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L. G. Moretto

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PAIRING FLUCTUATIONS IN EXCITED NUCLEI AND THE ABSENCE OF A SECOND ORDER PHASE TRANSITION†

L. G. Moretto

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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The statistical fluctuations in the nuclear gap parameter are studied. The difference between the average and most probable gap parameter is discussed and the absence of a sharp second order phase transition is shown.

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The standard theory of pairing as applied to excited systems is based on the grand partition function e^{Ω} obtained from the BCS Hamiltonian [1-5]. The function Ω is given by the expression:

$$\Omega = -\beta \sum (\varepsilon_k - \lambda - E_k) + 2 \sum \ln[1 + \exp - \beta E_k] - \beta \frac{\Delta^2}{G}$$

where $E_k = [(\varepsilon_k - \lambda)^2 + \Delta^2]^{1/2}$, $\beta = \frac{1}{T}$ is the inverse of the statistical temperature, λ is the chemical potential and G is the pairing strength. The gap parameter Δ, which is a measure of the pairing correlation, is defined by the gap equation:

$$\sum \frac{1}{E_k} \tanh \frac{1}{2} \beta E_k = \frac{2}{G} \quad .$$

Such equation predicts the existence of a critical temperature T_c at which Δ vanishes. Following this formalism, at the critical temperature, the system

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undergoes a second-order phase transition reverting back to a system of uncorrelated fermions. In the microcanonical ensemble, the gap equation is obtained by minimizing the energy at fixed quasi particle occupation numbers. In simpler words, the gap parameter is determined by maximizing the entropy of the system at constant energy. Similarly, in the canonical ensemble, the gap parameter can be determined by minimizing the free energy at fixed temperature. In fact, the gap equation can be also written as:

$$\left.\frac{\partial\Omega}{\partial\Delta}\right|_T = 0$$

Thus, the gap parameter values obtained from the gap equation are to be understood as most probable values. In the case of a system with a large number of particles, the use of the most probable value of Δ is completely justified. In this case, the fluctuations are very small, the probability distribution in the gap parameter approaches a delta function, and the most probable value coincides with the average value. However, nuclei are composed by a small number of particles and the fluctuations are expected to be very large indeed. The presence of fluctuations in the gap parameter is responsible for a difference between the most probable and the average value due to high order moments in the probability distribution. It will be shown that because of the presence of such fluctuations, it is not quite proper in the case of a nucleus to employ the most probable value of Δ. In fact, the second order phase transition and the associated discontinuity in the specific heat, which are present when the most probable value is used, disappear altogether when the average value is used.

The isothermal probability distribution [6] for the quantity Δ is given by:

$$P(\Delta) \propto e^{\Omega(\beta,\, \Delta)} \quad .$$

This expression assumes strong coupling between the Δ degree of freedom and all the intrinsic degrees of freedom, and disregards quantal effects. In fig. 1, a set of probability functions corresponding to different temperatures is plotted. The calculations have been performed for a uniform model. The density of the doubly degenerate single particle levels g is equal to 7 MeV^{-1} and the zero temperature gap parameter is equal to 1 MeV.

At very low temperatures, the probability curve is symmetrical and gaussian-like. The maximum of the distribution occurs at a value of Δ corresponding to the solution of the gap equation. Because of the symmetry of the distribution, the average value and the most probable value coincide. As the temperature increases, the distribution curves become broader and tend to become asymmetrical. While the maximum of the distribution still corresponds to the solution of the gap equation, the average value becomes distinctly different from the most probable value. At the critical temperature, the distribution has a maximum at $\Delta = 0$. Thus, the standard theory predicts the disappearance of the pairing correlations. A discontinuity in the specific heat of the system indicates a second order phase transition:

$$C = \frac{dE}{dT} = \frac{1}{2} \sum \mathrm{sech}^2 \frac{1}{2} \beta\, E_k [\beta^2 E_k{}^2 - \beta\Delta \frac{d\Delta}{dT}]$$

