well as enacted problem-solving processes. By capturing and analyzing rich qualitative data in Phase 2, I set the foundation for the subsequent phase by creating a holistic problem-solving framework that I utilized in Phase 3. Phase 3 addresses the other part of RQ3. During this phase of the study, I shifted the focus from rock climbing to mathematical problem-solving with the participants. The problems presented to the participants are described in detail later in this chapter. Table 1 below provides an overview of the data collection and analysis timeline.

Table 1: Overview of Study Data Collection and Analysis Timeline.

#	Phase	Data collection	Primary Data Analysis Methodology	Data Collection Timeline	Research Question	Analysis Timeline
0	Pilot Study	Preliminary Individual interview; Rock climbing session; Brief pre and post-climb interviews; Stimulated recall interview	N/A	Spring 2024	N/A	N/A
1	Environment Conditions	Semi-Structured focus group with route setters	Content analysis	Spring 2024	1	Spring 2024
2	Humanizing problem-solving framework creation	Preliminary Individual Semi-structured interview; Rock climbing session; Brief pre and post-climb interviews; Stimulated recall interview	Grounded Theory	Summer- Fall 2024	2 & 3	Summer 2024-Spring 2025
3	Doing mathematics	Individual task-based clinical math interviews; mathematics stimulated-recall interview	A priori with developed framework from Phase 1, then open coding	Spring & Summer 2025	3	Fall & Winter 2025

Emic-Etic Positioning and Intervention

Next, I will introduce two anthropological concepts that frame how knowledge is generated through research: emic and etic perspectives. *Emic* refers to the insider's perspective, that is, how individuals themselves understand, interpret, and describe their own experiences

(Pike, 1967). *Etic* refers to the outsider's perspective, analyzing a phenomenon using theoretical frameworks developed by researchers (Pike, 1967). In this study, disabled climbers hold *emic* knowledge about their experiences navigating disability-constrained problem-solving. In this study, my positioning is complex: I am both a rock climber and a neurodivergent person who has navigated educational marginalization, therefore I hold some *emic* knowledge. I deeply understand climbing culture and I have lived experience with constraint navigation, though my specific experiences differ from those of my physically disabled participants. I do not experience the same physical disability that my participants do which impacts both climbing and day-to-day activities in the same way. Concurrently, I bring an *etic* analytical lens shaped by enactivism, ecological psychology, and mathematics education scholarship in my interpretation of how problem-solving strategies transfer across domains.

This study operates within what Guba and Lincoln (1989) term *fourth generation evaluation*, which is centered in participants' lived experiences and co-constructing knowledge through dialogue. Unlike traditional evaluation approaches that position the researcher as an objective and detached judge, fourth generation evaluation is responsive (shaped by participants), constructivist (recognizes multiple valid realities), interactive (generates understanding through exchange), and emergent (design unfolds through progression). This framework grounds this study as an *intervention*: the research assessed existing problem-solving strategies and in doing so, it transformed participants' capacity to recognize those strategies as transferable across domains. Indeed, I explicitly invited disabled climbers to leverage their climbing expertise in mathematical contexts. Therefore, I cannot separate what transfer existed prior to the research from what transfer the research itself made possible. Fourth generation evaluation thus legitimizes a methodology where *emic* expertise and *etic* analysis coalesce through interaction in this study, transforming "observation" into intervention and making the research interaction itself a source of knowledge.

Context and Setting

The Climbing Gym

This dissertation focuses on rock climbing within the context of indoor climbing gyms. These facilities feature artificial climbing walls with color-coded routes that challenge climbers physically and mentally. The indoor climbing gym environment is distinct from outdoor climbing settings, offering a more controlled and accessible space for research purposes. Participants in this study navigate climbing walls, using harnesses and safety equipment to ascend and descend. The controlled setting allows for focused observations and data collection related to participants' problem-solving strategies and embodied experiences in a confined and standardized space. The indoor climbing gym setting ensured consistency across participant experiences. Route setters design these problems by manipulating constraints like hold type, placement, and orientation. Climbers rely on strategy, adaptation, and embodied problem-solving. In this sense, rock climbing integrates embodied, contextual interaction with planning and adjustment, much like mathematical problem-solving.

Both math and rock climbing operate within rigid parameters but allow for creativity and individual solutions shaped by prior experience. Indoor rock climbing routes have prescribed starting and finishing points, and climbers must use specific holds to progress. Despite these limitations, the experienced climber can adapt their technique, exploiting holds beyond the intended path for efficiency and creativity. Similarly, mathematics problems may present specific equations, formulas, initial values, and clear goals. However, experienced mathematicians can discover original problem-solving techniques, prove concepts in novel ways, or even develop entirely new methods. Thus, both domains balance rule-following with exploration and creative problem-solving.

The relatively stable environments of the climbing gym and the math problems have been selected because they present stable problem-solving environments and potentially,

control for the problem-solver. In contrast to sports where the focus can shift due to external factors like surfing or tennis, rock climbing and math offer challenges that require the individual to utilize their knowledge and skills actively while being in control of their environment. Holds remain fixed, offering the climber a static puzzle to solve; equations remain static until the solver acts on them. This stability is crucial, as it allows focus on a core cognitive process: continuously acting on affordance, creating and adapting plans, and assessing progress.

At their core, rock climbing and math share fundamental cognitive processes involving a cyclical pattern of planning, execution, and assessment (Rucińska, 2021). Rock climbing, like other embodied activities, trains the cognitive processes to refocus attention and mentally rehearse movement under constraints. This is evident when climbers step back to reassess their route approach. For the convenience of the reader, I have provided a glossary in Appendix C with some of the most commonly used specialized rock climbing terms.

Puzzles

Nguyen (2017) describes rock climbing as a "subtle, refined, and often hyper-intellectual sport. It's solving puzzles, with your body and mind" (p. 2). The normative primary objective is to reach the top, often through an elegant (i.e., most efficient and least number of errors) series of movements dictated by the unique challenges posed by the rock. The difficulty of the climb often enhances the experience, and precise and efficient movement is not just aesthetically pleasing, but often necessary for success. Even when imitating others, climbers must adapt those movements to their own bodies and the specific demands of the rock. This puzzle-solving aspect of climbing has even attracted mainstream attention, with a recent article in the New York Times highlighting the parallels between climbing and solving crossword puzzles (Vo, 2024).

Even outside the scope of rock climbing, movement philosophers conceptualize problem-solving in dance to be puzzle-oriented. Noland (2009) describes choreography in dance "...not as an aesthetic practice, but as the production of puzzles for the body to solve,

puzzles that require it to cope, to enact its kinesthetic and proprioceptive capacities, in unusual and taxing conditions" (p. 1). Indeed, these puzzles demand that dancers utilize their sense of movement and sense of body position in novel and challenging ways. This perspective highlights the problem-solving nature inherent in movement, where dancers (and climbers) must decipher and execute choreographic instructions — whether explicitly from a choreographer of dance or implicitly from a series of climbing holds in the rock climbing gym— often under physically demanding conditions.

Both rock climbing and solving puzzles are human activities that have evolved from our natural ability to solve problems in the environment. From an evolutionary psychology perspective, we needed to navigate complex terrain, manipulate objects, and figure out solutions to survive (Cosmides and Tooby, 1994). Rock climbing and puzzle-solving can be seen as modern-day manifestations of those skills, refined and transformed into recreational activities that challenge and engage our minds and bodies. Both the practices of rock climbing and solving puzzles are simply more explicitly embodied ways to engage in problem-solving that readily translate to the mathematical domain.

Recruitment

Participants were recruited through established relationships within local adaptive climbing communities, where I have been an active member for 5 years. Indeed, I created this study within a community that I helped to build and had rapport with (the local nonprofit adaptive climbing chapter). In other words, the participants in this study were all people with whom I had close relations with. Therefore, the type of data that emerged from this study was a product of trust built throughout several years.

Participants gave their informed consent and had the opportunity to withdraw from the study if they chose not to continue. Some recruited participants have since relocated to Salt Lake City, necessitating my travel between San Diego and Salt Lake City for data collection.

Validation Check

To validate the analytical framework and mitigate potential researcher bias, I enlisted a second coder to review and apply the coding scheme in the second phase of the study. The second coder (a doctoral student in the program) examined representative excerpts from the data to assess whether my code applications were defensible and whether alternative interpretations might be equally valid. Throughout this process, we discussed areas of divergence, which led to further refinement of code definitions.

Member Checking

Throughout the Phase 2 and 3 analysis process, I engaged in member checking to ensure interpretations honored participants' lived experiences. After developing preliminary findings for each participant's case study, I shared draft sections with each participant and invited their feedback on whether my interpretations resonated with their experiences. All six participants responded affirmatively in both phases, indicating that the analyses resonated with their experiences. Further, several participants noted that seeing their problem-solving visualized helped them recognize patterns they had not explicitly articulated before. This process ensured findings represented participants' lived experiences rather than solely from my interpretations.

Participants

All twelve participants (six route-setters and six disabled rock climbers) consented to participate in this study and signed the required IRB-approved consent forms. To participate in the focus group interview in Phase 1, individuals needed to be a route-setter at the main climbing gym where the study takes place (in San Diego) and have at least one year of experience with route setting. To participate in the climbing and math phases of the study (Phase 2 and 3), participants needed to self-identify as a disabled climber with at least one year of climbing experience and be 18 years of age or older. The recruited participants range in age

from 20 to 37. They have differentiated levels of expertise in mathematics. Below, I delineate the participants in Phase 1 (route-setters) and then the participants in Phase 2 and 3 (disabled rock climbers).

Route-Setters

The route-setters are the group of people who design and create the different climbing routes at the gym. Their work involves creatively placing and arranging holds to construct climbing problems for climbers to solve. For my focus group interview, I gathered insights from five male and one female route setter. Their experience levels ranged from one year to sixteen years in the field, offering a diverse set of perspectives on route-setting philosophies and practices. They all worked at the main gym the study took place in San Diego and all identify as non-disabled.

Taylor

Taylor is a 25-year-old male and is the Adaptive Program Specialist at the climbing gym. He has a spinal cord injury resulting in lower-body paralysis. He utilizes a manual wheelchair in his day-to-day life and climbs routes using solely his upper body, called "campusing," which involves a series of dynamic pull-ups as he navigates routes. Taylor began climbing five years ago, four months after his injury. Taylor also competes in rock climbing internationally. Taylor's experience in mathematics is self-proclaimed limited, and he took his last math class in high school.

Christopher

Christopher is a 23-year-old male and is a freshman at university, studying kinesiology. He is a Marine Corps veteran. He has a below-the-knee amputation and climbs with a prosthetic foot attachment specifically created for climbing. He has been climbing for two years and also competes in rock climbing internationally. He began climbing a couple of months after his

amputation. Christopher's last math class was in high school, though he describes having an affinity toward math and his mother is a high school math teacher.

Chad

Chad is a 37-year-old male who lives in Stratford, Canada. He is a competitive climber and was a diesel mechanic before he lost his vision. He lost 90 percent of his vision five years ago to a hereditary disease called Leber hereditary optic neuropathy. He has been climbing for almost five years, starting just a few months after his sight loss. Chad's last math class was in high school, but says that he enjoyed math and was rather good at it.

Kenneth

Kenneth is a 34-year-old male who lives in San Diego, California. Kenneth is currently the Adaptive Programs Lead at one of the climbing gyms in which the study took place. He is deeply involved in the adaptive climbing community and leads trips with disabled climbers. He previously worked in software development and prosthetics and has a three-year degree in software development from New Zealand, his last math class being in high school. He has a below-the-knee amputation and climbs with a prosthetic foot attachment specifically created for climbing. He has been climbing for over seven years.

Sofia

Sofia is a 27-year-old female who lives in Salt Lake City, Utah. She is a product developer at an outdoor company and has a Master's degree in Design and Innovation. She has Retinitis Pigmentosa, which has caused peripheral field loss, leaving her with only around 5 degrees of vision. Sofia has been climbing for eight years and competes in rock climbing internationally. Her last math classes were in her first year in college. Sofia only participated in the rock climbing phase of the study.

Ashley

Ashley is a 20-year-old female who lives in San Diego, California. She is currently a sophomore at university and is majoring in computer science, currently taking several mathematics classes. Ashley has Friedreich's Ataxia, a condition that has resulted in her using a wheelchair. She has been climbing for over a year.

Data Sources

In the first phase of the study, I conducted a focus group with route-setters. In the second phase, I conducted individual semi-structured interviews, rock climbing sessions and stimulated recall interviews for the rock climbing sessions with 6 participants (not included with route setters). In the third phase of the study, I conducted task-based clinical math interviews and stimulated recall for the task-based clinical math interviews with the Phase 2 participants.

Phase 1: Route-Setter Focus Group

To gain a deeper understanding of the climbing gym environment where my study will take place, and to answer RQ1, I conducted a semi-structured focus group interview with the route-setters, the individuals responsible for creating the climbing problems. This interview sought to elicit route-setters implicit methodology for designing climbing routes. See Appendix A for the focus group interview protocol. The data was recorded and transcribed using the Zoom transcription tool. Due to the limitations of the transcription software, I verified the transcripts through listening to the interviews.

Phase 2: Rock Climbing

In Phase 2, the data sources leveraged were a semi-structured initial interview, video recording of a rock climbing session, and stimulated recall interview for the rock climbing session. See Appendix A for specific protocols.

Semi-Structured Initial Interview

Each participant took part in one semi-structured interview before they went rock-climbing. The semi-structured interview protocol (Ginsburg, 1997) was designed to elicit their history and experiences with rock climbing, mathematics, problem-solving, and disability. The data was recorded and transcribed using the Zoom transcription tool. I then verified the interviews while listening to the interviews. After each interview, a contact summary form was completed that serves as an initial practical condensation of data (Miles et al., 1994). Additionally, this form served as a reference during the stimulated recall and task-based interview with the participant. Adhering to the guidelines set forth by Miles et. al., the contact summary form is designed to be concise, fitting on a single sheet of paper, and capture key concepts, themes, issues, and questions arising from interactions with the participant. See Appendix A for the semi-structured initial interview protocol and contact summary form.

Rock Climbing Session

The rock climbing routes selected differ for each participant due to different levels of experience. Some participants lead climbed and others top-roped (See Appendix C for definitions of each). Nonetheless, the methodology of route selection was relatively consistent across all the participants: Most of the participants climbed up to four routes each, where each climb was increasingly constrained and the final route was at their limit.

A phone was set-up on a tripod to videotape climbers from a third-person perspective while climbing. Before each climb, I conducted a brief, videotaped check-in with a phone where participants previewed the route and anticipated difficult sections. The route preview and discussion of the climb beforehand is a common practice in rock climbing. This allowed for targeted reflection during later analysis. After each climb, I checked in again to capture a brief self-assessment of the climb. Participants engaged in up to four climbs of increasing difficulty, culminating in a final route that pushed their individual limits.

Rock Climbing Stimulated Recall Interview

As a post-climb measure, I conducted semi-structured stimulated recall interviews (Dempsey, 2010) with each participant two to three days after their climbing session. To initiate the interview, I presented them with a picture of their completed climb and asked them to mark key elements while explaining their overall strategy and specific decisions made throughout the ascent. Next, I showed them select video clips from their climb, prompting a reflective discussion about their experiences, perceptions, and emotions during the activity. This stimulated recall approach aimed to capture not only in-the-moment decision-making but also post-climb insights, yielding a comprehensive understanding of how their problem-solving strategies connect with their reflections. The data was recorded and transcribed using the Zoom transcription tool. I then verified the interviews while listening to the interviews. See Appendix A for the stimulated recall interview protocol.

Phase 3: Mathematics

In Phase 3, the data sources leveraged were a task-based clinical math interview, and a stimulated recall interview for the math interview. All but one participant (Sofia) participated in the third phase of the study. See Appendix A for specific protocols and Appendix B for information on specific mathematical tasks.

Task-Based Clinical Math Interview

I conducted a one-hour individual task-based clinical math interview (Appendix A) for the second phase of the study. Clinical interviews are a type of semi-structured interview that help researchers more fully understand participants' thinking (Bernard, 1988). They are characterized by an initial standardization of a task, the use of "how did you do it" and "why" questions, immediate interpretations of responses, improvisation, and evolution of the interview process. I selected a clinical interview as an instrument because it allows for hypothesis testing,

revealing insight into problem-solving strategies. This approach enabled deeper exploration of participants' thinking and cognitive mechanisms, especially those they may not readily articulate. In rock climbing, climbers often rely on communication with their partner to solve problems and figure out sequences of movement. The clinical interview mirrored this collaborative dynamic, enabling me to probe and occasionally (sparingly) scaffold participants through the problem-solving process.

I used task-based questions in the clinical math interviews (Zazkis et al., 1998). This approach encouraged participants to construct solutions rather than simply 'finding' them. Given the varying levels of mathematical experience among participants, I selected puzzles that emphasized creativity and problem-solving rather than requiring foundational knowledge in areas such as algebra or calculus. As a commitment to the theoretical frameworks driving this study, I selected tactile mathematical tasks. Leveraging affordances and constraints and the conceptualization of both math and rock climbing as sensorimotor exploration of affordances, the tasks selected in this portion of the study directly engaged participants in physical manipulation and embodied problem-solving. This allowed me to investigate the interplay between sensorimotor exploration and the perception of affordances and constraints.

The theoretical framework driving this study, enactivism, argues that even "thinking in your head" and abstract symbol manipulation remain matters of embodiment. Thus, it was essential to note that analogous problem-solving strategies in this mathematical context need not manifest in explicit movement itself. The mathematics problems given to participants did not necessarily need to elicit overt movement but instead could induce similar problem-solving strategies (e.g., reassessing the problem when participants noticed themselves perceptually fixated on one particular solution path). However, the selection of tactile mathematical tasks created an analogue to problem-solving in rock climbing. This decision also held strong

axiomatic alignment with social justice principles, ensuring accessibility for all participants with diverse abilities.

I selected three puzzles for the mathematical task-based interviews: the Soma Cube, Towers of Hanoi, and Tangram (see Appendix B for images and task descriptions). The Soma Cube is a three-dimensional magnetic puzzle requiring participants to assemble seven pieces into a $3 \times 3 \times 3$ cube. The Towers of Hanoi consists of three pegs and a set of discs of different sizes that must be moved from one peg to another following specific rules. The Tangram is a puzzle consisting of polygons that must be arranged to form a specific shape. Each puzzle was chosen based on several dimensions: variety strategies, constraints/affordances, observability, and connection to rock climbing problem-solving (see Table 2).

All three of the mathematical tasks selected for this study originated across disciplines, and notably, outside of the mathematics classroom. The Soma Cube was developed during a physics lecture in 1933, where it was later adopted by psychologists to study problem-solving persistence and intrinsic motivation (Deci, 1971). Towers of Hanoi was developed in 1883 and has since become one the most cross-disciplinary mathematical tasks in existence (Hinz et al., 2013). Finally, the tangram is rooted in third-century geometric reasoning and was initially used to explain mathematical principles through embodied rearrangement (Shen et al., 1999). Ultimately, these tasks are consistent with the epistemological commitments of this study: that meaningful mathematical problem-solving does not need to be bound to the classroom, and that generative tasks such as these often live far beyond the classroom.

Mathematics Stimulated Recall Interview

After the mathematical problem-solving, I conducted semi-structured stimulated recall interviews (Dempsey, 2010) with each participant immediately after they did the puzzles. To initiate the interview, I presented them with either video footage of their solving or explicated

salient moments from their problem-solving. I then prompted a reflective discussion about their experiences, perceptions, and emotions during the activity. This stimulated recall approach aimed to capture not only in-the-moment decision-making but also post-problem-solving insights, yielding a comprehensive understanding of how their problem-solving strategies connect with their reflections. The data was recorded and transcribed using the Zoom transcription tool. I then re-transcribed the interviews while listening to the interviews. See Appendix A for the math stimulated recall interview protocol.

Table 2: Mathematical Task Design Rationale

Mathematical Tasks	Potential Strategies	Constraint/Affordance	What Can be Observed?
Soma Cube	<u>Trial & Error:</u> Testing different piece orientations and positions to find configurations that fit. <u>Spatial Reasoning:</u> Mentally rotating pieces to visualize how they might fit together in 3D space.	<u>Constraints:</u> Fixed pieces: Seven specific polycube pieces must all be used. Fixed dimensions: Final structure must be exactly $3 \times 3 \times 3$. <u>Affordances:</u> Multiple solutions: 240 possible configurations afford diverse pathways to success..	Overt physical manipulations; as they create new structures, I can tangibly see those structures being created.
Towers of Hanoi	<u>Breaking down:</u> Breaking the problem into smaller subproblems (e.g., moving $n-1$ discs, then the largest disc, then $n-1$ discs again). <u>Sequencing:</u> Determining the exact order of moves needed to transfer all	<u>Constraints:</u> Rigid rules: Only one disc can be moved at a time. <u>Affordances:</u> Clear rules: Simple, unambiguous constraints enable focused problem-solving.	Strategies are observable through deliberate, sequential actions and planning pauses.

Table 2: Mathematical Task Design Rationale Continued

Mathematical Tasks	Potential Strategies	Constraint/Affordance	What Can be Observed?
Tangram	<u>Trial & Error:</u> Testing different piece arrangements to find configurations that fit the frame <u>Backing Off:</u> Removing pieces when configurations don't work and starting over with a different approach.	<u>Constraints:</u> <u>Fixed frame:</u> All pieces must fit within the defined rectangular boundary. <u>Affordances:</u> <u>Visual feedback:</u> Gaps and overlaps are immediately visible, guiding adjustments.	Strategic approaches (starting with corners vs. edges vs. center). Recognition of "stuck" moments when configurations don't work.

Data Collection

Phase 1: Route-Setter Focus Group

The data that were collected from the focus group with route-setters were used to address RQ1 to understand the constraints and affordances designed into the rock climbing space. The data collected were a 1.5 hour audio recorded interview and corresponding cleaned-up transcription. The audio recording and transcription ensure a comprehensive record of the verbalization and interactions during the interview. Field notes and a contact summary form were completed as well.

Phase 2: Rock Climbing

For the rock climbing phase of the study, I collected data from a semi-structured initial interview, rock climbing session, and a stimulated recall interview each from 6 participants.

Semi-Structured Initial Interview

The data collected from the semi-structured initial interviews with each participant was used to answer RQ2. Each interview lasted approximately 60-100 minutes. The data collected from the interviews was audio recordings and cleaned-up transcriptions to ensure a comprehensive record of the participants' verbalizations and interactions during the interview. Field notes and a contact summary form was completed as well.

Rock Climbing Session

The data collected from the rock climbing session was used in part to answer RQ3. The data collected was video footage that captured all of their movement on the climbs, and brief pre and post-climb reflection videos. The rock climbing sessions ranged from thirty minutes to two hours of data. I also took a picture of each climb the participant did that will be used during the stimulated recall interview, allowing participants to mark key elements and explain their climbing strategies.

Rock Climbing Stimulated Recall Interview

The data that was collected from the rock climbing stimulated recall was used in part to answer RQ3. The data collected from the stimulated recall interviews was audio recordings and cleaned-up transcriptions to ensure a comprehensive record of the participants' verbalizations and interactions during the interview. I also collected the pictures of the climbs that they did with their markings on it. Field notes and a contact summary form was completed as well.

Phase 3: Mathematics

For the math phase of the study, I collected data from a task-based clinical math interview and a stimulated recall interview.

Task-Based Clinical Math Interview

The data that was collected from the task-based clinical interview will be used to answer part of RQ3. Each interview took about an hour. Additionally, the interviews were videotaped to capture participants' gestures and problem-solving, providing a visual element. Simultaneously, the audio was recorded and transcribed to ensure a comprehensive record of the participants' verbalizations and interactions during the problem-solving process. This multi-modal approach aimed to capture both the visual and auditory aspects of the participants' cognitive processes. Field notes and a contact summary form was completed as well.

Mathematics Stimulated Recall Interview

The data that was collected from the task-based clinical math interview was used to answer part of RQ3. The data that was collected from the mathematics stimulated recall interviews were audio recordings and transcriptions to ensure a comprehensive record of the participants' verbalizations and interactions during the interview. They were all about an hour long. Field notes and a contact summary form was completed as well.

Data Analysis

Analysis for RQ1

I conducted a content analysis (Weber, 1990) of the focus-group interview with the route setters to answer RQ, which aimed to understand the environmental conditions of the rock climbing environment in terms of constraints and affordances. In qualitative research, content analysis involves a systematic examination of textual data to identify and interpret patterns, themes, and meanings related to the research question. The goal was to identify specific instances where the concept of affordances and constraints were discussed, providing insights into how the setters create problems that give opportunities or limit opportunities for climbers.

Drawing on Gibson's (1979) concept of affordances and Newell's (1986) constraints framework, I developed an a priori coding scheme for the content analysis. Constraints are defined as limitations or restrictions that shape possibilities for action (Gibson, 1979; Newell, 1986). Newell identifies three types: organismic (individual capabilities), environmental (physical surroundings), and task-related (goal demands). Affordances are defined as action possibilities that emerge from organism-environment relations (Gibson, 1979; Chemero, 2013). In climbing contexts, research demonstrates that affordances are contingent on skill, with experts perceiving richer possibilities than novices (Rucińska, 2021).

To operationalize these concepts to route-setter interview data through analysis, I identified language patterns commonly associated with each construct: a) Constraint-related language is related to discussions of limits, restrictions, requirements, barriers, or what climbers cannot do; affordance-related language is related to discussions of possibilities, options, opportunities, or what climbers can do. I developed this coding mechanism by examining how constraints and affordances are discussed in climbing literature (Seifert et al., 2017; Hacques et al., 2021; Rucińska, 2021) and ecological psychology applications to learning (Abrahamson & Sánchez-García, 2016). Below is an example of the language associated with each concept:

- Constraint indicators: "limits," "restrictions," "barriers," "challenges," "difficulties," "obstacles,"
- Affordance indicators "opportunities," "possibilities," "enables," "allows," "affords," "multiple solution paths," "options," "different ways"

The coding scheme was systematically applied to the focus-group interview transcript using MAXQDA. Each instance of the identified keywords and phrases were coded as either constraint or affordance. The coded data were then categorized and organized to identify patterns and relationships. Instances of constraints were grouped together, and instances of affordances were also grouped together. See Table 3 for an example of this process for the

coding of affordances. As seen in the table below, the phrases associated with affordances were identified and bolded in each direct quote, illuminating a primary category within affordances: different solution paths.

While the overarching constraint/affordance binary was established deductively from operationalizing the literature, domain-specific sub-categories emerged inductively during analysis. For example, the affordance category revealed several different ways that route setters' consider multiple solution paths. These emergent patterns are fully detailed in Chapter 4.

Table 3: Example of Affordances Content Analysis

Affordances: These are opportunities or possibilities for action that the environment offers. In climbing, affordances include holds, features that enable resting or strategic body positioning, and the overall layout of the route that allows for different sequences of moves (different solution paths). "They have to solve it where their body is and everybody has, You know they have, There's different solutions for everybody, you know whether they're weak or they're strong or they're small or they're tall or they're that they're flexible or not. And then they just take all their tools and use it to come up with their own solution." <i>Different solution paths</i> "And I think that one. the one thing that might not be maybe so related to math where and maybe you can, maybe it's just my understanding in that. But there's never really one solution to things. You could think of it in a way, as there is one solution for you to do it, But there's not a wrong way in climbing" <i>Different solution paths</i> "The thing is we that actually is a philosophy that our team has, that there is no breaking beta It is using the tools that you have to do it in your own way" <i>Different solution paths</i> " as route setters know there are multiple ways to do things for the most part." <i>Different solution paths</i> "But I think that's what makes our team a little bit different is we don't try to force things. We try to make things feel, maybe intentional is not the word, but intuitive enough where it gets close to what we want them to do. But a lot of times when we see people do different things. We're like, oh, that's cool. Actually, I would rather people figure it out, figure out their solution to the problem" <i>Different solution paths</i>

Analysis for RQ2

To investigate the second research question, I analyzed participants' definitions of problem-solving across three contexts in order to categorize in terms of coherences and divergences. The data came from three specific questions in the first phase's semi-structured interviews: (a) How do you define problem-solving in general or in everyday life?; (b) How do you define problem-solving in mathematics?; and (c) How do you define problem-solving in rock climbing? The rationale for doing this was as a pre-intervention to understand the participants' existing relationships with problem-solving across contexts, which would later help distinguish between the pre-conceived ideas they brought to the study versus those that emerged through the intervention of positioning climbing expertise as relevant to mathematical thinking.

Though RQ3 does not explicitly mention everyday problem-solving (because I was not able to observe them outside of the mathematical or climbing context), I contend that it was crucial to understand how participants defined problem-solving in their day-to-day lives to establish a broader context for where climbing and mathematics fit within their overall problem-solving frameworks. For example, it was common for participants to orient climbing problem-solving toward day-to-day problem solving, whereas math problem-solving was more abstract and decontextualized. The data was initially organized by participants and their responses to the above questions. Next, I created a chart that organized the data in terms of coherences and divergences for each participant. See Table 4 for an example of this process. Finally, I looked across the coherences and divergences to identify any themes across participants.

Table 4: Example of coherences and divergences in categorization

Coherences	Divergences
Christopher: All three: obstacles or setbacks; making mistakes are mentioned in all three	Christopher: In math, he discusses the rigidity of it and how much he values that, specifically because he values there being “a specific answer.”
Chad: All Three: Efficiency, figuring it out on the fly (Even in math, playing soccer, kicking something with force)	Chad: In math: memorization, decontextualized, but once learned applicable and “good at it” then he began to like it In climbing: embodiment, terms such as feeling, anticipation of movement, resting In everyday: emotions are brought into the response, particularly grounded in societal comparison

Analysis for RQ3

RQ3 constituted the core investigation of this study and employed a rigorous, multi-phase analysis process. I utilized a grounded theory approach (Strauss & Corbin, 1990) with the primary goal of capturing a nuanced and holistic understanding of problem-solving and gaining insight into potential transfer across domains.

Phase 2 Analysis: Climbing Context

The second phase of the study (rock climbing) integrated three interconnected data sources: the initial semi-structured interview, the recorded climbing session, and the stimulated

recall interview. These components enabled robust triangulation, providing deep and nuanced insights into participants' problem-solving experiences.

Following grounded theory methodology, the analysis proceeded through three distinct coding phases. During *open coding*, I broke down the data analytically to gain initial insights, comparing events, actions, and interactions to understand similarities and differences. Over eighty codes emerged from this initial analysis of the Phase 2 data.

In the *axial coding* phase, I refined these initial codes into broader analytical categories, establishing relationships between concepts and testing them systematically against the data. This process condensed the original codes into six overarching categories: *problem-solving strategies, affect, dispositions, expertise, efficiency, and environmental factors*. Each category was further developed by examining its conditions, context, and consequences. See Table 5 for an example of the axial codes in relation to some open codes. During *selective coding*, I integrated these categories into a cohesive theoretical framework by identifying core concepts and their relationships. The analysis illuminated a complex and interconnected conceptualization of problem-solving, where strategies are not simply applied in isolation but emerge from a dynamic interplay of individual factors, environmental cues, and embodied experiences.

The coding in this study was entirely emergent and was grounded in concepts rather than targeting discrete keywords or phrases. A single unit of meaning could range from a single sentence or to an entire paragraph, depending on what was required to capture the complete expression of a concept. A single unit of meaning could carry multiple codes simultaneously, as the passage can often contain several subcodes at once. For example, one participant during the mathematical task said "I can feel myself getting excited as I move through this puzzle and the checkpoints." This passage was coded simultaneously as both affect (excitement) and environmental factors (checkpoints). Another participant said "A big theme in my life, not just in

climbing but in life, is knowing how to pick your battles. there isn't always an answer, and I have accepted that. I need to save my energy." Here, disposition (picking your battles) and efficiency (I need to save my energy) were coded because the participant is articulating an orientation and a prioritization of efficiency simultaneously. Indeed, the disposition is what motivates the desire to conserve energy. In all instances of coding, each code was counted as a discrete instance toward its respective category's frequency total.

Table 5: Example of Axial codes in relation to some Open codes

Axial Code	Open Codes
Problem-solving strategies	Resting, identifying constraints, identifying affordances, backing-off, breaking down problems, shifting perspective.
Affect	Fear, frustration, liberation, happiness.
Disposition	Societal perception, growth, failure, persistence, goals, purpose
Expertise Building	Improvement over time, pattern recognition, "problem-solving as a muscle."
Environment	Unaccommodating environment, high stakes environment, community.
Efficiency	Energy conservation, fatigue, fuel levels, wasting energy, speed, tempo.

Phase 3 Analysis: Mathematical Context

In the analysis of Phase 3 data, I operationalized this six-category framework as an a priori coding scheme while being open to emergent codes during analysis of Phase 3 data (mathematical tasks). The analysis combined the task-based interviews and stimulated recall interviews from the mathematical puzzle sessions. I allowed new codes and categories to emerge if the existing framework did not adequately capture the data, adapting or modifying codes as necessary based on emergent themes and patterns. I systematically examined connections and relationships between the a priori codes and any emergent ones.

Through this approach, I balanced structured analysis by leveraging the a priori framework with responsiveness to data nuances. No new broad categories emerged during Phase 3 analysis, though two new problem-solving strategies and several new open codes within existing categories were identified. I was thus able to refine the codebook based on this process. See Table 6 for an example of new subcodes that emerged during the analysis of Phase 3.

The process of immediately applying the framework from Phase 2 validated the framework's robustness and generalizability across domains, as the six categories developed from climbing data proved sufficient for capturing mathematical problem-solving as well. This cross-domain applicability added methodological rigor by enabling me to test and operationalize the framework, illuminating its utility for understanding problem-solving beyond the climbing context from which it emerged.

Table 6: Example of Emergent Axial codes in Phase 3

New Axial Code	Rationale for Addition
Strategy: Repeating for Refinement	Emerged in mathematical contexts where participants chose to redo completed puzzles to optimize their approach after initial success
Strategy: Starting Over	Emerged as distinct strategy when participants completely reset their strategy
Affect: Overwhelm	Captured affective responses to too much cognitive load in mathematical problem-solving
Affect: Sense of Progress	Reflected the discrete, measurable nature of mathematical puzzles
Disposition: Don't Force It	Where participants discussed not forcing puzzle pieces or approaches, reflecting a gentle, exploratory disposition
Environment: Feedback	little wins and visual checkpoints that helped participants know they were on the right track

Chapter 4: Findings

Chapter four discusses findings related to disabled rock climbers' problem-solving approaches across the domains of rock climbing and mathematics. I first present findings from Phase 1, examining the rock climbing gym environmental considerations. I then discuss the pre-intervention conceptualizations of problem-solving for each participant. Next, I present detailed individual profiles of each participant, establishing their unique problem-solving characteristics in both rock climbing and math across six dimensions: strategies, affect, dispositions, expertise, efficiency, and environmental factors. I then discuss the comparisons of problem-solving across rock climbing and math, illuminating the ways problem-solving characteristics move (or don't move) between climbing and mathematical contexts.

Phase 1: Environmental Considerations

To establish a foundation for understanding the experience of problem-solving of disabled rock climbers, I address RQ1 by examining the affordances and constraints built into the climbing environment. This analysis provides context and baseline validation for the subsequent exploration of participants' experiences of problem-solving. By identifying the specific opportunities and limitations designed into the problem-solving space in the rock climbing gym, the route-setters' methodology helps to ground the second phase of the study. The data comes from a one-hour focus-group interview with six route-setters. In the interview, they were asked questions regarding their route-setting methodology and philosophy, conceptualizations of problem-solving in rock climbing, and their considerations of disability in the climbing gym environment.

Affordances

As a reminder, in this dissertation, I define affordances as the opportunities or possibilities for action that the environment offers. The route setters in this study demonstrate an implicit

understanding of affordances, recognizing that different climbers will interact with the same route in unique ways based on their individual physical characteristics and skill levels. They emphasized the importance of providing a variety of affordances (options) within their route setting to achieve “fairness.” By manipulating certain sequences of movement on routes, they create a climbing environment rich with possibilities for action, allowing climbers to discover and utilize affordances that best suit their individual bodies and skills.

Different Solution Paths

Affordances emerged in the focus group interview through the discussion of different solution paths. One route setter asserted:

Climbers have to solve the problems with their body. You know...there's different solutions for every climber, whether they're weak or they're strong or they're small or they're tall or they're flexible or not. And then they just take all those tools and use it to come up with their own solution.

The route setter highlights that climbers must utilize their unique physical characteristics (“tools”) to solve the problems presented by the climbing wall. This means that there are multiple valid solutions depending on the climber's physical attributes, where “...there is no really wrong way to do it.” The route setters prioritize “fairness” in problem creation by intentionally designing routes that offer diverse solutions for climbers of varying body types (namely, short and tall climbers) that require the same amount of exertion. In other words, rather than creating a single correct solution that privileges certain body types, route setters design problem spaces where different bodies must exert comparable levels of energy.

We try to like, achieve this fairness for people of different body types, using things like efficiency and how efficient a move will feel like if a tall person and a short person climb through the same section, but they climb it differently. The most important thing is that we try to make it so both ways feel like the same level of efficiency for both short and tall climbers.

This quote demonstrates the route-setters' intention to achieve “fairness” for both short and tall individuals. The route setter seems to try to enact fairness through efficiency, ensuring that despite the height of the climber, they will have to expend similar levels of energy.

The fairness referred to above is often addressed through the concept of *spacing*— “In spacing, you don't want the holds to be the same distance apart, because that's boring to climb...So you want to vary your space between holds.” The lead route setter exemplifies this concept by discussing how a ladder is spaced: “Say you had a ladder right? And we had holds that were evenly spaced all the way through just like a ladder. Not everybody can climb that the same because every climber's body is different.” He elaborates on this metaphor by asserting that some people would be able to simply reach through and skip certain movements— “skipping the ladder rungs”— and some people wouldn't be able to do the moves at all. However, if the route-setters vary the spaces between each ladder rung so they are inconsistent and varied, the playing field is leveled to some degree. The route-setter argues that this concept of spacing enables more “intuitive movement, rather than intended movement... The goal is to have people experience a similar feeling by not forcing things, rather letting people figure out their own solutions to the problem.” This manipulation of hold spacing acts as an affordance, shaping the possibilities for action and guiding climbers towards a more intuitive and consistent experience on the route, unique to their own embodiment and skill set.

The concept of varying solution paths ties deeply into how the route setters talk about their ethos, as demonstrated with the following quote:

Our overarching shared idea is with what we're actually trying to achieve with setting the routes. And one of our big principles is that we want people to learn things. The way to do it is through movement. It's learning through movement so like we can't just tell people how to rock climb. They have to kind of figure it out and come up with their solutions through the climbing. So everybody's movement is different, because everybody's bodies are different. Our job is to make sure that there's enough ways for people to interact with the space for all different types of bodies, styles, and skill levels. We're teaching everybody, but everybody is going to learn in a different way.

The route setters prioritize creating a learning environment where climbers discover solutions through movement, recognizing that each climber's journey and problem-solving approach will be unique due to their individual physical characteristics and experiences. This philosophy aligns with the concept of affordances, as the route setters aim to provide a variety of possibilities for

action, allowing climbers to interact with the climbing wall and develop their skills in ways that are meaningful and relevant to their own bodies and learning styles.

Affordances as Relational, Not Universal

While route-setters prioritize creating multiple solution paths, their implicit conceptualization of affordances reveals an important theoretical insight: what functions as an affordance for one climber may function as a constraint for another. As one route-setter explained, "Some things are harder if you're smaller. Some things are harder if you're taller." For example, a foot chip (a small edge on the wall that is intended to be stepped on) that serves as a stable point to step on for one climber might be used to crimp with (a small surface for your fingers) by another. The route-setter's observation that "the orientation, like the actual directionality of the hold, is gonna determine how you move through the next sequence of holds... it opens doors and closes doors" illustrates how affordances are not inherent properties of the environment but emerge through interaction with specific bodies and skill sets.

Constraints

As a reminder, constraints are the limitations or restrictions that shape the possibilities for action. This section focuses on the constraints that route setters incorporate into their design process. They discuss intentionally limiting climbers' options for stable stances, forcing them to utilize more advanced techniques and problem-solving skills. The route-setters also acknowledged the limitations they face in accommodating disabled climbers due to resource constraints and the fixed nature of the climbing wall.

"Killing" Opportunities to Induce Discomfort

One of the ways in which constraints emerged is through the discussion of limiting, or "killing" opportunities for movement:

We all know, and most climbers will know, that to achieve the best balance, you want to create a triangle with your body. An equilateral triangle is the most stable position, so we try to eliminate or kill those opportunities to make that stable triangle. Because if your hand and foot are in a straight line, you're more likely to topple over. So, we try to force climbers to use their hips to create the triangle, rather than just relying on their hands and feet. And then, you have to consider all the angles on the wall, whether it's a right or left-facing wall, or if there's a volume [a large, amorphous and textured surface on the wall] that changes the angle. So, rock climbing is all about understanding and manipulating angles to find balance.

The route setter explained that climbers naturally seek balance by forming a triangular shape with their bodies, particularly an equilateral triangle for optimal stability. However, the route setters sometimes disrupt this natural tendency by constraining holds that would allow for easy triangle formation. This forces climbers to explore alternative body positions to maintain balance. This manipulation of route design is an intentional way to make climbers uncomfortable.

The route setters express a clear intention to challenge climbers by disrupting their comfort zones and pushing them to explore new movement possibilities. One route setter stated, "One place for my setting that I could grow is finding ways to throw people off that kind of perfect balance point." This sentiment is echoed by another setter who notes that "there's different ways to make people feel not comfortable." This intentional disruption of comfort is further illustrated by a setter who explains how they strategically place holds to make taller climbers "...a little bit more uncomfortable in the position." These quotes reveal a deliberate approach to route setting where constraints emerge through limiting movement to induce discomfort.

Space not for Disability

The route setters acknowledged the limitations they face in accommodating disabled climbers due to resource constraints and the fixed nature of the climbing wall. As the head route setter explains when asked if he set routes with a consideration for disabled climbers:

Basically, that has to do with whether the [the adaptive climbing program coordinator] asks me to do certain things, but often we just aren't able to. We're not able to do that because of limited resources. Sometimes things just need big volumes to change the wall angles... and if we don't have the resources to make that happen because the geometry doesn't work, then we just can't do it.

This quote reveals a critical gap between their route-setting philosophy and practical implementation. They cite resource constraints (such as holds or more overhanging real estate) and the fixed nature of the climbing wall as significant barriers toward designing for adaptive climbers. The emphasis on "if we had the resources" suggests that adaptive considerations are secondary to primary route-setting goals. Notably, the route-setters' discussion of "fairness" with respect to tall and short climbers suggests a relatively narrow conceptualization of bodily diversity.

Overall, the route-setters' philosophy reveals a designed environment rich with intended affordances—multiple solution paths, varied spacing, manipulated hold orientations—all aimed at creating "learning opportunities through movement," but limited to bodily variation of only short and tall climbers. Yet their recognition of resource limitations and therefore minimal adaptive considerations is now evident. Having established the affordances and constraints intentionally built into the climbing environment, I next look at how disabled rock climbers actually experience, negotiate, and sometimes transform these environmental features through their embodied problem-solving practices.

Conceptualizations of Problem-Solving: RQ2

I now turn to the lived experiences of my participants for the beginning of Phase 2 to address RQ2. The aim of addressing the second research question is to understand and interpret the lived experiences of disabled rock climbers as they describe problem-solving across rock climbing, mathematics, and their everyday life. It is essential to note that these definitions are tacit, meaning that though explicitly asked to define problem-solving across these domains, the

participants may not have had readily available language to articulate their nuanced understandings. Nonetheless, the following data are responses to the explicit questions in the semi-structured interview in the first phase of the study: (a) How do you define problem-solving in general or in everyday life?; How do you define problem-solving in mathematics?; and (c) How do you define problem solving in rock climbing?

Taylor

Taylor's perspectives on problem-solving reveal both coherences and divergences across the domains of rock climbing, mathematics, and everyday life. He consistently emphasized the importance of past experiences in shaping problem-solving approaches across all three domains and also sees similarities in the temporal aspect of problem-solving across rock climbing and every day. However, a key distinction arises in his descriptions of rock climbing problem-solving, where he utilizes embodied language and expresses the emotional impact of navigating a constrained environment.

Coherences

Across all three domains, Taylor grounds problem-solving in past lived experiences. In everyday problem-solving, he asserts that it means "taking past experiences and instances that have an impact on your life, reflecting on them and using them as puzzle pieces to complete the bigger picture." In math and climbing, he extends this notion of past experiences to the respective domain; that is, in math, the past experiences are translated into knowledge of formulas or mathematical building blocks, and in climbing past experiences are translated into knowledge of certain holds or foundational rock climbing skills.

In both rock climbing and everyday life, Taylor explicates a distinction between problem-solving in a "drawn-out way...meaning, sitting with your thoughts," and problem-solving "in the moment, where you cannot take more than half a second to think about it." When discussing

problem-solving in rock climbing, he contextualizes this concept when discussing projecting versus the onsight:

There's a big difference in problem-solving if you are projecting and have time to take a moment to think deeply about something versus when you are problem-solving on the onsight of a comp climb that you can only look at from the ground for about ten minutes and having to make quick decisions when you are on the wall.

On a project, Taylor discusses the nature of having to anticipate movement on terrain that is not designed for campusing (climbing with only upper body, not using your feet to climb) and limited sequences of movement (beta) that work for his body. On the onsight, he describes it as "following your gut...sort of like your body takes over. I've found times where you don't even give your mind enough time to think about it, you just follow your body." In sum, Taylor's similarities in his conceptualizations of problem-solving across all three domains are centered around past experiences and across rock climbing and every day, he marks a distinction between "drawn-out" and "in the moment" approaches to problem-solving.

