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UNIVERSITY OF CALIFORNIA, SAN DIEGO

**A Class of Allpass-based Filter Design Algorithms for Photonic Signal
Processors**

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Electrical Engineering (Signal and Image Processing)

by

Yujia Wang

Committee in charge:

Professor Truong Q. Nguyen, Chair
Professor Y. Shaya Fainman
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Professor Jiawang Nie
Professor Bhaskar D. Rao

2014

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The dissertation of Yujia Wang is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2014

EPIGRAPH

Be as you wish to seem.

—Socrates

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The author of this dissertation is the primary researcher and author of the above mentioned papers. Co-author Truong Q. Nguyen, listed in these publications, supervised the research, and other co-authors contributed to the papers in the form of writing and simulation.

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- Y. Wang, R. Aguinaldo, T. Nguyen, “Loss-tolerant allpass-based filter bank design suitable for integrated optics”, *Journal of Lightwave Technology*, (2014) [submitted].
- Y. Wang, R. Aguinaldo, T. Nguyen, “Realistic photonic filter design based on allpass substructures with waveguide loss compensation”, *Journal of Lightwave Technology*, 32.5 (2014): 1024-1031.
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- Y. Wang, A. Grieco, B. Slutsky, B. Rao, Y. Fainman, T. Nguyen, “Design and Analysis of Narrowband Filter for Optical Platform”, *Proc. 36th International Conference on Acoustics, Speech, and Signal Processing*, pages 1633-1636, Prague, Czech Republic, May 2011.

ABSTRACT OF THE DISSERTATION

A Class of Allpass-based Filter Design Algorithms for Photonic Signal Processors

by

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Doctor of Philosophy in Electrical Engineering (Signal and Image Processing)

University of California, San Diego, 2014

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Photonic systems have proven to be an integral element in the core areas of next generation communication infrastructures such as Wavelength division multiplexing, Discrete Time Optical Processing, Optical Time Division Multiplexing, and Optical Code Division Multiplexing. An optical setup can either replace, or be integrated into existing architectures to sidestep the inflexibility of conventional electronic circuitry used in telecommunications systems. In comparison to their electrical counterparts, photonic signal processors are able to outperform by providing superior sampling rates and crucial increase in bandwidth. Additionally, photonic devices benefit from very low transmission loss, and are impervious to the electromagnetic sources of interference that plague electronic devices.

However, realistic considerations arise when implementing photonic signal processors. Both innate material properties and capabilities of current fabrication technology result in a degradation in the system performance of photonic devices. Such unavoidable behavior prevents photonic devices from reaching their maximum potential, and in turn hinders the widespread distribution of all-optical infrastructures. From a signal processing perspective, the effects can be readily expressed as a power loss coupled with the signal's propagation through the system. The need for a set of DSP design techniques that specifically consider the unique characteristics of photonic devices is therefore immediately evident.

This dissertation explores the analysis and approach to a new class of all-pass based filter design algorithms specifically targeted for photonic implementation. Allpass filter based systems are ideal for photonic realizations because the behavior is naturally observed in a variety of nanoscale dielectric components, which can be considered as the basic building blocks to complex systems. We examine the waveguide power loss effect at the most fundamental level, and present a set of design algorithms for phase compensators, bandpass filters, and filter banks based on realistic characterizations of photonic allpass elements. To increase the robustness of practical deployment of the designs, stochastic models that can aid the evaluation of the post fabrication filter performances are also demonstrated.

Chapter 1

Introduction

With the promise to sidestep the limitations of current electronic based communication systems, developments in photonic integrated circuits have accelerated in recent years [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Optical component based systems are immune to the electromagnetic interference that plagues electronic networks [13], and their superior bandwidth and sampling rates make them ideal for next generation telecommunication infrastructures [14]. Photonics signal processors can be readily integrated into communication setups such as Wavelength division multiplexing (WDM), Optical Time Division Multiplexing (OTDM), and Optical Code Division Multiplexing (OCDM) [15]. While their presence in telecommunications is definitely undeniable and critical, research and development for photonic integrated circuits had been heavily self-contained. The vastly available communication theory and digital signal processing (DSP) techniques are still rather decoupled from the photonic physics [7]. To support data communication in an optical signal environment, digital filter designs that can be realistically implemented using photonic components are required.

1.1 Allpass-based Photonic Signal Processors

From a system integration perspective, the allpass filter is an ideal building block for realizing complex photonic signal processors. Allpass filters are desirable in assembling photonic integrated circuits [16] because they do not intrinsically

attenuate the optical signal power, indicating that no subsequent amplification is required and thus keeping power consumption and power dissipation to a minimum. They are also highly scalable, in that complex allpass filters can be readily realized in lattice or cascade assemblies of lower ordered sections. Furthermore, the allpass behavior is naturally observed in a variety of nanoscale dielectrics such as Bragg mirrors [17], ring resonators [18], and microdisks [19]. With a wide range of applications towards phase equalization [20, 21, 7], general frequency selective filtering [16, 22], and even advanced utilization such as sub-band processing [23, 8, 24], the digital allpass filter has demonstrated its solid connection to optical signal processing.

1.1.1 Phase Compensators

Allpass filters have long been used in digital communications and signal processing. These structures are excellent for applications that involve manipulation of the digital signal's phase, such as phase equalization, Hilbert transform, fractional delay, and dispersion compensation [25, 26]. In low loss photonic systems such as fiber optics (loss coefficient $\sim 2 \times 10^{-6}$ dB/cm [27]) the direct association of optical components with allpass elements is well justified. However, in the context of integrated optical systems such as dielectric waveguides (loss coefficient ~ 5 dB/cm [28, 29]) and plasmonic waveguides (loss coefficient $\sim 2 \times 10^3$ dB/cm [30]) the association becomes difficult to justify, particularly when effects such as bending loss are considered [31]. The implications for optical filter design are significant. In particular, as the devices are integrated and miniaturized it will become necessary to confront the issue of optical loss. The allpass assumption is also violated in situations that require long phase delay lengths and therefore long sections of waveguide, narrowband filters being an example [22]. These realistic considerations limit the utilization and performance of allpass filters in optical systems, especially in its direct phase related applications such as dispersion compensation in WDM systems [32, 33]. A design algorithm specifically targeted at photonic allpass phase compensators is therefore necessary for realistic implementations.

1.1.2 Bandpass Filters

Allpass filters are also critical in creating realizable photonic systems that provide processing on the input signal's power profile. A general bandpass filter can be realized using a combination of two allpass filters [34]. The design works by creating different phase profiles for the two allpass elements in desired frequency ranges. Being able to construct frequency selective filters using purely allpass substructures is highly beneficial in photonic applications. While a frequency selective filter can be directly realized using setups such as lattice filters [35, 36], a design based on allpass filters can lower power usage, reduce the effect of loss, and minimize footprint. Furthermore, a variety of basic photonic nanoscale elements used for constructing complex systems naturally behave as allpass filters.

With such promising features, the design and application of allpass-based filters from a signal processing perspective have found their ways into the photonic research community [16]. These techniques, however, are often a direct utilization of existing allpass design algorithms, assuming ideal materials and perfect fabrication processes. Although silicon waveguide bends can be as small as $R = 1 \mu\text{m}$ for a bending loss of less than 0.1 dB/turn [37], the conventional approach in silicon photonics is to use a much larger bending radius, typically $R = 10 \mu\text{m}$ [38, 39, 9], in order to ensure much lower loss, so that traditional filter design algorithms which are based on the assumption of near-lossless propagation may be used. While a significant amount of effort has been put into minimizing the scattering effect through optimization of the physical photonic structures, the performance of photonic filters will always be sub-optimal, even theoretically, due to material absorption [40]. The need for a set of photonic specific design techniques is again evident.

1.1.3 Filter Banks

A more advanced utilization of allpass filters is the DFT allpass filter bank, which is a fundamental factor for enabling sub-band processing. Sub-band processing is a powerful concept in communications and signal processing for efficient implementation of real time architectures. Its applications reach a wide range

of signal processing fields such as: speech coding [41], image coding [42], video coding [43], channel equalization [44], and high speed analog-to-digital conversion [45]. In this setup, a wideband signal is segmented into multiple frequency bands using a bank of filters. The system can then perform the signal processing on the sub-bands at a lower sampling rate, which significantly reduces the computational cost.

The idea of processing signals in sub-bands has also extended into the domain of photonics. Fundamental operations in WDM networks such as multiplexing rely on these architectures [46]. More advanced applications such as time-stretched ADC [47, 23, 48, 49] can also be readily connected to their DSP counterparts [45]. A wide range of photonic applications can therefore directly leverage off concepts in digital signal processing. Extending the sub-band processing idea beyond time stretched ADCs to more general, complex operations is critical for the development of all optical signal processing. For example, terahertz processing [50] could require ring resonators with bending radii smaller than $0.5\mu\text{m}$. An 8-channel sub-band processing setup, when combined with techniques such as time stretching, would enable the system to be realized with rings 8 times the size, which are far more realistic in terms of implementation [37].

The design of the filter bank used to separate the signal into frequency sub-bands is crucial in enabling the sub-band processing technology. Although it is possible to design the filters individually, a systematic approach to designing the entire set of filters is necessary for overall performance optimality. From a mathematical perspective, the design of a filter bank is a challenging problem due to the large number of optimization variables and prescribed requirements involved. A significant amount of effort has been put into the optimal design of various filter bank structures over the years [51, 52, 53, 54]. However, most of the proposed architectures are only optimal from a purely DSP perspective, and do not consider a number of design factors that are specific to photonic implementation. While photonic based filter bank design exists [55, 56], the algorithm is unsuitable for high performance filtering needs in architectures such as DTCOP and UDWDM because the qualities of the filters progressively degrade. When each filter is designed

individually, the channel-to-channel performances are not evenly balanced [57]. Furthermore, the effect of waveguide loss becomes increasingly prominent as the bandwidth of the filters become narrower. A design technique for high performance filter bank that considers the effect of loss is therefore highly beneficial.

1.2 Motivation

Loss characterization of both passive [58, 37] and active [59] silicon photonic components have become commonplace since such loss is fundamentally unavoidable due to fabrication limitations that induce scattering [60], additional scattering at waveguide bends and transitions [37], and free carrier absorption [61]. Design methods that should carry over from DSP are therefore plagued by the provision of non-ideal filter elements from the physical platform; this is especially true in the case of allpass filters. For example, a single ring resonator, coupled to a single bus waveguide, exhibits a rich spectrum of resonant nulls [58]. These resonances, however, occur because of the presence of loss in the ring. If the loss were absent, the ring resonator would impart a spectrally varying phase shift onto the light, but with unity transmission; i.e. an allpass filter would be realized.

The goal of this research is to derive a class of algorithms that can mitigate, and even harness the waveguide power loss effect for realistic allpass-based photonic signal processors. From a signal processing perspective, the waveguide power loss can be readily modeled as a variation on the delay element z^{-1} . It has been presented that the loss affects a transfer function by modifying the original response $A(z)$ into $A(\gamma^{-1}z)$, where γ captures the effect of the power loss [21, 16]. An algorithm that considers the compensation for a variation on the delay element z^{-1} is therefore crucial for the realistic implementation of photonic signal processors. Although significant efforts have been put into allpass analysis to demonstrate its utility and versatility [26] and robustness to coefficient perturbations [62, 25] in DSP implementations, its optimal design under this unique photonic filtering error has never been addressed.

Since the waveguide power loss can be considered as a predetermined effect

[16], we present a class of algorithms that directly integrate the loss effect γ into the filter design cost functions for various allpass-based systems. While a number of allpass phase compensator designs exist in both the least squares sense [63, 64, 65], and the minimax sense [25, 66, 67], there is no approach that directly considers the compensation for a variation on the delay element z^{-1} . Without any manipulation, the design of allpass phase compensator under waveguide loss is non-convex and NP hard. Instead, we first solve a relaxed problem to achieve an initial solution, and subsequently use a local search technique to arrive at filter coefficients that can mitigate the loss. The resulting phase compensator designs will prove to be crucial for next generation high capacity WDM architectures.

While a handful of papers aimed to address the issue of loss for direct form lattice implementation of photonic bandpass filters [35, 36], a loss compensating design algorithm for allpass-based structures has yet to be proposed. Here, we present the design principles for high-performance photonic filters based on allpass sub-structures with a finite degree of loss γ , which may result in reducing the footprint of filters by two orders of magnitude, and lead to much higher densities of optical components on a chip. Since the footprint of silicon photonic components is currently three orders of magnitude larger than of silicon electronic components, this reduction in size without significant degradation in performance is expected to have a large impact on the field of silicon optoelectronic integrated circuits.

Our work on waveguide loss compensating photonic filter designs extends to complex filter bank structures that guarantee identical, optimal performances in all channels. Although the idea of an allpass-based filter bank is first conceived in a signal processing context, the proposed setup is structurally dependent on additional processing to be functional [54]. We present a modified structure and design algorithm that is able to achieve the necessary filtering without extra hardware. Although the waveguide power loss is an inescapable source of error that is generally destructive to the filter performance, such effect also causes a photonic allpass filter to exhibit attenuation at certain controllable locations. We present a design algorithm that optimally utilizes the otherwise detrimental effect to mitigate the necessity for additional processing from the original allpass filter bank struc-

ture. By providing DSP design techniques that can compensate for the inescapable suboptimal behavior of photonic structures, we will facilitate the development of photonic signal processors.

This dissertation also briefly examines the translation of realizable photonic parameter values into design constraints. The filter design requirements in optical signal processors are often narrowband due to the nominal operational frequency of optical signals. A narrowband filter is difficult to design because in addition to requiring a high-order filter, it also demands poles that are extremely close to the unit circle—a characteristic highly undesirable in traditional digital signal processing research. A pole with magnitude close to unity implies that the system may demonstrate stability issues under the finite word length effect, which has been the subject of many research papers [26, 68, 69, 70, 71]. A photonic system has similar constraints because the pole magnitudes directly relate to the feasibility of realizing the filter using optical components. Although constraints for frequency selective filters algorithms exist [68, 72], allpass designs with constraints are scarce [66]. We derive a constraint based on the Argument Principle such that the allpass filters will be realizable using conventional technology under the cascade form.

A consideration that cannot be addressed during the design phase is the random phase error that arises due to tooling inaccuracy. Since such behavior is largely uncontrollable, the fabrication process changes the ideal first-order allpass transfer function $A(z)$ into $A(e^{-j\delta}z)$ where δ is a random variable based on the imperfection. A well-defined statistical model for the filter response under loss is necessary for performance analysis. However the effect is difficult to measure because the error is interconnected with both the feed-forward and feed-back paths of the allpass filter $A(z)$, and is structurally dependent. We present an innovative approach to extract the error term, and subsequently present a first definition of the stochastic model on the magnitude response of the error corrupted system. Specifically, we demonstrate the procedures to evaluating the expected performance of photonic allpass filters after being batch processed. This stochastic measurement of the filter under error can be readily used for large scale fabrication yield analysis.

