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ABSTRACT

The equilibrium energy spectrum of electrons in the outer radiation belt is determined by the injection spectrum and the loss processes which operate to remove the electrons or change their energy. The loss processes which have been considered here are ionization energy loss, multiple scattering, and electron-electron scattering; the injection spectra that have been considered are neutron beta-decay electrons, and monoenergetic electrons of 780-kev and 20-kev. The problem has been treated numerically. The results of the numerical calculation are compared to recent measurements of the outer-belt electron spectrum, and it appears that neutron decays produce a reasonable fraction of the outer-belt electrons, but other processes such as acceleration may be important.

Energy Spectrum of Electrons in the Outer Radiation Belt

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INTRODUCTION

One of the interesting problems about the Van Allen radiation belts is to determine the sources of particles that populate them. It is quite well established now that most of the protons in the inner belt come from the decay of neutrons leaking out of the atmosphere of the earth (Freden and White, 1960; Hess, 1959). The source of electrons is not as well determined. It would be surprising if the electrons in the inner radiation belt were not also from leakage neutron decay, but this is not certain currently.

We want to examine here what kind of an electron belt would be produced at two or three earth radii out from the surface of the earth by neutron decay alone. We have detailed quantitative knowledge of the neutron-decay source strength (Hess, Canfield, and Lingenfelter, 1961) and therefore can quantitatively study the equilibrium electron spectrum to be expected in this region. Then, by comparison with experimental information about the outer radiation belt, we can see whether neutrons produce an important fraction of the outer belt.

We know the neutron source strength adequately, but to determine the equilibrium electron spectrum we must understand the loss mechanisms operating to remove electrons. One such process is multiple small-angle Coulomb scattering (Christofilos, 1958; Welch and Whitaker, 1959; Wentworth, MacDonald, and Singer, 1959), which changes the pitch angle of the particles

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and lowers the mirror point so that the particles get lost in the atmosphere. Another process is ionization-energy loss which degrades the electron's energy. The former is proportional to Z^2 , where Z is the atomic number of the gas present; the latter is proportional to Z . Therefore, the dominant loss mechanism depends on the type of material present.

Particles in the inner radiation belt move most of the time in an atmosphere of oxygen and nitrogen. The higher Z of this material is such that multiple scattering is the principal loss mechanism. Particles in the outer belt exist essentially in a hydrogen exosphere where ionization energy loss is important.

A typical high-energy electron in the outer radiation belt might move on a magnetic line of force going out 24,200 km from the center of the earth at the equator. Let us consider a 780-kev electron which has a mirror point at 30-deg magnetic latitude. Let λ be the magnetic latitude of a field line, R be the distance from the center of the earth, and α be the pitch angle of the electron spiraling about the magnetic field line. $\tan \alpha = v_{\perp}/v_{\parallel}$ where $v_{\perp}$ and $v_{\parallel}$ are velocity components perpendicular and parallel to the magnetic field line, respectively. Let us approximate the earth's magnetic field by a dipole field for which the field lines are given by

$$R = R_0 \cos^2 \lambda \quad (1)$$

where R_0 is the radius of the field line at the equator ($\lambda = 0$). Its strength is given by

$$B = CR^{-3} (1 + 3 \sin^2 \lambda) \quad (2)$$

where $C = 8.1 \times 10^{10}$ gauss-km³. The pitch angle of the electron must satisfy the relation

$$B = B_m \sin^2 \alpha, \quad (3)$$

where B_m is the value of the magnetic field at the mirror point. Table I

gives values of α and λ that satisfy Eqs. 1, 2, and 3 for (a) a mirror point at 17,200 km or for (b) a mirror point at 8,000 km.

TABLE I. Values of α and λ satisfying Eqs. (1), (2), and (3) for a mirror point at (a) 17,200 km and (b) 8,000 km.