Divergences

A contrast in Taylor's descriptions of problem-solving across different domains is his use of embodied language and emotional expression when discussing rock climbing problem-solving, which is not evident in his descriptions of mathematical or everyday problem-solving. For example, he asserts "the terrain that I am on is for an able-bodied climber who can use their arms and legs. It's harsh and burly movements. Your arms and hands in very awkward positions. It's so painful because it's forcing particular movements." This demonstrates that Taylor's engagement with rock climbing may evoke a deeper connection to his embodiment and emotions of solving problems within an environment not made for him.

Christopher

Christopher's conceptualization of problem-solving reveals both consistencies and divergences across the domains of rock climbing, mathematics, and everyday life. He consistently views problem-solving as a process of navigating obstacles and setbacks to reach a solution, emphasizing the importance of identifying and correcting mistakes. However, his view of mathematical problem-solving diverges from his view of everyday and rock climbing problem-solving in its appreciation for rigidity and the existence of a single correct answer, suggesting a preference for structure and certainty in this domain.

Coherences

Christopher makes meaning of problem-solving across all three domains to be finding a solution amidst obstacles or setbacks, asserting in this definition of everyday problem-solving: "Problem-solving is the ability to come up with a solution....you have multiple setbacks along the way. It wouldn't be problem solving if you did not have set backs." Similarly, across all three domains, Christopher highlights the often inevitability of mistakes in problem-solving:

If you make errors on the bottom of a route, then you're gonna pay for them later. Getting to the top, or getting to the answer, like, obviously it doesn't always have to be perfect to make it the top. I guess it is the same in math. If you can think about and figure out why your answer doesn't make sense, you can fix it... no matter like how long a super long algebra takes, even if its 4 pages, you make mistakes, you figure out what went wrong, you will get the answer eventually.

Christopher consistently views problem-solving across all domains as a process of finding solutions amidst obstacles and setbacks. He emphasizes that mistakes are an inevitable part of this process, and that the key lies in identifying and correcting those errors to ultimately reach the solution.

Divergences

One main difference in Christopher's conceptualizations of problem-solving is seen in his characterization of problem-solving in mathematics. He discusses the value he finds in its "rigidity", particularly because there is often one correct answer: "I like that there's a specific answer. Whereas in other subjects, there can be multiple different answers, and that can be overwhelming... I have a one-track mind. Focusing on one solution path is easier for me...I think the appeal to math, it just makes sense." It seems that Christopher finds comfort in the structure and certainty of mathematics, where a clear solution exists amidst the nuance and ambiguities of other domains.

Chad

Chad's perspectives on problem-solving reveal both coherences and divergences across the domains of rock climbing, mathematics, and everyday life. He consistently emphasizes efficiency as a key element of problem-solving across all three domains. However, his descriptions of problem-solving vary by context. In mathematics, he highlights the importance of memorization and application of formulas. In rock climbing, he emphasizes the embodied and sensory-driven nature of the process. In everyday life, he expresses a desire to defy societal expectations and prove his capability. For example, as he moves through the world, he desires to not be perceived as blind and has adopted many tendencies to mitigate any assumptions about his visual impairment.

Coherences

Across rock climbing, mathematics, and everyday life, Chad succinctly defines problem-solving as "...the means of finding a solution to an obstacle." He consistently emphasizes the importance of efficiency ("trying to find the easiest way around a problem") across all three conceptualizations and adaptability ("figuring it out on the fly."). Chad views his problem-solving

approach as deeply intertwined with his disability, stating, "In my life, it's very taxing to act not blind, which is the core of my problem-solving day to day. It uses a lot of my energy."

Furthermore, Chad highlights the constraint of time in all aspects of his life, particularly in mathematics, asserting, "Do you want to spend 10 minutes or an hour on a math problem? We only have so much time. The clock is always ticking." He expresses a sense of urgency in maximizing his time to tackle new challenges, stating:

There is always a mountain to climb. The top of one mountain is just the start of the other. I feel like I'm never content with being. There's so much out there I wanna like do everything in this one lifetime.

This urgency is reflected in the way he talks about solving problems quickly and efficiently across all contexts.

Divergences

Chad demonstrated several divergences across his definitions of problem-solving: He sees problem-solving to be grounded in memorization and is often decontextualized. In climbing, he used more explicitly embodied language, and in everyday problem-solving his response is grounded in societal comparison. Chad describes problem-solving in mathematics:

I really enjoyed learning formulas, knowing them, I was super quick at learning them and it is super simple once you know how to do it. The challenge was memorizing formulas and wondering 'why do I need to learn this, I'm never going to use this?' And also, 'if this is hard, I don't want to do this.' But then, once I learned there was a reason and a purpose for it that I actually really enjoyed. Once I learned I could be good at it I began to enjoy it.

This suggests that his conceptualization of mathematical problem-solving is linked to his ability to memorize formulas and see their practical application. The initial challenge of decontextualization of problem-solving in mathematics created a barrier, but once he understood the purpose and felt competent, he began to enjoy it.

When describing problem-solving in rock climbing, Chad begins to use language around feeling and movement:

When I am rock climbing, I don't know rests, I don't know cruxes. I know none of those things. I have to solve problems on the fly... A lot of it is about feeling. I really like sensitive shoes for this reason because I am like 'wow, I can really feel everything with my foot' obviously I am touching everything with my hands, but I am also touching things with my foot, my knees.... Also, in rock climbing, you need to stay on balance. So I will touch holds before I move my foot to a hold, I will think to myself, it is like a triangle. If this goes here, this foot needs to go here, but if those hand goes here, that is not gonna work cause my center of gravity needs to stay here. So before I do things, I analyze my foot goes here and I am gonna move here, how is that gonna make me end? What is that gonna look like? And is that gonna be okay? And if not, maybe I will move my hand down, I will try certain angles and how that is going to fit. And in my mind, I imagine what I will look like in that finished position before I go to it.

Due to Chad's visual impairment, he relies heavily on feeling and proprioception to navigate the climbing wall. He actively explores the holds with his hands, feet, and knees, gathering information about their shape, texture, and position. Further, Chad visualizes his body in different positions throughout the climb. In sum, he sees problem-solving in rock climbing to be an embodied and sensory-driven process.

When Chad was asked about problem-solving in general, he describes it as a means to defy societal expectations and prove his capability despite his blindness. :

So I move through the world efficiently because I want to look like a sighted man. I don't want to be looked down on and I don't want pity from people either. I get that a lot... I thought I'd be so less than when I became blind, and I was at one point. Now, I've now made it my goal to be not just equal to an able bodied person, but better, so they can't look down on me.

He strives for efficiency not only to navigate the world effectively but also to challenge perceptions of inferiority often associated with disability. This suggests that his problem-solving on a day-to-day basis is deeply intertwined with his drive to overcome cultural stigmas.

Kenneth

Kenneth's perspective on problem-solving reveals both coherences and divergences across the domains of rock climbing, mathematics, and everyday life. He consistently emphasizes the importance of utilizing past experiences and personal motivation to achieve desired outcomes, highlighting the subjective and contextual nature of problem-solving. However, his view of mathematics diverges, as he describes it as abstract and decontextualized, lacking the personal relevance and connection to his lived experiences that he finds in other domains.

Coherences

Across all three domains, Kenneth defines problem-solving to be "...using the tools that I have that I've gained from my life experiences to create a specific outcome." He explicitly states that an obstacle is necessary in the process of problem-solving. He also asserts that in problem-solving, he has to be motivated to "...create an outcome that I want... It's like it has to have personal significance to me. Otherwise I might not really want to solve that problem." Whether discussing rock climbing, mathematics, or problem-solving in general, Kenneth's consistent thread is the importance of past experiences and motivation to solve the problem. In mathematics, Kenneth states:

You need to have, like the experience and skill set to know how to solve for X but in terms of it being something I really care about, I am not so sure. Unless solving for X is something that I need to do, because I'm trying to figure out how to build a garden bed to be in some specific space. And then it's like, 'okay, I can see I can see my needs to overcome this obstacle a little bit.'

Even in mathematics, it seems crucial to Kenneth to be able to leverage his past experiences to problem-solve. In rock climbing, Kenneth further elaborates on this point when he discusses the subjective nature of problem-solving, where the goal of the climb is context dependent.

I think the question is what do you desire? You might prefer to go the harder way on purpose when you are climbing. Maybe my goal another day is just to get to the top as fast as possible. It's *personal*. It's not right or wrong.

In other words, it seems that Kenneth sees problem-solving across all three domains as deeply intertwined with personal meaning and the utilization of past experiences to achieve desired outcomes.

Divergences

A contrast in Kenneth's descriptions of problem-solving across different domains is in his description of problem-solving in mathematics as abstract and decontextualized:

Everything was extremely abstract. And I was like, why do I need to know this... then fast forward to when I trained as a software developer. I took some math classes for that. And I absorbed those things very well, because I was being shown the relationship that I had to this other thing that I was learning. So I was being shown like, here's this math. And here's how it relates to the programming that you do, or here's how it relates to the way that a computer works, and, that made sense to me... Even those word problems, like the, you know, like down the street with 2 kittens in their bag, or whatever. Yeah, yeah. *laughs* I think it still just doesn't really matter to me.

This highlights Kenneth's struggle to find meaning and relevance in abstract mathematical concepts, contrasting with his ability to readily grasp and apply math when it's contextualized and connected to his personal interests and goals, such as software development. This suggests that Kenneth's engagement with problem-solving is deeply influenced by its perceived practicality and connection to his lived experiences.

Sofia

Sofia's views on problem-solving show both convergence and divergence across the domains of rock climbing, mathematics, and everyday life. She consistently emphasizes the importance of understanding constraints and resources, arguing that constraints can actually facilitate problem-solving by narrowing down possibilities and inducing a more focused approach. This perspective is evident in both her descriptions of everyday problem-solving and

her analysis of how climbers need to understand their embodied constraints to solve problems on the wall. However, her view of mathematics diverges, as she initially struggles to see it as a problem space, viewing it instead as a tool. This perspective shifts when she encounters applied mathematics in physics and dynamics, where she begins to appreciate math's role in solving real-world problems.

Convergences

Sofia describes her approach to problem-solving in both rock climbing and everyday life as a process resource management and considering constraints:

Problem-solving involves identifying a problem and all of its components and understanding the resources that you have available to you, and how those resources can be useful when addressing that issue. You also have to consider all the constraints involved in solving the problem.

She asserts that problem-solving requires a thorough understanding of the situation, including its individual parts, the available resources, and any limitations that might affect the solution. Sofia seems to believe that having more constraints can actually make problem-solving easier because it narrows down the possibilities and allows for a more focused approach. In contrast, open-ended problems with fewer constraints can feel overwhelming due to the sheer number of options and potential solutions. She also explicitly marks the distinction between problem-solving where you are “casually planning” and where you are “spontaneously solving problems in the moment” and brings in rock climbing to demonstrate this distinction. She discusses pre-planning in a competition climb when she has six minutes to talk about a route with her caller (refer to glossary in Appendix C) and the type of ongoing problem-solving that occurs when she is actively engaged in rock climbing. Further, for demonstrating what constraints can look like in problem-solving every day, she brings in rock climbing again to illustrate this concept:

So the problem solving in climbing comes down to like constraints. One of the constraints is like, how big or strong are you? How flexible are you ? How much do your legs weigh versus like your upper body and like, where's your center of gravity? Generally, just understanding the way your body moves is understanding your constraints which leads you to best solve problems that are on the wall.

This quote highlights that Sofia may believe that understanding your body's limitations and strengths is crucial for effective problem-solving in climbing. Factors such as height, strength, flexibility, and center of gravity all influence how a climber can solve problems on the wall. Sofia also argues that "...the more specific the constraints, the more specific solution. So I think because of that, it's easier to design with more constraints around you than when anything is a possibility, because you have too many options available to you." For indoor gym climbing, she talks about how the routes are designed with specific constraints to facilitate the development of certain resources or tools in a "mini course created by the route setters", like a certain kind of movement that can then be applied outside in a more organic context.

Divergences

Sofia's conceptualization of problem-solving in mathematics is a departure from her conceptualizations in rock climbing and everyday life. In rock climbing and everyday problem-solving, she sees these domains as spaces that are inherently equipped with a problem.

However, in the mathematics space, she struggles to see it as a problem space:

See, it's funny, because this is where I'm like, math is a specific tool. I was never good seeing math as inherently as the problem, it was more of a process. Problem solving in math is like treated as universal, but it's made by humans... I think that's why I struggled to see math itself as the problem. It's bullshit. We just like keep creating problems for ourselves to solve. and I am not good at thinking quite that that like.

Sofia views mathematics as a tool for problem-solving rather than the problem itself. She finds it challenging to see math as the problem because it is a human construct and not an inherent problem. However, her perspective shifted when she encountered physics and dynamics courses, where mathematical concepts were contextualized and applied to real-world phenomena: "But then, when it was part of like an actual problem that needed to be solved, like a free body diagram, I think it was easier for me to understand." This newfound relevance transformed her view of math, allowing her to see math as a problem space.

Ashley

Ashley's perspectives on problem-solving reveal both consistencies and divergences across the domains of rock climbing, mathematics, and everyday life. She emphasizes the role of past experiences and the individualized nature of problem-solving across all three domains. However, she perceives differences in the observability and social nature of problem-solving between everyday life, where she learns from watching others, and mathematics, which she views as a more internal and solitary process. In rock climbing, the personalized nature of problem-solving is amplified due to her unique disability, requiring her to develop highly individualized strategies.

Coherences

Ashley emphasizes the role of past experiences in shaping how she approaches problem-solving in all three domains:

In problem-solving, your past experiences really inform how you answer the question or how you figure out the situation. I feel like it's like the nature versus nurture question. I think problem solving is a lot about like nurture and how you were raised... But I also think that like it is how your brain is wired.

Ashley illustrates the influence of both nurture and nature on her problem-solving abilities. She recounts childhood experiences of road trips with her parents, where their emotional responses to unexpected situations shaped her own approach to problem-solving. She attributes her passion for problem-solving in mathematics, particularly her persistence and resilience, to an innate ability, recognizing that not everyone is wired to enjoy the challenges of the field.

Additionally, Ashley sees problem-solving as a personal and individualized process across all three domains. In her everyday life, she describes how her problem-solving is informed by her interactions with others and her observations of how they navigate the world. In mathematics, she acknowledges that everyone has their own way of working through problems in their head, even if it's not always visible. In climbing, she emphasizes the unique challenges she faces due to her rare disability, making her problem-solving highly personalized.

Divergences

Though Ashley sees problem-solving across all three domains to be grounded in lived experiences and to be a personalized process, there are divergences in her conceptualizations. She attributes her problem solving in day to day life to watching others engage in problem-solving: "Well, I think it's about how my past experiences interact with other people. And like, me specifically watching how they interact with other people. Me specifically watching how other people interact with the world." However, in mathematics, Ashley often does not feel like she has insight into other's processes: "I feel like for math, it's all like in your head. And I feel like, I guess you're not able to really see how other people are working through it in their head. It's not as much a lived experience." In contrast to her view of everyday problem-solving, Ashley perceives mathematical problem-solving as a more internal and solitary process. She notes that it's difficult to observe others' thought processes in mathematics, limiting the opportunity to learn from their approaches. However, Ashley's confidence in her own mathematical abilities mitigates the potential impact of the more "in your head" approaches.

I mean, yeah, like, I can't really see how other people are doing it. But if the teachers like 'Oh, like, put it this way', then, honestly like I feel like, because I have such a strong base in math that I can do it. Honestly, no matter what way they tell me like I'll be able to figure it out and do it.

Yet, in rock climbing, she experiences a highly personal approach to problem-solving, where not only do other's processes not benefit her, but she also has such a rare disability that induces a very specific type of problem-solving:

With rock climbing, problem solving is a lot more personal and my disability is so rare. I have to try it to like know it. I have to sit there and look at the route when I am stuck, sometimes i have to try the move ten times. Usually, if you try to tell me specific movement I'll try it, but I have make it my own. So yeah, overall I would say problem solving is a lot more personal in rock climbing.

Ashley finds the personalized nature of problem-solving amplified in rock climbing due to her unique disability, while in mathematics, she compensates for the lack of visible thought processes with her own strong foundation.

Themes Across Participants: Math Problem-Solving as Decontextualized

Across examining coherences and divergences within participants' conceptualizations of problem-solving, one prominent theme did emerge across participants: mathematical problem-solving as decontextualized. In other words, all the participants tended to discuss how mathematics problem-solving was not related to their everyday lives. Taylor described mathematics to be "experience-based, however, you have to translate formulas and building blocks to experiences" in order to make it more relatable. Similarly, Christopher expressed a preference for the rigidity and single correct answers in mathematics, contrasting it with the multiple solutions often found in everyday problems. Chad, Kenneth, and Sofia see mathematics as decontextualized and abstract, though all found meaning in it when they were able to relate it to something that they cared about, while Ashley viewed it as an "in your head," solitary process. These perspectives collectively suggest a perceived disconnect between mathematical problem-solving and the lived experiences of the participants. Rock climbing problem-solving and everyday problem-solving were much more similar in the eyes of participants. These findings suggest a potential disconnect between how participants perceive problem-solving in the explicitly embodied realm of climbing and the seemingly "abstract" and decontextualized domain of mathematics.

Table 7 summarizes these findings. Notably, these pre-intervention conceptualizations of problem-solving established an important baseline: participants entered the study with domain-specific framings that positioned climbing and mathematics as fundamentally different. In climbing, they saw embodiment, intuition, and personal meaning that was more connected to everyday problem-solving. In math, they saw abstractions, rigidity, and disconnect from bodily experience.

Table 7: Summary of convergences and divergences in conceptualizations of problem-solving

Coherences	Divergences
Taylor: Emphasizes experience-based learning across all three domains Values practical application in all domains	Taylor: Climbing: Highly embodied, emphasizes "body takes over"; climbing "without having to think about it;" Strong affective component in climbing not present in other domains Math: Requires translation of formulas to experiences to make experience-based
Christopher: Identifies obstacles/setbacks across all domains Values learning from mistakes in all domains	Christopher: Math: Appreciates rigidity and specific answers Climbing and Everyday: More flexible problem-solving approaches
Chad: Efficiency focus across domains "Figuring it out on the fly" in all domains	Chad: Math: Emphasizes memorization and decontextualized learning Climbing: Highly embodied, emphasize feeling, anticipation of movement, resting Everyday: Strong emotional and societal comparison component
Kenneth: Past Experiences and skillset emphasized across all three domains; problem-solving is personal, and has to have "personal significance to me."	Kenneth: Math: Abstract and decontextualized, mentions "disassociation", where in other domains it is more personal and contextual

Table 7: Summary of convergences and divergences in conceptualizations of problem-solving continued.

Coherences:	Divergences
Sofia: Every day and climbing: Focus on constraints and resources across domains; tool-based problem-solving approach	Sofia: Math: Seen as tool rather than problem space; viewed as too constrained Climbing: Highly embodied
Ashley: Personal and individualized across all three domains	Ashley: Math: Characterized as internal, "in your head" Every day: Emphasis on past experiences and social learning Climbing: Most personal due to unique disability

I have established participants' pre-intervention conceptualizations of problem-solving. I now shift from conceptualizations toward enaction, where participants actually engage in rock climbing sessions. This phase aimed to capture participants' problem-solving strategies in for what was for most participants (excluding Ashley), their domain of expertise, providing rich data on how disabled climbers navigate affordances and constraints in a context where they have developed significant competence. This portion of the study would later serve as the foundation for the a priori coding framework applied to mathematical problem-solving in Phase 3.

Phase 2: Rock Climbing

Emergent Codes

In order to develop a holistic framework through the study of the problem-solving of disabled climbers, I used grounded theory as the methodological approach to give rise to a holistic framework for problem-solving. To provide a comprehensive understanding of the participants' problem-solving attributes and orientations, the initial interviews in the second phase of the study were initially grounded in the context of rock climbing, serving as a launching pad for broader discussions about math and everyday life. While the primary focus was on capturing the nuances of problem-solving within the domain of rock climbing, the conversations expanded to encompass a range of experiences and insights. This fluid approach allowed for a more holistic exploration of problem-solving, as participants drew connections between their experiences in climbing and other areas of their lives, such as gardening, interpersonal relationships, or wrestling.

The analysis of the interview data revealed several emergent categories: *problem-solving strategies, disposition, affect, expertise building, efficiency, and environmental factors*. While these categories were contextualized within the realm of rock climbing, they were not always directly related to the activity itself. Participants frequently shifted their perspectives, drawing connections between their experiences in climbing and other domains. This shifting of perspectives provided valuable insights into the potential transferability of problem-solving strategies and the ways in which we draw upon our past experiences to make sense of the problems in front of us. Below, I discuss each category and their associated subcategories.

Problem-Solving Strategies

The category of Problem-Solving Strategies emerged from the interview data as a prominent theme, encompassing a wide range of strategies employed by participants. As a

reminder, I define problem-solving strategies to be mental and embodied tools individuals leverage to overcome challenges. This category was further refined into several subcategories, each representing a distinct aspect of the problem-solving process. One of the key subcategories within Problem-Solving Strategies is identifying constraints. This involves recognizing and understanding the limitations and challenges that shape the problem-solving process. Participants demonstrated identifying constraints in various ways, such as identifying cruxes (challenging sections in a climb), acknowledging their own physical limits and fatigue, and considering environmental factors that might impede their progress. For example, participants mentioned the importance of "knowing when to stop" and having "spatial and bodily awareness" to recognize their limits. They also discussed managing pain and fatigue, highlighting the need to adapt their strategies based on how their body feels. In each of these examples, participants were not simply experiencing limitations but actively perceiving, naming, and understanding these constraints as boundaries that shape which strategies remain viable and which approaches must be abandoned or modified.

In addition to identifying constraints, participants also demonstrated an awareness of affordances, or the opportunities and possibilities for action that the environment offers. This subcategory involves actively seeking and using these opportunities to facilitate problem-solving. Participants demonstrated this strategy through various approaches, such as identifying different solution paths and considering multiple options before making a decision. For example, participants mentioned the importance of "looking for options" and "trying different things" when faced with a challenge. Other subcategories of problem-solving strategies included resting, pacing, backing-off, trial and error, breaking down the problem, pattern recognition, and more. See Appendix D for a full list of problem-solving strategies with descriptions and examples.

Disposition

The category of *Dispositions* emerged as a significant theme in the data. I define dispositions to be the positioning, orientations, tendencies, or attitudes that individuals bring to their problem-solving in climbing. This conceptualization bridges mathematics education literature on productive dispositions (Mathematics Learning Study Committee, 2002) with philosophical work on dispositions as embodied tendencies. Perkins, Jay, and Tishman (1993) define thinking dispositions as consisting of ability, sensitivity, and inclination, so not just what one can do but the tendency to employ that ability when needed. Dispositions are not a personality trait (which are involuntary) or skill (which focuses on capability rather than tendency). Instead, dispositions can be thought of as "acquired patterns of behavior that are under one's control" (Ritchhart 2004, p. 21).

From an enactivist perspective, dispositions are not internal mental states separate from external situations but actually emerge through ongoing person-environment interactions. My analysis captures both dimensions: the more cognitive orientations participants brought to problem-solving (i.e., valuing process over outcomes, embracing mistakes as learning opportunities) and also how disposition was a response to (context-specific) affordances and cultural meanings, like the contrasting framings of climbing as embodied and intuitive versus mathematics as abstract and rigid.

Participants' past experiences, including their relationship with their disability, their background, cultural factors, and previous careers, were found to influence their dispositions towards problem-solving. These experiences shaped their beliefs, values, and expectations about the problem-solving process, as well as their preferred strategies and approaches. For example, participants who had experience in fields that emphasized structured problem-solving, such as mechanics or engineering, often demonstrated a preference for systematic and methodical approaches. In contrast, participants with backgrounds in creative or artistic fields often discussed problem-solving in a more flexible and fluid way.

Motivational orientations, such as achievement orientation, competitiveness, and desire for perfection, also emerged as key dispositions influencing participants' problem-solving behaviors. Competitive individuals often discussed the desire to outperform others or compare their performance to non-disabled individuals. Those with a desire for perfection often exhibited a meticulous and detail-oriented approach, striving to not make mistakes.

Participants' beliefs about the nature of problem-solving also played a significant role in shaping their dispositions. Some participants held a strong belief that there is always a solution, maintaining a firm desire to never give up and a persistent attitude even in the face of setbacks. Others acknowledged the possibility or even inevitability of there not being a solution. These beliefs about problem-solving influenced participants' resilience and their willingness to persist in problem-solving.

Temporal orientations, or how individuals perceive and relate to time, also emerged as a significant disposition. Some participants exhibited a "clock is ticking" mentality, emphasizing efficiency and speed in their problem-solving process. Others prioritized thoroughness and reflection, taking their time to carefully analyze the situation, think about others, and consider multiple options before making a decision. These temporal orientations influenced participants' pacing, their decision-making processes, and their overall approach to managing time and resources. See Appendix E for a detailed codebook for disposition.

The analysis also revealed significant individual differences in dispositions, highlighting the diverse and personal nature of problem-solving. Further, there was a potential connection between the nature of participants' disabilities and their dispositions towards problem-solving, particularly in terms of persistence and beliefs about finding solutions. For example, Taylor and Ashley (seated climbers), often demonstrated a greater awareness of their limitations and a willingness to "call it" when faced with challenges that "aren't worth it". In contrast, participants with different disabilities, such as Christopher (a lower leg amputee) and Chad (a blind climber),

often exhibited a greater degree of persistence than the rest of the participants and seemed to have a firm belief in the existence of there always being a solution, even when faced with seemingly insurmountable obstacles. This suggests that experiences with disability may shape individuals' orientations around problem-solving and their willingness to persist in the face of challenges.

Affect

The category of *Affect* emerged as a prominent category in understanding participants' problem-solving experiences. Affect, in this context, refers to the subjective experience of emotions, including a range of feelings such as happiness, frustration, fear, and empowerment expressed. Importantly, from an enactivist perspective, affect is not merely an internal response to problem-solving but is constitutive of the problem-solving process itself, and as Gallagher (2013) asserts "affective experience is not something that happens in addition to or separate from our perceptual and action-related engagement with the world" (p. 121) but is fundamentally embedded in our embodied sense-making. (Gallagher, 2013)

Participants' descriptions revealed a rich emotional landscape interwoven with their discussions around problem-solving. They frequently mentioned feelings of happiness and liberation, often associating climbing with a sense of joy and freedom. For many, climbing served as an escape, a "happy place" where they could experience a sense of liberation from daily embodied constraints. Problem-solving also elicited challenging emotions, such as frustration, fear, and panic. Chad explicitly noted the need to manage fear before problem-solving could proceed: "If I am afraid, I need to like hone that first before solving. There's no way I can solve this issue if I'm like shaking in my boots."

The analysis revealed several other affective themes. Participants emphasized the importance of being present, focusing on the moment, and fully engaging with the process of

problem-solving. Empathy also played a role, as participants considered how their decisions might impact others. Relief and a sense of safety were often experienced upon successfully completing challenging sequences. Finally, playfulness emerged as a theme, associated with the joy of exploration and play in problem-solving. Indeed, problem-solving for the participants is a deeply embodied, emotionally rich process. See Appendix F for a detailed codebook for affect.

Expertise Building

Expertise Building emerged as a category in understanding the participants' problem-solving processes. I define Expertise building as the development of skills and knowledge that contribute to proficiency in a certain domain. This development toward expertise is characterized by skill acquisition and refinement, the accumulation and integration of past experiences, and the transition to intuitive movement. Expertise is not merely the accumulation of knowledge, but by how that knowledge is organized and leveraged (Ericsson, 1991).

From the lens of ecological psychology, as individuals develop expertise, they no longer need to consciously deliberate about rules or procedures but engage in *absorbed coping* or *smooth coping*, action that emerges as a product of perceptual engagement with the environment rather than conscious planning (Dreyfus, 2002; Kronsted et al., 2022). This aligns with enactivist perspectives that position expertise as developing through accumulated history of structural coupling between organism and environment (Varela et al., 1991), where repeated engagement creates increasingly refined sensorimotor schemas that enable rapid and context-appropriate responses (Gallagher, 2005). In other words, it is easier for experts to experience the unveiling of affordances, leading to more intuitive problem-solving.

Notably, several participants likened problem-solving to a muscle that strengthens with use. Taylor articulated this explicitly:

Problem-solving is like a muscle. If you let yourself fall into a pit of intuition and assumption, it atrophies. But if you allow yourself to put it under consistent and

intentional stimuli (i.e. rock climbing) then it continues to grow and with that growth you can become more adept with navigating all the other facets within your life.

This metaphor captures expertise as developing through repeated engagement with challenges—what participants often attributed to navigating everyday life with disability, which provided constant training in adaptive problem-solving.

Participants describe a shift toward more embodied and intuitive problem-solving characterized by what they termed "mind-body flow." In this state, their bodies seemed to anticipate and react to challenges without conscious deliberation, enabling "on the fly" adaptations. Taylor captured this: "The body is switching over to the driver's seat when I'm climbing," while Chad described movement where "you're not even really thinking about what you're doing. You're kind of just doing it." Many participants explicitly use the verbiage of "flow" to describe this phenomenon. When Christopher described climbing where his "body knows how to react without me telling it how to," he is describing a consistent capacity for pre-reflective, sensorimotor-guided action that emerges through extensive practice, enabling him to smoothly cope as moves through the climbing route. The fact that participants frequently referred to this quality illuminates that it is a characteristic feature of their climbing expertise.

Finally, In climbing contexts, pattern recognition manifested as the ability to "read" routes from the ground, anticipate how holds would feel based on their orientation, and recognize when body positioning would be challenging. Kenneth described being able to identify "heel hooks and drop knees and stuff...from the ground." This pattern recognition capacity reflects what Rietveld and Kiverstein (2014) term "skilled intentionality," which is experts' sensitivity to affordances, developed through extensive engagement with a domain. .

Environmental Factors

The category of Environmental Factors emerged as a significant influence on participants' problem-solving experiences, highlighting how the surrounding environment shapes possibilities for action and engagement with challenges. This is foundational to ecological psychology's claim that behavior emerges through person-environment interactions (Gibson, 1979) Environments are not neutral vacuums for action and for disabled climbers, environmental factors encompassed physical features of climbing spaces and broader ableist structures that are inherently unaccommodating (Hamraie, 2017).

Safety emerged as a prominent environmental factor shaping problem-solving. Participants discussed protection systems, fall potential, and high-stakes consequences that influenced their decision-making, risk assessment, and therefore problem-solving process. Kenneth argues

I think there is far more problem-solving in the alpine because it carries a heavier weight in terms of consequence. So if you make a poor decision in that environment, your consequence could be life and death...And I think there are usually way fewer decisions we have to make in a more controlled environment, like the climbing gym.

Indeed, the specific context, like the gym climbing (controlled, predictable), competition climbing (time-limited, performance-oriented), or outdoor climbing (high-stakes) are highly influential in shaping problem-solving.

A crucial finding was the unaccommodating nature of environments that disabled individuals navigate. Participants consistently described environments "not made with them in mind," requiring constant problem-solving for activities non-disabled individuals take for granted. Participants framed this constant navigation as contributing to problem-solving expertise. Sofia explained disabled people are "used to having to solve problems in a creative way, or create solutions that don't exist." Less prominently, community emerged as an environmental factor

encompassing the social dimensions of problem-solving. Some participants described problem-solving as socially distributed, like learning from other climbers, receiving encouragement, and experiencing belonging. Ashley noted: "I learned from watching other people...you pick up on patterns." See Appendix H for a detailed codebook.

Efficiency

The final category, Efficiency, emerged as a pervasive theme throughout the data. Unlike the other aforementioned categories, it does not have a corresponding codebook with underlying subcodes. Uniquely, it is more of an underlying orientation shaping participants' entire approach to problem-solving. Efficiency refers to the ways in which participants sought to solve problems with the least amount of time, energy, and mental and physical exertion. This conceptualization aligns with ecological psychology's principle of "economy of action" (Sparrow & Newell, 1998).

Efficiency was captured by explicit mentions; this unique coding approach reflects efficiency's role as an underlying motivation and evaluative lens through which participants engaged with problem-solving across all domains. The salience of efficiency in participants' problem-solving likely reflects their embodied experiences navigating environments with constrained energy resources. Several participants explicitly discussed managing limited physical capacity, what one participant termed having "only so many spoons." This perspective positions efficiency not merely as preference for elegance but instead as a survival mechanism. When Chad stated "We only have so much time. The clock is always ticking," he articulated how temporal and energetic limitations make efficiency essential.

The route-setters' explicit consideration of efficiency in their design philosophy—aiming to ensure that "both ways feel like the same level of efficiency" for climbers of different heights—suggests efficiency is a shared value within climbing culture. The route setters explicit emphasis

of efficiency considerations in route-design may have amplified participants' attention to efficiency, making it a prominent lens for evaluating and narrating their problem-solving experiences. See Appendix I for detailed efficiency coding instances.

To illuminate how these dimensions operated for individual climbers with specific disabilities and personal circumstances, I now turn to detailed case studies of participants' climbing experiences. These case studies reveal how the six categories manifested uniquely for each participant, shaped by their particular bodily configurations, climbing expertise, and relationships with disability. By examining individual cases in depth, I demonstrate how problem-solving strategies, affective responses, dispositional orientations, expertise development, efficiency considerations, and environmental factors interweave in context-specific ways during actual engagement with climbing.

Rock Climbing Case Studies

Here, I develop a conceptual framework to illustrate the interconnectedness of problem-solving strategies, disposition, affect, expertise building, efficiency, and environmental factors in shaping individual problem-solving experiences grounded in the first phase of the study, namely with respect to rock climbing. Recognizing the inherently unique and personalized nature of problem-solving, this framework is adaptable to represent each participant's distinct positioning within the categories from above and the specific problem-solving strategies. This approach aligns with a case study methodology (Miles et al., 1994) where I first examine each participant's problem-solving narrative in detail before drawing comparisons and identifying overarching themes across the cases.

Taylor Rock Climbing Problem-Solving

Taylor's disposition towards problem-solving is significantly shaped by his experiences with navigating the world in a wheelchair and his journey of relearning how to interact with the world after life-altering traumas, which has cultivated a strong sense of self-awareness:

Over the course of the last six years and some change, my brain has changed immensely. Specifically, the way I approach everyday tasks and interact with new environments. When your life has been broken up into distinct chapters, divided by life-changing traumas, it feels like the stopwatch that unravels and coevolves alongside your personal timeline stops and restarts at the beginning of those chapters. Once that stopwatch begins again, it feels like you're given a chance to relearn yourself, your body, and how you interact with the world around you. Your preconceptions of how you're supposed to or used to operate gradually falls away and you have to begin remolding.

He describes these traumas as a "stopwatch" restarting and requiring a fundamental relearning of how he interacts with the world. This experience has fostered a heightened sense of curiosity, born out of a lack of intuitive understanding of unaccommodating environments. Taylor explicitly connects this curiosity to his problem-solving abilities, stating, "My curiosity has made me a better problem solver because I make an intentional effort to not assume anything, especially when it comes to my body and how it moves." This reflects a strong disposition characterized by intentionality and a resistance to relying solely on intuition, particularly in novel situations. His belief that "problem-solving is like a muscle" further highlights his active and intentional engagement with challenges. Furthermore, Taylor's reflections on his creative background—particularly in speculative design—reveal a preference for flexible and creative approaches to problem-solving, contrasting with a more rigid, formula-based approach. This inclination seems to inform his problem-solving style, encouraging him to explore novel solutions and think outside conventional approaches.

Pain is a pervasive influence on Taylor's climbing experience and shapes his problem-solving strategies. He frequently mentions managing pain as a key constraint, demonstrating a constant awareness of his body's limits and the potential for injury. This awareness informs his

decision-making process, often leading him to prioritize caution and long-term climbing goals over immediate gains. Taylor articulates this when discussing high-effort moves, stating that if his "body does not feel good in those positions," it is "just not worth an injury." He also describes situations where he has to stop climbing due to pain, illustrating how it directly limits his ability to push himself. The concept of "picking battles" is crucial to his approach, reflecting a strategic mindset aimed at preserving his body. This prioritization of self-preservation over pushing through pain differentiates his approach, as he acknowledges that "sometimes there isn't a solution, there is not a way, and that's okay," highlighting a realistic acceptance of limitations to avoid further injury.

Planning is crucial for Taylor, with a significant emphasis on pre-planning, where he carefully assesses the route from the ground, anticipating cruxes and spots to rest. This pre-emptive approach is particularly important due to his reliance on upper-body strength (campusing) and the limitations it imposes on his ability to adjust his strategy while on the wall. As Taylor explains: "My planning and strategizing of the beginning two thirds of the climbing is from the ground, and then the upper section of the climb is figured out on the wall." This highlights a distinct division in his planning: the initial portion of the climb is mapped out beforehand, while the latter part requires more in-the-moment adjustments. He does plan while climbing as well, adjusting his strategy as needed during the climb, though this is less frequent than his pre-planning. This on-the-wall planning is often limited however, as seen in his discussion of resting: "I think when I am resting, I really only look ahead to the next one or two moves." This suggests that while he can plan a bit on the wall, his overall strategy is largely determined before he begins climbing.

Taylor also expresses a nuanced understanding of confidence, noting that it diminishes as he approaches his physical limits on a climb. While he possesses a strong baseline confidence in his abilities, he acknowledges the uncertainty that arises when pushing his boundaries. Taylor's

description of climbing as an "outlet" and a space where he feels confident in his abilities, despite the daily reminders of his limitations, underscores the significant role of climbing in shaping his self-perception and fostering liberation and empowerment. He finds freedom in climbing, contrasting it with the "exhausting and limiting" considerations of navigating the world in a wheelchair, further highlighting how his relationship with an unaccommodating environment influences his disposition and approach to challenges.

Efficiency is a crucial consideration for Taylor, deeply intertwined with his management of energy, pain, and fatigue during climbing. He approaches problem-solving with an acute awareness of his physical resources, constantly adjusting his strategies to conserve energy and avoid injury. Taylor's emphasis on efficient rest and recovery, including prioritizing rest points and carefully managing the duration of rests, highlights his strategic approach to minimizing wasted energy. Further, he views "breaking beta" not just as finding alternative solutions, but as a strategy for efficiency, conserving energy when taking on challenging sequences. In essence, Taylor's problem-solving is molded by efficiency, balancing the need for power and strength with the necessity to conserve resources and climb sustainably.

Taylor's experience as a competitive climber also shapes his expertise orientation and problem-solving. In competitive settings, he emphasizes the need for in-the-moment decision-making and trusting his gut, contrasting this with the more deliberate and reflective approach he might take outside of competition. He describes a shift in his cognitive processes, moving away from extensive deliberation and towards a more intuitive, embodied response: "Your body knows what to do before your mind does." This reliance on intuition and embodied knowledge is essential in the time-constrained environment of competition, where climbers cannot "second guess" themselves or "sit there and think about all [their] options." Furthermore, Taylor's description of his body taking over, and his ability to see something from the ground and know how to do it,

highlights the development of expertise through experience, allowing him to anticipate and execute moves with more confidence and efficiency.

Taylor's problem-solving profile during rock climbing is visually represented in Figure 2, which displays both his positioning across five key dimensions (top radial chart) and his strategic approach frequencies, which is quantified by instances of emergence in the data (bottom bar chart). The radial chart reveals a diamond-shaped profile with his highest dimensions being *Efficiency* (28) and *Disposition* (26), followed by *Affect* (20), *Environment* (14), and *Expertise* (9). The emphasis of efficiency reflects Taylor's acute and consistent awareness of his physical resources, involving pain management and energy conservation. His high disposition score (26) captures his trauma-informed self-awareness and intentional approach to problem-solving, cultivated through "relearning yourself, your body, and how you interact with the world" after life-altering injuries. His high affect (20) and moderate environment (14) frequency illustrates his emotional connection to climbing as an "outlet" where he finds freedom and empowerment, contrasting with the "exhausting and limiting" considerations of navigating the world in a wheelchair. Though Taylor is an experienced climber, his lower expertise (9) score likely reflects his ongoing process of relearning intuitive movement after trauma. The strategy bar chart reveals *Constraints* as his overwhelmingly dominant approach (30 occurrences), demonstrating his constant negotiation with pain, upper-body limitations, and the need to "pick battles" rather than push through injury. His frequent use of *Sequencing* (12), *Rest* (10), and *Pre-Planning* (10) strategies illustrates a meticulous approach where he is "planning and strategizing of the beginning two thirds of the climbing is from the ground" and optimizing opportunities to recover. *Energy Conservation* (9) and *Break Down* (8) further reinforced his efficiency-driven decision-making, while moderate Trial and Error (6) and minimal Pattern Recognition (3) reflect his preference for pre-planned strategies over exploratory problem-solving.

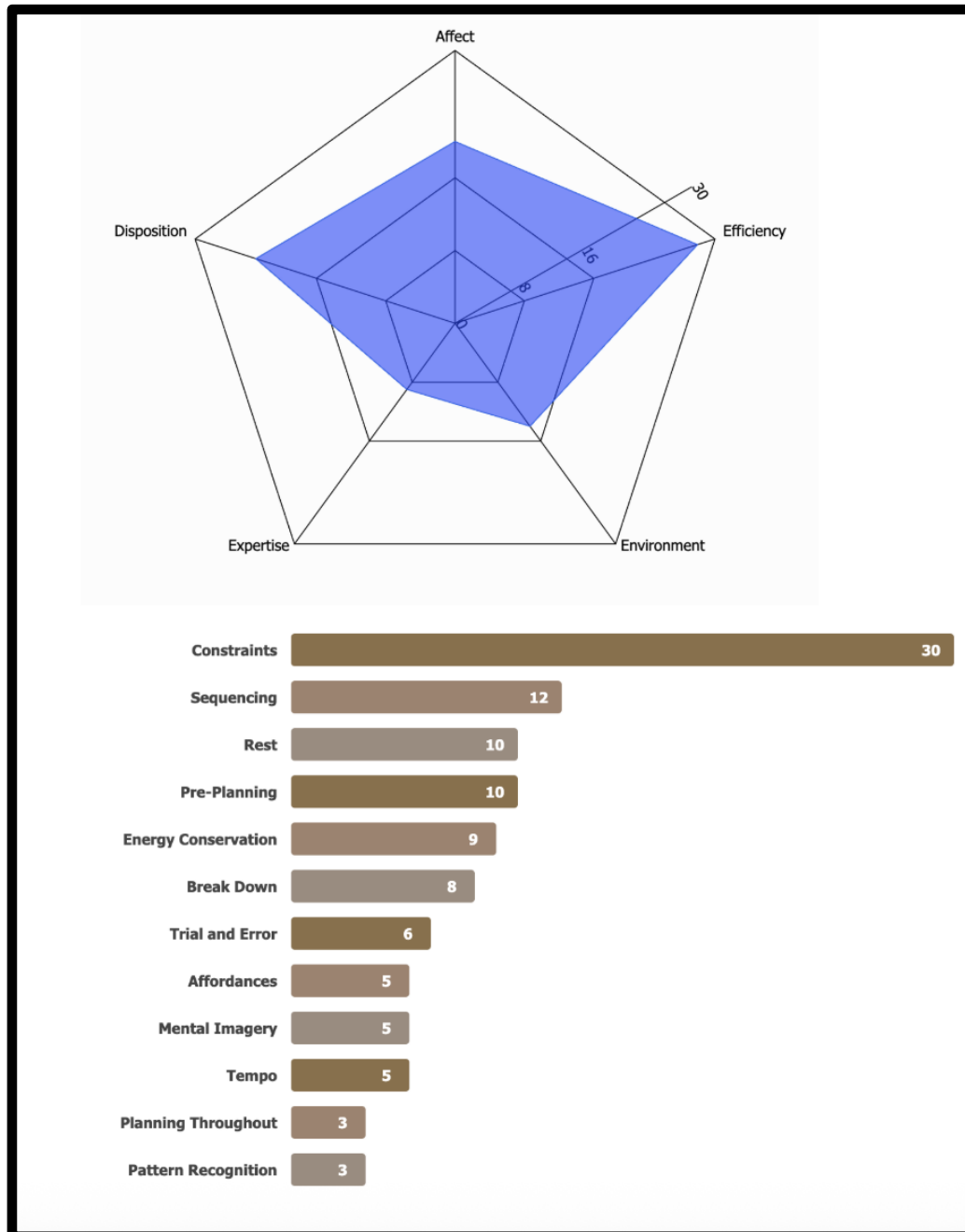


Figure 2: Taylor Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Taylor's radial chart on the top, depicting his positioning in problem-solving. A horizontal bar chart on the bottom provides a breakdown of Taylor's specific problem-solving strategies.

Christopher Rock Climbing Problem-Solving

Christopher's disposition towards problem-solving in climbing is strongly fueled by an ambition to excel, stating, "I want to be the best in the world." This aspiration is foundational to his self-belief and persistent, solution-oriented mindset. Though he is an amputee, he identifies more with non-disabled climbers and believes he can achieve any climb within his grade range, with some adaptations. He explicitly states, "if [non-disabled climbers] can do it, I know that I can do it," indicative of a core conviction that if a route is climbable by another person, he can and will find a way, even if it means just getting stronger. This is further emphasized when he says, "I really don't think there's a move in this gym that I can't do, and I have fully convinced myself about this." His upbringing instilled in him a persistent attitude, as he explains, "my parents instilled this from a very young age, especially in wrestling, don't ever quit" an idea he extends beyond sports to all aspects of his life. This belief in there always being a solution, in part influenced by his mother's profession as a math teacher, leads him to assert "there's always solutions to your problems. The problems that we do have in our day to day life just aren't that significant."