1.3 Organization

This dissertation carefully considers the design challenges faced by realistic implementations of photonic signal processors based on allpass filters. Optimal design algorithms for phase compensators, bandpass filters, and DFT filter bank are presented. The derivation of a stochastic measure for post fabrication performance analysis is also examined. The rest of the content is organized as follows:

Chapter 2 provides the foundation to photonic filters based on allpass substructures. We begin by introducing discrete time coherent optical signal processing (DTCOP), which is the fundamental concept to linking digital signal understandings to photonic systems. We then provide the details to photonic allpass structures by discussing the realization of first-order systems and high complexity allpass filters. This chapter concludes by demonstrating how allpass filters can be used to achieve frequency selective filters such as lowpass structures and filter banks. To further develop our understanding about the unique behaviors of photonic allpass filters, Chapter 3 analyzes the effect of waveguide power loss. Its physical origins as well as a detailed signal processing analysis on the effect are presented.

Chapter 4 derives the allpass phase compensator design algorithm that can compensate for the waveguide power loss. Our formulation and approach to solving the non-linear optimization problem are explained. Simulation results are provided to demonstrate the validity and performances of the proposed algorithms. Examples that demonstrate the benefit of the proposed design in WDM systems are used to motivate the development of high density WDM architectures. To expand the application of allpass-based photonic signal processors, Chapter 5 details on the derivation of a realistic photonic bandpass filter design algorithm. Simulation results are provided to demonstrate the superior performance from the proposed loss mitigating design algorithm under realistic fabrication conditions. Chapter 6 demonstrates an advanced application of allpass elements through the design of a photonic DFT filter bank. We present a design algorithm that intelligently utilizes the waveguide loss effect to create a novel filter bank structures exclusive to photonic implementation. Simulation results are provided to demonstrate the pro-

posed algorithm's utility in high efficiency time-stretched ADCs and high density WDM architectures.

To improve the assimilation of allpass-based photonic structures into realistic fabrication environments, Chapter 7 demonstrates how conventional photonic fabrication capability can be translated into a constraint for the filter design optimization setup. Example of how the constraint can be readily combined with existing Least Squares allpass design techniques are provided. Chapter 8 contains the signal processing models for post fabrication yield analysis in order to better connect the theoretical designs with realistic deployments. This chapter contains the derivation for the average performance of allpass filters under random waveguide dimension error. Because the effect of the tooling inaccuracy is different depending on the realization, the analyses for both the cascade and the lattice forms of high order allpass filters are provided. Expected performances for lowpass filters based on allpass substructures are also detailed. Simulation results are included to demonstrate the validity of the models. Chapter 9 outlines the proposed future work, and Chapter 10 concludes the dissertation.

1.4 Acknowledgments

The content of this chapter, in part, is a reprint of the published journal papers in OSA Optics Express, December 2013, IEEE Journal on Lightwave Technology, March 2014, the published conference paper in IEEE International Conference on Acoustics, Speech, and Signal Processing 2011, IEEE Global Communications Conference 2011, European Signal Processing Conference 2012, and a submitted paper to IEEE Journal on Lightwave Technology.

Chapter 2

Background

To consider the unique challenges faced by optical signal processing, we must first understand how a photonic system can be characterized. This Chapter provides the necessary details to establish mathematical models for the behaviors of ideal photonic allpass filters. For the principles of signal processing to apply, we will first briefly discuss the Discrete-Time Coherent Optical Processing regime.

2.1 Discrete-Time Coherent Optical Processing

While signal processing techniques can be related to photonic structures in a variety of setups [13], in this dissertation we focus on discrete-time coherent optical processing (DTCOP) devices [13]. In DTCOP, the RF signal of interest ($f \sim 2 \cdot 10^{10}\text{Hz}$) is imposed as amplitude modulation on the optical carrier ($f \sim 2 \cdot 10^{14}\text{Hz}$), and filtering is carried out in the optical domain. Photonic components such as couplers and partially transmissive mirrors perform arithmetic operations. An optical waveguide loop with traversal time on the order of the RF period serves as the discrete delay element z^{-1} , which can also be understood as the minimum amount of time between two observable instances of the signal in the system. Fine adjustment to the traversal time on the order of the period of the optical carrier multiplies the signal by a complex phase factor. Note that the free spectral range (FSR) of the device of interest needs only to correspond to the sampling rate of the RF signal. With the fundamentals of optical signal processing established, we

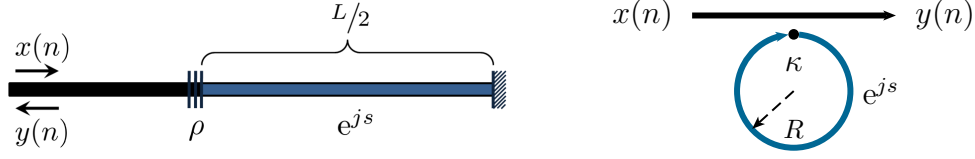


Figure 2.1: First-order Bragg reflectors allpass filter (left), where ρ is the reflection coefficient, L is the length of the waveguide, and s is the phase delay. First-order ring-based allpass (right), where κ is the coupling coefficient, R is the radius of the ring, and s is the phase delay.

can now examine photonic allpass filters.

2.2 Photonic Allpass Filters

In the context of digital filter design, a system is completely described by its transfer function. A transfer function $H(z)$ describes the system behavior under different frequencies, and is characterized by the complex variable $z = re^{j\omega}$, where r is the magnitude of the complex variable and ω denotes the frequency. An ideal allpass filter describes a system that exhibits no attenuation in the magnitude for all frequencies, but alters the phase of the input signal according to a prescribed profile. The following sections provide the details to ideal photonic allpass filters.

2.3 First-Order Photonic Allpass

The allpass filter is highly suitable for physical realization using optical components because it can be easily modularized through a cascade or lattice of first-order sections. A first-order allpass filter can be realized using a variety of nanoscale photonic elements such as Bragg reflectors [17], micro disks [19], and rings [18]. To demonstrate the operation of a photonic allpass filter, let us consider the first-order Fiber Bragg Gratings (FBG) setup and ring resonator based topology shown in Figure 2.1.

The FBG allpass consists of two Bragg mirrors as its primary elements, one with amplitude reflection coefficient ρ , and one with perfect reflection. A Bragg mirror is implemented as a periodic perturbation, for example, a small periodic

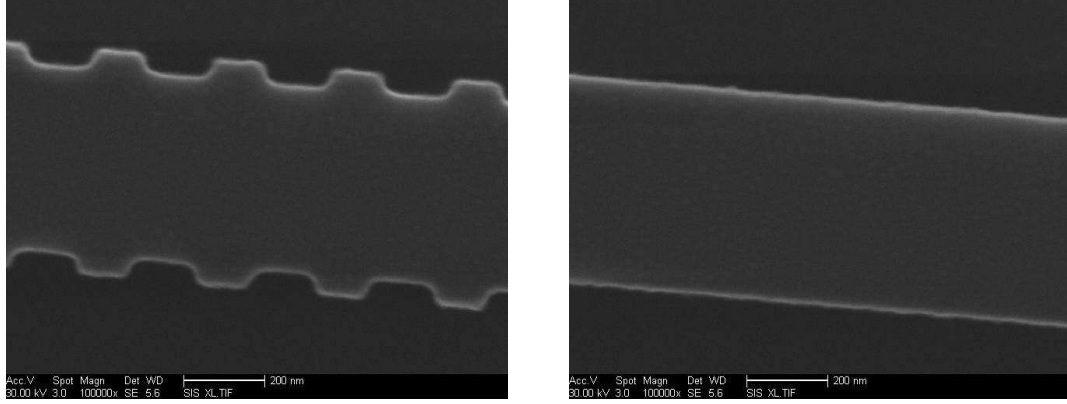


Figure 2.2: Top-down image of a Bragg mirror waveguide (left) created by introducing periodic perturbation. Top-down image of the plain waveguide (right) responsible for creating delay and phase shift.

modulation of waveguide width (Figure 2.2). The periodic structure reflects optical signals whose in-guide wavelength matches the modulation period. In the context of a DTCOP[13] application, the reflection bandwidth is wide enough to include the optical carrier along with any sidebands, but narrow enough to help suppress some of the optical noise at different wavelengths. The phase shift s is created by adjusting the dimensions of the waveguide on the order of the wavelength of the carrier signal. The ring-based photonic allpass is realized by engineering the placements of a plain waveguide and a ring resonator to induce coupling. The coupling coefficient κ and the reflection coefficients ρ satisfy

$$\kappa^2 + \rho^2 = 1 \quad (2.1)$$

In either setup, a fraction of the input signal $x(n)$ is coupled into the waveguide and experiences a delay dependent on the optical path length (L in the FBG and $2\pi R$ in the ring). Under ideal fabrication, the power of the input signal $x(n)$ is entirely contained within the structure, despite the obvious processing from the traversal of the optical signal and coupling. All photonic allpass structures can be represented by the signal flow diagram shown in Figure 2.3. The transfer function

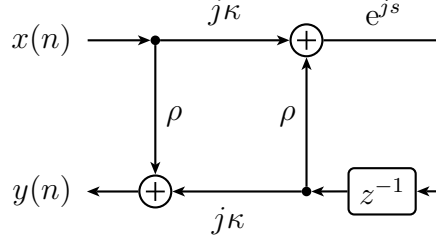


Figure 2.3: Signal flow diagram of the first-order photonic allpass filter. The coupling coefficient κ and the transmission coefficient ρ satisfy $\kappa^2 + \rho^2 = 1$. The phase delay s can be controlled by adjusting the dimensions of the waveguides.

of the first-order photonic allpass is therefore

$$A(z) = -e^{js} \frac{-\rho e^{-js} + z^{-1}}{1 - \rho e^{js} z^{-1}} \quad (2.2)$$

which differs from the canonical allpass form only by a phase shift. Note that while the negative sign does not pose significant difference to the filter response, it needs to be properly captured in higher-order realizations. Under realistic fabrication conditions, the actual response of a first-order photonic allpass filter is

$$A(z) = -e^{j(s+\delta)} \frac{-\rho e^{-j(s+\delta)} + \gamma z^{-1}}{1 - \gamma \rho e^{j(s+\delta)} z^{-1}} \quad (2.3)$$

where γ is the deterministic congregated waveguide power loss and is known *a-priori*, and δ is the random phase error resulting from tooling inaccuracies. Our aim in this dissertation is to compensate for the deterministic error through an intelligent design algorithm, and provide a statistical model for the random error in order to properly analyze yield.

2.4 High complexity allpass filters

In general, an allpass filter describes a system that exhibits no attenuation in the magnitude for all frequencies, but alters the phase of the input signal according to a prescribed profile. The transfer function for a canonical N th-order allpass filter

can be written as [26]

$$\hat{A}(z) = \frac{d_N^* + d_{N-1}^* z^{-1} + \dots + d_1^* z^{-(N-1)} + z^{-N}}{1 + d_1 z^{-1} + \dots + d_{N-1} z^{-(N-1)} + d_N z^{-N}} = z^{-N} \frac{D^*(1/z^*)}{D(z)} \quad (2.4)$$

where d_i 's are the filter coefficients. In this thesis, we will primarily consider the design of allpass-based systems with real coefficients for efficient implementation. The corresponding response of a photonic N th-order allpass is

$$A(z) = (-1)^N e^{j \sum_{i=0}^{N-1} s_i} \hat{A}(z) \quad (2.5)$$

where s_i is the phase shift from each individual section. It is advantageous to ensure $\sum_{i=0}^{N-1} s_i = 2k\pi$ for reducing implementation complexity, which is automatically satisfied when the system consists of only real coefficients. Notice that the system exhibits no processing on the input signal's power because the magnitudes of $D(z)$ and $D^*(1/z^*)$ are the same. The phase response of $A(z)$, however, is controllable and is governed by

$$\Theta_A(\omega) = -N\pi - 2 \arctan \frac{\Im\{D(e^{j\omega})\}}{\Re\{D(e^{j\omega})\}} \quad (2.6)$$

where we assume the overall $A(z)$ to have real coefficient. The phase response of an allpass filter is therefore dependent on the placements of the roots of $D(z)$. Note that for every root p_i of $D(z)$, there exists a zero z_i from $D^*(1/z^*)$ at $z_i = \frac{1}{p_i^*}$. Since precise placements of the roots of $D(z)$ depend on the values of the filter coefficients of $D(z)$, d_i 's are the ultimate control variables that need to be optimized for a given prescribed requirement. The allpass filter is highly suitable for physical realization using optical components because it can be easily modularized through a cascade or lattice of first-order sections.

2.4.1 Lattice Form

The lattice realization for the N th-order photonic allpass is shown in Figure 2.4. Similar to the first-order allpass, the structures can be represented by the signal flow graph shown in Figure 2.5. From the graph, the transfer function of

the overall system can be recursively constructed

$$A_{\text{lat}}^{(i)}(z) = S_{i-1}(zS_i^{-1}(z)) \quad (2.7)$$

with $S_i(z) = -e^{js_i} \frac{\rho_i e^{-js_i} + z^{-1}}{1 - \rho_i e^{js_i} z^{-1}}$ matching the response of a first-order photonic all-pass. Note that the negative sign from the first-order system will result in a scalar multiplier of $(-1)^N$ in the overall transfer function. The sign can be easily compensated for by minor adjustments in the filter design requirements. The phase terms e^{js_i} disappear from the overall transfer function under the real coefficient assumption. The overall transfer function therefore closely resembles the canonical form of an N th-order lattice allpass filter. The coefficients for the individual sections can be found from the overall N th-order transfer function by a modified Levinson Durbin recursion. The modification is needed to account for the sign and phase differences in the first-order sections, which can be captured by imposing the terms on the reflection coefficients in the iterations of the algorithm. Given the N th-order photonic allpass transfer function $A(z)$, Algorithm 1 shows the derivation for the reflection coefficients $\boldsymbol{\rho}$. The approach can be easily implemented by multiplying the output of the `MATLAB` function `tf2latc` with $(-1)^i$. The coupling coefficients can then be obtained by

$$\kappa = \sqrt{1 - \rho^2} \quad (2.8)$$

The coupling coefficients can then be used to parametrize the individual rings in an N th-order lattice allpass filter.

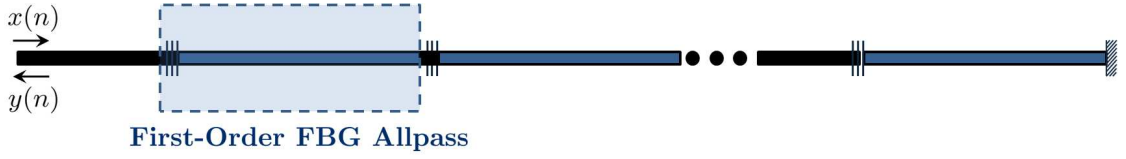


Figure 2.4: Lattice realization of an N th-order FBG allpass filter. Notice that each Bragg mirror interface serves as a four port interface.

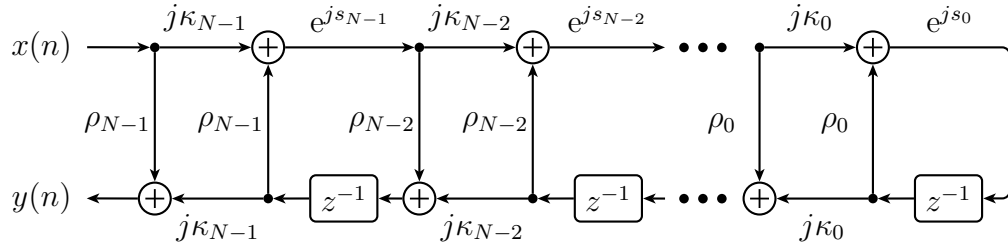


Figure 2.5: N th-order normalized latticed allpass filter signal flow.

