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**ESSAYS ON THE COSTS AND BENEFITS OF LONG TERM INFLATION**

A dissertation submitted in partial satisfaction of the requirements for the degree of

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in

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by

**Benjamín García**

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# Table of Contents

# List of Figures

# List of Tables

**Abstract**

Essays on the costs and benefits of long term inflation

by

Benjamín García

In this dissertation I empirically quantify some of the costs and benefits of a non-zero level of inflation. On the benefits side, I measure inflation's impact on reducing the probability of the ZLB constraining the central bank's decisions. Regarding the welfare costs, I focus on how the reduction of money holdings due to inflation can have real costs in terms of consumption, output and employment.

In the first essay, using a Time Varying Parameters Vector Auto Regression (TVP-VAR) framework, I construct an index that measures the probability of the nominal interest rate hitting the ZLB within the next 10 quarters. I show empirically how the probability of reaching the ZLB evolves over time and measure quantitatively how a rise of the inflation target can reduce this probability

In the second essay I find evidence of an asymmetric Taylor rule being in use, that as proposed by Reifschneider and Williams (2002 FOMC), respond more strongly to shocks when interest rates are close to zero. I find that a rule of this kind can have an effect on both the probability of hitting the ZLB, and also the sensitivity of this probability to changes on the inflation target. Therefore, using a linear model to evaluate the benefits - in terms of ZLB probability reduction - of an increase on the inflation target could induce biased results in those two fronts.

In the third essay I quantitatively measure the welfare costs of inflation using a monetary search model augmented in order to include an explicit form of imperfect competition between firms, where the share of the surplus going to the firms is determined endogenously. Under this framework the welfare cost of inflation is amplified by a feedback loop where a restricted money demand induces a reduction in the number of firms the market can support. This in turn increases the market concentration, reducing the consumer surplus and further decreasing the incentives to hold money. I find that a significant part of the estimated welfare costs of inflation can be derived by this interaction between money holdings and market concentration.

# Acknowledgments

To everyone who helped make this possible.

You know who you are, and how grateful I am of your help.

Thanks.

# Chapter 1

# Introduction

When choosing the optimal level of inflation, costs and benefits must be considered. Williams (2009) explained the concept by paraphrasing the mythological story of Daedalus, who advised his son Icarus to fly at moderate heights because if he flew too low the damp would clog his wings, and if too high the heat would melt them. Analogously, a too low inflation can be harmful by increasing the likelihood the zero lower bound(ZLB) constrains the central bank's ability to reduce interest rates in response to negative shocks to the economy. But other distortions are related with inflation, such as relative price distortions, sub optimal money holdings, or distortions related to imperfect competition and measurement bias. As the welfare costs of those distortions tend to be increasing in the level of inflation, a too high inflation can also be costly. Therefore, similar to the advice Daedalus gave to his son, the optimal policy would have to be one of a moderate inflation, not to high, not too low.

In this dissertation I contribute to the literature by empirically quantifying some of the costs and benefits of a higher level of inflation. On the benefits side, I measure inflation's impact on reducing the probability of the ZLB constraining the central bank decisions. Regarding the welfare costs, I focus on how the reduction of money holdings due to inflation can have real costs in terms of consumption, output and employment.

In chapter 2 , using a Time Varying Parameters Vector Auto Regression(TVP-VAR) framework, I construct an index, the Zero Probability Index (ZPI), based on the probability of the nominal interest rate hitting the zero lower bound within 10 quarters.

I show empirically how the probability of reaching the ZLB evolves over time and measure quantitatively how a rise of the inflation target can reduce this probability. In particular, raising the inflation target by 200 basis points, as suggested by Blanchard et al(2010), could significantly reduce this probability, in many cases by more than an order of magnitude.

I also find that high ZPI episodes tend to occur during recessions, and are determined by a combination of the initial state of the variables, and the estimated volatility of the shocks.

However, not in all episodes the causes of a high ZPI are the same. In the recessions of the 1980's, the probability is found to be highly influenced by an exceptionally volatile environment that overcome the dampening influence of the period's high nominal interest rates. On the other hand, the high ZPI estimated for the 2001 and 2007 recessions is mainly defined by an initial state of low interest rates. Because of this difference, an increase in the inflation target is found to be much more effective in reducing the estimated probability of the interest rate reaching the ZLB in the latter episodes.

The results from the TVP-VAR, due to the linear specification, implicitly assume that the response of economic variables is invariant to changes in the steady state level of inflation and nominal interest rates. However, the presence of non-linearities could introduce additional effects.

One of these possible non-linearities are asymmetric Taylor rules that, as proposed by Reifschneider and Williams (2002 FOMC), respond more strongly to shocks when interest rates are close to zero. In chapter 3 I find evidence, using three different methodologies, of an asymmetric Taylor rule being in use by the Fed. I find that a rule of this kind can have an effect on both the probability of hitting the ZLB, and also the sensitivity of this probability to changes on the inflation target.

Therefore, using a linear model to evaluate the benefits - in terms of ZLB probability reduction - of an increase on the inflation target could induce biased results in two fronts. Both the actual risk of reaching the ZLB and the possible reduction of this risk after an increase in the inflation level will be misrepresented.

In chapter 4, I quantitatively measure the welfare costs of inflation using a monetary search model augmented in order to include an explicit form of imperfect competition between firms. In the new monetarist literature, welfare costs of inflation can be derived from consumers restricting their monetary holdings. The reason of doing so comes in part because they pay the costs of holding money, but only get part of the welfare gains. Part of the transaction surplus goes to the sellers. In this chapter I build into these standard money-search models by introducing endogenous imperfect competition based on free entry decisions.

By introducing a Cournot type of imperfect competition with free entry, market concentration, and therefore the share of the surplus going to the firms, will be determined endogenously. The welfare costs of imperfect competition and inflation will not be independent, but jointly determined. I show that the welfare cost of a given inflation level will be endogenous to the market structure of the economy. At the same time, the competitive level that prevails in the economy will be also influenced by the level of inflation.

Under this framework the welfare cost of inflation will be amplified by a feedback loop where a restricted money demand will induce a reduction in the number of firms the market is able to support. This in turn will increase the market concentration, reducing the consumer surplus and further decreasing the incentives to hold money.

I find that a significant part of the estimated welfare costs of inflation can be derived by this interaction between money holdings and market concentration.

# Chapter 2

# Zero lower bound risk and long-term inflation in a time varying economy

After the last financial crisis, it appears that the so called *great moderation* – the period of low volatility of the business cycle fluctuations that started in the mid-1980's – has come to an end. Empirical evidence, as in Keating and Varcacel(2011, 2012) show an increase on the volatility of economic activity.

In presence of bigger shocks, the needed interest rates responses are also bigger, and the zero rate interest bound (ZLB) has becomes an issue of practical importance.

Blanchard et al (2010) argued that a way of giving the central banks more room for lowering interest rates without having to rely on alternative policies is to raise the inflation target. In that way the central bank will be able to respond to bigger shocks without reaching their policy limit.

In this paper, a time varying parameters vector auto regression (TVP-VAR) – as in Primicceri (2005) and Gali and Gambetti (2009) – is estimated. By modeling a time varying economy where the variance of the shocks and the response of variables to those shocks may change over time, I can compute for every period of time the probability of interest rates to hit the ZLB. I'm also able to asses the sensitivity of this probability to changes in the inflation target, and how this sensitivity also evolves over time.

Section 2.1 presents a discussion on the importance on the zero lower bound for the nominal interest rate, and how the inflation target can influence the probability of reaching the ZLB. In Section 2.2 the conceptual framework for the TVP VAR estimation and the empirical results is presented. The conclusions are in Section 2.3.

## 2.1 Volatility, inflation target and monetary policy

### 2.1.1 The end of the great moderation and the zero bound in nominal rates

The *great moderation* – a period of low volatility in the economy – appears to have ended. Clark (2009) documents an increase in volatility of the shocks after the recession that started in 2007.

A changing volatility can also have consequences on the likelihood of nominal interest rates reaching the zero lower bound. If the economy is entering a phase of increased volatility, it will also imply a period where monetary policy will be more frequently constrained by the zero bound, reducing their effectiveness in managing economic fluctuations.

There is, however, no conclusive evidence on how ineffective monetary policy may become in the presence of the zero bound. Chung et al (2012) suggest that while the Federal Reserve's quantitative easing policy improved macroecononomic conditions, it did not prevent the ZLB from having first order adverse consequences. Eggertson and Woodford (2003), on the other hand, argue that the zero bound, while it restricts possible stabilization, does so only by a modest degree.

But one fact is clear. When the interest rate is available as an instrument, it is the preferred way of doing monetary policy. Heterodox policies rarely happen when the interest rate is far from the zero bound. However, while potentially effective, alternative policies can be costly. Bernanke (2012) emphasizes that the use of nontraditional policies involves costs beyond those generally associated with more standard policies. Hamilton and Wu (2012) estimate that at the zero lower bound, buying $400 billion in long-term maturities could reduce the 10-year rate by 13 basis points.

Blinder (2000) advises to *"don't go there, prevention is far better than the cure"*. Bernanke, Reinhart and Sack (2004) conclude that despite some evidence that the use of nonstandard policies might be effective, policy makers should remain cautious as the effects of such policies remain quantitatively quite uncertain. After 4 years of QE policies, Bernanke (2012) assessed that the *"estimates of the effects of* *nontraditional policies on economic activity and inflation are uncertain".*

Jung, Teranishi and Watanabe(2005) emphasize the critical impact of CB credibility during zero bound level episodes. Adam and Billi (2004) make the same point on how the effectiveness of Monetary policy could be severely dampened without credibility. This is because given the lack of ability to change interest rates today, it is of crucial importance to be able to credibly affect future expectations.

It would logically follow then, that a CB that is not completely sure about his credibility, or that it is not completely sure about his credibility in the event of a crisis, would find a policy that avoids the zero bound as a good policy

### 2.1.2 Costs and benefits of inflation

There is consensus on the fact that high inflation is bad for the economy, and should be avoided. Walsh (2003) makes a good summary of the costs of inflation in a New Keynesian environment, emphasizing the loss of welfare derived from the deviation from the optimal consumption basket. In presence of sticky prices, when firms don't adjust prices simultaneously, inflation results in an inefficient dispersion of relative prices, inducing consumers to consume more of the cheaper goods and less of the most expensive ones. Because of diminishing marginal utility, the gains from consuming more of the cheaper goods are smaller than the loss from consuming less of the more expensive goods. The welfare costs under this framework can be eliminated under a zero inflation policy.

Then, why don't we observe zero inflation targets? The reasons are varied, but generally correlated. On one side there is the theory of inflation "greasing the wheels of the labor market" as in Tobin (1972) or Akerloff et al (1996) where the downward nominal rigidity of wages would make desirable some inflation as to allow reducing real wages in case of adverse shocks.

But most of the arguments go toward reducing the risk of deflation and liquidity traps that, as noted by Svensson (2003) among many others, can have severe negative consequences: as the real value of debt increases, commercial banks' balance sheet deteriorate, and unemployment rises, all this magnified by downward nominal rigidity that can further deteriorate aggregate demand.

So a too low inflation increases the risk of any shock causing deflation. Moreover if it is considered, as pointed out by Bernanke et al (2001), that because consumers tend to replace goods that become more expensive with more cheaper goods, there is likely an upward bias in measured inflation.

Also, in the case of heterogeneous productivity growths under the same monetary union, chasing a general low level of inflation may cause deflationary pressures on the countries with high productivity growth that should naturally have, because a Balassa-Samuelson Effect, a higher inflation. This channel is discussed by Masten (2008) and Rabanal (2009).

An additional benefit of higher inflation comes from avoiding reaching the zero bound limit. Summers (2001) make the point that low levels of inflation induce low levels of nominal interest rates, leaving the Central Bank with little room to lower interest rates in the event of a recession. Blachard, Dell'Ariccia and

Mauro(2010) argue that higher inflation before the crisis, and thus higher interest rates to begin with would have allowed the Fed to cut interest rates more and thus probably reduce the drop in output and the deterioration of fiscal positions. Blinder (2000) suggest setting a $p*$ sufficiently high to make the probability of encountering $r = 0$ extremely small. Adam and Billi (2004) and Williams (2009), among others, show in simulations how a higher inflation target can reduce the probability of reaching the zero bound. In this paper, I calculate how both the probability of hitting the ZLB and the sensitivity of that probability to the inflation level evolves over time.

## 2.2 Empirical Approach

To quantitatively asses the impact of an increase of the inflation target on the probability of hitting the ZLB, a TVP VAR is estimated. It allows for a time varying structure of the economy, and therefore a time varying risk of the interest rate reaching the ZLB.

The results of the estimation are used to compute, at each period of time, the variance of the shocks and the impulse response functions for the economic variables. For each period of time, multiple trajectories are then simulated in order to compute the probability of reaching the zero bound within a certain horizon.

Counterfactual scenarios are simulated in order to calculate the impact of a higher inflation target on the probability of reaching the ZLB, and also the required inflationary increase, at each period of time, to maintain this probability below an arbitrary threshold.

### 2.2.1 Estimation and Results

Following the methodology from Gali and Gambetti(2009), and similar to Primiceri (2005), Cogley and Sargent (2001,2005), and Cogley and Sbordone(2008), an $n$ variables and $p$ lags Bayesian VAR is estimated, with a specification given by:

$$x_t = A_{0,t} + A_{1,t}x_{t-1} + ... + A_{p,t}x_{t-p} + u_t \tag{2.1}$$

Where $x_t$ is a vector of endogenous variables, $A_{0,t}$ is a vector of time varying coefficients, and $A_{i,t}, i = 1, ..., p$ are matrices of time varying coefficients. The residuals $u_t$ are normally distributed with mean zero and var-cov matrix $\Sigma_t$ . Let $A_t = [A_{0,t}, A_{1,t}, ..., A_{p,t}]$ and $\theta_t = vec\,(A'_t)$ a vector that stack all elements of $A_t$. The parameters from $\theta_t$ are assumed to evolve as random walks subject to reflecting barriers that impose stability, ruling out explosive behaviors for the variables. The residuals $u_t$ are normally distributed with mean zero and a variance-covariance matrix $\Sigma_t$ that is also allowed to change over time[1].

Similar to Primiceri (2005), the VAR to be estimated has 2 lags and 3 endogenous variables that intend to replicate a small reduced form new Keynesian economic model: inflation, unemployment rate, and a short-term nominal interest rate. The sample size covers the period 1953Q3 to 2008Q3[2] . The first 40 quarters

[1] A full description of the estimation procedure is presented in the appendix, section A

[2] The end of the sample is restricted in order to exclude periods when the ZLB was binding

are used as a training period to initialize the priors. The model is estimated using data starting from 1964Q3.

All data is taken from the FRED Database of the Saint Louis Federal Reserve. Inflation is measured by the annual growth of the CPI. Unemployment is measured as the civilian unemployment of all workers over 16. The nominal interest rate is the effective federal funds rate. The data is presented in figure 2.1.

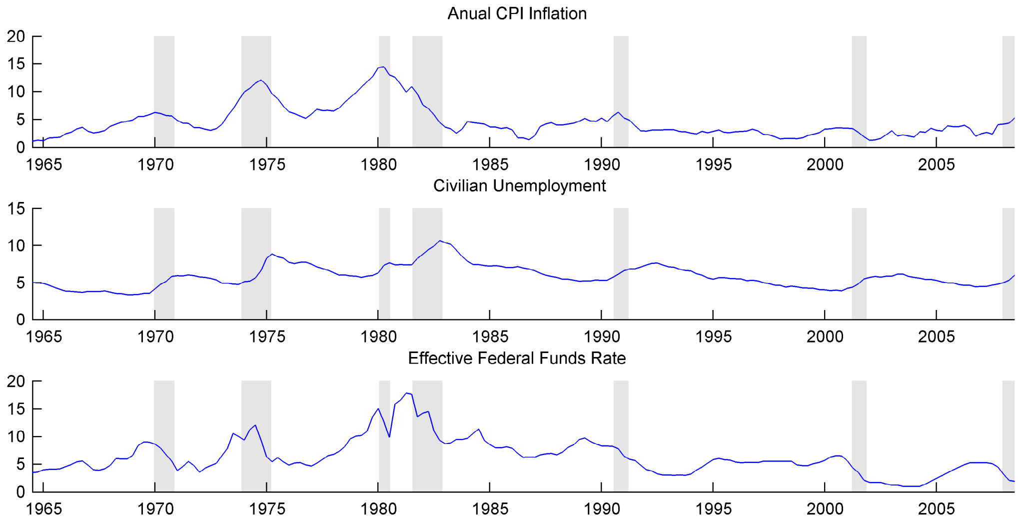


Figure 2.1: Sample period inflation, unemployement and interest rate

As pointed out by Williams (2014) two key factors affect the simulated probability of hitting the ZLB: the size and the duration of the shocks hitting the economy. In the context of this VAR estimation, the former will be represented by the estimated standard deviation of the residuals. A measure of how long a shock can influence a variable is obtained by summing up, for each equation, the own lag coefficients. The estimated results for both measures can be seen in figure 2.2.

