Title
TOO FOLK LEMMAS ON THE EXPANSION OF THE S-MATRIX OR THE OUT FIELD IN NORMAL ORDERED IN FIELDS

Author
Greenberg, O.W.

Publication Date
1961-07-27

UCRL-9800

UNIVERSITY OF CALIFORNIA

Ernest O. Lawrence

Radiation Laboratory

BERKELEY, CALIFORNIA

For publication in Mathematical Physics

UCRL-9800

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

TWO FOLK LEMMAS ON THE EXPANSION OF THE S-MATRIX OR THE OUT FIELD IN NORMAL ORDERED IN FIELDS

O. W. Greenberg

July 27, 1961

TWO FOLK LEMMAS ON THE EXPANSION OF THE

S-MATRIX OR THE OUT FIELD IN NORMAL ORDERED IN FIELDS

O. W. Greenberg

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 27, 1961

ABSTRACT

We prove that if the out field or the S-matrix is expanded in terms of normal ordered products of the in field, then either the expansion has infinite degree or it is the trivial case $A^{out} = A^{in}$, $S = 1$. From this fact it follows that any field theory model in which the Heisenberg field (local or not) has a terminating normal ordered expansion in terms of a (generalized) free field can not provide a non-trivial unitary S-matrix.

TWO FOLK LEMMAS ON THE EXPANSION OF THE S-MATRIX OR THE OUT FIELD IN NORMAL ORDERED IN FIELDS*

O. W. Greenberg†

Lawrence Radiation Laboratory††
University of California
Berkeley, California

July 27, 1961

1. INTRODUCTION

We prove two folk lemmas[1] concerning the impossibility that (1) a (non-trivial) finite degree expansion of the out field in terms of the in field, or (2) a (non-trivial) finite degree normal ordered expansion of the S-matrix in terms of the in field, can satisfy unitarity. The phrase "non-trivial" is inserted to exclude the equivalent trivial possibilities (1) $A^{out}(x) = A^{in}(x)$, and (2) $S = 1$. These lemmas remain valid if the in and out fields are replaced by a pair of generalized free fields[2] with the same Lehmann weight and the same relativistic no particle state, and the S-matrix is replaced by any unitary operator.

From these lemmas it follows that any field theory model in which the Heisenberg field (local or not) has a finite degree normal ordered expansion in terms of a free field or a generalized free field either has $S = 1$ or violates unitarity. This situation with respect to unitarity is in contrast with the possibility of constructing finite degree models with non-trivial locality.[3,4,2]

These lemmas are proved in Section 2; remarks about their physical content are made in Section 3.

2. IMPOSSIBILITY THAT A NON-TRIVIAL FINITE DEGREE NORMAL ORDERED EXPANSION OF A^{out} OR S CAN SATISFY UNITARITY

__Lemma 1.__[5] If A^{out} and A^{in} both belong to the usual irreducible respresentation of the mass m free field commutation relations,

$$[A^{in}(x), A^{in}(y)] = [A^{out}(x), A^{out}(y)] = i\Delta_{m^2}(x-y), \tag{1}$$

$$A^{in\,(+)}(x)|0\rangle = A^{out\,(+)}(x)|0\rangle = 0, \tag{2}$$

then either $A^{out} = A^{in}$, or the normal ordered expansion of A^{out} in terms of A^{in} (or vice versa) has infinite degree.

__Proof:__ Equations (1) and (2) imply

$$A^{in\,(-)}(x)|0\rangle = A^{out\,(-)}(x)|0\rangle. \tag{3}$$

If A^{out} has a finite degree normal ordered expansion in terms of A^{in}, this expansion must have the form

$$A^{out}(x) = A^{in}(x) + \sum_{n=2}^{N} \int d^4y_1 \cdots d^4y_n\, g^{(n)}(x-y_1, \cdots x-y_n) :A^{in}(y_1)\cdots A^{in}(y_n):, \tag{4}$$

where $g^{(n)}(y_1, \cdots y_n)$ is a (real, for neutral fields) symmetric, Lorentz invariant function of its arguments, and Eq. (3) requires that (a) the leading term be $A^{in}(x)$ and (b) the Fourier transform $f^{(n)}(k_1, \cdots k_n)$ of $g^{(n)}(y_1, \cdots y_n)$ vanish if all n k_i are in the same cone. Neutrality requires that $\overline{f}^{(n)}(-k_1, \cdots -k_n) = f^{(n)}(k_1, \cdots k_n)$. It is convenient to introduce Fourier transformed fields and to work in momentum space.