Algorithm 1 Transmission coefficients from allpass filter taps

Require: N th-order filter coefficients $\mathbf{d}$.

Initialize $\mathbf{d}^{(N)} = \frac{\mathbf{d}}{d_0}$, $\boldsymbol{\rho} = \mathbf{0}_{N \times 1}$

for $i = N \rightarrow 2$ **do**

$$\rho_i = (-1)^i d_i^{(i)}$$

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & (-1)^i \rho_i & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & (-1)^i \rho_i & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{d}^{(i-1)} \\ - \\ \vdots \\ \vdots \end{bmatrix} = (\mathbf{I} + \mathbf{P})^{-1} \mathbf{d}^{(i)}$$

$i \leftarrow i - 1$

end for

return $\boldsymbol{\rho}$

2.4.2 Cascade Form

The ideal N th-order allpass filter transfer function can be partitioned into first-order sections through polynomial factorization to derive the cascade form.

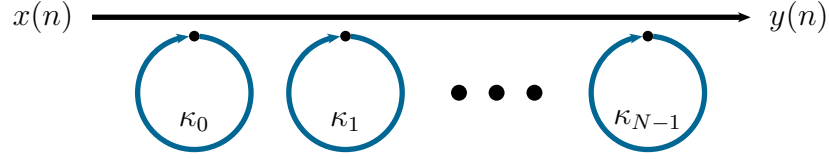


Figure 2.6: N th-order cascade ring allpass. Each coupling coefficient directly relates a pole and zero in the overall allpass response.

For photonic allpass, the cascade form can be expressed as

$$A_{\text{cas}}(z) = \prod_{i=1}^N -e^{js_i} \frac{-\rho_i e^{-js_i} + z^{-1}}{1 - \rho_i e^{js_i} z^{-1}} \quad (2.9)$$

Note again that the negative sign in the first-order section will result in a $(-1)^N$ multiplier in the N th-order transfer function, and the phase term e^{js_i} will cause an overall phase offset. While the concept is simple from a signal processing perspective, the realization of the structure is difficult for the FBG topology. Since the input and output are collected at the same port of the device, the first-order FBG allpass cannot be easily extended to a high-order cascade setup. The equivalent realization using rings is shown in Figure 2.6. Notice that in this setup, the poles and zeros of the overall system directly correlate with the coefficients of the individual sections.

2.5 Application in Frequency Selective Filters

The allpass filter processes all frequency components without any attenuation yet provides a change in the phase response. Such attributes of the allpass structure has been harnessed for designing filters of arbitrary frequency response. A frequency selective IIR filter $H(z)$ can be derived from the sum of two allpass filters [34]:

$$H(z) = \frac{1}{2} \left(\hat{A}_0(z) + \hat{A}_1(z) \right) \quad (2.10)$$

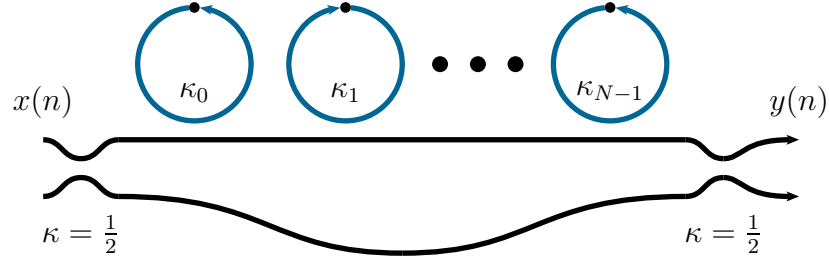


Figure 2.7: N th-order frequency selective filter built using allpass substructures. The delay arm is of length $2\pi R \cdot (N - 1)$, where R is the radius of the ring. Note that the example shows an N th-order allpass under the cascade form.

The two allpass filters $\hat{A}_0(z)$ and $\hat{A}_1(z)$ operate by phase matching and phase misalignment to produce changes in the magnitude response for the overall system [34]. Note that in the optical domain, all filters (lowpass, highpass, bandpass) are periodic when profiled against wavelength. The design of any photonic filter can therefore be reduced to the design of a lowpass filter centered at the baseband [73]. A lowpass filter $H(z)$ can be derived by requiring that $\hat{A}_0(z)$ and $\hat{A}_1(z)$ have the same phase response in the passband and a phase difference of π in the stopband. Note that a signal processing lowpass structure is analogous to a photonic bandpass filter when profiled against wavelength.

We can further simplify the form by empirically choosing one of the allpass sections as a delay:

$$H(z) = \frac{1}{2} \left(z^{-(N-1)} + \hat{A}(z) \right) \quad (2.11)$$

where N is the order of the allpass filter $\hat{A}(z)$. This form simplifies the design procedure by reducing the number of design parameters, and reduces the complexity of the analysis. In terms of photonic fabrication, (2.11) presents an ideal scenario for implementing high complexity systems because the delay element $z^{-(N-1)}$ can be easily realized using a plain waveguide. Therefore a high complexity frequency selective system simply requires the design and fabrication of the photonic allpass filter. The frequency selective filter $H(z)$ can then be readily realized as shown in Figure 2.7. The reduced form is ideal for photonic realization because the fabrication complexity lays solely on the allpass system. In this setup, a lowpass filter

$H(z)$ can be achieved by requiring $\hat{A}(z)$ to have the following phase behavior

$$\Theta_A(\omega) = \begin{cases} -(N-1)\omega & 0 \leq \omega \leq \omega_p \\ -(N-1)\omega - \pi & \omega_s \leq \omega \leq \pi \end{cases} \quad (2.12)$$

where ω_p and ω_s denote the passband and stopband frequencies, respectively. It can be readily seen from the prescribed phase response that this system will be also able to guarantee approximately constant group delay in the passband, which is highly desirable when implemented in optical communication systems. As readily seen by the construction of the lowpass form, allpass filters can be used to design systems with arbitrary passband and stopband requirements by phase manipulation. The versatility of the allpass filter makes it suitable for general signal processing applications in photonic communication systems.

Note that for realization using actual photonic allpass filters, the extra $(-1)^N$ term needs to be addressed. The sign difference can be captured into the phase requirement $\Theta_A(\omega)$ as an additional π shift, or into the formulation for $H(z)$ as

$$H(z) = \frac{1}{2} (z^{-(N-1)} + (-1)^N A(z)) \quad (2.13)$$

With the introduction of $(-1)^N$, we will ensure that the delay element is always in phase with the fabricated allpass rather than containing a π phase shift. Note that the sign element does not significantly affect the design or the analysis of allpass-based systems. We will therefore use $A(z)$ to express the response of both the canonical and photonic allpass response for notation simplicity for the rest of this dissertation. Although photonic realization of the lowpass filter can be easily accomplished, the performance is corrupted by the waveguide loss. One of the goals of this research is to present a design algorithm that provides filter coefficients for the allpass filter $A(z)$ such that the performances of $H(z)$ is optimized under power loss. In this dissertation, we directly consider the reduced form of (2.11) in regards to lowpass filter using allpass substructures.

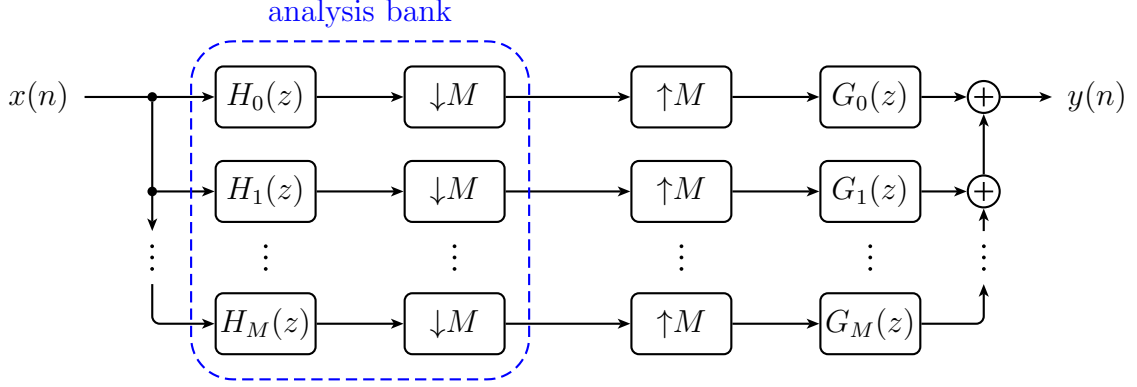


Figure 2.8: Signal flow graph for a M-channel filter bank. The sub-band signals can then be efficiently processed and recombined.

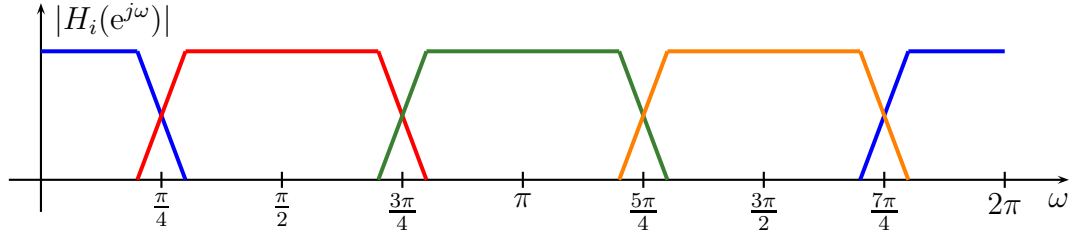


Figure 2.9: Ideal responses for the analysis filter in a 4-channel DFT filter bank.

2.6 Application in DFT Filter Banks

A bank of filters is an effective, and often necessary, approach to partitioning a wideband signal into multiple sub-bands for processing. Figure 2.8 shows the general multirate signal processing setup. The analysis bank is used to decompose the input signal into M frequency sub-bands with equal bandwidth, which can then be efficiently processed. The ideal responses for a 4-channel filter bank is shown in Figure 2.9. Note that while the filters can be derived individually knowing the desired responses, fidelity among the resulting designs cannot be guaranteed [57].

A more optimal approach is therefore to design the M filters as a whole. To facilitate the application of filter banks in photonic signal processors, we consider the allpass-based DFT filter bank [54] shown in Figure 2.10. One key aspect in this structure is the ability to switch the filtering and downsample operations. In signal processing, such feature is extremely beneficial because it implies that a wideband

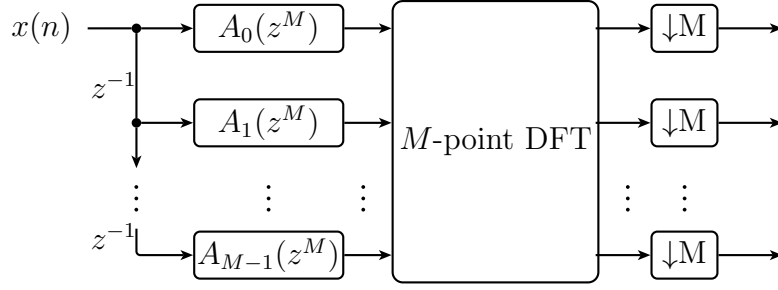


Figure 2.10: Four-channel allpass-based filter bank signal flow diagram. The downsample and filtering operations in this structure are interchangeable, allowing for efficiently implementation and processing.

signal can effectively be processed at sub-Nyquist sampling rates. This powerful aspect of the filter bank is explored in depth and proven to be crucial in analog-to-digital conversion applications in both digital and optical domains [23, 45]. In optical signals, the essence of downsampling can be observed in the process of time stretching. An optical signal slowed by a stretch factor of M has its bandwidth reduced by a factor of M , which indicates that the signal can be sampled by $\frac{f_s}{M}$, where f_s is the original Nyquist sampling rate. The operation directly maps to downsampling since the purpose of a downsample-by- M operation is exactly to reduce the sampling rate to $\frac{f_s}{M}$.

While the structure does not explicitly process the input signal through independent bandpass filters, the outputs are exactly the sub-band signals of $x(n)$. The implicit analysis filters of this structure can be expressed as

$$H_i(z) = \frac{1}{M} \sum_{k=0}^{M-1} e^{-j2\pi \frac{i \cdot k}{M}} A_k(z^M) z^{-k} \quad (2.14)$$

It is often desirable to ensure that $A_0(z^M)$ is a simple delay because such formulation would provide constant group delay in the passband regions for all bands.

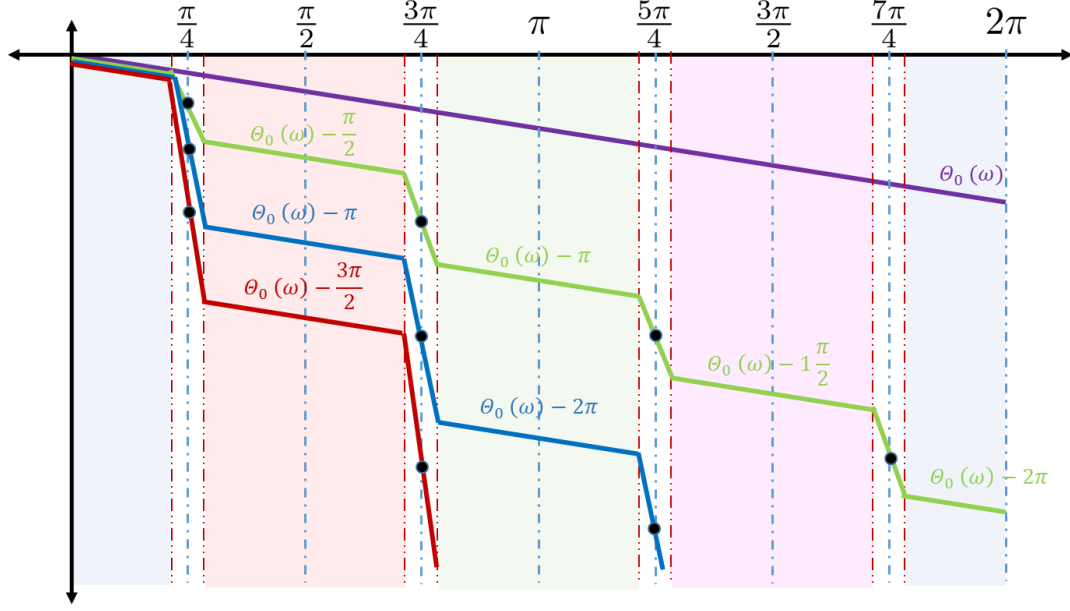


Figure 2.11: Phase requirements for $A_k(z^M)z^{-k}$ in a $M = 4$ -channel DFT filter bank. Note that $A_0(z^M)$ is assumed to be a delay.

For example, the analysis filters for a 4-channel filter bank are

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ H_3(z) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} z^{-4N} \\ A_1(z^4)z^{-1} \\ A_2(z^4)z^{-2} \\ A_3(z^4)z^{-3} \end{bmatrix} \quad (2.15)$$

where N is the order of the allpass filters after downsample.