Christopher's emotional state while problem-solving on the wall is shaped by confidence, empowerment, and presence. He expresses, "I just trust that I will get through it," particularly when facing challenging cruxes. Even when confronted with moves that at first seem impossible, his emotional response shifts from initial frustration, like thinking a move "feels impossible," to a determined disposition for alternatives, stating, "...but there's ways around everything." For Christopher, climbing carries emotional weight. He describes it as "empowering for me to get to the top of a climb," especially when reflecting on his initial feelings of being "genuinely broken after my amputation." While acknowledging fear, he mostly mentions joy and presence when climbing, noting, "...really, I just feel present in the moment."

Christopher's expertise in problem-solving in climbing reveals a developing sense of route reading. While he admits, "I don't think I am great at reading routes," he acknowledges

improvement and identifies specific areas for growth, asserting, "as far as like finding rests and knowing which holds are good and which holds are bad, I don't think I'm good with that." Christopher demonstrates an ability to learn and adapt, particularly in utilizing his prosthetic, which he believes sets him apart, saying, "I feel like I use my prosthetic in a way that a lot of amputees cannot, because I have forced myself to figure it out." His expertise is also shaped by learning from mentors, especially experienced amputee climbers like Kenneth, who have provided crucial insights and creative solutions to navigating the world with a prosthetic leg.

Christopher's approach to climbing demonstrates a dynamic planning process. He doesn't just read the route once at the bottom. He is continuously "breaking it down," as he says, "I'd say it's like a combination of both thinking of the climbing in totality and breaking it down." He focuses on reaching specific points, often after each crux, before reassessing and planning his next steps. When discussing a climb that he did, he asserts:

I did make a split moment decision in this weird section where I changed my beta because I knew where I needed my hands needed to end up, but I did not know how to get there. I can't even tell you how I did it; it was just instinct.

For Christopher, it's more about in-the-moment decisions and adaptations fueled by instinct rather than trying to follow through with rigid plans, and he says "... that is something I can't really know until I am in the moment in the position." Climbing efficiently is not about adhering to a predetermined plan but about continuously adapting on the wall. His methodology involves identifying rests and cruxes from the ground, planning for them, and then dynamically adjusting his strategy in the moment. When he does make it to a rest, he is able to look five to six moves ahead, and then iterates that process.

Christopher's problem-solving framework during rock climbing is visually represented in Figure 3, which displays both his positioning across five key dimensions (top radial chart) and his strategic approach frequencies (bottom bar chart). The radial chart reveals a profile with his highest dimensions being *Disposition* (24) and *Affect* (20), followed by notably lower scores in

Efficiency (8), Environment (8), and Expertise (4). High values of disposition and affect reflect Christopher's ambitious and confident problem-solving identity, anchored in his unwavering belief that "I want to be the best in the world." His strong affect illustrates his emotional engagement with climbing as empowering and inducing presence. expressing." His lower efficiency (8) and environment (8) scores suggest less concern with optimizing energy expenditure or environmental consideration, consistent with his dynamic, in-the-moment adaptation ("I'll figure it out when I get there") approach rather than rigid planning. His minimal expertise score (4) reflects his self-assessment that "I don't think I am great at reading routes," though he acknowledges developing competence, particularly in his unique ability to use his prosthetic in ways "that a lot of amputees cannot." The strategy bar chart reveals Affordances as his dominant approach (17 occurrences), demonstrating his core strategy of identifying what options the wall offers him—making "split moment decisions" based on real-time possibilities rather than predetermined sequences. His frequent use of Constraints (15) and Planning Throughout (14) strategies illustrate his continuous assessment process of "breaking it down" after each crux, while Rest (11), Tempo (8), and Energy Conservation (7) reflected his dynamic adjustments on the wall. Moderate Pre-Planning (6), Sequencing (6), and Pattern Recognition (6) alongside minimal Trial and Error (3) reinforced his preference for affordance-focused adaptation over systematic exploration.

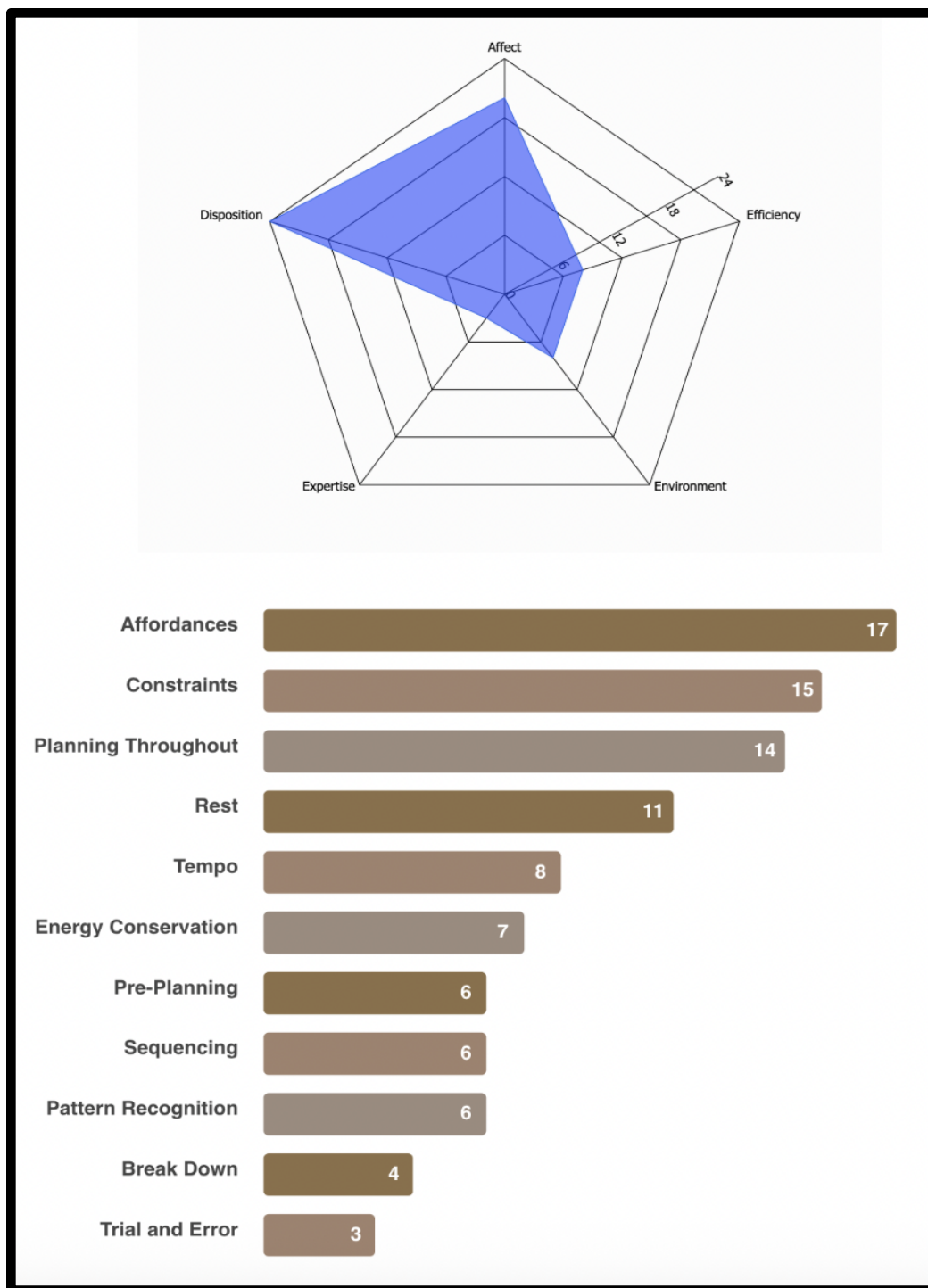


Figure 3: Christopher Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Christopher's radial chart on the top, depicting his positioning in problem-solving. A horizontal bar chart on the bottom provides a breakdown of Christopher's specific problem-solving strategies.

Kenneth Rock Climbing Problem-Solving

Kenneth's disposition as an adaptive climbing coach deeply shapes his approach to problem-solving, emphasizing self-discovery, personalized learning, and fluidity. He states his teaching philosophy: "I tend to lean much more towards self-discovery. Based on what I observe from the climber, I'll offer insights as tidbits to aid their learning. It's more of a self-directed journey where I'm just shaping the path a little." This philosophy of teaching highlights Kenneth's hands-off approach to teaching, prioritizing the student's own exploration and understanding over rigid or overbearing instruction. Kenneth demonstrates a fluid and adaptable approach to teaching, recognizing the diverse needs and goals of climbers, especially within the adaptive climbing community. He explains, "Everybody, especially in adaptive climbing, climbs very differently and may want different things from the experience. So, I'm fluid with my expectations and teach based on the individual I'm working with." This highlights a disposition that values individuality and tailors his teaching to meet each climber where they are, rather than imposing a one-size-fits-all solution.

Kenneth's disposition, informed by working with the adaptive community, is characterized by a rejection of rigid definitions. This is illustrated in his description of working with a child with autism for whom putting on a harness might be too overwhelming for a session. Kenneth argues that success for this child might be redefined as simply touching a specific hold. He states, "It's important to be really open to what it means to be a climber and what does climbing look like? Because I think it can just be very different to different people." This reflects a core belief that contrasts with the often narrow and "exclusionary attitudes" he observes in the non-disabled climbing community, which he criticizes for its emphasis on rigidity like, "Oh, you're not a climber unless you are lead climbing" or dismissive comments about other climbing styles. For Kenneth, his approach is grounded in the understanding that "the definition of climbing can be so much bigger than that," embracing a more expansive and nuanced perspective.

Kenneth is a playful and growth-oriented problem-solver. He explicitly states, "I try to keep centered in my mind that, like if I'm going to do this climbing thing, I should enjoy it, and I should do it for the right reasons. I'm not trying to take it too seriously!" This suggests that his disposition helps to foster a more playful approach to solving problems. Further, he emphasizes the importance of confidence in climbing, stating, "Yeah, I think that is probably why most people climb, whether they know it or not, is to experience growth and confidence." This intrinsic motivation to grow and the resulting confidence likely shape how he approaches and solves problems on the wall.

Kenneth explicitly contrasts different problem-solving styles, distinguishing between efficiency and creativity. When asked about disabled people better problem-solvers, he pushes back on the wording of 'better':

Better?! It depends on the goal. So, if you were trying to objectively measure problem-solving, you might ask: 'How did a person get from A to B, and what route did they take? Which of these routes was faster or used less energy?' For example, if you're an Olympic-level climber trying to break a speed record, then efficiency is your key measurement. However, I think there are also situations where creativity is more valuable in climbing. For instance, when you're not solely focused on performance and are out to have fun, explore, and grow, then creative climbing approaches become more important.

It is evident that Kenneth finds it important to contextualize the goal of the problem-solver, parsing out the difference between efficiency and creativity. Another distinction that Kenneth makes in his problem-solving is the difference between *Lateral* and *Progressive* thinking, framing them as distinct approaches to problem-solving. He describes thinking laterally as "drawing connections between things, like reading a news article, and then thinking about thinking about how that news article might affect some other piece of the world." This suggests that lateral thinking, for him, involves making connections across seemingly unrelated domains or considering the broader implications of information. He also acknowledges that this is something he finds challenging at times and says that climbing doesn't really help him to cultivate this kind of thinking.

In contrast, Kenneth says that climbing helps him with his "progressive" thinking. He explains, "I mean for progressive... I think, like it's more like you're looking up. And you're thinking, 'Here are my three options', and you ask yourself, 'which one do I want to take first?'" This portrays progressive thinking as a more linear process of evaluating a limited set of choices and selecting the most direct path forward, equating it to a "multiple choice question....versus laterally, it's like, draw upon everything in your brain to think about what could connect to this thing sideways....lateral is more creative, it's not a multiple choice question, it's an open-ended question." Refer to Figure 3 to see a visualization of Kenneth's personal framework of problem-solving.

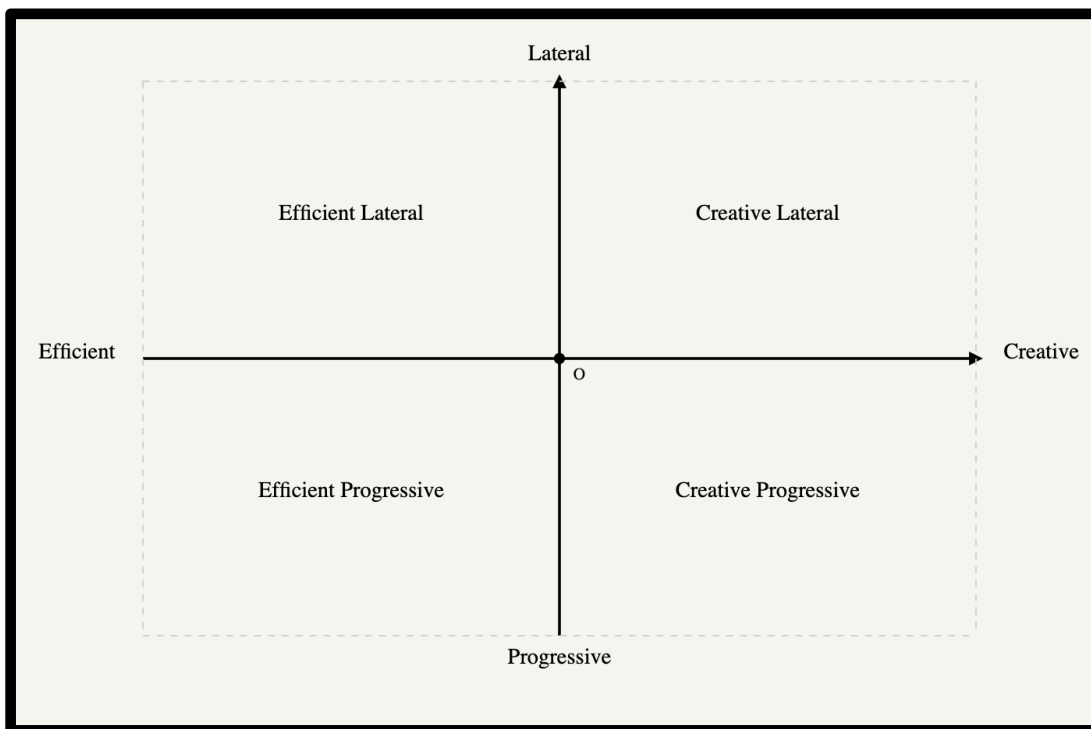


Figure 4: Kenneth's Personal Conceptualization of Problem-Solving.

Kenneth discusses the relationship between the number of options and how that relates to problem-solving. He observes on one climb with many holds, "Sometimes options can feel like constraints...it forces you to kind of know how to make the right choice or make a more efficient choice." This indicates that for Kenneth, having more choices doesn't automatically simplify the

problem; it requires a deeper level of understanding and experience to select an optimal path. Here, the concept of affordances becomes more nuanced for Kenneth: progressive thinking may contribute to the perception of immediate affordances (options), while lateral thinking may contribute to less obvious, more creative, and potentially more efficient ways to utilize those affordances.

Finally, Kenneth expresses an affinity for a more exploratory, “fuck around and find out” approach, which is his way of describing trial and error. This approach embraces the inherent uncertainty in climbing and encourages exploration of different affordances. Because he is not a competition climber, he is able to explore more rather than engaging in the rigorous process of planning both on and off the wall, asserting “I just don’t need to read the whole route, because I’m not a competitive climber. I’ll think about the first couple of clips and then I just figure out the rest.” Whenever he does actually plan, it’s with respect to safety considerations, particularly when he is climbing outside:

I think there is far more problem-solving in the alpine because it carries a heavier weight in terms of consequence. So if you make a poor decision in that environment, your consequence could be life and death...And I think there are usually way fewer decisions we have to make in a more controlled environment, like the climbing gym.

Ultimately, Kenneth’s approach to problem-solving is highly dependent on context, shifting from exploration and play in the gym to careful risk assessment and planning in environments where the stakes are higher.

Kenneth’s problem-solving profile during rock climbing is visually represented in Figure 5, which displays both his positioning across five key dimensions (top radial chart) and his strategy frequencies (bottom bar chart). The radial chart reveals a very high *Disposition* score (45), followed by *Affect* (22), *Environment* (15), *Expertise* (12), and notably low *Efficiency* (5). The high disposition value reflects Kenneth’s teaching-informed, philosophically grounded approach to climbing. His elevated affect (22) captures his playful, growth-oriented engagement. His

moderate environment score (15) reflects his context-dependent approach—shifting from exploration in the gym to risk assessment in alpine settings. His minimal efficiency score (5) embodies a more exploratory approach.. The strategy bar chart reveals *Affordances* as his dominant approach (15), demonstrating his ability to perceive opportunities on the wall. His frequent use of *Trial and Error* (8) and *Break Down* (7) strategies illustrate his exploratory mindset and ability to decompose problems into manageable progressive choices. Moderate *Constraints* (5) and minimal *Sequencing* (3), *Rest* (2), *Pattern Recognition* (2), *Tempo* (2), and *Energy Conservation* (1) reinforce his rejection of systematic planning.

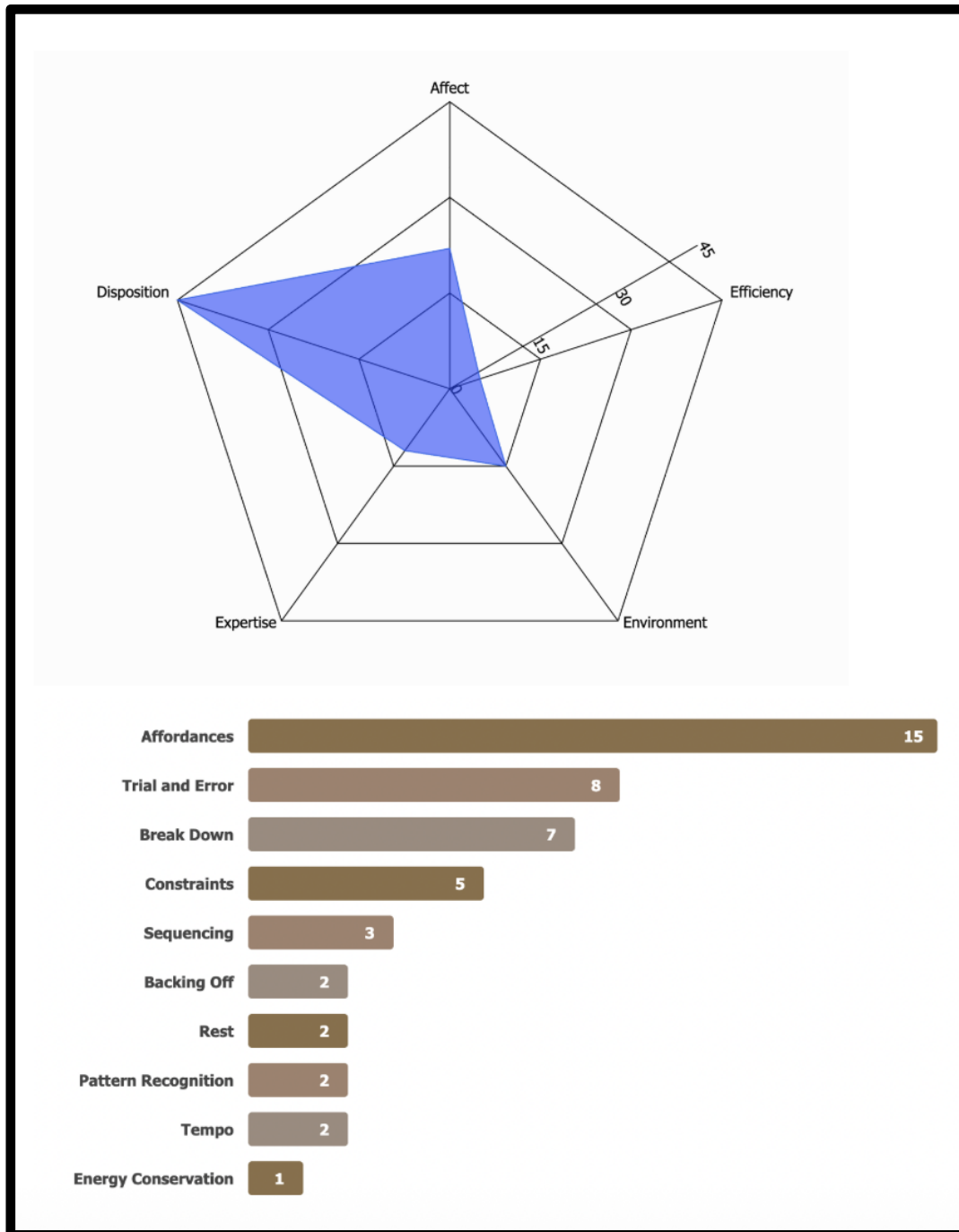


Figure 5: Kenneth's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Kenneth's radial chart on the top, depicting his positioning in problem-solving. A horizontal bar chart on the bottom provides a breakdown of Kenneth's specific problem-solving strategies.

Chad Rock Climbing Problem-Solving

Chad's disposition is shaped by resilience and a persistent approach to overcoming obstacles. He firmly believes, "There is always a solution to a problem," and this conviction fuels his personal mantra: "Don't say I can't say, how can I?" These stances inform the way he engages in problem-solving. He describes how he shifted his mindset when he lost his sight:

Whenever I would catch myself saying, I can't do this, I then took it... as I have two options here, I can get frustrated and pissed off or I can get excited that I'm now given a new problem to solve and make it fun.

This mentality demonstrates an explicit choice to embrace challenges, highlighting his orientation toward growth.

Chad's disposition is further characterized by a strong drive to prove himself. He is acutely aware of the societal stigma and pity that is associated with blindness, stating, "I don't want to be looked down on... I don't want pity from people either." This desire to be seen as capable, even "better than able-bodied individuals", motivates him to actively seek solutions and overcome hurdles, driving his problem-solving. It's not just about finding any solution; rather, it's about finding a solution that proves his capability and resilience. He also connects this drive to his role as a father, expressing, 'I really want them to look up at me. I always looked up to my father, and when I lost my sight, I was very sad thinking that my kids would never look up to me.' This demonstrates that Chad desires to be a role model and source of pride for his children. In other words, Chad's problem-solving is connected to his identity as a blind man, a father, and his desire to challenge negative perceptions of blindness.

Chad is constantly prioritizing efficiency. He articulates this clearly: "Efficiency is everything. Being blind, everything is so hard, you only have so many spoons, I want to save my spoons and use them for as many things as possible." This statement highlights his awareness of limited resources and the need to conserve them. Chad's pursuit of efficiency is not simply about doing things quickly; it's about learning how to do things in a way that will ultimately save him

time, employing trial and error until he figures it out. He explains his approach to learning: "So today I am gonna waste my time and energy doing something three ways. I will now know the best way of the three, so tomorrow when I do it, I use less energy." This demonstrates a willingness to invest time and energy upfront through trial and error to gain knowledge that will ultimately make him more efficient in the long run. He values the knowledge gained from experience, stating, "I've had to like learn through experience what works best for me."

A significant theme in Chad's problem-solving is his emphasis on learning through experience and doing things "on the fly" or "just going for it." He acknowledges that in climbing, he operates largely in the moment because he cannot plan from the ground: "I can't see rests, I don't know cruxes, I know none of those things. I do it on the fly." This suggests that Chad builds expertise by immediately engaging with the challenges and adapting as they come up in the moment. Beyond climbing, Chad's generally feels constrained by time. He states:

Do you want to spend 10 minutes or an hour on a problem? We only have so much time. The clock is always ticking. There is always a mountain to climb. The top of one mountain is just the start of the other.

This proverbial language reflects a broader philosophy of life, enabling him to engage in life with as much breadth as possible. This perspective on time as a constraint likely reinforces his "on the fly" approach in climbing and in life in general. He doesn't have time to overthink; he has to act and learn rapidly.

Chad's journey also highlights the process of cultivating skills and developing intuition. When Chad first went blind, he had to consciously and meticulously think about every movement, as he describes when learning to use a walking stick: "When I started using a walking stick, I had to literally think 'left foot, right hand', it was so slow and tedious and I hated it." However, with repeated practice and experience, these actions become ingrained: "It is a learned feeling or stored behavior within my body. I am not consciously thinking 'I'm gonna

move my foot here, then move my other foot here.' My body knows how to react without me telling it how to." Put another way, Chad is consistently *reacting* to his environment.

Chad is able to react so immediately because he is attuned to his environment and embodied knowledge. He explains that his body often takes over, particularly in situations requiring quick reactions: "So when I am hiking, I intuitively move. My body is problem solving for me, without me having to actively think... cause it is happening too quickly for my brain to react." This indicates that for Chad, problem-solving strategy is grounded in his body's learned experiences and intuitive responses.

To summarize, Chad has transformed his blindness into a catalyst for growth. He is extremely solution-oriented, encapsulated by his mantra of "Don't say I can't, say how can I?" When faced with the loss of his sight, Chad made an explicit decision to reframe frustration as excitement for new challenges. His problem-solving is deeply connected to his disability and his desire to disrupt negative societal perceptions about blindness. His disability induces a perspective of efficiency as essential due to limited energy resources ("spoons"). Chad has developed strong intuition and quick reaction time, asserting his body "problem solves for him" without conscious thought. Since he can't plan in climbing, he has developed exceptional skills for adapting "on the fly," making decisions in real-time. Ultimately, Chad's blindness is intrinsically connected to his daily approach to problem-solving. His disability isn't merely a factor in how he moves through the world—it's the core context that has shaped his entire problem-solving methodology.

Chad's problem-solving profile during rock climbing is visually represented in Figure 6, which displays both his positioning across five key dimensions (top radial chart) and his strategy frequencies (bottom bar chart). The radial chart reveals a profile with three very high dimensions—*Disposition* (40), *Efficiency* (30), and *Affect* (30), followed by *Expertise* (22) and notably minimal *Environment* (4). His high disposition score (40) captures his belief that "there is

always a solution to a problem" and this solution-oriented mindset is in part fueled by his desire to prove capability and challenge societal perceptions of blindness. His elevated efficiency (30) embodies his acute awareness that he only has so much energy, leading him to invest effort upfront through trial and error to discover optimal methods that conserve finite energy resources for future attempts. His high affect (30) illustrates his emotional reframing approach, transforming the loss of sight into growth opportunity and connecting his problem-solving to his identity as father who wants his children to look up to him. Relatively high expertise (22)—the highest among all participants—reflects years of adaptation where he has ultimately cultivated intuitive movement. His minimal environment score (4) captures his "on the fly" approach where environmental information becomes irrelevant compared to immediate sensorimotor feedback.

In Figure 6, The strategy bar chart reveals *Trial and Error* as his dominant approach (23 occurrences). His frequent use of *Constraints* (17) and *Affordances* (16) illustrates his constant negotiation with what he can perceive and utilize on the wall tactically. *Moderate Energy Conservation* (10), *Feeling the Problem* (9), *Tempo* (7), *Planning Throughout* (5), and *Pattern Recognition* (4) reinforce his real-time adaptation. Indeed, he can't plan from the ground so he has developed a vast skillset for making split-second decisions while climbing.

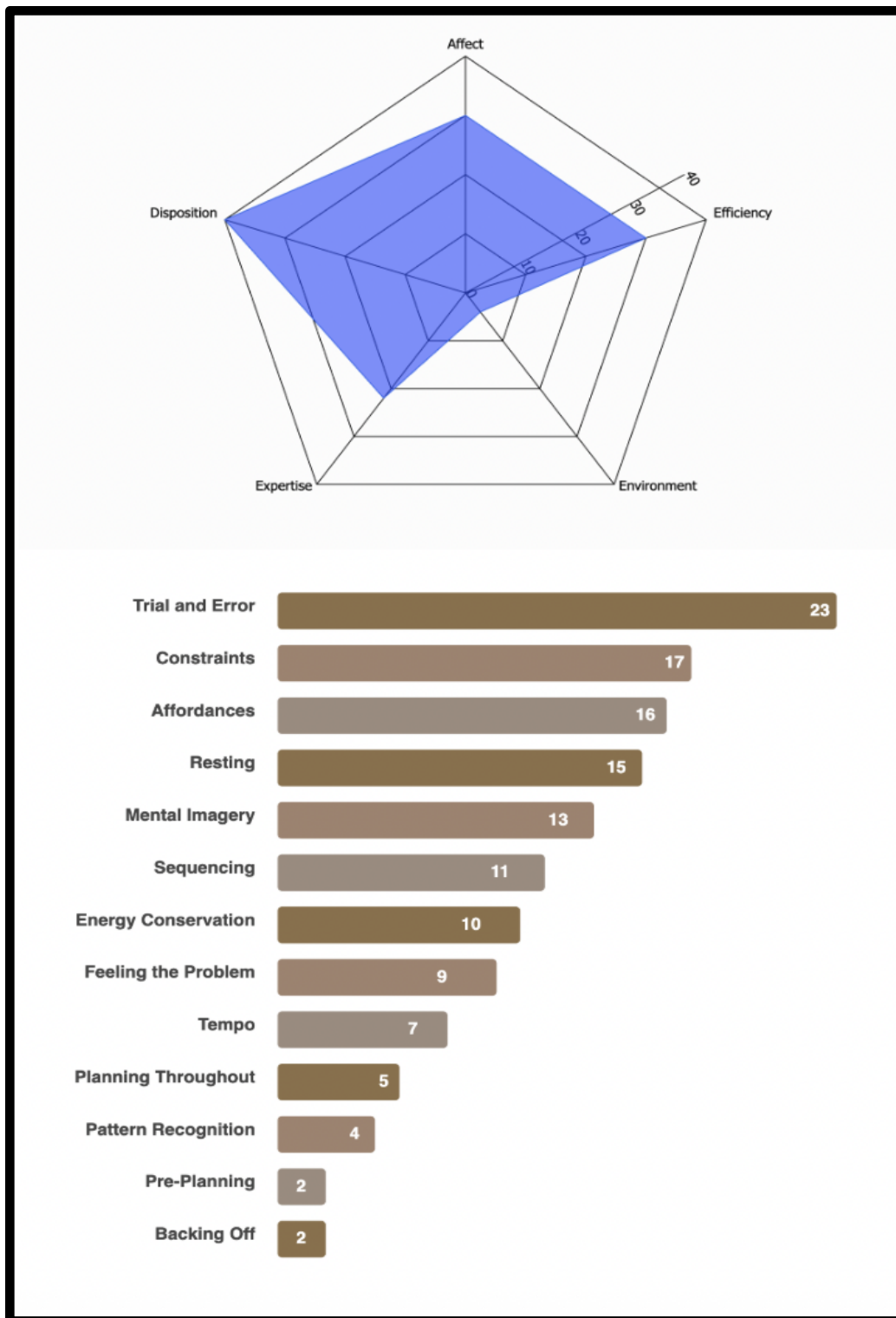


Figure 6: Chad's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Chad's radial chart on the top, depicting his positioning in problem-solving. A horizontal bar chart on the bottom provides a breakdown of Chad's specific problem-solving strategies.

Sofia Rock Climbing Problem-Solving

Sofia self-admittedly lives life in “complete onsight mode.” In climbing, *onsighting* encapsulates an approach to climbing where a climber goes in without any knowledge of the climb and completely improvises the climb in the moment (going in blind, so to speak). Therefore, her disposition is significantly shaped by her experience with vision loss, leading to an acceptance of uncertainty and a willingness to work with incomplete information, while also acknowledging that “planning is not really an option.” She explains, “As blind climbers, we’re using incomplete information to understand our world, and time is our enemy.” This reality shapes her approach not only to climbing but also to everyday life:

I'd say this applies to real life, too. When I'm walking down the street, I have to assume there isn't something in front of me, even if there is or isn't, because I have to proceed. Otherwise, I'd constantly be scanning and searching. And even in crowded spaces, while I might know where some things are, it's difficult to fully understand all the obstacles. So, I just have to move and deal with the consequences, because constraints like time and energy prevent me from waiting until I have complete visual information.

This suggests a realistic acceptance of uncertainty in her environment by acknowledging that she cannot always have a complete understanding of her surroundings due to the constraints of visual impairment.

This disposition of working off of incomplete information is not limited to her physical movement; she also applies it to her work in product development, where she “problem-solves for a living.” Describing the challenges of working across time zones, she says that she often has to rely on “a process of elimination and then a bit of deduction” to understand the cause of an issue, sometimes causing her to “leap to conclusions.” In other words, Sofia's disposition is characterized by a willingness to embrace uncertainty and move forward even when the full picture is not available, a skill cultivated through navigating a world with different perceptual input.

Sofia's affect is deeply related to the challenges and adaptations she faces as a blind person. A significant aspect of this is the “mental exhaustion and frustration” that can

accompany navigating the world. She explains, "I'm not sure how much Chad has talked about this, but being a blind person is really mentally exhausting. It's just exhausting to constantly feel the embarrassment and shame of breaking things, or making messes; it wears on you." This highlights the emotional burden of constantly having to be hyper aware of one's environment. She asserts that the "logical jumps" she constantly takes are in order to "save my brain from having to really think about every step, because it can be really overwhelming." In short, Sofia relies on those logical leaps as a key way to cope, both to conserve energy and to deal with the pressure of being blind.

Drawing from her background in design and her disability, she explicates the crucial role of empathy:

I'm certain, when it comes to design, the empathy you gain from understanding and empathizing is crucial. It's one thing to understand and give yourself space, but it's another thing to be so empathetic that you truly understand other people. And that's the base of design. It's understanding and viewing how other people see the world, and then solving problems from that perspective... and especially I think a lot of people in the disability space are empathetic because they themselves have been through really difficult things. Like, we exist in this world. We're people with your own depth and humanity. And yet the world doesn't see us or like, give us the same chances that it gives someone else.

For Sofia, problem-solving is grounded in a deep consideration of the needs of others, informed by her design background and her disability. Her own experiences of navigating a world that fails to accommodate those with disabilities have instilled a strong sense of awareness and concern for the perspectives of others. This emphasis on empathy has become a guiding principle for how she approaches problems — she desires to create solutions that are inclusive, acknowledging that while there may not be "always a solution for everyone, there should always be a solution for *someone*."

Throughout the interview, Sofia was constantly discussing constraints and the pivotal role they play in problem-solving. Sofia uses the idea of "stacked constraints" to describe how problems become more complex as constraints are accrued and require different problem-

solving strategies. She contrasts this with simpler problems that have a single variable: "So like, I studied systems and how I should break down problems to solve them into single variable problems— one at a time— there's like one variable right? Like that's the cleanest kind of problem." In her view, these single-variable problems are easier to handle with a "straightforward, structured, and linear" approach. However, she argues that many real-world problems, especially those encountered when dealing with disability, are not so simple.

Sofia explains, "I think with disability, you don't have any single variable problems. You have exclusively like multivariable problems that can't be solved one at a time necessarily. They are multidimensional and changing all the time." This is where she begins to dive into the concept of stacked constraints as she gestures the stacking of disks: "If this is your constraint, and then this is like another constraint stacked on top of it. You're already looking at such problems that, like someone may not have ever been forced to think about." Put another way, each new constraint adds another layer of complexity to the problem and forces the individual to think about problems in an entirely new way.

This stacking of constraints, according to Sofia, forces a shift in the way she thinks about problem-solving. She observes, "And I think because of being blind, I've been less and less effective at breaking down problems to individual components and now just deal with several things at once, which is sometimes, more efficient, time-wise. but isn't always necessarily as clean." She acknowledges that addressing multiple variables simultaneously can be less clean and structured — "the quick and dirty approach"— , but it is unavoidable when navigating the world with a disability.

To summarize, Sofia's problem-solving has been deeply shaped by her experience with blindness. She has embraced the necessity of moving through life without complete information, similar to how climbers approach an "onsight" climb without prior knowledge. Sofia is acutely aware of the mental exhaustion that comes with navigating the world as a blind person. Her use

of "logical jumps" represents a pivotal approach to conserving energy by avoiding overthinking every step. Her background in design combined with her disability has given her a profound capacity for empathy. She sees this as essential to problem-solving, particularly in creating inclusive solutions that acknowledge diverse perspectives and needs. Finally, she conceptualizes problems as having "stacked constraints" that amplify complexity. Unlike single-variable problems that can be solved sequentially, she recognizes that disability introduces multidimensional, constantly changing variables that must be addressed simultaneously. She acknowledges that her approach has evolved from breaking down problems into individual components (from before being disabled) to handling multiple variables at once—a method that may be less "clean" but is more efficient given her circumstances.

Sofia's problem-solving profile during rock climbing is visually represented in Figure 7, which displays both her positioning across four key dimensions (top radial chart) and her strategy frequencies (bottom bar chart). The radial chart reveals profile with Disposition (51) as her dominant dimension, followed by equally strong Affect (30) and Environment (30), with notably minimal Efficiency (5). Her exceptionally high disposition score (51)—the highest among all participants—captures her relationship with uncertainty and her willingness to operate with incomplete information due to vision loss, embodying what she calls "complete on-sight mode" where planning simply isn't an option. Her elevated affect (30) reflects the emotional layer of managing constant frustration and mental exhaustion from navigating an inaccessible world. She also maintains strong empathy that shapes how she approaches problems with others' needs in mind. Her high environment score (30) illustrates her acute attention to information in her environment and her framework of "stacked constraints," where she must simultaneously process multiple environmental variables. Her minimal efficiency score (5) reflects her acknowledgment that efficiency simply isn't an option as a blind individual.

In Figure 7, the strategy bar chart reveals Constraints as her dominant approach (26 occurrences), reflecting her constant navigation of layered limitations she describes as "stacking." Her frequent use of Trial and Error (16) reflects her "quick and dirty approach," and her Pacing (13) demonstrates she becomes critical for managing the "exhaustion" she speaks to. Mental Imagery (6), Feeling the Problem (5), and Resting (5) capture her strategies for operating without complete visual information. Affordances (5), Sequencing (3), Pre-planning (2), Pattern Recognition (2), and Break Down (2) round out her diverse strategic repertoire, though their lower frequencies reflect that blindness has shifted her away from vision-dependent approaches like pre-planning from the ground toward more exploratory, in-the-moment adaptation.

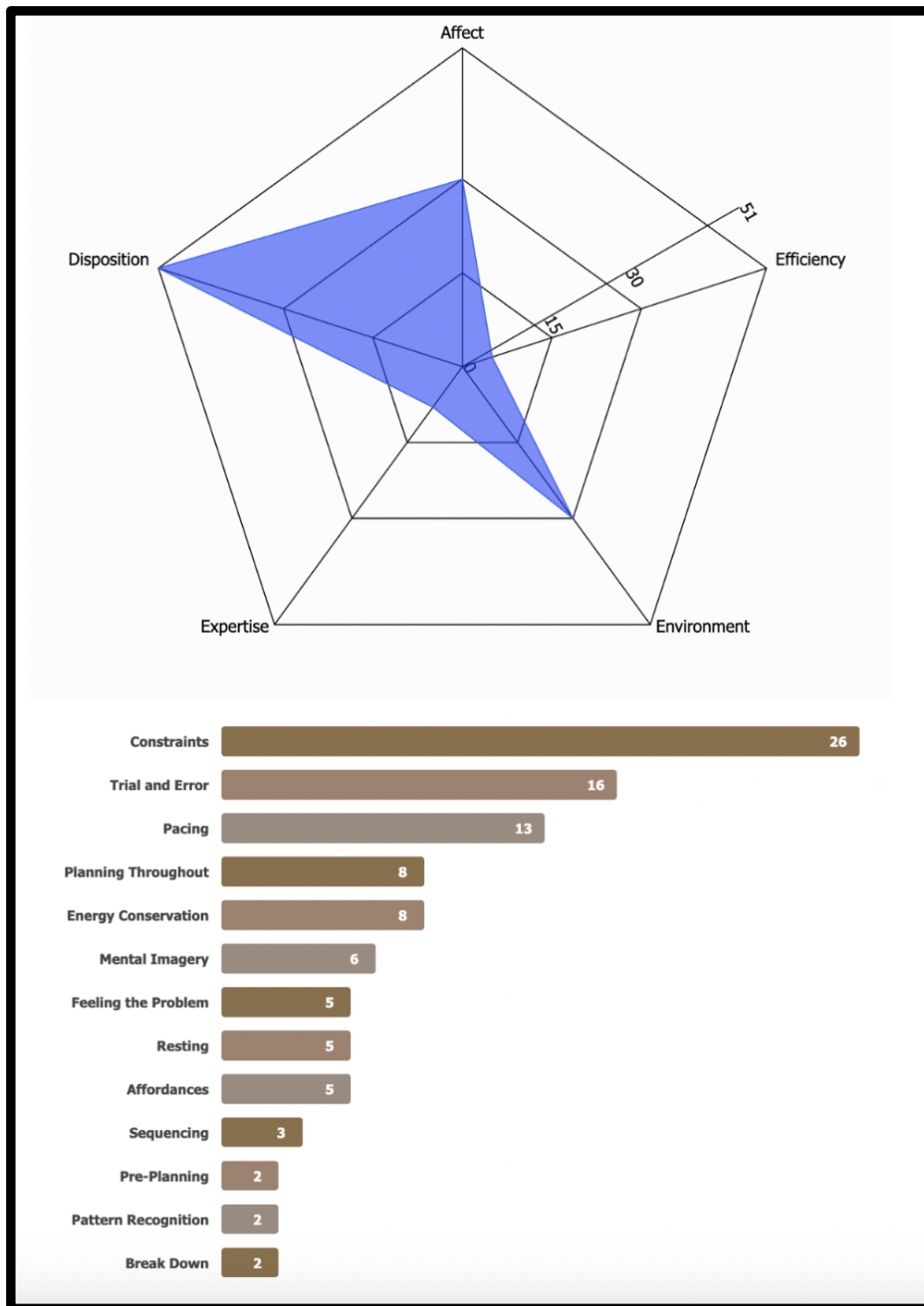


Figure 7: Sofia's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Sofia's radial chart on the top, depicting her positioning in problem-solving. A horizontal bar chart on the bottom provides a breakdown of Sofia's specific problem-solving strategies.

Ashley Rock Climbing Problem-Solving

Importantly, unlike other participants who were observed climbing, Ashley's data derives entirely from interview discourse about her climbing experiences rather than direct observation, as she did not physically climb during the study period. Ashley's disposition is characterized by a strong analytical and logical approach, particularly evident in her affinity for math and computer science. She explains her relationship with math: "I love math a lot! I've always liked it a lot more than English in school. Because there's always an answer to the problem. And you know for sure when the answer is right." This indicates a preference for structured domains with explicit and clear solutions, where she can apply rules and logic to arrive at a definitive answer. She also enjoys the objectivity of math, contrasting it with the subjective nature of subjects like English, where grading can feel arbitrary ("The teacher can just slap a grade on the paper and make up any reason for that."). This analytical mindset likely shapes her approach to problem-solving in other areas as well, with a preference for systematic processes to solutions.

Ashley is also "stubborn and competitive," which influence how she approaches challenges. She admits, "I am *really really* stubborn. So I'm still not the best at like giving up and walking away." While she acknowledges that this stubbornness can be a disadvantage, she also recognizes its benefits in enabling her to overcome obstacles, especially in the face of her disability: "But I would say [being stubborn] actually helps a lot with my disability, because it enables me to do more stuff, where other people may people may have given up." Her competitive spirit, evident in her desire to "get good grades and argue with the teacher," fuels her determination to solve problems and overcome obstacles, not allowing herself to feel confined.

However, Ashley's disability has impacted the way that she engages with problems, particularly in rock climbing, and "knowing when to call it." The nature of her disability constrains her environment because she is in a wheelchair, which she argues does not impact her

relationship with math: "Before my disability, I was a hundred percent stubborn and now it's like sixty percent. It comes down to trying different methods and learning when to call it. *Not in math*, but in other areas." She discusses how being in an unaccommodating environment induces her to "be more adaptable," and makes her constantly consider having "at least three back up options...and planning out like five different scenarios for how something will go, and like preparing myself mentally for, like all those different scenarios." This illustrates a key aspect of Ashley's problem-solving: the necessity of developing an adaptable and conditional approach in the face of constraints. The constraints imposed by a world not designed for her have forced her to move beyond a purely analytical or stubborn mindset. Now, she has learned to anticipate potential problems, generate multiple solutions in advance, and accept that some situations may require a shift in goals or to abandon the goal altogether.

Ashley draws a distinction between her problem-solving in math and rock climbing: in math, she has the ability to persevere through any problem and is not at all impacted by her disability. However, in climbing, she is constrained in how she can solve problems and must prioritize energy conservation: "I only have so much energy. When I go to the gym, know I only have three routes, so I need to be wise about what I choose to climb. I'm preserving my energy." This focus on conserving energy forces Ashley to adopt a problem-solving approach that emphasizes breadth rather than depth. Instead of focusing intensely on a single, complex problem for an extended period (as she might in math), she must carefully select which problems to engage with and how much energy to give to each. This implies a willingness to move on from a problem that is proving too energy-intensive, even if she hasn't fully solved it, in order to conserve resources for other climbs. This contrasts with the in-depth approach she might take in math, where she can dedicate extended time and effort to find a single, definitive solution.

Ashley's affect in problem-solving is strongly linked to her approach to persistence and managing frustration, particularly with respect to when she takes breaks. While she describes herself as "*really really stubborn*," she actively works to avoid becoming overwhelmed by frustration during the problem-solving process, drawing parallels between her strategies in coding and climbing:

I would say that is mostly approach how I code—I tweak one part over and over again. And then if I get too frustrated, I go back to know what I know worked and then try to add different pieces. I will only do that after taking a break though... Also, in climbing, I tweak one part of the climb at a time and keep doing that over and over again, and once I finally get the move, I will let myself come down from the wall. I take a ton of breaks and just hang there while I figure it out though.

This approach of taking deliberate breaks suggests that her "addictive personality" percolates into her problem-solving style. This characteristic doesn't translate to a head-down approach of not knowing when to quit; instead, it shows up as a persistent process of trial and error with necessary breaks to maintain clarity and prevent burnout or injury.