At fixed chemical potential, as in the case of the uniform model, the derivative of Δ with respect to T is given by:

$$\frac{d\Delta}{dT} = \frac{1}{2}\beta^2 \frac{\sum \mathrm{sech}^2 \frac{1}{2}\beta E_k}{\Delta\{\frac{1}{2}\beta \sum \frac{\mathrm{sech}^2 \frac{1}{2}\beta E_k}{E_k^2} - \sum \frac{\tanh \frac{1}{2}\beta E_k}{E_k^3}\}}$$

However, at the critical temperature, the average gap parameter is larger than zero, indicating that for small systems there is no discontinuity and phase transition. Only in the limit of an infinitely sharp distribution, typical of a macroscopic system, the average and most probable values do coincide and the discontinuity is present. For temperatures above the critical temperature, the most probable value of Δ remains zero, while the average value remains rather large and decreases slowly with increasing temperature. This can be seen in fig. 2, where the most probable and the average gap parameter are plotted as a function of temperature.

The very long high energy tail of the average gap parameter can be understood in a qualitative way; at any temperature (or excitation energy) and in particular even above the critical temperature, it is possible to have states characterized by a small number of high-energy quasi particles for which pairing may be very large indeed.

It may be interesting to evaluate the thermodynamical quantities on the basis of the average instead of the most probable gap parameter. The total energy of the system is given by:

$$E = \sum \varepsilon_k[1 - \frac{\varepsilon_k - \lambda}{E_k}\tanh(\frac{1}{2}\beta E_k)] - \frac{\Delta^2}{G} - (\Delta^2 + \beta\Delta\frac{\partial\Delta}{\partial\beta})(\sum \frac{1}{E_k}\tanh(\frac{1}{2}\beta E_k) - \frac{2}{G})$$

In the standard theory the last term is absent [5]. In fact, because of the gap equation, this term is zero for the most probable gap parameter and non zero for the average gap parameter. The temperature dependence of the excitation energy is shown in fig. 3. The thick line corresponds to the use of the average value of Δ; the thin line corresponds to the use of the most probable value of Δ; the dashed parabola corresponds to the behavior of the system in absence of pairing. At very low temperature, the two curves are very close. At the critical temperature, the standard theory predicts a kink which is nicely avoided by the present theory. The predicted function smoothly merges into the parabola, typical of the uncorrelated systems. It may be noticed that the largest deviations occur close to the critical temperature where they may become almost as large as 1 MeV. The entropy of the system can be calculated for the expression:

$$S = \Omega - \beta E + \beta\lambda N$$

In fig. 4, the entropy is presented as a function of temperature. As in the previous figure, the thick line corresponds to the present theory, the thin line corresponds to the standard theory and the dashed line corresponds to the unpaired system. Again, the present theory avoids the critical temperature kink, typical of the standard theory. The deviations are largest at the critical temperature and the high energy behavior of the uncorrelated system are asymptotically attained. The most dramatic changes are seen in the behavior of the specific heat as a function of temperature. In the case of the average gap parameter, the specific heat has been calculated by numerical differentiation of the energy as a function of temperature. This quantity is presented in fig. 5. The discontinuity seen in the standard theory prediction at the critical temperature is characteristic of the second order phase transitions. The present theory does not show any discontinuity. However, it is quite remarkable to notice that the specific heat does go through a maximum and a minimum. The lambdoid appearance of this curve tells that, close to the critical temperature, the nucleus undergoes a qualitative change although this change is not so sharp as in the case of a second order phase transition.

This description is in agreement with the experimental lack of discontinuities in the neutron evaporation spectra at an excitation energy region where the second order phase transition ought to occur. In fact, the dilution of the transition over a substantial energy range, coupled with the rapid increase of the level density, should make the transition itself very difficult to observe.

