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Finite Size Effects for Quark-Gluon Plasma Droplets [1]

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Abstract:

We calculate the partition function for color singlet quark-gluon plasma droplets analytically including shell effects and a total momentum constraint. Bulk properties become size-dependent and the entropy shows how the effective number of plasma degrees of freedom decreases with size. We derive the appropriate size-dependent phase-space distributions for quarks and gluons and point out phaenomenological consequences of our results.

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Ultra-relativistic nuclear collision experiments are soon to be realized at CERN and BNL with the goal of studying quark-gluon plasmas [1,2]. In order to understand properties of this new phase of matter as predicted by QCD it is necessary to investigate their relation to the variety of proposed experimental signatures. A popular theoretical approach has been to describe quark-gluon plasmas by equilibrium thermodynamical concepts, although the study of plasma evolution from the formation towards thermal and chemical equilibrium has only recently begun [2]. Thus, the application of equilibrium thermodynamics presents a strong assumption about the timescales of the underlying dynamics which deserves more work.

When studying the space-time evolution of plasma droplets and related observables one generally assumes an ideal or at most weakly interacting gas of quarks and gluons which is confined inside a finite size cavity by a phaenomenological vacuum pressure or volume energy as in all bag models. This assumption is based on the high-temperature behavior of QCD investigated in lattice calculations (see [1,2] and further references therein). Above a critical temperature $T_c \approx 150 - 250$ MeV, where a phase transition from hadronic matter to plasma occurs, one observes a rapid approach towards properties typical of an ideal quantum gas as described by the Stefan-Boltzmann law. However, there exist warnings in the literature that Stefan-Boltzmann like behavior could for example be mimicked by a cancellation of collective phonon and plasmon effects [3]. Even chromomagnetic monopoles might persist in the high-temperature phase [4] and eventually necessitate a different perturbation theory scheme.

From these remarks we conclude that it seems appropriate at present to describe the quark-gluon plasma as an assembly of quasi-particles with the same basic number of degrees of freedom as there are quarks and gluons. Effects of non-perturbative interactions are incorporated via the vacuum pressure and by requiring all physical states to be color singlets with respect to the SU(3) gauge group [5,6]. Previous studies of this model of the quark-gluon plasma, where in particular considerable finite size effects were observed [5,7], were incomplete in an important aspect: the spectrum of single-particle cavity modes was treated in the continuum approximation corresponding to the *thermodynamical limit* of an infinite system. Since finite size corrections can be expected to be quite large for experimentally accessible small plasma droplets, so-called shell effects due to the discreteness of single-particle energy levels should consistently be included.

The purpose of our present study is to calculate the partition function for color singlet spherical quark-gluon plasmas including average shell effects and fixing the total momentum of the droplet. We provide analytical results showing the *finite size corrections* which have to be applied in more realistic thermodynamical calculations related to plasma observables. We remark here that a numerical calculation of the energy of a plasma droplet with zero baryon number and SU(3)-color symmetry has been reported in ref.[8] which, however, did not realize the correct bag type boundary conditions and artificially assumed the fermion spectrum to be equal to the one for bosons. - In the second part of this paper we consider how the thermal phase-space distributions for quarks and gluons in plasma droplets are modified, again due to average shell effects and the color singlet and total momentum constraints on all many-particle states implicitly involved in the calculation.

To implement the color singlet constraint into the quantum statistical description of the system, we follow the group-theoretical projection method [9] which was recently extended to cover interacting systems as well [7,10]. Thus, the partition function for a color singlet quark-gluon plasma contained in a volume V at temperature T and quark chemical potential μ with a total momentum $\vec{p}$ is determined by:

$$Z(T,V,\mu;\vec{p}) \;=\; \int_{-\pi}^{+\pi} d(\phi/2)d(\psi/3)\; M(\phi,\psi) \int_{-\pi}^{+\pi} \frac{d^3\lambda}{(2\pi)^3}\; e^{i\beta\vec{\lambda}\cdot\vec{p}}\; \tilde{Z}(T,V,\mu;\vec{\lambda},\phi,\psi) \;, \tag{1}$$

where $M(\phi,\psi) \equiv (8/3\pi^2)[sin\frac{1}{2}(\psi+\phi/2)\;\; sin\phi/2\;\; sin\frac{1}{2}(\psi-\phi/2)]^2$ is the weight function (Haar measure) for the group integration. The *generating function* $\tilde{Z}$ is defined by

$$\tilde{Z}(T,V,\mu;\vec{\lambda},\phi,\psi) \;\equiv\; Tr\; e^{-\beta(\hat{H} + i\vec{\lambda}\cdot\hat{\vec{p}} - \mu[\hat{N}_q - \hat{N}_{\bar{q}}]) + i\phi\hat{I}_3 + i\psi\hat{Y}_8} \;, \tag{2}$$