Ashley also emphasizes the importance of community in climbing very explicitly, particularly in the context of learning to navigate her disability. She shares, "It's really helped me a lot. Kenneth was the first disabled person I really talked to and formed a friendship with... and he helped me to accept my disability a lot and be okay with being in a wheelchair." This highlights the value of connecting with others who have similar experiences and can offer guidance in solving problems. She further explains that "I think Taylor was the first wheelchair user I ever talked to, to learn a lot of stuff about navigating the world with a disability," solidifying her point that learning from others provides invaluable tools for problem-solving. Ashley recognizes that problem-solving is not solely an individualistic pursuit and that the support and shared experiences within a community can help her ability to navigate challenges and find solutions.

Ashley's problem-solving profile during rock climbing is visually represented in Figure 8, which displays both her positioning across five key dimensions (top radial chart) and her

strategic approach frequencies (bottom bar chart). As a reminder, unlike other participants who were observed climbing, Ashley's data derives entirely from interview discourse about her climbing experiences rather than direct observation, as she did not physically climb during the study period. The radial chart reveals *Disposition* (17) and *Environment* (15) as her highest dimensions, followed by *Efficiency* (9), *Affect* (7), and notably minimal *Expertise* (1). This shape reflects Ashley's adaptive approach to climbing that is likely in part shaped by her disability. Her elevated disposition (17) captures her "stubborn and competitive" nature tempered by pragmatic wisdom, reflecting her evolution from "100% stubborn to 60% stubborn" as she learned "when to call it" when faced with physical limitations. Her high environment score (15) illustrates her awareness of the role the environment plays in her problem-solving. Her relatively high efficiency (9) reflects a consistent consideration of how to conserve energy, and her minimal expertise (1) aligns with being a novice climber. The strategy bar chart reveals *Constraints* as her dominant approach (18), demonstrating a constant negotiation with physical limitations and energy conservation. Her frequent use of *Trial and Error* (9) illustrates her exploratory approach as a beginner and minimal *Planning Throughout* (3) suggests less developed ability to read routes or anticipate sequence.

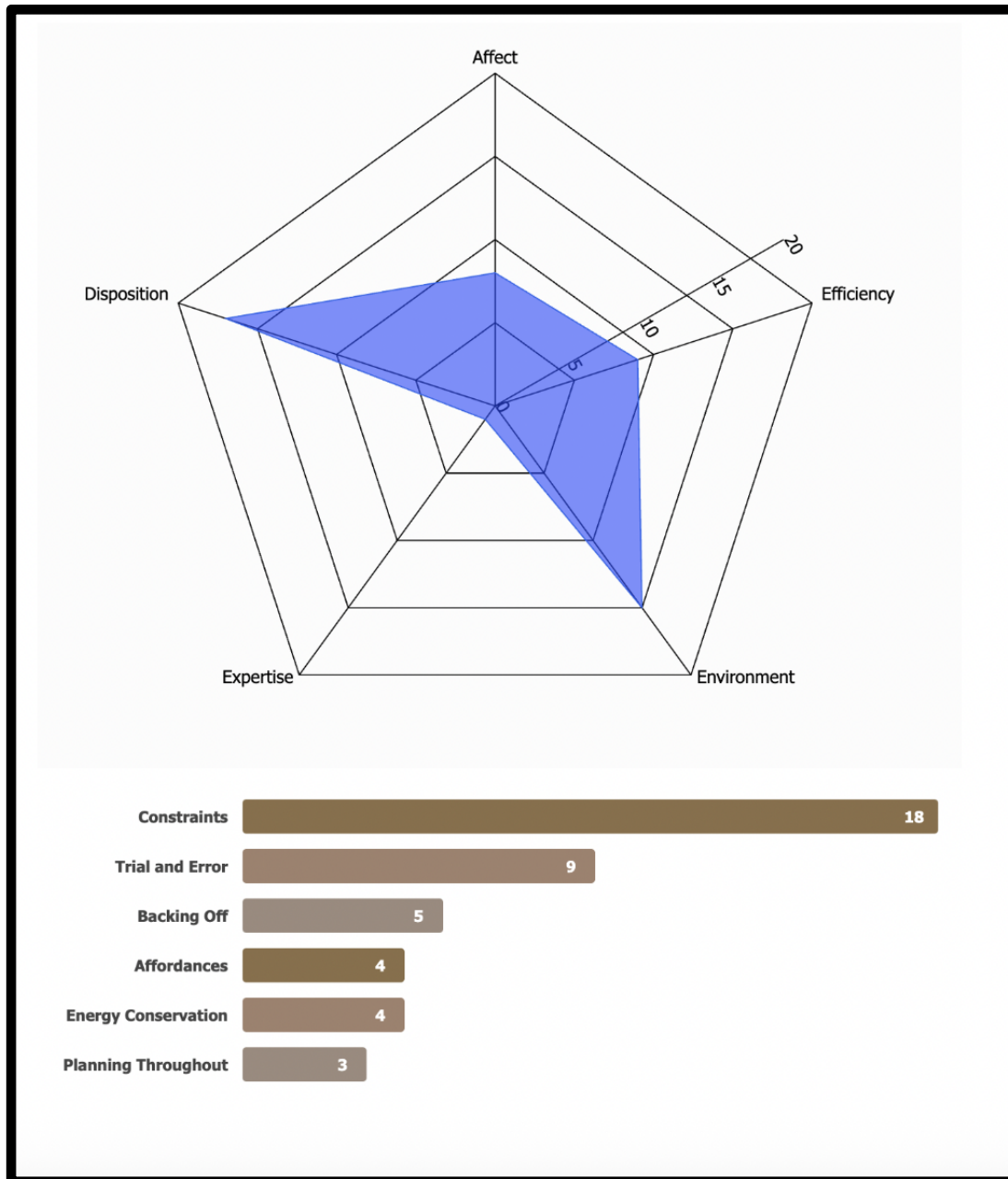


Figure 8: Ashley's Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Ashley's radial chart on the top, depicting her positioning in problem-solving. A horizontal bar chart on the bottom provides a breakdown of Ashley's specific problem-solving strategies.

Connections Across Participants

Efficiency Across Participants

Efficiency emerges as a critical consideration for all participants, but shows up differently based on individual circumstances. For Taylor, efficiency is deeply connected to pain management and energy conservation, leading him to carefully plan rest points and prioritize "breaking beta" to conserve resources. Chad similarly emphasizes that "efficiency is everything" due to limited energy resources (what he calls "spoons"), driving him to invest time upfront in trial and error to discover the most efficient approaches for future use. Ashley also prioritizes energy conservation, noting "I only have so much energy. When I go to the gym, I know I only have three routes, so I need to be wise about what I choose to climb." This common thread suggests that for many disabled climbers, efficiency is not merely about optimization but is about *necessity* due to energy limitations induced by disability.

Disposition Across Participants

The participants demonstrate varying dispositions toward problem-solving, often intrinsically shaped by their experiences with disability. Taylor's disposition is characterized by self-awareness and intentionality, developed through "relearning how to interact with the world after life-altering traumas." This has fostered a resistance to relying solely on intuition and maintaining a sense of curiosity. Christopher exhibits a strongly solution-oriented mindset, expressing that "if [non-disabled climbers] can do it, I know that I can do it," reflecting a firm conviction that every problem has a solution. Ashley's disposition is characterized by being "stubborn and competitive," which she acknowledges has evolved from "100% stubborn to 60% stubborn" as she's learned when to adjust her approach in response to disability. For Chad, his disposition is encapsulated in his mantra "Don't say I can't, say how can I?" which reflects his choice to

reframe frustration as excitement for new challenges. Kenneth's disposition as a teacher influences his disposition by emphasizing self-discovery, personalized learning, and fluidity in his approach to problem-solving. Sofia's disposition is shaped by her acceptance of uncertainty and necessity to operate with incomplete information due to vision loss, embracing what she calls "complete oversight mode" where planning isn't an option. Here, all participants demonstrate a generative disposition. In other words, their dispositions are framed around a transformation of constraints (associated with disability) into opportunities for problem-solving through inquiry, reframing, and engagement with alternative pathways.

Environment Across Participants

The participants' environmental contexts influence their problem-solving approaches in climbing and beyond in other domains. Sofia's relationship with her environment is formative for her problem-solving as she describes living life in "complete oversight mode" due to being blind, requiring her to accept uncertainty and work with incomplete information. She explains, "As blind climbers, we're using incomplete information to understand our world, and time is our enemy." Taylor contrasts the freedom he finds in climbing with the "exhausting and limiting" considerations of navigating the world in a wheelchair, highlighting how unaccommodating environments shape the way he perceives and responds to challenges. Kenneth's environmental perspective is grounded in different problem-solving approaches for different contexts—being playful and exploratory in the gym environment ("fuck around and find out") but more methodical in alpine settings where consequences can be life-or-death. Taylor, Christopher, Sofia, and Chad (all competition climbers) describe how the competitive environment induces completely different strategies than when they are climbing in casual contexts. Overall, the connection to the environment is a key factor in shaping their overall approach to problem-solving.

Affect Across Participants

Across interviewees, a critical component in each of their idiosyncratic problem-solving styles is affect. Taylor's affective profile centers on a dynamic relationship with confidence that changes as he approaches his limits on the wall, while also experiencing deep feelings of liberation when climbing compared to navigating the world in a wheelchair. Christopher's affective profile is characterized by imperturbable confidence, empowerment, and presence, shifting from initial frustration to determined persistence when facing challenges. Kenneth embraces a playful demeanor and growth-oriented mindset, prioritizing enjoyment and treating climbing as an opportunity for experiential learning rather than serious competition. Chad's emotional experience is grounded in his determination to challenge societal expectations about blindness, particularly motivated by the desire to be a role model for his children. Sofia's affective profile reflects her strategies for managing the mental exhaustion and constant frustration of navigating an inaccessible world with vision loss, while also maintaining a strong sense of empathy in her problem-solving. Ashley's affective tendencies show up in her balance between stubborn persistence and strategic breaks to prevent frustration from overwhelming her problem-solving process. Ultimately, affect plays a crucial role across participants in shaping their problem-solving.

Expertise Across Participants

Across the participants, expertise in problem-solving shows up in unique ways shaped by their specific disabilities and life experiences. Taylor's expertise can be seen in his ability to shift between deliberate pre-planning and intuitive, embodied knowledge in competitive settings where "your body knows what to do before your mind does." Christopher has developed specialized expertise in using his prosthetic leg in novel ways, combining this niche prosthetic knowledge with his understanding of route reading and "split moment decisions" based on

instinct. Kenneth's expertise— especially as a teacher— is more metacognitive, distinguishing between "lateral" and "progressive" thinking approaches and adapting his strategies based on environmental risk levels. Chad has cultivated expertise in "on the fly" problem-solving necessitated by his blindness, developing intuitive movement where "my body is problem solving for me." Sofia's expertise has evolved from linear single-variable problems to handling multiple "stacked constraints," or variables, simultaneously.

The common thread across participants is expertise as attunement to constraint; meaning, each participant has developed knowledge that enables them to fluidly perceive and respond to affordances within their individualized coupling with their environments. This concept can be seen in Taylor and Chad's intuitive embodied knowing, Christopher's prosthetic innovation, Kenneth's metacognitive flexibility, or Sofia's multivariable navigation

Strategies Across Participants

Across participants, problem-solving strategies differ— contingent on all the aforementioned characteristics. Taylor employs systematic pre-planning from the ground, emphasizing pain management as both constraint and strategic consideration through a "picking battles" mindset that prioritizes long-term sustainability over "sending." Christopher's strategy centers on identifying and leveraging affordances on the wall with the conviction that "if others can do it, I can find a way," making "split moment decisions." Kenneth distinguishes between different problem-solving approaches—using an exploratory method in low-risk gym environments but shifting to careful risk assessment in high-consequence alpine settings. Chad, unable to plan routes visually, has cultivated "on the fly" adaptation skills, relying on his body's intuitive problem-solving where "my body knows how to react without me telling it how to." Sofia, "living in uncertainty" due to blindness, makes "logical jumps" to conserve mental energy when dealing with multiple "stacked constraints" simultaneously. Ashley is a more conditional problem

solver, with "at least three back up options" and "five different scenarios," intentionally conserving energy by taking strategic breaks during problem-solving while leveraging knowledge from others with similar experiences. Ultimately, across participants, their strategies are a product of attributes listed above: efficiency concerns, disposition, environmental considerations, affect, and expertise development.

Phase 3: Mathematics

Applying the A Priori Framework from Phase 2

During Phase 3 analysis of mathematical puzzle tasks, the six-category framework developed from Phase 2 climbing data proved rather robust. Notably, no new categories emerged, indicating that the dimensions of Problem-Solving Strategies, Affect, Dispositions, Expertise, Efficiency, and Environmental Factors were sufficient for capturing mathematical problem-solving experience. However, there were several refinements within existing categories, with 18 new subcodes emerging across five of the six categories (all but expertise). For example, two new problem-solving strategies emerged during mathematical analysis. First, "Repeating for Refinement" captured participants' voluntary repetition of successfully completed puzzles to optimize their approach, find patterns, or achieve mastery. The second strategy that emerged was "Starting Over," which captures when participants completely abandoned their current approach, starting fresh.

An important development during Phase 3 analysis was the emergence of new subcodes within Environmental Factors that described task-level features rather than external contextual factors: Environmental Feedback; Decision Paralysis (Affordance overload); and Environmental Stability versus Dynamism. Newell's (1986) constraints framework that I leveraged earlier distinguishes organism, environment, and task as separate constraint categories. However, I use "environmental" more broadly to capture both external contextual factors (physical setting, social context, accessibility) and *task constraints*—the designed features of

problem spaces including goal structures, rules, and feedback mechanisms. In other words, I treat task design as a subset of environmental factors, reflecting how problem structure shapes possibilities for action parallel to broader contextual elements. This conceptualization aligns with Gibson's (1979) ecological approach, where affordances emerge from the relationship between organism and environment, and that relation is shaped by a goal. From this perspective, task features (rules, goals, feedback structures) are inherently part of the environment itself that affords or constrains action. Methodologically, this approach preserved the six-category framework established in Phase 2 while capturing the full range of environmental influences on participants' problem-solving.

Over all, the stability of the six-category structure made the process more methodologically rigorous by operationalizing and testing the framework across domains, illuminating its utility for understanding embodied problem-solving beyond the climbing context from which it emerged. See Appendix D-H for codebooks with refinements from this part of the study.

Next, I delineate these findings by exploring how each participant approached each of the three puzzles. I then discuss their attributes across the puzzles as well. Notably, all but one participant (Taylor) compared their problem-solving process of the puzzles to their experience of rock climbing, whether it was a specific type of rock or affective states evoked during climbing. Taylor's paraplegia constrains him to gym climbing exclusively and he therefore has less climbing reference points (outdoor rock types, different environmental contexts) that other participants drew upon when making sense of the puzzles. Five of the six Phase 2 participants completed Phase 3; Sofia was unable to participate in the mathematical portion of the study. Refer to Appendix B for pictures of the mathematical tasks.

Taylor Math Problem-Solving

Approach to SOMA

With the SOMA cube, Taylor had “no plan” and immediately adopted a trial-and-error approach. After several rounds of “playing around with it,” he eventually started “to think about it.” He eventually began to identify what he perceived to be the most complicated pieces—specifically the blue and purple and the potential corner pieces. These pieces were “more three-dimensional” for Taylor, and he felt “they were harder to work with” (See Appendix B for reference). He started over many times and when he ultimately solved it, asserted “I don’t think there was much strategy involved,” because “when you are trying out something totally new for the first time (no realistic parallel), you don’t have something to compare it to, no reference points, so you just have to go for it and see what happens.” Upon reflection, Taylor found it fun even though he couldn’t explicate any strategies because he “....had to go into a different part of my brain that [he] had not used before.”

Approach to Towers of Hanoi

The Towers of Hanoi represented Taylor’s “ideal” problem-solving experience because he said “it was similar to rock climbing.” He began by staring at the puzzle, mentally simulating movement, asserting “I am just gonna sit here and play it out in my head” before physically engaging—a planning strategy that worked for the four-disc version and he flowed into it seamlessly. However, attempting to pre-plan the five-disc version, Taylor asserted it “destroyed [his] brain,” forcing him to abandon that approach of mental simulation, and instead asserted that he was “just going to go for it.” With five discs, he stumbled more and backed off repeatedly to reassess his positioning, though he notably refused to start over completely. Instead, he relied on a key checkpoint: knowing he needed to get the biggest piece to the last rung became his reference point for tracking progress. He frequently assessed whether he was

"on the right track," using these cues to maintain momentum even when the intuitive flow broke down. He explicitly connects this to climbing:

In indoor climbing, there's obviously like a very clear reference...I've made X amount of progress from this route. You can be in those flow states, those brief check-ins, whether it's like your breathing or like fatigue or whatever it is, having those moments to reflect just help.

This puzzle exemplifies the balance Taylor often discusses — enough structure to provide feedback while still allowing space for intuitive movement through backing off and regrouping rather than completely resetting.

Approach to Tangram

With the Tangram puzzle, Taylor used a border-first strategy (completing the pieces of the border first, and then filling in the inner pieces after), equating it to a jigsaw puzzle. He broke down the problem by playing with pieces outside the frame before committing to putting them in the actual puzzle space. After filling in the borders, he identified a pattern—that the red and yellow pieces were parallel—and built his solution from that insight. Unlike the SOMA cube's complete novelty or the Towers' connection with climbing, Taylor found no direct parallel for the Tangram experience; however, he noted that it offered "creative freedom and clear bounds because the other pieces were in first."

Problem-Solving Attributes

During the mathematical portion of the study, Taylor's disposition towards problem-solving is shaped by what he terms "fluid perspective"—his self-proclaimed core philosophy. According to Taylor, "...it's the fluidity of just like giving yourself the grace and patience to accept the circumstances and the state of the situation that you are currently in." His approach centers on maintaining what he describes as a "blueprint with space to breathe." Meaning, he has a loose plan of attack with contingency options (Plan A and Plan B) while also allowing

space for intuitive flow because rigid planning would constrain creativity. He explicitly rejects over-planning. Indeed, Taylor asserts that when he has "...too many plans, too many what-ifs, then I can't flow" just as he avoids over-trusting intuition alone, seeking instead a balance between structure and spontaneity. This shows up in his patience and deliberateness and is evident when he asserts "the more patient and delicate that you are, the better chance you'll have to just flow."

Taylor's process-oriented mindset values small victories and flow state over goal achievement: "I like the small victories, it's not about achieving some huge goal. It is more about the process." He consistently connects problem-solving across domains, comparing mathematical puzzles to the game 2048, equating his problem-solving approach in Towers of Hanoi to rock climbing, and drawing parallels to his design work. His lifecycle analysis coursework in grad school exemplifies his "systems thinking" disposition, where he passionately discusses how "you tweak this one thing, how much it tweaks everything else." This indicates that he views problems as interconnected ecosystems rather than isolated obstacles. Taylor's holistic orientation extends to his fascination with materials as "building blocks of everything," revealing a disposition that seeks to understand not just immediate solutions but the entire cycle of relationships and implications within the problem space.

In the mathematical context, Taylor's environmental consideration centers on his need for what he calls "...bounds [in the environment] as reference points, brief moments of check-in, like am I making progress? I need them for flow state." The Towers of Hanoi exemplifies this notion, providing clear visual checkpoints that give him feedback that he is on the right track. He describes knowing "once you get the bottom piece over to the last section, you know that you're sort of going in the right direction." This environmental feedback serves as a reference point that allows him to move intuitively and quickly while maintaining a sense of progress. By contrast, the SOMA cube did not provide any feedback to Taylor, preventing him from

accessing flow, and he was ultimately unable to assess whether his movement was even productive. Therefore, Taylor's optimal problem-solving environment is one that offers just enough structure to provide feedback while preserving space for intuitive movement. This balance mirrors his broader disposition of maintaining loose plans with built-in checkpoints.

Taylor's affective engagement with problem-solving in the mathematical context is driven by his pursuit of enjoyment and flow. He found the Towers of Hanoi "really fun" specifically because of how it felt—the experience of moving intuitively and flowing through it. He prefers a “feelings-based approach rather than thinking about the algorithm.” His emotional satisfaction comes from the experience of the process rather than outcome. When he finished the SOMA cube without accessing flow, he asserted “...the process itself was not great.” The affective dimension also includes his self-compassion, giving himself “grace and patience to accept the circumstances and the state of the situation” he's currently in. This emotional self-acceptance allows him to navigate the oscillations in performance.

Taylor's expertise development is centered around accessing what he describes as flow state—a form of embodied knowing where “the algorithm just kind of becomes an innate part of a feeling rather than an algorithm.” When solving the Towers of Hanoi, he experienced this optimal grip most fully: moving intuitively, his brain “just kind of knows where to place something...after a while, you're not even really thinking about what you're doing. You're kind of just doing it.” For Taylor, flow state feels “most like climbing” specifically because both involve this embodied expertise—moving through sequences where the body seems to know exactly what to do “without having to really think about it.” Ultimately, expertise for Taylor is about developing the embodied knowledge that allows intuitive, fluid movement.

In sum, Taylor's approach to mathematical problem-solving is reflective of a philosophy he calls “fluid perspective”—balancing structured planning while leveraging intuition. His process-oriented mindset values the quality of engagement over outcomes. In the environment in which

he is problem-solving, he desires what he calls "bounds as reference points"—structural elements that provide progress feedback while facilitating and shaping intuitive movement. The Towers of Hanoi exemplified this ideal balance with clear visual checkpoints, while the SOMA cube's lack of feedback prevented flow. He is more feeling-based rather than algorithmic and his expertise development centers on achieving optimal grip—that embodied state where "the algorithm becomes an innate part of a feeling" and his brain "just kind of knows where to place something" without conscious deliberation. For Taylor, the expertise he values in the mathematical contexts mirrors climbing expertise: both involve developing knowledge that enables fluid, intuitive movement through problem spaces where the body seems to know exactly what to do.

Taylor's problem-solving profile during mathematical engagement is visually represented in Figure 9 below, which displays both his positioning across five key dimensions (top radial chart) and his strategic approach frequencies (bottom bar chart). His highest dimension is *Disposition* (16), followed by much lower scores across *Affect* (7), *Expertise* (4), *Environment* (4), and *Efficiency* (3). This figure demonstrates a problem-solving identity anchored in his disposition toward maintaining loose structure with space for intuitive movement, rather than heavy environmental consideration or efficiency maximization. His low expertise score aligns with his preference for a feelings-based approach rather than thinking about the algorithm. The strategy bar chart complements this positioning by revealing his tactical approach: *Constraints* and *Backing Off* were dominant strategies (both at 7 occurrences), illustrating his practice of identifying limitations in the puzzles while repeatedly reassessing his positioning. His frequent use of *Trial and Error* (5), *Pre-Planning* (5), and *Break Down* (4) strategies demonstrated the balance between preparation and exploration aligned with his framework of "blueprint with space to breathe." Notably, Taylor employed minimal *Starting Over** (2) and *Energy Conservation* (1) as well. Together, Figure 9 captures Taylor's problem-solving identity in the mathematical space: a disposition-driven, fluidity-oriented approach, favoring intuitive movement through problem spaces with just enough reference points to maintain flow

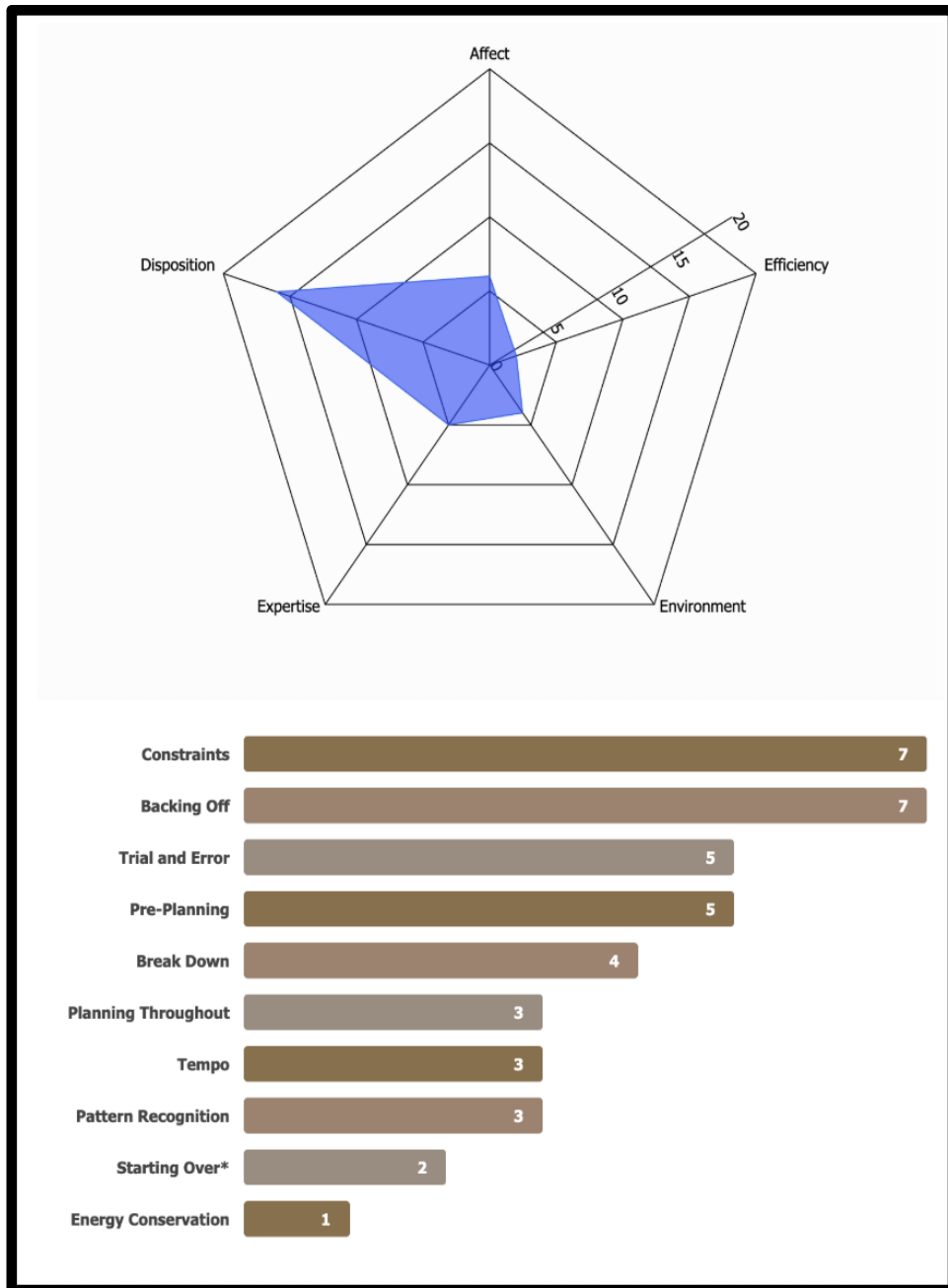


Figure 9: Taylor Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Taylor's radial chart at the top depicting his positioning in math problem-solving. A horizontal bar chart at the bottom provides a breakdown of Taylor's math specific problem-solving strategies.

Christopher Math Problem-Solving

Approach to SOMA

Christopher began the SOMA cube with an initial plan—asserting "...just knowing it's a 3 by 3, trying to get the bottom 3 by 3...and now trying to like, build up"—but quickly recognized this approach would not work. He then shifted to trial-and-error for several iterations, eventually identifying that the parts that stick out (blue and purple pieces) were his constraints. However, he repeatedly saved the blue and purple pieces for last, despite recognizing them as problematic, and started over multiple times. After 25 minutes, he began to develop a strategy: using the purple and blue pieces as bottom corner pieces to "...get them out of the way," rather than saving them for last. Upon completion, Christopher reflected that he "started off, I thought I had a strategy, but I guess I just was more patient...I was more patient and planned ahead to some degree, but it still felt like chance."

Christopher compared the SOMA cube to climbing in the Red River Gorge—a type of climbing (heavily features sandstone climbing with massive holds everywhere) that for Christopher, presents an overwhelming amount of options. Christopher laments "you have so many holds everywhere and they all suck. But you keep thinking that the next one would be better but it never gets better." Later in the interview, he also equated the SOMA cube to indoor climbing: "This one felt like indoor climbing cause there's only one way that you can do the moves...I face more just like complete shutdown moves because I can't figure out why it isn't working." Either way, whether Christopher feels overwhelmed with too many options or too few options, he equates the SOMA cube to types of climbing that feels constrained and is therefore challenging to figure out the beta that works for him.

Approach to Towers of Hanoi

Christopher immediately recognized the constraint of Towers of Hanoi: "I need to have the base over here eventually, which means...it needs to be the only one here...which means I don't need anything here, so...my strategy is to get these 3 to the middle." He completed the four-disc version efficiently, and when confronted with five discs, Christopher believed his exact algorithm from the four disc problem would transfer. He therefore struggled because he rigidly applied his first move pattern to the five disc problem— He held very tightly to making the same exact first move onto the middle rung and couldn't solve it efficiently that way. He then returned to the four disc problem to reinforce the pattern, and then, upon turning back to five discs and eventually recognized: "I need to get the whole thing over to the other side, not the middle. So maybe I need to change the first move." This realization allowed him to discover a generalizable algorithm based on odd versus even numbers: "for 4 you go here and 5 you go here, but that probably would work for all even and odds." He appreciated how he could "...keep checking in with myself and assess my progress. I kept having to pause at the cruxes along the way." Ultimately, Towers of Hanoi was Christopher's favorite puzzle because "It just makes sense, I know that there are set guidelines, there are rules." Finally, even after completing both the 4 and 5-disc problem, he insisted on repeating the problem again, to ensure optimal efficiency in the solution.

Christopher compared Towers of Hanoi to "a hard boulder problem because I started with a plan, but then I kind of got hit in the face, if you will, along the way." He appreciated how he could "keep checking in with myself and assess my progress. More broadly, Christopher connects the Towers of Hanoi to sequencing in climbing: "climbing is the same way...if I know I need to get my right hand on this one like particular hold, I'm gonna set up a sequence from the bottom in a way that makes sense." He elaborates that sequential thinking is essential "...at your

limit because you wouldn't want to all of a sudden be in a position that wouldn't enable the move you need." The puzzle mirrors his beta-reading process on the wall:

I knew in my head what I wanted to do, and sometimes with the climbing, I can run through the climb in my head cause I remember beta. But then, like as I've started to move the pieces...I kind of forgot that sequence. Climbing can be like that too...you're so focused on the hand you forget what foot you are supposed to use.

Christopher's parallel between Towers and climbing highlights the need for sequential adaptability. Just as he had to revise his approach when five discs didn't behave like four ("I need to get the whole thing over to the other side, not the middle. So maybe I need to change the first move"), climbers at their limit must adjust beta mid-sequence when executed movements don't match the anticipated movement. The difference between an easier climb where "you could just like match or something" and recover and limit climbing mirrors the difference between simple Towers problems and complex ones—at easier grades, you have flexibility to improvise, but at your limit, perfect sequencing becomes essential. The sequence must be intentional, internalized, and executed in the correct order, with strategic pauses at cruxes to evaluate how to best move forward.

Approach to Tangram

Christopher began the Tangram with an immediate strategy of identifying the corner pieces, recognizing that "...there are only certain pieces that can fit in certain places, like at the top right corner." He methodically tested every piece to determine what could fit, determining that there was only one piece that works for the corner, then proceeded to identify other problematic pieces. He recognized points where "this piece could go here, but so could this one...so I have options here which makes it trickier," but systematically ruled out possibilities until he could confidently commit: "this is the only piece that can go here." Christopher solved the Tangram quickly and efficiently. He appreciated this puzzle because "if you approach it in a

structured way, it comes together," contrasting it with the SOMA cube where he couldn't figure out how it worked or how to replicate a solution.

As a parallel to climbing, Christopher compared the Tangram to "a ladder jug haul...it felt easy...I started to figure things out along the way and then it just kept working." Christopher compared the Tangram to "a ladder jug haul...it felt easy...I started to figure things out along the way and then it just kept working." A "ladder jug haul" is a type of climbing sequence where large, positive holds (jugs) appear consistently like rungs on a ladder—each hold is obvious and leads naturally to the next without much explicit thought. Similarly, once Christopher identified the corner constraint on the Tangram, each subsequent piece placement became increasingly apparent as the remaining options diminished.

Problem-Solving Attributes

Christopher's disposition toward problem-solving in the mathematical context is characterized by his persistence, drive to excel, and competitive background, first drawing on his experience in wrestling:

I want to understand what's the limits of like what is humanly possible... That is probably from my experience with competitive wrestling...I was surrounded by a lot of world class wrestlers, ... and in general the way I was raised was just try to be the best at everything you do and to never give up.

Continuously throughout the interview, Christopher brought up rock climbing and referenced it to explain how he was orienting himself to the puzzles. During the span of time from the first and second phase of the study, Christopher stepped out of the competitive rock climbing space to focus on his bouldering career and had recently achieved a major feat in the space: climbing a V11—a grade that places him among elite climbers, as it typically takes years of dedicated training to achieve and represents a level that only a small percentage of climbers worldwide ever reach (the grades of bouldering range from V0-V16). This shift in focus equipped

Christopher with language in conceptualizing the puzzles: "Bouldering feels more like a puzzle than climbing ropes. On ropes, there's insignificant moves along the way, and they are not as engaging or challenging... like the moves are hard to remember. But in bouldering, every move matters." He also discusses the nuanced beta needed to achieve such a feat, meticulously verbally reciting and gesturing the entire sequence. Finally, he discusses disposition surrounding his disability: "Whether it's climbing or math, [my disability] is just another attribute that makes my problem solving personalized. But no matter what, I would still be problem solving, with or without disability" This stance reflects a disposition that suggests Christopher may see constraints not as limitations but as parameters that shape solution paths.

In the mathematical context, Christopher's environmental considerations center on feedback mechanisms and the ability to assess progress through checkpoints. The Towers of Hanoi exemplifies this kind of environment, where he could "keep checking in with myself and assess my progress." He describes "pausing at the cruxes" to evaluate his position, asking questions like "Was that the same amount of moves?" to verify he was on the right path, asserting "I liked that I was able to keep checking in with myself and assess my progress." By contrast, the SOMA cube's lack of clear feedback frustrated him because he was not able to figure out why it wasn't working and the excess of possibilities without clear constraint markers gave him no indication that he was going in the right direction.

Christopher's affect in mathematical problem-solving is tied to the sense of reward that comes from overcoming difficulty through sustained effort. He explicitly states that Towers of Hanoi was "more rewarding to complete than the Tangram because it was more difficult." His frustration with the SOMA cube stemmed not just from its difficulty but from the lack of clarity about why approaches weren't working—"it makes me mad when I don't know how I solved it. I got it, but it was complete chance!" His affective state improved throughout the study as he moved away from the SOMA cube and developed some confidence. Initially, after the SOMA

cube, he described his relationship with puzzles as frustrating: "They always make me mad, cause I can't do them as quick as I want to." This sentiment progressed after Towers of Hanoi to a sense of enjoyment, asserting "Okay, that was actually kind of fun, because it just makes sense...I know that there are set guidelines, there are rules."

Christopher's expertise in the mathematical context shows up as the ability to flow through problems—moving with confidence and momentum when structure and feedback align with his sequential thinking. This flow state appeared most clearly in the Tangram, where he described how "it just kept working" as he "started to figure things out along the way." In the Towers of Hanoi, Christopher's expertise emerged through his capacity to make sense of generalizable patterns (odd/even algorithm). The algorithm became internalized to the point where he was able to flow through the sequence. Ultimately for Christopher, expertise is about identifying what approaches are viable given your particular constraints, then flowing through those viable paths. When the Tangram offered clear constraints that guided him toward obvious moves, he could flow. When the SOMA cube offered too many possibilities without clear feedback, flow was impossible.

Christopher's problem-solving profile during mathematical engagement is visually represented in Figure 10, which displays both his positioning across five key dimensions (top radial chart) and his strategic approach frequencies (bottom bar chart). The radial chart reveals *Disposition* (10) as his highest dimension, followed by consistently low scores across *Affect* (5), *Efficiency* (4), *Environment* (4), and *Expertise* (4). His compressed geometric shape reflects a persistent, drive-oriented approach—his belief that he wants to "understand what's the limits of like what is humanly possible. His low affect score (5) captures his initial frustration with puzzles though this improved as he encountered structured problems like Towers of Hanoi. His minimal environment (4) and expertise (4) scores suggest less concern with environmental feedback and less developed mathematical knowledge, though he values checkpoints when available. " His

low efficiency (4) reflects his willingness to start over multiple times and repeat problems for refinement. The strategy bar chart reveals equal emphasis on *Constraints*, *Trial and Error*, and *Starting Over** (all at 9 occurrences), demonstrating his iterative approach of identifying limiting factors, playing around without a game plan, and completely resetting when needed. His moderate use of *Planning Throughout* (6), *Pattern Recognition* (6), *Sequencing* (5), and *Repeating for Refinement** (5) illustrated his developing ability to recognize generalizable patterns, particularly his discovery of the odd/even algorithm in Towers of Hanoi.

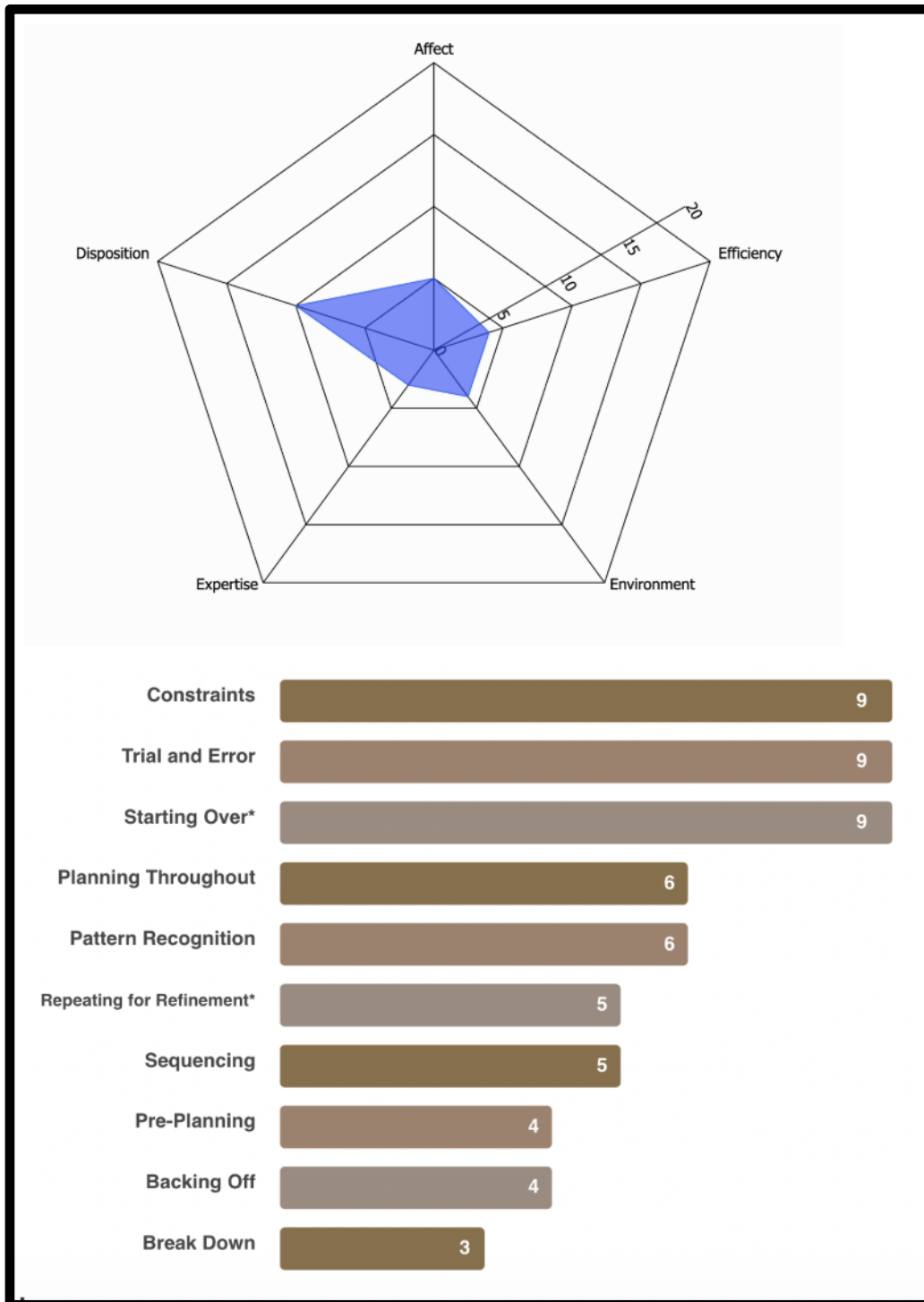


Figure 10: Christopher Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Christopher's radial chart at the top depicting his positioning in math problem-solving. A horizontal bar chart at the bottom provides a breakdown of Christopher's math specific problem-solving strategies.

Kenneth Math Problem-Solving

Approach to SOMA

Kenneth began the SOMA cube by identifying foundational pieces, asserting he was "looking at the ones that have three in some horizontal or vertical plane." He then shifted to what he describes as "just kind of fucking around"—engaging with the puzzle through extensive trial and error, playing with pieces to understand how they relate to each other since he couldn't "easily understand how they relate to each other by looking at them." He identified the blue and purple pieces as particularly challenging because they were more three dimensional, attempting to address them early by building them separately (see Appendix B for a picture of Kenneth working on this problem, particularly parsing out the "beta" for the blue and purple pieces). However, despite moving through multiple attempts and backing off repeatedly, saying "I started over a lot of times...and every time, I tried something different. I think I backtracked a lot" he ultimately solved it by abandoning his perceptual fixation on keeping blue and purple together, the only explicit strategy he recognized using. Upon completion, Kenneth felt neutral rather than accomplished: "I didn't really feel like I learned something and then did something where I applied it...it was almost just like I just banged my head against enough times that, by chance, how it came together." He acknowledged that if he took it apart, he wouldn't know how to put it back together, reflecting his sense that the solution emerged through persistence rather than understanding. The puzzle's open-ended nature created what he later described as "abstract and meaningless."

Like Christopher, Kenneth explicitly compared the SOMA cube to "climbing at the Red, or something where you're like climbing on that sandstone, and they're just like almost unlimited holds. Even on a hard climb it feels like there are unlimited things you can do." This parallel illuminates why the puzzle frustrated him: the abundance of options without clear constraints prevented him from establishing the foundation he needed to problem-solve in a way that he

enjoys. Unlike his preferred "progressive" problems where constraints shape decision-making, the SOMA cube's "lateral" nature—requiring him to make connections across unlimited possibilities without obvious structure—left him feeling without direction. The SOMA cube's requirement to start from scratch—building foundation without guidance—paralleled his broader distaste with open-ended beginnings in problems, whether in climbing or in his former work as a software developer where he "really really preferred to fix bugs in existing code than to create new [code] from scratch."

Approach to Towers of Hanoi

Kenneth approached the Towers of Hanoi with intentional pre-planning, spending over four minutes initially staring at the puzzle before making any moves. His first vocalized strategic insight was recognizing "...I think the big piece should only move once." He explicitly compared his thinking process to chess: "In chess, you need to think a few moves ahead but I am thinking probably like three moves ahead and then I get a bit fuzzy." Kenneth found the Towers of Hanoi more engaging than the SOMA cube. He valued this puzzle because "I can see it. I can't see the [SOMA], like I can't look at those three-dimensional shapes, and really see how they can come together...I can see the path to the solution. It's more straightforward." His initial approach was inefficient, but he revisited the puzzle mentally while away from the room ("while I was going to the restroom") and discovered a more efficient solution that he successfully completed upon returning. This breakthrough felt rewarding for Kenneth—"Yeah, yeah. It's super cool! It was a fun, little puzzle." He ultimately described the Towers as "more transparent in how "things interact with each other," allowing him to leverage a more progressive approach to problem-solving.

Kenneth explicitly identified the Towers of Hanoi as:

This puzzle is more like rock climbing than the other problems because it is more progressive. If you do a move in climbing, then there are only a couple of options of what you could do next. It's like a process of elimination, it's more constrained.

To Kenneth, this structural similarity to rock climbing helps to explain why he preferred this puzzle: "it's the type of problem solving that I a) more naturally enjoy and b) have practiced a lot more."

Kenneth positions both the Towers of Hanoi and rock climbing as "progressive" problems characterized by well-defined starting points, ending points, and intermediate checkpoints, what he called "middle points" that act as an "....achievement... so you don't have to get all the way to the end to feel success." More specifically, he equates this puzzle to the kind of problem-solving needed in slab climbing— climbing that is more like a steep ramp rather than a vertical wall that is very technical and necessitates very specific technique. Further, climbing with a prosthetic leg makes this type of climbing much more engaging, as Kenneth asserts: "What makes [slab] more creative is I can't climb slab in the same way that someone with two ankles can climb. I move a little bit differently, and I feel like I'm constantly like, kind of problem solving." Like slab climbing with a prosthetic, Towers requires him to work within clear constraints to find a creative path forward— a problem-solving approach he has practiced extensively and gravitates toward.

