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Publication Date
1950-08-22

UNIVERSITY OF CALIFORNIA

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BERKELEY, CALIFORNIA

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Kenneth M. Watson and Keith A. Brueckner

August 22, 1950

Berkeley, California

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THE ANALYSIS OF π-MESON PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

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I. Introduction

The data which has at present been obtained on the production of mesons in the collisions of two nucleons is very incomplete, yet it is sufficient to establish a number of interesting features of these processes. Indeed, there seems to be enough quantitative information to warrant the development of a rational, coherent interpretation of the processes of meson production in nucleon collisions, and it is the purpose of the present paper to sketch the outline of such a means of interpretation on the basis of a semi-phenomonological theory. Although this type of analysis if far less satisfying than one based on a fundamental theory of elementary particles, the lack of any satisfactory form of a basic theory makes it necessary to fall back on a phenomenological approach in the hope of obtaining a unified picture of the processes under consideration. The theory developed here should also be of assistance in the study of meson production in complex nuclei (which is not considered in the present paper, however) and in the comparison of the inverse processes of meson absorption.

Some of the qualitative experimental information on meson production which has been obtained at this laboratory is given in Table I. From the qualitative knowledge of these results, it is possible to deduce approximately the nucleon-nucleon cross sections for meson production. These results are given in Table II.

It was pointed out by Brueckner, Chew, and Hart[1] that the production of mesons in nucleon-nucleon collisions is strongly dependent on the interaction of the nucleons in the final state. In the course of applying the theory of the present paper, the calculations of these authors have been extended. A suggested means of deducing the nature of the meson-nucleon couplings for meson production by nucleon collisions is also given.

II. Formal Development

The method which we shall follow is to postulate a transition operator, T, which effects a transition from a state consisting of two incoming nucleons to a state with one meson present and the two nucleons scattered, and to attempt to deduce from the experimental results the form of this operator. In particular, T will be assumed to have the form[2] (units are chosen as $\hbar = c = 1$)

$$T = -2\pi i R \qquad (1)$$

R can be represented as a matrix in coordinate space of the form

$$(z\ x_1'\ x_2' \left| R \right| x_1\ x_2) \qquad (2)$$

where $\underline{x_1}$ and $\underline{x_2}$ (and $\underline{x_1}'$, $\underline{x_2}'$) are the coordinates of the two nucleons and $\underline{z}$ is the meson coordinate. Alternatively, R can be represented in momentum space by the variables $\underline{p}$, $\underline{p}'$ describing the relative momenta of the two nucleons before and after the collision, respectively, $\underline{P}$ and $\underline{P}'$ representing the respective total momentum of the two nucleons before and after the collision, and $\underline{q}$, the momentum of the created meson.

The collision is most simply described in the center of mass system, so we restrict ourselves to this coordinate system and set $\underline{P} = o$. Then R has the form

$$R = \delta(\underline{q} + \underline{P}')\ (\underline{q},\ \underline{p}' \left| R_o \right| \underline{p}) \qquad (3)$$

R_o in Eq. (3) can be expected, in general, to be a very complicated function of its arguments, so some condition must be found to impose simplifying restrictions on it. For energies sufficiently near the threshold for meson production (i.e., for energies presently available) such a condition obtains, for -- in the center of mass system -- the greater part of the available nucleon bombarding energy is found in the meson rest-energy in the final state. This implies relatively low kinetic energy for the particles in the final state ($\sim$20 Mev for the Berkeley cyclotron), so we shall assume that

$$q, \, p' \ll p \quad . \tag{4}$$

(This condition is better fulfilled than might at first appear, because it happens that most of the mesons have nearly all the available kinetic energy, leaving little for the two remaining nucleons.)

Condition (4) suggests that R_o can be expanded in a power series in $\underline{q}$ and $\underline{p}'$, with as few terms kept as are needed to explain the experimental results (as will be shown below, at most two terms in this expansion seem to be necessary to explain the present experiments). Thus we write R_o in the form

$$R_o = R_o^{(1)}(p) + q_i \, (R_o^{(2)}(p))_i + q_i \, q_j \, (R_o^{(3)}(p))_{ij} + \ldots$$
$$+ p_i' \, (R_o' \, (p))_i + \ldots \ldots \tag{5}$$

where the repeated vector indices i, j are summed from 1 to 3. Present experiments seem to indicate that the terms involving factors of p' are not important at the energies available, so such terms will not be considered in this paper. These terms introduce no essential complication in the present treatment and can readily be included if further experimental evidence warrants their use.

Dropping the terms in p', we can rewrite R_o in the somewhat more convenient form:

$$R_o = \sum_i \Gamma_i \, (p) \, O_i \, (\underline{\sigma}, \tau, \underline{q}, \underline{p}) \tag{5'}$$

where the summation is extended over as many terms as are found necessary. Here the $\Gamma_i(p)$ are real numerical functions of the magnitude of $\underline{p}$, and the O_i are operators involving products of the vectors $\underline{q}$ and $\underline{p}$ with the nucleon spin and isotopic spin matrices, $\underline{\sigma}$ and $\underline{\tau}$. We can assume that each O_i is homogeneous in both $\underline{q}$ and $\underline{p}$. The detailed structure of the O_i will be discussed in Section VII, but for the present we make no assumption as to their nature.

The non-occurrence of $\underline{p}'$ in Eq. (5') implies that its coordinate representation (Eq. (2)) involves the relative coordinates $\underline{x}_1'$ and $\underline{x}_2'$ as

$$\delta\,(\underline{x}_1' - \underline{x}_2') \quad . ^*$$

This, in turn, implies that the matrix element of R_o between an initial state, I, and a final state, F, will be of the form:

$$(F \mid R_o \mid I) = \sum_i \Gamma_i\,(p)\,(F \mid O_i \mid I)\ \ (2\pi)^{3/2}\,\psi_F^*\,(0)) \qquad (6)$$

where $\psi_F\ (0)$ is the coordinate wave function for the relative motion of the nucleons in the final state evaluated at the origin (i.e. $\psi_F\,(\underline{x}_1' - \underline{x}_2')\,\big|\,x_1' = x_2'$). We assume that the incoming nucleons are in plane wave states with relative momentum p, and the produced meson is in a plane wave state with momentum $\underline{q}$ (plane waves are normalized per unit volume in momentum space). Then $(F \mid O_i \mid I)$ is just the matrix element of the operators σ and τ occurring in O_i.

The cross section for meson production is then

$$d\,\sigma = (2\pi)^4\,d\,J \sum \left|(F \mid R_o \mid I)\right|^2 / \, v_r \qquad (7)$$

---

*If R_o were not expanded in powers of p', one would expect its effect in R_o to "smear out" the δ-function over a region of the order of 1/p. Unless the two nucleon potential is highly singular at the origin, this "smearing out" of the δ-function will not appreciably change the conclusions of the following sections. This provides an additional justification for neglecting the p' terms in Eq. (5).

where v_r is the relative velocity of the incoming nucleons, dJ is the volume in momentum space accessible to the particles in the final state, and $\sum$ means a summation over final spin states and an average over initial spin states. If the final nucleons are not bound to each other,

$$dJ = \sqrt{2}\ (2\pi)\ (M\,\mu)^{3/2}\ (1 + T/2\mu)^{1/2}\ (T\ (T_{max} - \tfrac{13}{12} T))^{1/2}\ d\,T\ d\,\Omega\,q \qquad (8)$$

where M is the nucleon mass, μ is the meson mass, T is the meson kinetic energy, $d\Omega q$ is an element of solid angle about the direction of q, and T_{max} is the initial kinetic energy of the nucleons minus the meson rest-energy. The factor of 13/12 results from taking $\mu/M = 1/6$. The final nucleons are treated non-relativistically in deriving Eq. (8). If the nucleons in the final state are bound (i.e., as a deuteron),

$$dJ = 12/13\ \ q\ (q^2 + \mu^2)^{1/2}\ d\Omega q \qquad (8')$$

In this case the mesons have a fixed energy.

Consistent with the form (5') for R_o, we assume that * (cf. Eq. (7))

$$\sum |(F\ |\ R_o\ |\ I)|^2 = (2\pi)^3 |\psi_F(0)|^2 \Big\{ \Gamma_1^2\ q^{2n}\ g_1\ (F,I)\ + \qquad (9)$$
$$+\ \Gamma_2^2\ q^{2n'}\ \cos^2\Theta\, g_2\ (F,I)\ +\ \Gamma_3^2\ q^{2n''}\ \cos^4\Theta\ g_3\ (F,I) + \ldots \Big\}$$

where the n's are positive integers and Θ is the angle between q and p. The $g_i(F,I)$ are numerical constants depending on the initial and final spin and isotopic spin states. The Γ's are numerical functions of p only. In the following sections we shall investigate individually the contributions of the following terms in Eq. (9).

*When the incoming nucleons are not identical, there will in general be odd powers of $\cos\Theta$ in Eq. (9). As we are primarily interested in p-p collisions here, we disregard such terms.

$$\begin{aligned}
&\text{Type IA} \longrightarrow \Gamma^2 \; g_{IA}(F,I) \\
&\text{Type IIA} \longrightarrow \Gamma^2 \; g_{IIA}(F,I)\; q^2 \\
&\text{Type IB} \longrightarrow \Gamma^2 \; g_{IB}(F,I)\; \cos^2\theta \\
&\text{Type IIB} \longrightarrow \Gamma^2 \; g_{IIB}(F,I)\; q^2 \cos^2\theta
\end{aligned} \tag{10}$$

Higher powers of q than the second do not seem necessary at present (indeed they predict a cross section incompatible with experimental results unless their contribution is small). There is also no present need for retaining powers of $\cos\theta$ higher than the second.

Finally, since the Γ's are numerical functions of p only, they are constant for any given beam energy. They are also, presumably, much more slowly varying functions of beam energy than are the other factors in the cross section. We thus assign them constant values compatible with the 340 Mev beam energy at Berkeley. Deviations of total cross sections at other energies from those here calculated will then give the dependence of these quantities on p.