Then the expansion has the form

$$\tilde{A}^{out}(k)\delta_m(k) = \tilde{A}^{in}(k)\delta_m(k) + \sum_{n=2}^{N}(2\pi)^{4n}\int d^4k_1\cdots d^4k_n$$

$$\times\,\delta(k-\Sigma_1{}^n k_i)\, f^{(n)}(k_1,\cdots k_n)\;:\tilde{A}^{in}(k_1)\delta_m(k_1)\cdots\tilde{A}^{in}(k_n)\delta_m(k_n): \quad (5)$$

where

$$A^{\substack{out\\in}}(x) = \int d^4k\; e^{-ikx}\,\tilde{A}^{\substack{out\\in}}(k)\,\delta_m(k),$$

$$\delta_m(k) \equiv \delta(k^2-m^2), \qquad \varepsilon\delta_m(k) \equiv \varepsilon(k)\,\delta(k^2-m^2),$$

and

$$g^{(n)}(y_1,\cdots y_n) = \int d^4k_1\cdots d^4k_n\; f^{(n)}(k_1,\cdots k_n)\,\exp\Big(-i\sum_1^n k_j y_j\Big).$$

In momentum space, the requirement of unitarity, Eq. (1), reads

$$[\tilde{A}^{out}(p)\delta_m(p),\ \tilde{A}^{out}(q)\delta_m(q)] =$$

$$= [\tilde{A}^{in}(p)\delta_m(p),\ \tilde{A}^{in}(q)\delta_m(q)] = (2\pi)^{-3}\varepsilon\delta_m(p)\,\delta(p+q). \quad (6)$$

Our proof consists in inserting the expansion Eq. (5) for $\tilde{A}^{out}$ into the commutation relation Eq. (6) and showing that because the term in the commutator with $2N - 2$ normal ordered $\tilde{A}^{in}$ operators must vanish, the coefficient $f^{(N)}(k_1,\cdots k_N)$ of the last term in the assumed expansion (Eq. (5)) must vanish. Repetition of this argument leads to the conclusion that $f^{(n)} = 0$, $2 \leqslant n \leqslant N$, and only the trivial case $\tilde{A}^{out}(k)\,\delta_m(k) = \tilde{A}^{in}(k)\,\delta_m(k)$ remains. We hope that the simplicity of this argument will not be obscured by the combinatorics associated with $2N - 2$ normal ordered operators.

The terms with 2N - 2 normal ordered operators in the commutator of the out fields, C_{2N-2}, is

$$C_{2N-2} = (2\pi)^{8N-3} N^2 \int d^4p_1 \cdots d^4p_{N-1}\, d^4q_1 \cdots d^4q_{N-1}$$
$$\times f^{(N)}(p_1, \cdots p_{N-1}, p - \Sigma_1^{N-1} p_i)\, f^{(N)}(q_1, \cdots q_{N-1}, q - \Sigma_1^{N-1} q_i)$$
$$\times \varepsilon \delta_m (p - \Sigma_1^{N-1} p_i)\, \delta(p - \Sigma_1^{N-1} p_i + q - \Sigma_1^{N-1} q_i)$$
$$\times :\prod_{i=1}^{N-1} \tilde{A}^{in}(p_i)\, \delta_m(p_i)\, \tilde{A}^{in}(q_i)\, \delta_m(q_i): ,$$

where we have made use of the symmetry of $f^{(N)}$; and at least one annihilator and one creator occurs in the normal ordered product. The condition on the function $f^{(N)}$ which follows from the vanishing of the operator C_{2N-2} can be found by taking the appropriate matrix element. We consider $M_{2N-2} \equiv \langle k_1, \cdots k_{N-1} \mid C_{2N-2} \mid k_N, \cdots k_{2N-2} \rangle$, where

$$\mid k_1, \cdots k_s \rangle = (s!)^{-1/2} \prod_1^s \tilde{A}^{in^*}(k_i)\, \delta_m(k_i) \mid 0 \rangle .$$