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Figure Captions

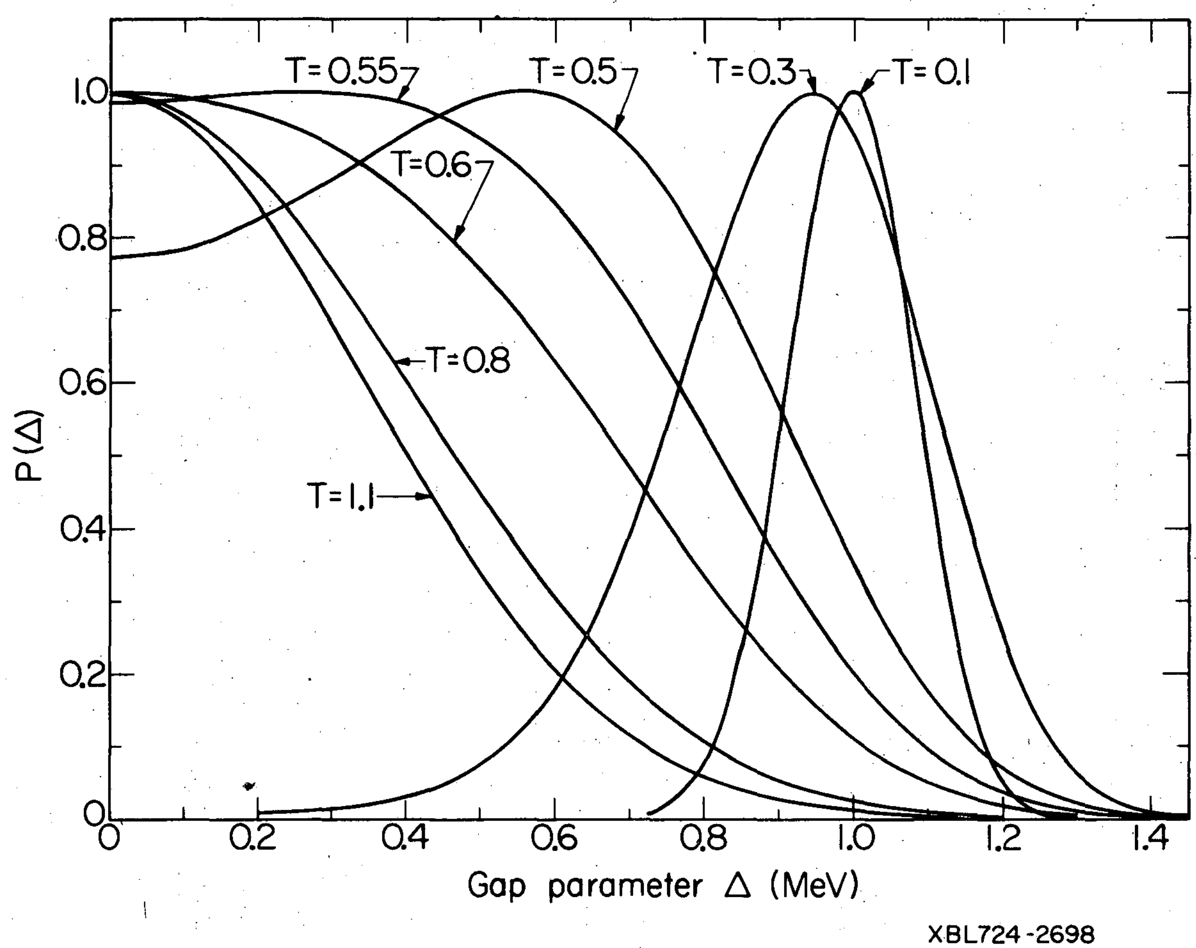
Fig. 1. Probability distributions for the gap parameter Δ at different temperatures. The value of Δ at the maximum corresponds to the solution of the gap equation. The critical temperature is $T = 0.57$.

Fig. 2. The average gap parameter (thick line) and the most probable gap parameter is a function of temperature.

Fig. 3. The excitation energy as a function of temperature. The thick and thin line correspond to the use of the average and the most probable gap parameter respectively. The dashed line corresponds to the unpaired system.

Fig. 4. The entropy as a function of temperature. The thick and thin line correspond to the use of the average and the most probable gap parameter respectively. The dashed line corresponds to the unpaired system.