Approach to Tangram

Kenneth approached the Tangram with a bit of pre-planning, stating "I can see that there are some angled pieces. So I suppose I'm gonna try to figure out how the angles go in. But really, I'm just gonna play with it." He began by exploring pieces outside the frame, noting observations like the width of certain pieces relative to one another. His strategy shifted toward constraint identification as he states "there are only certain pieces that fit in the corner," and continued to methodically test each piece in the corner position until he claims "I feel really comfortable that only purple can go there...I just tested all those different ones." However, he felt somewhat constrained by this assumption: "But I suppose I worry that that's somehow wrong, even though I just tested all those different ones. But that puts me in a corner now." He expressed concern that if his foundational assumption was incorrect, it could inhibit him from solving the

problem. Upon completion, Kenneth explicitly stated "I liked this one. I quite like solving problems that are half solved. Because I find beginning things to be challenging. Getting started feels really hard sometimes. Finding a solution feels more intuitive in that case." Ultimately, Kenneth positioned the Tangram as "creative, progressive...there's creativity in terms of irregular shapes,, but there's a progressiveness to it where I suspect there's only one way I can put these together because of its unique solution." The key distinction from the SOMA cube was that he could establish foundation: "I identified that there was one piece that fit in the corner, that is a progressive element. It was like, okay, I know that this goes there, so I can build off of that."

Like Chad, Kenneth equated the Tangram to technical face climbing: "I think the Tangram can equate to technical face climbing; Maybe it requires a little bit of piecing some things together to figure out." Technical face climbing involves navigating relatively flat surfaces using limited small hand and foot holds across the wall, requiring careful problem-solving to identify which holds will work in sequence—similar to testing which Tangram pieces fit specific positions. For Kenneth, the puzzle occupied a middle ground on his constraint continuum: more constrained than the SOMA cube's "climbing at the Red...where there are almost unlimited holds," but more open than the Towers of Hanoi's strict process of elimination.

Problem-Solving Attributes

Kenneth's disposition toward problem-solving in the mathematical context is characterized by his outcome orientation, pragmatic approach, and discomfort with abstract, open-ended problems. He approaches the mathematical tasks by asking "what's the shortest way I can get to the *what* that achieves my result?" This demonstrates prioritizing practical solutions over theoretical understanding. He explicitly states:

I definitely try to approach things from pretty basic standpoint. I don't really try to learn new math in my projects or to understand how I'm getting to the answer. It's minimal based on trying to get the result that I want, and so that might look like a trial and error approach or looking something up on the Internet, but not necessarily understanding why did I achieve the result that I want.

This disposition manifests across domains: in his former software development work, in his everyday mathematical problem-solving around measurements and garden building projects, and in his puzzle approaches where he gravitated toward problems with established foundations. His discomfort with open-ended beginnings may stem from perfectionism: "I have like a certain amount of perfectionism when it comes to building something from scratch. You have the opportunity...to have it be the best that it can be"—which creates pressure that can be mitigated when he states "...that foundation already exists, then I can just build off of what is there; it takes the pressure off." Ultimately, Kenneth's disposition reflects a pragmatic problem-solver who values arriving at functional solutions through trial and error, looking things up, building on existing a foundation, rather than investing time in understanding underlying principles that don't directly contribute to achieving his immediate goals.

Kenneth's affect in mathematical problem-solving is shaped by his ability to hold the process lightly and engage playfully. His Towers of Hanoi breakthrough elicited genuine enthusiasm—"Yeah, yeah. It's super cool! fun, little puzzle." Similarly, he appreciated the Tangram's half-solved structure: "I liked this one. I quite like solving problems that are half solved because finding solutions feels more intuitive.". In contrast, the SOMA cube left him feeling "neutral." Kenneth's affect thus may hinge on whether problems provide just enough structure to allow him to hold the process lightly and engage with playful experimentation. Finally, Kenneth's expertise in mathematical problem-solving manifests as intuition that emerges only when environmental constraints provide sufficient structure to guide his thinking. In the SOMA cube, the abundance of options without clear constraints inhibited an intuitive approach—where the sheer volume of possibilities overwhelms rather than guides problem-solving.

Kenneth's problem-solving framework during mathematical engagement is visually represented in Figure 11, which displays both his positioning across five key dimensions (top radial chart) and his strategic approach frequencies (bottom bar chart). The radial chart reveals *Disposition* (11) as his highest dimension, followed by consistently low scores across *Affect* (6), *Efficiency* (4), *Environment* (2), and *Expertise* (2). His minimal environment score (2) reflects his view of mathematical problem-solving as fundamentally abstract and decontextualized. His low expertise (2) aligns with his pragmatic stance of "looking something up on the Internet, but not necessarily understanding why did I achieve the result that I want," prioritizing functional solutions. His low efficiency (4) suggests less concern with optimizing initial approaches, instead favoring trial-and-error exploration. The strategy bar chart reveals *Backing Off* as his dominant approach (10 occurrences), demonstrating his iterative process of retreating and reassessing when approaches failed. His frequent use of *Trial and Error* (7) illustrated his "fucking around" exploratory mindset. Ultimately, these visual representations capture Kenneth's mathematical problem-solving identity: a pragmatic, foundation-seeking approach characterized by his discomfort with open-ended beginnings and his preference for progressive problems with clear constraints that allow intuition to emerge.

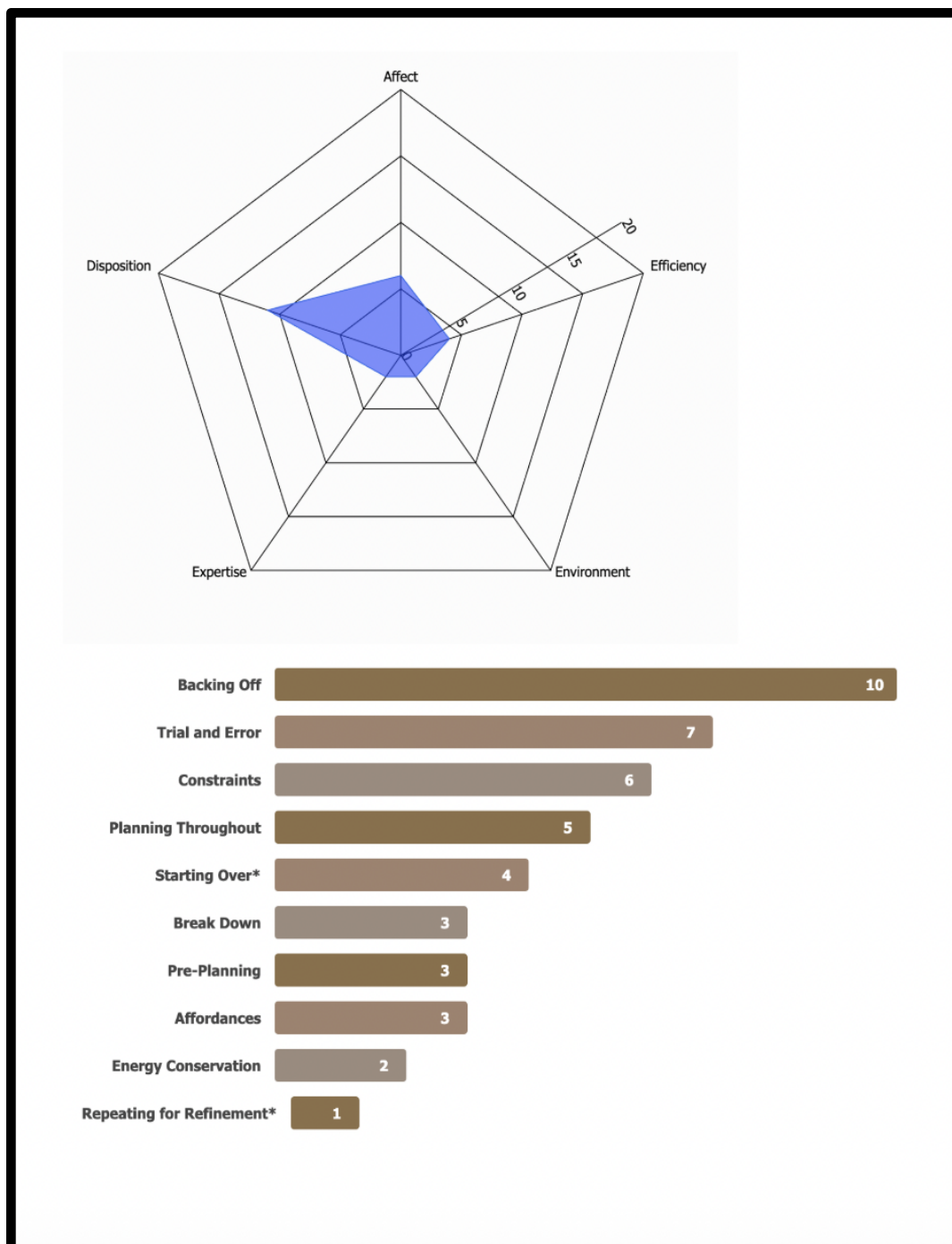


Figure 11: Kenneth Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Kenneth's radial chart at the top depicting his positioning in math problem-solving. A horizontal bar chart at the bottom provides a breakdown of Kenneth's math specific problem-solving strategies.

Chad Math Problem-Solving

Approach to SOMA

Chad began the SOMA cube with trial and error, eventually recognizing he needed "some sort of game plan" and pivoting to a base-building strategy. He knew he needed to create a 3x3 foundation, constantly feeling to verify the base dimensions: "I mean I am just trying to do this on the fly or trial and error, but it's not really working out for me." Initially attempting to leverage his sight, he says it "straining...like giving me a headache," so he then abandoned sight entirely and decided to solve it through solely feeling. He restarted multiple times and even after solving it, he immediately wanted to attempt it again, hoping to identify a replicable formula. He identified the three-dimensional, non-flat pieces as the most challenging because "they just seem like the most rigid shape, like non symmetrical." Upon reflection, Chad experienced no "aha moments" comparable to figuring out the beta on a climbing route—even after solving it twice and trying it several times after, he exclaimed "I still do not know how to do it!"

Chad made sense of his problem-solving of the SOMA cube by comparing it to different styles of rock climbing, landing on his experience of climbing conglomerate. Conglomerate rock climbing involves navigating rock composed of cemented rounded stones, creating a surface that is incredibly challenging to read for any climber, especially a blind climber. As Chad explained about the SOMA pieces: "it's like conglomerate, there's too many options...this can go like so many ways...it has to put this, it also has so many ways to go this way..." Each SOMA piece could be oriented and flipped in multiple directions, and each placement cascades into many more possibilities. This conglomerate metaphor captures his resistance to more dynamic problem spaces, where changing one variable creates "so many different scenarios" that overwhelm his desire for algorithmic execution.

Approach to Towers of Hanoi

Chad initially approached the Towers of Hanoi with trial and error, solving the four-disc version quickly and immediately wanting to repeat it for greater efficiency, asserting "I want to do it better and faster though...In an athletic realm, no one's ever been like, 'You're the slowest one to run a hundred meters. You're so great.'" When tackling the five-disc problem, he explicated his sequencing as he figured it out. He recognized early that his first move mattered strategically, though he struggled to articulate why. He became excited as he unlocked the pattern, transitioning into what he described as an intuitive flow state where his hands moved faster than his conscious thinking.

Chad explicitly compares the Towers of Hanoi to crack climbing (a type of climbing route that follows a system of cracks). In cracks, he experiences a deep knowing where "I move my arm just so, and the hold is exactly where I need to be. I don't have to search or adjust. It's so easy to flow with." This creates the building momentum he describes: "it's kind of like that feeling where I'm thinking, 'Oh my God, it's working, it's working, it's working!' And then, if I sink into that flow state, I don't have to stop."

Approach to Tangram

Chad approached the Tangram with minimal enthusiasm and quickly solved it without much elaboration. He relied heavily on feeling the pieces and identifying gaps: "I can see a blur of colors...I am just gonna start putting things down. You know, I have to drop it, flip it, spin it, feel for the gaps." He used trial and error and backed off several times, manipulating pieces through multiple orientations to find fits. He identified the angled piece as his constraint—calling it his "nemesis hold," borrowing climbing language for a particularly challenging grip. Unlike the other two puzzles, Chad felt no desire to solve the Tangram again.

Chad compared the Tangram directly to face climbing. Face climbing is when you climb up a relatively flat surface using small hand and foot holds that are scattered across the wall. In face climbing, he has to "... grab a hold and I have to feel around. Then I feel like, 'Okay, here's a foothold. I'm gonna put my foot there.' And then once I've got that locked in...I'm now like, 'Okay, I'm gonna use that. Next, I'm gonna use this.'" The Tangram parallels this process of iterative problem-solving "it's like, 'You have to try to fit this piece here. Did that work? No.' Then, 'I'm going to try to fit this here. Did that work? No.' It's a constant process." Chad asserts that face climbing is not conducive to memorization: "I never climb face routes the same way because I can't see or remember where the handholds are...the chance of my foot going to the same spot every time is very slim unless it's very beta-specific."

Problem-Solving Attributes

Chad's disposition toward problem-solving in the mathematical context is shaped by his belief that there is always a solution and by urgency; indeed, he is driven by an awareness that "the clock is ticking." He approaches problems with an algorithmic mindset, always seeking patterns and formulas: "I like doing problems that have a formula to it. Then once you learn the formula, you're just repeating it over and over again." His disposition is grounded in efficiency, asserting "slow is smooth, smooth is fast," illuminating a philosophy that values a systematic approach rather than rushed attempts that ultimately do not contribute to efficiency. His background as a mechanic reinforces this orientation, asserting "time is money," making efficiency crucial to his past profession. Chad constantly connects problem-solving across domains— diesel mechanics, hockey, rock climbing, navigating blindness— building capability, confidence, and trust by drawing on lessons learned across contexts.

Chad's affect with problem-solving in the mathematical context is shaped by his pursuit of rhythm, pattern recognition, and what he describes as the "mercury thermometer" experience—a steady rise of positive emotion as he unlocks algorithms. The Towers of Hanoi

exemplifies his ideal affective state: "This was more enjoyable because I could get into a rhythm; it's more like climbing. I got into a flow state doing this, I got so excited." He contrasts this with the SOMA cube's abrupt process, where he experienced frustration until sudden relief at completion: "...my energy was going down, and the longer I worked on it, the more frustrated I got. Then all of a sudden, the final piece went in, boom! It worked, and my mood just shot right up." For Chad, sustainable affect necessitates what he calls "little wins"—incremental progress markers that he relates to "when you get to a nice hold that you can rest on" in climbing.

In the mathematical context, Chad's expertise development is grounded in recognizing patterns and transforming trial-and-error exploration toward a generalizable, efficient, and intuitive algorithmic approach. "It can be frustrating until you learn a formula, and then it gets very easy, like when I first started climbing, it was really hard until I started learning the formula of like feeling around, even knowing what to use as a hold." His expertise is evident in Towers of Hanoi, where he began to explore and quickly identified a pattern: "Once the ball gets going, it's like, now I know exactly what I need to do and I start speeding up, and I'm almost like not even thinking about it. It's intuitive, I just can't tell you what I am doing." This transition from conscious problem-solving to intuitive movement represents the transition, when algorithms become so internalized that he cannot even articulate what is happening. Ultimately, Chad actively seeks generalizable formulas because mastery of these formulas enable intuitive execution later. Finally, he describes feeling sizes through his hands as he worked through the puzzles—"I'm seeing with this through my hands, like I'm seeing the size of the things. And I am good with my hands any way from when I was a mechanic"—demonstrating experiential knowledge developed through years of being a mechanic and navigating blindness.

Chad's experience with environmental factors in the second phase of the study illuminates attunement to environmental stability versus dynamism. He explicitly distinguishes between climbing's stable environment and the SOMA cube's dynamic nature:

The rock climb isn't moving...I mold around the rock when I climb. But with this puzzle, it needs to be molded around itself. This can change in so many ways—I move this one piece one way, and now it just created so many different scenarios. The rock climb was formed, it is what it is. Then I can remember: just move this way and then this way to make that work. And I can recreate that over and over again.

The stability from climbing gives space for pattern memorization and reproduction, aligning with his algorithmic disposition. However, the amount of affordances within the problem space of the SOMA cube led him to feel “overwhelmed.” Finally, environmental feedback emerges through his “mercury thermometer” metaphor—he needs feedback from his environment, which he calls “little wins,” to maintain positive affect and momentum, related to reaching rest holds in climbing.

Chad's problem-solving framework during mathematical engagement is visually represented in Figure 12, which displays both his positioning across five key dimensions (top radial chart) and his strategic approach frequencies (bottom bar chart). The radial chart shows *Affect* (14), *Disposition* (14), and *Efficiency* (14) forming a perfectly balanced triangle, followed by *Environment* (7) and *Expertise* (6). His balanced affect, disposition, and efficiency captures his integrated problem-solving philosophy where emotional engagement, solution-oriented approach, and efficiency-driven mindset work together in his problem-solving. His relatively moderate expertise (6) indicates developing algorithmic knowledge, demonstrating how expertise for Chad means internalizing formulas so completely they become intuitive. The strategy bar chart reveals *Feeling the Problem* as his dominant approach (9), demonstrating how he “sees with his hands.”. His frequent use of *Trial and Error* (8) illustrates his exploratory methodology and his balanced *Backing Off* (6), *Energy Conservation* (6), and *Resting* (6) reflect his resource management.

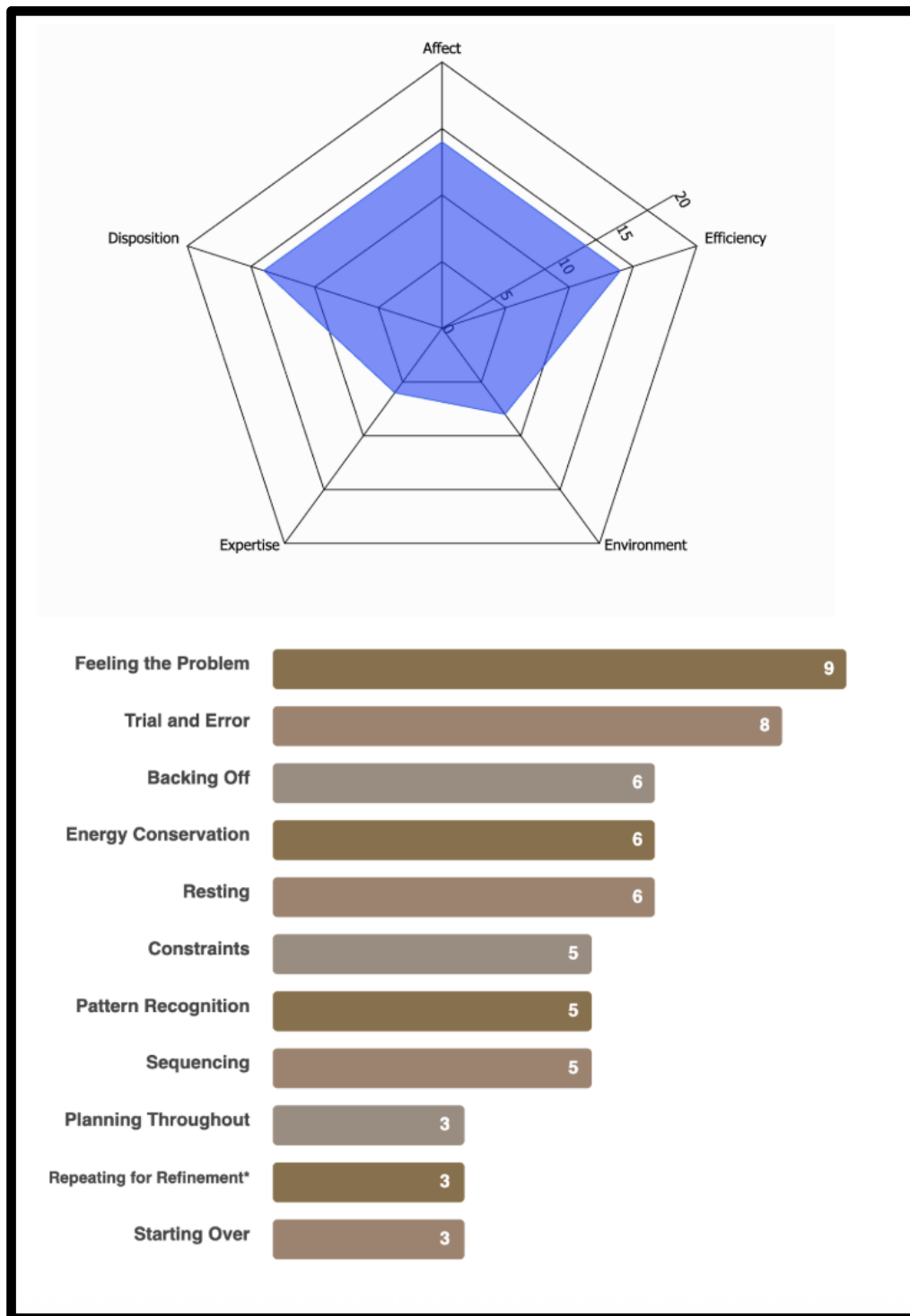


Figure 12: Chad Phase 3 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart

Note: This figure contains Chad's radial chart at the top depicting his positioning in math problem-solving. A horizontal bar chart at the bottom provides a breakdown of Kenneth's math specific problem-solving strategies.

Ashley Math Problem-Solving

Approach to SOMA Cube

At first, Ashley approached the Soma Cube optimistically, believing she could "just look at the pieces and know what to do." She immediately became fixated on the three-dimensional pieces and attempted to establish a foundational base with them, a strategy she tried across multiple attempts. However, she eventually found this approach ineffective as she discovered she needed to "leave spots open at the bottom" and then just "kind of just try every single piece" systematically. Unlike other participants who identified the corner pieces as complicated, Ashley gravitated toward them ("I love these pieces") because she found comfort in their predictability, stating "...at least I know what to do with them, they are definitely corner pieces!" She eventually developed an aversion to the green piece, which she described as "crooked...it messes up every single solution." Upon solving it and trying it again, Ashley explicitly stated she had no specific method, leading her to characterize the puzzle as:

I don't like this one because I just can't figure out the method to solve it.... Honestly, someone might find this one the easiest, but you have to be relaxed. You have to not really care about solving it, like maybe just mess around with it while you're watching TV or something. I think it's more mindless.... But maybe it just feels mindless because I don't know how to do it.

Ashley's frustration with the Soma Cube seemed to stem from its opacity. Indeed, the absence of a discernible systematic and replicable solution left her feeling unresolved.

Ashley compared the Soma Cube to:

...Trying a hard climb, like a climb that you're not enjoying. You're trying the same move over *and over and over again*, and it is not working...People are giving you beta, and it's just not helping, and I'm just like, 'What the heck, there is no way I can do this!'

Interestingly, Ashley emphasized the affective dimension in her comparison—The parallel wasn't about a type of rock or climbing style but rather a feeling—one of frustration and receiving unhelpful advice. Ashley noted that the Soma Cube provided no feedback, unlike climbing

where she experiences "tiny bit of progress" even when not actively moving higher on the wall. In climbing, she can feel when "that move feels right" or recognize incremental improvements like "my hand is getting a little further on the hold." The absence of this incremental feedback in the Soma Cube made it feel unsolvable and disconnected from the type of problem-solving that she prefers to and often engages in.

Approach to Towers of Hanoi

Ashley immediately recognized Towers of Hanoi, having encountered the algorithm in her coding classes. She immediately started the problem with no hesitation even though she could not explicitly recall the algorithm. As soon as she started, Ashley asserted "I definitely do think the first move matters." As she moved through the problem, she vocally mapped out a few moves ahead, sequencing some of her movements to come. When she made an error, she recognized it and restarted without hesitation. When presented with 5 discs instead of 4, Ashley immediately identified the key characteristic: "Oh, because n equals five is odd, so it's going to have a different starting point," and identified that odd versus even numbers would change the starting peg.

Ashley characterized Towers of Hanoi as "... that feeling of definitely doing a really easy climb, or not necessarily easy, but when I get more in a flow state." She emphasized: "it's not like having to try it multiple times. I know what to do." Again, her parallel highlights the affective experience of problem-solving. In this case, it is of competence, fluency, and smooth execution. She noted she could achieve this feeling on an easier outdoor climb with plentiful options or when doing "an easier one indoor, like more of a ladder." The flow state she described aligns with her experience of the recursive algorithm of Towers of Hanoi—once you understand the pattern, you can execute it reliably without cognitive strain. For Ashley, it seems that Towers of Hanoi represented automatic, proceduralized knowledge. Importantly, this experience is not

about the climb being "easy" in terms of grade but rather about having internalized the movement pattern such that execution feels fluid and repeatable.

Approach to Tangram

Rather than beginning with corner pieces like every other participant, Ashley immediately asked herself, "which ones are the diagonals?" and focused on the three pieces as her primary constraints. When asked about this focus, she explained her reasoning: "Maybe because there were less of them and the shapes are more unique." Her strategy centered on pieces having to be together rather than pieces in isolation or boundary pieces and she systematically tried combinations:

There were three diagonals, and two of them have to touch each other. Then I tried the remaining diagonal with another piece, but the piece that the two diagonals created was too long to fit in the remaining parts, so then I tried it with the next one. Then I just kept doing that.

In other words, Ashley created composite shapes from the diagonals and then assessed whether the residual spaces that remained could accommodate the remaining pieces. Her approach reflected her affinity for breaking down the problem into smaller parts by identifying the most restrictive constraints first, then working within those constraints.

Ashley described the Tangram as: "It's like trying a harder climb in the gym and I am getting to work a couple moves on it. The moves are doable and they are just out of my reach." She emphasized the incremental progress: "With each like progression, or each piece I put in, I'm like, 'Okay, I'm closer to getting this move.' And I want to work on it, I think cause I know it is possible and more fun." This parallel highlights the crucial role of feedback and the importance of tangible progress for her. Unlike the Soma Cube, where she received no feedback, the Tangram provided palpable signals of advancement. Each piece placed correctly brought her closer to the solution in a way that felt measurable. This incremental and felt sense of progress, even when not objectively advancing, keeps her engaged both off and on the wall.

Problem-Solving Attributes

Ashley is a highly systematic and methodical problem-solver in the mathematical problem-solving space, which is likely informed by being a computer science major. She seems to seek reproducible patterns and algorithmic approaches and describes her process as geared more toward procedural orientation, asserting "I like problem solving based on formulas." This disposition manifested most clearly in her approach to Towers of Hanoi, where she quickly recognized the algorithm and desired to explicate it in technical terms. Ashley's expertise in mathematics manifested most clearly during Towers of Hanoi when she was able to abstract the problem structurally and demonstrated a deeper conceptual understanding. Yet, Ashley's expertise showed up mostly with respect to algorithmic and computational contexts and she was more challenged when problems resisted systematic approaches, like the SOMA cube.

Ashley's affective relationship with mathematics is rather prominent. She explicitly states that staying calm is essential because she realizes "...when I get frustrated, I can't actually do anything. It's not productive at all." Her emotional regulation appears to be influenced by her observations of her parents' contrasting problem-solving styles—her father's calm approach versus her mother's tendency toward anxiety—leading her to recognize the benefits of both sides. Ashley also acknowledges that passion and engagement mattered, noting "you need to be passionate about what you're doing too though." Notably, when asked to relate the puzzles to climbing, Ashley consistently framed her responses through her affective experience rather than technical climbing characteristics. In fact, she explicitly stated "I don't relate these to different types of climbs at all." This is indicative that Ashley has a more affective lens for making sense of and articulating her experiences in problem-solving.

Ashley's efficiency in mathematical problem-solving manifests in heightened awareness of energy conservation that is likely shaped by her disability. Ashley's conception of efficiency is fundamentally tied to her stance regarding energy conservation "You prioritize what you need to worry about it, otherwise there's too many things to worry about. You can only spend so much

energy on certain things, because my energy is more precious." Ashley explains that disability has taught her to put things "...into perspective. Because I know I'm, okay, this is not the biggest deal in the world," allowing her to recognize when a problem (like the incomplete Soma Cube) isn't worth throwing energy into after a certain point.

The environmental factors shaping Ashley's mathematical problem-solving differed from those in climbing due to the absence of physical constraints. As she articulated, "...these puzzles don't have physical constraints. So I'm focused on that, but like in climbing, my disability is clearly holding me back." She argues that "...because of my disability, the makeup, my brain, and like my problem solving is most likely different...my toolset is different. It kinda feels like an advantage." This environmental alignment has likely positioned mathematics for Ashley as a domain where her disability-informed problem-solving approach has become an asset rather than a limitation. Indeed, in this space, she leverages strength in composure, strategic thinking, and energy management that had been cultivated and sharpened through navigating a world not built for her body.

Ashley's problem-solving framework during mathematical engagement is visually represented in Figure 13, which displays both her positioning across five key dimensions (top radial chart) and her strategic approach frequencies (bottom bar chart). The radial chart reveals her highest dimensions being both *Environment* (13) and *Disposition* (13), followed by *Affect* (11), *Expertise* (4), and notably low *Efficiency* (3). This figure reflects Ashley's systematic, computer science-informed approach where structured mathematical environments feel comfortable. Her elevated environment score (13) captures her attunement to environmental factors that help shape how she solves problems in the mathematical space. Her high disposition (13) illustrates her analytic nature that can flourish in mathematics without overt physical constraints. Her expertise (4) reflects domain-specific knowledge rooted in algorithmic thinking, evident in her immediate recognition of Towers of Hanoi's recursive structure, though her expertise remained limited when problems did not require systematic approaches like the SOMA

cube. Her low efficiency score (3) coexists with her profound awareness that "my energy is more precious," suggesting her efficiency manifested more as an implicit and unobservable discernment about when to disengage rather than explicit statements surrounding efficiency. The strategy bar chart reveals *Constraints* (8) and *Sequencing* (7) as her dominant approaches, demonstrating her focus on identifying limiting factors and mapping sequential moves ahead.

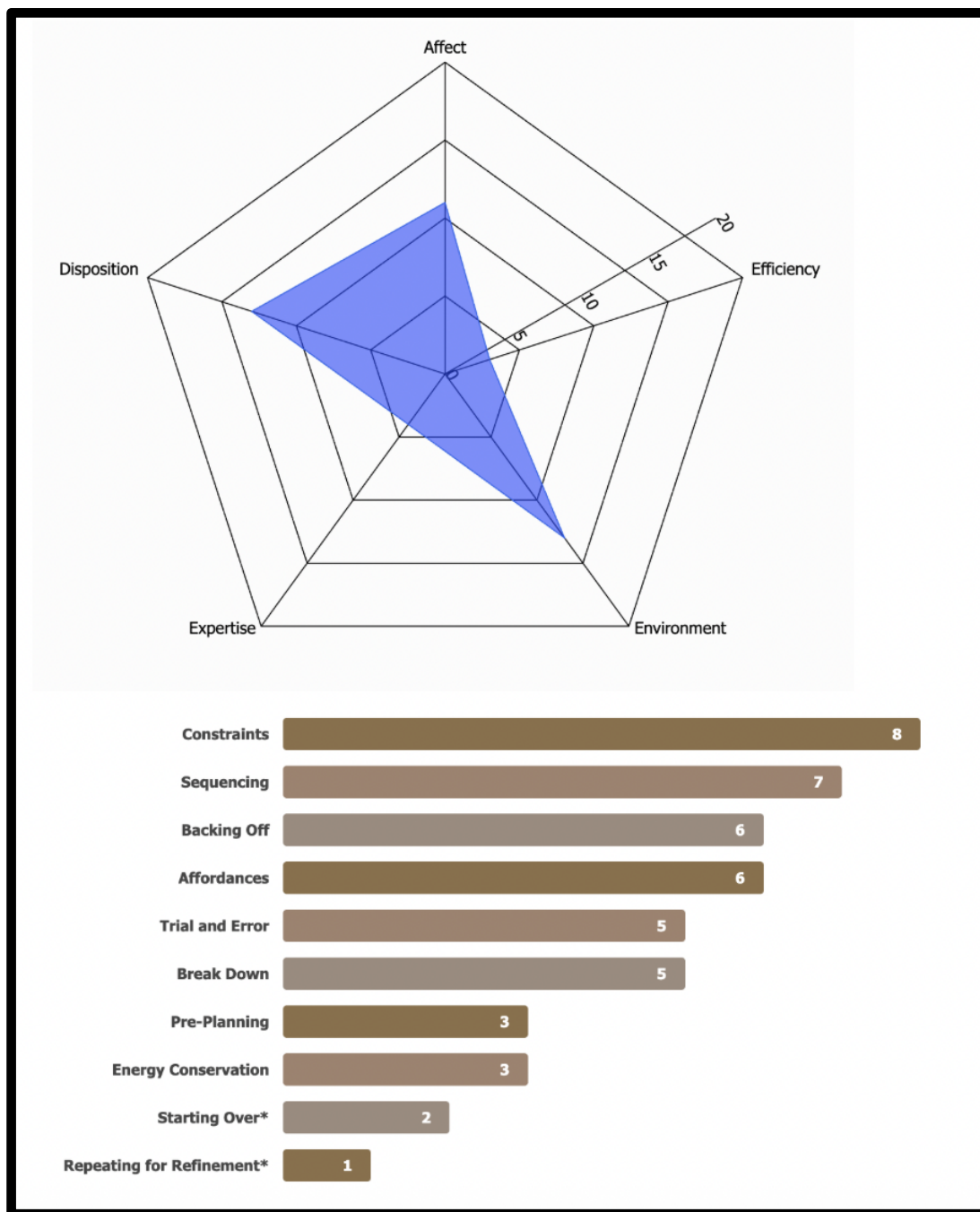


Figure 13: Ashley Phase 2 Problem-Solving Profile: Positioning Radial Chart and Strategy Bar Chart.

Note: This figure contains Ashley's radial chart at the top depicting his positioning in math problem-solving. A horizontal bar chart at the bottom provides a breakdown of Ashley's math specific problem-solving strategies.

Phase 2 and 3 Comparison: RQ3

This section addresses RQ3 by exploring the analogous orientations for each participant between problem-solving across the domains of rock climbing and mathematics. I first examine what remains consistent across contexts—the attributes and strategies that transfer between climbing and mathematics for each participant— then I next discuss what diverges in terms of orientations and strategies. Accompanied with each participants' analysis, I provide visual representations for each participants' attribute profiles (overlapping radar charts) and strategy frequencies (proportional diverging bar charts) for rock climbing and mathematics. The radial charts visualize the frequency of the five framework dimensions for each domain—Disposition, Affect, Efficiency, Environment, and Expertise—overlapping one another. The proportional diverging strategy bar chart quantifies different problem-solving strategies, with rock climbing frequencies on one side and mathematical problem-solving on the other, allowing direct comparison of approaches.

Taylor Similarities Across Domains

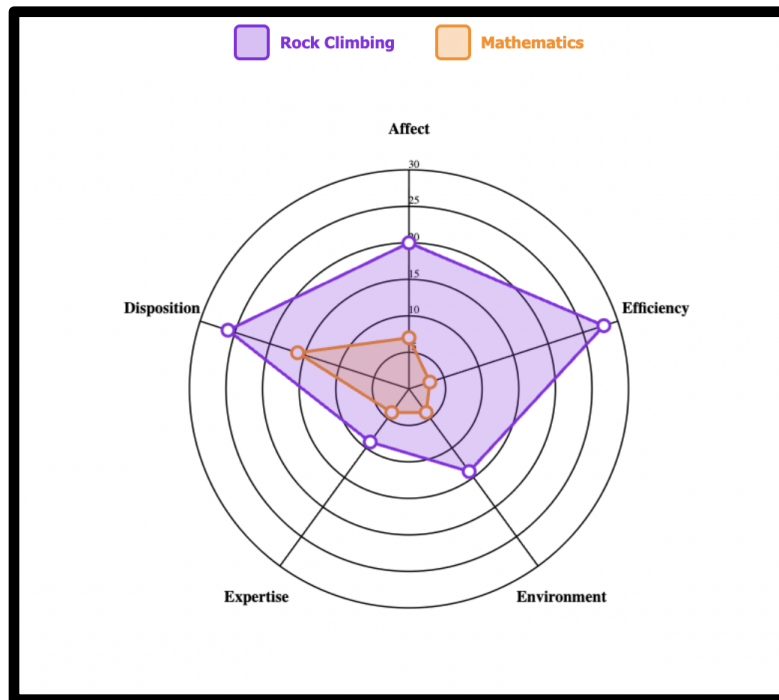


Figure 14: Taylor Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange).

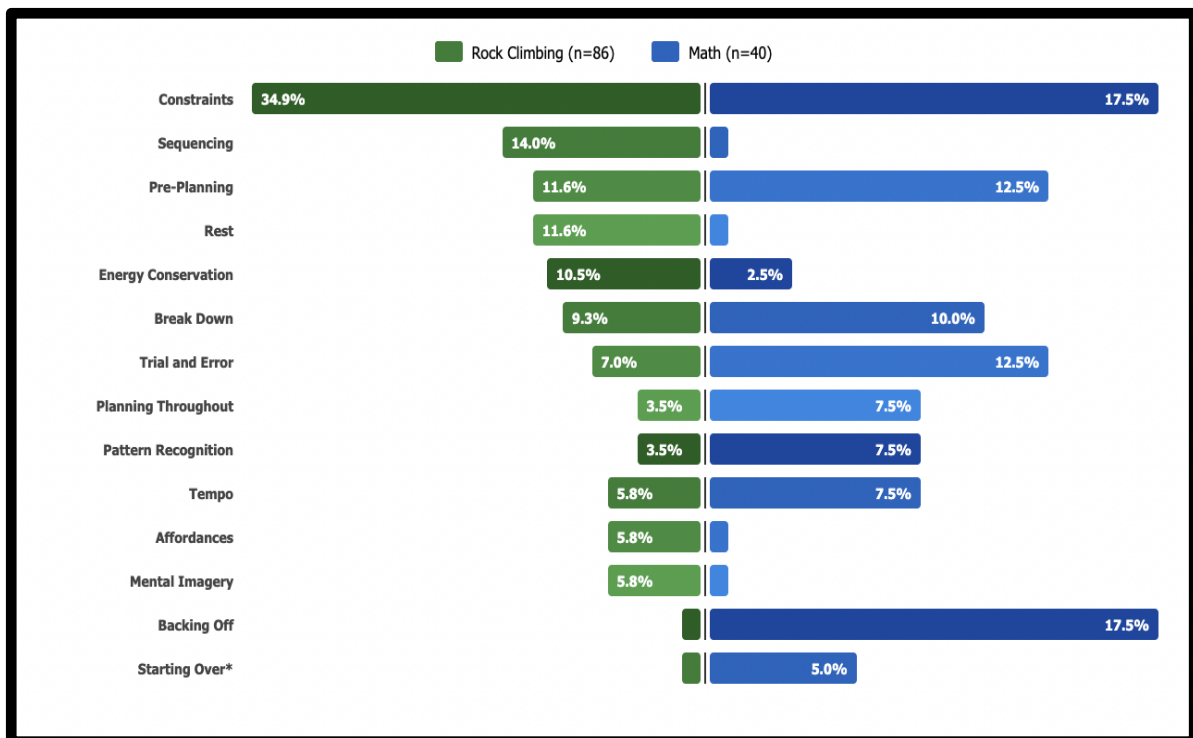


Figure 15: Taylor Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right).

Figure 14 presents Taylor's problem-solving framework across rock climbing and mathematical contexts through overlapping radial charts depicting his positioning across five key dimensions. Across domains, though considerably contracted, the most prominent attribute is Taylor's dimension of *Disposition*. In both contexts, his core philosophical orientations—fluid perspective, process over outcome, balance between structure and intuition—appears to be consistent. Further, his ability to connect both rock climbing and mathematical problem-solving to other domains, such as design, is consistent. This suggests that some of his core beliefs about problem-solving are transferable across domains. Similarly, his *Expertise* dimensions remain relatively comparable— while he's an expert in climbing, he also was able to leverage problem-solving competence to the mathematical tasks, namely in the form of intuition and flow state.

Figure 15 presents a diverging bar chart illustrating Taylor's strategy frequencies in each domain. The dominant strategy across both domains is *Constraints* (34.9% in climbing; 17.5% in math), which shows up due to Taylor's self-awareness about his body's limits in rock climbing and his need for "bounds as reference points" in the mathematics context). Indeed, both domains show his reliance on understanding the boundaries of the spaces where he is problem-solving. *Pre-Planning* is nearly proportionally equivalent (11.6% in climbing; 12.5% in math) reflecting consistently in his assessment of situations beforehand, whether it's planning a climbing route from the ground or planning before making any moves in Towers of Hanoi. *Break down* is also nearly proportionally equivalent (9.3% in climbing; 10% in math) *Trial and Error* is present in both (7% in climbing; 12.5% in math) particularly evident in his "playing around" approach to the SOMA cube and his willingness to explore moves on the wall.

Taylor Differences Across Domains

Taylor's mathematical profile shows a considerable contraction from rock climbing attributes across three critical dimensions: *Efficiency*, *Affect*, and *Environment* (see Figure 14). Efficiency in the mathematical context may collapse because the math puzzles don't

necessitate the same physical demands that climbing requires. Affect also is not as prominent in the mathematical context, indicative that for Taylor, problem-solving in the climbing context is a much more emotional experience. Finally, the disparity of the environmental dimension across both domains is prominent. In climbing, Taylor frequently discusses environmental barriers — he is constantly navigating spaces not designed for disabled bodies. The climbing environment is heavily constrained in ways that induce constant problem-solving. In the mathematical context, for Taylor, these environmental constraints are not as prominent, if at all. Tanner's expertise seems to have developed specifically through navigating constrained, often hostile environments and when those environmental pressures are not present, he loses the scaffolding that structures his approach. This suggests his problem-solving capacity is not just about having constraints within the task itself (like the Towers' rules) but about operating within environments that actively resist accommodation. Perhaps it is that the friction itself has actually facilitated his strategic thinking.

The math context shows *Backing Off* (17.5%) as a major strategy (see Figure 15) - this is seemingly unique to math and directly relates to Taylor's approach in the five-disc Towers of Hanoi where he stumbled more and backed off repeatedly to reassess, but notably refused to start over completely. *Starting Over* (5%) only appears in math, seen only when he had to scratch all of this progress in SOMA and start completely over. The most explicit differences are grounded in strategies tied to the overtly physically demanding practice of rock climbing. *Sequencing* (14%), *Rest* (11.6%), *Energy conservation* (10.5%) appear frequently in climbing but are minimal or absent in mathematics. This is representative of climbing's inherent physical demands. In climbing, Taylor, particularly due to his disability, must manage fatigue, meticulously pace his movements, and link sequences of movement. These strategies do not seem to be transferable in the mathematical context.

Notably, a key difference is the presence of *Affordances* (6.8%) in the rock climbing context, while being absent in the mathematical context. This is indicative of how each domain presents possibilities for action. In climbing, affordances are plentiful, particularly to the trained

eye—every hold offers different gripping options, body positioning creates varied possibilities, and routes are designed for tweaking beta across different climbers. The mathematical puzzles, by contrast, had more constrained solution spaces. Additionally, Taylor is an expert in the domain of climbing but is admittedly novel to the domain of mathematics; therefore, he has years of climbing expertise where affordance recognition is trained and explicit, while the new mathematical tasks veiled affordances to Taylor.

Christopher Similarities Across Domains

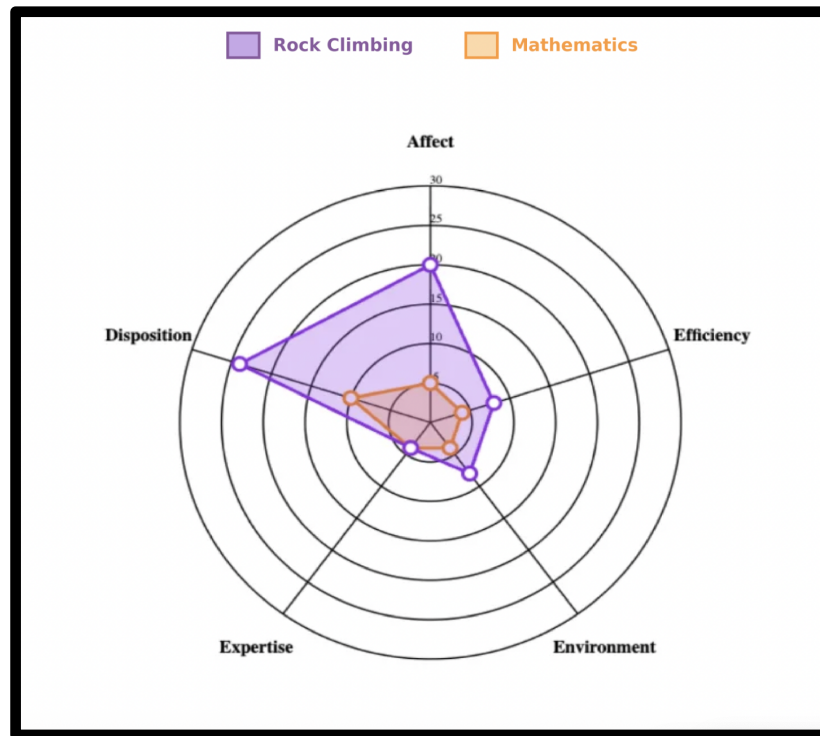


Figure 16: Christopher Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange)

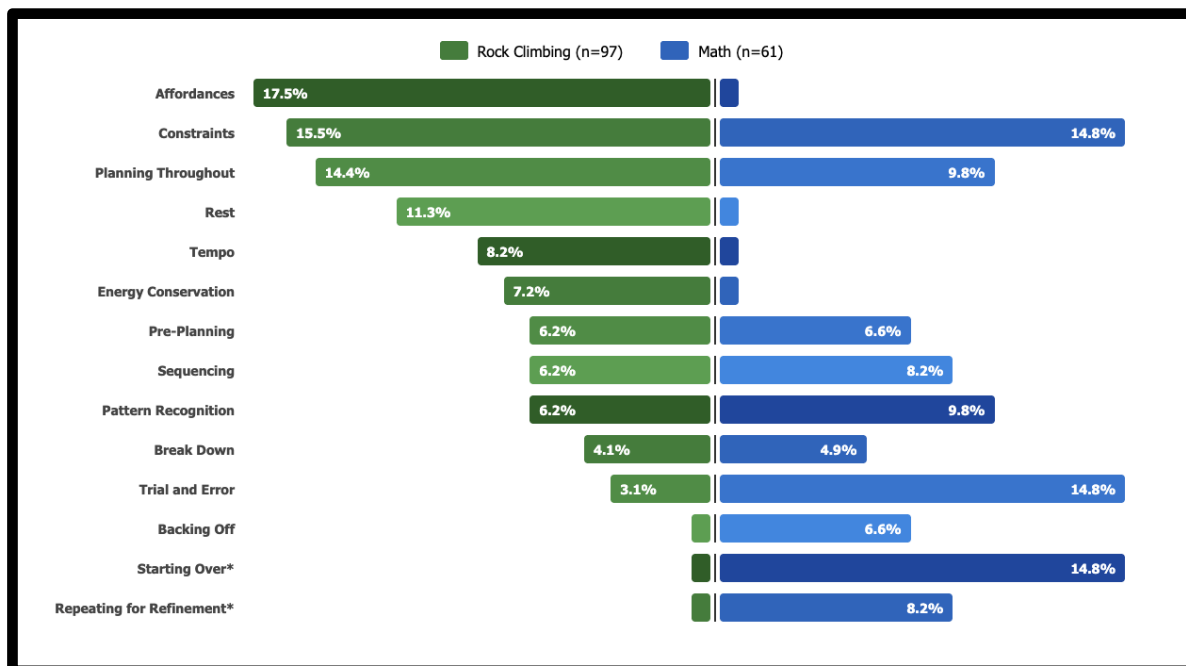


Figure 17: Christopher's Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)

Figure 16 presents Christopher's overlapping problem-solving profiles through radial charts depicting his positioning across five key dimensions in both rock climbing (purple polygon in Figure 16) and mathematics (orange polygon in Figure 16). Christopher's *Expertise* orientation is consistent across contexts and is not prioritized in the interviews—however, the depth of expertise differs: in climbing, his expertise draws on years of devotion and commitment, while mathematical expertise remained nascent with only isolated breakthrough moments in algorithmic development during Towers of Hanoi. Christopher's orientation toward *Efficiency* remains relatively stable across both climbing and mathematical contexts; efficiency is similarly not a priority for Christopher in either domain. Finally, Christopher's *Environmental* orientation is also relatively stable and is also not prioritized across both domains.