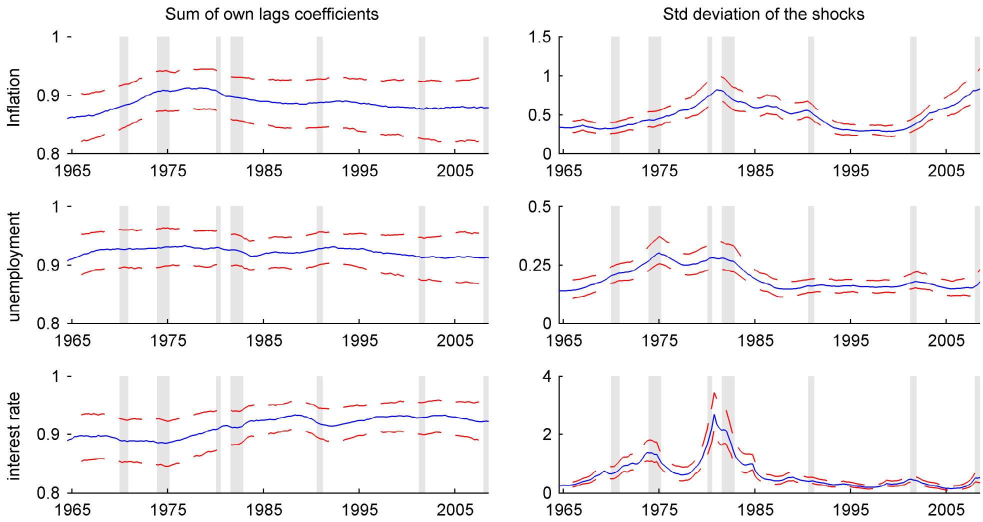


Figure 2.2: Estimated persistence of processes and volatility of shocks.

Regarding the estimated persistence of the processes, there are not any significant changes over the sample period. With respect to the estimated volatility of the shocks, the sample can be separated into three distinct periods. One of rising volatility starting in the first half of the 1970s until mid 1980s, followed by a period of markedly low volatility that is interrupted after the 2001 recession with a moderate increase of the volatility, that increases strongly again with the financial crisis. The price equation shows the biggest volatility during the financial crisis; on the other hand, the unemployment volatility presents comparable peaks during the recessions of 1974, 1980, 1982 and 2008. The interest rate equation dynamics are dominated by the volatility peak during the 1980 recession.

It is worth noting that in recessions the estimated volatility of the non systematic part of monetary policy tends to increase. This would be inline with

Calani, Garcia and Cowan(2011) in terms that facing large shocks, the linearity assumptions that permit the equivalence between simple policy rules and more complex optimal rules break down, and therefore the VAR identifies the changes in interest rates as being a non-systematic response to other variables.

From the TVP VAR estimation, the evolution over time of the systematic monetary policy response to economic shocks can be extracted. A measure of the strength of the policy response to a unitary shock is constructed by summing over the corresponding IRF. Figures 2.3 and 2.4 show both the response of the monetary policy to shocks in inflation and unemployment for different periods, and the evolution of the measure of aggregate response over time.

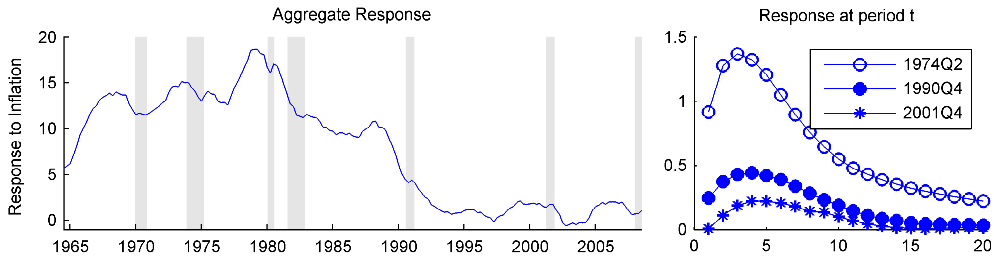


Figure 2.3: Estimated response of federal funds rate to a unitary shock in inflation

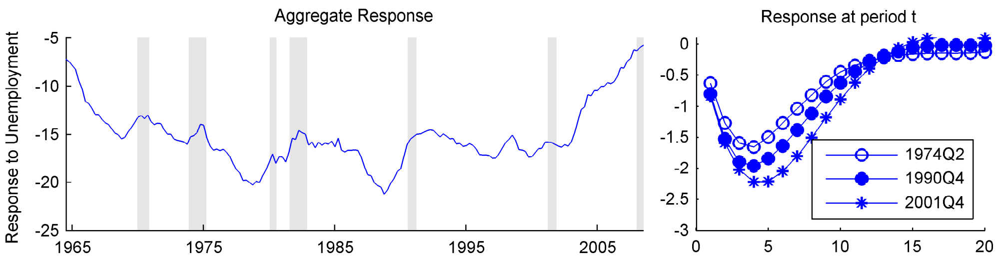


Figure 2.4: Estimated response of federal funds rate to a unitary shock in unemployment

This allows me to compute a time varying probability of reaching the zero lower bound. The *ZLB probability index* (ZPI) at time $t$ will be defined as the likelihood of reaching the zero lower bound within the next 10 quarters.[3]

In order to compute the index, at each period of time a forecast of the expected trajectory of the interest rate is calculated. Let equation (2.1) be expressed in companion form: $x_t = \mu_t + A_t x_{t-1} + u_t$, where $x_t \equiv [x_t', x_{t-1}', ..., x_{t-p+1}']'$, $u_t \equiv [u_t', 0, ..., 0]'$,$\mu_t \equiv [A_{0,t}', 0, ..., 0]'$, and $A_t$ is the corresponding companion matrix. As $\mu_t$ and $A_t$ evolve as random walks, $E_t(\mu_{t+j}) = \mu_t$ and $E_t(A_{t+j}) = A_t$, and the forecast for $j$ periods ahead can be recursively computed as $E_t\left(x_{t+j}\right) = \mu_t + A_t E_t\left(x_{t+j-1}\right)$. Note that this forecast will not necessarily converge monotonically towards the trend value, defined, as in Cogley and Sbordone (2008), as the level at which the variable is expected to settle after the short-run fluctuations die out, $\overline{x}_t = \lim_{j\to\infty} E_t\left(x_{t+j}\right)$. For example, Figure 2.5 shows how, by the end of the 2001 recession, the interest rate, while already below the trend, is expected to keep dropping for the next 3 quarters due to a combination of a relatively high unemployment (5.7%) and a low inflation (1.2%).

On top of the expected path for the interest rate, for each period 25,000 alternative trajectories are simulated based on the estimated impulse response functions, and a series of shocks drawn from the period VAR-COV matrix.

[3] Given the random walk nature of the time varying estimates, at each period $t$, all the parameters are expected to remain constant for the foreseeable future. Therefore, for every period considered, the simulated trajectories assume a constant parametrization of the economic structure.

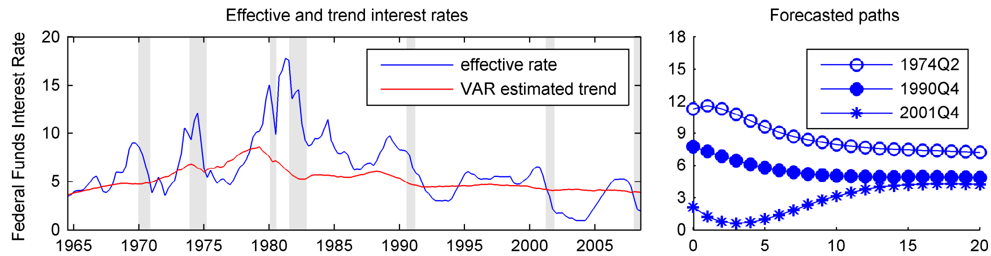


Figure 2.5: Estimated Federal Funds rate trend and convergence paths

The results in Figures 2.6 and 2.7 show some interesting results. First, high ZPI events tend to occur during NBER defined recessions. Special cases are the 1969-1970 recession, where the peak is reached afterwards, and the 1990-1991 recession, the only one without a relevant spike in the index.

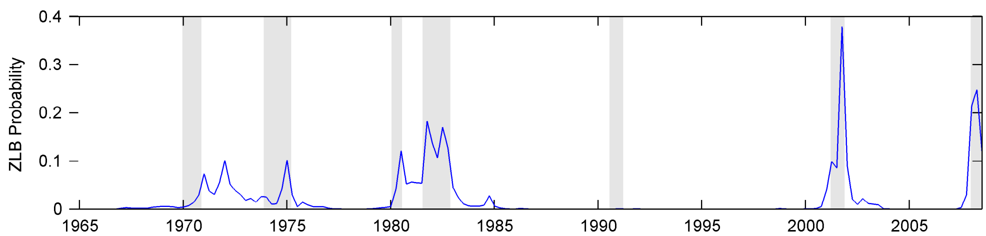


Figure 2.6: Estimated probabilities of reaching the ZLB within 10 quarters: ZPI index

The three main high risk episodes, where the ZPI passed the 10% mark, happened during the recessions of early 1980s, 2001 and after the financial crisis. As presented in Figure 2.7, compared with 1970Q2, the 1980s episode has a comparable starting point and expected convergence path, while the 2001 and 2008 episodes have similar dispersion on the simulated paths. However, all three of the episodes have considerably higher estimated probabilities. I postulate the hypothesis that

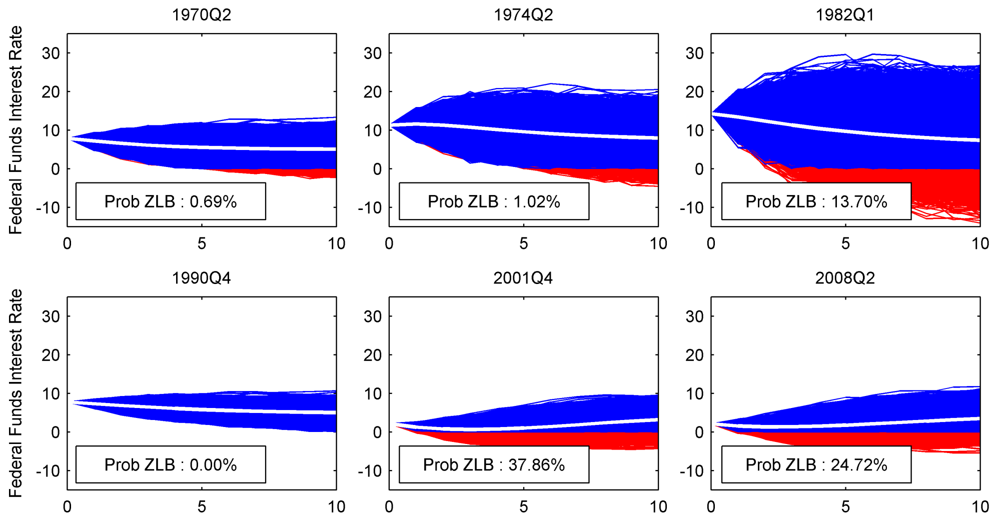


Figure 2.7: Simulated interest rate trajectories during different NBER defined recessions. In red trajectories that at some point cross the ZLB, in white trajectory without shocks

the high estimated ZPI in the case of the 1980s episode is mainly due to a high economic uncertainty, while in 2001 and 2008 it is due to a low initial state.

The validity of this hypothesis is tested by simulating two counterfactuals. To isolate the impact of varying starting points, the ZPI is simulated assuming that at each point of time, the initial state is the long term trend. In order to deal with the implications of a changing volatility, the ZPI is computed while maintaining a constant level of uncertainty equal to the average from 2001.

The results shown in Figures 2.8 and 2.9 seem to validate the previous hypothesis. Choosing a starting point equal to the trend values significantly reduces the estimated ZPI for the 2001 and 07-08 episodes, while at the same time increases the ZPI for the 1980s recessions, where the effective interest rates where above the estimated trends. When computing the ZPI assuming a constant variance equal to the 2001 average, the ZPI is completely wiped out for the 1980s reccesions. Interestingly, this counterfactual also increases the ZPI for the period after the 2001 recession, a period characterized by very low interest rates.

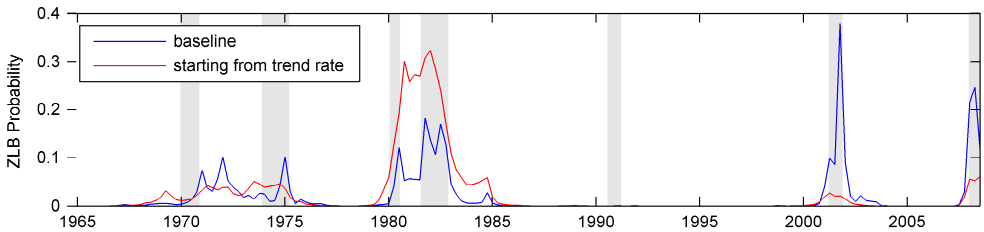


Figure 2.8: Effect of the initial state on the estimated ZPI

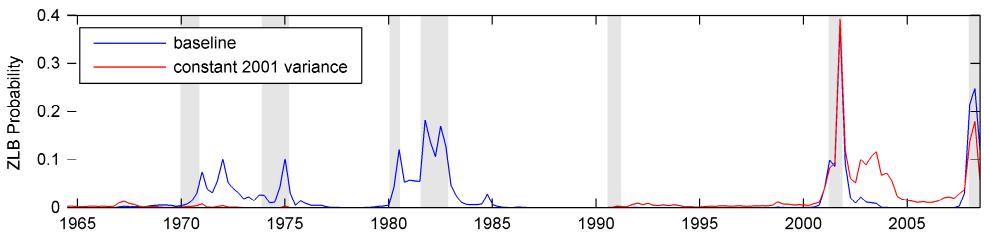


Figure 2.9: Effect of volatility on the estimated ZPI

### 2.2.2 A change on the inflation target counter-factual

Its been argued by Blanchard et al (2010), among others, that a rise in the inflation target could help reduce the likelihood of reaching the ZLB.

Under the assumption of super-neutrality of money – that is, a change in the trend inflation level should not have an impact on real variables – it is straightforward to asses the impact to a change in the inflation target on the probability of hitting the ZLB, as the change would simply imply a shift in the nominal interest rate of the same magnitude.

Figure 2.10 presents the counter-factual of following Blanchard's suggestion and using an inflation target 200 basic points higher than the baseline.

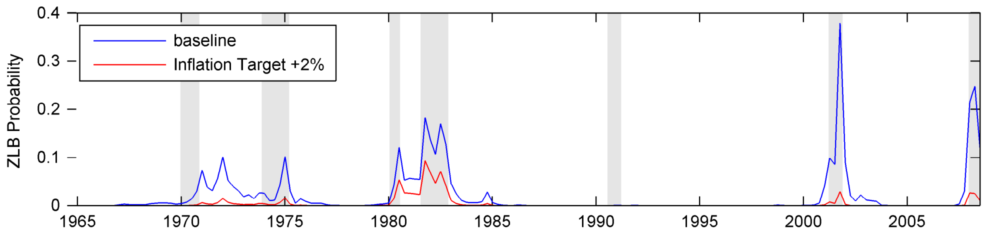


Figure 2.10: Effect of an increase on the inflation target on the estimated ZPI

I find a considerable decrease in the estimated probability of reaching the ZLB in all relevant episodes. When the inflation target increases by 2% the estimated ZPI almost vanishes for all episodes but the 1980s recessions. The ZPI in the 1980s is still greatly reduced, dropping to approximately half of the one estimated in the base scenario. This is consistent with the results suggested in the previous section regarding a higher relative impact of volatility on the estimated ZPI for the 1980s episodes. On the other hand, the bigger impact of a higher inflation target in the latter episodes is consistent with the diagnosis of a ZPI greatly affected by a low initial interest rate.

I also look at an alternative approach to asses the impact of an increased inflation trend on the estimated ZPI. I ask the question of what would be the inflation increase required, at each period of time, in order to maintain the ZPI below some threshold level. Figure 2.11 show that the increase in inflation required to keep the ZPI below 5% and 1% within the whole sample period is 270 and 650 basis points respectively.

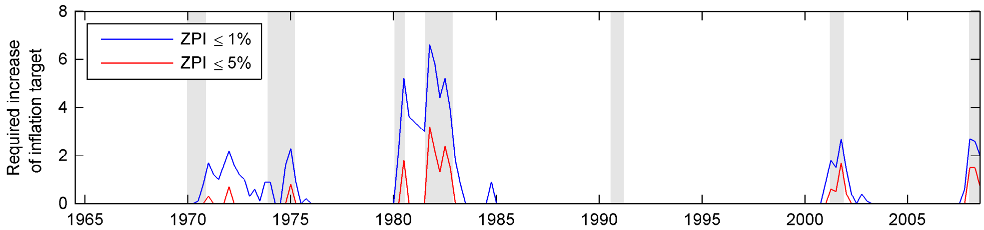


Figure 2.11: Required increase in inflation target in order to attain a lower ZPI

Even if the estimated ZPI is lower for the 1980s recessions than the 21st century ones, the increase in the inflation level required to reduce the probability is much higher. This due to the higher estimated volatility in the period compared to other recessive periods.

## 2.3 Conclusions

Using a TVP VAR framework, I construct an index, the Zero Probability Index (ZPI), based on the probability of the nominal interest rate to hit the zero lower bound within 10 quarters.

I show empirically how the probability of reaching the ZLB evolves over time, and how an increase in the inflation target can help reduce this probability. In particular, raising the inflation target by 200 basis points, as suggested by Blanchard, could significantly reduce this probability, in many cases by more than an order of magnitude.

I also find that high ZPI episodes tend to occur during recessions, and are determined by a combination of the initial state of the variables, and the estimated volatility of the shocks.