Since the allpass filters have no controllable behavior in the magnitude, the system operates based on the principle of phase matching. Given the phase profile $\Theta_0(M\omega)$ of $A_0(z^M)$, a passband in $H_0(z)$ can be created by requiring the rest of the allpasses to have the same phase in $0 \leq \omega \leq \omega_p$, where ω_p is the passband frequency. A stopband can be created in a similar fashion by requiring a phase offset of π . An example phase requirement for a 4-channel filter bank is shown in Figure 2.11. Notice that there is a required phase increment in each of the allpass filters at $\frac{(2k+1)\pi}{4}$. In general, the prescribed phase $\Theta_k(\omega)$ for each $A_k(z^M)z^{-k}$ pair

in a M -channel filter bank is

$$\Theta_k(\omega) = -MN\omega - \frac{2\pi(l+1)k}{M}, \quad \omega_{p,l} \leq \omega \leq \omega_{s,l} \quad (2.16)$$

where the start and end points of the l -th band are

$$\omega_{p,l} = \begin{cases} 0 & l = 0 \\ \frac{2(l-1)\pi}{M} + \omega_s & l = 1, 2, \dots, M \end{cases} \quad (2.17)$$

$$\omega_{s,l} = \begin{cases} \frac{2l\pi}{M} + \omega_p & l = 0, 1, \dots, M-1 \\ 2\pi & l = M \end{cases} \quad (2.18)$$

The passband and stopband frequencies also have to satisfy the M -band condition

$$\omega_p + \omega_s = \frac{2\pi}{M} \quad (2.19)$$

These stringent requirements on the phase profile are in place to guarantee consistent performance across all M -channels.

2.7 Summary

The photonic allpass filter is versatile and can be easily utilized in a wide range of applications. We will examine the design of allpass-based signal processors that can tolerate for errors that are known *a-priori*, and model the performance of the structures based on statistical measures post fabrication. Although the actual response of a photonic allpass differs from the canonical transfer function by a sign change, we will ignore this effect throughout the rest of this dissertation. The effect does not significantly affect either the design or the analysis because it can be easily absorbed into the prescribed requirements. For example, we can simply introduce an additional $N\pi$ into the prescribed phase requirements when the allpasses are to be used for phase equalization. We will further assume real coefficients for the allpass filters to reduce design and realization complexity.

2.8 Acknowledgments

This chapter, in part, is a reprint of the published journal papers in OSA Optics Express, December 2013, IEEE Journal on Lightwave Technology, March 2014, the published conference paper in IEEE International Conference on Acoustics, Speech, and Signal Processing 2011, IEEE Global Communications Conference 2011, European Signal Processing Conference 2012, and a submitted paper to IEEE Journal on Lightwave Technology.

Chapter 3

Waveguide Loss Analysis

Although waveguide loss is a well understood effect in the photonics research community, the details of its significance from an allpass design perspective is absent. In the signal processing context, the effect of loss can be modeled as a variation on the unit delay z^{-1} . The waveguide loss corrupts the ideal allpass response $A(z)$ by $A(\gamma^{-1}z)$, where γ is the grouped loss effect [21]. Note that the power loss from absorption and scattering can be grouped and expressed as a power attenuation coefficient γ , and ultimately results in a degradation on the optical signal captured by $\gamma = e^{-\alpha L/2}$, where L is the length of the waveguide. This section provides the physical origins of the loss γ and an extensive study of its effect on the filter structure.

3.1 Physical Origins

There are two mechanisms responsible for loss within an optical waveguide: scattering loss and material absorption. Scattering loss is a consequence of artifacts like roughness in the waveguide sidewalls [60] or excessive bending of the waveguide [74]. It is therefore amenable to reduction through improved fabrication precision. In contrast, absorption is an intrinsic material property and is unavoidable without abandoning the material system. In the context of digital signal processing, all loss has the same effect regardless of provenance. As such, it is most pertinent to consider the aggregate absorption coefficient γ of the waveguide. This is also

convenient in the sense that loss characterization methods generally determine the aggregate absorption coefficient [75]. While state-of-the-art fabrication techniques have reported waveguides with minimal loss [76], the effect is still prominent from a signal processing perspective. For example, a ring resonator that exhibits a loss of 2.6% per 90 degree bend will still result in $\gamma = 0.9$ and significantly affect the filter behavior.

A generalized allpass design algorithm that incorporates the maximum allowable waveguide loss is extremely valuable. It will allow relatively lossy waveguides that are inexpensive and simple to fabricate to replace low loss waveguides that are costly and complicated to fabricate [40, 77]. The eased fabrication constraints will also increase the compatibility of photonics with other platforms, such as CMOS. Finally, it will allow the realization of new photonic filter capabilities.

3.2 Signal Processing Analysis

Since the power loss is the result of an optical signal traversing through a non-ideal waveguide, it can be considered to be coupled with the unit delay element z^{-1} [21]. The effect on an ideal first-order allpass transfer function with real coefficients from γ is

$$A_{err}(z) = \frac{d_1 + \gamma z^{-1}}{1 + \gamma d_1 z^{-1}} \quad (3.1)$$

where d_1 is the first-order allpass filter coefficient and $0 \leq \gamma \leq 1$ captures the waveguide power loss. Note that the power loss results from imperfections in fabricating a waveguide, the value of γ may vary depending on the dimensions of the unit. The effect can, however, be generalized to N th-order systems regardless of the form used for realization. Because the dimensions of any individual section within an N th-order allpass needs to correlate with the sampling rate of the system, all individual waveguide segments within a given system can be assumed to be identical. The waveguide loss can therefore be considered as a systematic error independent of the realization and exists in each first-order section of a higher-order allpass filter. The actual N th-order allpass transfer function with waveguide

power loss is then

$$A_{err}(z) = \frac{d_N + \gamma d_{N-1}z^{-1} + \dots + \gamma^N z^{-N}}{1 + \gamma d_1 z^{-1} + \dots + \gamma^N d_N z^{-N}} = A(\gamma^{-1}z) \quad (3.2)$$

Note that we are assuming the loss effect is uniform throughout the photonic filter. In general, the waveguide loss is not only a stochastic parameter dependent on the degree of roughness and inaccuracies in the waveguide dimensions, but also a function of the wavelength [60, 58]. However, we can expect the deviation of the loss to be small for a given application since all of the individual sections within the device must be similar in dimensions. Furthermore, a stable, nominal value of γ that is known *a-priori* is expected under stable fabrication conditions [35].

Notice that by changing the argument of $A(z)$ from z to $\gamma^{-1}z$, we are effectively scaling the poles and zeroes radii by γ as shown in Figure 3.1. The root scaling effect is particularly alarming in an allpass filter, because a precise relation between each pole and zero pair is needed to maintain a constant magnitude response. Figure 3.2 shows the effect of $\gamma = 0.9$ on the magnitude and phase for a 6th-order allpass. As shown in the figure, because the relationship of having zero pole pairs at z_i and $\frac{1}{z_i^*}$ is violated, the magnitude response can no longer be maintained at unity for all frequencies. The root scaling results in a notch effect (note that the zeroes move closer to the unit circle in Figure 3.1) at every pole and zero pair location, causing attenuations in the magnitude response. Also notice that the loss causes significant phase distortion near the transition band edges.

The effect of waveguide loss propagates to the overall system when used to construct frequency selective filters. Figure 3.3 shows the result from using the same lossy allpass filter in a lowpass architecture according to

$$H(z) = \frac{1}{2} (z^{-(N-1)} + A_{err}(z)) \quad (3.3)$$

where $N = 6$ and $A_{err}(z)$ is the N th-order γ corrupted allpass filter. Note that while the overall response still resembles a lowpass filter, the attenuation in the passband and stopband are significantly distorted and the transition band is widened. Although the attenuation offsets in the passband and stopband cannot

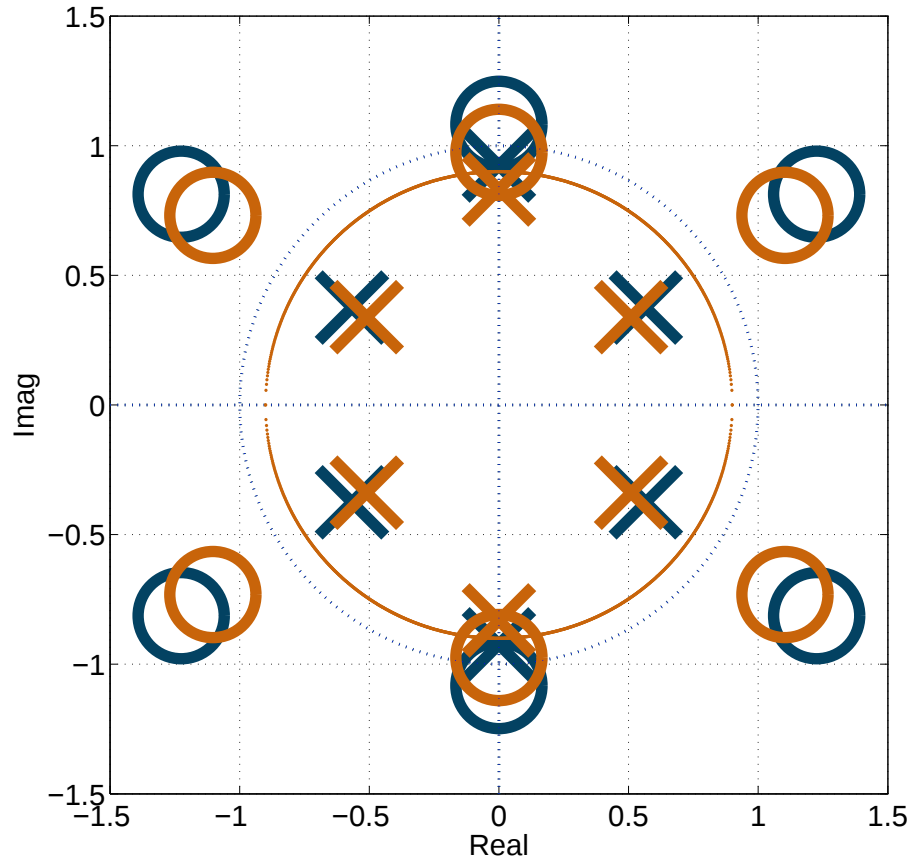


Figure 3.1: Effect of waveguide loss on a 6th-order allpass with $\gamma = 0.9$. The ideal poles and zeroes placements are shown in blue.

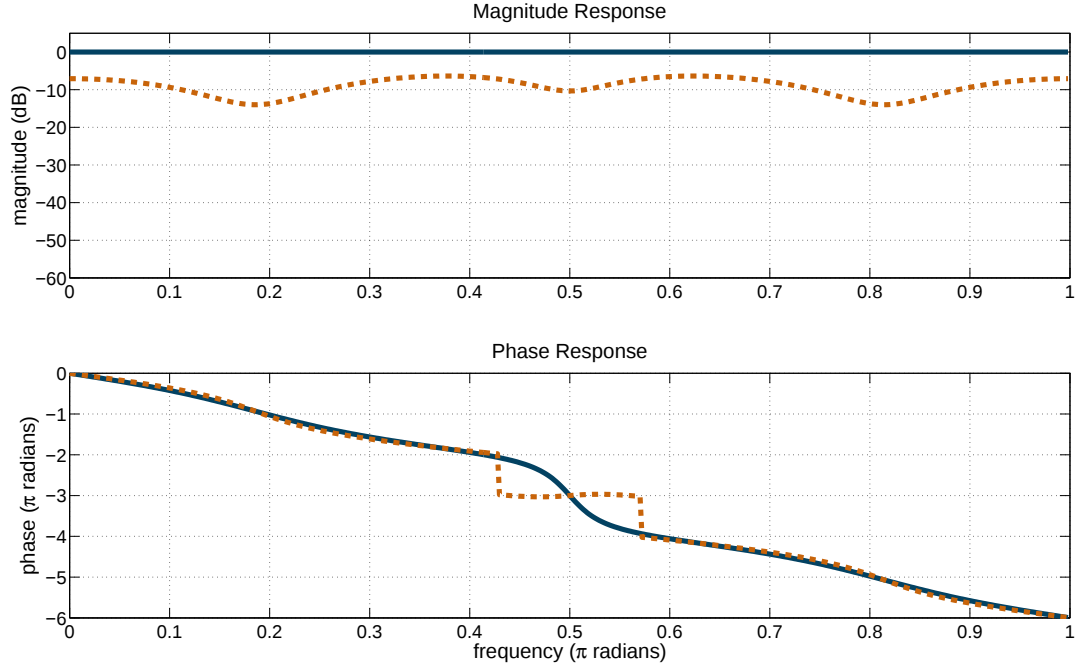


Figure 3.2: Effect of $\gamma = 0.9$ on the magnitude and phase responses of a 6th-order allpass. The solid lines are the ideal responses.

be adjusted due to losses in signal power, the overall performance can be improved by considering the effect of γ directly in the problem formulation. As a result, the ripples and widened transition band that result from large phase errors can be reduced.

The waveguide loss corrupts the overall frequency response, yet the requirement for an allpass filter can only be specified in the phase. An immediate afterthought is then to reformulate the prescribed phase response $\Theta_{pre}(\omega)$ to capture the distortion effect, which would allow existing allpass design methods to be still valid. If we can derive a direct relationship between the corrupted phase response $\Theta_{err}(\omega)$ and the phase response of the ideal allpass $\Theta(\omega)$, then the loss effect can be inferred to be independent of the uncorrupted response. However, recall $A_{err}(z) = A(\gamma^{-1}z)$, which implies that in the time domain we have

$$a_{err}(n) = \gamma^n a(n) \quad (3.4)$$

where $a_{err}(n)$ and $a(n)$ are the time domain responses for the corrupted allpass

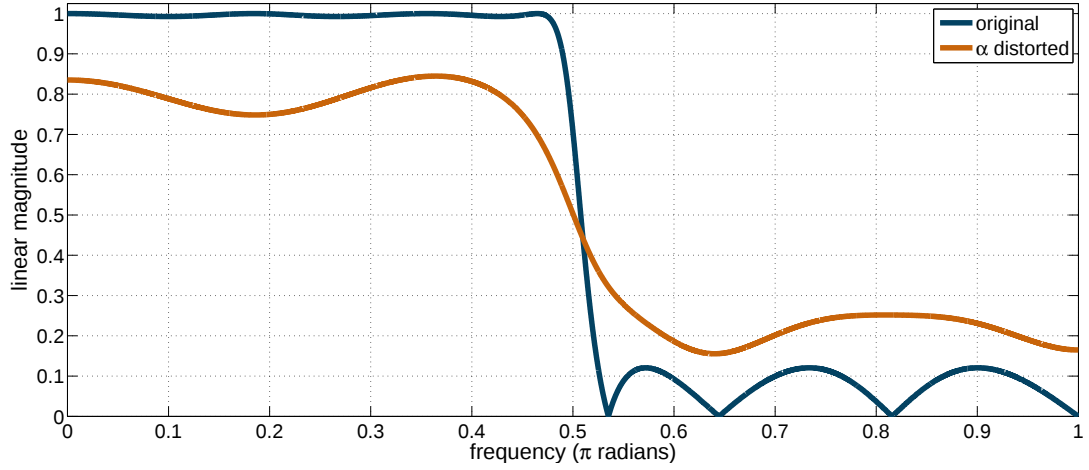


Figure 3.3: Effect of $\gamma = 0.9$ on the magnitude response of a lowpass filter using a distorted 6th-order allpass filter, the second allpass filter is a simple delay z^{-5} .

filter and the ideal system, respectively. If we let $\Lambda(z)$ be the z -transform of γ^n , then we can also relate $A_{err}(z)$ and $A(z)$ by

$$A_{err}(z) = \Lambda(z) * A(z) \quad (3.5)$$

and the phase response is

$$\Theta_{err}(\omega) = \arctan \frac{\Lambda_R(\omega) * A_I(\omega) + \Lambda_I(\omega) * A_R(\omega)}{\Lambda_R(\omega) * A_R(\omega) - \Lambda_I(\omega) * A_I(\omega)} \quad (3.6)$$

where $\Lambda_R(\omega)$, $\Lambda_I(\omega)$ are the real and imaginary parts of $\Lambda(e^{j\omega})$ and $A_R(\omega)$ and $A_I(\omega)$ are the real and imaginary parts of $A(e^{j\omega})$. The distortion function cannot be decoupled from the uncorrupted phase response, which implies that the severity of the distortion is dependent of the ideal allpass response under the given requirements. This is an expected conclusion because the root scaling would affect the roots that are closer to the unit circle than the pairs that are further away.