After doing some combinatorics, making use of the symmetry of $f^{(N)}$ to combine terms where possible and performing the dp and dq integrations with the delta functions which result from the commutators of the in fields, we find

$$M_{2N-2} = (2\pi)^{2N+3} N^2 (N-1)! \prod_{h=1}^{2N-2} \theta(k_h)\, \delta_m(k_h)\, \delta(p + q - \Sigma_1^{N-1} k_i + \Sigma_1^{N-1} k_{N-1+i})$$
$$\times \Big\{ f^{(N)}(k_1, \cdots k_{N-1}, p - \Sigma_1^{N-1} k_i)\, f^{(N)}(-k_N, \cdots -k_{2N-2}, q + \Sigma_1^{N-1} k_{N-1+i})$$
$$\times \varepsilon \delta_m (p - \Sigma_1^{N-1} k_i) +$$

$$+\sum_{\alpha=1}^{N-1}\sum_{\beta=1}^{N-1} f^{(N)}(k_1,\cdots k_{\alpha-1},-k_{N-1+\beta},k_{\alpha+1},\cdots k_{N-1},p-\sum_1^{N-1\,/\alpha} k_i + k_{N-1+\beta})$$

$$\times f^{(N)}(-k_N,\cdots -k_{N-1+\beta-1},k_\alpha,-k_{N-1+\beta+1},\cdots k_{2N-2},q+\sum_1^{N-1\,/\beta} k_{N-1+i}-k_\alpha)$$

$$\times\,\varepsilon\,\delta_m(p-\sum_1^{N-1\,/\alpha} k_i + k_{N-1+\beta}) + \cdots +$$

$$+\sum_{\alpha_1>\cdots>\alpha_s=1}^{N-1}\ \sum_{\beta_1>\cdots>\beta_s=1}^{N-1} f^{(N)}(k_{\alpha_{s+1}},\cdots k_{\alpha_{N-1}},-k_{\beta_1},\cdots -k_{\beta_s},p-\sum_{s+1}^{N-1} k_{\alpha_i}+\sum_1^s k_{\beta_i})$$

$$\times f^{(N)}(-k_{\beta_{s+1}},\cdots -k_{\beta_{N-1}},k_{\alpha_1},\cdots k_{\alpha_s},q+\sum_{s+1}^{N-1} k_{\beta_i}-\sum_1^s k_{\alpha_i})$$

$$\times\,\varepsilon\,\delta_m(p-\sum_{s+1}^{N-1} k_{\alpha_i}+\sum_1^s k_{\beta_i}) + \cdots +$$

$$+ f^{(N)}(-k_N,\cdots -k_{2N-2},p+\Sigma_1^{N-1} k_{N-1+i})\, f^{(N)}(k_1,\cdots k_{N-1},q-\Sigma_1^{N-1} k_i)$$

$$\times\,\varepsilon\,\delta_m(p+\Sigma_1^{N-1} k_{N-1+i}) \Big\} \qquad (7)$$

Here the notation Σ'^{α} means that $i = \alpha$ is omitted from the sum. In the general term, the momenta $(k_1, \cdots k_{N-1})$ are divided into two groups $(k_{\alpha_{s+1}}, \cdots k_{\alpha_{N-1}})$ and $(k_{\alpha_1}, \cdots k_{\alpha_s})$, where $\alpha_1, \cdots \alpha_{N-1}$ are some permutation of $1, \cdots N-1$, in all possible combinations, but disregarding permutations which do not exchange momenta between the two groups. A similar division is performed on the momenta $(k_N, \cdots k_{2N-2})$. The total number of terms in which both sets of $N-1$ momenta are divided in groups of $N-1-s$ and s is

$$[(N-1)!]^2 \left[\frac{(N-1)!}{(N-1-s)! \quad s!} \right]^2 ,$$

where the factor $[(N-1)!]^2$ represents the number of terms which are equivalent since they differ only by permutation of the first $N-1$ arguments in each $f^{(N)}$. One of these factors $(N-1)!$ is removed by the normalization of the states used in forming the matrix element M_{2N-2}; the other such factor appears as a common factor on the right-hand side of Eq. (7).

From the commutation rules of A^{out}, Eq. (1), we know that

$$M_{2N-2} = 0. \tag{8}$$

We will prove that Eq. (8) requires $f^{(N)} = 0$. Our first step is to show that the cases with different numbers of the momentum arguments of $f^{(N)}$ in each cone can be treated separately. In Eq. (7) there are N different sets of terms corresponding to the values $s = 0, 1, \cdots N-1$. Each set of terms has as a factor a mass shell delta function whose argument is $p - \sum_{s+1}^{N-1} k_{\alpha_i} + \sum_{1}^{s} k_{\beta_i}$. Since for all k_i , $k_i^2 = m^2$, $k_i^o > 0$, these mass shell delta functions do not, in general, contribute simultaneously for different values of s except on a set of lower dimension in the space of the k_i (i.e., except on a set of measure zero). Sets of measure zero can be neglected since the $f^{(N)}$, like S-matrix elements,[6] must be finite and thus cannot contain delta functions. Thus, effectively, the mass shell delta functions in Eq. (7) isolate terms with different values of s, i.e. with different numbers of the momenta in $f^{(N)}$ in each cone.