R (km)	λ (deg)	α (deg)	
		Case a	Case b
24,200	0	34.4	8.3
17,200	30	90	14.8
8,000	54.9	--	90

In order to essentially lose a particle from the radiation belt by scattering, its angle must be altered so that the mirror point is lowered to about 8,000 km, where it will encounter a relatively dense atmosphere. From Table I we see that, if the scattering takes place at the equator, a change of pitch angle of 26.1 deg is necessary to change the mirror point from 17,200 km to 8,000 km; if the scattering takes place at 17,200 km, then a change of pitch angle of 75.2 deg is necessary. As an average intermediate situation, we will consider that a change of pitch angle of one radian will remove a particle from the outer belt. This corresponds to a scattering through a projected scattering angle of one radian, or a total scattering angle of $\sqrt{2}$ radians.

To determine which loss mechanisms are dominant in the outer belt, we must consider a change of pitch angle by Coulomb scattering and energy loss by ionization. For particles traveling in unionized hydrogen gas, Table II lists the probabilities of multiple scattering and single scattering to projected angles greater than one radian, and scattering from an electron which produces a recoil electron of greater than 30 kev energy. This calculation is summarized in Table II as a function of the particle's range.

TABLE II. Summary of scattering probabilities as a function of particle range

E (kev)	R (gm/cm ²)	ΔR (gm/cm ²)	$\bar{E}$ (kev)	$\overline{\theta_p^2}$ (rad ²)	$\overline{\Sigma \theta_p^2}$ (rad ²)	ΣP_{MS}	ΣP_{SS}	ΣN_{ee}
780	0.153	0.020	740	0.016	0.016	0.0	0.0005	0.11
700	0.133	0.025	650	0.026	0.042	0.0	0.0012	0.24
600	0.108	0.024	550	0.033	0.075	0.0003	0.0023	0.37
500	0.084	0.023	450	0.045	0.12	0.0031	0.0037	0.50
400	0.061	0.0213	350	0.065	0.19	0.024	0.0059	0.63
300	0.0397	0.0187	250	0.10	0.29	0.064	0.0097	0.74
200	0.0210	0.0144	150	0.19	0.48	0.16	0.018	0.83
100	0.00657	0.00463	75	0.20	0.68	0.22	0.029	0.85
50	0.00194							

Here E is the electron kinetic energy in kev, R is its range in hydrogen in gm/cm² (National Bureau of Standards, 1956), ΔR is the amount of hydrogen necessary to lower the electron energy from the top to the bottom of the energy group indicated, $\bar{E}$ is the average kinetic energy of the group in kev, $\overline{\theta_p^2}$ is the square of the r.m.s. projected multiple-scattering angle in radians² (Bethe and Ashkin, 1953), $\overline{\Sigma \theta_p^2}$ is the sum of these angles to get the additive effect, ΣP_{MS} is the probability that this value of $\overline{\Sigma \theta_p^2}$ will scatter the particle downward one radian or more, ΣP_{SS} is the probability that the electron will single scatter one radian or more in the projected angle downwards, and ΣN_{ee} is the number of recoil electrons from electron-electron collisions which have energy 30 kev or higher (Bethe and Ashkin, 1953).

Inspection of this table shows that an electron will slow down from 780 to about 50 kev before the scattering gets large enough that a significant number of particles will be lost from the outer belt ($\Sigma P_{MS} = 0.22$

at 50 kev). Below 50 kev, the scattering increases sharply, and a substantial fraction of the particles will be lost by scattering. Only about a quarter of the particles in the outer belt will scatter out of the belt when a 780-kev electron slows down to 50 kev. About one secondary electron of energy greater than 30 kev will be made per 780-kev electron slowing down. Thus in the numerical calculation to follow we must consider both slowing down and scattering as important loss mechanisms in the outer zone, with the additional production of low-energy electrons by electron-electron collisions.

NUMERICAL CALCULATIONS

The scattering calculations were coded for an IBM 709 computer. The program was run with several input spectra. The first problem was started by having all particles in one energy group (770 to 778 kev) and was stopped when all particles were slowed down below 30 kev. This case of a mono-energetic pulse source can be checked analytically and demonstrates the kinds of physical processes that take place. This problem is moderately similar to what happened in the Argus experiment when a pulse of electrons was suddenly injected into the earth's magnetic field by a nuclear explosion (Argus, 1959).