3.3 Summary

Since we cannot isolate the effect of the waveguide loss, we must capture the effect into the filter design problem formulation. The rest of this dissertation details on the formulation and the solution to designing the allpass coefficients for phase equalizers, bandpass filters, and filter banks that can tolerate the waveguide loss. Note that although allpass filter specification are only in the phase, the magnitude response is evidently also affected. We will carefully consider the effect in both the allpass magnitude and phase distortions and present algorithms that can not only mitigate, but also harness the effect of waveguide power loss.

3.4 Acknowledgments

This chapter, in full, is a reprint of the published journal papers in OSA Optics Express, December 2013, IEEE Journal on Lightwave Technology, March 2014.

Chapter 4

Loss Compensating Phase Equalizers

To create a class of filter design algorithms with waveguide loss compensation, we begin by setting up the optimization problem for an allpass phase compensator under loss. Note that while a distortion in the magnitude also exists, it can be corrected by a follow-up finite impulse response (FIR) linear-phase filter in phase equalization applications. Such structure can be easily fabricated using a tap delay line architecture with negligible loss [21]. Furthermore FIR linear-phase filters presents no additional processing on the phase, thus preserving the output of the allpass filter. We will therefore focus our design efforts on providing a desirable phase behavior from the waveguide loss corrupted allpass filter, and assume that any magnitude distortion can be compensated by additional post filtering operations.

Specifically, we are interested in finding the coefficient vector $\mathbf{d}$ that minimizes the error between the actual response under waveguide loss and a prescribed requirement. The connection between photonic parameters of the individual first-order sections and the design variable $\mathbf{d}$ can be established through a modified Levinson Durbin recursion in the lattice setup, or polynomial factorization in the cascade. In this dissertation, we are primarily interested in allpass filters with real coefficients. Given these considerations, we present a minimax allpass filter design algorithm that is able to compensate for the delay element variation in

terms of phase performance. Without any manipulation, the design problem is mathematically challenging because it involves an optimization problem that is non-convex and NP hard. Instead, we solve a relaxed problem by employing an iterative approach in conjunction with a Branch and Bound global optimization technique. The proposed algorithm is expected to improve the performance and increase the utilization of allpass filters for optical signal phase based applications such as dispersion compensation and group delay equalization.

4.1 Problem Setup

To formulate the filter design optimization problem under waveguide power loss, let us first examine the transfer function incorporating the effect of α . Since the effect can be captured as $A_{err}(z) = A(\gamma^{-1}z)$, we can write the transfer function as

$$A_{err}(z) = \gamma^N z^{-N} \frac{D(\gamma z^{-1})}{D(\gamma^{-1}z)} \quad (4.1)$$

where $D(z) = 1 + d_1 z^{-1} + \dots + d_{N-1} z^{-(N-1)} + d_N z^{-N}$. Even under waveguide loss, the principle of an allpass filter still lies in its phase behavior. Now the phase response of this filter is

$$\Theta_{err}(\omega) = -N\omega + \arctan \frac{\sum_{k=0}^N d_k \gamma^{-k} \sin k\omega}{\sum_{k=0}^N d_k \gamma^{-k} \cos k\omega} + \arctan \frac{\sum_{k=0}^N d_k \gamma^k \sin k\omega}{\sum_{k=0}^N d_k \gamma^k \cos k\omega} \quad (4.2)$$

or in vector notations

$$\Theta_{err}(\omega) = -N\omega + \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{s}(\omega)}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{c}(\omega)} + \arctan \frac{\mathbf{d}^\top \mathbf{A} \mathbf{s}(\omega)}{\mathbf{d}^\top \mathbf{A} \mathbf{c}(\omega)} \quad (4.3)$$

where

$$\begin{aligned}
\mathbf{d} &= \begin{bmatrix} 1 & d_1 & \cdots & d_{N-1} & d_N \end{bmatrix}^T \\
\mathbf{s}(\omega) &= \begin{bmatrix} 0 & \sin \omega & \cdots & \sin(N-1)\omega & \sin N\omega \end{bmatrix}^T \\
\mathbf{c}(\omega) &= \begin{bmatrix} 1 & \cos \omega & \cdots & \cos(N-1)\omega & \cos N\omega \end{bmatrix}^T \\
\mathbf{A} &= \text{diag} \left\{ 1 \quad \gamma \quad \cdots \quad \gamma^{N-1} \quad \gamma^N \right\}
\end{aligned} \tag{4.4}$$

Our goal is then to have $\Theta_{err}(\omega)$ best match a prescribed phase response $\Theta_{pre}(\omega)$. In other words, we would like to minimize

$$\Delta\Theta(\omega) = \Theta_{pre}(\omega) - \Theta_{err}(\omega) \tag{4.5}$$

For notation simplicity, let us define $\beta(\omega) \triangleq \Theta_{pre}(\omega) + N\omega$, then we have

$$\Delta\Theta(\omega) = \beta(\omega) - \arctan \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{s}(\omega)}{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{c}(\omega)} - \arctan \frac{\mathbf{d}^T \mathbf{A} \mathbf{s}(\omega)}{\mathbf{d}^T \mathbf{A} \mathbf{c}(\omega)} \tag{4.6}$$

Notice that for the uncorrupted allpass design, the arguments of the first and second arctan terms are the same. In such case we can reformulate the cost function into a generalized Rayleigh Quotient, which can be solved using existing algorithms [63, 65]. In the corrupted case, however, the symmetry in the numerator and denominator is lost, and we have to utilize the following property to combine the arctan terms

$$\arctan u + \arctan v = \arctan \frac{u + v}{1 - uv} \tag{4.7}$$

With trigonometric manipulation, we can arrive at

$$\Delta\Theta(\omega) = \arctan \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}} \tag{4.8}$$

where the (n, m) -th element in matrices $\mathbf{P}(\omega)$ and $\mathbf{Q}(\omega)$ are

$$\mathbf{P}_{n,m}(\omega) = \sin(\beta(\omega) - n\omega - m\omega) \tag{4.9}$$

$$\mathbf{Q}_{n,m}(\omega) = \cos(\beta(\omega) - n\omega - m\omega) \tag{4.10}$$

We can further simplify by first considering Taylor series expansion on $\arctan$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad (4.11)$$

and discard all the non-linear terms for small approximation errors

$$\Delta\Theta(\omega) \approx \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}} \quad (4.12)$$

Our goal from here is to minimize $\Delta\Theta(\omega)$ in either the least squares sense or minimax. For least squares, our cost function is

$$J_{LS} = \int_R W(\omega) \left| \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}} \right|^2 d\omega \quad (4.13)$$

where $W(\omega)$ is a weight function and R denotes the frequency ranges of interest. Recall that our optimization variable is $\mathbf{d}$, and if we take the square of the quantity, we would have to optimize over a cost function that contains a fourth-order polynomial in $\mathbf{d}$. Together with the fractional form, the cost function would be highly non-convex. Even if we were to neglect the squaring operation, the integration and absolute value operation remain since $\mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A}$ and $\mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A}$ are not symmetric and not semidefinite, again the problem is non convex and NP hard. Such limitations prevent us from using existing methods in which the matrices are symmetric and positive semidefinite. All of these factors make the least squares approach particularly difficult to pursue, and instead we will consider the $\mathcal{L}_\infty$ -norm cost function.

4.2 Approach

We can approach the loss compensating allpass design problem in the minimax sense by solving the following optimization problem

$$\underset{\mathbf{d}}{\text{minimize}} \quad \max_{\omega} W(\omega) \left| \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}} \right| \quad (4.14)$$

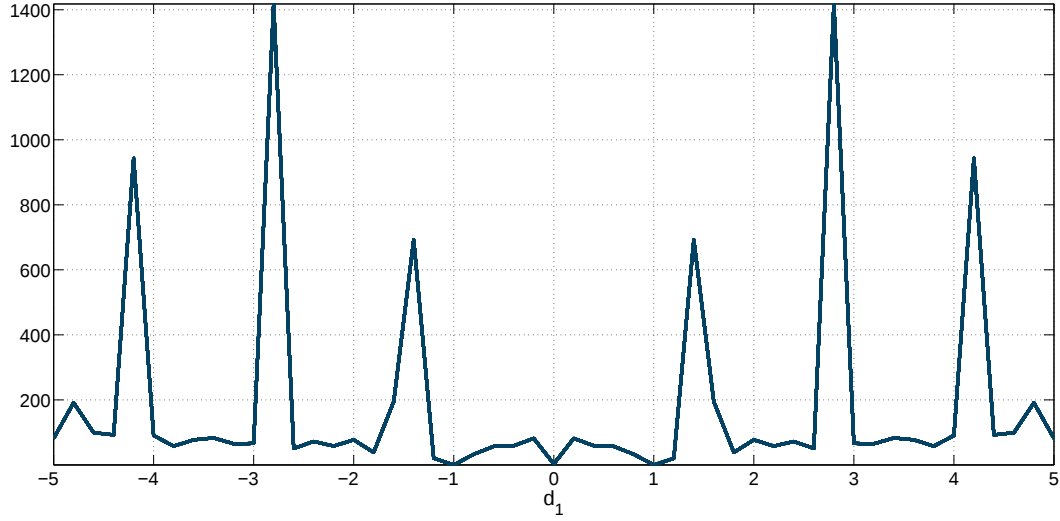


Figure 4.1: Cost function for $N = 1$ and a waveguide loss of $\gamma = 0.9$. The solution vector is again normalized by d_0 .

Notice that this optimization is challenging due to both the fraction form and non semidefinite matrices $\mathbf{A}^{-1}\mathbf{P}(\omega)\mathbf{A}$ and $\mathbf{A}^{-1}\mathbf{Q}(\omega)\mathbf{A}$. To begin our analysis, let us examine the cost function for $N = 1$ case. Figure 4.1 shows the cost function for

$$\Theta_{pre}(\omega) = \begin{cases} -(N-1)\omega & 0 \leq \omega \leq 0.4\pi \\ -(N-1)\omega - \pi & 0.6\pi \leq \omega \leq \pi \end{cases} \quad (4.15)$$

First notice that aside from the trivial solution of $d_1 = 0$, $d_0 = d_1 = 1$ also appears to be an optimum. However, this is another result that we would like to avoid. To generalize, any solution for symmetric $\mathbf{d}$ needs to be discarded because it would result in simple cancellation in the overall transfer function $A_{err}(z)$.

We can now consider the cost function for higher dimensions. Figure 4.2 shows the cost function for $N = 2$ with the $\Theta_{pre}(\omega)$ in (4.15). Notice that the graph is highly irregular with closely packed local minimums. The graph implies that not only would the standard convex approach fail, even global approaches are not applicable because they can easily get trapped in a local minimum. However, if the denominator is fixed, then we can simply consider the following optimization

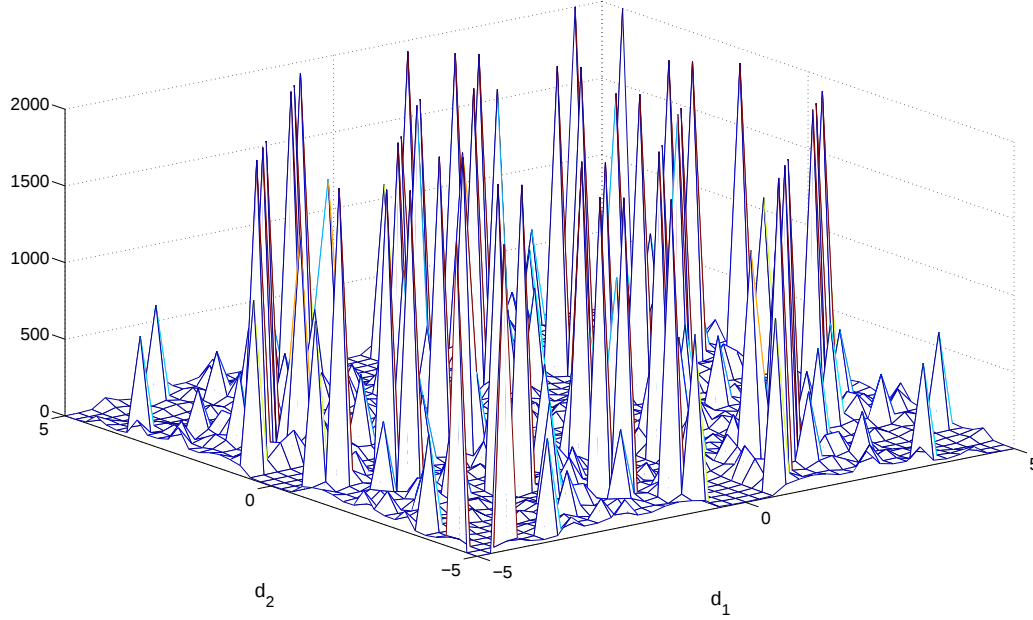


Figure 4.2: Cost function for $N = 2$ and a waveguide loss of $\gamma = 0.9$. Recall that in this case $d_0 = 1$.

with quadratic constraints

$$\begin{aligned} & \underset{\mathbf{d}}{\text{minimize}} && \max_{\omega} W(\omega) |\mathbf{d}^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}| \\ & \text{subject to} && \mathbf{d}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d} = 1 \end{aligned} \tag{4.16}$$

Figure 4.3 shows the cost function for the $N = 2$ case with the same form of $\Theta_{pre}(\omega)$. Although the objective is still not convex, it is much smoother and there are relaxation techniques and global optimization methods that can be applied.