Fig. 5. The specific heat as a function of temperature. The thick and thin line correspond to the use of the average and the most probable gap parameter respectively.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T=0.1
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T=1.1
1.0
0.8
0.6
0.4
0.2
0
P (Δ)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Gap parameter Δ (MeV)
XBL724-2698

Fig. 1

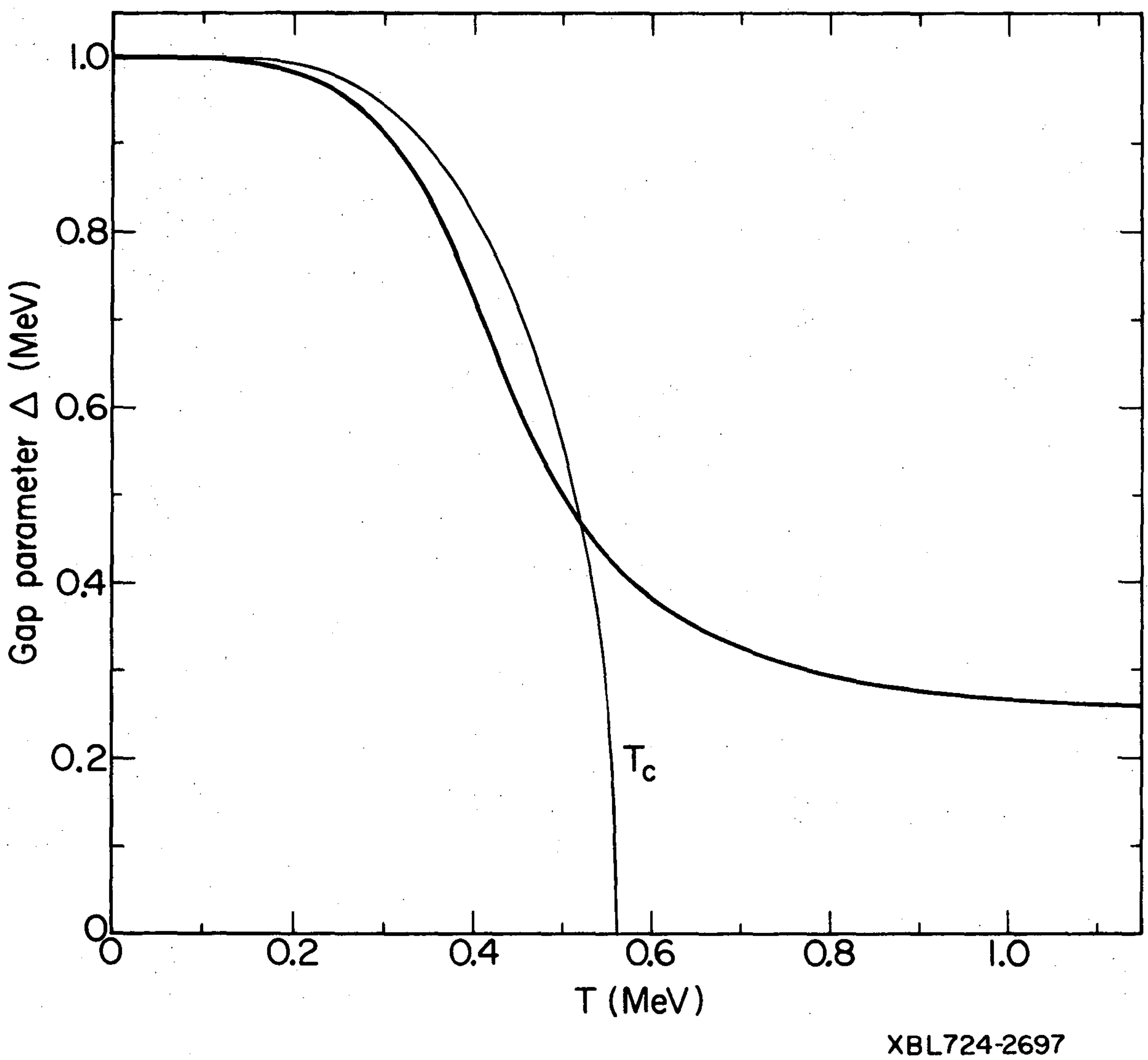

1.0
0.8
0.6
0.4
0.2
0
Gap parameter Δ (MeV)
0
0.2
0.4
0.6
0.8
1.0
T (MeV)
T_c
XBL724-2697