This paragraph gives the argument that $f^{(N)}(k_1,\cdots k_{N-1}, -k_N) = 0$, $k_i^o > 0$, $1 \leq i \leq N$, for the special case when exactly N-1 of the momentum arguments of $f^{(N)}$ are in one cone. Consider the case s = 0, for which there is just one term. Eliminate q using the four dimensional momentum conservation delta function, choose $k_1 = k_N$, $k_2 = k_{N+1}, \cdots k_{N-1} = k_{2N-2}$, and use the neutrality condition $f^{(N)}(k_i) = \bar{f}^{(N)}(-k_i)$. Then, dropping irrelevant factors, we find

$$|f^{(N)}(k_1, \cdots k_{N-1}, p - \Sigma_1^{N-1} k_i)|^2 \varepsilon \delta_m (p - \Sigma_1^{N-1} k_i) = 0. \qquad (9)$$

Since $f^{(N)}$ vanishes if all k_i are in the same cone, we choose p so that (a) $p^2 = m^2$, $p^o > 0$, (b) the delta function in Eq. (9) contributes, and (c) the ε gives a negative sign. Since we can obtain any $k_N = -(p - \Sigma_1^{N-1} k_i)$, $k_N^2 = m^2$, $k_N^o > 0$ in this way, we conclude that

$$f^{(N)}(k_1, \cdots k_{N-1}, -k_N) = 0$$

Consideration of the next case, in which there are $N - 2$ k_i in one cone and 2 in the other, leads us to the terms with $s = 1$. Here there are $(N - 1)^2$ different terms instead of just one. After repeating the considerations above Eq. (9), we find that the following sum of terms must vanish:

$$\sum_{\alpha=1}^{N-1} | f^{(N)}(k_1, \cdots k_{\alpha-1}, -k_\alpha, k_{\alpha+1}, \cdots k_{N-1}, p - \sum_1^{N-1,\alpha} k_i + k_\alpha)|^2$$
$$\times \varepsilon \delta_m (p - \sum_1^{N-1,\alpha} k_i + k_\alpha) +$$
$$+ \sum_{\alpha \neq \beta = 1}^{N-1} f^{(N)}(k_1, \cdots k_\beta, \cdots k_{\alpha-1}, -k_\beta, k_{\alpha+1}, \cdots k_{N-1}, p - \sum_1^{N-1,\alpha} k_i + k_\beta)$$
$$\times \bar{f}^{(N)}(k_1, \cdots k_\alpha, \cdots k_{\beta-1}, -k_\alpha, k_{\beta+1}, \cdots k_{N-1}, p - \sum_1^{N-1,\alpha} k_\lambda + k_\beta)$$
$$\times \varepsilon \delta_m (p - \sum_1^{N-1,\alpha} k_i + k_\beta) = 0. \qquad (10)$$

If the left-hand side of Eq. (10) consisted of a sum, with positive coefficients, of terms of the form $| f^{(N)} |^2$, we could conclude that each $f^{(N)}$ in Eq. (10) vanishes. However the terms in Eq. (10) having the form $f^{(N)} \bar{f}^{(N)}$ with different arguments upset this conclusion and demand further study. Since these terms contain k_α and $-k_\alpha$ in a single $f^{(N)}$, they are a special case of the term with $N - 2$ creators and 2 annihilators (or vice versa) which we are now considering.