The second calculation starts with the neutron β -decay spectrum and follows these particles down to 30 kev. This problem has a bearing on the make up of the outer radiation belt.

Each of these calculations have been studied for (1) a pulse source and (2) a continuous source. In Case (1) a squirt of particles is started all at one time and slowed down together. In Case (2) the same number of particles are injected at each interval in time and the problem carried forward until an equilibrium spectrum is obtained.

The computer program starts with an initial spectrum and calculates the spectrum after a short time Δt . The time step is taken short enough so that the probability of a particle with $E = 30$ kev scattering twice in one time step is very small. The process is repeated for many time steps until the problem is completed. Three loss mechanisms have been considered: (a) the usual ionization-energy loss for a moving charged particle having distant collisions with electrons (dE/dx); (b) close collisions with electrons which occur less frequently but are important in that larger momentum transfers are involved and new, fast particles can be made in this way; (c) the loss by scattering which removes particles from the region by

changing the pitch angle.

Loss processes (a) and (b) are handled together while (c) is considered separately. To handle (c), we calculate how many particles $S(E)$ are lost from each energy group in each time step by scattering. The fraction of particles scattering in each step is obtained from

$$S(E) = \frac{\Delta t \bar{\rho}}{[\bar{\rho} \tau(E)]},$$

where $\bar{\rho}$ is the number of protons per cubic centimeter traversed by the electron, and $\bar{\rho} \tau(E)$ is the product of proton density and electron lifetime as calculated by Wentworth, MacDonald, and Singer (1959).

The energy loss processes (a) and (b) have been represented by

$$N^i(m+1) = N^i(m) + \sum_{l=i+1}^{\max} n^l(m) P_l^i - N^i(m) \sum_{l=1/2}^{i-1} P_l^i \quad (5)$$

$$- N^i(m) \left| \frac{dE^i}{dx} \right|_c \frac{\Delta x^i}{\Delta E} + N^{i+1}(m) \left| \frac{dE^{i+1}}{dx} \right|_c \frac{\Delta x^{i+1}}{\Delta E}$$

which relates the number of particles in the i^{th} energy group at time step $m+1$, $N^i(m+1)$, to those in that group at time step m , $N^i(m)$.

Here P_l^i is the probability that an electron be scattered from group l (superscript) to group i (subscript) by an electron-electron scatter.

The first term on the right side of Eq. (5) is the number of particles in group i just before time step $m+1$. The second term is the number that scatter into group i from energies above it; this electron may be a recoil electron or a higher-energy electron that has been degraded in energy (these two electrons are indistinguishable). The third term is the number of particles that scatter out of group i into lower energy groups. The fourth term is the number of particles that lose enough energy by ionization to be degraded from the i^{th} group to the next lower energy group. The

The last term is the number of particles in the $i = 1$ group that lose enough energy to be degraded into the i th group.

The probability P_1^l is

$$P_1^l = (v \rho_e \Delta t) \left(\frac{d\sigma_{ee}}{dW} \right) \Delta E^1, \quad (6)$$

where ρ_e is the electron density. In Eq. (6) the electron-electron scattering cross section has been calculated by C. Møller (1932);

$$\frac{d\sigma_{ee}}{dW} = \frac{2\pi r_0^2}{\beta^2} (mc^2) \left[\frac{1}{W^2} + \frac{1}{(E-W)^2} + \frac{1}{(E+mc^2)^2} - \frac{2E+mc^2}{(E+mc^2)^2} \frac{1}{W(E-W)} \right], \quad (7)$$

where $2\pi r_0^2 = 2\pi(e^2/mc^2)^2 = 0.50 \times 10^{-24} \text{ cm}^2$; $\beta = v/c$ for the incident electron; m is the rest-mass energy of the electron; E is the incident-electron kinetic energy; and W is the kinetic energy of either of the two final electrons. All quantities are expressed in the laboratory coordinate system.