Fig. 2

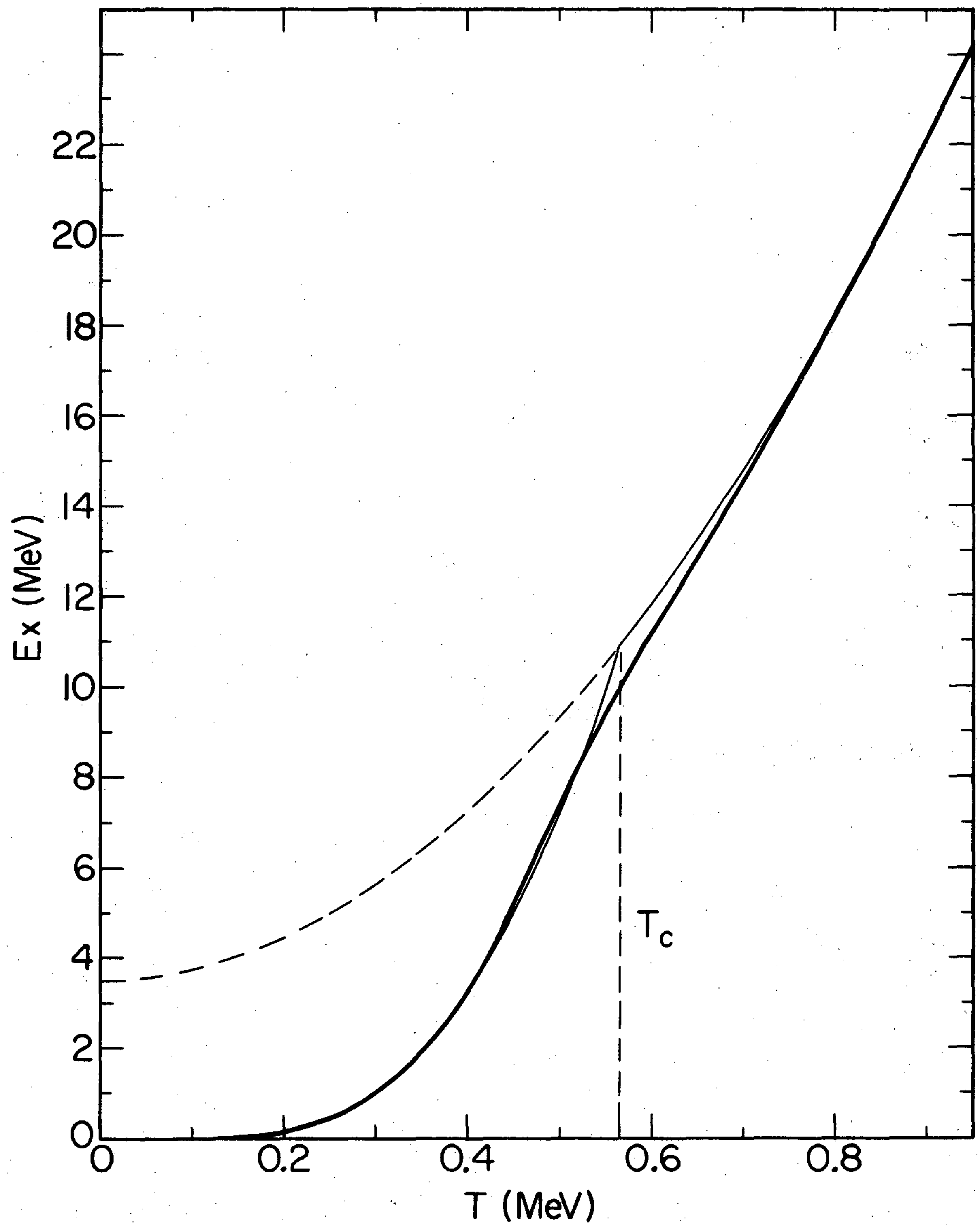
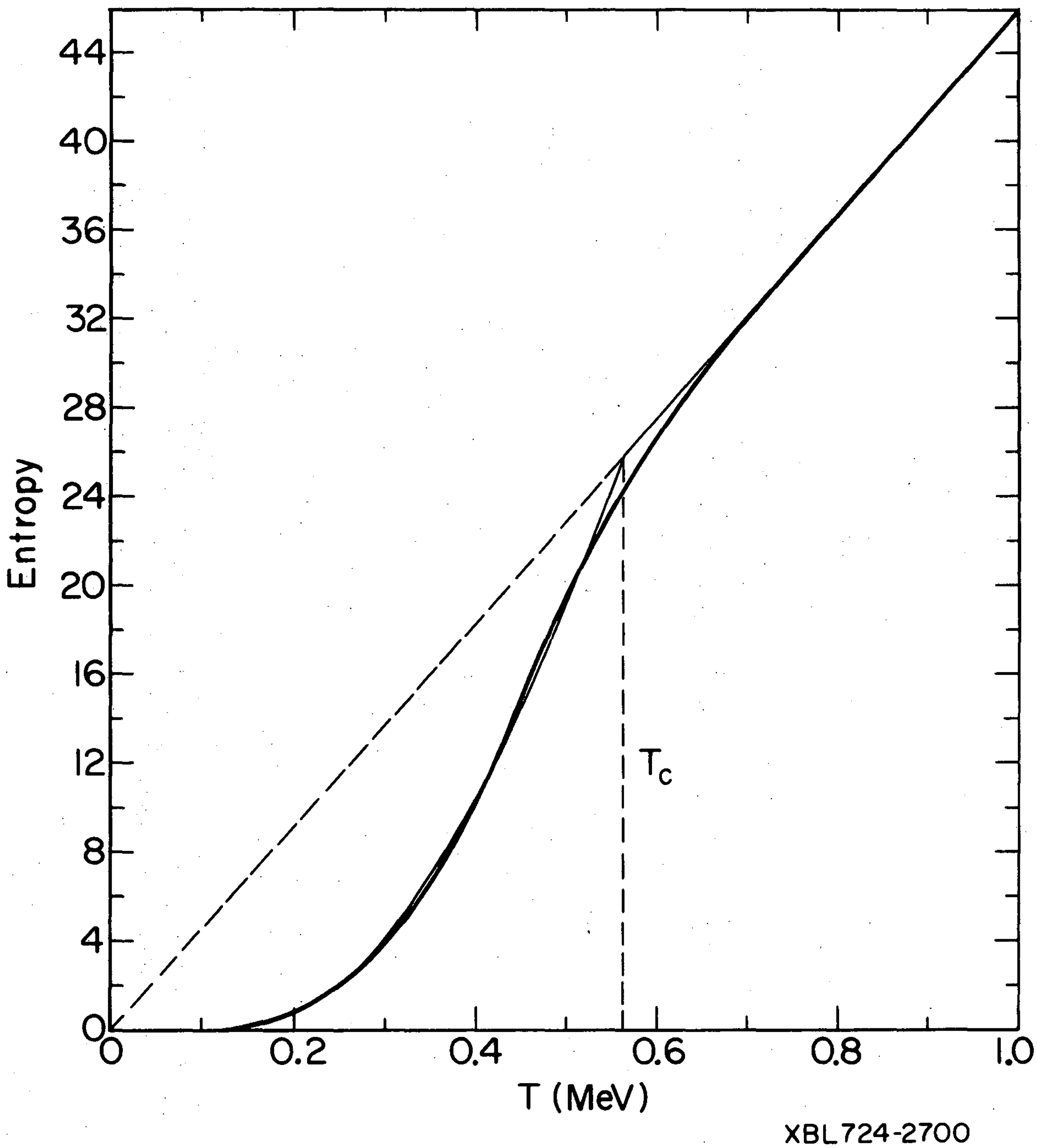


Fig. 3



Entropy
44
40
36
32
28
24
20
16
12
8
4
0
0
0.2
0.4
0.6
0.8
1.0
T (MeV)
Tc
XBL724-2700