III. Evaluation of $|\psi_F(o)|^2$

If the energy of the final nucleons were sufficiently high, we could take $(2\pi)^3|\psi_F(o)|^2$ as unity (two, for identical particles). In actual fact, for the energies presently available this quantity is effectively much larger than unity and must be calculated on the basis of some assumed force for the two nucleon system. In particular, we need only the partial wave corresponding to zero angular momentum. Also, if the final state is a neutron-proton system with some triplet spin state present, there is a large probability that a deuteron will be formed.

If the nucleons in the final state are not bound, the wave function $\psi_F(o)$ is a function of their relative momentum, p', which is related by energy conservation to the meson energy, T, by

$$p' = (M(T_{max} - 13/12\ T))^{1/2} \qquad (11)$$

(the symbols have been defined above).

Unfortunately, $\psi_F(o)$, depends more strongly upon the assumed shape of the two nucleon potential than does the low energy scattering data. Calculations have thus been made for both square well and exponential potentials (tensor forces have not been included) whose parameters were chosen to fit the low energy scattering. The n-p (neutron-proton) triplet scattering and effective range were taken from Christian and Hart[3]. In accordance with an assumed charge symmetry of nuclear forces, the n-n (neutron-neutron), p-p (proton-proton), and n-p singlet potentials were assumed to be the same and taken from Jackson's and Blatt's work[4]. The effect of the Coulomb interaction in the p-p case was included for the square well, but not for the exponential well, and amounted to only a 10 percent reduction in the total cross section for 340 Mev collisions (laboratory frame of reference)--this would give an even smaller correction at higher energies.

The total cross sections calculated for the exponential well were somewhat more than 50 percent greater than those calculated for a square well (for 340 Mev energy in the laboratory system); however, the relative cross sections for various different final states were in fairly good agreement for the two assumed well shapes. For instance, for a final state involving an n-p system in a triplet spin state, the ratio of the number of deuterons formed to the number of unbound particles agreed to within about 5 percent for these two well shapes. Also the cross section for forming a deuteron using an exponential well agreed to within a few percent with that calculated using the deuteron wave function of Chew and Goldberger[5].

The exponential well was chosen for the ensuing calculations as being the more physically reasonable, and all numerical results to be presented were obtained from it. It is felt that the greatest uncertainty arising from this assumed well shape lies in the total calculated cross sections and that the angular distributions and relative cross sections given are sufficiently reliable until much more accurate and complete experimental results are available than at present.

IV. Calculated Cross Sections

It will be convenient to introduce the following type of notation to designate the various meson production processes:

$$(T,\ S;\ p\ p,\ \pi^+)$$

This is taken as indicating the collision of two protons in an initial triplet spin state to produce a π^+ meson, leaving the resulting two nucleons in a singlet spin state. Similarly (s, s; n p, π^0) indicates a singlet to singlet scattering of a neutron and proton to produce a π^0 meson, etc.

In the present section we will give the cross sections corresponding to the four types of terms occurring in expressions (10), and in later sections will consider what linear combinations give the best agreement with experiments. Then for the present we need only enumerate the final nucleon states in calculating meson cross sections, since an examination of expressions (10) shows that the various initial nucleon states (i.e., charge and spin states) enter only through the multiplicative constants g (F, I). Values of $\Gamma^2 g$ were arbitrarily chosen to normalize the total cross sections (disregarding deuteron formation) to $2(10)^{-28}$ cm^2. The values used for Γ^2 g are given in Table III for the four cases of expressions (10).

There are then three types of final states to be considered. The first is an n-p triplet state with the neutron and proton left as free particles. The corresponding cross section will be designated as

$$d\,\sigma^{t}_{IA} = \sigma^{t}_{o\,IA}\ (\theta\ ,T)\ d\Omega_q\ d\ T \qquad (12)$$

where IA refers to type IA of the expressions (10), etc. The second is an n-p triplet state with the neutron and proton bound to form a deuteron, with a cross section:

$$d\,\sigma^{d}_{IA} = \sigma^{d}_{o_{IA}} (\theta, T)\; d\Omega_q\; dT \tag{13}$$

etc., where

$$\sigma^{d}_{o_{IA}} (\theta, T) = \delta\,(T - 12/13\,(T_{max} + \varepsilon_d))\; \sigma^{d}_{o_{IA}} (\theta, T)\,, \tag{14}$$

since the mesons created by deuteron formation have constant energy. Here E_d is the deuteron binding energy. The third case is an n-p or n-n or p-p singlet state (as we are assuming the same singlet potential for all nucleons and are neglecting the small Coulomb correction for the p-p final state). This cross section is designated for type IA, etc., as

$$d\,\sigma^{s}_{IA} = \sigma^{s}_{o_{IA}} (\theta, T)\; d\Omega_q\; dT \tag{15}$$

These cross sections are defined with the values of $\Gamma^2 g$ given in Table III. The units of σ_o in each case are assumed to be cm^2 (Mev-steradian)$^{-1}$.

To facilitate comparison with the experiment, the differential cross sections (12) and (15) (i.e. σ_o) have been transformed to the laboratory system and are given in Figs. 1 thru 6 for 340, 400, and 450 Mev beam energies. They are plotted as the meson energy spectrum at various angles. The results for 400 and 450 Mev beam energies are included because of the present existence of cyclotrons of higher energies than that of the Berkeley cyclotron, even though experimental results from these are not as yet available. The corresponding values of $\sigma^{d}_{o_o}$ (Eq. (14)) are given in Table IV with their respective meson energies. For comparison with experiment the δ-function in Eq. (14) should be replaced by a function of finite extent corresponding to the energy resolution of the detection apparatus and the spread in beam energy, and the resulting values of $d\sigma^{d}$ (Eq. (13)) added to $d\sigma^{t}$ (Eq. (12)). Experiments at Berkeley seem to indicate that about one-half the mesons for the (pp, π^{+}) process are accompanied by deuteron formation with a beam energy of 340 Mev.

The tendency of the curves in Figs. 1 thru 6 to have a peak near the maximum possible meson energy is due to the rapid increase in $|\psi_{F}(o)|^{2}$ with increasing meson energy--a dependence with predominates over the variation of the phase space factor, dJ, (Eq. (7)) for large meson energies.

The variation of the total cross section with energy is given in Figs. 7 and 8 for final singlet and triplet (deuteron formation included) states for types I and II (constant and q-dependence, respectively, in the transition operator R_{o}). The cross sections fall off much more slowly with energy than would be expected on the basis of phase space arguments alone. The production with deuteron formation for a final triplet state causes the triplet cross section to be appreciably larger than the singlet at low energies. The cross sections were arbitrarily normalized to $8(10)^{-28}$ cm^{2} at 340 Mev and constant values of $\Gamma^{2}g$ were assumed. Deviations in observed cross sections at higher energies can be used to deduce the dependence of Γ^{2} on p, the relative momentum of the initial nucleons in the center of mass system.

V. Fit to the Experimental Data on (pp, π^+) Production

The most detailed experimental data available is for (pp, π^+) production with a beam energy of 340 $\pm$ 2 Mev. The experiments of Cartwright and Whitehead[6] and Cartwright, Richman, Whitehead, and Wilcox[7] give the meson energy spectrum at zero degrees ($\pm$ 5^0) with respect to the beam direction. Further results concerning the energy spectrum as 30^0 have been obtained by Peterson[8]. Here the energy resolution is not as yet so very good and only the total cross section (i.e., integral of spectrum over meson energy) at 30^0 is used.

We shall use the meson energy spectrum at zero degrees to determine the relative amount of singlet and triplet final states. The experimental results are indicated by the points in Fig. 9. The fit was made by drawing a curve through the experimental points and taking the ratio, r, of the area for meson energies greater than 65 Mev to the area for meson energies less than 65 Mev. The relative admixture of singlet and triplet final states was obtained from the curves of Figs 1 and 2 and the values of $d\sigma^d$ from Table IV, by choosing this admixture to give the same value for the ratio, r.

The resulting cross section for Type IA (expressions (10)--also IB, as no distinction can be made between IA and IB in the forward direction) is

$$d\,\sigma_{IA} = d\,\sigma^t_{IA} + d\,\sigma^d_{IA} \qquad (16)$$

where $d\,\sigma^t_{IA}$ and $d\,\sigma^d_{IA}$ are the expressions of Eqs. (12) and (13). For this case no singlet cross section was necessary, but as much as 20 percent is not incompatible with the experiments.

For type IIA (or IIB) the cross section is

$$d\,\sigma_{IIA} = (.459)\ (d\,\sigma^t_{IIA} + d\,\sigma^d_{IIA} + 4.36\ d\,\sigma^s_{IIA}) \qquad (17)$$

where $d\sigma^{s}_{IIA}$ is the singlet differential cross section of Eq. (15). The factor (.459) gives Eq. (17) the correct absolute magnitude (this factor was unity for Eq. (16), due to the choice of the Γ^2g's in Table III).

The cross sections of Eqs. (16) and (17) are plotted in Figs (9) and (10), respectively, for $\theta = 0^{\circ}$ with the cross section for mesons of energy greater than 65 Mev averaged uniformly over the interval of 65 to 75 Mev. The experimental points are plotted in the same figures to facilitate comparison. The actual shape of the meson spectrum arising from deuteron formation depends on the characteristics of the detection apparatus, so comparison of the peak shapes in the drawings is meaningless. The general effect of the actual detector characteristics will be to raise the center of the theoretical peaks and to spread its base.

From a comparison of Figs. (9) and (10) it appears impossible to rule out either type I or type II cross sections, although type I (transition operator independent of meson momentum) seems to give a better fit. A combination of the two is also quite possible. This point can be further clarified only by additional experimental results of greater accuracy and preferably at other beam energies.

As mentioned previously, calculations assuming a q^2 dependence for R_o (Eq. (3)) indicate that this type of term can be ruled out except as a small correction in the series of Eq. (9).