While introducing a constraint on the denominator reduces the irregularities in the original cost function, (4.16) remains a nonconvex quadratic programming problem and NP hard. One approach is to lift the problem into a matrix optimization through semidefinite relaxation [78]. Notice that the quantity $\mathbf{d}^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}$ is equivalent to $\text{trace} \{ \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d} \mathbf{d}^T \}$. We can therefore reformulate (4.16) into

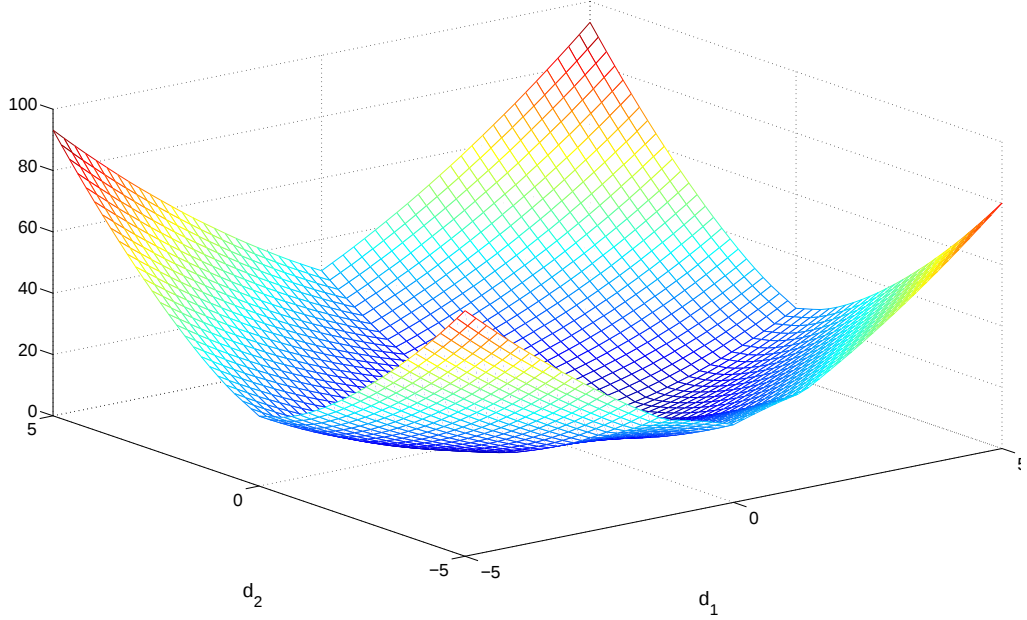


Figure 4.3: Relaxed cost function for $N = 2$ and a waveguide loss of $\gamma = 0.9$. Note that $\mathbf{d}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d} = 1$ in this case.

a matrix optimization problem

$$\begin{aligned} & \underset{\mathbf{D}}{\text{minimize}} && \max_{\omega} W(\omega) | \text{trace} \{ \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{D} \} | \\ & \text{subject to} && \mathbf{D} = \text{rank } 1 \end{aligned} \quad (4.17)$$

The problem becomes a linear programming in $\mathbf{D}$ once the rank constraint is dropped. Given the optimal $\mathbf{D}$, we can obtain the coefficient vector $\mathbf{d}$ by simply preserving the eigenvector corresponding to the largest eigenvalue, which would give the best rank-1 approximation in the least squares sense. This relaxation, however, suffers from the rank approximation.

A more direct approach is therefore to utilize a global optimization technique such as branch and bound [79]. Figure 4.4 demonstrates the concept of branch and bound at a given iteration. In branch and bound, the original feasible set to the optimization problem is branched into smaller subsets. Within each subset, a lower bound is computed based on a relaxed problem. A branch is discarded when its minimum is greater than the existing upper bound computed based on

known cost values. This process continues until a termination criterion is met, or when a single solution is found. In application for nonlinear programming, the original optimization problem is partitioned into subsets, and a convex envelope is found within each subset. The algorithm continues to branch and terminates when the upper and lower bounds meet. We will use this approach to solve the following optimization problem

$$\begin{aligned} & \underset{\eta, \mathbf{d}}{\text{minimize}} && \eta \\ & \text{subject to} && -\eta < W(\omega_i) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{P}(\omega_i) \mathbf{A} \mathbf{d} < \eta \end{aligned} \quad (4.18)$$

where we discretize the problem by considering $\omega_i = \frac{2\pi i}{L}$, $0 \leq i < L$. Also, instead of the $\mathcal{L}_\infty$ -norm, we consider the equivalent problem of enforcing L constraints for computational savings. To demonstrate the need for a global optimization approach, Figure 4.5 shows the phase errors from using semidefinite relaxation and branch and bound for the $\Theta_{pre}(\omega)$ in (4.25) with $N = 5$, $\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$, and $\gamma = 0.9$. As expected, the relaxation fails because the rank approximation error is too significant to ignore.

The final step in our algorithm is to combine (4.18) with the constraint on the denominator. The complete optimization problem is therefore

$$\begin{aligned} & \underset{\mathbf{d}, \eta, \epsilon}{\text{minimize}} && \eta + \epsilon \\ & \text{subject to} && -\eta \leq W(\omega_i) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{P}(\omega_i) \mathbf{A} \mathbf{d} \leq \eta \\ & && -\epsilon \leq \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Q}(\omega_i) \mathbf{A} \mathbf{d} - 1 \leq \epsilon \end{aligned} \quad (4.19)$$

Note that we have introduced a new optimization variable ϵ to relax the original equality constraint. The proposed algorithm is able to compensate for waveguide power loss in photonic filters because the effect is directly captured in the optimization problem by the variable $\mathbf{A}$.

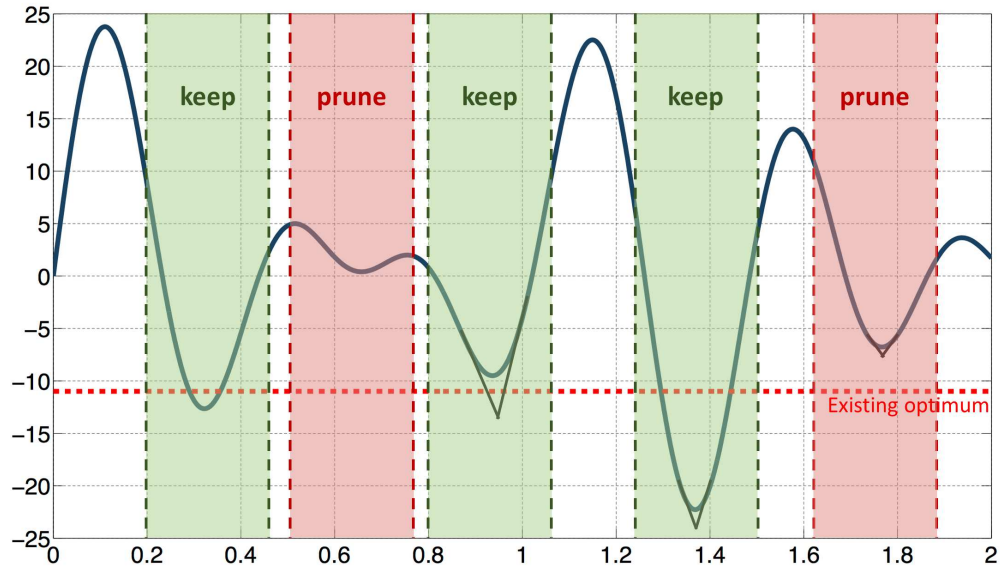


Figure 4.4: Concept demonstration for Branch and Bound. The blue line shows the actual objective function. The vertical dotted lines show the locations of known cost values, and the red dotted line shows the existing upper bound based on the known values. The solid black lines show the convex relaxations.

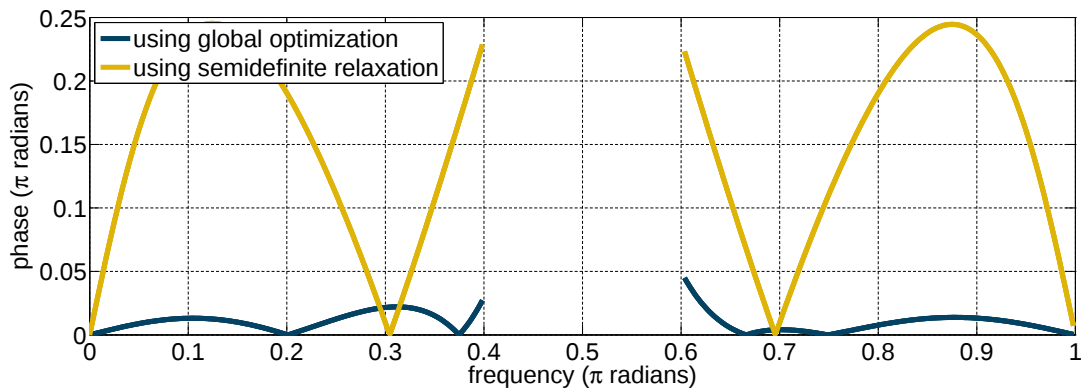


Figure 4.5: Phase errors from using the branch and bound approach and from using semidefinite relaxation.

4.3 Refinement

While (4.19) can already provide loss compensating designs, we can further improve the algorithm by introducing an iterative refinement step. One approach would be to consider the current solution as a initialization to a descent optimization algorithm such as quasi Newton or gradient descent [80]. Alternatively, we can employ a more direct recursion to improve the resulting coefficients [81]. In (4.19), the optimal result is obtained when we examine a further confined feasible set. However, the quantity $\mathbf{d}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}$ does not necessarily have to be 1 to achieve minimal cost in the original problem. We can therefore refine the result by searching in a small neighborhood near the solution from (4.19).

To implement the local search, we consider an iterative approach to remove the denominator in the original optimization problem (5.14) through

$$\underset{\mathbf{d}}{\text{minimize}} \quad \max_{\omega} W(\omega) \left| \frac{\mathbf{d}_k^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}_k}{\mathbf{d}_{k-1}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}_{k-1}} \right| \quad (4.20)$$

where k is the iteration variable. The procedure is analogous to treating the numerator and denominator as two separate optimization problems. Note that when the denominator is a fixed quantity, we can simply modify (4.19) to solve the numerator optimization problem.

Since the denominator is solved using only a naive update, we must control the step size to search only within a small neighborhood from the original guess $\mathbf{d}_0$. To ensure the denominator does not differ significantly from iteration $k-1$ to iteration k , we introduce the following constraint

$$|\mathbf{d}_k^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}_k - \mathbf{d}_{k-1}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}_{k-1}| < \lambda \quad (4.21)$$

where λ is the step size control, and can be either manually chosen or introduced as an optimization variable. The overall algorithm with the refinement step is outlined in Algorithm 2.

Note that the number of samples L determines the resolution in frequency. Increasing L would result in smaller error between the given and prescribed phase

Algorithm 2 Allpass loss refinement

Require: Filter order N , prescribed phase response to match $\Theta_{pre}(\omega)$, waveguide power loss γ , number of samples L .

Compute $\mathbf{d}_0$ using Branch and Bound on (4.19).

Formulate $\mathbf{A} = \text{diag}\{1 \ \gamma \ \cdots \ \gamma^{N-1} \ \gamma^N\}$.

Formulate $\omega_i = \frac{2\pi i}{L}$, $0 \leq k < L$.

Calculate $\mathbf{P}(\omega_i)$, $\mathbf{Q}(\omega_i)$ according to (4.10).

k=1;

repeat

 Formulate weight function

$$W_k(\omega_i) = \frac{1}{|\mathbf{d}_{k-1}^\top \mathbf{A}^{-1} \mathbf{Q}(\omega_i) \mathbf{A} \mathbf{d}_{k-1}|} \quad (4.22)$$

 Solve using Branch and Bound

$$\begin{aligned} & \underset{\mathbf{d}_i, \eta, \lambda}{\text{minimize}} && \eta + \lambda \\ & \text{subject to} && -\eta \leq W_k(\omega_i) \mathbf{d}_i^\top \mathbf{A}^{-1} \mathbf{P}(\omega_i) \mathbf{A} \mathbf{d}_k \leq \eta \\ & && -\lambda \leq \mathbf{d}_k^\top \mathbf{A}^{-1} \mathbf{Q}(\omega_i) \mathbf{A} \mathbf{d}_k - \mathbf{d}_{k-1}^\top \mathbf{A}^{-1} \mathbf{Q}(\omega_i) \mathbf{A} \mathbf{d}_{k-1} \leq \lambda \end{aligned} \quad (4.23)$$

 k++;

until $\mathbf{d}$ converges

responses at the cost of increased computational complexity. The convergence of the algorithm can be governed by the difference between $\mathbf{d}_k$ and $\mathbf{d}_{k-1}$. In our approach we require

$$\|\mathbf{d}_k - \mathbf{d}_{k-1}\|_2^2 \leq 1 \times 10^{-6} \quad (4.24)$$

for the algorithm to terminate. Note that the refinement step can also be applied independently to existing allpass designs. We present a design algorithm that not only can be used to achieve loss compensating allpass filters, but also applied to improve existing implementations.

4.4 Simulation Results

To demonstrate the performance of our algorithm, let us first consider the equalization of the phase response shown in Figure 4.6. This phase profile exemplifies the response of a bandpass filter used in WDM that introduces nonlinear chromatic dispersion near the band edges. With the current phase response, only a narrow bandwidth at the center of the filter is usable because other frequency components within the passband will experience non-uniform group delay (negative derivative of the phase). The purpose of a followup allpass filter is then to equalize the filter's phase to be linear, which subsequently increase the usable region of the passband [33]. We will therefore consider the design of a 4th-order allpass filter that is able to linearize the given phase response in the passband region of the bandpass filter from 0.15π to 0.6π . In this example we will examine the resulting allpass responses for a congregated loss effect of $\gamma = 0.95$. For a $1\mu\text{m}$ by $0.5\mu\text{m}$ SiNx strip waveguide cladded in SiO_2 with $\alpha = 25\text{dB/cm}$ [31, 82, 5], this would correspond to a bending radius of $\sim 11\mu\text{m}$, or equivalently $\sim 1\%$ loss per 90 degree turn. Reducing the allowable bend radius is important for filter miniaturization, since the footprint occupied by a ring resonator unit cell is proportional to the square of the bend radius.

Figure 4.7 shows the desired response and those of the traditional and proposed methods. The branch and bound method in the proposed algorithm is implemented in MATLAB using the YALMIP [83] toolbox. To derive the response from

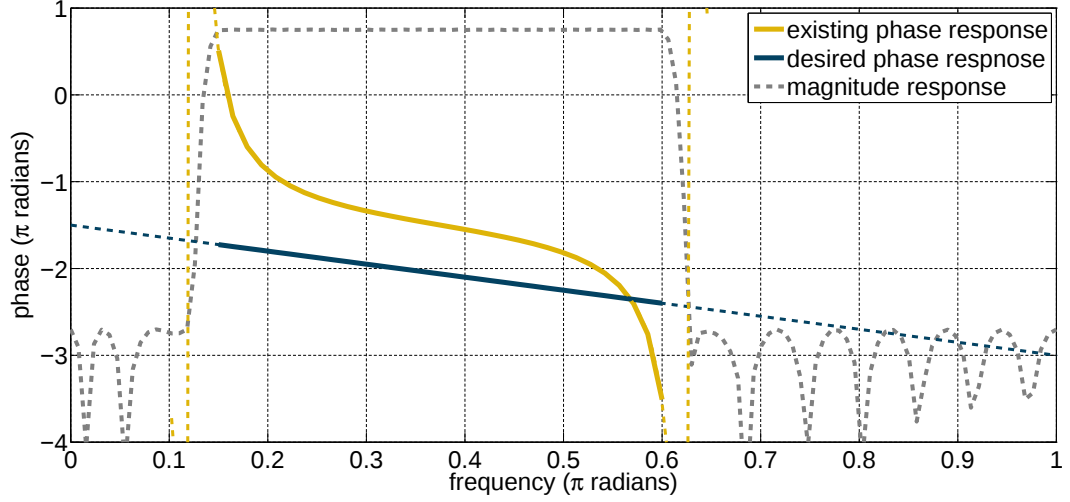


Figure 4.6: Actual and desired phase responses for a bandpass structure with nonlinear chromatic dispersion near the pass band edges. The solid portions represent the frequency range of interest. The profile of the magnitude response is also shown for reference.

a traditional approach, an ideal allpass filter is first designed using the minimax criterion and then subsequently corrupted according to (3.2). From the graph, it is immediately evident that since traditional method does not consider the effect of γ in the optimization, the phase profile under waveguide loss significantly digresses from the desired profile. Since we directly consider the effect in the optimization setup, the phase response from the proposed method is able to closely track the desired profile even under waveguide loss.