with $\beta \equiv T^{-1}$ and where $\hat{H}$, $\hat{\vec{p}}$, $\hat{N}_q$, $\hat{N}_{\bar{q}}$, $\hat{I}_3$, and $\hat{Y}_8$ respectively denote the Hamiltonian, total linear momentum, quark number, antiquark number, color "isospin", and color "hypercharge" operators. The unrestricted trace in eq.(2) includes *all states* of the many-particle Hilbert space. Note that, although the trace is unrestricted for $\tilde{Z}$, the various integrations in eq.(1) effectively project out a *canonical partition function* with respect to color and momentum quantum numbers. This was shown in detail for an internal SU(N) symmetry in refs.[7,10] and the projection onto a definite total momentum was applied previously e.g. in the context of early particle production models [11]. Furthermore, notice that instead of introducing a quark chemical potential corresponding to a grand-canonical ensemble one could also treat the baryon number in a canonical ensemble as in ref.[8]. However, for a quark-gluon plasma surrounded by hadronic matter one cannot exclude baryon number fluctuations even in the central region with zero average baryon density, which favors our mixed canonical/grand-canonical ensemble [5,10].

To calculate $\tilde{Z}$ for quarks and gluons, which are noninteracting according to the model discussed above, we use the Fock space representation and obtain (cf. [5,10])

$$\tilde{Z} \;=\; \prod_{i=r,g,b} \tilde{Z}_Q^{(i)} \prod_{j=\lambda,\mu,\nu,\delta} \tilde{Z}_G^{(j)} \;. \tag{3}$$

Here the quark and gluon contributions respectively are

$$ln\tilde{Z}_Q^{(i)} \equiv \sum_{\vec{k}} \left\{ ln(1+e^{-\beta[\epsilon(\vec{k}) + i\vec{\lambda}\cdot\vec{k} - \mu] + i\alpha_i}) \;+\; ln(1+e^{-\beta[\epsilon(\vec{k}) + i\vec{\lambda}\cdot\vec{k} + \mu] - i\alpha_i}) \right\} \;, \tag{4}$$

where $\alpha_r \equiv \phi/2+\psi/3$, $\alpha_g \equiv -\phi/2+\psi/3$, $\alpha_b \equiv -2\psi/3$, and

$$ln\tilde{Z}_G^{(j)} \equiv -\sum_{\vec{k}'}\left\{ln(1-e^{-\beta[\epsilon'(\vec{k}')+i\vec{\lambda}\cdot\vec{k}']+i\alpha_j}) + ln(1-e^{-\beta[\epsilon'(\vec{k}')+i\vec{\lambda}\cdot\vec{k}']-i\alpha_j})\right\} , \tag{5}$$

where $\alpha_\lambda \equiv \alpha_r - \alpha_g$, $\alpha_\mu \equiv \alpha_g - \alpha_b$, $\alpha_\nu \equiv \alpha_b - \alpha_r$, $\alpha_\delta \equiv 0$. The single-particle energies $\epsilon(\vec{k}) = k$, $\epsilon(\vec{k}') = k'$ in the massless limit only depend on the momenta of cavity modes $\vec{k}, \vec{k}'$ according to the relevant boundary conditions for quark and gluon fields. At this point one usually would turn to numerical techniques in order to further evaluate $\tilde{Z}$ from eqs.(3,4,5), since in most cases the cavity modes are not known explicitly and even with the simple approximations possible for a spherical M.I.T. bag one cannot perform the above mode sums in closed form. Instead in our derivation for a spherical cavity of radius R we make use of *asymptotic density of states formulae* which reflect the average shell effects. For the transverse radiation field gluons the bag surface boundary conditions, $\vec{n}\cdot\vec{E} = 0 = \vec{n}\times\vec{B}$ ($\vec{n}$ normal to the surface), by duality transforming E into B and vice versa are analogous to the QED case of photons in a cavity with a perfectly conducting boundary. For this problem the density of states ρ is known [12] and thus yields the result for gluons with $d_G = 2$ polarization states

$$\rho_G(k') \; = \; d_G\left(\frac{Vk'^2}{2\pi^2} - \frac{4R}{3\pi} + \kappa\delta(k') + O(R^{-1}k'^{-2})\right) \; \equiv \; A_G k'^2 + B_G + \dots \; , \tag{6}$$

where κ is an unknown constant and we keep only the first two terms in the following. For quarks one can deduce a similar density of states from a calculation of their zero-point energy in the M.I.T. bag [13], $E_0(\varsigma) \equiv -2\int_0^\infty d\epsilon\rho_Q(\epsilon)\epsilon e^{-\varsigma\epsilon} = -2[3V/\pi^2\varsigma^4 - R/3\pi\varsigma^2 + O(R^{-1})]$, where an exponential cut-off is used. Then, the level density for quarks with a degeneracy factor $d_Q = 2(spin)\times 2(isospin)$ is

$$\rho_Q(k) \; = \; d_Q\left(\frac{Vk^2}{2\pi^2} - \frac{R}{3\pi} + O(R^{-1}k^{-2})\right) \; \equiv \; A_Q k^2 + B_Q + \dots \; . \tag{7}$$

Notice the absence of surface terms in ρ_Q and ρ_G corresponding to the absence of surface energy contributions below. Both results, eqs.(6,7), were previously used for an accurate calculation of the asymptotic mass spectrum of bag model states in refs.[14].