However, the causes of a high ZPI are not the same in all episodes. In the 1980's recessions the high ZPI is a consequence of an exceptionally volatile environment, that overcome the dampening influence of the period's high nominal interest rates. On the other hand, the high ZPI estimated for the 2001 and 2007 recessions is mainly determined by an initial state of low interest rates. Because of this, an increase in the inflation target is found to be much more effective at reducing the estimated probability of the interest rates reaching the ZLB in the latter episodes.

# Chapter 3

# Asymmetric monetary policy response and the effects of a rise of the inflation target

As a recommendation for monetary policy in a low inflation environment, Reifschneider and Williams (2002 FOMC) proposed an asymmetric Taylor Rule with a threshold level that automatically goes to zero whenever interest rates go below 1 percent. I test whether monetary policy has been already working in a similar asymmetric fashion, with a negative correlation between the level of interest rate and the strength of the monetary policy responses. The consequences of such a policy are analyzed, in particular regarding the expected effects of a rise in the inflation target that in this scenario would have the side effect of having a monetary policy that would be, on average, less responsive to both unemployment and inflation.

## 3.1 Asymmetric monetary policy and the zero bound on nominal interest rates

Reifschneider and Williams' argument favoring an asymmetric Taylor rule is justified in terms of avoiding excessive welfare costs in cases where the zero bound on nominal interest rates constrains the possible responses of the monetary authorities against negative shocks. When policy is constrained by the zero bound, prices tend to fall more rapidly, causing an unintended policy tightening as real interest rates raise. As they argue, this ends up exacerbating the rise in unemployment, and under extreme conditions, can even turn into a self-reinforcing spiral, with falling output pushing down inflation, and falling inflation pushing real interest rates higher, putting even more pressure on unemployment.

One possible solution to ameliorate the welfare costs during zero interest rate episodes is to have rules that are more responsive to fluctuations of inflation and output. This kind of rule has two main benefits. First, it reduces the probability inflation is below the target when a major disturbance hits the economy, taking away one of the ingredients of a deflationary spiral. The second benefit is that as it reduces interest rates quickly when the economy weakens, the severity of major downturns is reduced, lowering the risk of deflation.

However, a more aggressive behavior can also have some drawbacks. It can increase the volatility of interest rates and the frequency of policy reversals. It also amplifies the risk and magnitude of policy mistakes in presence of faulty data and mismeasurement of the economy's productive capacity. Finally, Reifschneider and Williams argue that a quick drop of interest rates could trigger confidence crisis if investors become worried the interest rate could become constrained by the zero bound.

In this context, they propose an asymmetric rule that only responds stronger when the interest rate is close to the zero bound. As the argument goes, this allows for the potential drawbacks of strong monetary responses to only appear when the costs of maintaining a relatively weak policy response grow in magnitude. When interest rates are relatively high, the benefits of a stronger policy in terms of avoiding the welfare costs during zero interest rate episodes are smaller compared to these costs. Therefore, an increased response to output and inflation isn't justified.

## 3.2 Empirical evidence for asymmetry on the Taylor rule responses

The hypothesis of an asymmetric monetary policy is tested through three empirical approaches: A vector autoregression with time varying policy responses, a single equation Taylor rule regression with interaction terms, and a DSGE model with a non-linear monetary policy rule.

### 3.2.1 Single equation Taylor rule estimation with interaction terms

The Taylor rule is estimated as a single equation regression, where every variable but the interest rate is considered exogenous. In order to test the asymmetry of monetary policy responses, interaction terms are added to the standard rule. In the equation estimated equation, interest rates $r$ are a function a of last period interest rate, inflation $\pi$, unemployment $u$ and the interaction between the level of interest rate and inflation and unemployment.

$$r_t = \beta_0 + \beta_i r_{t-1} + \beta_\pi \pi_t + \beta_u u_t + \beta_{\pi,i} \pi_t r_t + \beta_{u,i} u_t r_t + \varepsilon_t \tag{3.1}$$

Given the specification, if significant parameters accompanying the interaction between interest rate and exogenous variables are found, the magnitude of the monetary policy response to inflation and unemployment depends on the level of the interest rate.

A sign of the interaction term parameter equal to the one of the direct effect means that the estimated monetary policy response becomes weaker as interest rates approach zero. Opposite signs, on the other hand, signify that smaller interest rates are correlated with stronger policy responses.

Contemporary shocks to the interest rate equation are expected to influence current and expected future values of inflation and unemployment. The potential bias in the estimated parameters due to endogeneity is dealt by two different approaches, an OLS estimation using past periods' market expectations as proxy for the potentially endogenous variables, and a GMM estimation that uses, as suggested by Favero (2001), lags of the possibly endogenous variables as instruments.

For the first approach, data from the *Survey of Professional Forecasters* from the Federal Reserve Bank of Philadelphia is used. The quarterly survey started in 1968, with additional variables added in 1981. It asks participants for their forecasts on the evolution of selected economic variables. The mean of the respondents answers will be used for the regressions. The proxy for the expectation of the interaction terms is computed by taking the mean of the multiplication of the correspondent answers of each surveyed agent, then $\widehat{E(xy)} = \sum_1^J x_j y_j / J$. Where $x_j$ and $y_j$ are the answers of each agent $j$ on their expectation for variables $x$ and $y$.

The Taylor rule is then estimated using the answers given at *t-1* for the expected value of the variables for time $t$ or *t+1*, depending if the specification tested is forward looking. As expectations were formed last period, they won't be affected by contemporary shocks to the interest rate equation, and will therefore be expected to be uncorrelated with the error term, eliminating the parameter bias by endogeneity.

As a measure of economic activity, the expectation for the unemployment rate is used. For prices, the expected CPI inflation rate, and for the interest rate, the expected rate of the 3-months treasury bill. While unemployment expectation answers are available from 1968, questions regarding CPI and interest rates were only added on the 1981 revision of the survey. The initial date for the sample size of the regression will therefore be restricted to 1981Q4. The end of the sample is set at 2008Q3, the last quarter before reaching the ZLB.

For the GMM estimation, the endogeneity problem is tackled by using lagged values of the variables as instruments. Not relying on survey data also allows for for an extended sample size. The GMM sample will span from 1966Q1 to 2008Q3.

The regression results are in Table 3.1. The interaction terms between interest rate and unemployment are consistently significant and of opposite sign as the coefficient accompanying the unemployment rate. This would indicate a stronger monetary policy response to unemployment shocks when interest rates are close to zero.

Regarding inflation, the OLS specification interaction terms are always significant and of opposite sign from the direct effect. For the GMM estimation, however, both the direct effect of inflation on interest rates and the interaction terms only appear significant in the specification that is forward looking and also includes dummies for the different chairs of the FED. In the specification where inflation and the interaction term appears to be significant, the coefficients are also comparable in magnitude to their OLS counterparts.

Overall, the results from single equation Taylor rule estimation support the hypothesis of a stronger monetary policy response to inflation and output when interest rates are closer to zero, although the results are more robust in the case of unemployment than inflation.

| Dependent variable: Federal Funds Rate | OLS | | | | GMM | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| const | 1.548** | 2.094*** | 2.040** | 2.674*** | 3.909*** | 4.136*** | 3.886*** | 2.101** |
| | (0.723) | (0.769) | (0.784) | (0.920) | (0.613) | (0.540) | (0.679) | (1.071) |
| $r_{t-1}$ | 0.629*** | 0.493*** | 0.545*** | 0.406** | 0.327*** | 0.308*** | 0.275*** | 0.414*** |
| | (0.121) | (0.138) | (0.125) | (0.161) | (0.075) | (0.081) | (0.073) | (0.128) |
| $\pi_t$ | 0.779*** | | 0.922*** | | 0.084 | | 0.107 | |
| | (0.249) | | (0.266) | | (0.108) | | (0.126) | |
| $r_t \cdot \pi_t$ | -0.066*** | | -0.071*** | | -0.003 | | -0.009 | |
| | (0.019) | | (0.018) | | (0.012) | | (0.011) | |
| $\pi_{t+1}$ | | 1.048*** | | 1.216*** | | 0.051 | | 0.972*** |
| | | (0.261) | | (0.302) | | (0.131) | | (0.274) |
| $r_t \cdot \pi_{t+1}$ | | -0.069*** | | -0.078*** | | -0.001 | | -0.052** |
| | | (0.015) | | (0.020) | | (0.014) | | (0.020) |
| $u_t$ | -0.520*** | -0.705*** | -0.690*** | -0.888*** | -0.712*** | -0.739*** | -0.786*** | -0.752*** |
| | (0.157) | (0.183) | (0.177) | (0.232) | (0.125) | (0.132) | (0.129) | (0.148) |
| $r_t \cdot u_t$ | 0.070*** | 0.083*** | 0.076*** | 0.093*** | 0.114*** | 0.118*** | 0.135*** | 0.086*** |
| | (0.024) | (0.025) | (0.023) | (0.027) | (0.020) | (0.022) | (0.020) | (0.020) |
| DUM_Martin | | | | | | | 0.205 | -0.443 |
| | | | | | | | (0.254) | (0.577) |
| DUM_Burns | | | | | | | 0.177 | -0.745 |
| | | | | | | | (0.191) | (0.620) |
| DUM_Miller | | | | | | | 0.109 | -1.869** |
| | | | | | | | (0.303) | (0.800) |
| DUM_Volcker | | | 0.847** | 0.845** | | | -0.302 | 1.102 |
| | | | (0.408) | (0.407) | | | (0.379) | (0.707) |
| DUM_Greenspan | | | 0.339 | 0.245 | | | 0.258 | 0.596 |
| | | | (0.282) | (0.255) | | | (0.163) | (0.525) |
| Sample | 1981.4 - 2008.3 | 1981.4 - 2008.3 | 1981.4 - 2008.3 | 1981.4 - 2008.3 | 1966.1 - 2008.3 | 1966.1 - 2008.3 | 1966.1 - 2008.3 | 1966.1 - 2008.3 |
| N obs | 108 | 108 | 108 | 108 | 171 | 171 | 171 | 171 |
| Adj. $R^2$ | 0.948 | 0.952 | 0.950 | 0.953 | | | | |
| GMM Q-crit | | | | | 0.026 | 0.030 | 0.021 | 0.025 |

*Standard errors in parentheses - * indicates significance at 10% - ** indicates significance at 5% - *** indicates significance at 1%*

Table 3.1: Estimated effect of the interest rate level on the Taylor rule response to inflation and unemployment

### 3.2.2 VAR with time varying coefficients

The hypotheses of an asymmetric monetary policy is also tested through a time varying parameters methodology. A three equation time varying parameters vector autoregression (TVP-VAR) is estimated. Following the methodology from Gali and Gambetti(2009), and similar to Primiceri (2005), Cogley and Sargent (2001,2005), and Cogley and Sbordone(2008), an $n$ variables and $p$ lags Bayesian VAR is estimated, with a specification given by:

$$x_t = A_{0,t} + A_{1,t}x_{t-1} + ... + A_{p,t}x_{t-p} + u_t \tag{3.2}$$

Where $x_t$ is a vector of endogenous variables, $A_{0,t}$ is a vector of time varying coefficients, and $A_{i,t}, i = 1, ..., p$ are matrices of time varying coefficients. The residuals $u_t$ are normally distributed with mean zero and var-cov matrix $\Sigma_t$ . Let $A_t = [A_{0,t}, A_{1,t}, ..., A_{p,t}]$ and $\theta_t = vec\,(A_t')$ a vector that stack all elements of $A_t$. The parameters from $\theta_t$ are assumed to evolve as random walks subject to reflecting barriers that impose stability, ruling out explosive behaviors for the variables. The residuals $u_t$ are normally distributed with mean zero and a variance-covariance matrix $\Sigma_t$ that is also allowed to change over time[1].

The endogenous variables incorporated into the VAR are the federal funds rate, the inflation rate, and the output gap. From the estimation results, a time varying interest rate response to inflation and output can be obtained

$$r_t = A_t + \rho_t\,(L)\,r_t + \phi_{\pi,t}\,(L)\,\pi_t + \phi_{y,t}\,(L)\,y_t + \upsilon_t \tag{3.3}$$

[1]A full description of the estimation procedure is presented in the appendix, section A

As in Primiceri (2005), the time varying long term responses to inflation and output can be expressed as

$$\Phi_{\pi,t} = (1 - \rho_t(1))^{-1} \cdot \phi_{\pi,t}(1) \tag{3.4}$$

$$\Phi_{y,t} = (1 - \rho_t(1))^{-1} \cdot \phi_{y,t}(1) \tag{3.5}$$

In order to test for monetary policy asymmetric responses, the time series of $\Phi_{yt}$ and $\Phi_{it}$ are regressed against the interest rate. In order to interpret the results as percent changes in the strength of monetary policy responses, the dependent variables are expressed as logarithms.

Table 3.2 shows the regression results. A significant coefficient accompanying the interest rate implies the level of the interest rate has an effect on the strength of the response. If the coefficient is negative, then a lower interest rate will be correlated with stronger responses. When no dummies for FED chairs are present, the model is not able to identify any effect. However, after introducing them, the results show consistent negative coefficients for the asymmetry parameter for both inflation and output responses, suggesting that the monetary policy tends to react stronger to both variables when the interest rate approaches zero. Introducing trend variables does not affect the results in a significant manner.

Interestingly, the magnitude of the effect is very similar for the estimated responses of inflation and output, although the estimates for the output response appear to be more precise, with an estimated standard deviation for the interest rate parameter in the inflation regression between 30% to 40% larger than in the

| Dependent variable: | $\log(\vartheta_{\pi,t})$ | | | $\log(\vartheta_{y,t})$ | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| const | -0.005 | -0.022** | -0.028 | -0.014** | -0.050*** | -0.099*** |
| | (0.005) | (0.009) | (0.025) | (0.006) | (0.011) | (0.035) |
| $\log(\vartheta_{\pi,t-1})$ | 0.988*** | 0.957*** | 0.943*** | | | |
| | (0.010) | (0.017) | (0.019) | | | |
| $\log(\vartheta_{y,t-1})$ | | | | 0.984*** | 0.947*** | 0.928*** |
| | | | | (0.005) | (0.013) | (0.018) |
| $r_t$ | 0.182 | -0.705** | -0.880*** | 0.079 | -0.740*** | -0.790*** |
| | (0.218) | (0.301) | (0.296) | (0.179) | (0.224) | (0.214) |
| DUM_Martin | | 0.016*** | 0.012 | | -0.009 | 0.004 |
| | | (0.006) | (0.020) | | (0.008) | (0.015) |
| DUM_Burns | | 0.019*** | 0.009 | | -0.003 | 0.003 |
| | | (0.006) | (0.017) | | (0.006) | (0.012) |
| DUM_Miller | | 0.030*** | 0.019 | | 0.009* | 0.009 |
| | | (0.006) | (0.014) | | (0.004) | (0.011) |
| DUM_Volcker | | 0.049*** | 0.040*** | | 0.030*** | 0.030*** |
| | | (0.009) | (0.013) | | (0.004) | (0.010) |
| DUM_Greenspan | | 0.017*** | 0.008 | | 0.013*** | 0.010 |
| | | (0.006) | (0.007) | | (0.003) | (0.007) |
| trend | | | 4.3E-04 | | | 6.2E-04** |
| | | | (3.4E-04) | | | (2.9E-04) |
| $trend^2$ | | | -2.4E-06 | | | -2.5E-06** |
| | | | (1.5E-06) | | | (1.2E-06) |
| Sample | 1966.1-2008.3 | 1966.1-2008.3 | 1966.1-2008.3 | 1966.1-2008.3 | 1966.1-2008.3 | 1966.1-2008.3 |
| N obs | 171 | 171 | 171 | 171 | 171 | 171 |
| Adj. $R^2$ | 0.991 | 0.993 | 0.993 | 0.996 | 0.997 | 0.997 |

*Standard errors in parentheses - * indicates significance at 10% - ** indicates significance at 5% - *** indicates significance at 1%*

Table 3.2: Estimated effect of the interest rate level on the TVP-VAR implied time varying Taylor rule coefficients

output regression.

### 3.2.3 DSGE model with a non linear Taylor rule

A third approach used to test the presence of asymmetries in the monetary policy responses is based on a bayesian dynamic stochastic general equilibrium model estimation. The specification is based on widely used medium scale DSGE model developed by Smets and Wouters(2007) but with a modified Taylor rule that incorporates the possibility of asymmetric responses.

The original model, belonging to the New Keynesian or New Neoclassical Synthesis class of monetary business cycle models[2], has 14 endogenous variables: output $y_t$, consumption $c_t$, real value of capital stock $q_t$, capital services used in production $k_t^s$, installed capital $k_t$, capital utilization rate $z_t$, rental rate of capital $r_t^k$, inflation $\pi_t$, wages $w_t$, markups for the goods and labor markets $\mu_t^p$ and $\mu_t^w$, worked hours $l_t$, and nominal interest rate $r_t$. It features many frictions that affect both nominal and real decisions of households and firms, including sticky nominal price and wage settings, habit formation in consumption and investment adjustment costs, and variable capital utilization and fixed costs in production. It also includes seven orthogonal structural shocks: total factor productivity shocks, two shocks that affect the intertemporal margin (risk premium shocks and investment-specific technology shocks), two shocks that affect the intratemporal margin (wage and price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks).