Let us consider a second example where the prescribed phase response is of the following form

$$\Theta_{pre}(\omega) = \begin{cases} -(N-1)\omega & 0 \leq \omega \leq \omega_p \\ -(N-1)\omega - \pi & \omega_s \leq \omega \leq \pi \end{cases} \quad (4.25)$$

Here N is the filter order, ω_p is a normalized frequency value that governs the passband, and ω_s is the stopband. In comparison to the previous example, this prescribed response is more tolerating because it only contains a discontinuity. However, the effect of waveguide loss is still prominent, and can be easily identi-

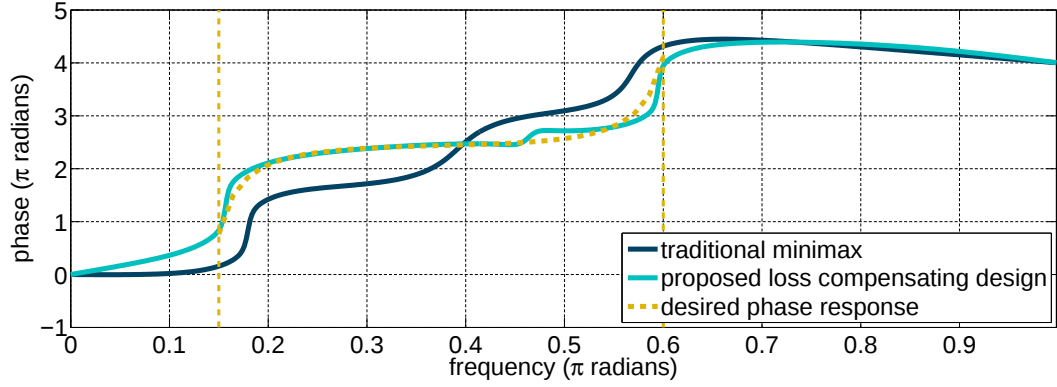


Figure 4.7: Phase responses of 4th-order allpass filters that are designed using the traditional minimax approach and the proposed algorithm for $\gamma = 0.95$. Note that the region of interest are confined to within the passband of the example bandpass filter (0.15π to 0.6π).

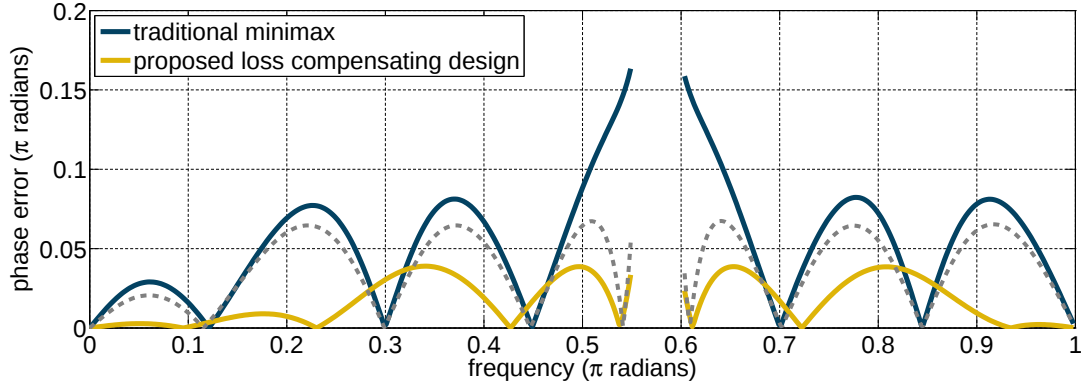


Figure 4.8: Phase errors for the traditional minimax allpass and that of the proposed design. The ideal response of the minimax is also plotted for reference.

fied when we directly examine the phase error. Figure 4.8 shows the phase error comparison results for $N = 7$, $\gamma = 0.9$, $\omega_p = 0.55\pi$, $\omega_s = 0.6\pi$. Notice that using the traditional design, the corrupted allpass response shows large errors near the band edges. The proposed design shows significant improvement in terms of maximum $\Delta\Theta(\omega)$.

Because the loss effect is directly captured in the optimization setup, the result phase error from the proposed method is evenly distributed throughout the frequencies. The designed allpass will therefore not only have an minimal maximum error, but also reduced variations throughout. To demonstrate the overall

Table 4.1: Normalized maximum phase differences for $N = 7$ allpass filters under various waveguide loss. Note that $\gamma = 0.8$ would correspond to a photonic allpass filter whose individual sections are composed of Bragg reflectors and $1\mu\text{m} \times 0.5\mu\text{m}$ SiNx strip waveguides cladded in SiO_2 that are $485\mu\text{m}$ long.

Method	loss	passband, stopband			
		0.4, 0.6	0.5, 0.6	0.55, 0.6	0.575, 0.6
corrupted minimax	$\gamma = 0.93$	0.0119	0.0406	0.0902	0.1975
	$\gamma = 0.9$	0.0163	0.0561	0.1542	0.2598
	$\gamma = 0.8$	0.0626	0.1758	0.2599	0.2955
compensating design (no refinement)	$\gamma = 0.93$	0.0230	0.0582	0.1322	0.2232
	$\gamma = 0.9$	0.0231	0.0494	0.1075	0.2010
	$\gamma = 0.8$	0.0252	0.0298	0.0209	0.0701
compensating design (with refinement)	$\gamma = 0.93$	0.0142	0.0266	0.0583	0.0804
	$\gamma = 0.9$	0.0127	0.0220	0.0378	0.0487
	$\gamma = 0.8$	0.0129	0.0147	0.0162	0.0313
minimax refined	$\gamma = 0.93$	0.0186	0.0412	0.0732	0.1120
	$\gamma = 0.9$	0.0184	0.0355	0.0578	0.0919
	$\gamma = 0.8$	0.0135	0.0097	0.0172	0.0560

effectiveness and robustness of our method, Table 4.1 summarizes the comparison results for various design parameters of the form shown in (4.25). Notice that the proposed method does not display an improvement when the effect of γ is negligible under highly tolerant requirements and low loss. In all other conditions, the proposed design is able to outperform because traditional approaches do not consider the effect of waveguide power loss. In general, the proposed algorithm's superiority is most obvious for $0.8 < \gamma < 0.99$. The variance measurements demonstrate that our method is not only able to reduce the maximum phase error, but also improve the phase matching across all frequencies. As shown by the tables, even minor changes in the waveguide power with $\gamma = 0.93$ causes significant phase differences. The proposed design and the refinement technique are both aimed to minimize this effect.

It has been demonstrated that allpass filters are excellent for photonic system realization of arbitrary magnitude filter [16]. Under ideal fabrication, traditional frequency selective filter (Butterworth, Chebyshev, Elliptic) can be decomposed into allpass substructures without losing optimality [34]. Specifically, a general bandpass filter can be obtained by placing two allpass filters in a parallel

Table 4.2: Variance measurements across the frequencies for $N = 7$ allpass filters under various waveguide loss. Units are in 1×10^{-4}

Method	loss	passband, stopband			
		0.4, 0.6	0.5, 0.6	0.55, 0.6	0.575, 0.6
corrupted minimax	$\gamma = 0.93$	0.148	1.469	7.098	20.11
	$\gamma = 0.9$	0.237	2.371	11.94	28.27
	$\gamma = 0.8$	1.672	12.10	32.72	54.48
compensating design (no refinement)	$\gamma = 0.93$	0.148	0.611	2.550	8.864
	$\gamma = 0.9$	0.158	0.422	1.645	6.562
	$\gamma = 0.8$	0.498	0.508	0.268	0.639
compensating design (with refinement)	$\gamma = 0.93$	0.228	0.746	2.605	4.858
	$\gamma = 0.9$	0.214	0.476	1.288	2.310
	$\gamma = 0.8$	0.223	0.252	0.315	0.632
minimax refined	$\gamma = 0.93$	0.330	1.496	5.102	7.523
	$\gamma = 0.9$	0.264	1.254	2.937	5.403
	$\gamma = 0.8$	0.222	0.073	0.230	2.262

setup. Recall that we can further require one of the allpass filters to be a simple delay for reduced design and fabrication complexity

$$H(z) = \frac{1}{2} (z^{-(N-1)} + A_{err}(z)) \quad (4.26)$$

where N is the order of the allpass filter $A_{err}(z)$. Ideally, the system operates by requiring the phase response of $A_{err}(z)$ to be $-(N-1)\omega$ in the passband regions to creating complex addition, and $-(N-1)\omega - \pi$ in the stopband regions for subtraction. Under waveguide power loss, the phase response of $A_{err}(z)$ is distorted, and traditional designs can no longer provide optimal performance. Figure 4.9 shows the comparison result of lowpass filters created using the traditional minimax approach and the proposed method for $\gamma = 0.9$. Note that the phase requirement for the allpass filter takes the form in (4.25). Without accounting for the loss effect, the magnitude response displays large ripples due to the increased peak phase error and large variations. By reducing the phase error of the loss corrupted allpass, the proposed design is able to provide smoother magnitude responses in the passband and stopband. However, the attenuations in the passband and stopband have been affected by the loss, and the need for a design technique specifically for bandpass filters based on allpass substructures is immediately evident.

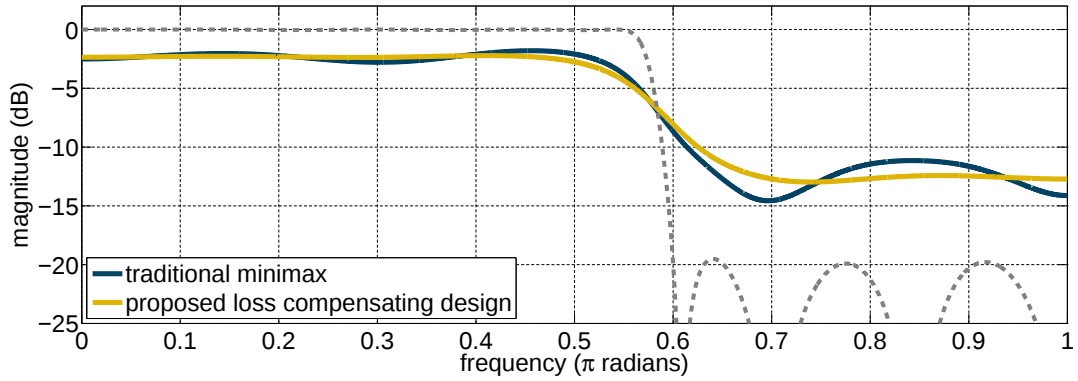


Figure 4.9: Linear magnitude responses of lowpass filters generated using waveguide loss corrupted minimax and the proposed compensating design. The ideal magnitude response from a minimax design is also shown for reference.

4.5 Summary

This chapter presents an allpass design method that can mitigate the effect of waveguide power loss. We propose a two part optimization algorithm that can be used individually, or applied to an existing design to improve the results in the phase. By directly considering the waveguide loss in the algorithm, we are able to produce allpass filter coefficients that can provide optimal response given a prescribed phase requirement even under non-ideal fabrications. Comparison results with the minimax approach demonstrate that the proposed algorithm is able to provide performance gains for both high and low loss devices. The design method will be able to lift the performance limitation of allpass filters used in WDM applications such as dispersion compensation and group delay equalization.

4.6 Acknowledgments

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Chapter 5

Loss Compensating Bandpass Filters

While an algorithm that minimizes the effect of waveguide power loss in the phase is crucial for a variety of photonic applications such as dispersion compensation, it neglects the fact that the magnitude of the allpass filter is also affected. It is therefore important to consider the frequency response of the corrupted response as a whole for certain utilizations of the allpass filter such as gain equalization. Traditionally, allpass design approaches depend only on a requirement for the phase response, even when the applications are magnitude based. The basis for these approaches is that the magnitude of the ideal allpass is unity for all frequencies, and therefore the behavior of an overall system can be directly manipulated by the phase. Under waveguide loss, however, the magnitude response of the allpass has ripples at the locations of the roots. Such behavior means that design of the corrupted allpass needs to be application specific, because the distorted magnitude response can be optimally utilized to satisfy the needs of a certain system. In particular, we will demonstrate how a lowpass filter using suboptimal allpasses can yield better performance comparing to traditional approaches such as the minimax design approach.

5.1 Allpass-based Bandpass Filters

In the optical domain, all filters (lowpass, highpass, bandpass) are periodic when profiled against wavelength. The design of any photonic filter can therefore be reduced to the design of a lowpass filter centered at the baseband [73]. Recall that under ideal conditions, a lowpass filter based on a single allpass filter and a delay as shown in (2.11) can be constructed by requiring the allpass phase to satisfy (2.12). The principle behind such structure is that the allpass has unity magnitude response for all frequencies, and different passband and stopband regions can be created by matching its phase to that of the delay. Although the photonic realizations are unable to provide the ideal unit magnitude response, they can still be readily used in constructing lowpass filters. In this chapter we will consider design of the coefficients d_i 's of the canonical allpass filter $A(z)$, such that the corrupted response

$$H(z) = \lambda z^{-(N-1)} + (1 - \lambda)A(\gamma^{-1}z) \quad (5.1)$$

yields the best possible performance under waveguide power loss. The term $0 \leq \lambda \leq 1$ governs the distribution of the input signal's power between the two arms of the system. Under ideal fabrication conditions, the input's power is evenly distributed between the two possible signal paths because the allpass substructures have equal magnitude contributions. Realistically, however, the effect of loss is more prominent in the photonic allpass than the simple delay path, resulting in different attenuation levels between the two subsystems. By allowing flexibility on the weights, we can intelligently utilize the magnitude distortion in the allpass to achieve increased performance in the overall structure.

5.2 Approach

Since the overall lowpass filter operates based on the phase manipulation of the allpass substructures, it would be reasonable to simply consider the error between the phase response of $A(\gamma^{-1}z)$ and the prescribed phase requirement. How-

ever, because the corrupted allpass can no longer provide distortion-free magnitude response, the magnitude of the overall filter is also affected. Simply considering the phase is then inadequate because the distorted allpass magnitude response can be intelligently utilized to produce an overall optimal behavior. Therefore, we must examine the frequency response of $H(z)$ as an entirety, and design the algorithm such that both the magnitude and phase errors are minimized.