The ordinary ionization-energy loss which has been corrected for that part of the energy loss that has already been included as large-angle scattering in the second and third terms in Eq. (5) is

$$\left| \frac{dE^1}{dx} \right|_c = \left| \frac{dE^1}{dx} \right| - \sum_{l=1/2}^{1-l} \frac{d^2 P_l^1}{dE dx} (E^1 - E^l) \Delta E, \quad (8)$$

where (Bethe, 1933)

$$\left| \frac{dE}{dx} \right| = \rho_e mc^2 \frac{2\pi r_0^2}{\beta^2} \left[\log_e \frac{mc^2 E \beta^2}{2I^2 (1-\beta^2)} - (2 \sqrt{1-\beta^2} - 1 + \beta^2) \log_e 2 + 1 - \beta^2 + \frac{1}{8} (1 - \sqrt{1-\beta^2})^2 \right]. \quad (9)$$

Here ρ_e is the number of electrons per cm^3 , I is the ionization energy constant, and the rest of the symbols are defined in the preceding paragraph.

Combining Eqs. (5) and (8), we get

$$\begin{aligned}
 N^i(m+1) = N^i(m) + \sum_{l=i+1}^{l_{\max}} N^l(m) P_l^i + N^i(m) \sum_{l=i/2}^{i-1} (1-l-1) P_l^i \\
 + N^{i+1}(m) \left| \frac{dE^{i+1}}{dx} \right| \frac{\beta^{i+1}_c \Delta t}{\Delta E} - N^i(m) \left| \frac{dE^i}{dx} \right| \frac{\beta^i_c \Delta t}{\Delta E} \\
 - N^{i+1}(m) \sum_{l=\frac{i+1}{2}}^{l_{\max}} (1-l+1) P_l^{i+1} .
 \end{aligned} \tag{10}$$

In the energy region from 30 to 60 Mev, additional approximations have improved the convergence of the sums. The character of the spectrum on either side of 60 kev lend validity to these approximations.

The calculation for each time step is repeated until all particles have slowed down below 30 kev. The time interval Δt has been selected to be small enough so that there is a very small chance that a particle be scattered twice within that time step. Since the chance of scattering goes up with a decrease in particle energy, a compromise must be made between computer time and the lowest energy of interest. On this basis, the lowest energy of interest was chosen to be 30 kev.

As

RESULTS

As stated before, in the first problem we started with a pulse of particles at $E = 780$ kev and followed them as they produced secondary electrons, were slowed down by ionization energy loss, and were scattered out. The starting data of the second problem was the neutron decay spectrum.

A. The 780-kev electron problem. The first problem was undertaken to demonstrate the physics in the slowing-down problem. Starting with all the particles at 780 kev, the calculation proceeds until the particles all slow down to less than 30 kev. Figure 1 shows several steps in the slowing-down process. In a short time, the initial pulse of particles, point A in Figure 1, turns into a U-shaped distribution, curve B, by producing knock-on electrons. Curves B, C, D and E are drawn for equal time intervals where the energy loss by ionization as well as the build-up of low-energy recoil electrons can easily be seen. Curve C' (dashed line) is the solution for the problem ignoring the scattering-out term. One sees that the effect of this scattering-out term becomes large only for energies below about 100 kev. Eventually all the particles are lost by scattering or pass below the energy cut-off of the problem. It should be pointed out that these figures give the density of particles at a given energy (electrons/cm³). If a flux of particles is desired (electrons/cm²-sec), the spectra must be multiplied by the appropriate velocity.

The Argus nuclear explosion (Argus Experiment, 1959) was carried out at about 300 miles above the earth and formed a shell of trapped electrons in space. The electrons were produced by β -decay of the fission fragments from the explosion. These electrons were lost eventually by multiple small-angle Coulomb scattering (Welch and Whitaker, 1959) on the oxygen and nitrogen atoms in the atmosphere. However, if the Argus explosions had been at an altitude of 2000 km or more and had therefore been in the

hydrogen exosphere, the electron spectrum would have behaved roughly as we have shown in Fig. 1 because of the three loss mechanisms considered. There is, of course, a spectrum of energy for electrons, born of a nuclear explosion, with some having energies of more than 1 Mev. The physical processes remain the same, however, and the above solution for a δ -function input should give a qualitative feeling for what would be expected for a high-altitude Argus experiment.