Using eqs.(6,7) we obtain the generating functions for quarks and gluons, eqs.(4,5), by replacing $\sum_{\vec{k}} \to \int d^3k\rho_{Q,G}(k)/4\pi k^2$. Of course, the approximation with $\rho_{Q,G}(k) \equiv A_{Q,G}k^2 + B_{Q,G}$ can only be valid for sufficiently large average momenta, i.e. for sufficiently high particle number density in a plasma droplet. Then we have to calculate the integrals

$$I_\pm \; \equiv \; \pm\int d^3k\frac{\rho_{Q,G}(k)}{4\pi k^2}\left\{ln(1\pm e^{-\beta[k+i\vec{\lambda}\cdot\vec{k}]+z}) + \; z\to -z\right\} , \tag{8}$$

where z is a complex parameter representing $z_Q^{(i)} \equiv \beta\mu + i\alpha_i$ for quarks (I_+) and $z_G^{(j)} \equiv i\alpha_j$ for gluons (I_-) and the second logarithmic term is indicated which differs from the first one only by $z \to -z$. The angular integrations are elementary giving

$$I_\pm = \pm\frac{A_{Q,G}}{\beta^3}(1+\lambda^2)^{-2}\int_0^\infty dx\, x^2\left\{ln(1\pm e^{-x+z}) + z\to -z\right\}$$
$$\pm\frac{B_{Q,G}}{\beta}\frac{arctg\,\lambda}{\lambda}\int_0^\infty dx\left\{ln(1\pm e^{-x+z}) + z\to -z\right\} . \quad (9)$$

The remaining integrals constitute special cases of a general result [7,15] calculated by contour integration:

$$I_n(z)_\pm \equiv \int_0^\infty dx\, x^n\left\{\frac{1}{e^{x-z}\pm 1} + (-1)^{n+1}\frac{1}{e^{x+z}\pm 1}\right\} , \quad n = 0,1,2,\ldots ,$$
$$= \pm\frac{(2\pi i)^{n+1}}{n+1}B_{n+1}\left([\pi(1\pm 1) - 2iz]/4\pi\right) , \quad 0 < Re[\pi(1\pm 1) - 2iz] < 4\pi , \quad (10)$$

where B_{n+1} denote Bernoulli polynomials [16] and expressions on the r.h.s. of eq.(10) can be continued to neighbouring strips by 2π-*periodicity* in $Im\ z$. Thus, we obtain the final results:

$$ln\tilde{Z}_Q^{(i)} = \frac{4}{3}\pi^4\frac{A_Q}{\beta^3}(1+\lambda^2)^{-2}B_4((\pi - iz_Q^{(i)})/2\pi) - 2\pi^2\frac{B_Q}{\beta}\frac{arctg\,\lambda}{\lambda}B_2((\pi - iz_Q^{(i)})/2\pi) , \quad (11)$$

with the restriction $-i\pi < Im\ z < i\pi$, and

$$ln\tilde{Z}_G^{(j)} = -\frac{4}{3}\pi^4\frac{A_G}{\beta^3}(1+\lambda^2)^{-2}B_4(-iz_G^{(j)}/2\pi) + 2\pi^2\frac{B_G}{\beta}\frac{arctg\,\lambda}{\lambda}B_2(-iz_G^{(j)}/2\pi) , \quad (12)$$

with the restriction $0 < Im\ z < 2i\pi$. If one sets $B_Q = B_G = 0$ in eqs.(11,12), one recovers the continuum limit of these generating functions studied previously [5,6,7]. If one sets $\lambda = z_Q^{(i)} = z_G^{(j)} = 0$ instead, one obtains partition functions for free fermions and bosons including finite size corrections due to shell effects.

With the total generating function determined according to eqs.(3,11,12) we now proceed to calculate the partition function by eq.(1). For that purpose we expand the weight function M to leading order in the angular variables and expand the argument of the exponential representing the generating function to second order in ϕ, ψ, or λ (*Gaussian approximation*). Using $\sum_i \alpha_i = 0 = \sum_j \alpha_j$ with α_i, α_j as defined after eqs.(4,5) respectively and consistently extending the integrations to infinity we obtain

$$Z = \int_{-\infty}^{+\infty} d(\phi/2)d(\psi/3)\,\frac{1}{6\pi^2}(\phi/2)^2[\psi^2 - (\phi/2)^2]^2\, e^{-[(\phi/2)^2 + 3(\psi/3)^2]C}$$
$$\times \int_0^\infty d\lambda\,\frac{4\pi\lambda}{\beta p}\,sin\beta\lambda p\, e^{-\lambda^2 D} \times Z_0 , \quad (13)$$

where we already performed the elementary angular λ-integrations, and where

$$C \equiv \frac{1}{6}VT^3(d_Q[1+3(\frac{\beta\mu}{\pi})^2]+6d_G) - \frac{1}{3\pi}RT(d_Q-12d_G) \ , \tag{14}$$