To investigate the angular distribution of the mesons, the ratio of the area under the experimental meson-spectrum curves at 0° to that at 30° was found to be

$$2.60\ (\pm\ 20\ \text{percent})$$

The same ratio calculated from the curves in Figs. (1) and (2) with the singlet to triplet ratios found above is for Type A (i.e., no angular dependence in the center of mass system):

$$\sigma(0^\circ) / \sigma(30^\circ) = \quad 1.29 \quad (IA)$$
$$1.26 \quad (IIA)$$

For type B ($\cos^2 \theta$ dependence in the center of mass system) we have:

$$\sigma(0^\circ) / \sigma(30^\circ) = \quad 5.36 \quad (IB)$$
$$4.95 \quad (IIB)$$

Each of these ratios is in bad disagreement with the experimental ratio, 2.60. We thus conclude that more than one term in the series of Eq. (9) is needed. We keep the first two terms in Eq. (9), giving an angular distribution in the center of mass system of the form

$$a + b \cos^2 \theta$$

(a, b are independent of θ.) Further experiments are necessary to determine if still more terms are required.

We can fit the experimental total cross sections at 0° and 30° for $n = n' = o$ (i.e. transition operator independent of meson momentum) by assuming the expression:

$$d\sigma_I = 0.336 \left[d\sigma_{IA} + 0.66\, d\sigma_{IB} \right] \tag{18}$$

where $d\sigma_{IA}$ is given by Eq. (16) and $d\sigma_{IB}$ is obtained from the same equation by replacing A by B in it.

For $n = n' = 1$ in Eq. (9) (i.e. linear dependence on meson momentum in the transition operator) the experimental results can be fitted by

$$d\sigma_{II} = 0.307 \left[d\sigma_{IIA} + 0.76\, d\sigma_{IIB} \right] \tag{19}$$

where $d\sigma_{IIA}$ is given by Eq. (17) and again $d\sigma_{IIB}$ is obtained by replacing A by B in that equation. These correspond roughly to a center of mass angular distribution of the form

$$1 + 2\cos^2\theta$$

The most reasonable combination of terms is perhaps to take n = o, n' = 1 in Eq. (9). The proper cross section is (we designate this as type III):

$$d\sigma_{III} = .476 \left[d\sigma_{IA} + 0.70\, d\sigma_{IIB} \right] \qquad (20)$$

The ratio of singlet to triplet admixture in the final states cannot be determined for the two terms of Eqs. (18), (19), and (20) individually because of the limited experimental data, so we have here used the definitions of Eqs. (16) and (17) somewhat arbitrarily.

The foregoing analysis, although appearing to be fruitful, indicates the real need for further experimental results at both other angles and other beam energies. It is felt that when such information becomes available, the present calculations will provide a framework on which to interpret them and that then a unique choice between the possible types of cross section here discussed will be possible. The information for such analysis should be obtainable from Figs. 1 to 8 and Tables III and IV.

VI. Other Types of Processes

Little is known of the cross sections for meson production by other than p-p collisions. Experiments by Bjorkland, Crandall, Moyer, and York[9] indicate that the cross section for π^0 meson production in p-p collisions, if nonvanishing, is less than 1/30 the cross section for p-p production of π^+ mesons at 340 Mev. (They observed no production in p-p collisions, the factor 1/30 representing their estimated experimental uncertainty.) The present calculations indicate p-p cross sections not much less than about 1/3 the (p-p, π^+) cross section can be expected on the basis of the interaction of the particles in the final state if the transition operators are the same. It thus appears that there is some selection rule prohibiting π^0 production in p-p collisions. Very little is known about the cross section for meson production in n-p collisions. As such information becomes available, the curves given in Figs. 1 thru 6 with the meson-deuteron cross sections given in Table IV should prove adequate for an analysis similar to that made in the last section for (pp, π^+) production.**

** R. Jastrow (Phys. Rev., in press) has suggested a strong repulsive core for singlet state nuclear forces. As the process of meson production in nucleon collisions seems to provide a means of probing nuclear forces at close distances, the existence of such a repulsive core may lead to differences in the meson production cross section for those processes involving identical nucleons in the final state (e.g. (np, π^+ or π^-) production. A preliminary study of this effect indicates that the peak of high energy mesons may be less pronounced if there is a repulsive core. The experimental production of charged mesons in n-p collisions may throw considerable light on this question.

VII. Analysis of the operator $O(\sigma, \tau, q, p)$

The discussion of the forms of the operator $O(\sigma, \tau, q, p)$ (Eq. (5')) as deduced from meson theory has been given by Brueckner[1]. That none of these was adequate to explain quantitatively the experimental results, suggests that possible forms of O should be discussed on the basis of more general arguments. We shall restrict the possible forms of O by considering the necessary symmetry properties of the expressions and the simplifications of the possible forms of the operator due to the smallness of the final momenta compared with the initial momenta, as discussed in Section II.

If the meson is scalar--or pseudoscalar[9]--, O must transform under coordinate reflection as a scalar or pseudoscalar, respectively. Then, since we are using the isotopic spin formalism in Eq. (5'), O must be symmetric with respect to an interchange of the two nucleons. Finally, since R in Eq. (1) can be considered as a perturbing potential in the Schrodinger[2] equation for the nucleons, we can restrict it to be Hermitean.

For a pseudoscalar meson the following products of $\underline{\sigma}$, $\underline{p}$, $\underline{q}$ are possible, if we designate one nucleon by the superscript "1" and the other by "2" (as $\sigma^{(1)}$ and $\sigma^{(2)}$, etc.):

$$\begin{aligned} A_1^{ps}(1) &= (p \cdot q)^{n-1} \sigma^{(1)} \cdot p \\ A_2^{ps}(1) &= (p \cdot q)^{n} \sigma^{(1)} \cdot q \\ A_3^{ps}(1) &= (p \cdot q)^{n-2} \sigma^{(1)} \cdot p \, \sigma^{(2)} \cdot (p \times q) \\ A_4^{ps}(1) &= (p \cdot q)^{n-1} \sigma^{(1)} \cdot q \, \sigma^{(2)} \cdot (p \times q) \end{aligned} \qquad (21)$$

For a scalar meson, we have:

$$A_1^s\ (1) = (p \cdot q)^n \quad 10)$$
$$A_2^s\ (1) = (p \cdot q)^{n-2}\ \sigma^{(1)} \cdot p\ \sigma^{(2)} \cdot p$$
$$A_3^s\ (1) = (p \cdot q)^{n-1}\ \sigma^{(1)} \cdot p \times q$$
$$A_4^s\ (1) = (p \cdot q)^{n-2}\ \sigma^{(1)} \cdot (p \times q)\ \sigma^{(2)} \cdot (p \times q)$$
$$A_5^s\ (1) = (p \cdot q)^n\ \sigma^{(1)} \cdot q\ \sigma^{(2)} \cdot q$$
$$A_6^s\ (1) = (p \cdot q)^{n-1}\ \sigma^{(1)} \cdot p\ \ \sigma^{(2)} \cdot q \qquad (22)$$

Similar quantities A(2) can be obtained by interchanging superscripts "1" and "2" and replacing p by -p . Further factors of the form $(p \times q)^2$ may be introduced, but are excluded from contributing significantly to the cross section because they introduce too high a power of q (see Section V).

A_1^{ps} (with n = o) is characteristic of pseudoscalar meson theory with pseudoscalar coupling while a particular linear combination of A_1^{ps} with n = 1 and A_3^{ps} with n = o is characteristic of pseudoscalar theory with pseudovector coupling. A_1^s is obtained from scalar meson theory with scalar coupling. For further details, the paper of Brueckner[1] should be consulted.

For the isotopic spin dependence, we choose the following combination of τ-operators:

$$\tau_i^{(1)}\ T \text{ and } \tau_i^{(2)}\ T \ (\ i = 1,2,3) \qquad (23)$$

where

$$T \equiv \tau_1^{(1)} \tau_1^{(2)} + \tau_2^{(1)} \tau_2^{(2)} + \beta \tau_3^{(1)} \tau_3^{(2)} + \beta\ d\ \tau_4^{(1)} \tau_4^{(2)} \qquad (24)$$

The τ_i (i = 1,2,3) are the usual isotopic spin operators and τ_4 is the unit two-dimensional matrix. The index "i" on the τ_i in Eqs. (23) represents the isotopic spin type of the meson emitted (or absorbed).

β and d are arbitrary paramenters.

We now give O the following form:

$$
\begin{aligned}
O = & \left[\tau_i^{(2)}\, T\, A^{(1)} + \tau_i^{(1)}\, T\, A^{(2)}\right] \\
& + p\left[\tau_i^{(1)}\, T\, A^{(1)} + \tau_i^{(2)}\, T\, A^{(2)}\right] \\
& + e\left\{\left[T\, \tau_i^{(2)}\, A^{(1)} + T\, \tau_i^{(1)}\, A^{(2)}\right]\right. \\
& \left. + \rho\left[T\, \tau_i^{(1)}\, A^{(1)} + T\, \tau_i^{(2)}\, A^{(3)}\right]\right\}
\end{aligned} \tag{25}
$$

Here the index "i" corresponds to the type of meson emitted or absorbed. ρ is an arbitrary parameter and e is restricted to be ± 1 in order that R be Hermitean. (O in Eq. (25) is defined to within a factor $i = \sqrt{-1}$. This factor is irrelevant for calculating cross sections, so will not be explicitly included.) The form of O in Eq. (25) is invariant with respect to an interchange of "1" and "2"-- that is, an interchange of the nucleons. In accordance with Eq. (5'), R_o can be expected in general to be a sum of O's of the type given by Eq. (25) with various different A's of the form given in Eqs. (21) or (22).

Eq. (25) implies an assumed symmetry with respect to an interchange of nucleon isotopic spin states which is implied by meson theory. If this is not borne out by experiments, a more general assumption as to the isotopic spin dependence in Eq. (25) will be necessary.