If we consider a situation where a stream of 780-kev electrons is continuously introduced into the magnetic field, an equilibrium spectrum will result. The solid line A in Fig. 2 shows the spectrum that would result after equilibrium is established. The total number of particles as a function of the time after initiating the injection (the build-up toward equilibrium) is given in Fig. 3, Curve B. An example of this physical process might be a particle accelerator in space continuously emitting a beam of 780-kev electrons (Christofilos, 1959).

Several checks have been made to insure that the computer program was doing the desired calculation. One such check has been indicated in Fig. 2 by a dashed line, curve B. This line indicates the equilibrium spectra that would be attained if there were a continuous source of 780-kev electrons and the only loss mechanism was ionization-energy loss. This case can be calculated analytically quite easily, and the results agree with the machine calculations. To illustrate the nature of the analytic solution, first assume that dE/dx (ionization-energy loss per centimeter of stopping material) has the simplified form

$$\frac{dE}{dx} = kv^{-2}, \quad (11)$$

where k is a constant and v the electron velocity. The probability (dP/dE) of finding a particle between energy E and dE , is directly proportional to the time dt that the particle spends in this energy interval. Therefore,

we can write

$$\frac{dP(E)}{dE} = \left[\frac{dt}{dE} \left[\frac{dE}{dx} \frac{dx}{dt} \right]^{-1} \right] \approx \frac{v}{k} = \frac{1}{k} \frac{(E^2 + 2Em)^{1/2}}{E + m} \quad (12)$$

where m is the electron mass, and E is its kinetic energy. Starting with the relativistic dE/dx formula, Eq. (9), for the ionization-energy loss of electrons, a correct relativistic calculation of dP/dE has been done following the above outline. Results of this analytic calculation agree very closely with those of the corresponding machine calculation, which is shown by the dotted line (b) in Fig. 2.

B. Neutron β -decay electrons. The main purpose of this paper is to calculate what electron spectrum would exist in the outer radiation belt resulting from neutron β -decay electrons alone. Starting with a pulse of neutron β -decay electrons (curve A, Fig. 4), the calculations proceed as before. The slowing down and scattering out is shown in several stages in Fig. 4. Curves B, C, and D are drawn for equal time intervals.

The equilibrium spectrum (curve A, Fig. 5) is compared with the original β -decay spectra, curve B in Fig. 5. Curve A in Fig. 5 is the electron energy spectrum that would exist in the outer radiation belt if neutron decay was the only source of electrons and we had considered all of the loss processes. These data are also electron density rather than flux. We see that the equilibrium spectra is much the same shape as the original β -decay spectrum except for a slight shift toward lower energy. The build-up toward equilibrium as a function of time is shown in Fig. 3, curve A. This figure gives the total number (N) of particles with energies greater than 30 kev as a function of time t . The units on the abscissa are $f\bar{p}t$, where f is a dimensionless number that characterizes the increased effectiveness of a partially ionized medium to multiple scatter or to lose energy by

ionization energy loss, $\bar{\rho}$ is the average density in gm/cm^3 , and t is the time in seconds. The units on the ordinate are $f\bar{\rho}N/n$, where N is the number of particles/ cm^3 in the radiation belt and n is the rate of injection in electrons/ $\text{cm}^3\text{-sec}$.

For example, assume a particle mirrors at 30 deg magnetic latitude on the line that reaches a maximum distance of 3.8 earth radii from the center of the earth. Assume that the hydrogen is 50% ionized and has an average density of 10^{-21} gm/cm^3 (Johnson, 1960; Lenchek, Singer, and Wentworth, 1961). From Fig. 3 we see that the electron spectra reaches 50% of its equilibrium value for $f\bar{\rho}t = 0.8 \times 10^{-12} \text{ gm-sec/cm}^3$.

Here f is

$$f = \frac{\left| \frac{dE}{dx} \right|_{\text{ion}} f_{\text{ion}} + \left| \frac{dE}{dx} \right|_{\text{neut}} f_{\text{neut}}}{\left| \frac{dE}{dx} \right|_{\text{neut}}} \quad (13)$$

where $\left| \frac{dE}{dx} \right|_{\text{ion}}$ is the ionization energy loss by completely ionized hydrogen, and f_{ion} is the fraction of the hydrogen that is ionized. Subscripts "neut" denote the corresponding quantities for neutral hydrogen. The same kind of an equation can be written for the multiple scattering angle (Spitzer, 1956).