Z_0 denotes the unprojected partition function for quarks and gluons including shell corrections,

$$\begin{aligned} lnZ_0(T,R,\mu) &= \frac{\pi^2}{12}VT^3(2\cdot 3\cdot d_Q[\frac{7}{60}+\frac{1}{2}(\frac{\beta\mu}{\pi})^2+\frac{1}{4}(\frac{\beta\mu}{\pi})^4]+8\cdot d_G\cdot\frac{8}{60}) \\ &\quad - \frac{\pi}{18}RT(2\cdot 3\cdot d_Q[\frac{1}{2}+\frac{3}{2}(\frac{\beta\mu}{\pi})^2]+8\cdot d_G\cdot 4) \equiv X-Y \ , \end{aligned} \tag{15}$$

and,

$$D \equiv 2X - \frac{1}{3}Y \ . \tag{16}$$

Note the particular factorization in eq.(13) of Z_0 being multiplied by two separate correction factors due to the color and momentum projection, which only applies for the Gaussian approximation. We do not discuss improvements beyond this approximation here, since the accuracy of our results is limited by the validity of the asymptotic density of states formulae, eqs.(6,7). Therefore, the leading term $\sim V$ in eqs.(14-16) should always be larger than the next one $\sim R$. For $\mu = 0$ this yields the condition $RT > .9$. From a previous comparison between the Gaussian approximation and a particular numerical calculation [7] (cf. also the second of refs. [5]) we learned that for $\mu = 0$ and $RT > 1$ this approximation was qualitatively correct with an error of $\approx 30\%$, decreasing towards a few percent at $RT \approx 2$, and becoming negligible for larger RT-values. Also for increasing μ the error rapidly decreased ($\ll 5\%$ for $\mu \approx 3T$, $RT > 1$). Therefore, we expect the Gaussian approximation consistently to fit together with the density of states formulae within the present description of hot quark-gluon plasma droplets. Finally, we perform the remaining Gaussian integrations to obtain the *color singlet fixed-momentum partition function*:

$$Z(T,R,\mu;p) = \tfrac{1}{2}\sqrt{\tfrac{\pi}{3}}\, C^{-4}D^{-3/2}\, e^{-(\beta p)^2/4D}\, e^{-\beta BV}\, Z_0 \ , \tag{17}$$

with C, Z_0, D respectively as in eqs.(14,15,16). We inserted into eq.(17) a factor for the vacuum pressure B discussed in the introduction with its phaenomenological value in the range 145 MeV$\leq B^{1/4} \leq$235 MeV [1,2]. Notice that one simply has to omit either the factor $(2\pi\sqrt{3})^{-1}C^{-4}$ or $\pi^{3/2}D^{-3/2}exp - (\beta p)^2/4D$ from eq.(17) if one wants to drop either the exact constraints on color or momentum quantum numbers.

Including the vacuum pressure, one derives from eqs.(1,2) for $p = 0$ (or from eqs.(14-17)) the equation of state typical of a plasma droplet of non-interacting massless constituents [5,7], $P = (E/V - 4B)/3$. Here P, E denote its pressure and energy density given by $P \equiv T\partial_V lnZ$ and $E \equiv -\partial_\beta lnZ$. There are, however, finite size effects due to the constrained ensemble underlying our calculations as well as the average shell effects. As we observed earlier [5,7] the *effective number of degrees of freedom* d_{eff} in a small plasma droplet is reduced drastically from the number corresponding to the Stefan-Boltzmann limit with $2 \cdot 3 \cdot d_Q \cdot 7/8$ (anti)quark plus $8 \cdot d_G$ gluon degrees of freedom (cf. eq.(15)). The relevant parameter is RT. Furthermore, increasing the chemical potential increases d_{eff}. Thus, the plasma pressure is reduced and the *limit of stability* of a droplet where $P = 0$ is shifted towards higher temperature or higher chemical potential as compared to the Stefan-Boltzmann limit depending on droplet size [5,7].

To complement our previous results we now calculate the *entropy density* s given by $s \equiv V^{-1}\partial_T(TlnZ)$. Using eqs.(14-17) with $p = 0$ we obtain:

$$s = s_0 + V^{-1}\left(ln\tfrac{1}{2}\sqrt{\tfrac{\pi}{3}} - 4lnC - \tfrac{3}{2}lnD - 4C^{-1}T\partial_T lnC - \tfrac{3}{2}T\partial_T lnD\right) , \qquad (18)$$

where the remaining derivatives follow directly from eqs.(14,16) and s_0 is the entropy density corresponding to Z_0, eq.(15),

$$s_0/T^3 = s_{SB}/T^3 - \tfrac{1}{12}(RT)^{-2}(3d_Q + 32d_G) , \qquad (19)$$

with $s_{SB}/T^3 \equiv \frac{\pi^2}{6}(3d_Q[\frac{7}{15}+(\beta\mu/\pi)^2]+\frac{32}{15}d_G)$ corresponding to the Stefan-Boltzmann limit. In *Fig.1a* we show the deviation from the Stefan-Boltzmann limit, s_0/s_{SB} according to eq.(19), due to the finite size average shell effects alone. *Fig.1b* illustrates the result for $s/s_{SB} \equiv d_{eff}$ given by eq.(18) which additionally includes the color singlet and fixed-momentum constraints. These ratios measure the effective number of degrees of freedom in the plasma and show the essential RT- and μ-dependence mentioned above. For example, in *Fig.1b* we observe respectively $\approx 40, 20, 10\%$ reduction of d_{eff} for a $10, 27, 65$ fm^3 baryon number zero droplet at $T = 200$ MeV. Omitting the fixed-momentum constraint would result in a shift of the curves in *Fig.1b* by approximately .1 RT-units to the left. The value $s_\pi/s_{SB} = 3/37$ for a free massless pion gas is also indicated in *Fig.1b* (dashed line).