To investigate the allowed types of transitions predicted by the various A's in Eq. (25), we keep but one term at a time of this type in R_o. Then writing

$$
Q \equiv \Sigma \left|(F \mid R_o \mid I)\right|^2 \tag{26}
$$

where the sum represents a summation over final spin states and an average over the initial spin states, we have for pseudoscalar mesons:

$$Q_1 = \Gamma_1^{\,2}\ p^{2n}\ q^{2(n-1)}\ (\cos\theta)^{2(n-1)}\ g_1\ (F,I)$$
$$Q_2 = \Gamma_2^{\,2}\ p^{2n}\ q^{2(n+1)}\ (\cos\theta)^{2n}\ g_2(F,I)$$
$$Q_3 = \Gamma_2^{\,2}\ p^{2n}\ q^{2(n-1)}\ (\cos\theta)^{2(n-2)}\ \sin^2\theta\ g_3\ (F,I.)$$
$$Q_4 = \Gamma_4^{\,2}\ p^{2n}\ q^{2(n+1)}\ (\cos\theta)^{2(n-1)}\ \sin^2\theta\ g_4\ (F,I) \qquad (27)$$

For scalar mesons we have:

$$Q_1^{\,1} = \Gamma_1^{\,12}\ p^{2n}\ q^{2n}\ (\cos\theta)^{2n}\ g_1^{\,1}(F,I)$$
$$Q_2^{\,1} = \Gamma_2^{\,12}\ p^{2n}\ g^{2(n-2)}\ (\cos\theta)^{2(n-2)}\ g_2^{\,1}\ (F,I)$$
$$Q_3^{\,1} = \Gamma_3^{\,12}\ p^{2n}\ q^{2n}\ (\cos\theta)^{2(n-1)}\ \sin^2\theta\ g_3^{\,1}\ (F,I)$$
$$Q_4^{\,1} = \Gamma_4^{\,12}\ p^{2n}\ q^{2n}\ (\cos\theta)^{2(n-2)}\ \sin^4\theta\ g_4^{\,1}\ (F,I)$$
$$Q_5^{\,1} = \Gamma_5^{\,12}\ p^{2n}\ q^{2(n+2)}\ (\cos\theta)^{2n}\ g_5^{\,1}\ (F,I)$$
$$Q_6^{\,1} = \Gamma_6^{\,12}\ p^{2n}\ q^{2n}\ (\cos\theta)^{2(n-1)}\ g_6^{\,1}\ (F,I) f_{F,I}\ (\theta) \qquad (28)$$

In these equations the subscript on the Q's refers to the subscript on the A's of Eqs. (21) and (22). The Γ's are numerical functions of p only and the g's are numerical constants depending on the initial and final states and are the g's of Eqs. (9) and (10). The g's fall into two classes, depending upon whether n(the power of p in Eqs. (21) and (22), which is assumed large enough that the exponents of q and cos Θ in Eqs. (27) and (28) are never negative) is even or odd. We can designate this evenness or oddness of n by an additional subscript on the g's, i.e., "e" or "o", respectively. Then we write g_1,e, g_1,o, $g_1^{\,1}$, e, etc. The angular dependence of $Q_6^{\,1}$ depends upon F and I, so is designated just as a function of θ. The values of the g's are given in Table V.[12] In Table V the following abbreviations are used:

$$a \equiv 8/3\ \beta^2\ (1 + d)^2\ (1 + e)^2$$
$$b^{\pm} \equiv 8/3\ \left[2 \mp \beta\ (1 - d) \pm e\ \beta\ (1 + d)\right]^2 \qquad (29)$$
$$c \equiv 8/3\left[\beta\ (1 - d)\ (1 + e) + 2\ (1 - e)\right]^2$$

Those g's having the same value are grouped in one column in Table V.

The cross sections for meson production are obtained by substituting the Q's of Eqs. (27) and (28) for $\sum|(F|R_o|I)|^2$ in Eq. (7). For an unpolarized incident beam, one must of course take 1/4 the cross section for an initial singlet state plus 3/4 that for an initial triplet state.

None of the Q's in Eqs. (27) and (28) is satisfactory alone because of their angular distributions (cf. Eqs. (18), (19), and (20)). We also need consider none with a power of q higher than the second. There are also difficulties with most of the g's in Table V, even in light of the meager known information on these processes. Thus those g's in Table V, column (c), predict π_o production in p-p, but not n-p collisions, contrary to observation. Those in column (d) correspond to a q^2 dependence of the Q's, but do not permit a mixture of singlet and triplet final states for (pp, π^+) production as seemed necessary in Section V.

In the case of the pseudoscalar type of interactions, the g's of column (a) can be made to yield no (pp, π^o) production by taking a in Eq. (29) equal to zero. Since d = -1 would appear to be a numerical coincidence, it seems perhaps more reasonable to take e = -1. Also these types of interaction can only be of Type I (since we have excluded higher powers of q than the first in the transition operator). This then implies, by the results of Section V, that the amount of singlet final state in (pp, π^+) production must be small, which in turn implies a relatively small (np, π^+ or π^-) cross section, (as is apparent from an examination of Table V, column (a)). Pseudoscalar interactions leading to column (b) suffer from the difficulty that they imply meson-momentum dependent matrix elements (Type II), but give no final state singlet admixture for the (pp, π^+) process--in disagreement with Eq. (17).

The angular distribution deduced in Section V seems to imply that R_o must contain the sum of at least two interactions of the sort given by Eqs. (21) and (22). (The interference terms in the cross section resulting from adding two types of O's in R_o vanish in most cases, but may be easily calculated when non-vanishing.) The arguments of the preceding paragraph when applied to linear combinations of two terms are quite restrictive. In fact, for the pseudoscalar interactions there seems to be only one combination of two terms that gives the correct (pp, π^+) cross section, as deduced in Section V. This involves a sum of O_1^{ps} (n = 1) and O_1^{ps} (n = 2) (the notation on the O's corresponding to that of the A's of Eq. (21)). This gives a center-of-mass angular dependence of the form $(a + b\, q^2 \cos^2\theta)$ --i.e., of the general form of Eq. (20). Here the constant term contains a singlet-triplet final state mixture and the $q^2 \cos^2\theta$ term contains only a triplet final state. The parameters of Eq. (29) can be used to adjust the amount of singlet state as was done in Section V. Further experimental information on cross sections for other processes will be necessary to fix these parameters uniquely. There are several possible combinations of terms for the scalar type of interactions.

It seems, however, necessary to await further experimental results before attempting to make a unique choice between interaction types. That such a choice can be made seems very likely, due to the very different predictions concerning selection rules, angular dependence, and energy spectrum made by the various interactions of Eqs. (21) and (22).

Further information concerning these interactions can be obtained from experiments on the inverse processes of meson absorption. For instance, the absorption of a π^--meson by a deuteron with the π^- energy going into the kinetic energies of the final neutrons, is the exact inverse of the (nn, π^-) process to form a deuteron. If the π^- is absorbed from the lowest Bohr orbit, however, some care must be exercised in interpreting Table V, since then those interactions of Eqs. (21) and (22) containing an odd power of q will very nearly vanish. Indeed, all the scalar interactions of Eq. (22) vanish for this process.

VIII. Conclusions

We have given the outline for an analysis of meson production in nucleon collisions which, it is hoped, will be of use in better understanding these processes. The necessary ambiguities in the present considerations can undoubtedly be greatly reduced in the near future by the constantly increasing amount of experimental material becoming available.

Such difficulties as the dependence of the cross section on the interactions of the nucleons in the final state may be troublesome when very accurate and detailed data become available--but may actually be of great value in obtaining further information on the nature of nuclear forces themselves due to the somewhat critical dependence of $\psi_F(o)$ on the shape of these forces. That is, the large momentum transfer involved gives a means of probing the forces at small distances.

We must finally recognize that the deductions in Sections V and VI are based on very limited experimental data and thus may have to be modified as new and more precise data becomes available. It is felt, however, that the general framework of the present theory is well established by present experimental results.

There are several noteworthy features of the predictions made by the present theory. The first is that the cross sections do not decrease with decreasing beam energy--or increase with increasing beam energy--as rapidly as has been thought (Figs. (7) and (8)). This implies that it will be easier to do experiments at lower bombarding energies. The rather sharp peak of high energy mesons--indicated by Figs. (1) to (6) and Table IV--will be of considerable assistance in obtaining a source of nearly mono-energetic mesons for such experiments as meson scattering. Also, the observed angular distributions depend rather sensitively on the assumed center-of-mass angular distribution, which should make deductions as to the form of the latter quite straightforward.

A study of the curves in Figs. (1) to (6) with the cross sections of Table IV should be useful in planning further experiments on the production of mesons, in that they give an indication of the angles and energies at which the most mesons should be found.

Acknowledgments

We wish to thank Professor Robert Serber for interesting discussions of the theory of this paper. We also wish to thank Dr. Chaim Richman, Mr. William Cartwright, Miss Marian Whitehead, and Dr. Vince Z. Peterson for aid in interpreting their experiments and for permission to quote their results in advance of publication. We wish to express further appreciation to Mr. David Clark and to Mr. William Noh for their considerable assistance in performing the laborious calculations of Sections IV and V.

The work described in this report was performed under the auspices of the Atomic Energy Commission.

Information Division
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(In particular, see appendix by Brueckner, Chew, and Hart)

2. Since R represents a weak interaction, it can also be considered as a perturbing potential in the Schrodinger equation of the two nucleons.

3. R. Christian and E. Hart, Phys. Rev. 77, 441 (1950)

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5. G. F. Chew and M. L. Goldberger, Phys. Rev. 75, 1637 (1949)

6. W. Cartwright and M. Whitehead, private communication

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8. V. Peterson, private communication

9. Bjorkland, Crandall, Moyer, and York, Phys. Rev. 77, 213 (1950)

10. A similar analysis to that given here is possible for vector and pseudovector mesons. That the neutral meson cannot have spin one, however, follows from its decay into two γ-rays (C. N. Yang, Phys. Rev. 77, 242 (1950)). Present evidence would seem to indicate that the charged mesons are of the same type as the neutral mesons because of their nearly equivalent mass and production cross sections.