As illustrated in the diverging bar chart (Figure 17), the most prominent consistency in strategies across domains appears in Christopher's attention to *Constraints*, which is proportionally high in both domains (15.5% in climbing, 14.8% in math), though this emphasis on constraint identification shows up differently in each context: in climbing, constraints emerge from bodily limitations and cruxes on the wall; in mathematics, they appear as constraints around complicated pieces. Similarly, his use of *Planning Throughout* shows comparable frequency (14.4% in climbing, 9.8% in math), illuminating his sequential thinking across domains. *Pattern Recognition* also remains relatively consistent (6.2% in climbing, 9.8% in math). Finally, *Pre-Planning* strategies are similarly analogous across domains (6.2% in climbing, 6.6% in math), reflecting a consistent approach of mentally rehearsing sequences before execution—whether running through climbing beta in his head or mapping out the disc movements in Towers of Hanoi.

Christopher Differences Across Domains

As seen in Figure 16, Christopher's *Disposition* is considerably contracted from climbing to math. In both domains, his disposition is characterized by his persistence, a strong conviction,

and a solution-oriented mindset. However, his years of climbing may have deepened his dispositional engagement, while the mathematical context represents newer terrain. Christopher's *Affective* engagement is not as prominent in the mathematical context, indicating that for Christopher, problem-solving in the climbing context may be a more emotional experience.

Christopher's strategies across domains differ in several dimensions— most notable is Christopher's heavy reliance on *Affordance* recognition in climbing (17.5%) compared to mathematics (0%). This disparity is indicative of Christopher's ability to perceive continuous affordances on the wall as well as through body positioning, while the mathematical puzzles offered far fewer obvious affordances to guide problem-solving. In the climbing context, Christopher uses *Trial-and-Error* minimally (3.1%), however in mathematics, trial-and-error increases to 14.8%. Indeed, in mathematics, his relative inexperience forced heavy trial-and-error exploration, particularly visible in his SOMA cube struggles. Finally, the mathematics context shows extremely elevated use of *Starting Over* (14.8% versus 0%) and *Backing Off* strategies (6.6% versus 0%). Christopher used these strategies with the SOMA cube and Towers of Hanoi, where he "started over multiple times" because he "couldn't figure out why it wasn't working." In climbing, his expertise and the affordance recognition minimize the need for complete resets or even backing off; instead, he can usually adjust mid-sequence.

Broadly, these strategic differences underscore how Christopher's domain expertise shapes not just the frequency of strategy use but also a difference between experts and novices: experts navigate with confidence leveraging affordances while novices cycle through trial-and-error and frequent restarts in order to find viable approaches.

Kenneth Similarities Across Domains

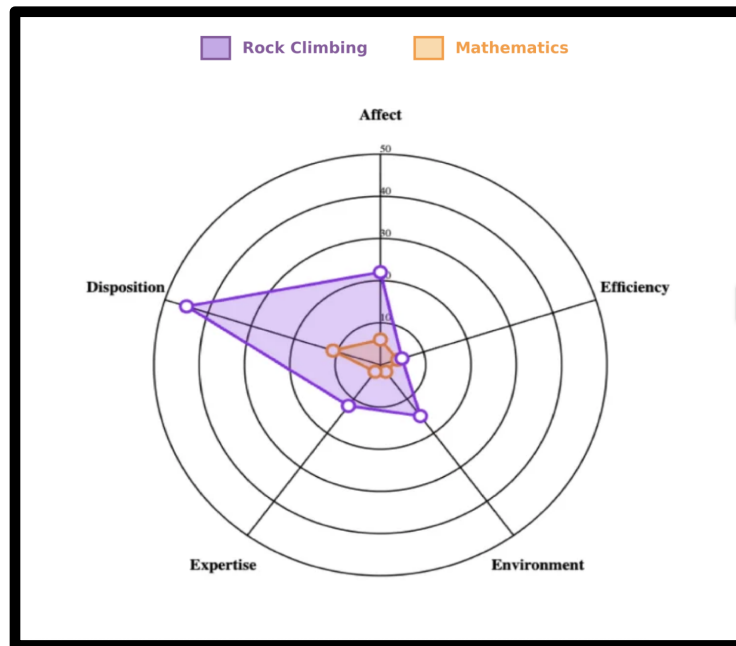


Figure 18: Kenneth Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange)

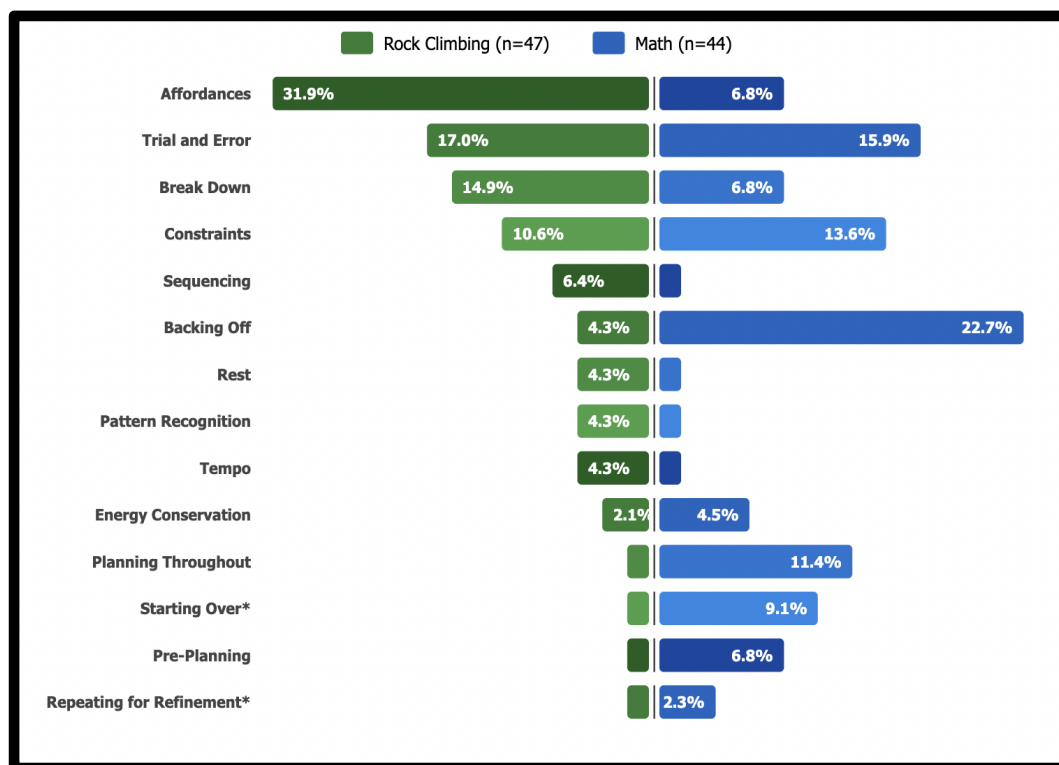


Figure 19: Kenneth's Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)

Figure 18 provides a visualization of Kenneth's overlapping problem-solving profiles through radial charts depicting his positioning across five key dimensions in both rock climbing (purple polygon) and mathematics (orange polygon). Though Kenneth's problem-solving orientations reveal prominent contraction across nearly all five dimensions from rock climbing to mathematical contexts, *Disposition* remains Kenneth's strongest orientation in both climbing and mathematics, though it contracts substantially. Across domains, he is pragmatic and takes an exploratory approach to problem-solving. Further, in both contexts, he draws parallels across several domains, ranging from gardening to coding. *Efficiency*, though consistent across both domains, shows relatively low priority in both contexts. Though he explicitly discusses efficiency's importance throughout both interviews, his prioritization of efficiency seems to be highly context-dependent, emerging primarily in high-stakes environments like alpine climbing rather than in the climbing gym or with puzzles where playful exploration takes precedence.

As illustrated in the diverging bar chart (Figure 19), Kenneth employed nearly identical numbers of total strategies across both domains (47 in climbing; 44 in math)—a unique pattern among all participants. This consistency may reflect his pragmatic, outcome-oriented approach that prioritizes "getting to the result" regardless of the domain. Kenneth explicitly stated he doesn't invest in "understanding how I'm getting to the answer." Instead, he would rather focus on trial-and-error or just "looking something up" to achieve the desired solution. This pragmatic disposition may have produced a relatively domain-general (at least, between climbing and math) strategic repertoire that transferred in some capacity.

The most notable consistency within his strategies appears in *Trial and Error*, which remains high in both domains (17.0% in climbing, 15.9% in math), reflecting his self-described "fuck around and find out" exploratory approach that characterizes his problem-solving philosophy. *Constraints* are also relatively consistent across domains (10.6% in climbing, 13.6% in math).

Kenneth Differences Across Domains

Kenneth's *Environment* orientation collapses prominently from climbing to mathematics (Figure 18); in climbing, the rock provides continuous cues for problem-solving. However, in the mathematical context, Kenneth was not as highly sensitive to his environment. *Expertise* similarly contracts from climbing to mathematics, reflecting years of experience in climbing and coaching in climbing, versus his explicit stance in mathematics where he does not "try to learn new math" or understand underlying principles; instead, he desires to achieve results through looking things up rather than developing deep knowledge. *Affect* also contracts across domains, suggesting that climbing engages him emotionally in ways that mathematical problem-solving does not.

In Figure 19, Kenneth's heavy reliance on *Affordance* recognition in climbing (31.9%) compared to mathematics (6.8%) is reflective of the climbing environment readily providing more options than in the mathematical context. This may be reflective of his expert to novice orientation from climbing to math. *Backing Off* (22.7% in math versus 4.3% in climbing), *Planning Throughout* (11.4% in math versus 0% in climbing), *Starting Over* (9.1% in math versus 0% in climbing), and *Pre-Planning* (6.8% in math versus 0% in climbing) were all strategies that were far more present in math than in climbing. In mathematics, it seems that he shifted to more effortful and deliberate strategies. Notably, strategies like *Sequencing* (6.4% climbing; 0% math) and *Breaking Down* (14.9% in climbing; 6.8% in math) were more pronounced in climbing, which may be indicative of the "progressive" problem-solving he associates with climbing, where clear starting points, endpoints, and intermediate "checkpoints" naturally unveil how to break problems into parts and how to move through them.

Chad Similarities Across Domains

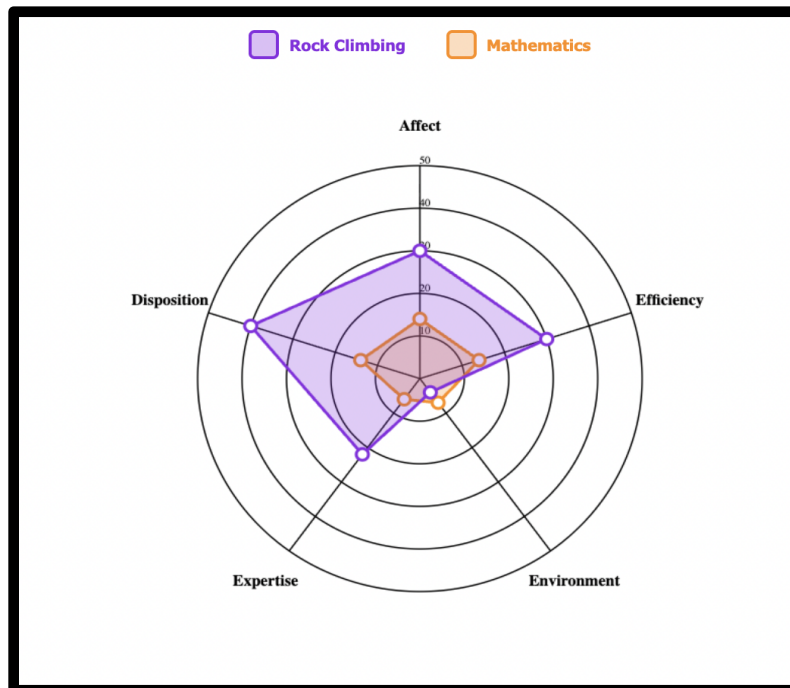


Figure 20: Kenneth's Phase 1 & 2 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)

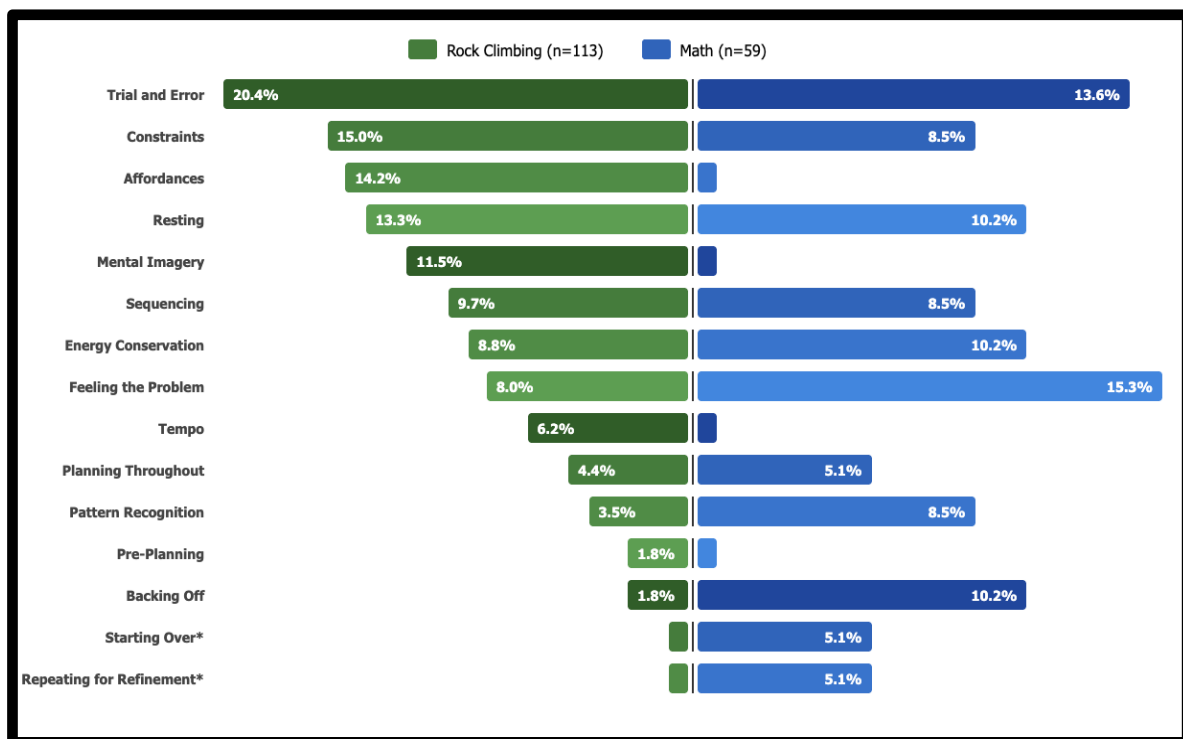


Figure 21: Chad's Phase 2 & 3 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)

Figure 20 provides a visualization of Chad's overlapping problem-solving profiles through radial charts depicting his positioning across five key dimensions in both rock climbing (purple polygon) and mathematics (orange polygon). Across domains, *Disposition*, though relatively contracted, remains the most prominent attribute. His disposition is generally characterized by an unwavering belief that every problem and his personal mantra: "Don't say I can't, say how can I?" When Chad lost his sight, he made an explicit choice to reframe frustration as excitement for new challenges, and this reframing is evident in how he approaches obstacles in both contexts. In climbing, his disposition manifests as a commitment to what he deems "figuring it out on the fly" because he cannot see rests or cruxes from the ground; indeed, he cannot plan beforehand. Similarly, in mathematical problem-solving, he begins immediately engaging with the task—starting the SOMA cube with trial-and-error, jumping directly into the Towers of Hanoi, approaching the Tangram by moving around the pieces immediately. Across these contexts, his disposition is grounded in action, driven by his awareness that "the clock is always ticking."

Across both domains, *Affect*, *Efficiency*, and *Expertise* are contracted but maintain a similar structure (Figure 20). In both rock climbing and mathematics, Chad exemplifies an affective orientation, seeing both as spaces for emotions, such as frustration and excitement. His prioritization of efficiency and expertise development seems to also be analogous across domains—the goal of efficiency drives pattern-seeking; this builds expertise through transforming conscious effort into automatic and intuitive execution.

As illustrated in the diverging bar chart (Figure 21), across domains, *Trial and Error* (20.4% in climbing; 13.6% in math) is the dominant strategy. This consistency is reflective of his general approach in problem-solving: trial and error is a necessary pathway toward repeatable and generalizable algorithms. Understood another way, trial and error acts as his discovery mechanism, a way in which he identifies patterns that can then be systematized. *Constraints* (15.0% in climbing; 8.5% in math) is also prominent across domains, demonstrating his focus on limitations and boundaries that shape the problem space, or narrowing the space for

exploration. Resting (13.3% in climbing; 10.2% in math) and energy conservation (8.8% in climbing; 10.2% in math) show up similarly, reflecting his efficiency-driven approach. These four strategies—trial and error, constraints, resting, and energy conservation—form the core of Chad's problem-solving identity across domains, accounting for 57% of his climbing strategies and 42% of his mathematical strategies. This reveals a consistent approach centered on experimentation, awareness of limitations, and resource management to ultimately contribute to efficiency and generalizable solutions.

Chad Differences Across Domains

Chad's profile shows an increase in rock climbing relative to math attributes across only one dimension: *Environment* (see Figure 20). In rock climbing, Chad encounters what he describes as a stable environment—"The rock climb isn't moving...I mold around the rock when I climb"—where the route remains fixed and allows for pattern memorization and reproduction. Indeed, the rock's permanence seems to align with his preference for environments that support pattern recognition and replicable solutions. On the other hand, the three mathematical puzzles presented varying degrees of environmental stability. Thus, Chad's environmental considerations are more pronounced in the mathematical space than in the climbing space and it may reflect the mathematical context forcing explicit environmental awareness that remained in the background in gym climbing's consistently stable terrain.

Chad also contracted in *Efficiency*, *Expertise*, and *Affect* in mathematics from climbing. Due to being a novice in the mathematical domain (whereas he is an expert in the climbing domain), efficiency was far less a priority. Without discovering repeatable algorithms (particularly in SOMA cube), he could not achieve the intuitive flow ("on the fly approach") that generates positive affect. His problem-solving stalled in the exploratory phase rather than reaching the algorithmic execution that, in climbing, produces his "mercury thermometer" experience of rising momentum and satisfaction.

As seen in Figure 21, Affordances and Mental Imagery appear almost exclusively in climbing (14.2% and 11.5% respectively). This may be reflective of climbing's explicitly physical demands where Chad must tactilely identify what holds afford and mentally simulate full-body movements through space. These qualities are not readily evident in the mathematical space. Backing off is proportionally much higher in mathematics (10.2% vs 1.8% in climbing), and *Pattern Recognition*, though used in both contexts, is proportionally leveraged more in mathematics (8.5% vs 3.5%). *Starting Over* and *Repeating for Refinement* appear exclusively in mathematics (5.1% each), indicating that the mathematical context provided more opportunities for deliberate iteration and refinement compared to the climbing problem space.

Ashley Similarities Across Domains

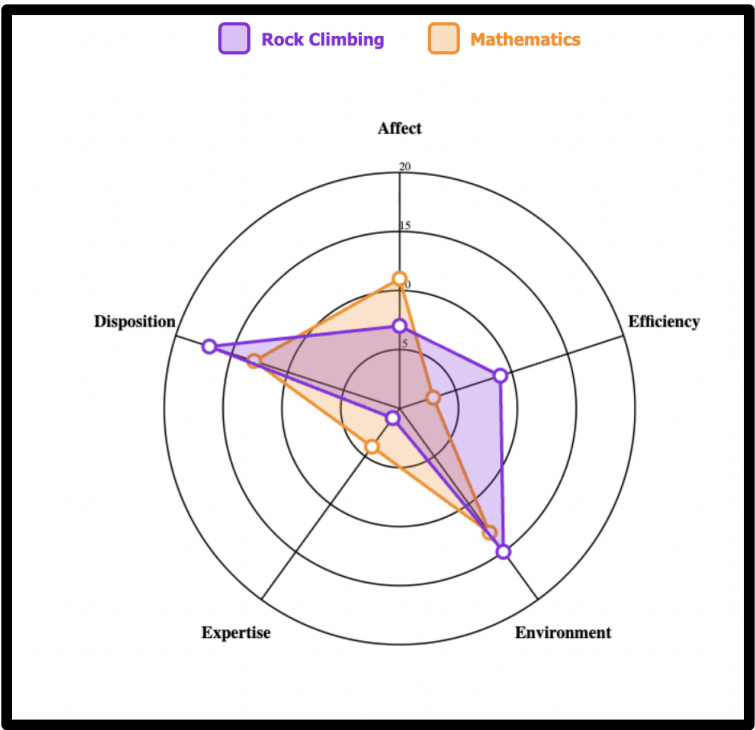


Figure 22: Ashley Phase 2 & 3 Radial Comparison: Rock Climbing (purple) and Math (orange)

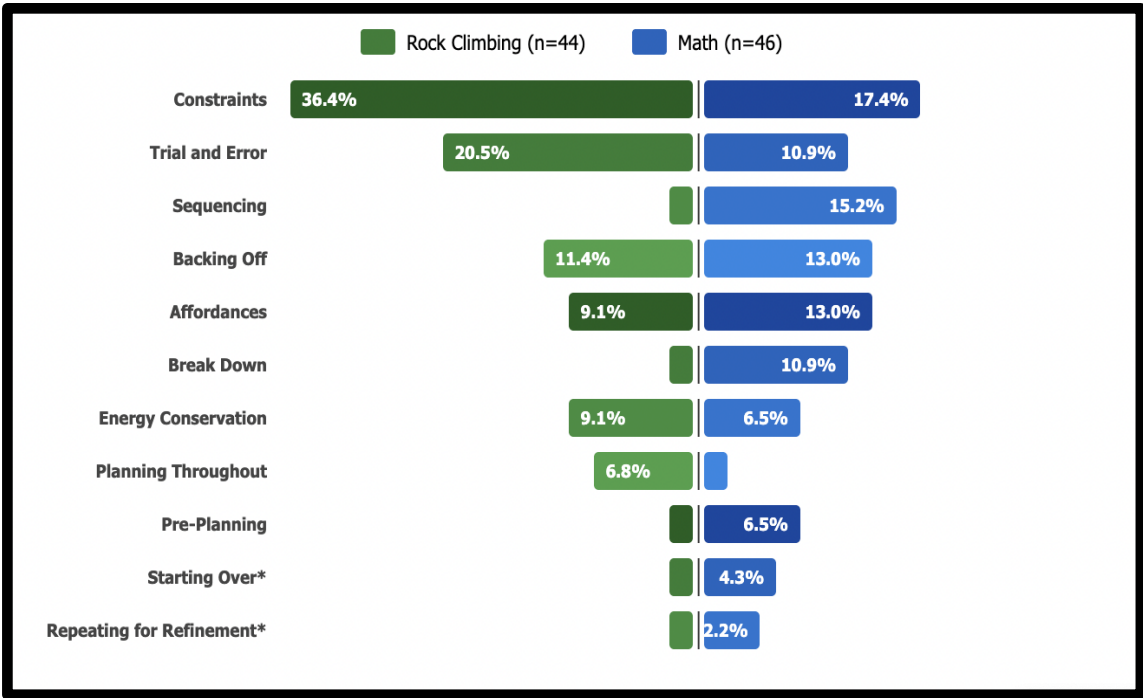


Figure 23: Ashley's Phase 1 & 2 Proportional Diverging Bar Chart: Rock Climbing (left) and Math (right)

Figure 22 provides a visualization of Ashley's overlapping problem-solving profiles through radial charts depicting her positioning across five key dimensions in both rock climbing (purple polygon) and mathematics (orange polygon). Ashley's *disposition* remained fairly consistent across contexts—her analytical and persistent disposition is apparent in both contexts and she is able to engage with problems productively while also recognizing when to step back. Her focus on *Environmental* considerations are comparable across contexts as well— Ashley is acutely attuned to how her environment influences her ability to engage with problems. Whether navigating the embodied constraints on the climbing wall or the instructional structures of the mathematics classroom, she demonstrates heightened environmental awareness. Though Environment remained high in both domains, the nature of environmental factors diverge: climbing's slightly higher frequency may reflect a constant negotiation with physical barriers where Ashley asserts "...my disability is clearly holding me back and beta is not gonna help," while for Ashley, math's environmental dimension is a more accessible and collaborative space where "these puzzles don't have physical constraints" and instructional guidance is actually usable because she argues "my brain can do it."

As illustrated in the diverging bar chart (Figure 23), several strategies appeared consistently for Ashley across both climbing and mathematics. *Backing Off* was nearly equivalent across contexts (11.4% in climbing; 13.0% in math), reflecting her proclivity for disengagement from unproductive problems, whether physically lowering from a particular sequence on the wall or taking a step back from a puzzle. *Energy Conservation* appeared in both domains (9.1% climbing; 6.5% math), demonstrating her concern with resource management, whether conserving physical stamina for a particular number of climbing routes or allocating more cerebral energy efficiently. *Affordances* were relatively balanced (9.1% climbing; 13.0% math), indicating that across domains Ashley consistently sees opportunities in the problem-solving space. *Trial and Error* appeared substantially in both contexts (20.5%

climbing, 10.9% math), revealing her willingness for exploration when clear systematic methods weren't immediately apparent.

Ashley Differences Across Domains

As seen in Figure 22, The most dramatic divergence occurs in *Expertise*, expanding from climbing to mathematics. This is reflective of Ashley being a novice climber while demonstrating moderate competence in math through her computer science background. On the other hand, *Affect* contracted from rock climbing to mathematics, making her the only participant with higher affective engagement in mathematics than climbing. Finally, *Efficiency* contracted sharply from climbing to math. This disparity is reflective of the overtly physical demands of climbing that make efficiency considerations particularly salient for Ashley.

As Figure 23 shows, *Constraints* were Ashley's prominent climbing strategy (36.4%) far more than mathematics (17.4%), reflecting how she constantly navigates physical limitations in climbing. Three strategies appeared exclusively in mathematics: *Sequencing* (15.2%), *Break Down* (10.9%), *Starting Over* (4.3%) and *Repeating for Refinement* (2.2%). The emergence of these strategies demonstrates a more comprehensive repertoire Ashley draws from in a domain she may feel more expertise. *Planning Throughout* appeared only in climbing (6.8%), suggesting that the overtly physical nature of climbing may require ongoing assessment and adjustment in ways that mathematics—where she could pre-plan algorithmic approaches—did not. *Pre-Planning* showed the inverse pattern, appearing only in math (6.5%), reflecting her ability to map out solution strategies before execution.

RQ3 Themes Across Participants

Next, I zoom out from individual findings from participants and consider the broader findings from the six categories. Through grounded theory analysis of climbing sessions, stimulated recall interviews, and mathematical task-based clinical interviews, I developed a

holistic problem-solving framework consisting of six interconnected dimensions: *Disposition*, *Affect*, *Efficiency*, *Environment*, *Expertise*, and *Problem-Solving Strategies*. Each participant's problem-solving profile was then mapped across both rock climbing and mathematical domains, revealing their relationship across domains and how they might transfer. In this section, I examine the connections that emerged across participants, partitioned by the six aforementioned categories. I am then able to identify generalizable principles that can constitute a holistic framework while also accounting for the nuance and flexibility in accommodating for the richness of individual problem-solving experiences.

Disposition is Foundational

Across all six participants, and across both domains, *Disposition* emerged as a prominent dimension of problem-solving. Certainly, while disposition was present throughout participants and contexts, how it emerged varied significantly. Further, every participant demonstrated some degree of a shift in disposition when moving from climbing to mathematics. The prominence of disposition suggests that it forms a crucial lens through which individuals engage with problems and the fact that all participants leveraged this lens differently across domains points to disposition as both a stable attribute and contextually-dependent. This dualistic nature suggests that individuals carry and leverage certain orientations with them while also adapting these orientations based on environmental demands, and personal histories within specific domains.

Affect is Constituent to Problem-Solving

Affect emerged as constitutive of problem-solving across all participants. In other words, it was fundamental to engagement, persistence, and evaluation of problem-solving experiences. Participants' affective states shaped which strategies felt viable, whether they should persist with the problem, and whether solutions felt satisfying. The relationship operated bidirectionally: emotional states influenced which strategy to leverage while problem-solving

experiences generated emotional responses that participants were able to speak to upon reflection. Affect thus functions as a condition for problem solving, where participants need appropriate and productive affective states to access their strategic repertoires to sustain engagement over time.

Efficiency Is Almost Always A Consideration

Efficiency emerged prominently across all participants, particularly in climbing contexts where every participant demonstrated heightened awareness of energy conservation, fatigue management, and effort optimization. Participants (especially seated and blind climbers) articulated efficiency not as preference for elegance but as necessity born from constrained energy reserves—what disability studies scholars term "spoon theory" (Miserandino, 2003). However, efficiency proved contingent on more than disability. It is also related to domain-specific demands and developed expertise within the domain. Participants whose disabilities created primarily physical constraints (seated and amputee climbers) showed dramatic efficiency contractions from climbing to mathematics as the physical demands from climbing dissipated. Critically, even perceptual disability that transcends domains (blindness) did not demonstrate consistent efficiency concerns across domains—Chad's efficiency also contracted substantially because, while his blindness persisted across domains, he lacked the mathematical expertise to recognize and pursue efficient approaches. Efficiency thus may become a consistent feature across domains when individuals possess sufficient domain knowledge to distinguish efficient from inefficient paths and conversely may diminish when lack of expertise leaves individuals unable to make such distinctions, regardless of whether disability persists across domains.

Environmental Attunement is Key

Environmental attunement emerged as a critical dimension across all participants,

manifesting as heightened sensitivity to contextual features that shaped problem-solving possibilities. Participants demonstrated acute awareness of how their specific bodies related to surrounding environments: some attended primarily to accessibility and energy costs for navigation, framing problem-solving through resource scarcity; others emphasized pressure related to time and information access, positioning environmental engagement as time-constrained and necessitating strategic gathering of information in order to problem-solve. Therefore, in any situation where someone feels a mismatch with their environment and their capabilities, one must develop the ability to identify affordances, constraints, risks, demands, and so on.

Expertise as Flow

Expertise development emerged uniquely across participants (contingent upon disability experience, domain familiarity, and beliefs about competency development), but one thing remained constant: in rock climbing and mathematical domains, expertise manifested as an embodied knowledge where there is a transition from conscious, deliberate problem-solving to intuitive, flow-state movement. Interestingly, participants frequently described "problem-solving as a muscle" that strengthens through use, indicating that consistent engagement with challenges induced by disability cultivated more robust approaches to solving problems in their everyday life. However, this embodied expertise from navigating constrained environments did not necessarily transfer seamlessly to mathematics, particularly because this domain lacked affordances for that type of knowing to emerge.

Constraints and Affordances Across the Board

Each and every participant employed constraint identification and navigation as a prominent (and usually dominant) problem-solving strategy across both domains. In climbing, constraints were shaped by tangible, embodied forms that participants actively identified and

had to work with. In the mathematical puzzles, constraint identification remained prominent but shifted away from embodied constraints to more structural and spatial constraints. Some participants explicitly articulated the desire for boundaries and reference points in shaping their problem-solving. Ultimately, these findings show that constraints are not just obstacles but can be reframed as structural features that shape and guide problem-solving. The constant experience of navigating constraints in an unaccommodating world may actually shape the participants' cognitive infrastructure to seek out and utilize boundaries as scaffolding for problem-solving.

Affordance-seeking emerged as a prominent strategy for most participants, particularly in climbing contexts. What is especially salient is that the perception of affordances seems to be developed through expertise development. These findings suggest that years of navigating different types of holds, body positions, and exposure to various movement sequences trained participants to perceive affordances that novices cannot see. However, this capacity does not automatically transfer to new domains; it must be cultivated through experience and expertise development in each problem-solving context.

Chapter 5: Discussion

Connecting Back to Research Questions

In this section, I revisit and discuss the three research questions that structured this study, connecting the findings to the aforementioned literature. RQ1 examined the environmental affordances and constraints that shape the climbing context; RQ2 captured participants' pre-intervention perceptions of problem-solving across domains; and RQ3 explored enacted problem-solving strategies through an interventional design that positioned climbing expertise as relevant to mathematical engagement. Together, these findings validate the central theoretical commitment—that problem-solving is fundamentally embodied and relational.

Research Question 1: Constraints and Affordances

The route-setters' focus group findings validate and provide a different way to think about Gibson's (1977) theory of affordances while simultaneously revealing critical tensions with the ecological-enactive framework this dissertation is built upon. The route-setters are able to articulate an implicit understanding of affordances as relational possibilities. They recognize that different climbers perceive and utilize different action opportunities based on their embodiment. Their intentional manipulation of hold spacing to create "fairness" for different bodies through varied solution paths directly operationalizes Gibson's insight into the subjectivity of affordances.

The route-setters deliberate "killing" of stable affordances (i.e., eliminating triangle-forming shapes between holds to induce discomfort and make the climbing more challenging) aligns with Abrahamson and Sánchez-García's (2016) discussion of constraints in mathematics education from an ecological dynamics perspective, where task constraints can foster new movement possibilities and therefore conceptual development. The setters' philosophy that "we can't just tell people how to rock climb...they have to figure it out through movement" further

mirrors Abrahamson's principle of creating conditions where learners must discover solutions themselves.

The finding that route setters make routes without explicitly considering disabled climbers (unless requested to) reinforces the ecological shrinkage (Dokumaci, 2023) embedded in the climbing environment's design. Despite disabled climbers' presence and participation in the gym, the gym environment remains fundamentally designed for non-disabled bodies, creating a reduced field of affordances (Kiverstein et al., 2021; Rietveld & Kiverstein 2014) for disabled climbers. This finding provides empirical evidence of the study's central premise: disabled climbers navigate environments made without them in consideration, and therefore cultivate creative adaptations born from necessity rather than abundance of possibilities. The route-setters' focus on "fairness" between short and tall climbers but not disabled climbers illuminates ableist bias through design decisions that render disability invisible while attending to other types of variation within bodies. In sum, attending RQ1 and considering the environment of the study, I validated conditions that make disabled climbers' problem-solving expertise visible and valuable.

Research Question 2: Pre-Intervention Assessment

RQ2 functioned as a type of pre-intervention assessment, capturing participants' perceived similarities and differences in problem-solving across rock climbing, mathematics, and everyday life before they took part in the rock climbing and mathematics parts of the study. These conceptualizations, gathered through the first clinical interview prior to enacted problem-solving, reveal how participants have internalized dominant narratives about mathematics—narratives that the subsequent intervention (Phase 2) eventually disrupted to some degree. The consistent finding across all participants was the perception of mathematics as decontextualized, abstract, and disconnected from their lived experience.

This perception amongst participants aligns with researchers who argue that traditional mathematics education strips problems of authentic context (Brown et. al, 1989; Lave, 1988). This disconnect between math and the real world directly reflects the Platonic roots of mathematics education (Lakoff & Núñez, 2000; Núñez, 2008). Participants seem to have internalized this Platonic ideology through their mathematics education, leading them to conceptualize mathematical problem-solving as cerebral and disembodied in contrast with describing rock climbing in embodied terms (Chad's "feeling," Taylor's "body takes over," Ashley's "I have to try it to know it").

These findings suggest that traditional mathematics pedagogy may be still at odds with enactivist theory, where mathematical cognition is fundamentally embodied and grounded in sensorimotor engagement (Varela et al., 1992; Abrahamson & Bakker, 2016). However, participants' statements also revealed that the decontextualized nature of math is more of a pedagogical implication rather than an inevitability. Kenneth, Chad, and Sofia all noted that when mathematics was connected to meaningful contexts, it became accessible, useful, and even enjoyable. Therefore, the problem seems to not be mathematics itself but in the lack of connection of content to students' lived experiences (Wagner & Borden, 2020).

This pre-intervention data helped me to understand the perceptions that eventually allowed the study to function as an intervention. By explicitly inviting participants to connect the domains of rock climbing and math, Phase 2 created conditions for participants to discover dimensions of mathematical problem-solving that their prior education had obfuscated. The finding that participants perceived mathematics problem-solving as fundamentally different from climbing and everyday life problem-solving positioned RQ3 to investigate whether enacted problem-solving would reveal cross-domain coherences that participants' prior mathematics education had obscured.

Research Question 3: Domain Bridging Between Math and Climbing

RQ3 examined observed similarities and differences in problem-solving strategies across rock climbing and mathematical domains. In the previous research question, participants had uniformly characterized mathematics as decontextualized and fundamentally unlike climbing. However, in the third phase of the study, participants readily borrowed climbing terminology to articulate mathematical experiences. This language transfer was not merely metaphorical but represented participants drawing on embodied problem-solving schemas (Lakoff and Nunez 2000) cultivated through climbing to make sense of the mathematical puzzles.

Constraint and affordance identification emerged as the dominant strategy across both domains for nearly all participants, validating Gibson's (1977) and Chemero's (2013) theories that affordances and constraints structure all problem-solving. However, affordance recognition, which appeared prominently in climbing, often contracted in mathematics where participants lacked equivalent expertise. This aligns with Rucińska's (2021) finding that expert climbers perceive vastly more affordances than novices.

My methodological commitment to foregrounding and utilizing climbing expertise bridges the apparent gap between embodied and seemingly abstract cognition. I did not treat physical problem-solving as merely a stepping stone on the way to true mathematical thinking. In other words, I positioned movement-based knowledge as fundamentally constitutive of (rather than merely supportive of) more seemingly abstract domains (Varela et al. 1992). This commitment revealed that the progressive, constraint and affordance-navigating schemas cultivated through climbing shaped cognitive structures that were transferable across problem-solving contexts. This finding validates the enactivist position that even “abstract” domains like mathematics remain grounded in the embodied patterns through which organisms couple with their worlds (Varela et al., 1992; Di Paolo et al., 2017).

Conceptual Bridges

The Role of Language in Articulating about Unfamiliar Domains

When participants encountered mathematical puzzles (a relatively unfamiliar domain for most) they often borrowed language from rock climbing to articulate their problem-solving experiences and to make sense of the puzzles. This phenomenon aligns with embodied cognition research demonstrating that people understand abstract concepts through metaphorical mappings from concrete, bodily experiences (Lakoff & Johnson, 1980; Lakoff & Núñez, 2000). Participants' use of climbing language to make sense of mathematical puzzles exemplifies the process of grounding seemingly abstracted concepts to their experiences with sensorimotor engagement: they mapped the mathematical structures onto their embodied experiences of rock climbing to help make them comprehensible.

It is important to note that this did not emerge completely spontaneously; the very design of the study created auspicious conditions for this cross-domain sense-making. As a researcher who is also a rock climber, I explicitly situated the study within climbing contexts. Participants were recruited from the climbing community and the first phase of the study centered around climbing problem-solving. This functioned as an intervention, where I primed climbing as the available conceptual framework for making sense of mathematical puzzles. I will next explore what happened when climbing expertise was explicitly positioned as relevant to mathematical problem-solving.

Participants readily adopted climbing terminology to describe mathematical experiences. One example was in their use of *crux*—climbing terminology for the hardest part of a route—to identify the hardest part of the mathematical tasks. In Towers of Hanoi, Christopher stated “I kept having to pause at the cruxes along the way,” and continued to incorporate that language throughout the rest of the puzzles. He seemed to find the *crux* concept useful moving through the puzzles, transforming a rather amorphous mathematical puzzle into a recognizable

structure with identifiable peak difficulty. Chad's language particularly illuminated how climbing concepts structured his mathematical sense-making. He described needing environmental feedback—"little wins"—to maintain momentum, directly paralleling the experience of getting to rest holds in climbing ("when you get to a nice hold that you can rest on"). Here, Chad was able to articulate a problem-solving need (incremental progress feedback) through comparison to resting in climbing that might otherwise have remained implicit.

Comparing Mathematical Puzzles to Different Styles of Climbing

The stimulated recall interviews following mathematical puzzles initially allowed participants to freely reflect on their problem-solving experiences with the puzzles. When the first two participants organically began using climbing language and comparisons to styles of climbing to make sense of the puzzles, I recognized this as a significant pattern and modified the interview protocol. For remaining participants, I explicitly asked them to consider how their mathematical experiences related to different styles of climbing. All participants readily engaged in this comparative activity, drawing parallels between puzzle characteristics and climbing contexts. They invoked specific rock types (conglomerate, sandstone) to describe affordance landscapes, climbing styles (crack climbing, face climbing, slab) to characterize movement demands, and affective states experienced on particular routes to articulate their emotional engagement with puzzles. See Appendix J for pictures of the different types of climbing. Ultimately, these findings demonstrate that when domain expertise (particularly movement-oriented) is explicitly positioned as relevant to novel and more seemingly abstract problem-solving, participants can leverage that expertise to better articulate their experience. This speaks to the interventional flavor of the methodological form of this study. Even if participants would not spontaneously compare mathematics to climbing outside this study context, the fact that they readily made such connections when invited suggests a potentially powerful pedagogical approach: deliberately activating problem-solving expertise from one

domain of practice as a resource for cultivating productive orientations toward mathematical problem-solving.

Towers of Hanoi and “Progressive” Thinking

One finding in the rock climbing and mathematical puzzle parallel was particularly salient and unexpected: across all participants, the Towers of Hanoi puzzle was their preferred mathematical task. When participants encountered Towers of Hanoi, they felt a sense of familiarity: "this just makes sense," "this is like climbing," "this is the type of problem solving I naturally enjoy." This preference seems to be connected to structural similarities between Towers of Hanoi and what Kenneth termed "progressive thinking"—a problem-solving orientation shaped by a constrained and stable environment, eliciting a more sequential approach to problem-solving. This "progressive" quality—described by Kenneth by well-defined starting points, ending points, and intermediate checkpoints that Kenneth called "middle points"—distinguished both Towers of Hanoi and rock climbing from more open-ended problems like SOMA. This notion of incremental success is comparable to rock climbing, where simply moving away from the ground, reaching resting holds, or moving past cruxes provides incremental feedback and motivation. Similarly, on Towers of Hanoi, successfully moving the largest disc to the target rung or reducing the problem down to an $n-1$ disc problem acts as tangible progress markers.

The preference for Towers of Hanoi became more clear when contrasted with participants' aversion to the SOMA cube—a puzzle all participants expressed frustration with due to a lack of a tangible and repeatable solution path. Further, many participants compared SOMA to climbing contexts characterized by overwhelming options or lack of clear progression and feedback. This distinction aligns with research distinguishing well-structured problems (clear initial states, goal states) from ill-structured problems (ambiguous goals, multiple solution paths, vague constraints) (Simon, 1973).

It is possible that years of climbing have not just given participants language but has molded their cognitive preferences or schemas toward sequential, constrained-guided problem-solving (Chi, Feltovich, & Glaser, 1981). Indeed, they have practiced "progressive" problem-solving extensively and when encountering Towers of Hanoi, this ingrained orientation from climbing found a matching structure in the puzzle, enabling what feels like intuitive, natural problem-solving. This pattern aligns with research on *problem isomorphs* demonstrating that structural features of problems beyond surface content shape problem-solving success and preference (Kotovsky, Hayes, & Simon, 1985).

In conclusion, in this part of the discussion, I have discussed some powerful key insights: a) the role of linguistic and cognitive shaping from embodied expertise that can be transferable when productively activated, acting as conceptual bridge, and; b) embodied domain expertise may train individuals to be favorably disposed to particular types of problems across contexts. Next, I will zoom out and explore problem-solving process structure from rock climbing and its analogous orientation to mathematics problem-solving, while also relating it to Carlson and Bloom's taxonomy.