Thus, d_{eff} may approach the pion gas value for small quark-gluon plasmas accessible to experiment. There is particular uncertainty as to whether the plasma will be formed in a single flux tube like region or in the form of several randomly distributed small droplets. In the widely adopted two-phase picture for the transition between a baryonless plasma and a pion gas hadronization properties depend on the ratio $r \equiv s_\pi/s$ which may vary considerably according to our above results. For the deflagration model [17] we conclude that the *deflagration shock front proceeds faster with increasing r* and the maximum possible energy flux of plasma into hadron gas conversion is increased. In the hydrodynamic model of plasma evolution [18,19] including the transition through a mixed phase the proper time τ_h when all plasma is converted is determined by the relation $\tau_h = r^{-1}\tau_q$ (1+1-dimensional expansion). Here at $\tau_q \approx 2 - 20$ fm/c the first hadrons appear. Therefore, the *mixed phase becomes shorter with increasing r*. These remarks may suffice to show the importance of finite size corrections for thermodynamical quark-gluon plasma properties. Simple analytical expressions for all equilibrium quantities entering more realistic hydrodynamical calculations of plasma evolution can easily be obtained from the partition function Z of eq.(17).

We now turn to the study of distribution functions for quarks and gluons in a color singlet plasma droplet with specified total momentum. Using general expressions for the expectation value of a number operator in the constrained (canonical) ensemble [10] one obtains the average occupation number $n_\pm$ (cf. eq.(1)),

$$n(\vec{k})_\pm = Z^{-1}\int_{-\pi}^{+\pi} d(\phi/2)d(\psi/3)\; M(\phi,\psi)\int_{-\pi}^{+\pi}\frac{d^3\lambda}{(2\pi)^3}\; e^{i\beta\vec{\lambda}\cdot\vec{p}}\;\frac{\tilde{Z}(\vec{\lambda},\phi,\psi)}{e^{\beta(k-\mu+i\vec{\lambda}\cdot\vec{k})-i\alpha}\pm 1}\;, \tag{20}$$

where $+$ $(-)$ stands for quarks/fermions with $\alpha\equiv\alpha_i$ (gluons/bosons with $\alpha\equiv\alpha_j$) with α_i,α_j as defined after eqs.(4,5) and $\mu\equiv 0$ for gluons. $n_\pm$ is independent of the vacuum pressure, since its contributions to $Z,\tilde{Z}$ cancel. Similarly to the prove that Z generally is real [7], it follows by the symmetry properties of M and $\tilde{Z}$ that $n(\vec{k})$ is real. Expanding the denominator in eq.(20) and applying the *Gaussian approximation* as before with $\tilde{Z}$ as given by eqs.(11,12) we arrive at the result $(\vec{p}\equiv 0)$:

$$\begin{aligned} n(k)_\pm &= Z^{-1}\sum_{m=0}^{\infty}(\mp 1)^m e^{-\beta(k-\mu)(m+1)}\, Z_0 \\ &\cdot\; \pi^{3/2}D^{-3/2}e^{-(m+1)^2(\beta k)^2/4D}\cdot\frac{1}{2\pi\sqrt{3}}C^{-4}e^{-(m+1)^2\kappa_\pm/C}\cdot(1+\mathrm{O}(C^{-1})) \\ &= \sum_{m=0}^{\infty}(\mp 1)^m e^{-\beta(k-\mu)(m+1)}\; e^{-(m+1)^2(\beta k)^2/4D}\; e^{-(m+1)^2\kappa_\pm/C} \\ &\cdot\; (1+\mathrm{O}(C^{-1}))\;, \end{aligned} \tag{21}$$

with C,Z_0,D,Z respectively as in eqs.(14-17), $\kappa_+\equiv 1/3$, $\kappa_-\equiv 1$ except for "chargeless" gluons with $\alpha=\alpha_6\equiv 0$ where $\kappa_-\equiv 0$. As indicated in eq.(21) we calculated the Gaussian integrals to $\mathrm{O}(C^{-1})$ $\sim\mathrm{O}((RT)^{-3})$. $n_\pm$ replace ordinary Fermi and Bose $(\mu=0)$ distributions in a quark-gluon plasma droplet. We displayed here the expansion of the fermionic distribution valid for $k>\mu$ only (for $k<\mu$ one has to replace $(k-\mu)\to(\mu-k)$ and $(m+1)\to m$ in the exponentials).