11. Further terms of the type $(\underline{p} \cdot \underline{q})^n \underline{\sigma}^{(1)} \cdot \underline{\sigma}^{(2)}$ lead to results essentially equivalent to those obtained from A_1^s.

12. Additional interaction of the forms:

$$A_5^{ps}(1) = \frac{1}{2}(\underline{p} \cdot \underline{q})^{n-1}\, \underline{\sigma}^{(1)} \times \underline{\sigma}^{(2)} \cdot \underline{p}$$

$$A_6^{ps}(1) = \frac{1}{2}(\underline{p} \cdot \underline{q})^{n}\, \underline{\sigma}^{(1)} \times \underline{\sigma}^{(2)} \cdot \underline{q}$$

$$A_7^{s}(1) = \frac{1}{2}(\underline{p} \cdot \underline{q})^{n-1}\, \underline{\sigma}^{(1)} \times \underline{\sigma}^{(2)} \cdot \underline{p} \times \underline{q}$$

lead to g's which fit into columns "1" and "2" of Table V with the exception that their T → T values vanish.

A. Free nucleons	B. Complex nuclei
1) $P + P \rightarrow \pi^+$ (allowed)[(a)]	1) $P + (P,N) \rightarrow \pi^+$ (allowed)[(c)]
2) $P + N \rightarrow \pi^+$ (unobserved)	2) $P + (P,N) \rightarrow \pi^-$ (allowed)[(c)]
3) $N + N \rightarrow \pi^-$ (unobserved)	3) $P + (P,N) \rightarrow \pi^o$ (allowed)[(b)]
4) $N + P \rightarrow \pi^-$ (unobserved)	4) $N + (P,N) \rightarrow \pi^+$ (allowed)[(d)]
5) $N + N \rightarrow \pi^o$ (unobserved)	5) $N + (P,N) \rightarrow \pi^-$ (allowed)[(d)]
6) $N + P \rightarrow \pi^o$ (unobserved)	6) $N + (P,N) \rightarrow \pi^o$ (unobserved)
7) $P + P \rightarrow \pi^o$ (forbidden)[(b)]	

Table I. Qualitative experimental results for processes involving production of charged and neutral mesons by nucleons bombarding free nucleons or complex nuclei.

(a) Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. 78, 823 (1950)

(b) Bjorkland, Crandall, Moyer, and York, Phys. Rev. 77, 213 (1950)

(c) Richman and Wilcox, Phys. Rev. 78, 496 (1950)

(d) Bradner, O'Connell, and Rankin, private communication

I. Process		II. Experiment	III. Total cross sections ($\times 10^{28}$ cm^{-2})	
			Complex nuclei	Free
1) $P + P \rightarrow \pi^+$	allowed	A(1), B(1)	3.3 ± 1.0	6 ± 2
2) $P + P \rightarrow \pi^0$	forbidden	A(7)	?	0.1 ± 0.1
3) $N + P \rightarrow \pi^+$	allowed	B(4), B(2)	0.8 ± 0.4	?
4) $N + P \rightarrow \pi^-$	allowed	B(2)	0.8 ± 0.4	?
5) $N + P \rightarrow \pi^0$	allowed	A(7) + B(3)	1.7 ± 0.9	?
6) $N + N \rightarrow \pi^-$	possible	B(5)	?	?
7) $N + N \rightarrow \pi^0$	unobserved		?	?

Table II. Nucleon-nucleon cross sections for production of charged and neutral mesons. The experimental results given in Table I from which the cross sections are deduced are indicated in column II.

$\Gamma^2 g$	I A (Triplet) $1.94(10)^{-44}(\mathrm{Mev})^{-5}\mathrm{cm}^2$	I (A) Singlet $2.01(10)^{-44}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^5}$	II (A) Triplet $3.68(10)^{-48}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^7}$	II A Singlet $3.28(10)^{-48}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^7}$
$\Gamma^2 g$	I(B) Triplet $5.82(10)^{-44}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^5}$	I (B) Singlet $6.03(10)^{-44}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^5}$	II B Triplet $1.10(10)^{-47}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^7}$	II B Singlet $9.84(10)^{-48}\frac{\mathrm{cm}^2}{(\mathrm{Mev})^7}$

Table III. Values of the arbitrary constants $\Gamma^2 g$ adjusted to give a total cross section (neglecting deuteron formation) at 340 Mev of 2×10^{-28} cm^2.

θ	IA	IB	IIA	IIB	Meson energy (Mev)	Bombarding energy (Mev)
0°	1.46	4.38	2.53	7.59	73	
30°	1.24	1.09	2.16	1.90	56	340
60°	0.66	0.16	1.16	0.27	28	
0°	1.79	5.39	6.25	18.75	128	
45°	1.12	0.04	3.91	0.13	83	
90°	0.35	0.58	1.23	2.03	27	400
135°	0.13	0.12	0.47	0.44	13	
0°	2.18	6.54	12.3	36.9	158	
45°	1.32	0.26	7.47	1.47	105	450
90°	0.48	0.52	2.69	2.93	42	
135°	0.19	0.50	1.08	2.83	19	

Table IV
$d\sigma/d\Omega$ deuterons

Differential cross section in units of 10^{-28} cm^2 per unit solid angle for production of a positive π-meson and a deuteron in a p-p collision. θ is the angle between the directions of the nucleon and meson momenta in the laboratory system. The columns headed IA, IB, etc. are for transition operators of the type IA, IB, etc. respectively (cf. expressions (10)).

		$g_1,o;g_2,o$ $g_3,o;g_4,o$ $g_3'\ o$	$g_1,e;g_2,e$ $g_3,e;g_4,e$ g_3',e	$g_1',e;g_2',e$ $g_4',e;g_5',e$	$g_1',o;g_2',o$ $g_4',o;g_5',o$	$g_6',o\ (F,I)\ f_{FD}(\theta)$
$NN \rightarrow \pi^-$ $PP \rightarrow \pi^+$	$T \rightarrow T$	$2b^-(1-\rho)^2$	0	0	$3b^-(1-\rho)^2$	$2b^-(1-\rho)^2(1+1/2\cos^2\theta)$
	$T \rightarrow S$	$b^+(1+\rho)^2$	0	0	0	$b^+(1+\rho)^2\sin^2\theta$
	$S \rightarrow T$	0	$3b^-(1-\rho)^2$	0	0	0
	$S \rightarrow S$	0	0	$3b^+(1+\rho)^2$	0	0
$PP \rightarrow \pi^0$ $NN \rightarrow \pi^0$	$T \rightarrow T$	0	0	0	0	0
	$T \rightarrow S$	$a(1+\rho)^2$	0	0	0	$a(1+\rho)^2\sin^2\theta$
	$S \rightarrow T$	0	0	0	0	0
	$S \rightarrow S$	0	0	$3a(1+\rho)^2$	0	0
$NP \rightarrow \pi^+$ $NP \rightarrow \pi^-$	$T \rightarrow T$	0	0	0	0	0
	$T \rightarrow S$	$\frac{1}{2}b^+(1+\rho)^2$	$\frac{1}{2}b^-(1-\rho)^2$	0	0	$\frac{1}{2}b^+(1+\rho)^2\sin^2\theta$
	$S \rightarrow T$	0	0	0	0	0
	$S \rightarrow S$	0	0	$\frac{3}{2}b^+(1+\rho)^2$	$\frac{3}{2}b^-(1-\rho)^2$	$\frac{3}{2}b^-(1-\rho)^2\cos^2\theta$
$NP \rightarrow \pi^0$	$T \rightarrow T$	$c(1-\rho)^2$	0	0	$\frac{3}{2}c(1-\rho)^2$	$c^-(1-\rho)^2(1+1/2\cos^2\theta)$
	$T \rightarrow S$	0	$2c(1-\rho)^2$	0	0	0
	$S \rightarrow T$	0	$\frac{2}{3}c(1-\rho)^2$	0	0	0
	$S \rightarrow S$	0	0	0	$\frac{3}{2}c(1-\rho)^2$	$\frac{3}{2}c(1-\rho)^2\cos^2\theta$
		(a)	(b)	(c)	(d)	(e)

Table V. Values of the constants g and g' for various processes of meson production. The definitions of a, $b^{\pm}$, and c are given in equation (29). The first column gives the type of process, the second column gives the initial and final spin states as singlet (s) or triplet (t).