Fig. 4

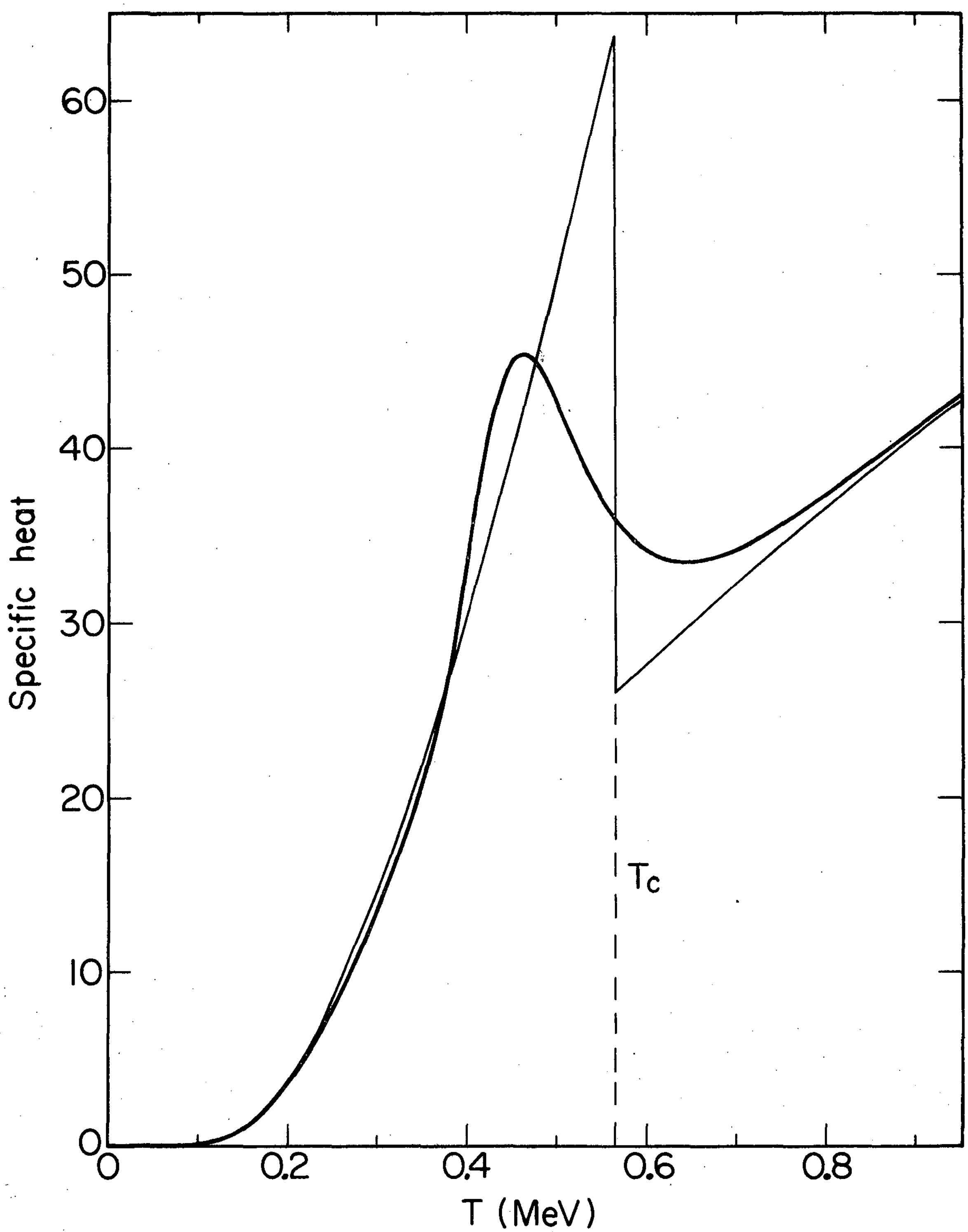

60
50
40
30
20
10
0
Specific heat
Tc
0
0.2
0.4
0.6
0.8
T (MeV)
XBL724-2699

Fig. 5