Embodied Problem-Solving Cycle

The following framework emerged from analyzing how disabled climbers navigate challenges in rock climbing and subsequently transfer their problem-solving approaches to mathematical puzzles. The cyclical nature of this framework distinguishes it from traditional linear models (Polya, 1945) by recognizing that problem-solving is iterative, involves refinement and adjustment based on feedback from each attempt. I will later parallel this cycle to Carlson and Bloom's (2005) taxonomy.

The Four-Phase Cycle

The problem-solving cycle that emerged from this study consists of four interconnected phases: *Beta* (Orientation and Self-Assessment), *Sequence* (Execution), *High Point* (Maximum Progress), and *Feedback* (Evaluation and Adjustment). Each phase follows into the next, with the feedback phase circling back to beta, creating a continuous loop that goes until a solution is arrived at. Refer to Figure 24 to see a representation of this cycle.

Phase 1: Beta (Orientation and Assessment)

The beta phase represents the initial orientation to a problem and involves gathering information, recognizing affordances (what actions or approaches are possible), identifying constraints (what limitations exist), identifying what skills or tools are needed, and conducting self-assessment (i.e., stacking yourself up against the problem) relative to the problem. Here, it is common to visualize yourself doing the problem and to do some pre-planning. As a reminder, the term beta derives from rock climbing terminology, where it refers to the specific method for completing a route. In this framework, beta is more broad—it encompasses the decision-making process of determining "what will I try?," which can be interpreted as a type of commitment to a particular approach before execution begins.

Phase 2: Sequence (Execution)

The sequence phase involves the actual implementation or execution of the chosen beta. Having decided upon an approach in the previous phase, one can now execute their plan through sequential action. This phase represents the transition from planning to actually doing it. This phase embodies the principle of "I've decided what to do—now I will do it," but there is space for adaptation and changing plans mid-sequence, as seen often in rock climbing (Rucińska, 2021)

Phase 3: High Point (Maximum Progress)

The high point represents the furthest extent of progress achievable with the current beta and execution, even with some in-the-moment micro-adjustments along the way. The high point is not synonymous with solution; rather, it marks what a particular approach can accomplish before one either encounters insurmountable difficulty or reaches completion. A high point may occur when the one becomes stuck, makes an error, or successfully completes the task. This phase serves as a moment of evaluation, revealing how far the chosen beta can carry oneself toward the desired outcome. The high point provides crucial information about the viability of the current approach and helps to inform the subsequent feedback phase.

Phase 4: Feedback (Evaluation and Adjustment)

In the feedback phase, one assesses what happened during their attempt, examining both what worked and what didn't work to extract important information. This phase generates insight that informs the next iteration, enabling one to decide whether to use the same beta again, refine their existing beta, or abandon the current beta entirely in favor of a different approach. This phase completes the cycle by feeding directly back into the beta phase, where one can then reorient to the problem with enhanced understanding of how to better solve the problem. See Figure 24 for a visual representation for this process.

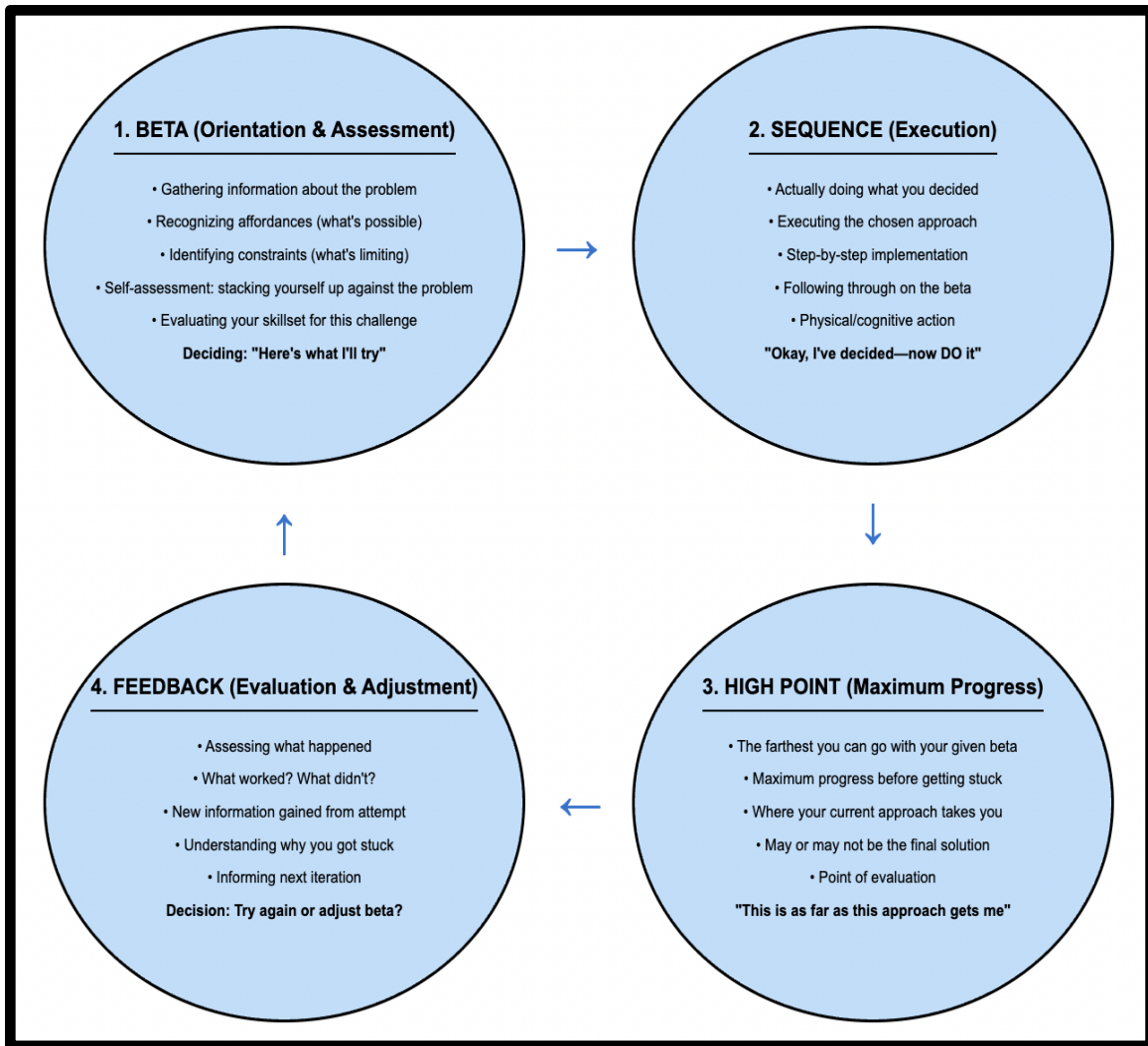


Figure 24: Four-Phase Embodied Problem-Solving Cycle

Relating to Carlson and Bloom and Beyond: The High Point

Carlson and Bloom's (2005) multidimensional problem-solving framework arose from studying expert mathematicians solving math problems, while the cyclical framework from this dissertation (moving forward, I will refer to it as the Embodied Problem-Solving Cycle, EPSC) emerged from studying disabled climbers' problem-solving across climbing and mathematical domains. Both frameworks share fundamental structural similarities while diverging as well. Carlson and Bloom identified four phases that expert problem-solvers cycle through: a) Orientation (sense-making, organization, construction); b) Planning (devising a plan or strategy); c) Executing (carrying out the plan); and d) Checking (is it reasonable? Accept solution?

Reject?) It is crucial to mention that within this framework, the meta-cognitive process of monitoring is embedded within every step, evaluating if the emerging results fit within one's current understanding, if the approach is productive, and so on.

Carlson and Bloom's taxonomy and the EPSC map onto each other in the following ways:

- Orientation → Beta
- Planning → Beta
- Executing → Sequence
- Checking → Feedback

However, where the frameworks diverge is in the "high point" phase. Their "executing" phase leads directly to "checking," with an implicit assumption that execution produces something to check. The EPSC instead introduces "high point" as a distinct phase between execution and feedback. This addition explicates an important ingredient in the execution/sequence phase—that one way may get stuck somewhere along the way or they immediately complete the problem, or somewhere in between. The high point explicitly names the boundary of what a particular beta can accomplish, reframing "getting stuck" not as failure but as information that generates feedback. By naming the high point, the framework validates partial progress and reframes it as legitimate phases of problem-solving rather than failures. Students who reach a high point that is short of an adequate solution are not failing. Instead, they are generating information. The high point phase extends the productive failure theory (Kapur, 2008): when students reach a high point short of solution, they are building knowledge structures that productive failure research shows prepares learners for deeper understanding.

Disability Unveils the High Point

The EPSC shares Carlson and Bloom's (2005) fundamental insight that problem-solving is iterative and cyclical. However, by grounding the framework in embodied cognition and

centering disabled climbers' cross-domain expertise, it extends beyond Carlson and Bloom's taxonomy. It is through working with disabled climbers that the concept of the high point was made visible; if solely looking at expert rock climbers (analogous to expert mathematicians), the concept of high point would not have been as salient. Instead, the variance in what it means to complete a route illuminated an important component: reaching one's maximum achievable point with a given approach—whether or not it constitutes “completion”—provides essential feedback for subsequent attempts. Disabled climbers' experiences made explicit what often remains invisible in researching expert performance.

Disabled climbers frequently encounter routes designed for non-disabled bodies, and therefore their high points vary significantly. This insight reframes “incomplete attempts” from failures toward information-generating events. In other words, students' partial progress is too often dismissed as inadequacy or even failure rather than being recognized as a natural phase in the cyclical refinement toward solution. This exemplifies why centering marginalized populations in research reveals dynamics that remain invisible when studying only those for whom systems were designed (Garland-Thomson, 2011; Gutiérrez, 2018; Hamraie, 2017). As Gutiérrez (2018) argues, “mathematics education cannot truly improve until it adequately addresses the very students who the system has most failed” (p. 2), precisely because their experiences expose constraints that remain invisible for those the system privileges. In order to better understand the EPSC and see it in practice, Figure 25 is a visual representation of the manifestations of the cycle across rock climbing and math.

Rock Climbing	Mathematics
1. BETA (Orientation) • Looking at the route from the ground • Identifying holds, cruxes, rests • Recognizing what movements are possible • Assessing: "Can I reach that? Do I have the strength?" • Deciding on approach: "I'll use heel hooks on that overhang" 2. SEQUENCE (Execution) • Actually climbing • Executing the planned movements • Move by move: left hand to that hold, right foot on that chip • Following through on the beta 3. HIGH POINT (Progress Limit) • Get stuck at a certain hold • "I made it to the rest but can't reach the next hold" • As far as this beta takes you • May fall or take tension here 4. FEEDBACK (Evaluation) • "My foot slipped—I need different footwork" • "I'm too pumped—need to rest earlier" • Learning from the attempt • Deciding: try same beta again or adjust? • → Return to BETA with new information SOLUTION: When your high point reaches the top hold and completes the route successfully.	1. BETA (Orientation) • Reading the problem: $x + y = 2$ and $2x + 3 = y$ • Recognizing: system of equations • Identifying constraints: two unknowns, two equations • Seeing affordances: could use substitution, elimination, or graphing • Self-assessment: "I know how to do substitution" • Decision: "I'll try substitution method" 2. SEQUENCE (Execution) • Actually working through the math: 1. Solve first equation for y: $y = 2 - x$ 2. Substitute into second equation 3. Simplify and solve • Following through on the chosen method 3. HIGH POINT (Progress Limit) • Might get: $2x + 3 = 2 - x$, then $3x = -1$ • "I got this far but I'm stuck on what to do next" • Or: successfully get $x = -1/3$ • Or: make an error and get impossible result • As far as this attempt takes you 4. FEEDBACK (Evaluation) • Check: does this make sense? • Verify by plugging back in • "I made an algebra error—need to redo that step" • Or: "Substitution is messy, maybe try elimination?" • Learning what works and what doesn't • → Return to BETA with refined approach SOLUTION: When your high point gives $x = -1/3$, $y = 7/3$ and verification confirms both equations are satisfied.

Figure 25: Manifestations of the four-phase cycle across rock climbing and math.

Chapter 6: Implications

Epistemic Climb-it

A core implication of this study is about the epistemic climate of mathematics classrooms—the cultural norms, discourse practices, and tacit assumptions that shape what counts as legitimate mathematical knowledge and valid problem-solving approaches (Feucht, 2010.). Traditional mathematics classroom environments cultivate an epistemic climate where abstract, disembodied reasoning is valued. The findings from this dissertation suggests that deliberately reconceptualizing the epistemic climate of the mathematics classroom has the potential to expand mathematical engagement by positioning diverse forms of embodied knowledge as legitimate resources for sensemaking.

Before diving into shifts in epistemic climate, however, it is crucial to address the rapport that must be established by instructors in order to help students feel safe enough to bring their whole selves into the classroom. On the first day, this could mean being very explicit about what kind of classroom this is: one where students' expertise is positioned as valuable, where embodiment is welcomed, where names are learned and used, where mistakes are okay and part of the process. Therefore, building rapport in the math classroom is the very first pedagogical act of shifting an epistemic climate.

Traditionally in the mathematics classroom, students are denied exploratory space (Boaler 2002). When instructors immediately provide prescriptive solution methods rather than creating conditions for students to grapple with problems, it trains students to see themselves as passive receivers of mathematical knowledge rather than active problem-solvers (Schoenfeld 2016). Many students in the mathematics classroom have been taught to feel embarrassed by exploratory struggle. Unfortunately, what should be their greatest strength becomes a source of shame (Dweck 2006). Creating a mathematical epistemic climate that supports genuine

problem-solving requires giving students space to explore rather than prematurely intervening. This aligns with productive failure theory (Kapur, 2008) but extends it by emphasizing that exploration must be culturally mediated through explicit linguistic resources that positions struggle and partial progress as legitimate phases of mathematical engagement (like the high point).

When the cultural context systematically rejects opportunities for exploration and individualized decision-making, problem-solving becomes impossible, regardless of individual capacity. In the mathematics classroom, when an epistemic climate denies students space to grapple with problems or dismisses their embodied expertise as irrelevant to mathematical reasoning, the culture eliminates the opportunity for genuine problem-solving. In order to desettle the traditional mathematics epistemic climate, we must first consider all the ways in which we can give students permission to actually problem-solve.

Supporting Disabled Learners in the Mathematics Classroom

The findings of this study highlight that disabled learners may bring with them a particular fluency with constraint navigation that is directly relevant to mathematical problem-solving. Educators can actively create opportunities for students to draw on and activate that knowledge through inviting them to connect their lived experience of navigating constraints to the constraints and affordances they encounter in mathematical tasks. Educators can ask them *how* they read the problem space (not just what they did), what they tried first, where they may have hit a wall, and how they adapted. These are mathematically rich questions for disabled students who have spent their lives engaging in adaptive problem-solving. This is, at its heart, what dignity affirming pedagogies like culturally responsive and sustaining pedagogy have advocated for: centering students' lived experiences as a legitimate and generative resource in the mathematics' classroom.

A central implication of this study is the ways in which it offers math educators who will inevitably have disabled students in their classrooms. Concretely, this means educators should deeply examine where ableist assumptions are ingrained in their pedagogy: in the framing of struggle as a deficit, in the privileging of singular solution paths, etc. Instead, they can design tasks that invite multiple solution paths and they can normalize adaptive strategies by reconceptualizing what it means to engage in legitimate mathematical practices. Universal Design for Learning (UDL) offers a framework for this implication, where instead of retrofitting accommodations for disabled students, more accessible learning environments are considered from the very start (Meyer et al., 2017). The findings of this study extend and deepen what UDL has given us language to understand: when the epistemic climate positions embodied and adaptive expertise as mathematically legitimate, disabled students are included as full participants in mathematical sense making.

Sociomathematical Norms and Classroom Practices

This study suggests that mathematics classrooms could benefit from establishing a pan-domain problem-solving discourse that draws on and validates students' diverse embodied experiences. So, for example, rather than insisting students first learn formal mathematical terminology divorced from their experience, instructors could invite students to bring language from domains where they have cultivated problem-solving expertise—sports, arts, music, crafts, gaming—and use that language as conceptual scaffolding for mathematical sense-making.

Rasmussen and colleagues' (2005) work on sociomathematical norms and classroom mathematical practices illuminates how mathematics classroom communities develop an emergent and shared understanding of what counts as legitimate mathematical activity. Consider the following example: On the first day of math class, the instructor poses the question:

"Think about an activity where you're a competent problem-solver, maybe a sport, type of art, game, job, or hobby. What kind of language do you use to describe your problem-solving process?" Students share terminology from their domains:

- Climbers contribute: "crux," "send," "project," "rest point" "high point," "onsight"
- Musicians offer: "practice," "fingering," "phrasing," "dynamics," "tempo change," "sight reading"
- Gamers suggest: "upgrading," "checkpoint," "grinding"
- Chess players suggest: "opening moves," "calculations," "end game"
- Painters share: "caricature," "Plein air," "composition," "abstraction" "economy of brushstrokes"
- Ballet dancers offer: "choreography," "technique," "intention," "improvisation," "marking"
- Surfers contribute: "eyeing a section," "drawing your line," "knifing a drop," "reading the wave"
- B.A.S.E. jumpers suggest: "exit strategy," "checkpoint," "angle of attack"

Following this conversation, the class collaboratively negotiates on a shared repertoire to draw from moving forward. The instructor poses questions: "Which terms capture ideas that show up across your different domains? What language feels most useful for describing mathematical problem-solving?" Through discussion, students may come to the consensus that a gamer's "checkpoint" and a climber's "rest point" describe similar experiences: they are both describing moments to assess progress and reorient to the problem. The class then collectively decides which terms to incorporate into their negotiated problem-solving vocabulary. This list is documented publicly (shared document; course website; etc.) and is constantly referenced throughout the course. Over time, the class develops pan-domain problem-solving discourse

that bridges math and domains that they are passionate about, creating sociomathematical norms that activates and leverages diverse expertise.

Problem-Solving is not a Monolithic Capacity

It is essential to recognize that climbing-cultivated progressive thinking (at least, this is what we see in this study) represents only one cognitive orientation among many that can support one way of mathematical reasoning. I argue that different embodied practices foster different cognitive orientations, each valuable for different types of mathematical problems. For example, mountain biking cultivates in-the-moment decision-making under unpredictable conditions, which is reminiscent of lateral thinking that are characteristics of open-ended problems. In other words, different embodied practices foster different problem-solving capabilities.

In the classroom, I believe this means that the collective pool of problem-solving orientations that students bring from their varied life experiences is a strength to the community of learning. A community college classroom may include a student whose carpentry experience has cultivated spatial reasoning, a chef whose cooking develops proportional thinking, a caregiver whose caregiving builds adaptive planning, or a gamer whose gaming fosters algorithmic thinking. Indeed, these students may become experts in a particular genre of mathematical problem-solving. So students can learn not only from the instructor but from each other's expertise, developing more robust and flexible mathematical capacity through exposure to multiple ways of thinking about problems. All in all, problem-solving is not a monolithic capacity, and if we want students to develop a capacity to solve a diversity of mathematical problems, they need exposure to diverse forms of cultural practice.

The Periodic Table Of Embodied Activities

When Mendeleev constructed the periodic table in 1869, he organized known elements by weight and properties, revealing patterns that predicted elements not yet discovered. The gaps told scientists exactly where to look—leading to the discovery of several elements. This study attempts something analogous for embodied problem-solving: examining rock climbing revealed one "element" in what could become a comprehensive organizational system mapping how different embodied practices cultivate different cognitive orientations applicable to different mathematical structures.

Climbing represents only one position in this larger structure. The gaps in our current understanding tell us where to look next. So, what embodied practices cultivate the lateral, exploratory thinking that open-ended problems demand? Perhaps surfers, who navigate dynamic, chaotic ocean conditions likely feel rather comfortable with uncertainty and real-time adaptation—filling a gap for more open-ended problems. Maybe B.A.S.E. jumpers' split-second risk calculations might illuminate how students navigate high-stakes testing environments or time-constrained problem-solving. The list goes on.

Future studies could help to fill in a systematic framework, organizing embodied practices by the cognitive orientations they cultivate: sequential vs. exploratory, constrained vs. open-ended, individual vs. collaborative, high-consequence vs. low-stakes, pattern-based vs. improvisational, static vs. dynamic environments. Each practice would occupy a position based on multiple dimensions, and the gaps would reveal which orientations remain underexplored—pointing researchers toward practices that might cultivate currently unmapped problem-solving approaches. Future research can systematically examine diverse embodied practices to fill the current gaps: Each study adds an "element" to the emerging structure, gradually revealing the full periodic table of embodied problem-solving orientations and their relationships to mathematical cognition.

Applying the Humanistic Problem-Solving Framework

The overlapping radial chart and diverging bar chart can act as a useful tool in understanding how participants engage in problem-solving. The visual representations provide a palpable and digestible way of making sense of individuals' approach to problem-solving across domains. Moving forward, it is possible that the framework established in this study can do the same for researchers who want to continue on this strand of research, aiming to understand how problem-solving transfers across domains. Initially, I aimed to understand how strategies differed across domains, but throughout the analysis I began to realize there was much more at play, such as the five attributes (affect; disposition; expertise; environment; efficiency) that deeply impacted the way participants problem-solved in each context.

Researchers investigating transfer can leverage these visualization tools to make comparable analyses across different domains and participant groups. The radial chart enables researchers to systematically compare how individuals position themselves across the five attribute dimensions in different contexts, making patterns of coherence and divergence immediately visible. The diverging bar chart allows researchers to track which strategies are consistent across domains versus which remain context-specific. Together, these tools provide a replicable methodology for examining problem-solving as a situated and relational phenomenon.

Applying the Embodied Problem-Solving Cycle

The four-phase problem-solving cycle (Beta, Sequence, High Point, and Feedback) offers implications for understanding and supporting problem-solving in the classroom. The framework reframes "incomplete" attempts not as failures but instead as information-generating processes. The High Point represents the maximum progress achievable with a given approach, making visible what that particular beta can and cannot accomplish (i.e., do you want to try the same beta again or do you want to "tweak" it slightly?). Students who reach a High Point without

solving the problem (i.e generating the exact solution $x=2$) are not "stuck" or "failing"—they are at a critical point where the Feedback phase can generate new understanding about constraints, affordances, and alternative approaches. By explicitly teaching students to recognize and reflect on their High Points, educators and students can cultivate more metacognitive awareness and adaptive problem-solving strategies. The framework also suggests that effective learning environments should provide opportunities for multiple iterations through the cycle, allowing learners to refine their beta based on accumulated feedback rather than expecting immediate solutions. For example, in climbing, we call the refining of beta “dialing your beta,” and it is a crucial part of the problem-solving process by helping to cultivate maximum efficiency and deepen understanding.

Burning Questions

I am left with many questions that I have not yet addressed in this dissertation, especially regarding the pervasive yet nebulous concept of *flow*. Here, I list a few that I am excited to further explore:

1. What conditions enable flow state in mathematical problem-solving?
 - a. Participants identified clear feedback, checkpoints, and progressive structure as facilitating flow. How can mathematical tasks be designed to provide these affordances without reducing problems to algorithmic execution?
 - b. If flow emerges from person-environment coupling rather than individual skill alone, what does this mean for mathematics classrooms? Should instruction focus on task design that creates flow-conducive conditions, or on helping students recognize and sustain flow when it emerges?
2. What role does attention play in accessing flow and perceiving affordances?

- a. Is the perception of affordances solely related to expertise, or can it also come down to attention? Certainly, one can be an expert in a domain but when they are not paying attention, affordances cannot be perceived.

Chapter 7: Limitations

A fundamental limitation of this study is its lack of intersectionality among participants. The participant pool was predominantly composed of white men, and all participants possessed socioeconomic privilege that afforded access to climbing gyms and adaptive equipment. This study involved six primary participants which is limiting in generalizability. Participants were recruited through convenience sampling from a climbing gym. The study focused on specific disability types (limb difference, visual impairment, participants who use wheelchairs) and did not include cognitive disabilities, chronic illness, neurodivergence, or Deaf/hard of hearing experiences.

Participants rock climbed for only one 90-minute session, and similarly engaged with mathematical puzzles for only one 90-minute session. Longitudinal engagement might reveal whether observed strategies persist, evolve, or are no longer used over time. The puzzles (SOMA cube, Towers of Hanoi, Tangram) were spatial and manipulative rather than symbolic, algebraic, or proof-based. Findings may be harder to extend to abstract algebra, formal proofs, or computational mathematics where it is more difficult to observe strategies physically. Ashley did not physically climb during the study, meaning her Phase 1 data derives solely from interview data rather than observation, limiting direct comparison with other participants. Sofia did not participate in the second phase of the study, limiting cross-domain analysis of her problem-solving strategies.

As a climber and with my engagement with the adaptive climbing community, my close relationship with the community shaped every aspect of this study. I explicitly asked participants to compare puzzles to climbing and invited climbing terminology, meaning the study functioned as an intervention. Findings demonstrate what becomes possible when embodied expertise is deliberately activated but cannot claim these connections arise spontaneously without pedagogical prompting. Additionally, many of the participants were close personal friends of mine, which served as both an affordance and a constraint throughout the study. While these

relationships facilitated trust and access to vulnerable expressions throughout interviews, they also introduced methodological challenges in maintaining research boundaries. Certainly, at times, it was challenging to manage the ethical complexities of studying individuals with whom I care deeply for.

Most significantly, I am not an adaptive climber myself. Though many participants were close personal friends, I am an outsider to the lived experience of navigating climbing (and the world more broadly) with a physical disability. This positionality requires me to infer more and make more interpretations of participants' experiences rather than being able to draw from my own embodied understanding of disability. My non-disabled perspective limits the depth and nuance with which I can comprehend and therefore articulate the strategies, constraints, and activist affordances that emerge from disabled individuals' lived experiences.

Chapter 8: Conclusion

I learned throughout this study that framing disabled climbers as "better" or "more creative" problem-solvers perpetuates harmful fetishization of disability. I learned that the value of centering disabled climbers lies not in any inherent superiority. Instead, exploring how disabled individuals navigate constrained environments provides a magnifying glass into how all learning emerges from constraint. As Garland-Thomson (2011) argues, both the deficit model of disability and "supercrip" narratives are harmful. This study attempts to navigate that binary by studying disabled climbers' problem-solving expertise without unnecessarily uplifting or degrading—simply as knowledgeable informants about navigating constraints. The insights gained from this study benefit mathematics education because disabled individuals' experiences make visible the constraint-navigation dynamics that operate for all learners, but often remain invisible when studying only those for whom environments were designed. Indeed, the mathematics environment is alienating—and thus shrinks—for many students who are not deemed explicitly disabled, but whose ways of learning and knowing do not align with traditional mathematical culture. By studying how disabled individuals navigate explicitly constrained environments, we gain insight into creating mathematics classrooms that expand rather than shrink affordances for all students navigating spaces not designed with them in mind. Ultimately, this research demonstrates that learning from disabled people—and from people in general—requires positioning them as experts in their own experience rather than objects of inspiration or pity, recognizing that every body's coupling with environments generates valuable knowledge about how cognition works, both in the classroom, and out in the wild.

In our current political climate, where marginalized groups are being systematically dismissed, erased from curriculum, and denied access to equitable education, and where artificial intelligence (AI) is increasingly positioned as a "replacement" for human thinking, it is more crucial than ever to center the human in problem-solving research. In this dissertation, I

foreground the embodied, situated, and deeply personal nature of problem-solving and I center disabled individuals whose ways of knowing emerge from navigating a world not designed for them. I argue that human diversity is not a fault or something to be eliminated, but rather a rich source of insight and innovation, much like how an ecosystem thrives with diversity.

Monocultures, whether in agriculture or education, are fragile and vulnerable systems susceptible to collapse, while diverse ecosystems demonstrate durability and adaptation that cannot be understood from studying any single species in isolation (Tilman et. al, 2002).

The richness of human problem-solving lies in its intrinsic entanglement with bodies, emotions, environments, and lived experience. This research illuminates that human problem-solving is deeply embodied, and I argue that no AI can replicate it because it emerges from being in a body in the world. AI may be able to help us problem-solve across various contexts and can be a valuable tool when used correctly, but is inevitably an abstraction. Rather than asking how we can make students think more like machines (faster, more efficient, more standardized) and reducing down our complexity, we should be asking how we can honor and cultivate the messy, embodied, relational ways humans actually make sense of problems. This dissertation argues for a mathematics education and a broader educational paradigm that recognizes learners as *whole people* whose problem-solving capabilities are inseparable from who they are, where they come from, and what their bodies know.

Real problems do not come in neat little packages. The silos that we have built in education are not only unproductive, but are harmful. We separate mathematics from movement, cognition from embodiment, academic knowledge from lived experience. This dissertation argues against the compartmentalization of knowledge that characterizes traditional education models. This became apparent through working with disabled climbers. They are simultaneously navigating physical constraints, social constraints, cognitive constraints, and emotional constraints. Indeed, disabled individuals are expert interdisciplinary problem-

solvers. Getting through an inaccessible world requires synthesizing knowledge from multiple domains constantly.

All in all, I argue that our educational models must shift their focus away from siloed subjects and toward authentic problems to be solved. When we strip mathematics of context, movement, and embodiment, we are impoverishing it. Then, we are left with what participants in this study initially described: something abstract, disconnected, and alienating. Ultimately, learning that is truly transformative is not just academic; it is inherently cross-disciplinary and deeply embodied.

References

- Abrahamson, D. (2024). Enactivist learning: Grounding mathematics concepts in emergent perception for action. *Inellectica*, 80, 149-174.
- Abrahamson, D. (2021). Grasp Actually: An Evolutionist Argument for Enactivist Mathematics Education. *Hum. Dev.*, 65(2), 77-93.
- Abrahamson, D., & Bakker, A. (2016). Making Sense of Movement in Embodied Design for Mathematics Learning. *Cogn. Research*, 1(1), 33.
- Abrahamson, D., & Sánchez-García, R. (2016). Learning Is Moving in New Ways: The Ecological Dynamics of Mathematics Education. *Journal of the Learning Sciences*, 25(2), 203-239.
- Abrahamson, D., & Shulman, A. (2018). Co-constructing movement in mathematics and dance: An interdisciplinary pedagogical dialogue on subjectivity and awareness. *FRJ*.
- Acar, O. A., Tarakci, M., & van Knippenberg, D. (2019). Creativity and Innovation Under Constraints: A Cross-Disciplinary Integrative Review. *J. Manage.*, 45(1), 96-121.
- Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *J. Learn. Sci.*, 21(2), 247-286.
- Anderson, J. A., & Hinton, G. E. (2014). Models of Information Processing in the Brain. *Parallel Models of Associative Memory* (pp. 33-74). Psychology Press.
- Anderson, M. L. (2021). *After Phrenology: Neural Reuse and the Interactive Brain*. MIT Press.
- Annamma, S. A., Connor, D., & Ferri, B. (2013). Dis/ability critical race studies (DisCrit): Theorizing at the intersections of race and dis/ability. *Race ethnicity and education*, 16(1), 1-31
- Araújo, D., Davids, K., & Hristovski, R. (2006). The Ecological Dynamics of Decision making in Sport. *Psychol. Sport Exerc.*, 7(6), 653-676.
- Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *J. Exp. Psychol. Gen.*, 130(2), 224-237.
- Baggs, E., & Chemero, A. (2020). Thinking with Other Minds. *Behav. Brain Sci.*, 43-92.
- Barbour, K. (2011). *Dancing Across the Page: Narrative and Embodied Ways of Knowing*. Intellect.
- Barnes, C. (2012). Understanding the Social Model Of Disability: Past, Present And Future. Abingdon: Routledge.
- Battey, D., & Marshall, B. L. (2023). Math Logics: Perpetrators of Whiteness in STEM Educational Spaces. In J. Ravulo, K. Olcoñ, T. Dune, A. Workman, & P. Liamputtong (Eds.), *Handbook of Critical Whiteness: Deconstructing Dominant Discourses Across Disciplines* (pp. 1-11). Springer Nature Singapore.
- Baynton, D. C. (1996). *Forbidden signs: American culture and the campaign against sign*

- language. University of Chicago Press.
- Becker, K. M. (2023). *Learning Through Movement in the K-6 Classroom: Integrating Theater and Dance to Achieve Educational Equity*. Taylor & Francis.
- Berne, P., & Raditz, V. (2019). To Survive Climate Catastrophe, Look to Queer and Disabled Folks. *Yes! Magazine*.
- Bernard, H. R. (1988). Research methods in cultural anthropology in relation to scientific criteria. *Psychological Review*, 57(6), 334–344.
- Bicudo, M. A. V., Czarnocha, B., Rosa, M., & Marciniak, M. (2023). *Ongoing Advancements Philosophy of Mathematics Education*. Springer Nature.
- Blanchard-Fields, F. (2007). Everyday problem solving and emotion. *Current Directions in Psychological Science*, 16, 26–31.
- Boaler, J. (2022). *Mathematical Mindsets: Unleashing Students' Potential through Creative Mathematics, Inspiring Messages and Innovative Teaching*. John Wiley & Sons.
- Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning*. Lawrence Erlbaum Associates.
- Bokhove, C., & Jones, K. (2018). Stimulating Mathematical Creativity through Constraints in Problem-Solving. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the Scope of Research on Mathematical Problem Solving: A Focus on Technology, Creativity and Affect* (pp. 301–319). Springer International Publishing.
- Borasi, R. (1986). On the Nature of Problems. *Educational Studies in Mathematics*, 17(2), 125–141.
- Brown, L. X. Z. (2011). *The significance of semantics: Person-first language: Why it matters*. Autistic Hoya.
- Brown, J. S., Collins, A., & Duguid, P. (1989). *Situated cognition and the culture of learning*. 1989, 18(1), 32-42.
- Bruineberg, J., Chemero, A., & Rietveld, E. (2019). General ecological information supports engagement with affordances for “higher” cognition. *Synthese*, 196(12), 5231–5251.
- Burgstahler, S. E., & Cory, R. C. (Eds.). (2010). *Universal design in higher education: From principles to practice*. Harvard Education Press.
- Byrne-Haber, S. (2019). Want an employee who is a natural problem solver? Hire someone with a Disability. In *Medium*. <https://sheribyrnehaber.medium.com/want-an-employee-who-is-a-natural-problem-solver-hire-someone-with-a-disability-ef498b3e91b8>
- Calvo-Merino, B., Glaser, D. E., Grèzes, J., Passingham, R. E., & Haggard, P. (2005). Action observation and acquired motor skills: An fMRI study with expert dancers. *Cereb. Cortex*, 15(8), 1243–1249.
- Campbell, F. K. (2009). *Contours of ableism: The production of disability and abledness*. Palgrave Macmillan.

- Carlson, M. P., & Bloom, I. (2005). The Cyclic Nature of Problem Solving: An Emergent Multidimensional Problem-Solving Framework. *Educ Stud Math*, 58(1), 45–75.
- Chamberlin, S. A. (2008). What is Problem-Solving in the Mathematics Classroom? *Philosophy of Mathematics Education Journal*, 23(1), 1-25.
- Chemero, A. (2013). Radical Embodied Cognitive Science. *Rev. Gen. Psychol.*, 17(2), 145–150.
- Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive science*, 5(2), 121-152.
- Communicating About People with Disabilities. (2020). In NDRN.
<https://www.ndrn.org/resource/communicating-about-people-with-disabilities/>
- Cornelius, S. W., & Caspi, A. (1987). Everyday problem solving in adulthood and old age. *Psychology and aging*, 2(2), 144.
- Cosmides, L., & Tooby, J. (1994). Better than Rational: Evolutionary Psychology and the Invisible Hand. *The American Economic Review*, 84(2), 327–332.
- Cuffari, E. C., Di Paolo, E., & De Jaegher, H. (2015). From Participatory Sense-Making to Language: There and Back Again. *Phenomenol. Cognitive Sci.*, 14(4), 1089–1125.
- D'Alessio, S. (2011). Inclusive Education and Special Needs Education. In *Inclusive Education in Italy* (pp. 23–42). Brill.
- Deci, E. L. (1971). Effects of externally mediated rewards on intrinsic motivation. *Journal of Personality and Social Psychology*, 18(1), 105–115.
- Dempsey, N. P. (2010). Stimulated Recall Interviews in Ethnography. *Qual. Sociol.*, 33(3), 349–367.
- Dewey, J. (1986). Experience and Education. *Educ. Forum*, 50(3), 241–252.
- Di Paolo, E. A. (2023). F/acts Ways of Enactive Worldmaking. *Journal of Consciousness Studies*, 30(11–12), 159–189.
- Di Paolo, E., Buhrmann, T., & Barandiaran, X. (2017). *Sensorimotor Life: An enactive proposal*. Oxford University Press.
- diSessa, A. A. (2007). An Interactional Analysis of Clinical Interviewing. *Cogn. Instr.*, 25(4), 523–565.
- Dokumaci, A. (2017). Vital affordances, Occupying Niches: An Ecological Approach to Disability and Performance. *Research in Drama Education: The Journal of Applied Theatre and Performance*, 22(3), 393–412.
- Dokumaci, A. (2023). *Activist Affordances: How Disabled People Improvise More Habitable Worlds*. Duke University Press.
- Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics Anxiety: What Have We Learned in 60 years? *Front. Psychol.*, 7, 508.

- Dreyfus, H. L. (2002). Intelligence without representation – Merleau-Ponty's critique of mental representation The relevance of phenomenology to scientific explanation. *Phenomenol. Cogn. Sci.*, 1(4), 367–383.
- Dunagan, C., Fenton, R. L., & Dorn, E. D. (2019). *Modelling Improvisation as Emergence: A Critical Investigation of the Practice of Cognition*. Oxford University Press.
- Dunn, D. S., & Burcaw, S. (2013). Disability Identity: Exploring Narrative Accounts of Disability. *Rehabil. Psychol.*, 58(2), 148–157.
- Dweck, C. S. (2006). *Mindset: The new psychology of success*.
- Ernest, J. B., Reinholz, D. L., & Shah, N. (2019). Hidden competence: Women's mathematical participation in public and private classroom spaces. *Educ. Stud. Math.*, 102(2), 153–172.
- Ericsson, K. A., & (Eds.), S. J. (1991). *Toward a general theory of expertise: Prospects and limits*.
- Feiten, T. E., Peck, Z., Holland, K., & Chemero, A. (2023). Constructive Constraints: On the Role of Chance and Complexity in Artistic Creativity. *Possibility Studies & Society*, 1(3), 311–323.
- Feucht, F. C. (2010). Epistemic climate in elementary classrooms. *Personal epistemology in the classroom: Theory, research, and implications for practice*, 55-93.
- Fischer, M. H., & Brugger, P. (2011). When digits help digits: Spatial-numerical associations point to finger counting as prime example of embodied cognition. *Front. Psychol.*, 2, 260.
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231-235). Hillsdale, NJ: Lawrence Erlbaum.
- Fodor, J. A. (1983). *The Modularity of Mind*. MIT Press.
- Foley, A. E., Herts, J. B., Borgonovi, F., Guerriero, S., Levine, S. C., & Beilock, S. L. (2017). The Math Anxiety-Performance Link: A Global Phenomenon. *Curr. Dir. Psychol. Sci.*, 26(1), 52–58.
- For School Mathematics, N. C. of T. of M. C. on S. (1989). *Curriculum and evaluation standards for school mathematics*. National Council of Teachers of Mathematics.
- Freire, P. (1978). Pedagogy of the Oppressed. In *Toward a Sociology of Education* (pp. 374–386). Routledge.
- Gallagher, S. (2013). The socially extended mind. *Cognitive Systems Research*, 25–26, 4–12.
- Gallagher, S. (2017). *Enactivist Interventions: Rethinking the Mind*. Oxford University Press.
- Gandell, R. (2024). Students' Mathematical Thinking in Movement. *Int. J. Res. Undergrad. Math. Ed.*
- Garofalo and Lester. (1985). *Metacognition, Cognitive Monitoring, and Mathematical Performance*.
- Gerofsky, S. (2015). Approaches to Embodied Learning in Mathematics. In *Handbook of International Research in Mathematics Education* (pp. 60-97). Routledge.

- Gibson, J. J. (1977). The Theory of Affordances. *Hilldale, USA*, 67–82.
- Giguere, M. (2011). Dancing Thoughts: An Examination of Children's Cognition and Creative Process in Dance. *Res. Dance Educ.*, 12(1), 5–28.
- Ginsburg, H. (1997). *Entering the Child's Mind: The Clinical Interview In Psychological Research and Practice*. Cambridge University Press.
- Goldin, G. A. (1997). Chapter 4: Observing Mathematical Problem Solving through Task-Based Interviews. *Journal for Research in Mathematics Education. Monograph*, 9, 40–177.
- Goldin-Meadow, S., Nusbaum, H., Kelly, S. D., & Wagner, S. (2001). Explaining math: Gesturing lightens the load. *Psychological Science*, 12(6), 516–522.
- Gorey, C. (2019). 'People with Disabilities are some of the Best Problem-Solvers in our Society'. Inspirefest.
- Gutiérrez, J. (2018). Math: It's Not What You "Think". In Kay, J. and Luckin, R. (Eds.) Rethinking Learning in the Digital Age: Making the Learning Sciences Count, 13th International Conference of the Learning Sciences (ICLS) 2018, Volume 2. London, UK: International Society of the Learning Sciences.
- Gutiérrez, R. (2013). The Sociopolitical Turn in Mathematics Education. *JRME*, 44(1), 37–68.
- Gutiérrez, R. (2018). Rehumanizing Mathematics: A Vision for the Future. *Invited Speaker at the Latinx in the Mathematics Sciences Conference*.
- Hacques, G., Komar, J., Dicks, M., & Seifert, L. (2021). Exploring to Learn and Learning to Explore. *Psychol. Res.*, 85(4), 1367–1379.
- Halmos, P. R. (1980). The Heart of Mathematics. *Am. Math. Mon.*, 87(7), 519–524.
- Hamraie, A. (2017). *Building access: Universal design and the politics of disability*.
- Harnad, S. (1990). The Symbol Grounding Problem. *Physica D*, 42(1), 335–346.
- Hernandez, G., & Kronsted, C. (2025). Integrating rock climbing into the mathematics curriculum- An enactive affordance-based approach. *The Philosophy of Mathematics Education Journal*, 42.
- Hinz, A. M., Klavžar, S., Milutinović, U., & Petr, C. (2013). *The tower of Hanoi: Myths and maths*.
- Hottinger, S. N. (2016). *Inventing the Mathematician: Gender, Race, and Our Cultural Understanding of Mathematics*. State University of New York Press.
- Hutchins, E. (1996). *Cognition in the Wild*. MIT Press.
- Initiative, C. C. S. S. & Others. (2010). Common Core State Standards for Mathematics. [Http://Www. Corestandards. Org/Assets/CCSSI_Math% 20Standards. Pdf](http://www.Corestandards.Org/Assets/CCSSI_Math%20Standards.Pdf).
- Jasmin, K., & Casasanto, D. (2012). The QWERTY Effect: How Typing Shapes the Meanings of Words. *Psychon. Bull. Rev.*, 19(3), 499–504.