[2] A full description of the equations of the model is presented in the appendix, section B.1

Households maximize a nonseparable utility function with two arguments (goods and labor effort) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit variable. Labor is differentiated by a union, so there is some monopoly power over wages. Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment increasing costs. Firms produce differentiated goods, decide on labor and capital inputs, and set prices. The model also features an exogeneous spending process, and a monetary policy reaction function: $r_t = \rho r_{t-1} + (1-\rho)\left[\overline{r}_\pi \pi_t + \overline{r}_y \widehat{y}_t\right] + \overline{r}_{\Delta y} \Delta \widehat{y}_t + \varepsilon_t^r$. The monetary authority adjusts the interest rate $r_t$ in response to inflation $\pi_t$, the output gap $\widehat{y}_t$ and the change in the output gap $\Delta \widehat{y}_t$.

In this paper's version of the model, the Taylor rule is augmented to allow for asymmetric responses, where the level of the interest rate determines the strength of the monetary policy's response to variables:

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_{\pi,t}\pi_t + r_{y,t}\widehat{y}_t\right] + r_{\Delta y,t}\Delta \widehat{y}_t + \varepsilon_t^r \tag{3.6}$$

where

$$r_{y,t} = \overline{r}_y \exp\{m_y r_t/100\} \tag{3.7}$$

$$r_{\Delta y,t} = \overline{r}_{\Delta y} \exp\{m_y r_t/100\} \tag{3.8}$$

$$r_{\pi,t} = 1 + (\overline{r}_\pi - 1)\exp\{m_\pi r_t/100\} \tag{3.9}$$

The parameters $m_\pi$ and $m_y$ define respectively the level of asymmetry for the responses to inflation and output. Under this specification, the effect of $r_t$ on the strength of the responses is constant in percentage terms. A coefficient of 1 on the asymmetry parameter implies that an interest rate that is one percent above its steady state level is correlated with a 1% stronger response to the corresponding variable.[3] Conversely, a negative value of the parameter means that monetary policy responses are stronger when interest rates are lower. If $m_\pi = m_y = 0$, then $r_{\pi,t} = \overline{r}_\pi, r_{y,t} = \overline{r}_y$, and $r_{\Delta y,t} = \overline{r}_{\Delta y}$, and the Taylor rule collapses to the original from Smets and Wouters.

The structure for $r_{\pi,t}$ is slightly modified with respect to equations (3.7) and (3.8) in order to guarantee the fulfillment of the Taylor principle. As the effect of $r_t$ on $r_{\pi,t}$ is defined only on the part of $\overline{r}$ above unity, $r_{\pi,t}$ will always be greater than 1, regardless of the interest rate level.

Given the nonlinear nature of the Taylor rule, a second order approximation of the model is estimated. Seven observable variables are used for the estimation: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Measurement error is allowed for all observables but the interest rate. A particle filter procedure is used for the nonlinear estimation, with 50,000 particles and 25,000 MCMC replications. Uninformative flat priors are set for the asymmetry parameters. The rest of the priors are kept as in the base model specification. The model is estimated for two samples. The first one starts — as in Smets and Wouters(2007) — in 1966Q1. The second sample is set to begin at 1987Q4, as Alan Greenspan took

[3] In the model the interest rate is specified in quarterly terms. If the asymmetry parameter is 1, an interest rate increase of 1% in annual terms implies a 0.25% stronger response.

| | Prior Distribution | | | Posterior Distribution | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Dist. | $\mu$ | $\sigma$ | $\mu$ | | $\sigma$ | | Prob(x<0) | |
| | | | | 1966Q1 2008Q3 | 1987Q4 2008Q3 | 1966Q1 2008Q3 | 1987Q4 2008Q3 | 1966Q1 2008Q3 | 1987Q4 2008Q3 |
| $\rho$ | Beta | 0.75 | 0.10 | 0.87 | 0.85 | 0.01 | 0.03 | 0.00 | 0.00 |
| $\bar{r}_\pi$ | Normal | 1.50 | 0.25 | 2.03 | 1.92 | 0.03 | 0.16 | 0.00 | 0.00 |
| $\bar{r}_y$ | Normal | 0.13 | 0.05 | 0.17 | 0.19 | 0.01 | 0.03 | 0.00 | 0.00 |
| $\bar{r}_{\Delta y}$ | Normal | 0.13 | 0.05 | 0.26 | 0.17 | 0.01 | 0.03 | 0.00 | 0.00 |
| $m_\pi$ | Uniform | 0.00 | 173 | -132 | -50.2 | 25.4 | 87.9 | 1.00 | 0.69 |
| $m_y$ | Uniform | 0.00 | 173 | -37.2 | -18.2 | 7.44 | 23.9 | 1.00 | 0.77 |

Table 3.3: Estimated monetary policy parameters of a DSGE model with an augmented Taylor Rule

charge of the Fed. For both specifications, the end of the sample is set at 2008Q3, just before the federal funds rate reached the zero lower bound.

Table 3.3 presents the estimation results for the Taylor rule parameters.[4] Evidence of an asymmetric monetary policy response to both output and inflation can be observed. The negative values of the asymmetry parameters $m_\pi$ and $m_y$ are indicative of a monetary policy that acts stronger when the interest rate is lower. This is specially marked in the extended sample estimation, with higher estimated coefficients, and virtually all the mass of the parameter distribution on the negative side.

The implications of the estimated parameters on the strength of the monetary policy responses are summarized on Table 3.4. Focusing on the full sample estimation where the steady state nominal interest rate is estimated to be 3.8% in annual terms, a movement in the interest rate from 1% to 10% is correlated with a

[4]The prior and posterior distributions for all the model parameters are presented in the appendix, tables B.1 and B.2.

| | 1966Q1-2008Q3 | | | 1987Q1-2008Q3 | | |
|---|---|---|---|---|---|---|
| | r=1% | r=5% | r=10% | r=1% | r=5% | r=10% |
| $r_\pi$ | 3.60 | 1.69 | 1.13 | 2.34 | 1.81 | 1.43 |
| $r_y$ | 0.22 | 0.15 | 0.10 | 0.22 | 0.18 | 0.14 |
| $r_{\Delta y}$ | 0.33 | 0.23 | 0.14 | 0.20 | 0.16 | 0.13 |

Table 3.4: Sensitivity of Taylor rule parameters to the annual interest rate level

change in the Taylor rule response with respect to inflation from 3.60 to 1.13. The same variation in interest rates correlates respectively with a change in the response to the output gap and the output gap growth from 0.22 to 0.10 and from 0.33 to 0.14.

## 3.3 Economic implications of an asymmetric monetary policy

In order to asses the economic impact of an asymmetric monetary policy, the DSGE model is simulated under the estimated parameters from both samples. The simulation considered both the baseline cases with the estimated asymmetry parameters, and also a counterfactual with the asymmetry parameter values set at zero.

The model is simulated using an extended path algorithm, as proposed by Fair and Taylor (1983) and further developed by Adjemian and Juillard(2011, 2013). It allows for occasionally binding constraints, such the non negativity of the nominal interest rate. One advantage of the extended path algorithm compared to other popular algorithms that consider these kind of constraints, like the one by Guerrieri and Iacovello(2015), is that it can also handle nonlinear models, avoiding linearization altogether[5]. This is a necessary feature due to the nonlinear nature of the model's asymmetric Taylor rule.

The sample starting from 1987Q4 contains only the great moderation era, with very low volatility shocks. This causes the simulated interest rate paths to seldom reach the zero boundary. In order to make both specifications more

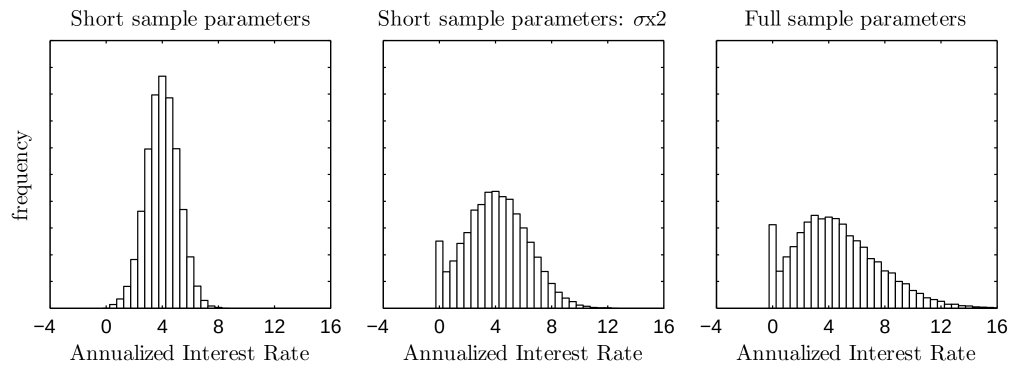


Figure 3.1: Distribution of simulated interest rates for different parametrizations

comparable, the simulations for the shorter sample are made with shocks having twice their estimated standard deviation. Figure 3.1 show the distribution of the simulated interest rates under these different parametrizations.

The statistics to be analyzed are the frequency of deep recessions and zero interest rate events, and the sensitivity of the ZLB frequency to changes in

[5]A comprehensive description of the extended path simulation algorithm is presented in Adjemian and Juillard(2013)

| | Short sample parameters | | | Full Sample parameters | | |
|---|---|---|---|---|---|---|
| | m=$\overline{\text{m}}$ | m=0 | Δ | m=$\overline{\text{m}}$ | m=0 | Δ |
| | | | Deep Recessions | | | |
| Quarters per 100 years | 10.9 | 13.4 | -2.5 | 32.4 | 56.4 | -24.1 |
| | *(0.79)* | *(0.87)* | *(0.21)* | *(2.10)* | *(3.13)* | *(1.88)* |
| Episodes per 100 years | 2.39 | 2.82 | -0.43 | 3.92 | 5.29 | -1.37 |
| | *(0.14)* | *(0.14)* | *(0.07)* | *(0.23)* | *(0.25)* | *(0.16)* |
| Mean Episode Duration | 4.32 | 4.55 | -0.23 | 8.04 | 10.33 | -2.29 |
| | *(0.19)* | *(0.18)* | *(0.14)* | *(0.41)* | *(0.47)* | *(0.44)* |
| E(y\|r=0) | -4.04 | -4.73 | 0.69 | -5.70 | -9.74 | 4.04 |
| | *(0.17)* | *(0.19)* | *(0.03)* | *(0.77)* | *(1.02)* | *(0.85)* |
| | | | ZLB likelihood | | | |
| Quarters per 100 years | 15.7 | 13.8 | 1.88 | 20.1 | 24.3 | -4.17 |
| | *(0.90)* | *(0.83)* | *(0.15)* | *(1.63)* | *(1.84)* | *(0.52)* |
| Episodes per 100 years | 4.70 | 4.17 | 0.53 | 6.45 | 7.34 | -0.90 |
| | *(0.23)* | *(0.22)* | *(0.09)* | *(0.45)* | *(0.49)* | *(0.23)* |
| Mean Episode Duration | 3.27 | 3.23 | 0.04 | 3.09 | 3.27 | -0.17 |
| | *(0.11)* | *(0.12)* | *(0.06)* | *(0.13)* | *(0.13)* | *(0.10)* |
| | | | +1% inflation → Δ ZLB likelihood | | | |
| Δ ZLB Quarters per 100 years | -9.62 | -8.79 | -0.83 | -10.5 | -10.7 | 0.19 |
| | *(0.52)* | *(0.51)* | *(0.15)* | *(0.74)* | *(0.72)* | *(0.53)* |
| Δ ZLB Episodes per 100 years | -2.66 | -2.43 | -0.23 | -2.94 | -3.10 | 0.15 |
| | *(0.16)* | *(0.17)* | *(0.12)* | *(0.26)* | *(0.29)* | *(0.27)* |
| Δ ZLB Mean Episode Duration | -0.35 | -0.40 | 0.05 | -0.39 | -0.09 | -0.30 |
| | *(0.15)* | *(0.15)* | *(0.11)* | *(0.11)* | *(0.14)* | *(0.16)* |

*Bootstraped standard deviation in parentheses*

Table 3.5: Economic implications of an asymmetric monetary policy rule

the long term inflation level. In order to compute these numbers, each model specification is simulated for 50,000 quarters. Simulations are subdivided into 125 subsamples of 100 years each. For every subsample each statistic is computed. Reported in table 3.5 are the sample means and the standard deviation of those means, calculated by bootstraping methods. In order to have a proper comparability between specifications, and to correctly incorporate the covariance into the standard errors of the difference between models, the shocks for every simulation are generated using the same seed for the random generation process. The same is done for the bootstraping random sampling.

### 3.3.1 Severity of Downturns

The main justification behind Reifschneider and Williams' proposal for an asymmetric policy rule is to avoid highly recessive episodes. When interest rates are low, monetary policy preemptively start reacting in a stronger manner. By the time the interest rate hits zero and its unable to stimulate the economy, recessive and/or deflationary events will be of lower magnitude. The rule avoids an excessive downturn that can potentially be amplified if the interest rate hits its minimum and loses its ability to aid on the economic recovery.

Model simulations support this idea. Episodes of deep recessions are defined as periods with a negative output gap larger than 5 percent. A monetary policy with an asymmetric response to variables decreases the number of episodes by a magnitude between 15 to 26 percent depending the sample period of the estimation. The mean duration of each one of those episodes decreases by 22 percent if the full sample period is considered. Considering the restricted sample estimation results, the asymmetry of the Taylor rule reduces the duration by 5 percent.

Overall, the frequency at which the economy is expected to be under this definition of recession decreases by 19 to 43 percent. Also of interest is the fact that, with an asymmetric rule, the mean output gap during ZLB episodes is between 0.7 and 4.0 percent of GDP closer to zero. In the simulated model, when the interest rate is unable to go lower, the economy tends to be in better shape if an asymmetric rule is in place.

### 3.3.2 Likelihood of Reaching the ZLB

The flipside of an asymmetric rule that responds more strongly to shocks when interest rates are lower, is that the likelihood of actually hitting the ZLB could increase. Stronger responses can lead to higher volatility of interest rates and can therefore increase the probability of interest rates reaching their minimum. However, in forward looking models, just the threat to respond strongly can potentially stabilize the variable, leading to an interest rate that actually moves less than would be the case with a weaker response.

Depending the sample used for the estimation, both of these scenarios appear. With the full sample parameters, an asymmetric Taylor rule response decreases the frequency at which the interest rate is at zero level by 17%. On the other hand, with the restricted sample parameters, the zero lower bound frequency increases by 14%. In both cases, the change is driven almost completely by a variation in the number of episodes. The variation in the mean duration of the episodes is negligible.

### 3.3.3 Inflation level and ZLB frequency

Higher inflation levels are expected to reduce the likelihood of reaching the ZLB. As the nominal interest rates will also be higher on average, the restriction on the nominal interest rate will be farther away from the steady state, and therefore will reach its limit less frequently.

The sensitivity of the ZLB frequency to changes in the long term inflation level is also affected by the level of asymmetry in the Taylor rule. A higher inflation target is associated with nominal interest rates that on average will be further from zero. With an asymmetric policy rule this also changes the expected strength of the monetary responses. In forward looking models the effect on the volatility of inflation and output — and thus interest rates — is not clear, and will depend on the particular structure of the economy.

In the case of the full sample estimation when an asymmetric Taylor rule is in place, an increase in the inflation target is, at the same time, more effective in reducing the duration of the ZLB episodes, and less effective in reducing the number of episodes. In terms of the total reduction of the frequency the ZLB being binding, both effects cancel out. With the restricted sample parameters, an increase in the inflation target is more efficient into reducing the frequency of ZLB episodes but just as effective into reducing the mean duration of each episode. Overall, when an asymmetric rule is in place, the reduction on the frequency the interest rate is at zero is 9.5% bigger when the Taylor rule presents asymmetric responses to variables.

## 3.4 Conclusions

The recent financial crisis turned the possibility of the interest rates hitting their minimum level into a real concern, beyond a theoretical curiosity. Some authors, such as Blanchard *et al*(2010), suggested increasing the inflation target as a mechanism that can reduce the likelihood of the zero lower bound being hit.

In order to evaluate the convenience of implementing such a policy, both the likelihood of ZLB episodes and the sensitivity of that likelihood to changes of the inflation level must be quantified. By allowing for asymmetric responses of the Taylor rule, I am able to compute additional effects that would be absent if the analysis were performed using linear systems such as VARs or linearized DSGEs.

If an asymmetric monetary policy is in place — as suggested by the empirical evidence — the use of nonlinear models appear as an important tool to properly evaluate the potential benefits — in terms of the reduction of the ZLB probability — of an inflation target increase.

# Chapter 4

# Welfare costs of inflation and imperfect competition in a monetary search model

In the new monetarist literature, welfare costs of inflation can be derived from consumers restricting their monetary holdings. The reason of doing so comes mainly because they pay the costs of holding money, but only get part of the welfare gains, while part of the transaction surplus goes to the sellers. This share is assumed to be constant and defined by an exogenous bargaining power parameter. In this chapter I endogenize the share of surplus going to consumers. In order to do so, an explicit form of imperfect competition is introduced into a monetary search equilibrium model. By introducing a Cournot type of imperfect competition with free entry, the share of the surplus going to the firms will be determined endogenously.

The welfare costs of imperfect competition and inflation will not then be independent, will be jointly determined within the model.