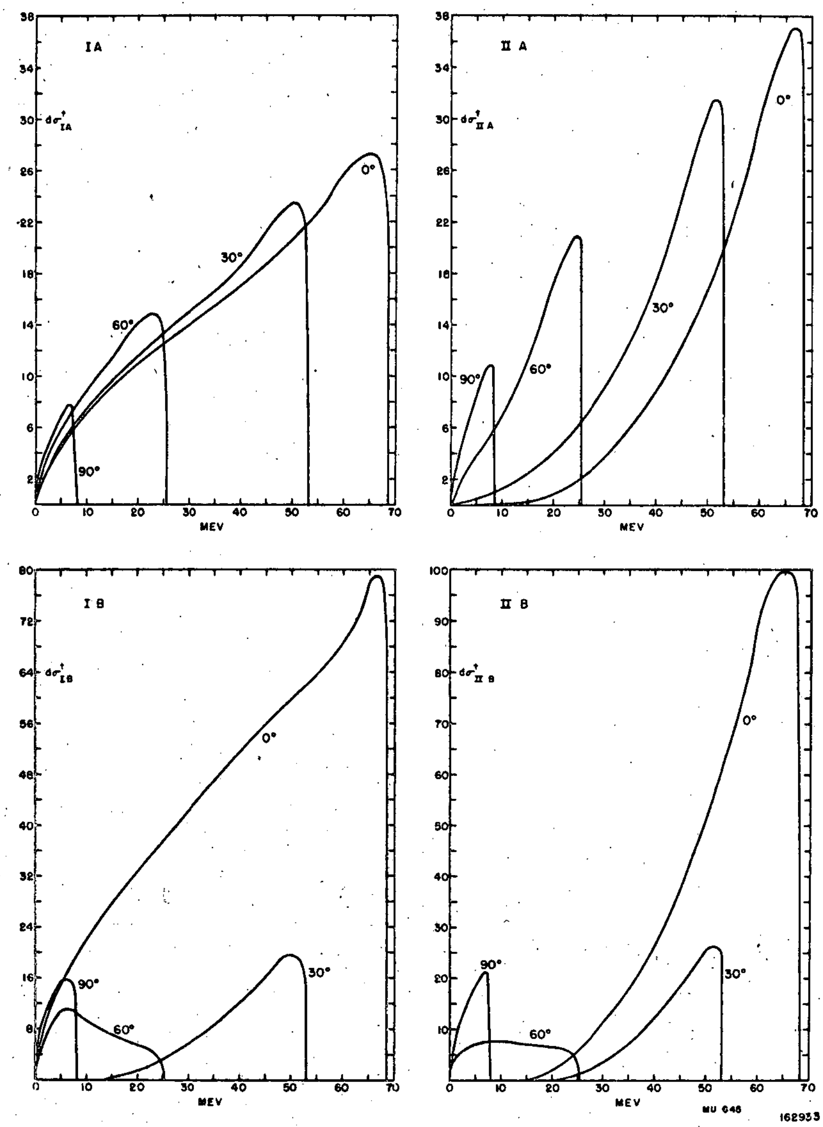

Fig. 1. Differential cross section for meson production in the laboratory system in units of 10^{-31} cm^2 per Mev per unit solid angle (designated as σ_0 in Eqs. 12 and 15), at 340 Mev for the incident nucleon. The final nucleons are assumed to be in a triplet spin state. The energy scale in Mev refers to the meson kinetic energy; the angles indicated are the angle between the meson and incident nucleon momenta. The labeling as IA, IIA, etc. refers to the type of transition operator defined in Equation 10. In the following Figs. 2-6, the units and definitions will be the same except where indicated.

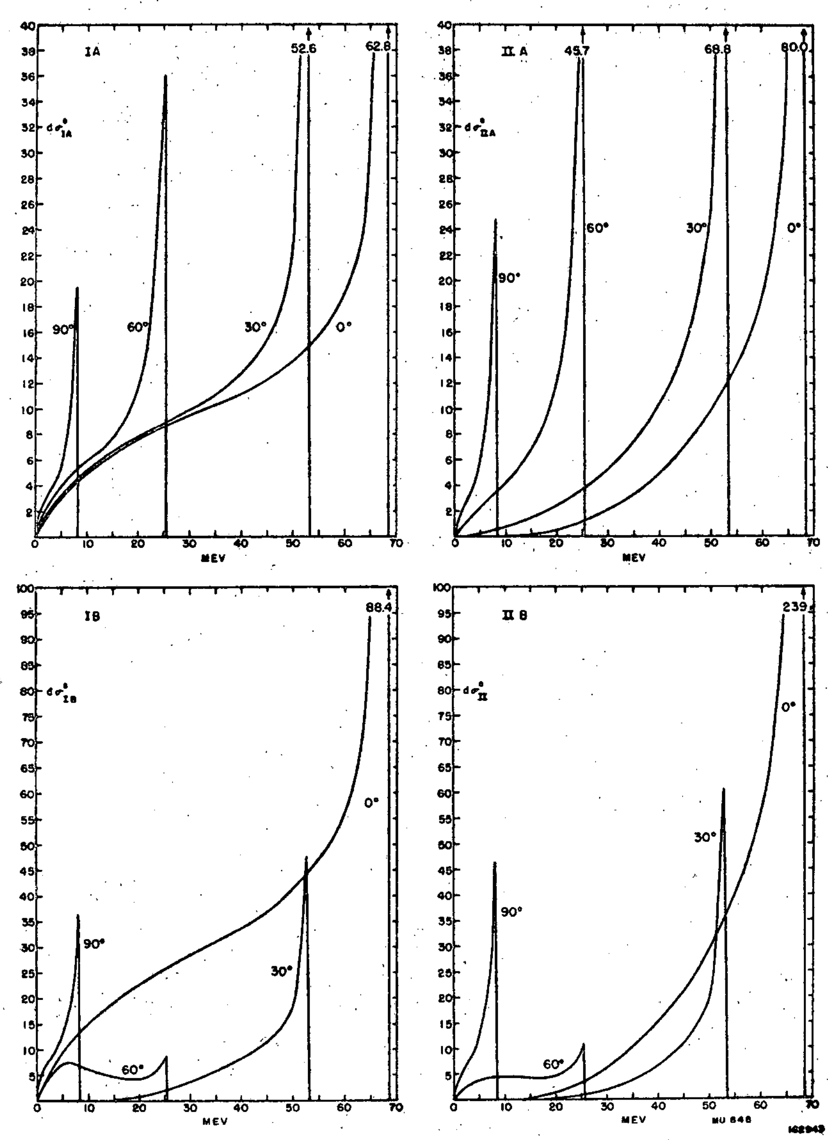

Fig. 2. Differential cross section for meson production. The incident energy is 340 Mev, the final nucleons are assumed to be in a singlet spin state.

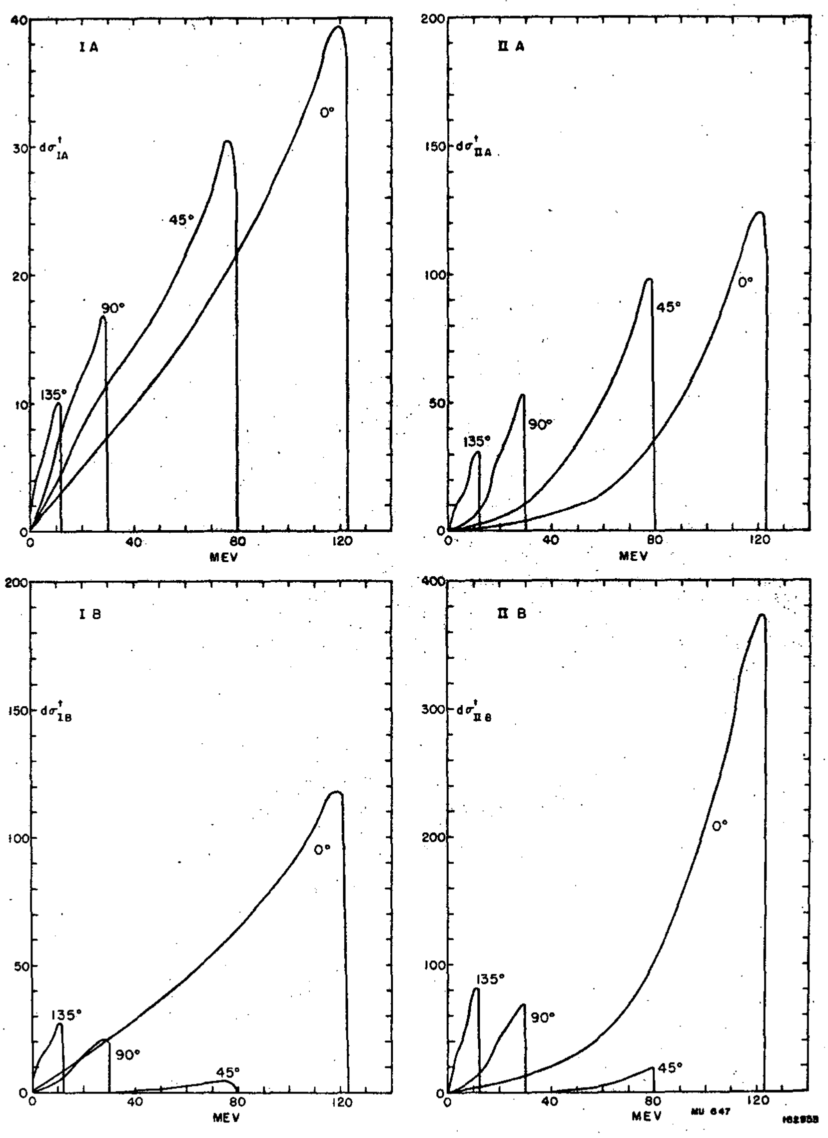

Fig. 3. **Differential** cross section for meson production. The incident energy is 400 Mev, the final nucleons are assumed to be in a triplet spin state.