Multiplying $n(k)_\pm$ with the respective density of states according to eqs.(6,7) we obtain the *phase-space distributions* $f_{Q,G}$ *for confined quarks and gluons*:

$$f_Q(k) = d_Q\frac{k^2}{2\pi^2}(1-\frac{1}{2}(kR)^{-2})\sum_{m=0}^{\infty}(-1)^m e^{-\beta(k-\mu)(m+1)}\; e^{-(m+1)^2[(\beta k)^2/4D\;+\;1/3C]}\;, \tag{22}$$

where for $k<\mu$ again the abovementioned replacement has to be made,

$$f_G(k) = d_G\frac{k^2}{2\pi^2}(1-2(kR)^{-2})\sum_{m=0}^{\infty}e^{-\beta k(m+1)}\; e^{-(m+1)^2[(\beta k)^2/4D\;+\;\kappa_-/C]}\;. \tag{23}$$

$f(k)dk$ gives the average number of particles (e.g. "red" quarks) per unit volume having momenta between k and $k+dk$. Eqs.(22,23), which are accurate to $O(R^{-3})$, replace the well-known fermion and boson blackbody radiation formulae for the case of a color singlet total momentum zero ensemble in a finite cavity with M.I.T. bag model boundary conditions. We observe that eq.(23) requires $kR = \beta k \cdot RT > \sqrt{2}$. Omitting the exponential corrections in eq.(23) we reproduce the result of ref.[12], which only takes the average shell effects into account.

The various correction factors due to the average shell effects and the color and momentum projection again can be clearly identified if one compares our results with Planck's law. Or, particularly by looking at the Boltzmann limit ($m = 0$) in eqs.(22,23) we qualitatively expect three effects to arise in comparison with the usual distribution functions: i) low-momentum modes are suppressed due to the underlying discreteness of the single-particle spectrum, ii) there is an overall reduction caused by the restriction to color singlet states, iii) high-momentum modes are suppressed by the requirement of zero total momentum. This is illustrated by *Fig*.2 where we show the dimensionless functions $\beta^2 f_Q$ (for $\mu = 0$) and $\beta^2 f_G$ according to eqs.(22,23) respectively with $d_Q = 4$ and $d_G = 2, \kappa_- = 1$. They depend on the dimensionless variable βk and the size parameter RT. For comparison the distributions without any finite size corrections corresponding to Planck's law are also given (dashed curves). One observes considerable deviations from this infinite volume limit. Notice also the size-dependent shift of the maximum of the gluon/boson distribution. Finally, we want to draw attention to the fact that the momentum-dependent exponential correction factors in eqs.(22,23) lead to a size-dependent slope change for $\log f_{Q,G}$ at high momenta. This is not obvious from *Fig*.2, but easily can be recognized in a logarithmic plot of the high-momentum tails.

Since phase-space distributions of quarks or gluons enter all calculations of cross sections for particle production out of a plasma droplet, for example the dilepton probe studied in refs.[20] (and refs. therein), we conclude that finite size effects should be taken into account to improve these tools for quark matter diagnosis. Summarizing we may say that bulk as well as single-particle properties in quark-gluon plasma droplets depend sensitively on the size of the system which will only be determined by experiment.

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Figure captions

Fig.1 Size-dependent deviation of the entropy density from its Stefan-Boltzmann limit value for various chemical potentials (lowest curve with $\mu = 0$ etc.): a) due to shell effects, s_0/s_{SB} from eq.(19); b) due to shell effects and color and momentum constraints, s/s_{SB} from eq.(18) (π's: dashed).

Fig.2 Phase-space distributions for quarks (upper three curves), $\beta^2 f_Q$ from eq.(22), and gluons (lower three curves), $\beta^2 f_G$ from eq.(23), respectively with $RT \to \infty$ (dashed), $RT = 1.5$ (upper full curve), and $RT = 1$ (lower full curve).