In that sense, the welfare cost of a given inflation level will be endogenous to the market structure of the economy. At the same time, the level of competition prevailing in the economy will also be influenced by the inflation level. The model is based on Rocheteau and Wright 2005(RW05) and Lagos and Wright(LW05), but modifying the structure of the decentralized market to allow for each buyer to face more than one seller. The equilibrium prices and quantities will be derived from a Cournot style competition between the sellers. As in LW05 and RW05, inflation will be costly because it increases the cost of holding money. This has the effect of reducing the amount of money held by agents in the economy.

In this framework the welfare loss from inflation gets amplified by an *"imperfect competition multiplier"*, with a mechanism similar to Jaimovich and Floetotto (2008). The reduced demand reduces firms' operational profits, which gives an incentive for firms to leave the market, increasing market concentration and lowering output as well as the share of surplus going to consumers. The reduction in consumer surplus due to firms exiting decreases the marginal benefit of carrying money, which in turn reduces money holding even more, further amplifying the welfare loss. Figure 4.1 illustrates this mechanism.

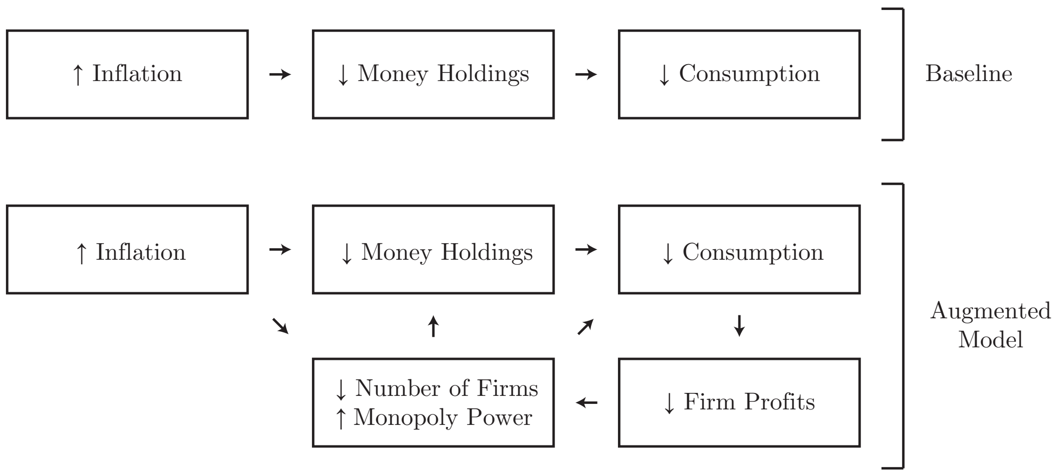


Figure 4.1: Consequences of a rise in the inflation level

## 4.1 A monetary search model with imperfect competition and free entry

The model basic structure is built as a modified version of the framework outlined in Lagos and Wright (2005) and Rocheteau and Wright (2005). In this version of the model, there are 2 types of agents: *buyers* and *sellers*. There is a continuum of buyers of mass 1. There is also an infinite amount of potential sellers that are able to participate in the economy, from which an amount $N$ chooses to do so.

Time is discrete, and each period is subdivided in two subperiods, day and night. For every type of agent, the discount factor is $\beta_d$ between the day and night, and $\beta_n$ between the night and the next period's day. $\beta_d, \beta_n \in [0, 1]$ and $\beta = \beta_d \beta_n$. During the day, all agents consume, but labor is only supplied by buyers. During the night, sellers supply labor, and only buyers are able to consume. During the day there will be a frictionless centralized market, while at night buyers will search for any of $S$ submarkets, and only if they are able to find one, will they be able to consume. Each submarket will be populated by $n = N/S$ sellers that will compete among themselves in a Cournot fashion in order to serve the buyers. The number of submarkets will be normalized to 1. The friction during the night, were sellers are able to produce but can't consume, and buyers want to consume but cannot produce, combined with the assumption that agents are anonymous generates a role for money as a trade facilitator.

In general, for agents type $i$, preferences are $U_i\left(x_i, h_i, Q_i, H_i\right)$, where $\left(x_i, h_i\right)$ and $\left(Q_i, H_i\right)$ represent consumption and labor during the day and night respectively:

$$U_{i,t}\left(x_{i,t}, h_{i,t}, Q_{i,t}, H_{i,t}\right) = v_i\left(x_{i,t}\right) - \varsigma_i\left(h_{i,t}\right) + \beta_d\left[u_i\left(Q_{i,t}\right) - \varsigma_i\left(H_{i,t}\right)\right] \tag{4.1}$$

For buyers, utility of consumption is defined by $v_b\left(x_{b,t}\right) = \omega \ln\left(x_{b,t}\right)$ during the day and $u_b\left(Q_{b,t}\right) = Q_{b,t}^{1-\eta_1} / \left(1-\eta_1\right)$ during the night. Define $y_t = \int^1 x_{b,t} + \int^N x_{s,t}$ as the total amount of consumption goods produced by the buyers. During the day, production technology converts labor into consumption goods one to one, with a constant and unitary marginal cost of labor. Cost of production for buyers during the day is then $c_b\left(y_t\right) = y_t$. Sellers' marginal utility of consumption during the day will be linear and unitary: $v_s\left(x_{s,t}\right) = x_{s,t}$. During the night, sellers' labor disutility will be $\varsigma_i\left(H_{i,t}\right) = H_{s,t}^{1+\eta_2} / \left(1+\eta_2\right)$. The amount of labor required to produce a unit of a night consumption good is 1. The cost function for night consumption goods can then be written as $c_s(q_{s,t}) = q_{s,t}^{1+\eta_2}/(1+\eta_2)$. At the night market , $q_{s,t}$ is the production of each seller, where $\int^N q_{s,t} = \int^1 Q_{b,t}$.

The preferences of each type of agent can be expressed as

$$U_{b,t} = v_b(x_{b,t}) - y_t + \beta_d u_b(Q_{b,t}) \tag{4.2}$$

$$U_{s,t} = v_s(x_{s,t}) - \beta_d c_s(q_{s,t}) \tag{4.3}$$

In order to simplify the notation, the subscripts denoting time and the type of agent during the night market will be dropped.

The real value of an amount of money $m$ in the hands of an agent at date $t$, is defined by $z_t = \phi_t m_t$. With a constant gross growth rate of money of $\gamma$, the evolution of the price of money should follow $\phi_{t+1}/\phi_t = 1/\gamma$ in the steady state.

Let $V_b$ and $W_b$ be the value functions of a buyer in the night and day market respectively. In the centralized day market, the problem of a buyer $j$ with $z_j$ real money holdings is

$$\begin{aligned} W_b(z_j) = & \max_{z'_j, x_j, y_j} \quad v_b(x_j) - y_j + \beta_d V_b\left(z'_j\right) \\ & \quad st \qquad z'_j + x_j = z_j + T_j + y_j \end{aligned} \tag{4.4}$$

Where $z'_j$ are the real money balances taken into the night market and $T_j$ are real transfers due to changes in the aggregate money supply. By plugging the restriction into the maximization problem the value function becomes $W_b(z_j) = z_i + T_j + \max_{z'_i, x_i} \left\{v_b(x_j) - x_j - z'_j + \beta_d V_b\left(z'_j\right)\right\}$. Two features of the value function become clear: First, $W_b$ is linear in $z_j$. Second, the optimal values for $z'_j$ and $x_j$ are independent from the amount of money that is brought to the day market.

In the night market, the value function of buyer $j$ who is carrying an amount on money $z_j$ is

$$\begin{aligned} V_b(z_j) = \quad & \max_{Q_j, d_j} \quad \alpha_b(S)\left[u(Q_j) + \beta_n W_b\left(\frac{z_j - d_j}{\gamma}\right)\right] + (1 - \alpha_b(S))\left[\beta_n W_b\left(\frac{z_j}{\gamma}\right)\right] \\ & st \quad pQ_j = d_j \\ & \qquad z_j - d_j \geq 0 \end{aligned} \tag{4.5}$$

where $z_j$ correspond to the money brought to the day market, while $d_j$ represent the total monetary spending if a match is made. The probability a buyer will find a suitable match is $\alpha_b\left(S\lambda^{-1}\right) = S\lambda^{-1}\left(1 - e^{-S^{-1}\lambda}\right)$, where $S\lambda^{-1}$ is the product between the possible successful matches (in this case the number $S$ of markets to be found) , and $\lambda^{-1}$ – which represents search efficiency – and is related to search technology or search effort. Under this specification, $\alpha_b'(S/\lambda) > 0$, $\alpha_b''(S/\lambda) < 0$ , $\lim_{(S/\lambda)\to 0} \alpha_b(S/\lambda) = 0$ and $\lim_{(S/\lambda)\to\infty} \alpha_b(S/\lambda) = 1$. As the number of markets $S$ is normalized to 1, the probability of a match will depend exclusively on the search efficiency parameter $\lambda^{-1}$.

As the value function in the night market is linear in money, the utility maximization problem on the night market can be expressed, conditional on a match, as one of quasilinear utility, where, given money holdings $z_j$, buyers maximize utility considering that spending money on the night market means not being able to do it on the day market next period.

$$
\begin{aligned}
\max_{Q,d} \quad & U_j = u\left(Q_j\right) + a\left(z_j - d_j\right) \\
st \quad & \qquad pQ_j = d_j \\
& \qquad z_j - d_j > 0
\end{aligned}
\tag{4.6}
$$

Where $a = \beta_n/\gamma$ represent the utility of carrying money to the next period, and, as in RW, depends on the discount factor and the inflation rate.

If buyers carry enough money so the non negativity restriction on cash holdings is not binding, the first order conditions imply that

$$u'\left(Q_j\right) - ap = 0 \tag{4.7}$$

This gives us the indirect demand function when money holdings do not restrict the buyer decision

$$p_U = \frac{u'\left(Q_j\right)}{a} \tag{4.8}$$

### 4.1.1 A decentralized market with Cournot competition

After searching, $B_s$ buyers will find a market populated by $n$ sellers, where $0 \leq B_s \leq 1$ .

Each seller maximizes profits subject to an inverse demand function $p\left(Q/B_s\right)$, a cost function $c\left(q_i\right)$, and a fixed cost of entry $\tau_f$ .

It will be assumed that all buyers search together, so they either all find the night market with probability $\alpha_b$, or they all fail to find anything. An alternative specification could be totally independent search, where every buyer with infinitely small mass has a random chance to have a successful search. In this case, the market will receive with certainty a mass $\alpha_b$ of buyers. The night market will never receive more, and will never receive less.

These two specifications can be thought of the two polar cases of a binomial distribution characterization of the probability of a market to receive customers. Assume that buyers organize their search by grouping into $G$ clusters, where $G \in \mathbb{N}_{\geq 1}$. For every given $G$, the set containing all possible amounts of buyers the market can receive, is denoted by $\psi = g/G$, where $g \in \mathbb{N}_0^G$. The chance that each cluster of buyers will find the market is $\alpha_b$. The probability that the market receives a mass $\psi$ of buyers is then $pr\left(B_k = \psi\right) = \binom{G}{g}\left(\alpha_b\right)^g\left(1-\alpha_b\right)^{G-g}$, while $pr\left(B_k = \theta \mid \theta \notin \psi\right) = 0$. For example, assume $\alpha_b = 0.5$, and $G = 2$, meaning there is a 50% chance a buyer can find the market, and buyers are grouped into two clusters. Then $g = \{0, 1, 2\}$, $\psi = \{0, 0.5, 1\}$, and $pr\left(B_k = \psi\right) = \{0.25, 0.5, 0.25\}$. With 25% chance the market will receive no customers, with 50% chance will receive a mass 0.5 of buyers, and with 25% will receive a mass 1 of buyers. The assumption of all buyers searching together correspond to the case of $G = 1$, while the alternative of completely independent search correspond to $G \to \infty$. The main difference between these parameterizations is on the volatility of the expected customers received. When $G = 1$, uncertainty is maximized, and when $G \to \infty$ uncertainty goes to 0. In particular, the volatility of expected customers can be expressed as $\sigma^2_{B_s} = \alpha_b\left(1-\alpha_b\right)/G$. While $G$ could be calibrated to match some empirical volatility moment, a unitary value is assumed as it greatly simplifies the mathematical derivation and solving of the model. Under this assumption, if a match is made, $B_k = 1$, and $Q = Q_i$.

When maximizing profits, sellers take into account that while they have to pay the cost of producing in the night, they can only use the obtained revenue at the next period's day market. Therefore, the relevant value for revenues is not $pq_i$, but $apq_i$, where $a = \beta_n/\gamma \leq 1$.

$$\begin{array}{ll} \max\limits_{q_i} & Profit_i = aq_i p\left(Q\right) - c\left(q_i\right) - \tau_f \\ st & p = \frac{u'(Q)}{a} \\ & Q = \sum\limits_{i=1}^{N} q_i \end{array} \tag{4.9}$$

The first order condition with respect to $q_i$ simply shows that the optimal firm behavior requires the discounted marginal revenue to be equal to the marginal cost of the last unit sold.

$$\underbrace{u'\left(Q\right) + q_i u''\left(Q\right)}_{Mg\ revenue_U} = \underbrace{c'\left(q_i\right)}_{Mg\ cost} \tag{4.10}$$

The first term on the LHS corresponds to the quantity effect on revenue. The second term corresponds to the price effect of an extra unit put on the market. Assuming positive and marginally decreasing utility of consumption, the first term is expected to be positive, and the second negative.

If every seller in the market has the same preferences and cost functions, $q_i = Q/n$, and the first order condition becomes

$$\underbrace{u'\left(Q\right)}_{ap} + \frac{Q}{n} u''\left(Q\right) = c'\left(q_i\right) \tag{4.11}$$

As the number $n$ of firms increases, the influence of each firm's output on prices goes down. In the limit, if $n \to \infty$, then the second term disappears, and the result converges to the perfect competition solution where the discounted price $ap$ equals the marginal cost.

### 4.1.2 Restricted money holdings

The inverse demand function from equation (4.8) is only valid when the buyers carry enough money to pay for that quantity of goods. If a point in the demand function is not feasible because of the lack of sufficient money holdings, the only way to make the buyer willing to absorb that quantity will be to lower the price enough so that she is able to buy it.

From the point of view of the seller, the relevant inverse demand function, defined as the price at which the buyer is willing (and able) to acquire any specific amount of goods will be the minimum between the unrestricted demand and the maximum units the money holdings are able to buy.

$$p = \min \left\{ \underbrace{u'(Q)/a}_{p_U}, \underbrace{z/Q}_{p_R} \right\} \tag{4.12}$$

Figure 4.2 illustrates this idea. When money holdings are not enough to buy a particular set $(p, Q)$, the demand function is depressed so as to comply with the constraint of non-negativity of money holdings. If money holdings $z$ are less than $\widehat{d}$, the spending that would occur if money holdings were not binding, then the equilibrium of the market will be determined by firms maximizing profits subject to

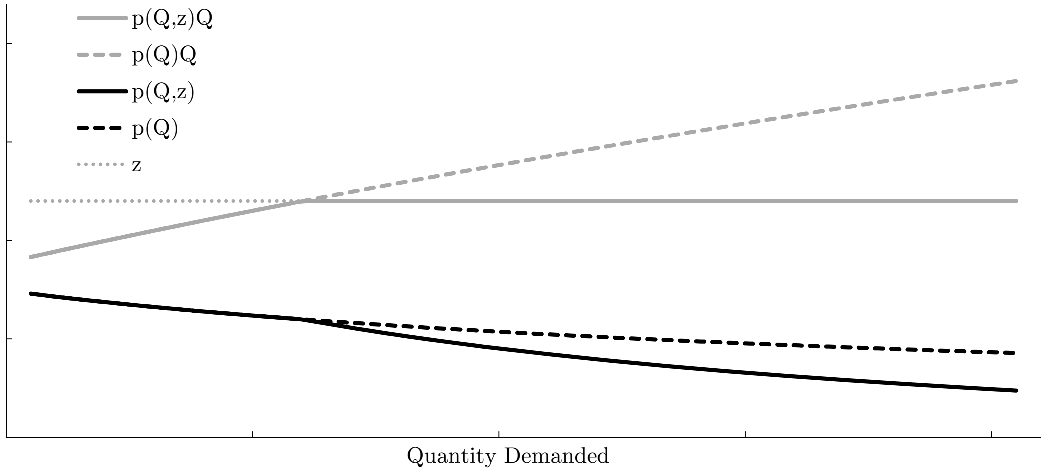


Figure 4.2: Demand in the decentralized market with restricted money holdings

$p = z/Q$.

$$\begin{aligned} \max_{q_i} \quad & \mathit{Profit}_i = apq_i - c\left(q_i\right) - \tau \\ st \quad & p = \tfrac{z}{Q} \\ & Q = \sum_{i=1}^{N} q_i \\ & \tau = \tau_f + \tau_m \end{aligned} \tag{4.13}$$

Again, the first order condition with respect to $q_i$ shows that the optimal firm behavior requires the discounted marginal revenue to be equal to the marginal cost of the last unit sold.

$$\underbrace{(az/Q)\left(1 - q/Q\right)}_{Mg\ revenue_R} = \underbrace{c'\left(q_i\right)}_{Mg\ cost} \tag{4.14}$$

In equilibrium $Q = nq$ and the discounted price is closer to the marginal cost as the number of firms increases.

$$\underbrace{\frac{az}{Q}}_{ap}\left(1 - \frac{1}{n}\right) = c'\left(q_i\right) \tag{4.15}$$