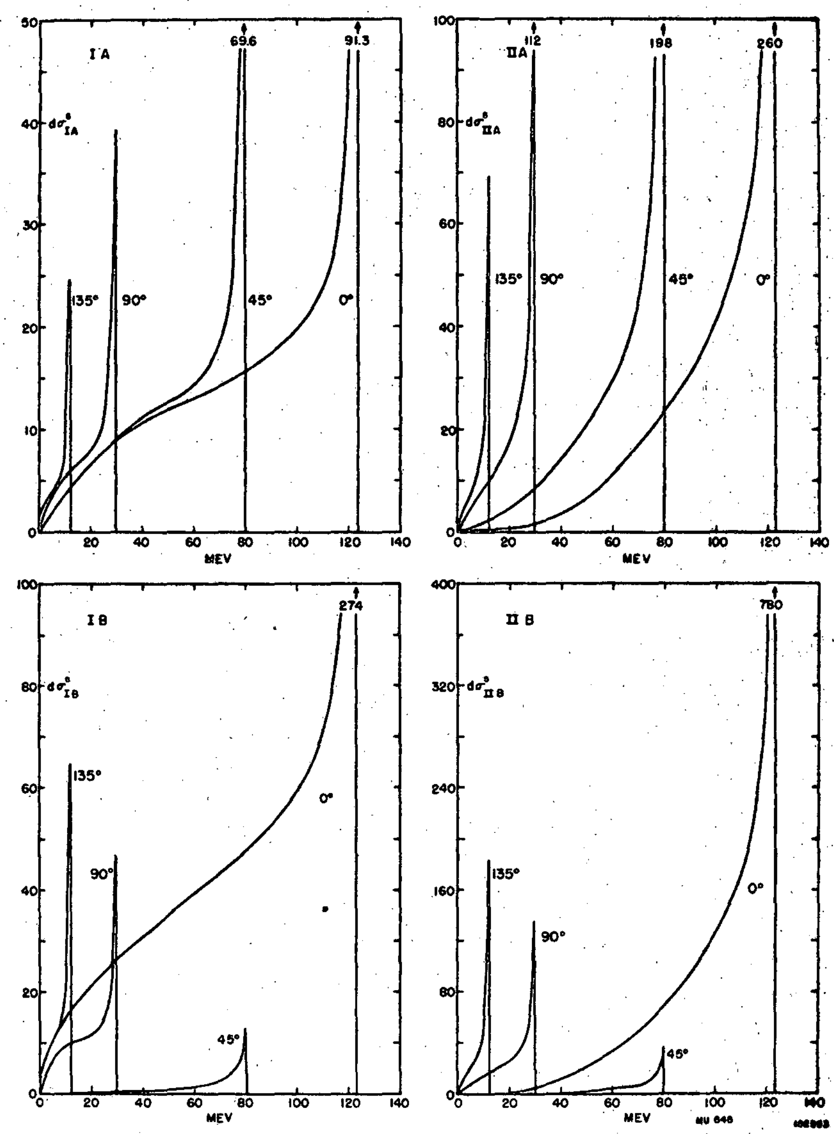

Fig. 4. Differential cross section for meson production. The incident energy is 400 Mev, the final nucleons are assumed to be in a singlet spin state.

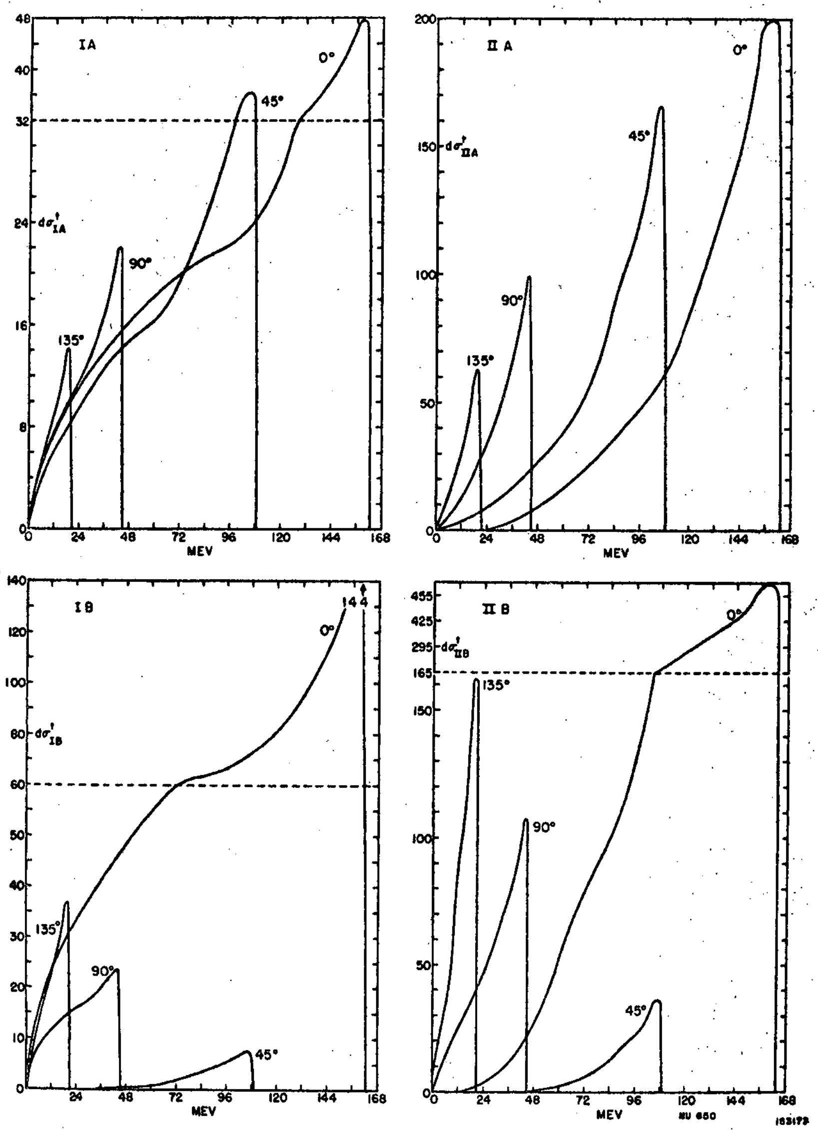

Fig. 5. Differential cross section for meson production. The incident energy is 450 Mev, the final nucleons are assumed to be in a triplet spin state.

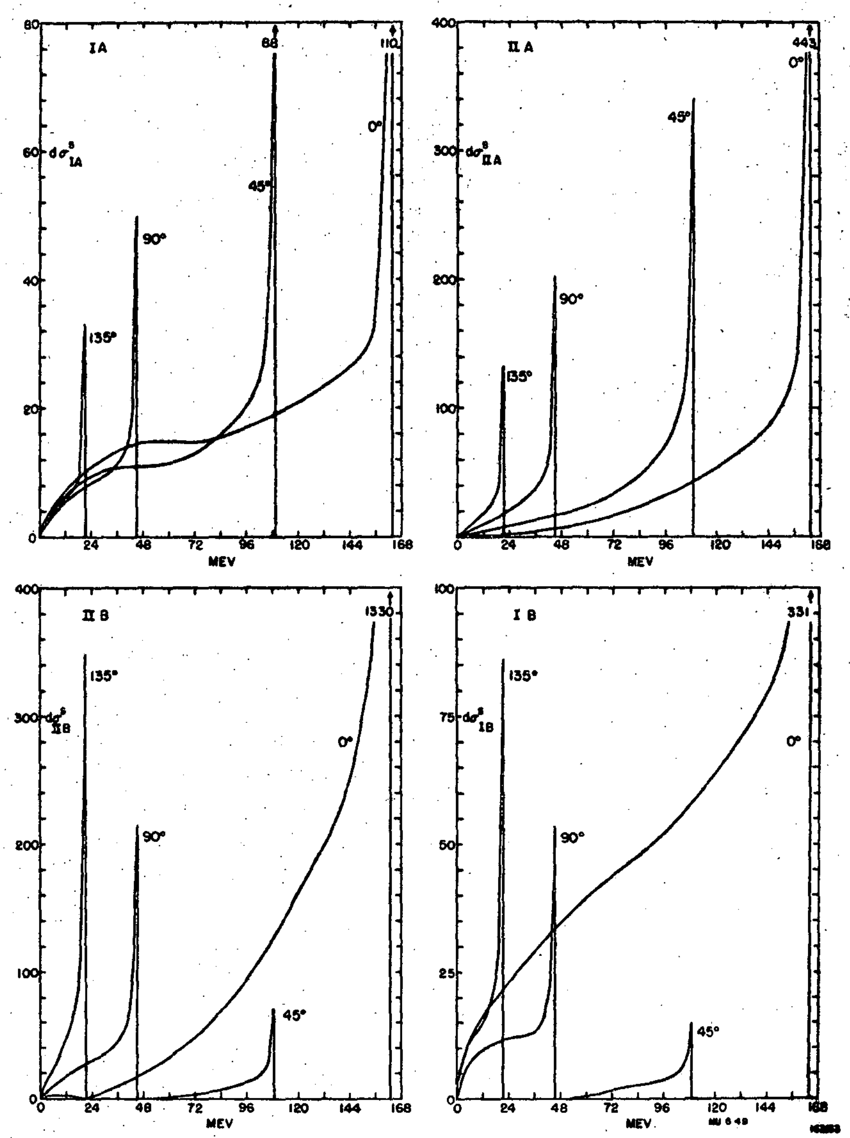

Fig. 6. **Differential cross section for meson production.** The incident energy is 450 Mev, the final nucleons are assumed to be in a singlet spin state.