For any value of f_{ion} , f has values that range from one when $f_{\text{ion}} = 0$ to about three when $f_{\text{ion}} = 1.0$. If we assume $f = 2$, $\bar{\rho} = 10^{-21} \text{ gm/cm}^3$, then the time for 50% equilibrium of neutron decay electrons is $4 \times 10^8 \text{ sec}$ or about 12 years.

The equilibrium intensity may be obtained from $Nf\bar{\rho}/n = 10^{-12} \text{ gm-sec/cm}^3$ (curve A, Fig. 3). If we assume the electron injection rate $n = 3 \times 10^{-13} \text{ electrons/(cm}^3\text{-sec)}$ (Hess, Canfield, and Lingenfelter, 1961) and values of f and $\bar{\rho}$ as above, then $N = 1.5 \times 10^{-4} \text{ electrons/cm}^3$. An approximate flux can be obtained by multiplying this figure by the most probable velocity of $2.2 \times 10^{10} \text{ cm/sec}$, yielding a flux of $3.4 \times 10^6 \text{ electrons/(cm}^2\text{-sec)}$ for neutron-decay electrons

above 30-kev energy as a representative value for the outer belt. It should be noted that electron-electron scattering does not scale as above since it is independent of f at these energies. However this effect is not as large as that of ionization energy loss or multiple scattering, and f is bounded between 1.0 and 3.0. Therefore the scaling error should not be large. The numerical problem of this paper was performed with $f = 2.0$, so that this problem does not affect the example above. In Fig. 3, then, f should not be a variable, but should be set equal to 2.0.

The possibility has been suggested that there are a large number of low-energy electrons in the outer radiation belt (Arnoldy, Hoffman, and Winckler, 1960). We have made a further calculation which bears on this low-energy group of electrons. A lifetime of about 1.5×10^7 sec is all that can be expected of a 20-kev electron in a hydrogen atmosphere of 1200 atoms/cm^3 average density because of its range and multiple-scattering loss. Thus 20-kev electrons must be injected at a rate of 7×10^{-7} electrons/ $(\text{cm}^3\text{-sec})$ to achieve equilibrium fluxes of 10^{11} electrons/ $(\text{cm}^2\text{-sec})$. This source strength is about 10^7 times that of the neutron β -decay source.

The spectrum calculated here is for neutron β -decay electrons trapped in the outer Van Allen radiation belt. One would expect, however, that the spectrum at lower altitudes would be different, even on magnetic field lines that extend into the upper belt. We attempted to see how this spectrum at lower altitude would differ from that in the upper altitudes by keeping track of the number and energy of those electrons which were Coulomb-scattered out of the upper radiation belt. These particles are the ones whose mirror altitude is lowered, and therefore they feed the spectrum at lower altitudes. Using these scattered particles as an input spectrum for lower altitudes and using a lifetime proportional to E^2 for electrons in the nitrogen-oxygen atmosphere of lower altitudes (Welch and Whitaker, 1959), we obtain a flux spectrum as shown in Fig. 6. It should be emphasized that

the geometry we have used is not that of the radiation belts. We have essentially put electrons in a box of hydrogen and calculated the equilibrium spectrum inside the box. We also calculated the energy spectrum of particles scattered out of this box and weighted them with the lifetime they would experience in the lower atmosphere. Kellog (1960) has also calculated this leakage spectrum using an input spectrum of neutron β -decay (our scattered spectrum has many more lower-energy particles), and a loss mechanism of ionization-energy loss (the E^2 lifetime of Welch and Whitaker (1959) favors higher-energy particles more strongly than ionization-energy loss mechanisms). As a result of these two balancing differences, about the same result is obtained.