In this case the markups, defined as the ratio between prices and marginal costs, can be expressed as an explicit function of the discount factor, inflation level, and the equilibrium number of firms: $Mkup = \left(\frac{N}{N-1}\right)\frac{\gamma}{\beta_n}$

Depending on the level of money holdings $z$, the equilibrium quantities sold in the night market will be given by eqs (4.11) or (4.15). Define $\widehat{Q}$ as the quantity where money holdings became restrictive, the quantity such that $u'(\widehat{Q})/a = z/\widehat{Q}$. It can be seen see in figure 4.3 that at $\widehat{Q}$ the inverse demand function has a sudden change in slope, going from the unrestricted case from the left to the restricted case to the right. This causes a discontinuity in the marginal revenue function, defined as $\partial pq/\partial q$. Given a discontinuous marginal revenue function there are 3 possible

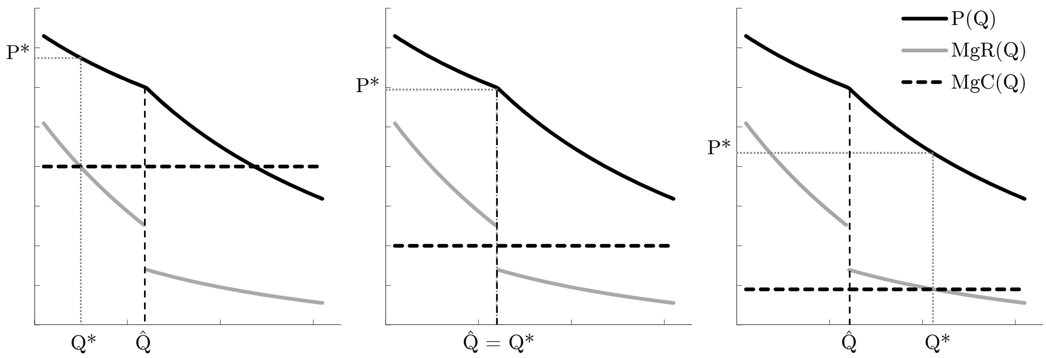


Figure 4.3: Possible equilibria in the decentralized market with restricted money holdings

equilibria, depending on the relationship between marginal cost and marginal revenue evaluated at $\widehat{Q}$ :

$$if \quad C'(\widehat{Q}) > MgR_U(\widehat{Q}) \qquad \begin{cases} MgR_U\left(Q^*\right) = C'(Q^*) \\ P^* = P_U\left(Q^*\right) \end{cases}$$

$$if \quad MgR_R(\widehat{Q}) < C'(\widehat{Q}) < MgR_U\left(\widehat{Q}\right) \quad \begin{cases} Q^* = \widehat{Q} \\ P^* = P_U\left(Q^*\right) = P_R\left(Q^*\right) \end{cases} \tag{4.16}$$

$$if \quad C'(\widehat{Q}) < MgR_R(\widehat{Q}) \qquad \begin{cases} MgR_R\left(Q^*\right) = C'(Q^*) \\ P^* = P_R\left(Q^*\right) \end{cases}$$

With these equations the equilibrium prices and quantities for the night market can pinned down for any given $z$ and $N$. Let $P^M(z)$ be the price that will prevail in the market when consumers bring an amount $z$ of money.

### 4.1.3 Equilibrium money holdings

The amount of money holdings that a buyer brings to the night market is derived from maximizing the value function. Assume that each individual buyer is small enough such that she is incapable of modifying the equilibrium prices[1]. From eq (4.4), the FOC of $W^b(z_b)$ with respect to $z_b'$ is

$$\beta_d V^b_{z_b'}\left(z_b'\right) - 1 = 0 \tag{4.17}$$

[1]This assumption can also potentially generate suboptimal money holdings, as individual buyer won't take into consideration the possible benefits that their money holding could bring to other buyers and sellers

Equation (4.5) shows that if $z_b > \widehat{d}$, and the money holdings are larger than the equilibrium spending, then $\beta_d V^b_{z'_b} = \frac{\beta_n}{\gamma}$. Assuming, as in RW05, that $\gamma \geq \beta > 0$, with $\beta = \beta_d \beta_n$, it is clear then that bringing money holdings larger than $\widehat{d}$ will never be optimal. This is because holding money is costly, and therefore holding money that's not going to be spent implies a welfare loss. If $z_b < \widehat{d}$, then $z_b = d$, and all money holdings are going to be spent if a match occurs, so $Qp = z$. In this case the FOC with respect to money holdings can be expressed as

$$\beta_d V^b_z (z) - 1 = \beta_d \alpha_b \left[ \frac{\partial u (Q)}{\partial Q} \frac{\partial Q}{\partial z} \right] + \frac{\beta}{\gamma} (1 - \alpha_b) - 1 = 0 \tag{4.18}$$

For any given price that is expected to clear the market in case there is a match, the amount of money buyers will be willing to hold will be denoted by the function $Z^B (p)$, that correspond to the money holdings such that

$$u \left( p/Z^B \right)' /Z^B = (\beta_d \alpha_b)^{-1} - a \left( \alpha_b^{-1} - 1 \right) \tag{4.19}$$

The equilibrium can be thought of as the intersection of two best response functions: on one hand the market, for any amount of money holdings $z$, is responding with a market clearing price $P^M (z)$. On the other hand, for each price $p$ buyers will respond with money holdings equal to $Z^B (p)$. Figure 4.4 shows how, for any given number of firms $n$, both reaction functions are affected from an increase in the inflation level $\pi = \gamma - 1$. If a match is made, higher inflation will make sellers demand a higher compensation for their labor, as by the time they will be able to spend their monetary payments, the value of money will be lower. This causes an upward shift in the $P^M (z)$ function, implying that at any given $z$, the market will

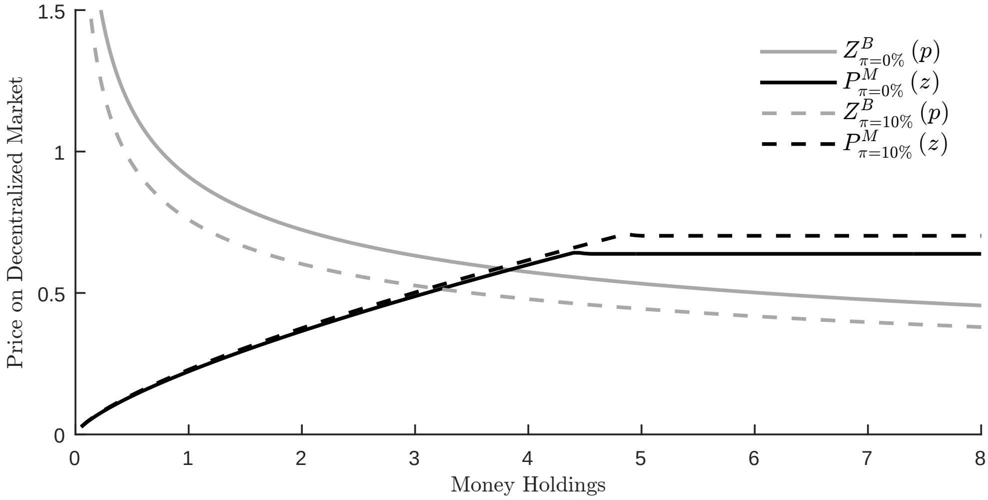


Figure 4.4: Inflation and the equilibrium money holdings with fixed number of sellers

clear with a higher price. The best response functions $P^M(z)$ appear flat to the right because after money holdings stop being binding a buyer carrying extra money doesn't change the equilibrium prices or quantities.

The other, and by far more influential way inflation affects equilibrium money holdings, is by shifting the best response function $Z^B(p)$. Higher inflation implies that when no match is made, buyer's monetary holdings loss of value will also be higher. As a consequence, for any price $p$, buyers will be willing to bring less money to the night market, shifting $Z^B(p)$ to the left.

Assuming an equilibrium with restricted money holdings (it will be for all chosen parameterizations), analytical expression can be found, for any number of firms, for the equilibrium defined by $z^*, p^*$ and $Q^*$ :

$$z^* = A_1 A_2^{A_3+A_4} \tag{4.20}$$

$$p^* = A_1 A_2^{A_4} \tag{4.21}$$

$$Q^* = A_2^{A_3} \tag{4.22}$$

Where $A_1 = \frac{\gamma\beta_d\alpha_b}{\gamma+(\alpha_b-1)\beta}$, $A_2 = \left(\frac{A_1\beta_n(n-1)}{\gamma n\hat{}1-\eta_2}\right)^{\frac{1}{1+\eta_2}}$, $A_3 = \frac{1+\eta_2}{\eta_1+\eta_2}$, and $A_4 = -A_3\eta_1$.

### 4.1.4 Free Entry and Number of sellers

The equilibrium number of firms will be determined by the free entry condition, where firms will enter until their expected profits equal zero.

$$E\left[Profit_i\right] = \alpha_k \cdot \left(ap^* q_i^* - c\left(q_i^*\right)\right) - \tau = 0 \tag{4.23}$$

If all firms behave the same, $q_i = Q/n$, and the equilibrium number of firms will be the $n$ such that

$$n = \frac{\left(\left(1+\eta_2\right)ap^* - \left(Q/n\right)^{\eta 2}\right)\alpha_k Q^*}{\tau\left(1+\eta_2\right)} \tag{4.24}$$

The effect of inflation on the number of firms will come from its effect on profits. First, inflation directly affects the discount factor $a$, and therefore the discounted income. Also, higher inflation restricts buyers' money holdings and therefore total spending $pq$. The consequent reduction on expected profits will mean the market will clear with fewer firms, as less competition increases per seller expected profits, counteracting the impact of a reduced demand. Figure 4.5 shows the effect of inflation on expected profits and how it induces a reduction in the number of sellers, increasing market concentration.

By solving equations (4.20), (4.21), (4.22) and (4.24) the equilibrium can be fully characterized for any set of parameters.

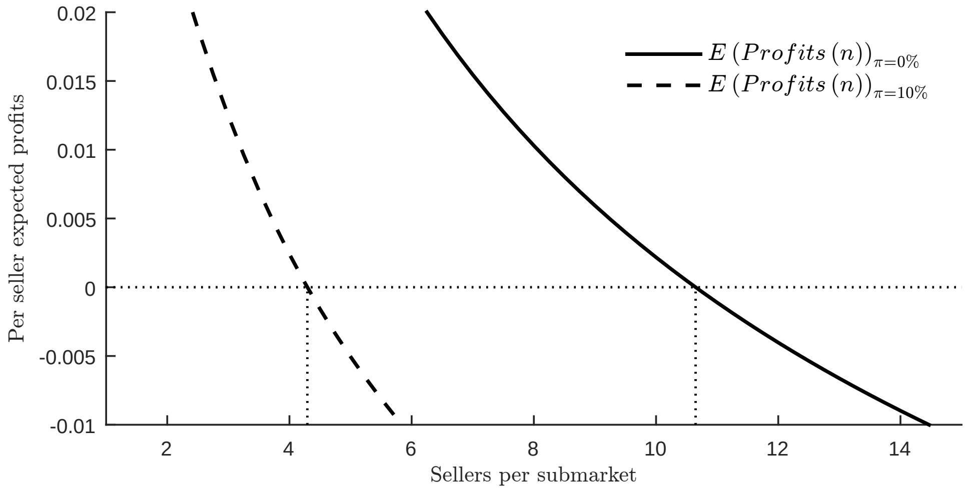


Figure 4.5: Inflation and the equilibrium number of sellers under free entry

## 4.2 Calibration and Results

The parameters in the baseline model will be calibrated as it follows: The labor disutility parameter $\eta_2$ is set at 2.503, as in Smets and Wouters (2003). $\beta$ is set at an annual rate of 0.96, with $\beta_d = 1$ and $\beta_n = 0.96$. This assumes no discount between subperiods. The fixed cost of participation for the seller $\tau_f$ is calibrated so that with an inflation rate of 2%, the markups in the decentralized market equal 20%, as in Craig and Rocheteau 2008 (CR08). In a similar vein as LW05 and CR08, parameters $\omega, \lambda,$ and $\eta_1$ are chosen to match empirical money demand.

Following Lucas (2000), the real money holdings over income ratio $L = M/PY$ is defined as a function of interest rates $i$, where the ratio between real balances and income depends on the cost of holding cash. In this model specification, $M$ is analogous to $z$. Nominal GDP will be constructed by multiplying output in each subperiod by its price. During the night, prices and output will be given by equations (4.21) and (4.22). In the centralized frictionless day market, prices will be equal to marginal costs, and therefore buyers will equalize marginal utility with the unitary marginal cost. Buyers optimal consumption $x_b$ will then equal $\omega$. Sellers, on the other hand, will have money holdings from the previous night market, although its value will have diminished due to inflation. Given the assumption of $\beta \leq \gamma$, it will be optimal for sellers to spend all their cash. This will allow them, in case they were matched last period, to buy $z/\gamma$ units of the day's good. Similar to LW05, and CR08 the model's counterpart to Lucas money demand $L(i) = M/PY$ is defined as

$$L(\gamma) = \frac{M}{PY} = \frac{z^*}{x_b + x_s + \alpha_b pQ} = \frac{z^*}{\omega + z^* \alpha_b \cdot (1 + \gamma^{-1})} \tag{4.25}$$

Figure 4.6 shows how the model fits the downward slope of the data for the 1915-2014 sample[2] .

This baseline parameterization allows for the analysis of the consequences of a rise in inflation. The focus will be on the impact of going from 0% to 10% on four variables: number of sellers participating in the economy $N$, money holdings $z$,

[2]As in Lucas(2000), the interest rate is the short rate commercial paper. From 1915-1975 its taken from *Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition*(HSUS), compiled from Friedman and Schwartz(1982). Between 1976-2014 is from the Federal Reserve Bank of St. Louis FRED Database.

The Money Supply is M1 in Billion of Dollars. Between 1915-1958 its from HSUS, compiled from Friedman and Schwartz(1982) and Rasche(1987).Between 1959-2014 is from the FRED database

GDP from 1915-1946 comes from HSUS's *GDP Millennial Edition Series*, compiled from varied sources. Between 1947-2014 is from the FRED Database

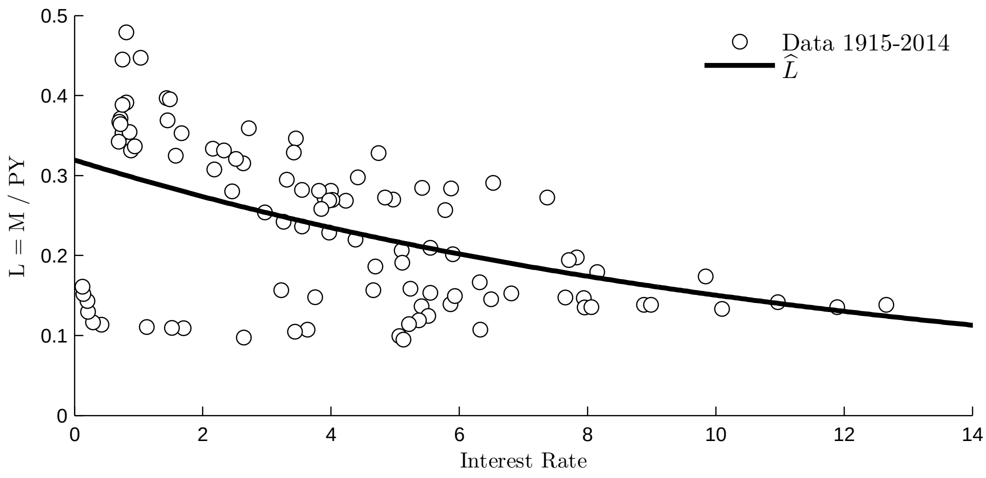


Figure 4.6: Money demand and model fitted values

consumer surplus share during the night market, and welfare losses from inflation.

As in LW05 and CR08, let $(1 - \Delta_0^\pi)$ be the welfare loss, defined as the percentage of consumption agents would be willing to give up to go from an inflation rate of $\pi$ to zero[3]. Defining the long term total utility of buyers and sellers given an inflation rate of $\pi$ as

$$U_\pi = v_b\left(x_{b,\pi}\right) + v_s\left(x_{s,\pi}\right) - \left(x_{b,\pi} + x_{s,\pi}\right) + \alpha_b\beta_d\left[u\left(Q_\pi\right) - n_\pi c\left(q_\pi\right)\right] - N_\pi\tau_f \quad (4.26)$$

The welfare cost of inflation will be the value of $(1 - \Delta_0^\pi)$ such that

$$U_\pi = v_b\left(x_{b,0}\cdot\Delta_0^\pi\right) + v_s\left(x_{s,0}\cdot\Delta_0^\pi\right) - \left(x_{b,0} + x_{s,0}\right) + \alpha_b\beta_d\left[u\left(Q_0\cdot\Delta_0^\pi\right) - n_0 c\left(q_0\right)\right] - N_0\tau_f \quad (4.27)$$

[3]Lucas(2000) computes the welfare cost in a slightly different, but mainly equivalent way. He ask, given an inflation of $\pi$, how much extra consumption should be given to agents in order to make them indifferent to a zero inflation equilibrium

The effect of inflation on the analyzed variables is decomposed into 4 parts. First, the direct effect of inflation on prices and quantities sold, maintaining both $z$ and $n$ at their zero inflation values. Second, the additional effect due to the role inflation plays in determining the equilibrium number of sellers. The share will be obtained by computing the model with a level $\pi$ of inflation, but keeping $z_\pi = z_0$. To avoid double counting, the direct effect of inflation on prices and quantities is subtracted. The direct effect of inflation on the equilibrium money holdings will be computed the same way as the previous one, but keeping $n_\pi = n_0$. Finally, the additional effect of the feedback loop between reduced money holdings and less participating firms will be computed by subtracting all three contributions previously obtained from the total impact of inflation on the model variables. Figure 4.6 show this decomposition for the three analyzed variables.