We can examine the terms we get if we choose, for example, $k_1 = k_2$. We then find a sum of terms, including terms of the types

$$| f^{(N)}(k_1, -k_1, k_3, \cdots k_{N-1}, p - \Sigma_3^{N-1} k_i) |^2 ,$$

$$| f^{(N)}(k_1, k_1, -k_3, k_4, \cdots k_{N-1}, p - \Sigma_4^{N-1} k_i - 2k_1 + k_3) |^2 ,$$

$$f^{(N)}(k_1, k_1, k_3, -k_3, k_5, \cdots k_{N-1}, p - \Sigma_5^{N-1} k_i - 2k_1)$$
$$\times \bar{f}^{(N)}(k_1, k_1, k_4, -k_4, k_5, \cdots k_{N-1}, p - \Sigma_5 \; k_i - 2k_1) ,$$

and

$$f^{(N)}(k_1, k_3, -k_3, k_4, \cdots k_{N-1}, p - \Sigma_4^{N-1} k_i - k_1)$$

$$\times \bar{f}^{(N)}(k_1, k_1, -k_1, k_4, \cdots p - \Sigma_4^{N-1} k_i - k_1) .$$

Notice that the $f^{(N)}$ which previously appeared in a term of the type $f^{(N)} \bar{f}^{(N)}$ now appears as $|f^{(N)}|^2$; however there are still terms of the type $f^{(N)} \bar{f}^{(N)}$. These last terms contain $f^{(N)}$ or $\bar{f}^{(N)}$ evaluated at a still more special set of arguments than any of the earlier terms which we have encountered.

Rather than continuing our discussion by setting more and more sets of momenta k_i equal to each other, we go at once to the extreme case and set all the k_i equal to k. We then find the equation

$$(N-1)^2 \, | f^{(N)}(k, \cdots k, -k, p - (N-3)k) |^2$$
$$\times \, \varepsilon \, \delta_m (p - (N-3)k) = 0 ,$$

and conclude that

$$f^{(N)}(k, \cdots k, -k, -k') = 0.$$

Having shown that this most special case of $f^{(N)}$ vanishes, we now allow more and more of the k_i to differ and obtain a set of equations in which at each step the "special" terms which do not contain $|f^{(N)}|^2$ have already been shown to vanish so that we can conclude that the $f^{(N)}$ which occur in absolute values squared vanish. Finally we again reach Eq. (10), this time having proved earlier that the terms in the sum $\sum_{\alpha \neq \beta}$ vanish, and conclude that

$$f^{(N)}(k_1, \dots k_{N-2}, -k_{N-1}, -k_N) = 0.$$

The argument for the other cases $s = 2, 3, \cdots N-2$, corresponding to the other possible distributions of the k_i between the two cones, proceeds in analogy to that of the case $s = 1$ above. Thus we can conclude that

$$f^{(N)}(k_1, \dots k_{N-1-s}, -k_{N-s}, \dots -k_N) = 0, \quad 0 \leq s \leq N-2,$$

and that the last term in the expansion of Eq. (5) vanishes. Repetition of our entire argument for $n = N - 1$, $n = N - 2$, $\cdots$ $n = 2$, leads us to conclude that only $\tilde{A}^{out}(k)\, \delta_{in}(k) = \tilde{A}^{in}(k)\, \delta_{in}(k)$ is consistent with the commutation relations (i.e. with unitarity) and completes the proof of lemma 1.

<u>Lemma 2.</u> If the S-operator (which relates A^{out} and A^{in} by $A^{out}(x) = S^{-1} A^{in}(x) S$) is unitary, then either $S = 1$, or the normal ordered expansion of S in terms of A^{in} (or A^{out}) has infinite degree.

<u>Proof:</u> We deduce lemma 2 as a corollary to lemma 1. If S is unitary and has a finite degree normal ordered expansion in terms of A^{in} then $A^{out}(x) = S^* A^{in}(x) S$ would have a finite degree normal ordered in field expansion. However lemma 1 excludes this possibility except for the trivial case $A^{out} = A^{in}$ which corresponds to $S = e^{i\varphi}\, 1$. The requirement $S\,|\,0\,\rangle = |\,0\,\rangle$ fixes $\varphi = 0$, which completes the proof of lemma 2.

From the methods of proof of these lemmas, it is clear that they remain valid for generalized free fields. We state them in a form appropriate for this case.

Lemma 1a. If ϕ_1 and ϕ_2 are generalized free fields which both have the same relativistic no particle state[7]

$$\phi_1^{(+)}(x)|0\rangle = \phi_1^{(s)}(x)|0\rangle = \phi_2^{(+)}(x)|0\rangle = \phi_2^{(s)}(x)|0\rangle = 0, \qquad (11)$$

and the same Lehmann weight

$$[\phi_1(x), \phi_1(y)] = [\phi_2(x), \phi_2(y)] = i\int da^2 \rho(a^2) \Delta_{a^2}(x-y), \qquad (12)$$

then either $\phi_2 = \phi_1$, or the normal ordered expansion of ϕ_2 in terms of ϕ_1 (or vice versa) has infinite degree.