Kellog argues that neutron decay is not an important source of electrons in the radiation belts. His argument, however, is based in large measure on electron flux measurements which have, since then, been revised downward by as much as a factor of 10^3 . O'Brien, Van Allen, Laughlin, and Frank (1961) have recently determined that there were $\approx 10^8$ electrons/(cm²-sec) of energy greater than 40 kev in the outer belt on September 5, 1961. This was their highest reported flux; a time average of their values might be several times lower than this figure. The observed flux at these energies is about a factor of ten larger in magnitude than our calculated value. Since exospheric densities are quite uncertain and acceleration processes may substantially increase the lifetime of the neutron decay electrons, it is not at all compelling that sources other than neutron decay are required.

Lenchek and Singer (1961) have recently calculated the equilibrium flux of outer-belt electrons to be expected from neutron β -decay in a manner rather similar to our present paper. They consider coulomb scattering

and ionization-energy loss of the electrons. The shape of the energy spectrums they calculate agrees quite well with ours. But they arrive at different conclusions than we do about the origin of the electrons observed in the radiation belt. They conclude that less than 5% of the electrons seen by Holly and Johnson (1960) in the inner radiation belt at ≈ 1100 -km altitude are due to neutrons, and even a smaller fraction of the electrons seen by Cladis, Chase, Imhof, and Knecht (1961) at ≈ 1000 km in the outer belt are due to neutron decay. Lenchek and Singer state that neutron albedo contributes most if not all of the electrons above 400 kev, but that few of the electrons of lower energy are made by neutrons.

We feel it is dangerous to base a general conclusion about the origin of all particles in the radiation belt on a few measurements at quite low altitudes. The analysis of these measurements depends on detailed information about the magnetic field of the earth's surface, the mirror-point altitude as a function of longitude, and the details of the atmosphere in a region where it is changing rapidly.

CONCLUSIONS

In this paper we have calculated the energy spectrum of electrons to be found in the outer belt if it were made of neutron β -decay electrons. Several experiments have given some information about the outer-belt spectrum, but the problem is not yet solved. The experiments that involve bremsstrahlung will not be considered here because their interpretation is so difficult.

The Mehta experiment (Vernov, 1959 and 1960), indicated that there were about 1% as many electrons with $E > 500$ kev than with $E > 50$ kev. Our calculated spectrum gives about 8% for the ratio of particles with $E > 500$ as with $E > 50$ kev. This would indicate that experimentally there may be fewer electrons in the few-hundred-kev range than we calculated, but this is quite sensitive to the energy-threshold setting. If the Mehta-experiment threshold had actually been 600 kev instead of 500, we would agree with the 1% figure.

The leakage-electron energy spectrum measured by Cladis, Chase, Imhof, and Knecht (1961) shows a steeper energy spectrum and more low-energy electrons than can be explained by neutron decay. The fact that this measurement was carried out at quite low altitudes may mean that processes or electron sources are involved here that do not contribute at high altitudes.

O'Brien et al. (1961) have recently reported on the results of their experiment on Explorer XII. They have measured the electron spectrum in the middle of the outer belt by using several detectors sensitive in different energy ranges. The spectrum derived from these measurements for September 5, 1961, is shown in Fig. 7. Essentially, no electrons with $E > 5$ Mev were seen, but some electrons in the range $1.6 < E \leq 5$ Mev

were probably observed, although the interpretation of these experimental results is not unique. These high-energy electrons are very important for two reasons. First, they help explain results from several previous experiments that had seemed incompatible. Second, they cannot be made directly by neutron decay. There are not very many of these 1.6 to 5-Mev electrons; there were less than 1% of the total of September 5, and even less on other days.

We must decide how similar the neutron β -decay electron energy spectrum is to the observed electron spectrum. There are two features of the spectrum to compare: (a) the shape and (b) the height, which amounts to comparing the flux. From Fig. 7 comparing (a) the shape of the two spectra (aside from the electrons above 1.6 Mev), it is not obvious whether the spectral shapes agree or not. There may be too many low-energy electrons observed to agree with the calculated spectrum. One thing is certain--there is not the very large low-energy peak that earlier measurements indicated.