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Fig.1 a

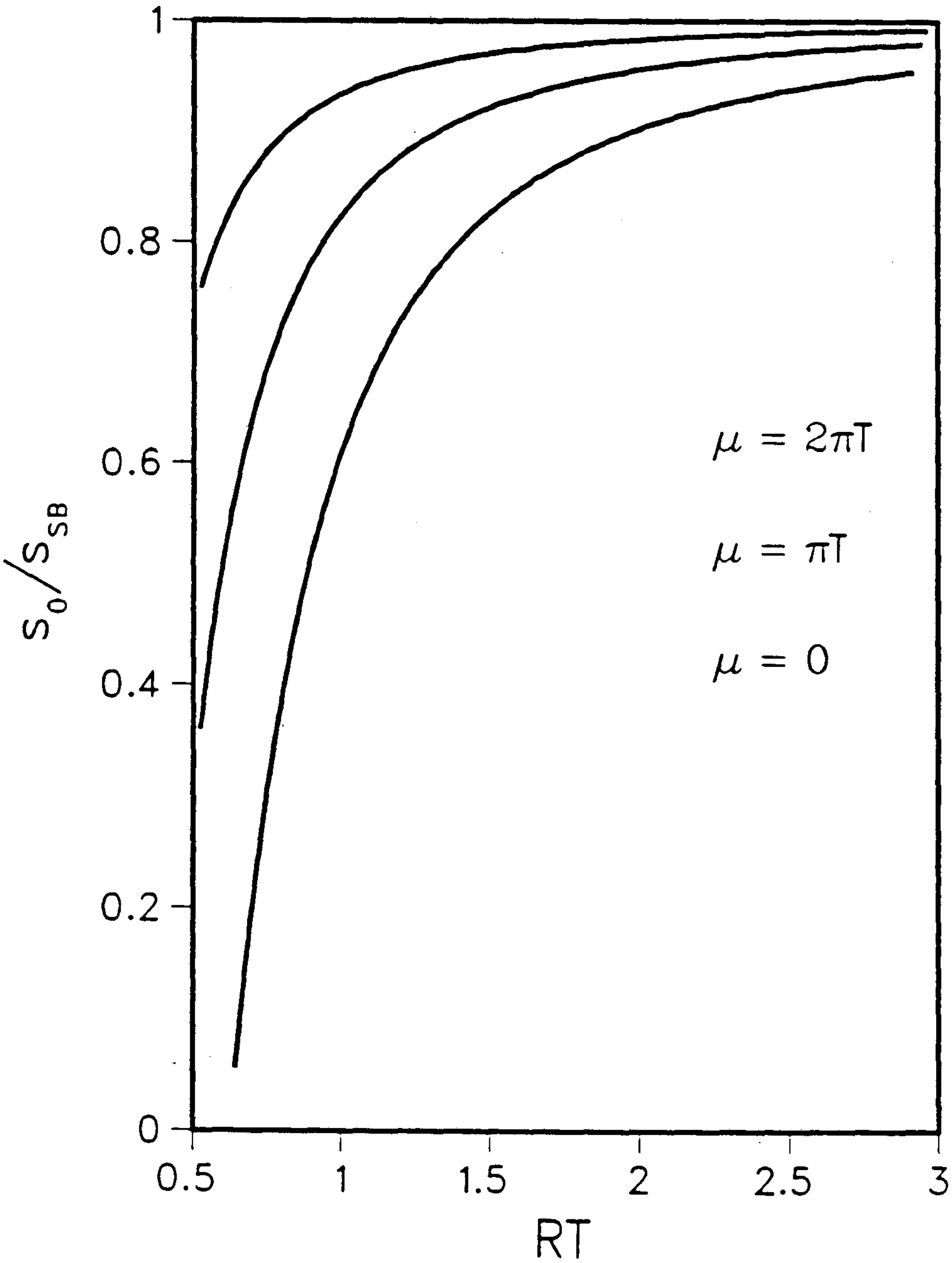


Fig.1 b