As expected, higher inflation, by increasing the cost of money, causes a decrease in the desired holdings. However, less than a fifth of the effect can be attributed directly to the rise in inflation. The second round effects causes most of the impact, where the decrease in money holdings decreases the number of sellers, which further reduces the desired holdings. In the case of the number of sellers willing to participate in the economy, the proportion of the variation due to second round effects is similar.

The share of total surplus going to the buyers during the night is a key variable, as it contributes to the suboptimality of money holdings. Buyers only consider the private benefits of money. As the benefits to the sellers are not taken

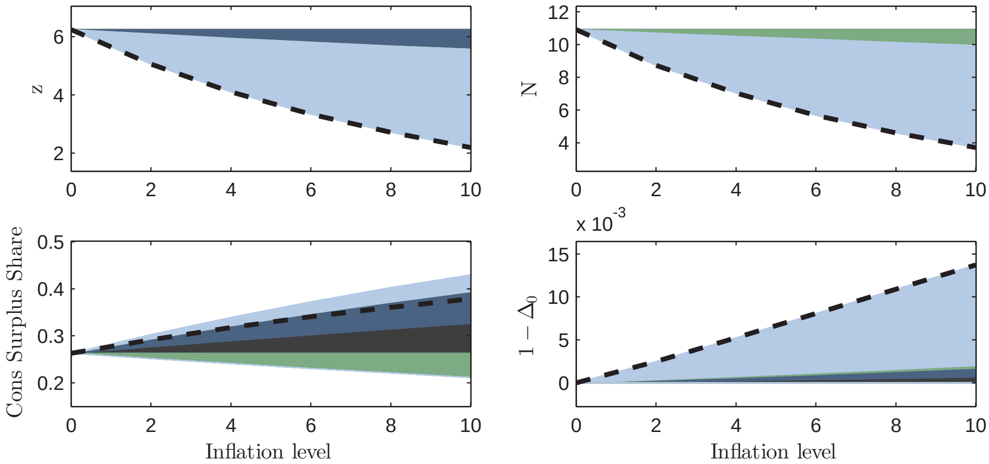


Figure 4.7: Consequences of an increase in the inflation level, different contributions. In black the direct effect on prices. The blue area represents the direct effect of inflation on money holdings. Green is the direct effect on the number of sellers willing to participate. The light blue area corresponds to the feedback loop between money holdings and number of sellers.

into account while making the decision, less money will be brought than what is optimal for the economy as a whole.

The direct impact of inflation on prices and quantities increases the buyer's share of the surplus, as this effect mainly affects the sellers. With higher inflation, the monetary payments sellers receive for their labor is worth less in terms of next sub-period purchasing power. This causes a reduction in the sellers' surplus due to the night market's transactions.

The effect of inflation on the desired money holdings increases the buyer's share, as the demand reduction decreases sellers' profits.

The reduction in the number of participating sellers due to inflation tends to decrease the buyers' surplus share, as the higher monopoly power allows sellers to extract a higher amount of surplus from the buyers.

The feedback loop between fewer firms and reduced money holdings doesn't have a clear predicted consequence on the buyer's surplus share, as it is a combination between the positive effect of the money holdings reduction and the negative effect due to less firms participating. For the baseline calibration, the overall effect is an increase in the buyers surplus share.

Overall, the increase in inflation tends to increase the consumer surplus share. This help dampen the welfare costs of inflation, as with a higher consumer surplus a higher proportion of the benefits from holding money will be internalized, reducing the inefficiency in the equilibrium money holdings.

Regarding welfare costs, the overall cost of 10% of inflation is 1.37%, meaning that agents would be willing to give up 1.37% of consumption to have zero[4] instead of ten percent of inflation. Again, the majority of the effect come from the interaction of the effects arising from changes in money holdings and number of firms, which accounts for 85% of the total cost.

As a robustness check, several alternative parameterizations are chosen. Regarding the money demand empirical match, the parameters $\omega, \lambda,$ and $\eta_1$ are re-estimated to fit more restricted samples in order to have a better comparison with

[4]The baseline measure for the cost of inflation is defined as the cost from going from 0% inflation. As a benchmark for inflation costs its also possible to compute $(1 - \Delta_f^{\pi})$, the cost of deviating from the Friedman rule for optimal inflation, that sets nominal interest rates at 0%. With this measure, the cost of a 10% inflation rate rises to 1.44%

Lucas and LW05 who, respectively, end their sample on the years 1994 and 2000. Also, as the elasticity parameter $\eta_1$ just barely affects the fit for moderate inflation, I fixed it at the values of 0.1, 0.5 and 0.9, choosing the parameter $\omega$ and $\lambda$ that better fit the data. Figure 4.8 presents the fit with the different parameterizations.

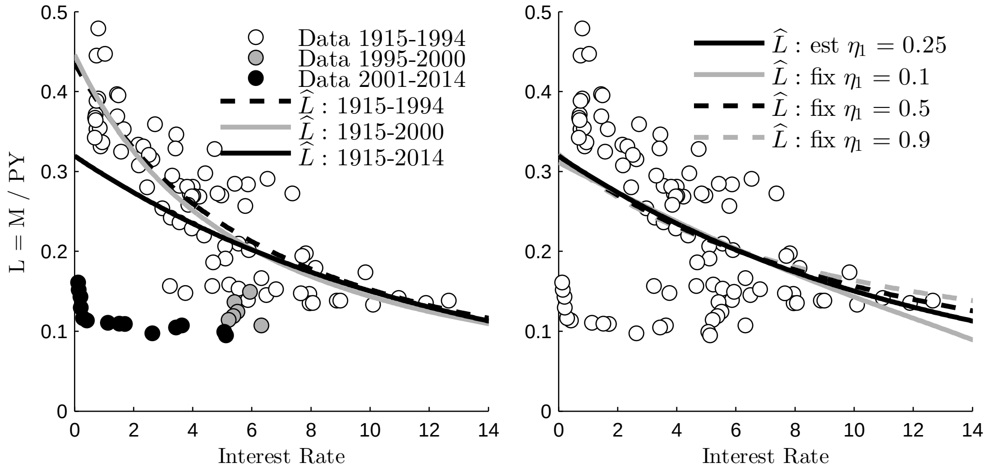


Figure 4.8: Money demand and model fitted values for different samples and parametrizations

Also chosen are alternative balances for the day and night discount factors, and different values for the labor disutility parameter.

Table 4.1 presents the estimated welfare costs of a 10% rate of inflation across different specifications. The estimated cost, ranging from 0.9% to 1.7%, sits on the lower part of the range of 1.2%-4.6% found by LW05 and 0.8%-10.4% found by CR08, and closer to Lucas' estimated range of 0.5%-1.5%.

Across specifications, the total welfare cost is mainly derived through interacting effects between changes in money holdings and number of firms. The

| | | | | | |
|---|---|---|---|---|---|
| | Sample: 1915-2014 | Sample: 1915-2014 | Sample: 1915-2014 | Sample: 1915-2014 | Sample: 1915-2014 |
| | $\beta_d=1.00$ $\eta_1=0.15$ | $\boldsymbol{\beta_d=0.98}$ $\eta_1=0.17$ | $\boldsymbol{\beta_d=0.96}$ $\eta_1=0.19$ | $\beta_d=1.00$ $\eta_1=0.17$ | $\beta_d=1.00$ $\eta_1=0.19$ |
| | $\omega=19.8$ $\eta_2=2.50$ | $\omega=16.6$ $\eta_2=2.50$ | $\omega=14.2$ $\eta_2=2.50$ | $\omega=14.3$ $\boldsymbol{\eta_2=1.50}$ | $\omega=14.3$ $\boldsymbol{\eta_2=1.00}$ |
| | $\lambda=1.29$ $\tau_f=0.23$ | $\lambda=1.51$ $\tau_f=0.20$ | $\lambda=1.73$ $\tau_f=0.18$ | $\lambda=1.53$ $\tau_f=0.15$ | $\lambda=1.71$ $\tau_f=0.10$ |
| | (1) | (2) | (3) | (4) | (5) |
| $1-\Delta_0$ | 0.014 | 0.014 | 0.014 | 0.014 | 0.014 |
| % $\pi\to q$ | 0.049 | 0.049 | 0.049 | 0.062 | 0.072 |
| % $\pi\to z$ | 0.076 | 0.086 | 0.095 | 0.138 | 0.206 |
| % $\pi\to n$ | 0.023 | 0.023 | 0.023 | 0.010 | 0.000 |
| % $z \leftrightarrow n$ | 0.853 | 0.843 | 0.833 | 0.791 | 0.722 |
| | Sample: 1915-2014 | Sample: 1915-2014 | Sample: 1915-2014 | Sample: 1915-**2000** | Sample: 1915-**1994** |
| | $\beta_d=1.00$ $\boldsymbol{\eta_1=0.10}$ | $\beta_d=1.00$ $\boldsymbol{\eta_1=0.50}$ | $\beta_d=1.00$ $\boldsymbol{\eta_1=0.90}$ | $\beta_d=1.00$ $\eta_1=0.67$ | $\beta_d=1.00$ $\eta_1=0.62$ |
| | $\omega=21.0$ $\eta_2=2.50$ | $\omega=8.53$ $\eta_2=2.50$ | $\omega=3.55$ $\eta_2=2.50$ | $\omega=4.63$ $\eta_2=2.50$ | $\omega=5.21$ $\eta_2=2.50$ |
| | $\lambda=0.58$ $\tau_f=0.37$ | $\lambda=5.45$ $\tau_f=0.03$ | $\lambda=10.63$ $\tau_f=0.006$ | $\lambda=13.4$ $\tau_f=0.006$ | $\lambda=11.1$ $\tau_f=0.008$ |
| | (6) | (7) | (8) | (9) | (10) |
| $1-\Delta_0$ | 0.017 | 0.010 | 0.009 | 0.012 | 0.012 |
| % $\pi\to q$ | 0.048 | 0.039 | 0.038 | 0.028 | 0.029 |
| % $\pi\to z$ | 0.045 | 0.193 | 0.260 | 0.228 | 0.221 |
| % $\pi\to n$ | 0.011 | 0.056 | 0.073 | 0.055 | 0.055 |
| % $z \leftrightarrow n$ | 0.896 | 0.713 | 0.630 | 0.689 | 0.696 |

Table 4.1: Estimated welfare cost of a 10% rate of inflation for different sample sizes and parametrizations. The parameters that differ from the baseline specification are highlighted

feedback loop 63% to 90% on the total cost of inflation.

## 4.3 Conclusions

By augmenting a monetary search model with imperfect competition and free entry, I show that the cost of inflation can be derived not only from the reduction of money holdings, but also from the effect inflation can have on the equilibrium number of firms the market can support. Moreover, these two effects reinforce each other, the reduction inn money holdings causes a reduction in the number of firms a market can support, and also a reduction of the number of firms reduces the incentives to carry money, by increasing the firms monopoly power. I find that under this framework, the vast majority of the welfare costs of inflation can be traced back to the feedback loop between the reduction of money holdings and the increase in market concentration.

# Appendix A

# Time varying parameters VAR estimation procedure

Following the methodology from Gali and Gambetti(2009), and similar to Primiceri (2005), Cogley and Sargent (2001,2005), and Cogley and Sbordone(2008), an $n$ variables and $p$ lags Bayesian VAR is estimated, with a specification given by:

$$x_t = A_{0,t} + A_{1,t}x_{t-1} + ... + A_{p,t}x_{t-p} + u_t \tag{A.1}$$

Where $x_t$ is a vector of endogenous variables, $A_{0,t}$ is a vector of time varying coefficients, and $A_{i,t}, i = 1, ..., p$ are matrices of time varying coefficients. The residuals $u_t$ are normally distributed with mean zero and var-cov matrix $\Sigma_t$ . Let $A_t = [A_{0,t}, A_{1,t}, ..., A_{p,t}]$ and $\theta_t = vec\,(A'_t)$ a vector that stack all elements of $A_t$. The parameters from $\theta_t$ are assumed to evolve as random walks subject to reflecting barriers that impose stability, ruling out explosive behaviors for the variables. Then, apart from the reflecting barrier, $\theta_t$ evolves as

$$\theta_t = \theta_{t-1} + \omega_t \tag{A.2}$$

Where $\omega_t \sim N(0, \Omega)$. The variance-covariance $\Sigma_t$ is also assumed to change over time. Let $\Sigma_t = F_t D_t F_t'$, where $F_t^{-1}$ is the lower triangular matrix

$$F_t^{-1} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \gamma_{2,1,t} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \gamma_{n,1,t} & \dots & \gamma_{n,n,t} & 1 \end{bmatrix} \tag{A.3}$$

and $D_t$ is the diagonal matrix

$$D_t = \begin{bmatrix} \sigma_{1,t} & 0 & \dots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sigma_{n,t} \end{bmatrix} \tag{A.4}$$

The evolution of $\Sigma_t$ is determined by the evolution of $\gamma_t$ and $\sigma_t$, where the first is a vector of the non-zero and non-one elements of $F_t^{-1}$, and $\sigma_t$ is the vector of diagonal elements of $D_t$.

$$\gamma_t = \gamma_{t-1} + \zeta_t \tag{A.5}$$

$$\ln(\sigma_t) = \ln(\sigma_{t-1}) + \xi_t \tag{A.6}$$

Where $\zeta_t \sim N(0, \Psi)$ and $\xi_t \sim N(0, \Xi)$

Let $\theta^T, \gamma^T$ and $\sigma^T$ be the sequence of the corresponding variables up to time $T$. The conditional prior density is assumed to be given by

$$p\left(\theta^T \middle| \gamma^T, \sigma^T, \Psi, \Xi, \Omega\right) \propto I\left(\theta^T\right) f\left(\theta^T \middle| \gamma^T, \sigma^T, \Psi, \Xi, \Omega\right) \tag{A.7a}$$

where

$$I\left(\theta^T\right) = \Pi_{t=0}^{T} I\left(\theta_t\right) \tag{A.7b}$$

$$f\left(\theta^T \middle| \gamma^T, \sigma^T, \Psi, \Xi, \Omega\right) = f\left(\theta_0\right) f\left(\theta^T \middle| \gamma^T, \sigma^T, \Psi, \Xi, \Omega\right) \tag{A.7c}$$

And $f\left(\theta^T \middle| \gamma^T, \sigma^T, \Psi, \Xi, \Omega\right)$ is consistent with (A.2). The index function $I\left(\theta_t\right)$ equals one if the absolute value of every root from the associated VAR polynomial are larger than one, and zero otherwise. It ensures that the estimated system won't have an explosive behavior by setting the likelihood of those parameters equal to zero. Let $\widehat{z}_{OLS}$ be the estimated parameter $z$ from a time invariant VAR using a training sample with $T_0$ observations. As in Benati and Mumtaz(2007) and Primiceri(2005), the prior densities and parameters take the form of

$$p\left(\theta_0\right) \propto I\left(\theta_0\right) N\left(\widehat{\theta}_{OLS}, \sigma^2_{\widehat{\theta}_{OLS}}\right) \tag{A.8a}$$

$$p\left(\log \sigma_0\right) = N\left(\log \widehat{\sigma}_{OLS}, 10 \times I\right) \tag{A.8b}$$

$$p\left(\gamma_0\right) = N\left(\widehat{\gamma}_{OLS}, \left|\widehat{\gamma}_{OLS}\right|\right) \tag{A.8c}$$

$$p\left(\Omega\right) = IW\left(\frac{1}{0.005 \times \sigma^2_{\widehat{\theta}_{OLS}}}, T_0\right) \tag{A.8d}$$

$$p\left(\Psi\right) = IW\left(\frac{1}{0.001 \times \left|\widehat{\gamma}_{OLS}\right|}, 2\right) \tag{A.8e}$$

$$p\left(\Xi_{i,i}\right) = IG\left(\frac{0.0001}{2}, \frac{1}{2}\right) \tag{A.8f}$$

The realizations from the posterior density are drawn using an Markov chain Monte Carlo(MCMC) algorithm – the Gibbs sampler – which works in an iterative way. Each iteration is done in four steps. In each step, realizations of a subset of the parameters are drawn conditional on a particular realization of the reamaining coefficients. In the next step, another subset of parameters is drawn conditional on the draws from the previous step. Under regularity conditions, the iteration on these four steps produce draws from the joint density.