In comparing (b) the heights of the spectra, we probably should not take the experimental spectra in Fig. 7, because it was the highest observed flux. We should decrease this a factor of three or more. Using a representative time-average flux of $\sim 3 \times 10^7$ electrons/(cm²-sec) for $E > 40$ kev (O'Brien et al., 1961), we find that our values are low by about one order of magnitude.

In this paper we have considered only neutrons made by galactic cosmic-ray protons. It is now quite certain that polar-cap protons contribute substantially to the low-energy trapped proton spectrum (Armstrong et al., 1961, Naugle and Kniffen, 1961), by generating neutrons in the polar atmosphere. Polar-cap protons will also, in the same way, add considerably

to the trapped-electron flux. Lenchek (1962) has recently estimated that 100 times as many neutrons are generated by polar cap protons as by galactic cosmic-ray protons. If our calculated electron flux were increased by a factor of 100, the calculated flux would exceed the measured flux.

It appears from this that neutron decays produce a reasonable fraction of the outer-belt electrons, but other processes such as acceleration may be important. Electrons with greater than 1-Mev energy may be neutron β -decay electrons accelerated to these energies. We still need more definitive experiments to answer the question about whether any electron sources other than neutrons are needed to explain, in detail, the characteristics of the outer belt.

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FIGURE LEGENDS

Fig. 1. A single group of electrons injected at 780 kev (point A) are slowed down, produce knock-on electrons, and scatter out of the region of interest in the outer belt. Curve B shows a time soon after the injection time; curves C, D, and E are at equal time intervals after injection. The dotted portion of curve C shows the effect of not including the Coulomb out-scattering term in the calculation.

Fig. 2. Curve A is the equilibrium spectrum produced by a continuous injection of electrons of 780-kev energy into the outer radiation belt. The dashed curve B is the equilibrium spectrum if we assume that ionization-energy loss is the sole loss mechanism.

Fig. 3. The build-up toward equilibrium by a continuous injection of electrons in the outer belt. Curve A is, in units defined below, the total number of electrons (above 30 kev) as a function of time after the start of uniform injection of electrons with a β -decay energy spectrum. Curve B is the same quantity for a delta-function source of electrons at 780-kev kinetic energy. On the abscissa we have plotted the quantity $(f\bar{\rho}t)$ in gm-sec/cm^3 , where t is the time in seconds, $\bar{\rho}$ is the average density of hydrogen in gm/cm^3 , and f is a factor that describes the increased effectiveness of a partially ionized medium to multiple scatter and absorb energy by ionization energy loss (see text). The quantity $(Nf\bar{\rho}/n)$ is plotted on the ordinate, where N is the number of electrons/ cm^3 at equilibrium and n is the number of electrons/ $(\text{cm}^3\text{-sec})$ which are injected.

Fig. 4. Curve A is the neutron β -decay energy spectrum. Curves B, C, and D show the changes that would occur at equal time intervals after a single burst of β -decay electrons.

Fig. 5. Curve B is the electron energy spectrum from neutron β -decay. With this as the energy spectrum of the electrons injected into the outer belt, the equilibrium energy distribution of curve A was obtained after considering the loss processes discussed in the text. The plot is electron density (not flux) per kev on a linear scale.

Fig. 6. Electron-leakage flux spectrum. This curve shows, as a function of energy, the sum of all particles that are lost from the upper Van Allen radiation belt by multiple-Coulomb scattering, weighted by their lifetime at lower altitudes and multiplied by their velocity to yield a flux. The curve has been drawn arbitrarily to pass through the second experimental point of Cladis, Chase, Imhof, and Knecht (1961).

Fig. 7. Equilibrium flux of electrons attained by trapping 3×10^{-13} electrons/(cm³-sec) in an atmosphere of hydrogen of average density 10^{-21} gm/cm³. The incident spectrum is assumed to be that of a neutron decay electron; the hydrogen is assumed to be about 50% ionized. The experimental points and limits are those of O'Brien, Van Allen, Laughlin, and Frank (1961) for September 5, 1961, when the electron flux was higher than average.

Fig. 1

MU-25788

Fig. 2

MU-25783

Fig. 3

MU-25785

Fig. 4

MU-25787

Fig. 5

MU-24782

Fig. 6j

MU-25784

Fig. 7₁

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