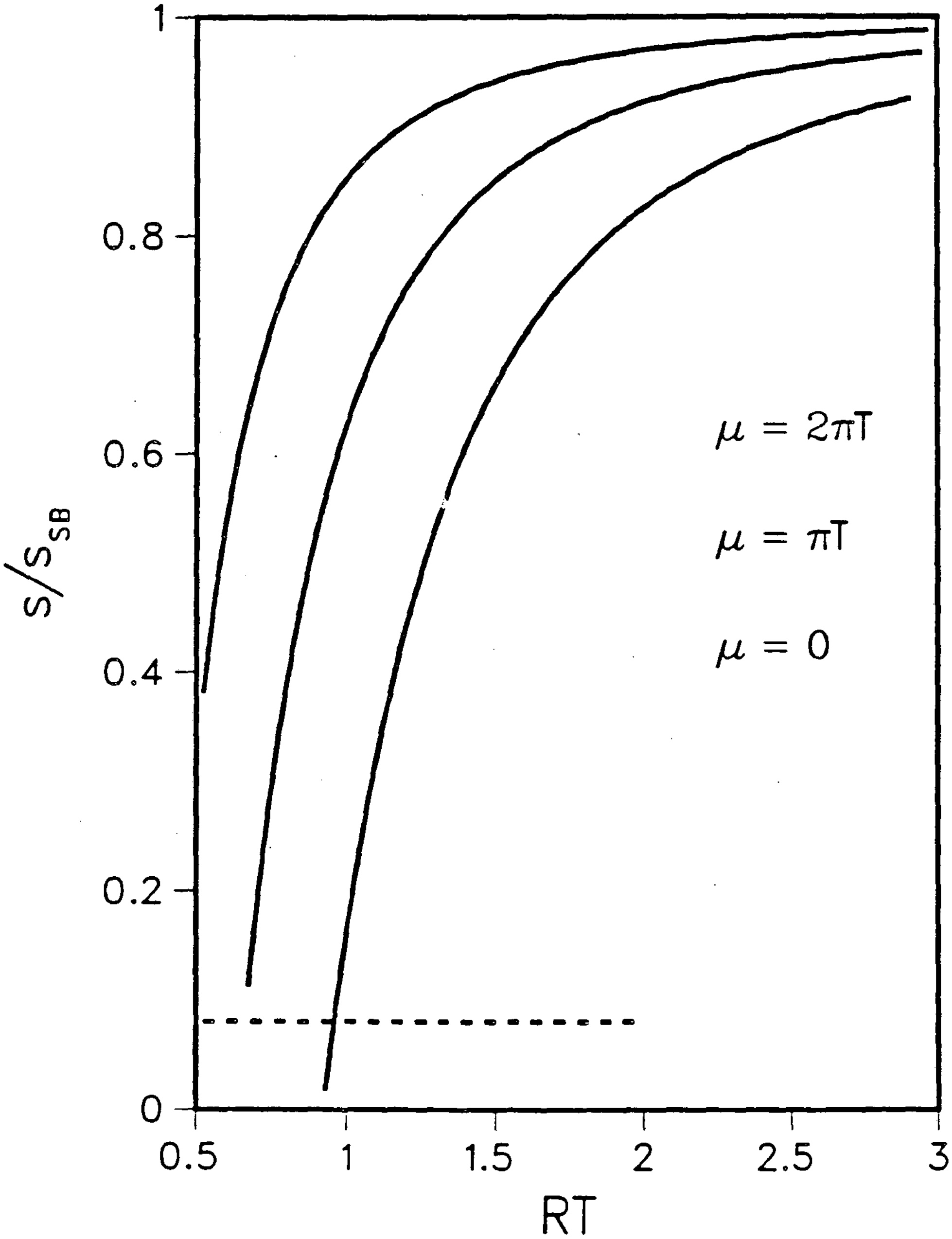
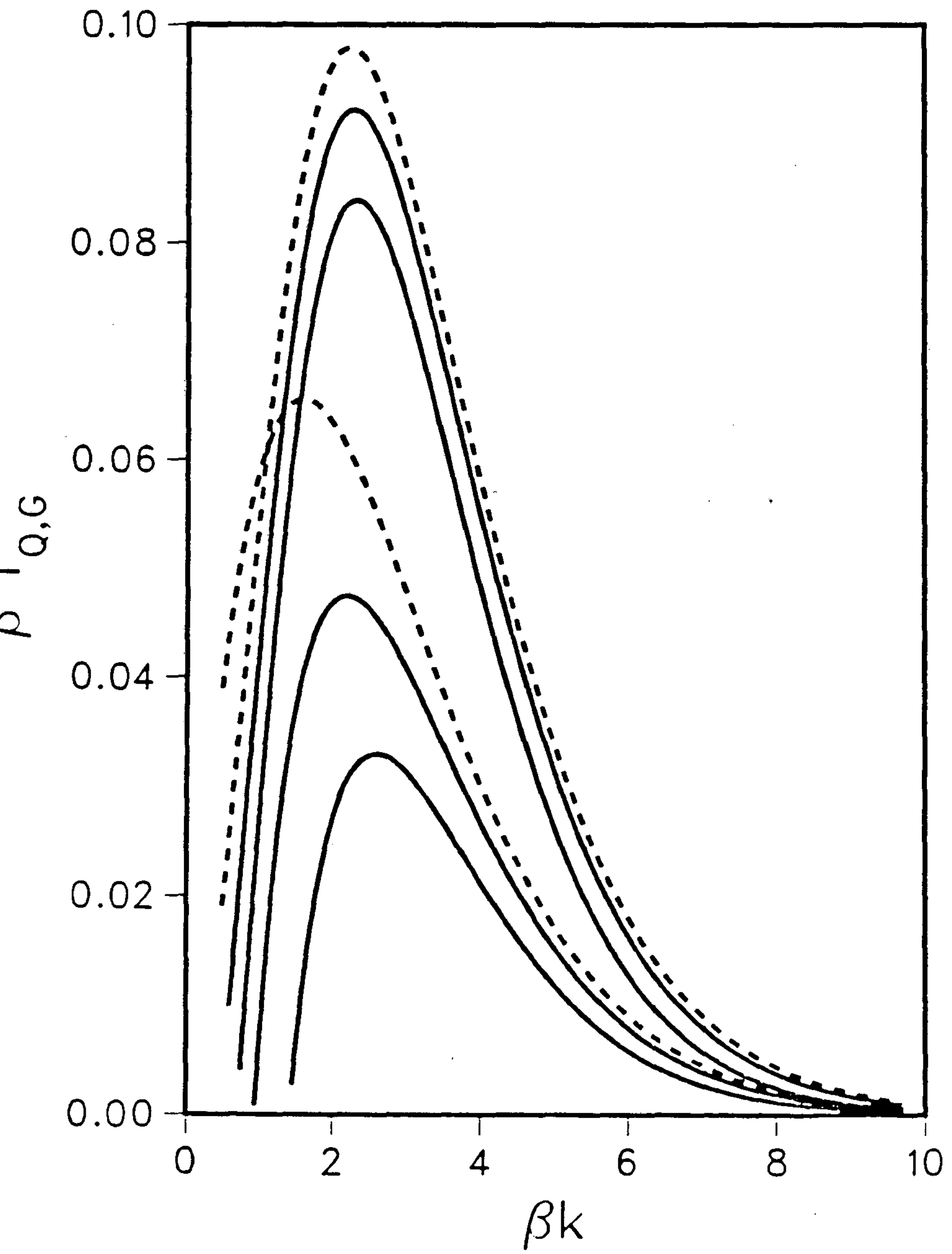


Fig.2



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