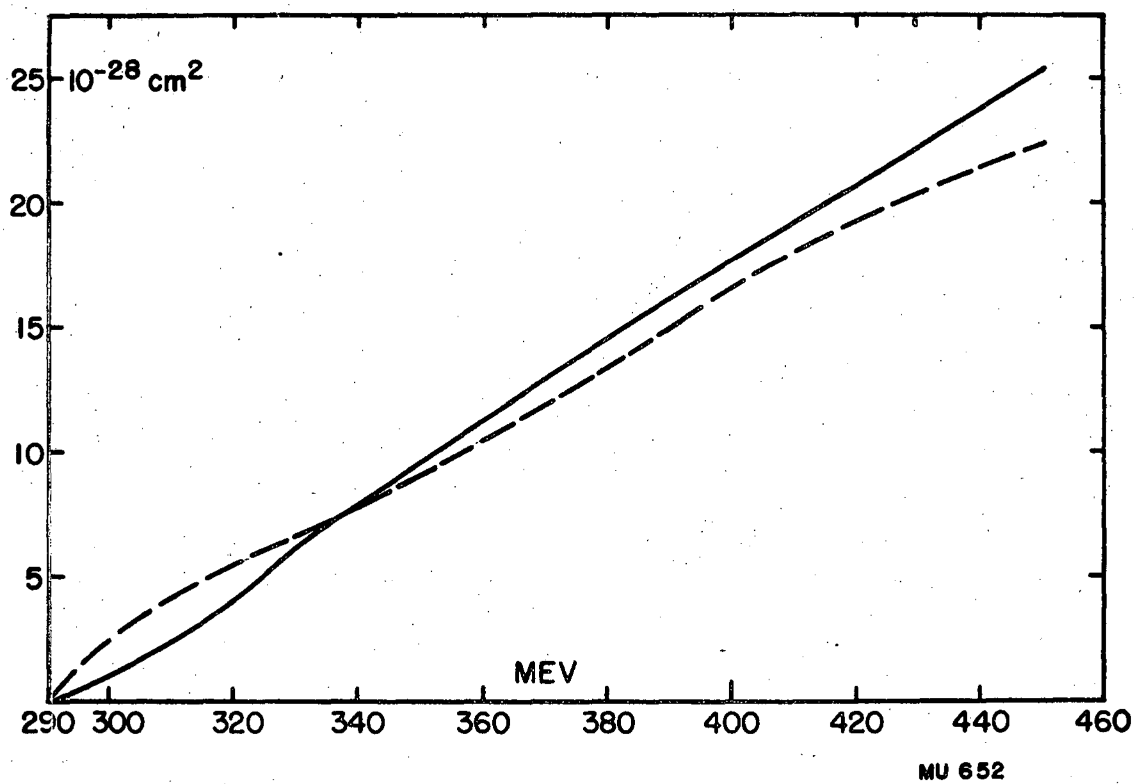

Fig. 7. Variation of the total cross section for meson production with the energy of the incident nucleon. The transition operator R_o (see Eq. 3) is assumed to be independent of meson momentum. The cross sections are arbitrarily normalized to 8×10^{-28} cm^2 at 340 Mev. The solid curve is for a final nucleon singlet state, the dashed curve is for a final nucleon triplet state and includes the possibility of deuteron formation.

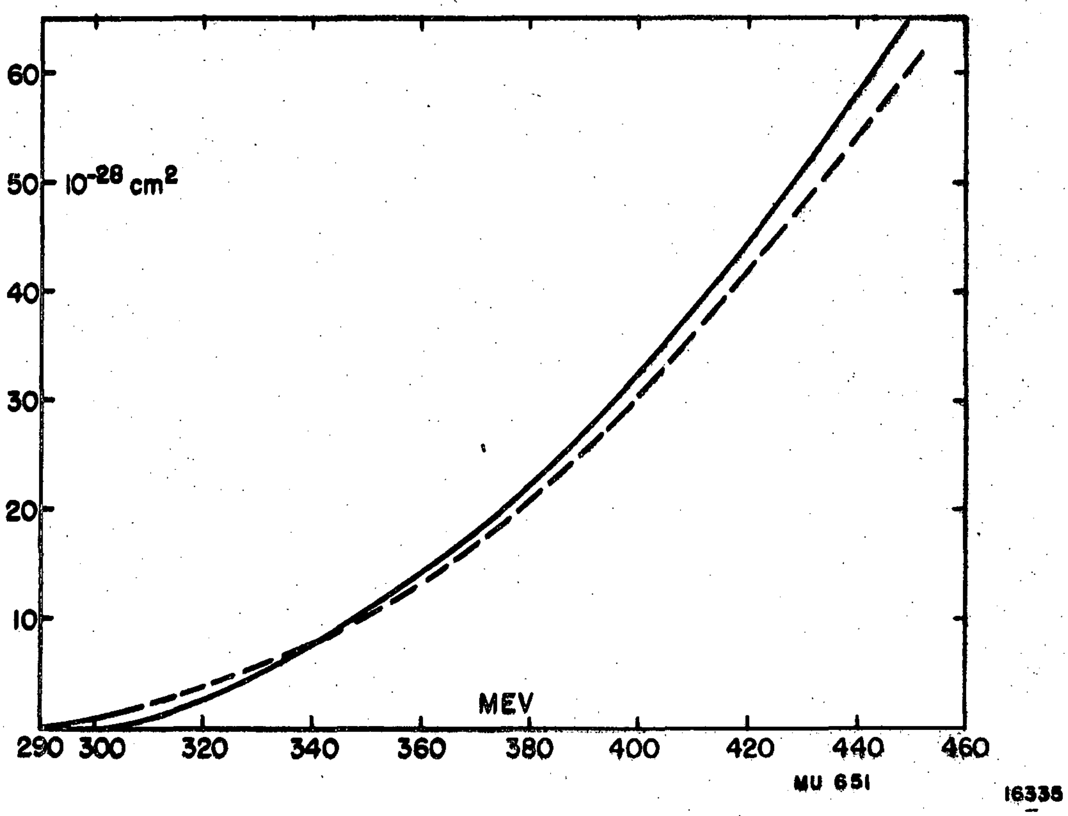

Fig. 8. Variation of the total cross section for meson production. The definitions and symbols are the same as for Fig. 7 except that the transition operator R_0 is assumed to depend linearly on the meson momentum.

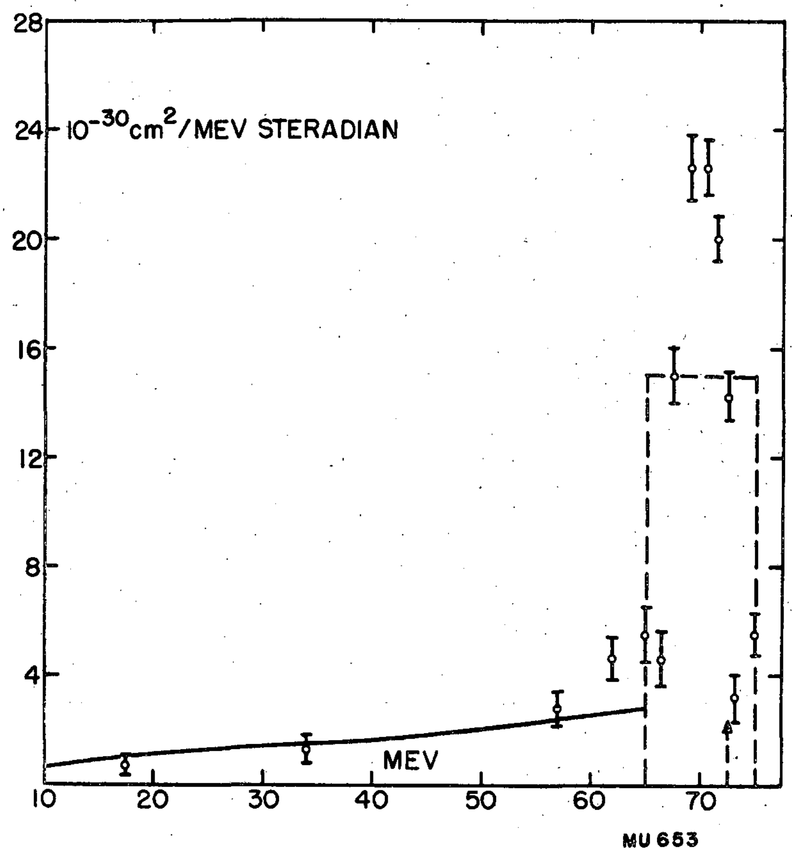

Fig. 9. **Differential cross section for meson production in the direction of the beam at 340 Mev.** The transition operator R_o (see Eq. 3) is assumed to be of the form given in Eq. 16, i.e. independent of meson momentum and leading to a final nucleon triplet spin state. The cross section for mesons with energies greater than 65 Mev (including the delta function contribution for deuteron formation at 72 Mev) is averaged uniformly over the energy interval of 65 to 75 Mev. The points indicated are from the experimental results of Cartwright, Richman, Whitehead, and Wilcox[7] and Cartwright and Whitehead.[6]

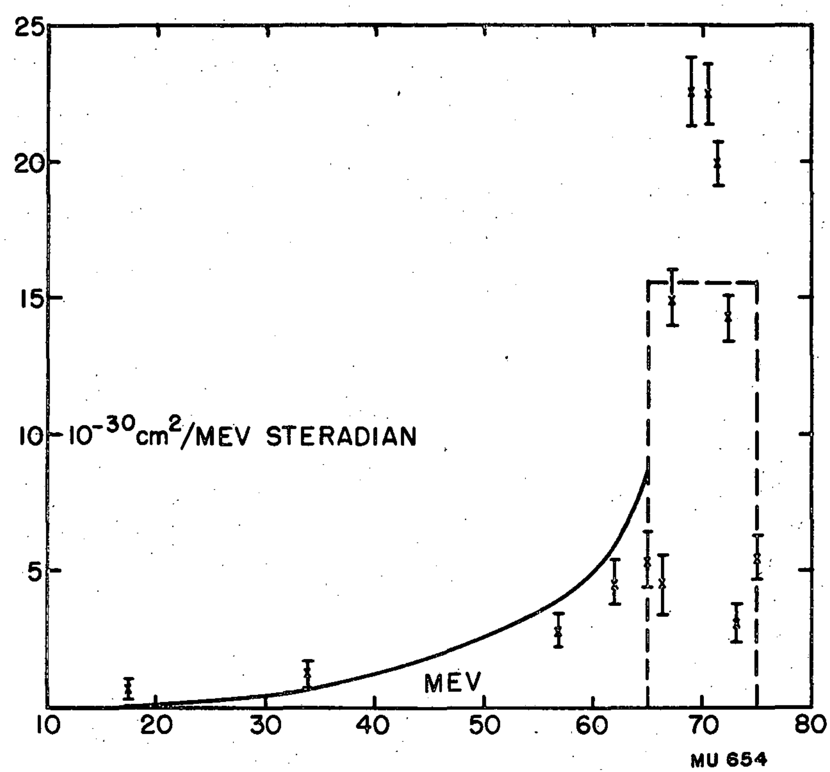

Fig. 10. Differential cross section for meson production at 340 Mev. The definitions and symbols are the same as Fig. 9, except that the transition operator is assumed to be of the form given in Eq. 17, i.e. linearly dependent on meson momentum and leading to a mixture of singlet and triplet spin states.