- Karin, M., & Bollinger, L. (2020). Disability rights: Past, present, and future — “A roadmap for disability rights.” *Veterans & Military Law & Policy eJournal*. https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/udclr23§ion=5&casa_token=c-g_pfi6T9MAAAAA:0THedQMCTOdXBHUXVSXXpONYyJJePiAUX0-on-hw7NFgiY_N8bZ0JU2A4hJlgSnszd0GcvnJUE
- Kapur, M. (2008). Productive failure. *Cognitive Instruction*, 26(3), 379–424.
- Kemp, A. (2021). Nature's Problem Solvers Should Be on Your Hire List. In *Diversiti.uk*. <https://diversiti.uk/2021/09/16/natures-problem-solvers-should-be-on-your-hire-list-disability>
- Kiverstein, J., van Dijk, L., & Rietveld, E. (2021). The Field and Landscape of Affordances: Koffka's Two Environments Revisited. *Synthese*, 198(S9), 2279–2296.
- Kotovsky, K., Hayes, J. R., & Simon, H. A. (1985). Why are some problems hard? Evidence from Tower of Hanoi. *Cognitive psychology*, 17(2), 248–294.
- Kronsted, C., & Gallagher, S. (2021). Dances and Affordances: The Relationship between Dance Training and Conceptual Problem-Solving. *Journal of Aesthetic Education*, 55(1), 35–55.
- Kronsted, C., Kugele, S., Neemeh, Z. A., Ryan, K. J., Jr, & Franklin, S. (2022). Embodied intelligence: Smooth coping in the Learning Intelligent Decision Agent cognitive architecture. *Front. Psychol.*, 13, 846931.
- Lakoff, G., & Johnson, M. (2024). *Metaphors we live by*. University of Chicago press.
- Lakoff, G., & Nuñez, R. E. (2000). *Where Mathematics Come From How The Embodied Mind Brings Mathematics Into Being*. Basic Books.
- Lambert, R. (2018). “Indefensible, illogical, and unsupported”; countering deficit mythologies about the potential of students with learning disabilities in mathematics. *Education Sciences*, 8(2), 72.
- Lambert, R., & Tan, P. (2020). Does disability matter in mathematics educational research? A critical comparison of research on students with and without disabilities. *Math. Educ. Res. J.*, 32(1), 5–35.
- Landy, D., & Goldstone, R. L. (2007). How Abstract is Symbolic Thought? *J. Exp. Psychol. Learn. Mem. Cogn.*, 33(4), 720–733.
- Lave, J., Smith, S., & Butler, M. (1988). Problem solving as an everyday practice. In *The teaching and assessing of mathematical problem solving* (pp. 61–81). Routledge.
- Lester, F. K., & University-Bloomingt, I. (1994). *Musings about Mathematical Problem-Solving Research: 1970–1994*.
- Lewis, T. (2022). Working Definition of Ableism. In *Talialewis.com*. <https://www.talialewis.com/blog>
- Leyva, L. A., McNeill, R. T., Marshall, B. L., & Guzmán, O. A. (2021). “It Seems Like They Purposefully Try to Make as Many Kids Drop”: An Analysis of Logics and Mechanisms of Racial-Gendered Inequality in Introductory Mathematics Instruction. *J. Higher Educ.*, 92(5), 784–

- Leyva, L. A. (2021). *Black Women's Counter-Stories of Resilience and Within-Group Tensions in the White, Patriarchal Space of Mathematics Education*.
- Liu, R. (2011). Attitudes toward math, memorization, and math proficiency (Doctoral dissertation, Kean University).
- LópezLeiva, Carlos A., and Lena Licón Khisty. (2014). "Juntos Pero no Revueltos": *Microaggressions and Language in the Mathematics Education of Non-Dominant Latinas/os*.
- Macrine, S. L., & Fugate, J. M. B. (Eds.). (2022). *Movement matters: How Embodied Cognition Informs Teaching and Learning*. The MIT Press.
- Mammarella, I. C., Caviola, S., & Dowker, A. (2019). *Mathematics Anxiety: What Is Known, and What is Still Missing*. Routledge.
- Mason, J. (1989). Mathematical abstraction as the result of a delicate shift of attention. *For Learn. Math.*, 9(2), 2–8.
- Mason, J., & Spence, M. (1999). Beyond Mere Knowledge of Mathematics: The Importance of Knowing-to Act in the Moment. In D. Tirosh (Ed.), *Forms of Mathematical Knowledge* (pp. 135–161). Springer Netherlands.
- Mason, J. (2023). Specifying Defining, Generalising and Abstracting Mathematically All Seen as Subtly Different Shifts of Attention. In M. A. V. Bicudo, B. Czarnocha, M. Rosa, & M. Marciniak (Eds.), *Ongoing Advancements in Philosophy of Mathematics Education* (pp. 103–124). Springer International Publishing.
- Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, National Research Council, & National Academy of Sciences. (2002). *Helping children learn mathematics* (J. Kilpatrick & J. Swafford, Eds.). National Academies Press.
- McGee, E. O., & Martin, D. B. (2011). "You would not believe what I have to go through to prove my intellectual value!" Stereotype Management among Academically Successful Black Mathematics and Engineering Students. *Am. Educ. Res. J.*, 48(6), 1347–1389.
- Meyer, A., Rose, D. H., & Gordon, D. (2014). *Universal design for learning: Theory and practice*. CAST Professional Publishing.
- Miles, M. B., Michael Huberman, A., & Saldana, J. (1994). *Qualitative Data Analysis*. SAGE.
- Mingon, M., & Sutton, J. (2021). Why Robots Can't Haka: Skilled Performance and Embodied Knowledge in the Māori Haka. *Synthese*, 199(1–2), 4337–4365.
- National Research Council. (2012). *Education for life and work: Developing transferable knowledge and skills in the 21st century*. The National Academies Press.
<https://doi.org/10.17226/13398>

- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*.
- National Council of Teachers of Mathematics. (2020). *Catalyzing change in middle school mathematics: Initiating critical conversations*. Author.
- National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics*.
<https://www.corestandards.org/Math/>
- Nemirovsky, R. (2003). Three conjectures concerning the relationship between body activity and understanding mathematics. *Proceedings of PME*.
- Newell, K. (1986). Constraints on the development of coordination. *Motor Development on Children: Aspects of Coordination and Control*, 341–360
- Newen, A., De Bruin, L., & Gallagher, S. (2018). *The Oxford Handbook of 4E Cognition*. Oxford University Press.
- Neveu, M., Schwartz, C., Vossius, L., & Rousselle, L. (2023). Contribution of finger gnosis and fine motor skills to early numerical and arithmetic abilities: New insights from 3D motion analyses. *Dev. Psychol.*, 59(12), 2356–2366.
- Nguyen, C. T. The Philosophers' Magazine. (2017). The aesthetics of rock climbing. *Philosophical Magazine*, 78, 37–43.
- Noland, C. (2009). *Coping and Choreography*.
- Núñez, R. E. (2008). 17—Mathematics, the Ultimate Challenge to Embodiment: Truth and the Grounding of Axiomatic Systems. In P. Calvo & A. Gomila (Eds.), *Handbook of Cognitive Science* (pp. 333–353). Elsevier.
- Oliver, M. (1990). *The politics of disablement*. Macmillan.
- Padilla, A., & Tan, P. (2019). Toward Inclusive Mathematics Education: A Metatheoretical Reflection about Countering Ableism in Mathematics Standards and Curriculum. *Int. J. Qual. Stud. Educ.*, 32(3), 299–322.
- Papert, S. (1980). *Mindstorms: Children, Computers, and Powerful Ideas*. Basic Books.
- Perkins, D., Jay, E., & Tishman, S. (1993). Beyond abilities: A dispositional theory of thinking. *Merrill-Palmer Quarterly*, 39(1), 1–21.
- Philipp, R. A. (1992). The Many Uses of Algebraic Variables. *MT*, 85(7), 557–561.
- Pike, K. L. (1967). *Etic and emic standpoints for the description of behavior*.
- Polya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method*. Princeton University Press.
- Preston, L. (2001). Seeing the Unseen: A Rock Climbing Experience. *Journal of Outdoor and Environmental Education*.

- Rasmussen, C., Stephan, M., & Allen, K. (2004). Classroom mathematical practices and gesturing. *The Journal of Mathematical Behavior*, 23(3), 301-323.
- Resnick, L. B. (Ed.). (1976). Metacognitive Aspects of Problem-Solving. *The Nature of Intelligence* (p. Flavell, J. H.). Lawrence Erlbaum Associates.
- Retief, M., & Letšosa, R. (2018). Models of Disability: A Brief Overview. *HTS Teol. Stud. / Theol. Stud.*, 74(1).
- Rietveld, E., & Kiverstein, J. (2014). A Rich Landscape of Affordances. *Ecol. Psychol.*, 26(4), 325–352.
- Ritchhart, R. (2004). *Intellectual character: What it is, why it matters, and how to get it*.
- Rucińska, Z. (2021). Enactive planning in rock climbing: Recalibration, visualization and nested affordances. *Synthese*, 199(1–2), 5285–5310.
- Saia, T. (2019). *Disability as an identity: Disability cultural centers in higher education* [PhD Thesis]. search.proquest.com.
- Schoenfeld, A. H. (1987). *Cognitive Science and Mathematics Education*. Psychology Press.
- Schoenfeld, A. H. (2016). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Reprint). *Journal of Education*, 196(2), 1–38.
- Schoenfeld, A. H. & Shelley V (Eds.). (2013). MAKING MATHEMATICS AND MAKING PASTA: FROM COOKBOOK PROCEDURES TO REALLY COOKING. In *Thinking Practices in Mathematics and Science Learning* (0 ed., pp. 309–330). Routledge.
- Seifert, L., Cordier, R., Orth, D., Courtine, Y., & Croft, J. L. (2017). Role of Route Previewing Strategies on Climbing Fluency and Exploratory Movements. *PLoS One*, 12(4), e0176306.
- Shakespeare, T. (2013). The Social Model. In L. J. Davis (Ed.), *The Disability Studies Reader* (4th ed, pp. 214–221). Routledge.
- Shapiro, L. (2019). *Embodied Cognition*. Routledge.
- Sheets-Johnstone, M. (2011). The primacy of movement. *The Primacy of Movement*, 1–606.
- Sheets-Johnstone, M. (2012). From movement to dance. *Phenomenol. Cognitive Sci.*, 11(1), 39–57.
- Shen, K., Crossley, J. N., & Lun, A. W. C. (1999). *The nine chapters on the mathematical art: Companion and commentary*. Oxford University Press.
- Siebers, T. (2008). *Disability theory*. University of Michigan Press.
- Simon, H. A. (1973). The structure of ill structured problems. *Artificial intelligence*, 4(3-4), 181-201.
- Sixtus, E., Krause, F., Lindemann, O., & Fischer, M. H. (2023). A sensorimotor perspective on numerical cognition. *Trends Cogn. Sci.*, 27(4), 367–378.

- Smith, M. U. (2012). *Toward a Unified Theory of Problem Solving: Views From the Content Domains*. Routledge.
- Sojitra, V. (2023). "I say the word "Disabled" loud and proud. In *Instagram*.
<https://www.instagram.com/p/CvPoWYOr3TI/>
- Sowder, L. (1985). Cognitive Psychology and Mathematical Problem Solving: A Discussion of Mayer's Paper 1. In *Teaching and Learning Mathematical Problem Solving* (pp. 139–145). Routledge.
- Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real-world connections: Metacognitive characteristics of secondary students. *Educational Studies in Mathematics*.
- Stains, M., Harshman, J., Barker, M. K., Chasteen, S. V., Cole, R., DeChenne-Peters, S. E., ... & Young,
- Strauss, A., & Corbin, J. M. (1990). *Basics of Qualitative Research: Grounded Theory Procedures and Techniques*. SAGE Publications.
- Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 273 – 285). Thousand Oaks, CA: Sage Publications.
- Suchman, L. A. (1987). *Plans and situated actions: The problem of human-machine communication*. Cambridge university press.
- Swain, J., French, S., & Cameron, C. (2003). *Controversial Issues In A Disabling Society*. McGraw-Hill Education (UK).
- Tan, P., Lambert, R., Padilla, A., & Wieman, R. (2019). A disability studies in mathematics education review of intellectual disabilities: Directions for future inquiry and practice. *Journal of Mathematical Behavior*, 54(100672), 100672.
- Tilman, D., Cassman, K. G., Matson, P. A., Naylor, R., & Polasky, S. (2002). Agricultural sustainability and intensive production practices. *Nature*, 418(6898), 671-677.
- Tobias, S. (1993). *Overcoming math anxiety*. WW Norton & Company.
- Toro, J., Kiverstein, J., & Rietveld, E. (2020). The Ecological-Enactive Model of Disability: Why Disability Does Not Entail Pathological Embodiment. *Front. Psychol.*, 11, 1162.
- Travis, M. (2024). 3 Lessons For Employers On Disability Inclusion In A Post-Pandemic Workplace. *Forbes Magazine*.
- van Dijk, L., & Rietveld, E. (2016). Foregrounding Sociomaterial Practice in Our Understanding of Affordances: The Skilled Intentionality Framework. *Front. Psychol.*, 7, 1969.
- Sepehr Vakil, Maxine McKinney de Royston, Na'ilah Suad Nasir & Ben
- Kirshner (2016) Rethinking Race and Power in Design-Based Research: Reflections from the Field, *Cognition and Instruction*, 34:3, 194-209, DOI: 10.1080/07370008.2016.1169817

- Tancredi, S., Abdu, R., Balasubramaniam, R., & Abrahamson, D. (2022). Intermodality in multimodal learning analytics for cognitive theory development: A case from embodied design for mathematics learning. In *The Multimodal Learning Analytics Handbook* (pp. 133–158). Springer International Publishing.
- Varela, F. J., Rosch, E., & Thompson, E. (1992). *The Embodied Mind: Cognitive Science and Human Experience*. MIT Press.
- Vilar, L., Araújo, D., Davids, K., & Button, C. (2012). The role of ecological dynamics in analysing performance in team sports. *Sports Med.*, 42(1), 1–10.
- Vo, L. T. (2024, June 17). Across or Up, Rock Climbers and Crossword Solvers Puzzle Alike. *The New York Times*. Retrieved from <https://www.nytimes.com/2024/06/17/crosswords/rock-climbing-crossword-puzzles.html>
- Wagner, D., & Borden, L., (2020). Conceptions of equitable mathematics teaching practices. In E.A. Silver & F.K. Lester (Eds.), *Reimagining equitable mathematics instruction for K-12 teachers: The 93rd Yearbook of the National Council of Teachers of Mathematics* (pp. 45-59).
- Weber, R. P. (1990). *Basic Content Analysis*. SAGE.
- Welch, S. (2022). *Choreography as Embodied Critical Inquiry*. Springer International Publishing.
- Wendell, S. (1996). *The rejected body: Feminist philosophical reflections on disability*. Routledge.
- Willoughby, M., Hudson, K., Hong, Y., & Wylie, A. (2021). Improvements in motor competence skills are associated with improvements in executive function and math problem-solving skills in early childhood. *Dev. Psychol.*, 57(9), 1463–1470.
- Woods, H., & Thomas, D. (2003). *Working with People with Learning Disabilities: Theory and Practice*. Jessica Kingsley Publishers
- Wong, A. (2020). Random House.
- Yin, R. K. (2008). *Case study research: Design and methods (4th ed.)*. SAGE Publications.
- Zazkis, R., & Hazzan, O. (1998). Interviewing in mathematics education research: Choosing the questions. *The Journal of Mathematical Behavior*, 17(4), 429–439. doi: 10.1016/S0732-3123(99)00006-1

Appendix A: Interview Protocols

Route-Setter Focus Group Protocol

Introduction

- Welcome all participants to the focus group.
- Briefly introduce the purpose of the session: to explore and understand the problem-solving methodology of route setters in rock climbing. As you may know, I am doing my dissertation on the parallels between problem-solving in rock climbing and problem-solving in mathematics. Specifically, I aim to understand the analogies strategies through the lens of disabled climbers.
- Emphasize the importance of their insights in understanding how they set routes. They are the best setters I have ever encountered and I want to learn from them.
- Assure participants of confidentiality and encourage open and honest discussion.

Warm-Up Questions

- Could each of you briefly introduce yourselves, mentioning your experience as route setters and climbing in general?
- What initially attracted you to route setting in rock climbing?

Main Discussion

- What is your philosophy of climbing?
- What are your thoughts on gym climbing and outdoor climbing? What are the parallels and what are the differences?
- As a route setter, you create problems that other people must solve. What's your process like?
- What factors do you consider when selecting holds and determining the difficulty level of a route? What distinguishes an easier route from a harder route?

- I think you all do this very well... How do you adapt your approach when setting routes for different skill levels or body types?
- What do you look for when you are forerunning routes?
- Think about an advanced climber— what qualities do you associate with them? How do you characterize an expert versus a novice climber?
- How do you incorporate feedback from climbers into your route-setting practice?
- What strategies do you use to keep routes fresh and engaging for repeat climbers?
- What makes a route unique?
- What characterizes a difficult route?
- Do you consider what you guys do connected to math problem-solving to any degree?
- Do you see any connections between math problem-solving and problem-solving in climbing?
- Modifications (changing something about what you are doing) vs accommodations-?
- As route setters, how do you consider the needs and abilities of the disabled climbing community when setting routes in the gym? Are there specific adjustments or considerations you make to ensure inclusivity and accessibility for climbers with disabilities?
- In what ways could you imagine a more accessible climbing environment?

Closing

- Is there anything else you would like to share about your approach to route setting or any insights you believe are important for me to consider?
- Thank the participants for their valuable contributions and conclude the focus group.

Semi-Structured Climber Protocol

Introduction

- Welcome the participant and thank them for their willingness to participate.
- Explain the purpose of the interview: to better understand their experiences with disability, math, rock climbing, and problem-solving.
- Assure confidentiality and explain how the information will be used.

Participant Background

Disability

- Can you briefly describe your disability and how it affects your daily life?

Experience with Math

- What is your relationship with math? How do you feel about your relationship with math?
- Can you share any positive experiences or challenges related to learning math?
- How are you using math in your day-to-day life?

Rock Climbing Experience

- How long have you been climbing? How often do you climb? Mostly indoors or outdoors?
- How did you become interested in rock climbing?

Problem-Solving in Climbing

- How do you define problem-solving in general?
- How do you define problem-solving in math?
- How do you approach problem-solving while climbing?
- Can you describe your thought process when planning a climbing route?

Integration of Math and Problem-Solving in Climbing

- Now that we have discussed problem-solving in rock climbing, Do you see any connections between math problem-solving and problem-solving in climbing?

- Do you think your mathematical skills influence your approach to climbing?
- Have you ever used mathematical thinking to solve climbing-related problems?

Reflections on Disability, Math, and Climbing

- In what ways do you feel supported in the climbing gym? In what ways do you not?
- In what ways has climbing impacted your life?
- In what ways has climbing impacted your relationship with your disability or your perception of it?
- How might your disability affect your problem-solving abilities in your day-to-day life?
- Do you believe that climbing has influenced your problem-solving abilities outside of the climbing context?
- What are a few words that would describe the lessons that climbing has taught you?

Closing

- Is there anything else you would like to share about your experiences with disability, math, rock climbing, or problem-solving?
- Thank the participant for their time and valuable insights and offer any further assistance or resources if needed.

Stimulated Recall Climber Protocol

Introduction

- Welcome the participant and thank them for their willingness to participate.
- Explain the purpose of the interview: to better understand their experiences with disability, math, rock climbing, and problem-solving.
- Assure confidentiality and explain how the information will be used.

Introduction to Task

- Here is a picture of the climb you did. Let's talk about it.
- What were your expectations or goals going into this climb?
- How did you feel physically and mentally before starting the climb?
- **play salient moments from video.

Video Playback

- As you watch this part of the climb, what thoughts come to mind?
- What were you focusing on during this particular moment?
- Did anything unexpected happen during this part of the climb?

Stimulated Recall

- Can you describe what you were feeling or thinking as you encountered [specific moment] in the video?
- Where was your focus/attention at this moment?
- How did you approach the crux presented in that moment?
- Were there any particular strategies or techniques you used to overcome it?

Probe for Further Information

- Can you elaborate on the decision-making process behind [specific action]?
- What factors influenced your choices during [specific moment]?

- How did you adapt your approach based on the feedback you were receiving from your body or from the hold?

Reflective Questions

- Looking back on this climb, what aspects do you feel particularly proud of?
- Are there any areas where you felt you could have performed better or differently?
- How did this climbing session compare to your previous experiences?
- Is there anything else you would like to add or discuss about your climbing experience?
- Thank you for sharing your insights and experiences. Your feedback is incredibly valuable for my study!

Math Task-Based Clinical Math Interview Protocol

Introduction

- Welcome the participant and thank them for their willingness to participate.
- Explain the purpose of the interview: to better understand their experiences with disability, math, rock climbing, and problem-solving.
- Assure confidentiality and explain how the information will be used.

Introduction to the Task

- Can you describe your experience with problem-solving in mathematics?
- How do you typically approach mathematical problems that require creative thinking?
- Are there any strategies or techniques you find particularly effective in solving mathematical problems?
- *** Give the participant the math problem

Mathematical Problem-Solving

- As you work through this problem, can you walk me through your thought process?
- What steps are you taking to approach the problem?
- Are there any alternative methods or approaches you're considering?

Reflection and Discussion

- What challenges did you encounter while solving this problem?
- How did you decide which strategies to use?
- Were there any moments where you had to adapt your approach?
- Can you identify any similarities between your problem-solving approach in mathematics and your approach to climbing routes?

Repeat Process

- How does your problem-solving approach differ when faced with a problem of a different type or complexity?

- Are there any specific problem-solving strategies you tend to use more frequently?
- Can you recall a time when you applied a particularly creative solution to a mathematical problem?
- Do you notice any similarities in the way you approach different types of mathematical problems and climbing routes?

Analogies to Rock Climbing Routes

- Can you describe your thought process when planning a climbing route?
- Are there any similarities between the challenges you face in solving mathematical problems and navigating climbing routes?
- How do you adapt to unexpected obstacles or changes in a climbing route, and do you see parallels to problem-solving in mathematics?
- Can you provide specific examples where insights from one domain (mathematics or climbing) have informed your approach in the other domain?

Closing

- Is there anything else you would like to share about your experiences with mathematical problem-solving?
- Thank you for your valuable insights and participation in this interview. Your reflections will contribute to our understanding of cognitive processes in mathematics and climbing.

Math Stimulated Recall Interview Protocol

- Briefly review the purpose of the study and the stimulated recall interview.
- Reiterate confidentiality and consent procedures.
- Explain that they will be solving a math puzzle and then discussing their experience.

Stimulated Recall Interview

- Review salient moments with video recording with the participant.
- Pause the video at key moments to ask the participant about their thoughts, feelings, and decisions during the problem-solving process.
- Use probing questions to encourage them to elaborate on their strategies, challenges, and any connections to their rock climbing strategies.

Interview Guide (Example Questions):

- Can you describe what you were thinking and feeling when you first encountered the puzzle?
- What were your initial impressions of the puzzle? Did it remind you of any other experiences?
- What strategies did you use to approach the puzzle?
- Can you describe any specific moments where you felt stuck or unsure of how to proceed? How did you overcome those challenges?
- Did you notice any patterns or relationships within the puzzle? How did you use them to solve it?
- Did you find yourself using any strategies similar to those you use in rock climbing?
- Did you experience any "aha" moments during the process?
- What did you learn from this experience?

Contact Summary Form

Interview Contact Summary Form

Interviewer Name: Gabriela Hernandez

Date of Interview: [Date]

Interviewee Information:

Name: [Interviewee's Name]

Contact Information: [Phone Number/Email Address]

Role/Occupation: [Interviewee's Position or Affiliation]

Interview Details:

Interview Topic: [Brief Description of the Interview Topic]

Purpose of Interview: [Objective or Goal of the Interview]

Duration of Interview: [Estimated Duration]

Location: [Where the Interview Took Place, e.g., In-Person, Phone, Video Call]

Scheduled Date and Time: [Scheduled Date and Time of the Interview]

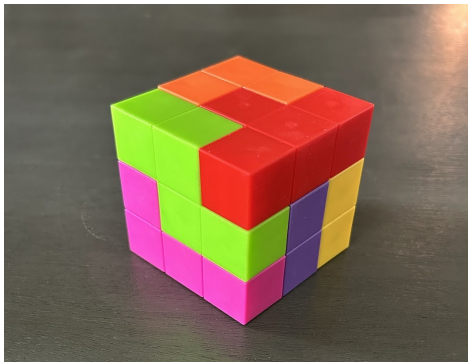
Salient Themes/ Notes

Appendix B: Mathematical Tasks

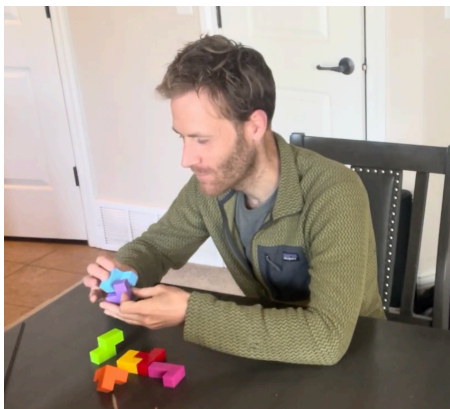
Soma Cube



Refer to above picture. This is how the problem was presented to participants: six magnetic pieces spread out. They were told that the final solution would be $3 \times 3 \times 3$. Participants were told all pieces must be used.

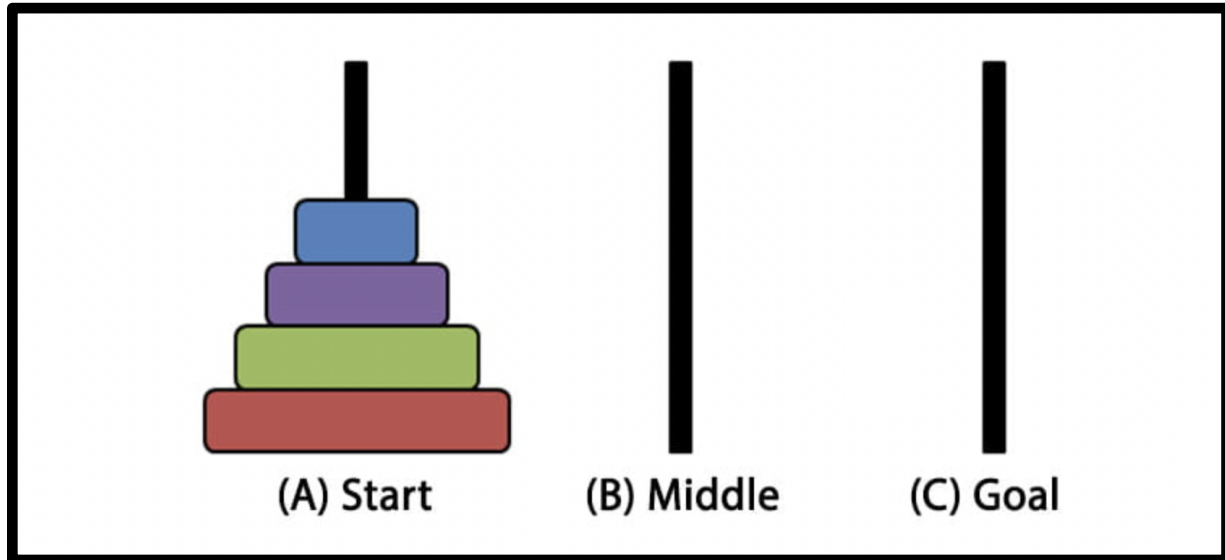


Refer to above picture. This is one of 240 possible solutions for the SOMA cube.



Kenneth solving the SOMA cube by first working with the blue and purple pieces

Towers of Hanoi



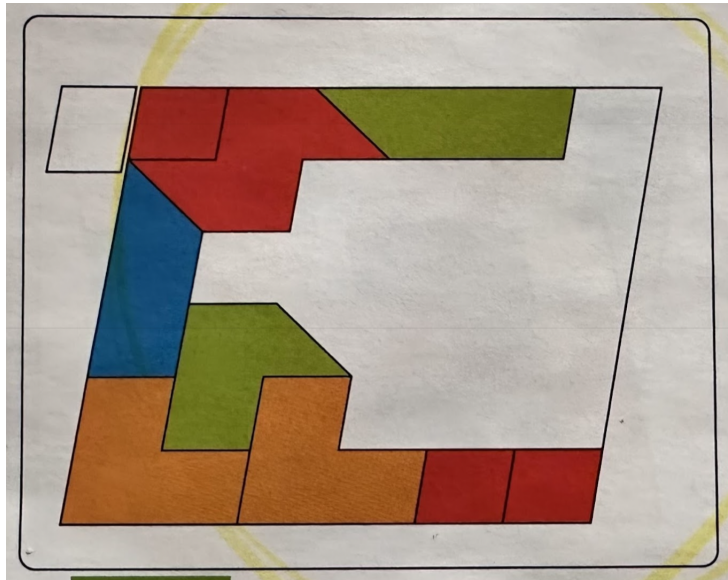
Participants were given a four disc problem to start. All disks are in different sizes and stacked in a way that a smaller disk is on top of the larger.

The goal is to move all the disks from the first spot to the third spot so they are stacked in the same way as in the beginning (all discs from Start rung to Goal rung) However, there are several constraints.

- First is that the disks can be moved only one at the time.
- Second is that only the disk at the top can be moved on another spot
- Third is that the a larger disk can't be moved on top of the smaller disk (just on top of disk larger than it) or on the empty spot.)
- You want to accomplish this in the minimal number of moves

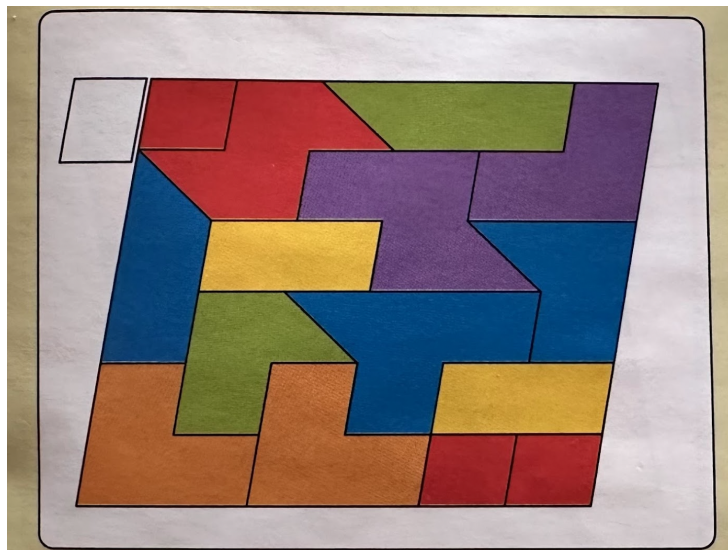
Solution: The formula used to calculate the minimum number of moves is $(2^n)-1$, where n is a number of disks.

Tangram



Participants were presented with a wooden frame containing seven geometric pieces arranged in an initial configuration (see above picture).

Participants were given the remaining six pieces to complete the puzzle. The goal was to arrange all pieces to fill the frame completely without gaps or overlaps (bottom image is completed solution).



Appendix C: Climbing Glossary

Term	Explanation
Backing-off	Taking a step back from a climbing sequence to reassess the movement. This is often done in a somewhat restful position.
Belaying	Managing the safety rope system to protect a climber from falling while they ascend or descend the wall.
Beta	The specific/subjective sequence for a section of a route (one possible way of solving of many for a problem).
Bouldering	A type of climbing where climbers tackle short, challenging routes, known as problems. Bouldering is done without a harness or rope. In a gym, these routes are often 10-15 feet high, and safety is ensured by thick pads below that cushion falls.
Caller	A sighted guide who provides verbal instructions to a blind climber, describing the route, holds, and necessary movements.
Campusing	Climbing with only upper body, not using your feet to climb
Crux	The hardest parts of the route.
Cruxing Out	When the climber is actively at the hardest part of the climb.
jug	A type of climbing hold that is easy to grip, hang off of, and conserve energy on. A jug provides possibilities for further movement upwards.
jug-able	To grab a hold optimally to conserve energy while grabbing it.
Lead- climbing	A type of climbing where climbers clip into quickdraws for safety. If an individual falls while lead climbing, they will fall to their last point of safety.
Onsight	When a climber tries to climb a route without knowing any of the sequences of

	movement
Projects/Projecting	A route at a climber's limit that takes an extended amount of time to send.
Route	A pre-established climb/path in the gym, designated by different colored holds and varying in difficulty.
Send	Completing a route without falling.
Take	The act of the belayer taking slack in the rope which is often followed by resting.
Top-roping	A type of climbing where a rope runs through an anchor at the top of the route, and the climber is attached to the rope via a harness.

Appendix D: Problem-Solving Strategy Codebook

Note: codes in pink at the bottom are from the mathematics portion of the study

Code	Description	Example(s)
PROBLEM-SOLVING STRATEGIES	mental and physical tools individuals leverage to overcome challenge	
Resting	When they explicitly refer to rests or recovery	"I stayed at that rest as a long as possible until I could get something back"
Pacing	this refers to instances when they talk about speed, tempo, changing pace	"And yeah, the rest of it was just like sprint to the finish and get as much back as you can" "I'd say the second thing I look for is knowing where the tempo changes."
Subset of Pacing: Taking your time	mentions of patience; not rushing; stopping and having to think about your next sequence	"I mean, it forces me to like: like slow down and like feel the move"
IDENTIFYING Constraints: Cruxes	Explicit mentions of cruxes (the challenging parts)	"And then I knew that bad hold would be the crux"
IDENTIFYING Constraints: in general	General mentions of constraints in problem-solving	"Every problem has constraints, I just need to identify them."
IDENTIFYING CONSTRAINTS: Knowing limits	the notion of knowing when to stop	".... knowing there's like a certain kind spatial and like bodily awareness, to know like where my limits are."
IDENTIFYING CONSTRAINTS: pain	When they mention they are in pain as they are climbing	" I had to stop because the move hurt too much."
IDENTIFYING CONSTRAINTS: fatigue	very explicit mentions of fatigue or of getting tired on the wall; or "fighting the pump"	"When I was warming up, I got super pumped right away."
IDENTIFYING CONSTRAINTS: time	Mentions of only having so much time; time as a constraint	"We only have so much time, so it impacts how I approach

		the route."
Mind taking over // Higher-order thinking	Mentions of this moment where you cannot rely on your intuition any more— - you have to start being more <i>intentional</i> and think about it more	"then you really have to start thinking about it" "if I can get past like the first half the climb, I don't know exactly like how much power and like energy I'll have left. But like, that's when my my brain really takes into like a higher gear."
IDENTIFYING AFFORDANCES: seeking/considering options	explicit mentions of options in problem-solving	"In the cruxes I do, especially when it comes to clipping stances, : because, like sometimes, there's there's clips where you have to like, really pull in with your right foot or something, but those are a lot more taxing on me. And so sometimes like figuring out a different foot option for my left foot." " Climbing is slow for me— 99 percent of the time you are thinking, am I doing this move or am I doing that move?"
IDENTIFYING AFFORDANCES: different solution paths	When participants mention that there are multiple ways at arriving to a solution	" that is the appeal to climbing, you know, ten people can climb one route and every one does it differently and it might not be noticeable at first. It's in the subtle differences, even if it is not apparent at first. We can all do it differently."
Energy Conservation	mentions of conserving energy, saving energy (before getting to fatigue!)	" I knew I had to save my energy for the next sequence."

Backing Off	mentions of down climbing or backing off to re-evaluate movement or needing to regroup; more broadly, tactical retreat within same attempt	" And then I'm maybe thinking, oh, I can't quite reach the hold that I was going to go to, or I can't hit it at quite the right angle, but I can see that there's a foothold that I could use that is different. But I have to go down and reposition my feet before I move up." "I thought, let's go back, let's regroup, let's see if we can get our act together."
Mental Imagery	mentions of painting a picture or filling in holes; making decisions off incomplete information; simulating movement in your head	"it helps me to fill in the blanks in my environment." "I am imagining the sequence."
Break Down the Problem	when participants explicitly discussing breaking down the problem into smaller parts	"I am breaking up the different crux sections."
Planning Ahead	mapping out the problem before hand, figuring out movement from the ground	"Planning really happens on the ground." " But like, heel hooks and drop knees and stuff, I can read those from the ground"
Planning Throughout	Mentions of planning on the wall and figuring out movement while climbing; changing plans while climbing	"I think when I am resting, I really only look ahead to the next one or two moves. " "Flags are only something that I notice when I am on the wall, actually in the movement itself, not something I can anticipate from the ground."
Subset of planning throughout: collect information	When they mention needing to gather information on wall	"When I run into something that's unexpected, I have to stop and wait. Figure out how to proceed. I need to collect information, and sometimes it

		takes time, just kind of waiting and scanning my eyes back and forth. "
Shifting Perspective	when they mention having some sort of lens and shifting that to solve a problem	"I mean, we are looking at the climb through a different lens, therefore solving problems in different ways."
Feeling the problem	Feeling your way through a problem (how blind people solve rock climbing problems because they cannot plan) ; "searching" "scanning"; seems to be unique to visual impairment	" There's a term we call the yellow paging, where I'm just like searching left like I scan the wall left to right. It's like you like a sweeping motion."
Pattern Recognition	recognizes salient sequences, patterns (this can also be a part of expertise building)	" eventually, I know what holds are better and can identify them immediately."
Reflection	Mentions of looking back on a problem and desiring to understand it better (deliberate recall)	"Yeah, I would say, it's more similar, like, you know, if you make you make errors on the bottom of a route, then you're gonna pay for them later. Getting to the top, or getting to the answer, like, obviously it doesn't always have to be perfect to make it the top. I guess it is the same in math. If you can think about and figure out why your answer doesn't make sense, you can fix it."
Sequencing	linking up very specific movement in a description; phase transitions	Strings of very specific sequencing of movement
Trial & Error	A more exploratory approach; trying something, it didn't work, try again; mentions of "just going for it;	"F around and find out" ; "just playing around and see what happens"
Starting Over	Abandoning current attempt to begin fresh; Distinguished from	"I'm just gonna start completely over from

	"Backing Off" (tactical retreat within same attempt) and "Repeating for Refinement" (voluntary repetition after success).	scratch"
Repeating for Refinement	Voluntary repetition after successful completion to improve performance, find patterns, or achieve mastery	"I want to do it again but try and do it more efficiently"

Appendix E: Disposition Codebook

Note: codes in pink at the bottom are from the mathematics portion of the study

Code	Description	Example(s)
Disposition	Orientations, positioning, beliefs, tendencies, or attitudes that individuals bring to their problem-solving (more temporally stable than affect)	
Connecting to other domains	When participants bring in other experiences from other domains	When participants bring in their experiences from biking, gardening, being a mechanic, product developer, when talking about problem-solving
Survival Mode	When they position themselves in a mode that is grounded in survival	"I am constantly in flight or fight mode, just trying to survive."
Trust	Reliance on your feet, your strength, your endurance, and more generally, your self	"I trust myself that in the moves that are out of the crux sequence, I can find a comfortable position to clip when I am there. "
Adaptability/ flexibility/ fluidity	this idea of going with the flow, being adaptable, malleable	"definitely, I can't think of the word for it right now, but like creativity, and like willing to change your answer, learning to try different ways to go through something. Being adaptable. I guess the opposite of stubborn — - flexible!! I think their demeanor when they're solving problems."
Confidence	When they mention feelings of confidence as they engage in problem-solving	"Yeah, it's a trust thing I've also been learned to just move through the world with confidence, because i literally have to, I have confidence that I am going

		to be able to react accordingly on the fly."
Starting over	when they discuss a stopwatch beginning again//going back to learning how to walk again	"I was reborn when I went blind. and I also am very happy and lucky and fortunate that I have children who also had to like, learning to do things. Cause then, because then well, I'm learning alongside of that. and I have someone to play with me as a little kid learning how to walk, run, ride, a bike, catch a ball." "Once that stopwatch begins again, it feels like you're given a chance to relearn yourself, your body, and how you interact with the world around you. Your preconceptions of how you're supposed to or used to operate gradually falls away and you have to begin remodeling."
Clock is ticking	mentions of losing time, you only have so much time	"I don't have time for it." "Do you want to spend 10 minutes or an hour on a problem? We only have so much time. The clock is always ticking"
Societal Perception/ Comparison	comparisons to non-disabled folk and considering how people perceive them.	"because I thought I'd be so less than, and I was at one point, I've now like made it my goal to be not just equal to an able bodied person, but better, so They can't look down on me."

The belief that there is always a solution	there is always an answer. doesn't need to be done perfectly, but it can ALWAYS be done	"don't say I can't say, how can I?" "so I learned that as long as you keep trying eventually there will be a solution" " And I guess, just known that there's always solutions to your problems. The problems that we do have in our day to day life just aren't that significant.
The belief that there is not always a solution	Not always an answer, and that is okay	" A big theme in my life, not just in climbing but in life, is knowing how to pick your battles. There isn't always an answer, and I have accepted that."
Growth	When participants see problem-solving as an opportunity for growth	"And I would say, that's, when you, when you are not so focused on performance based outcome, where it's just like we're out here to have fun and to explore and to grow."
Acceptance of failure	When they mention failure being a part of the process; not fearing it	"So where you're learning how to solve a problem... I think central to that is like not being afraid to fail."
Persistence	trying over and over again, not giving up (not necessarily related to the belief that there is always a solution)	" And because climbing is so adaptable, it has helped me to figure out how to work around things and keep trying."
Freedom	When participants engage in problem-solving as a means to break away from the constraints of their disability in everyday life	"Spending every waking second in a wheelchair and navigating the world in a wheelchair requires so much more thinking and creativity and problem solving, when i can leave my chair on the ground and leave that part

		of me behind. “
Don't Sweat the Small Stuff	Belief in not worrying excessively about minor details or setbacks	“I just know this isn't that big of a deal, I have bigger things to worry about”
Don't Force It	Belief in being delicate and patient rather than forcing solutions	“There's no reason to try and force it to try and make it happen. I like to take a more gentle approach”
Process Orientation	Valuing the journey and process over the final outcome	I like the small victories, it's not about achieving some huge goal. It is more about the process
Outcome Orientation	Focusing on outcomes rather than understanding why a solution works	“I'm not interested in how we get the solution--if it works, it works.”
Preference for Structure	Preference for guidelines, rules	“I like guidelines when I solve problems”

Appendix F: Affect Codebook

Note: codes in pink at the bottom are from the mathematics portion of the study

Code	Description	Example(s)
Affect	The subjective experience of emotions	
Empathy	Mentions of empathy or considering the needs of others when problem-solving	"Okay, if if we're thinking about this like problem-solving in design, right? Design is based on empathy and like your ability to to understand where someone else is coming from."
Patience	When participants mention patience being a part of the problem-solving process	"Problem-solving in a comp to me looks like being patient and listening to my caller."
Relief	When participants mention particular moments of relief when completing a sequence; desiring relief when in a sequence	"...I can't see relief, cause there's a few times that I'll even say, do I have relief coming soon? Because I'm dying!"
Fear	Mentions of fear when problem-solving; overcoming fear	"I have always been afraid of heights, so climbing was a great challenge for me."
Frustration	When participants express frustration or moments of being disgruntled in their process of problem-solving	"I now have a problem to solve. I then took it, as I have 2 options here, I can get frustrated and pissed off, that I have a new problem to solve, or I can get excited, that I'm now given a new problem to solve and make it fun. Like math, math can be fun or you can be irritated by it."
Play/joy/fun	mentioning the fun or playfulness of	"How I feel after climbing a

	problem-solving; seeing the joy in it	route... if it's fun and challenging, I am happy."
Being Present	Mentions of presence/emotion while engaging in problem-solving	"I think something I learn from climbing...it's a concentrated and isolated experience of being on the wall. There's a lot there to learn in that present moment." "I think Towers of Hanoi is more engaging than Soma"
Empowerment	When participants discuss moments of empowerment when overcoming a challenge	"It is empowering for me to get to the top of a climb"
Overwhelm	Feelings of being overwhelmed by the task or situation	"I just need a minute. I am feeling super overwhelmed by all the options right now."
Pride	Feelings of pride or accomplishment; Captured participants' emotional responses to successful completion of a task	"Yeah, I am actually proud of myself for doing that!"
Excitement	Explicit mentions of excitement or enthusiasm	"I can feel myself getting excited as I move through this puzzle and and the checkpoints."

Appendix G: Expertise Building Codebook

Code	Description	Example(s)
Expertise Building	The development of knowledge that contribute to proficiency in a certain domain.	
Skill Acquisition	The development of a certain set of skills over time.	"I would say, the first thing I look for is like any really hard right foot moves, specifically like right foot clips...this is something that comes with time with route reading"
Improvement	Mentions of getting better over time; improving performance over time	"I don't think I do great at reading routes... I have gotten better over time though." "I have developed strength now, so I don't need a walking stick any more. My ankles are so strong now."
Past Experiences	Drawing from past experiences; an accumulation of experiences that contribute to knowing	"Yeah, because I think if you don't have the experience, it's very difficult to figure out the movement... Yeah, you need that knowledge base to pull from." "So yeah, solving problems and situations that I'm unfamiliar with or even like topics, I'm unfamiliar with just having this really big like deep experience base of of having solved a lot of problems"
Intuition/ Optimal Grip/ Smooth Coping	A form of knowing or understanding something without conscious reasoning. It can involve quick judgments or insights that arise "without thought"	"on the fly...Without even thinking about it"; "My body is problem-solving for me"; "Mind-body-flow" "My body knows how to

		react without me telling it how to" "The body is switching over to the drivers seat when I'm climbing."
Pattern Recognition / Memorization	Quickly identifying relevant information that aids with problem-solving	"... But now my brain has memorized, when I see this hold, I now know how to use it."
Problem-Solving is a Muscle	When participants explicitly connect problem-solving as a muscle that can strengthen over time	"Problem-solving is like a muscle. If you let yourself fall into a pit of intuition and assumption, it atrophies." "Being disabled... I have to flex the problem-solving muscle more..." "Yeah, for example, like I did so many bicep curls, now i can lift anything I want."

Appendix H: Environmental Factors Codebook

Note: codes in pink at the bottom are from the mathematics portion of the study

Code	Description	Example(s)
Environmental Factors	The role of the surrounding environment in shaping their engagement with problem-solving	
Safety	Concerns about protection on routes, the potential for falls, and the high stakes associated with certain climbing situations	"When I am climbing outside, I'm assessing rock quality, like just assessing safety, too. I think. Like, when I'm climbing outside, I'm definitely thinking more about safety." "In those moments in climbing when you are so terrified, you then have to use that fear as fuel to get you through to safety. It definitely impacts the way I make decisions on the wall."
Unaccommodating Environment	When participants mention that the environment they are in is made without them in mind; when they mention that the environment is constrained due to their disability	"I think also people with disabilities are usually already quite good at problem solving, because they, because they have to figured out how to move themselves through the world because the world is not designed for them"
Community	When participants talk about problem-solving as a social process; when they discuss the importance of community in their domain	"Problem-solving is about how my past experiences interact with other people. And like me, me, specifically watching how they interact with other people and the world. Me specifically watching how they interact

		with the world.”
Context Matters	When participants discuss how problem-solving differs in different contexts (i.e outside, competition climbing, and gym climbing)	“ I think it's much more difficult to break beta bouldering than it is on rope climbing. I also feel like I boulder way harder outside than I do inside I think that is because I am an amputee and the setting of boulders inside force a specific movement, whereas climbing outdoors, there is space for adaptation. “
Environmental Feedback	Progress markers, checkpoints, and verification mechanisms provided by the problem environment	"there's like, obviously like a visual checkpoint knowing that like once you get like the bottom piece over to like the last section, you know that you're sort of on the right track"
Decision Paralysis (Affordance overload)	Environmental condition created when problem space presents overwhelming, undifferentiated options; focuses on how the environment's structure—or lack thereof—inhibits action selection.	“There's too many options, I don't know what to do next!"
Environmental Stability vs Dynamism	Characterization of whether the problem space remains fixed (stable) or changes (dynamic) during solving.	The rock climb isn't moving, I mold around the rock when I climb. But with this puzzle, it needs to be molded around itself. This can change in so many ways—I move this one piece one way, and now it just created so many different scenarios. “

Appendix J: Pictures of Different Types of Climbing



Rock Climbing Type.

Top left to bottom right: Conglomerate; Technical Face Climbing; Red River Gorge; Slab Climbing; Jug Haul