Lemma 2a. If a unitary operator[8] U relates ϕ_2 and ϕ_1 by $\phi_2(x) = U^{-1}\phi_1(x)U$ then either $U = 1$, or the normal ordered expansion of U in terms of ϕ_1 (or ϕ_2) has infinite degree.

Finally, these lemmas provide a proof that any field theory model[3,4,2] in which the Heisenberg field has a finite degree normal ordered expansion in terms of a (generalized) free field cannot have a non-trivial unitary S-matrix. No assumption about the locality of the Heisenberg field is necessary for this conclusion.

3. REMARKS

We make some remarks on the physical content of these lemmas. If lemma 2 were not true then it would be possible that there be, for example, elastic scattering between pairs of particles,

$$ {}_{in}\langle p_1, p_2 | S | q_1, q_2 \rangle_{in} \neq {}_{in}\langle p_1, p_2 | 1 | q_1, q_2 \rangle_{in} \quad , $$

but no elastic scattering between--say--N + 1 particles,

$$ {}_{in}\langle p_1, \cdots p_{N+1} | S | q_1, \cdots q_{N+1} \rangle_{in} = {}_{in}\langle p_1, \cdots p_{N+1} | 1 | q_1, \cdots q_{N+1} \rangle_{in} \; . $$

On intuitive grounds (or on the basis of Feynman diagrams), we expect that N + 1 incoming particles must at least produce that elastic scattering which would result from the elastic scattering between all pairs of incoming particles. Thus lemma 2 seems obvious intuitively.

Since the demonstrations of lemmas 1 and 2 require no statements about the interpolating Heisenberg field, these lemmas are independent of the assumption of locality. It is an open question whether an S-matrix which allows only a finite set of intrinsic processes is consistent with locality; clearly such a possibility is consistent with unitarity alone. For example, an S-operator of the form $S = e^{i\eta}$,

$$ \eta = \int d^4k_1 \cdots d^4k_4 \; \delta(k - \Sigma_1{}^4 k_i) \, f(k_1, \cdots k_4) $$
$$ \times \; : \tilde{A}^{in}(k_1)\delta_m(k_1) \cdots \tilde{A}^{in}(k_4)\delta_m(k_4) : \quad , \qquad (13) $$

where $f(k_i) = \bar{f}(-k_i)$, and f is totally symmetric, leads to elastic scattering only. Such an S-operator is unitary since η has been chosen Hermitian. Lemma 2 is not violated since the normal ordered expansion of this S-operator does not terminate. We do not know whether local field theory allows such an S-matrix. It is interesting that the elastic scattering amplitude which follows from Eq. (13) cannot have the form of the Mandelstam representation.[9]

ACKNOWLEDGMENTS

We thank A. S. Wightman for suggesting the consideration of lemma 1 and for several helpful discussions, and P. G. Federbush and M. T. Grisaru for asking a question which led to lemma 2. It is a pleasure to thank G. F. Chew for the hospitality of the Lawrence Radiation Laboratory.

FOOTNOTES AND REFERENCES

* Supported in part by the United States Atomic Energy Commision.

† National Science Foundation Postdoctoral Fellow.

†† Now at Physics Department, University of Maryland, College Park, Maryland.

1. These folk lemmas have been proved by many people, but as far as we know do not appear in the literature. The present author proved lemma 1 in 1956 upon the suggestion of A. S. Wightman.

2. O. W. Greenberg, Annals of Physics (to be published).

3. A. S. Wightman, Problèmes Mathematique de la Théorie Quantique des Champs, University of Paris Lecture Notes (1957), pp. 57-64.

4. K. Bardakci and E. C. G. Sudarshan (to be published).

5. We consider only neutral scalar fields. However with appropriate changes, such as using anticommutators for Fermi fields rather than commutators, the results obtained can be extended straightforwardly to a finite number of charged and neutral fields with arbitrary spin and internal quantum numbers.

6. Lehmann, Symanzik and Zimmermann, Nuovo cimento 6, 319 (1957).

7. $\phi^{(s)}(x)$ is the Fourier transform of $\tilde{\phi}(k)$ restricted to space-like momenta.

8. In Reference 2 it was proved that Eq. (11) and (12) determine uniquely all the vacuum expectation values of a generalized free field. Therefore there exists such a unitary operator relating ϕ_2 and ϕ_1.

9. We are indebted to M. Froissart and A. Scotti for this remark.