For each iteration $i$ of the Gibbs sampler, in the first step realizations for $\theta_i^T$ are drawn conditional on $x^T, \gamma_{i-1}^T, \sigma_{i-1}^T, \Psi_{i-1}, \Xi_{i-1}$ and $\Omega_{i-1}$ by using the Carter and Kohn(1994) algorithm. In the second step, using the same procedure described in Primiceri(2005), draws of $\gamma^T$ are obtained conditional $x^T, \theta_i^T, \sigma_{i-1}^T, \Psi_{i-1}, \Xi_{i-1}$ and $\Omega_{i-1}$. In the third step the draws from $\sigma_i^T$, conditional to $x^T, \theta_i^T, \gamma_i^T, \Psi_{i-1}, \Xi_{i-1}$ and $\Omega_{i-1}$, are obtained using the algorithm by Jaquier et al(2004). Finally, in the fourth step, draws from $\Psi_i, \Xi_i$ and $\Omega_i$, conditional to $x^T, \theta_i^T, \gamma_i^T$ and $\sigma_i^T$ are obtained as in Gelman et al(1995). The parameters $\gamma_0^T, \sigma_0^T, \Psi_0, \Xi_0$ and $\Omega_0$ are initialized using the correspondent parameters of the training sample estimated VAR.

# Appendix B

# Smets and Wouters DSGE model

## B.1 Model description

This section describes the equations from the Smets and Wouters (2007) DSGE model that is going to be estimated. The model has 14 endogenous variables: output $y_t$, consumption $c_t$, real value of capital stock $q_t$, capital services used in production $k_t^s$, installed capital $k_t$, capital utilization rate $z_t$, rental rate of capital $r_t^k$, inflation $\pi_t$, wages $w_t$, markups for the goods and labor markets $\mu_t^p$ and $\mu_t^w$, worked hours $l_t$, and nominal interest rate $r_t$. All variables are log-linearized around their steady state balanced growth path. Starred variables denote steady-state values.

The aggregate resource constraint is given by:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \tag{B.1a}$$

where

$$c_y = 1 - i_y - g_y \tag{B.1b}$$

$$i_y = (\gamma - 1 + \delta) k_y \tag{B.1c}$$

$$z_y = R_*^k k_y \tag{B.1d}$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{B.1e}$$

Output is absorbed by consumption, capital utilization costs that are function of the capital utilization rate, and exogenous spending. $c_y$,$i_y$,$g_y$ and $z_y$ are the steady state share of output that is absorbed by the corresponding variables.

The Euler equation for consumption is given by:

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) + c_3 (rr_t + \varepsilon_t^b) \tag{B.2a}$$

where

$$rr_t = r_t - E_t \pi_{t+1} \tag{B.2b}$$

$$c_1 = \frac{\lambda/\gamma}{1 + \lambda/\gamma} \tag{B.2c}$$

$$c_2 = \frac{(\sigma_c - 1)\left(W_*^h L_*/C_*\right)}{\sigma_c (1 + \lambda/\gamma)} \tag{B.2d}$$

$$c_3 = \frac{1 - \lambda/\gamma}{\sigma_c (1 + \lambda/\gamma)} \tag{B.2e}$$

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^g \tag{B.2f}$$

The ex-ante expected real interest rate is $rr_t$. $(1 - \sigma_c)$ and $\lambda$ come from the underlying utility function and represent respectively the intertemporal elasticity of substitution for consumption, and a consumption habit parameter. The parameter

$\gamma$ represent is steady state growth, and the disturbance term $\varepsilon_t^b$ is a wedge between the interest rate controlled by the central bank and the return on assets held by the households.

Euler equation that defines the dynamics of investment is given by

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i \tag{B.3a}$$

where

$$i_1 = \frac{1}{1 + \beta\gamma^{1-\sigma_c}} \tag{B.3b}$$

$$i_2 = \frac{i_1}{\gamma^2 \varphi} \tag{B.3c}$$

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i \tag{B.3d}$$

$\varphi$ is the steady-state elasticity of the capital adjustment cost function, and $\beta$ is the discount factor applied by households. $\varepsilon_t^i$ represents a disturbance to the investment-specific technology process.

The arbitrage equation for the real value of the capital stock is:

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - rr_t \tag{B.4a}$$

where

$$q_1 = \beta\gamma^{-\sigma_c} (1 - \delta) = \frac{1 - \delta}{R_*^k + (1 - \delta)} \tag{B.4b}$$

The depreciation rate is denoted by the parameter $\delta$.

The aggregate production function comes from a Cobb Douglas function with capital and labor services as inputs:

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a) \tag{B.5a}$$

where

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a \tag{B.5b}$$

The parameter $\alpha$ is the share of capital in production, while the parameter $\phi_p$ is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

Capital services are a function of the capital stock installed last period and the degree of capital utilization, as it is assumed that capital becomes effective with a one quarter lag.

$$k_t^s = k_{t-1} + z_t \tag{B.6}$$

The degree of capital utilization is a positive function of the rental rate of capital:

$$z_t = z_1 r_t^k \tag{B.7a}$$

where

$$z_1 = \frac{1-\psi}{\psi} \tag{B.7b}$$

The parameter $\psi$ relates to the capital utilization adjustment cost and it is normalized to be between zero and one.

Capital accumulation depends on the net investment and also on the efficiency of the investment expenditure as captured by the investment specific technology disturbance $\varepsilon_t^i$:

$$k_t = k_1 k_{t-1} + (1-k_1)\, i_t + k_2 \varepsilon_t^i \tag{B.8a}$$

where

$$k_1 = \frac{1-\delta}{\gamma} \tag{B.8b}$$

$$k_2 = \frac{1-k_1}{i_2} \tag{B.8c}$$

The price mark-up on the goods market is defined as the difference between the average price and the nominal marginal cost. Is equal to the difference between the marginal product of labor $mpl_t$ and the real wage:

$$\mu_t^p = mpl_t - w_t \tag{B.9a}$$

where

$$mpl_t = \alpha\left(k_t^s - l_t\right) + \varepsilon_t^a \tag{B.9b}$$

Profit maximization by price-setting firms gives rise to a New-Keynesian Phillips curve:

$$\pi_t = \pi_1\pi_{t-1} + \pi_2 E_t\pi_{t+1} - \pi_3\mu_t^p + \varepsilon_t^p \tag{B.10a}$$

where

$$\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p} \tag{B.10b}$$

$$\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p} \tag{B.10c}$$

$$\pi_3 = \frac{\left(1-\xi_p\right)\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)}{\left(1+\beta\gamma^{1-\sigma_c}\iota_p\right)\xi_p\left(\left(\phi_p-1\right)\varepsilon_p+1\right)} \tag{B.10d}$$

$$\varepsilon_t^p = \rho_p\varepsilon_{t-1}^p + \eta_t^p - \mu_p\eta_{t-1}^p \tag{B.10e}$$

The price setting frictions are defined by the parameters $\xi_p$ and $\iota_p$, that respectively define the fraction of firms that are unable to re-optimize prices, and the degree of indexation to past inflation. $\varepsilon_p$ define the curvature of the Kimball goods market aggregator. $\varepsilon_t^p$ is a price mark-up disturbance.

The rental rate of capital is related to the real wage and the capital-labor ratio:

$$r_t^k = w_t - (k_t - l_t) \tag{B.11}$$

The labor market's wage mark-up is defined as the difference between the real wage and the marginal rate of substitution between labor and consumption $mrs_t$:

$$\mu_t^w = w_t - mrs_t \tag{B.12a}$$

where

$$mrs_t = \sigma_l l_t + (1 - \lambda/\gamma)^{-1} (c_t - (\lambda/\gamma) c_{t-1}) \tag{B.12b}$$

$\sigma_l$ is the elasticity of labor supply with respect to the real wage.

Due to wage stickiness and partial indexation of wages, real wages only gradually adjust to their desired levels:

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \tag{B.13a}$$

where

$$w_1 = \frac{1}{1 + \beta\gamma^{1-\sigma_c}} \tag{B.13b}$$

$$w_2 = \frac{1 + \beta\gamma^{1-\sigma_c}\iota_w}{1 + \beta\gamma^{1-\sigma_c}} \tag{B.13c}$$

$$w_3 = \frac{\iota_w}{(1 + \beta\gamma^{1-\sigma_c}\iota_p)} \tag{B.13d}$$

$$w_4 = \frac{(1-\xi_w)\left(1 - \beta\gamma^{1-\sigma_c}\xi_w\right)}{(1 + \beta\gamma^{1-\sigma_c})\,\xi_w\left((\phi_w - 1)\,\varepsilon_w + 1\right)} \tag{B.13e}$$

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{B.13f}$$

The level of wage stickiness and the wage indexation is represented respectively by $\xi_w$ and $\iota_w$. The Kimball market aggregator is $\varepsilon_w$, the steady state labor market mark-up is $(\phi_w - 1)$, and $\varepsilon_t^w$ is a wage mark-up disturbance.

In the Smets and Wouters model, the Taylor rule gradually adjust the interest rate in response to inflation and output. There is also a short-run feedback from the change in the output gap.

$$r_t = \rho r_{t-1} + (1-\rho)\left[\overline{r}_\pi \pi_t + \overline{r}_y \widehat{y}_t\right] + \overline{r}_{\Delta y} \Delta \widehat{y}_t + \varepsilon_t^r \tag{B.14a}$$

where

$$\widehat{y}_t = y_t - y_t^p \tag{B.14b}$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^w \tag{B.14c}$$

The output gap $\widehat{y}_t$ is the difference between actual and potential output. Potential output $y_t^p$ is defined as the level of output that would prevail under flexible prices and wages.

As described in section (3.2.3) the Taylor rule is modified to allow for an asymmetric response to variables. Equation (B.14a) then becomes:

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_{\pi,t}\pi_t + r_{y,t}\widehat{y}_t\right] + r_{\Delta y,t}\Delta\widehat{y}_t + \varepsilon_t^r \tag{B.15a}$$

where

$$r_{\pi,t} = 1 + (\overline{r}_\pi - 1)\exp\{m_\pi r_t/100\} \tag{B.15b}$$

$$r_{y,t} = \overline{r}_y \exp\{m_y r_t/100\} \tag{B.15c}$$

$$r_{\Delta y,t} = \overline{r}_{\Delta y} \exp\{m_y r_t/100\} \tag{B.15d}$$

With this modification the strength of the response to output and inflation is not fixed at $\overline{r}_y, \overline{r}_y$, and $\overline{r}_{\Delta y}$, but changes over time as a function of the level of the interest rate $r_t$.

## B.2 Estimation

A second order approximation of the model is estimated. Seven observable variables are used for the estimation: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Measurement error $\varepsilon_t^{obs} \sim N(0, \sigma_{obs})$ is allowed for all observables but the interest rate. The corresponding measurement equation is:

$$Y_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{yobs} \\ \varepsilon_t^{cobs} \\ \varepsilon_t^{iobs} \\ \varepsilon_t^{wobs} \\ \varepsilon_t^{lobs} \\ \varepsilon_t^{\pi obs} \\ 0 \end{bmatrix} \tag{B.16}$$

A particle filter procedure is used for the non linear estimation, with 50,000 particles and 15,000 MCMC replications, from which the first 1,000 are discarded as a burn-in period. Uninformative flat priors are set for the asymmetry parameters. For the rest of the parameters, the priors are kept as in the base model specification. The model is estimated for two samples. The first one starts, as in Smets and Wouters (2007), in 1966Q1. The second sample is set to begin at 1987Q4, as Alan Greenspan took charge of the Fed. For both specifications, the end of the sample is set at 2008Q3, just before the federal funds rate reached the zero lower bound. Tables B.1 and B.2 present the results of the estimation.

| | Prior Distribution | | | Posterior Distribution | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | $\mu$ | $\sigma$ | $\mu$ | | $\sigma$ | |
| | | | | 1966Q1<br>2008Q3 | 1987Q4<br>2008Q3 | 1966Q1<br>2008Q3 | 1987Q4<br>2008Q3 |
| $\varphi$ | Normal | 4.00 | 1.50 | 2.36 | 4.28 | 0.160 | 1.26 |
| $\sigma_c$ | Normal | 1.50 | 0.38 | 0.96 | 1.06 | 0.024 | 0.12 |
| $h$ | Beta | 0.70 | 0.10 | 0.78 | 0.79 | 0.007 | 0.05 |
| $\xi_w$ | Beta | 0.50 | 0.10 | 0.82 | 0.74 | 0.025 | 0.10 |
| $\sigma_l$ | Normal | 2.00 | 0.75 | 1.11 | 0.70 | 0.244 | 0.31 |
| $\xi_p$ | Beta | 0.50 | 0.10 | 0.57 | 0.74 | 0.008 | 0.05 |
| $\iota_w$ | Beta | 0.50 | 0.15 | 0.44 | 0.47 | 0.038 | 0.14 |
| $\iota_p$ | Beta | 0.50 | 0.15 | 0.24 | 0.55 | 0.075 | 0.11 |
| $\psi$ | Beta | 0.50 | 0.15 | 0.70 | 0.64 | 0.039 | 0.10 |
| $\phi$ | Normal | 1.25 | 0.13 | 1.64 | 1.54 | 0.045 | 0.10 |
| $\bar{r}_y$ | Normal | 1.50 | 0.25 | 2.03 | 1.92 | 0.030 | 0.16 |
| $\rho$ | Beta | 0.75 | 0.10 | 0.87 | 0.85 | 0.012 | 0.03 |
| $\bar{r}_y$ | Normal | 0.13 | 0.05 | 0.17 | 0.19 | 0.009 | 0.03 |
| $\bar{r}_{\Delta y}$ | Normal | 0.13 | 0.05 | 0.26 | 0.17 | 0.010 | 0.03 |
| $m_y$ | Uniform | 0.00 | 173 | -37.2 | -18.2 | 7.44 | 23.9 |
| $m_\pi$ | Uniform | 0.00 | 173 | -132 | -50.2 | 25.38 | 87.9 |
| $\bar{\pi}$ | Gamma | 0.63 | 0.10 | 0.69 | 0.66 | 0.027 | 0.04 |
| $\bar{l}$ | Normal | 0.00 | 2.00 | -0.56 | 0.47 | 0.166 | 0.35 |
| $\bar{\gamma}$ | Normal | 0.40 | 0.10 | 0.40 | 0.47 | 0.015 | 0.03 |
| $a$ | Normal | 0.30 | 0.05 | 0.32 | 0.32 | 0.008 | 0.04 |

Table B.1: Prior and posterior distribution of the DSGE structural parameters

| | Prior Distribution | | | Posterior Distribution | | | |
|---|---|---|---|---|---|---|---|
| | Dist. | μ | σ | μ | | σ | |
| | | | | 1966Q1 2008Q3 | 1987Q4 2008Q3 | 1966Q1 2008Q3 | 1987Q4 2008Q3 |
| $\sigma_a$ | Inv Gamma | 0.10 | 2.00 | 0.33 | 0.17 | 0.011 | 0.06 |
| $\sigma_b$ | Inv Gamma | 0.10 | 2.00 | 0.04 | 0.04 | 0.008 | 0.01 |
| $\sigma_g$ | Inv Gamma | 0.10 | 2.00 | 0.17 | 0.07 | 0.024 | 0.03 |
| $\sigma_l$ | Inv Gamma | 0.10 | 2.00 | 0.05 | 0.08 | 0.012 | 0.03 |
| $\sigma_r$ | Inv Gamma | 0.10 | 2.00 | 0.19 | 0.04 | 0.007 | 0.01 |
| $\sigma_p$ | Inv Gamma | 0.10 | 2.00 | 0.05 | 0.05 | 0.007 | 0.01 |
| $\sigma_w$ | Inv Gamma | 0.10 | 2.00 | 0.05 | 0.06 | 0.003 | 0.02 |
| $\rho_a$ | Beta | 0.50 | 0.20 | 0.90 | 0.82 | 0.005 | 0.09 |
| $\rho_b$ | Beta | 0.50 | 0.20 | 0.86 | 0.80 | 0.023 | 0.05 |
| $\rho_g$ | Beta | 0.50 | 0.20 | 0.35 | 0.36 | 0.066 | 0.16 |
| $\rho_l$ | Beta | 0.50 | 0.20 | 0.38 | 0.74 | 0.036 | 0.18 |
| $\rho_r$ | Beta | 0.50 | 0.20 | 0.09 | 0.57 | 0.017 | 0.10 |
| $\rho_p$ | Beta | 0.50 | 0.20 | 0.43 | 0.42 | 0.076 | 0.13 |
| $\rho_w$ | Beta | 0.50 | 0.20 | 0.98 | 0.82 | 0.004 | 0.09 |
| $\mu_p$ | Beta | 0.50 | 0.20 | 0.79 | 0.59 | 0.035 | 0.16 |
| $\mu_w$ | Beta | 0.50 | 0.20 | 0.91 | 0.54 | 0.019 | 0.16 |
| $\rho_{ga}$ | Normal | 0.50 | 0.25 | 0.57 | 0.54 | 0.025 | 0.16 |
| $\sigma_{\Delta yobs}$ | Inv Gamma | 0.10 | 2.00 | 0.28 | 0.24 | 0.040 | 0.10 |
| $\sigma_{\Delta cobs}$ | Inv Gamma | 0.10 | 2.00 | 0.50 | 0.35 | 0.018 | 0.05 |
| $\sigma_{\Delta iobs}$ | Inv Gamma | 0.10 | 2.00 | 1.03 | 0.85 | 0.046 | 0.12 |
| $\sigma_{\Delta wobs}$ | Inv Gamma | 0.10 | 2.00 | 0.61 | 0.68 | 0.012 | 0.06 |
| $\sigma_{\pi obs}$ | Inv Gamma | 0.10 | 2.00 | 0.14 | 0.08 | 0.011 | 0.02 |
| $\sigma_{lobs}$ | Inv Gamma | 0.10 | 2.00 | 0.10 | 0.06 | 0.010 | 0.02 |

Table B.2: Prior and posterior distribution of the DSGE shock processes and measurement errors

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