5.2.1 Cost Function

Because the average value of the waveguide loss can be estimated prior to fabrication [21], we can absorb the term γ directly into the cost function of the filter design optimization. To derive the error between the frequency response of $H(e^{j\omega})$ and a prescribed requirement, let us first define

$$\mathbf{d} = \begin{bmatrix} 1 & d_1 & \cdots & d_N \end{bmatrix}^T \quad (5.2)$$

$$\mathbf{e}(e^{jMw}) = \begin{bmatrix} 1 & e^{-j\omega} & \cdots & e^{-jN\omega} \end{bmatrix}^T \quad (5.3)$$

$$\mathbf{A} = \text{diag} \left\{ 1 \quad \gamma \quad \cdots \quad \gamma^N \right\} \quad (5.4)$$

Note that the waveguide loss effect is captured by the diagonal matrix $\mathbf{A}$, and can be considered as a non-linear weighing effect on the filter coefficients. The allpass frequency response under waveguide loss is then

$$A(\gamma^{-1}e^{j\omega}) = e^{-jN\omega} \gamma^N \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{e}^*(e^{j\omega})}{\mathbf{d}^T \mathbf{A} \mathbf{e}(e^{jMw})} \quad (5.5)$$

and the corresponding lowpass filter is

$$\begin{aligned} H(e^{j\omega}) &= \lambda e^{-j(N-1)\omega} + (1 - \lambda) e^{-jN\omega} \gamma^N \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{e}^*(e^{j\omega})}{\mathbf{d}^T \mathbf{A} \mathbf{e}(e^{jMw})} \\ &= \frac{\hat{\mathbf{d}}^T \mathbf{Y}(e^{j\omega}) \hat{\mathbf{d}} + \mathbf{b}^T(e^{j\omega}) \hat{\mathbf{d}}}{\mathbf{c}^T(e^{j\omega}) \hat{\mathbf{d}}} \end{aligned} \quad (5.6)$$

where we let $\hat{\mathbf{d}}^\top = \begin{bmatrix} \mathbf{d}^\top & \lambda \end{bmatrix}$ for notation simplicity. The matrix $\mathbf{Y}(\omega)$ is

$$\mathbf{Y}(e^{j\omega}) = \begin{bmatrix} \mathbf{0}_{N+2 \times N+1} & \mathbf{y}(e^{j\omega}) \end{bmatrix} \quad (5.7)$$

and

$$\begin{aligned} \mathbf{y}^\top(e^{j\omega}) &= \begin{bmatrix} e^{-jN\omega} (\mathbf{e}^{j\omega} \mathbf{e}^\top(e^{j\omega}) \mathbf{A} - \gamma^N \mathbf{e}^H(e^{j\omega}) \mathbf{A}^{-1}) & 0 \end{bmatrix} \\ \mathbf{b}^\top(e^{j\omega}) &= \begin{bmatrix} \gamma^N e^{-jN\omega} \mathbf{e}^H(e^{j\omega}) \mathbf{A}^{-1} & 0 \end{bmatrix} \\ \mathbf{c}^\top(e^{j\omega}) &= \begin{bmatrix} \mathbf{e}^\top(e^{j\omega}) \mathbf{A} & 0 \end{bmatrix} \end{aligned} \quad (5.8)$$

Our goal is then to match $H(e^{j\omega})$ to a prescribed frequency response $P(e^{j\omega})$ under a given waveguide loss γ . For the design of analysis filters for two-channel filter bank, $D(e^{j\omega})$ takes the form

$$P(e^{j\omega}) = \begin{cases} e^{-j(N-1)\omega} & 0 \leq \omega \leq \omega_p \\ \delta e^{-j(N-1)\omega} & \omega_s \leq \omega \leq \pi \end{cases} \quad (5.9)$$

where δ is a small allowable error in the stopband. Note that the phase requirement $\Theta_P(\omega) = -(N-1)\omega$ is necessary for the allpass-based design when $A_0(z) = z^{-(N-1)}$. The error function is then

$$\begin{aligned} E(e^{j\omega}) &= W(\omega)(H(e^{j\omega}) - P(e^{j\omega})) \\ &= W(\omega) \frac{\hat{\mathbf{d}}^\top \mathbf{Y}(e^{j\omega}) \hat{\mathbf{d}} + \hat{\mathbf{b}}^\top(e^{j\omega}) \hat{\mathbf{d}}}{\mathbf{c}^\top(e^{j\omega}) \hat{\mathbf{d}}} \end{aligned} \quad (5.10)$$

where

$$\hat{\mathbf{b}}(e^{j\omega}) = \mathbf{b}(e^{j\omega}) - P(e^{j\omega}) \mathbf{c}(e^{j\omega}) \quad (5.11)$$

and $W(\omega)$ is a weighing function that decides relative importance at the different frequency bands. We can immediately notice that the error function is formulated as a quadratic function over a linear term. However, since $\mathbf{Y}(e^{j\omega})$ is not symmetric and not semi-definite, we cannot directly utilize any standard convex optimization based approach.

For implementation, we can further consider only a discrete set of frequency points of interest, and formulate the cost vector $\mathbf{x}$, where the n th element of $\mathbf{x}$ is

$$x_n = E(e^{j\omega_n}) \quad (5.12)$$

with ω_n as a discretized frequency value. The objective is then to minimize the $\mathcal{L}_2$ -norm, $\mathcal{L}_\infty$ -norm, or in general $\mathcal{L}_p$ -norm of $\mathbf{x}$. Note that p should be chosen as a power of 2 in order to produce a differentiable cost function [84]. Under this condition, powerful global optimization techniques such as the gradient descent and the quasi-Newton approach can be applied. Our overall objective function is therefore the following unconstrained optimization

$$\underset{\hat{\mathbf{d}}}{\text{minimize}} \quad \|\mathbf{x}\|_p \quad (5.13)$$

To reach an optimal $\mathbf{d}$ and λ that yield loss compensating result, we will apply the quasi-Newton technique [80]. In this descent minimization algorithm, the estimated gradient $\mathbf{g}_k$ and the estimated Hessian $\mathbf{H}_k$ of the cost function are iteratively calculated. The optimization variable $\hat{\mathbf{d}}_k$ is then continuously updated by $\delta_k \approx -\mathbf{H}_k^{-1}\mathbf{g}_k$ until a stationary point is reached. Because our cost function is highly non-linear, the choice of an initial solution becomes critical. To establish the basis for the global search, we consider the following reduced optimization problem

$$\underset{\mathbf{d}}{\text{minimize}} \quad \max_{\omega_n} \mathbf{d}^T \hat{\mathbf{y}}(e^{j\omega_n}) \hat{\mathbf{y}}^H(e^{j\omega_n}) \mathbf{d} \quad (5.14)$$

where

$$\hat{\mathbf{y}}(e^{j\omega}) = e^{-j(N-1)\omega} \mathbf{A} \mathbf{e}(e^{jM\omega}) + e^{-jN\omega} \gamma^N \mathbf{A}^{-1} \mathbf{e}^*(e^{j\omega}) - 2P(e^{j\omega}) \mathbf{A} \mathbf{e}(e^{jM\omega}) \quad (5.15)$$

The reduced problem is derived by assuming $\lambda = \frac{1}{2}$, and ignoring the denominator term. Note that the denominator term in the original cost function can be considered as a nonlinear weighing function over the frequencies [85]. By ignoring the effect of the denominator, we are effectively optimizing over a re-weighted cost function of the original problem. This setup directly corresponds to the minimax

formulation, which is convex and can be efficiently solved.

5.2.2 Stability

Without any constraint on the form of the denominator, the resulting transfer function could contain poles that are outside of the unit circle. While the overall response may provide the best performance, a pole radius larger than 1 cannot be realistically implemented and can cause unexpected behaviors in the response. Several forms of stability constraints exist for filter design algorithms [80]. In particular, we will consider the following requirement to ensure that the system is stable [85]

$$\mathcal{Re}\{D(\gamma^{-1}e^{j\omega})\} > 0 \quad (5.16)$$

Recall that $D(\gamma^{-1}z)$ is the corrupted denominator response of the allpass. The stability constraint can therefore be written as

$$\hat{\mathbf{d}}^T \mathbf{g}(\omega) > 0 \quad (5.17)$$

where

$$\mathbf{g}^T(\omega) = \begin{bmatrix} 1 & \gamma \cos \omega & \cdots & \gamma^N \cos N\omega & 0 \end{bmatrix} \quad (5.18)$$

The constraint is therefore of linear form, and can be easily introduced into the optimization setup.

5.2.3 Overall Algorithm

The overall algorithm is outlined in Algorithm 4. Note that a linear equality constraint for $\lambda = \frac{1}{2}$ can be easily introduced if symmetric analysis filters are needed.

5.3 Simulation Results

To demonstrate the performance of the proposed algorithm, we compare the filter performances between the proposed method and a design obtained from

Algorithm 3 Loss compensating lowpass filter design based on allpass substructures

Require: Filter order N , waveguide power loss γ , passband frequency ω_p , stopband frequency ω_s , allowable stopband error δ .

Formulate the desired frequency response $P(e^{j\omega})$ according to (5.9).

Formulate loss matrix $\mathbf{A} = \text{diag}\{1 \ \gamma \ \cdots \ \gamma^N\}$, and frequency vector $\mathbf{e}(e^{j\omega_n}) = [1 \ e^{-j\omega_n} \ \cdots \ e^{-jN\omega_n}]^T$.

Create $\hat{\mathbf{y}}(e^{j\omega_n})$ according to (5.15) and solve the following unconstrained convex optimization

$$\underset{\mathbf{d}}{\text{minimize}} \quad \max_{\omega_n} \mathbf{d}^T \hat{\mathbf{y}}(e^{j\omega_n}) \hat{\mathbf{y}}^H(e^{j\omega_n}) \mathbf{d} \quad (5.19)$$

Set the initial solution $\hat{\mathbf{d}}_0 = [\mathbf{d} \ \frac{1}{2}]$.

Formulate $\mathbf{x}$ according to (5.12) and (5.10) and $\mathbf{g}(\omega_n)$ according to (5.18).

Solve the following constrained optimization problem using quasi-Newton

$$\begin{aligned} &\underset{\hat{\mathbf{d}}}{\text{minimize}} \quad \|\mathbf{x}\|_p \\ &\text{subject to} \quad \hat{\mathbf{d}}^T \mathbf{g}(\omega_n) > 0 \end{aligned} \quad (5.20)$$

if symmetric filters are needed **then**

Introduce constraint

$$\hat{d}_{N+1} = \frac{1}{2} \quad (5.21)$$

end if

Table 5.1: Intensity Coupling Coefficients for a 6th-order lowpass filter based on allpass substructures

	κ_0^2	κ_1^2	κ_2^2	κ_3^2	κ_4^2	κ_5^2	λ
Proposed	0.9988	0.9999	0.9816	0.9998	0.6665	0.9992	0.373
Traditional	0.9899	1	0.9752	1	0.6360	1	0.5

the minimax approach [25]. In our proposed method, the effect of loss is directly considered in the cost function, whereas the minimax approach first designs an ideal allpass and is subsequently corrupted according to $A(\gamma^{-1}z)$. The initialization convex problem is solved using the convex optimization program YALMIP [83], and the quasi-Newton steps are accomplished through the MATLAB optimization toolbox.

Let us first consider the design of a lowpass filter $H(z)$ with the following requirements: $N = 6$, $w_p = 0.45\pi$, $\omega_s = 0.55\pi$, and $\gamma = 0.9$. Figure 5.1 shows the filter design results using $p = 1$ in comparison to the uncompensated case. Recall that by allowing flexibility on the weight λ , we can optimally distribute the power of the input signal. The resulting responses demonstrate higher performance in terms of passband flatness and stopband rejection. Also notice that the traditional approach shows a small amount of phase ripple in the passband, whereas the proposed algorithm is able to ensure a linear-phase response and thus constant group delay. Table 5.1 lists the coupling coefficients of the proposed and traditional approaches; the labeling corresponds to that of Figure 2.7. Notice that the proposed algorithm heavily adjusts the value of λ to minimize the effect of power loss from the allpass branch.

To verify the outputs of our algorithm, we use the ρ , κ , and γ coefficients as inputs into an optical transfer-matrix model of coupled ring resonators [86]. The transfer function derived from the matrix model corresponds to the ring-loaded branch of the Mach-Zehnder interferometer in Figure 2.7. This intermediate transfer function is coherently added with the linear-delay signal of the bottom branch, as per (5.1), to yield the full transfer function of our filter.

In addition, to evaluate our algorithm's robustness in terms of a physically realizable device, we include a realistic modal refractive index dispersion in the filter simulation. The dispersion of the mode index was calculated from a 2-D

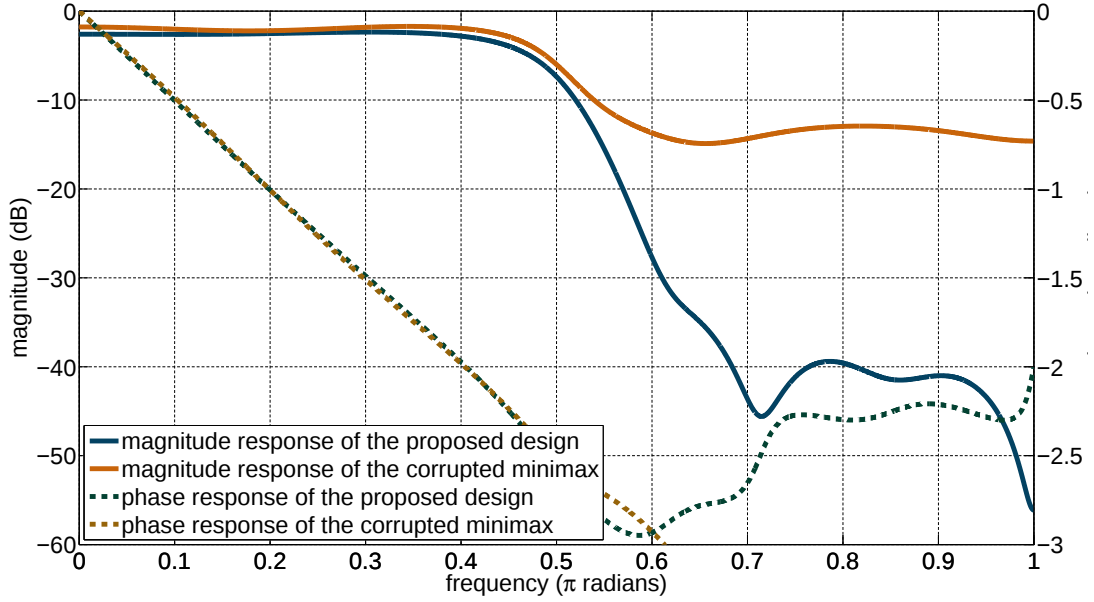


Figure 5.1: Comparison results for the 6th-order allpass-based lowpass filter for $\gamma = 0.9$. The second allpass is chosen as a delay z^{-5} .

cross-sectional mode analysis using the finite-element method (FEM). We used a strip waveguide width $\times$ height geometry of $400 \times 230 \text{ nm}^2$; silicon core and SiO_2 cladding. This design yields a zero group-delay dispersion of the waveguides in the telecommunications C-band.

Similarly, FEM analysis of the directional couplers yields the design space for the coupling coefficients. In the coupling region, we use a constant waveguide-to-waveguide gap of 200 nm; the length of each coupler is varied as per the the output of the design algorithm (Table 5.1). Due to refractive-index dispersion, these coupler lengths are nominal only at a single wavelength (chosen here to be 1550 nm). Thus, there will be wavelength-dependent variation of the coupling coefficients that our design algorithm does not account for [58].

In this example, we choose a ring length of $54.7 \mu\text{m}$, which yields an FSR of 10 nm and requires bend radii between 4 - $9 \mu\text{m}$ depending on the coupler lengths. To show how our algorithm can be used to design compact high-performance filters despite some bending loss, we impose a realistic round-trip attenuation of 0.445 dB in each ring (i.e. $\gamma = 0.9$) [58]; this number represents the combination of all

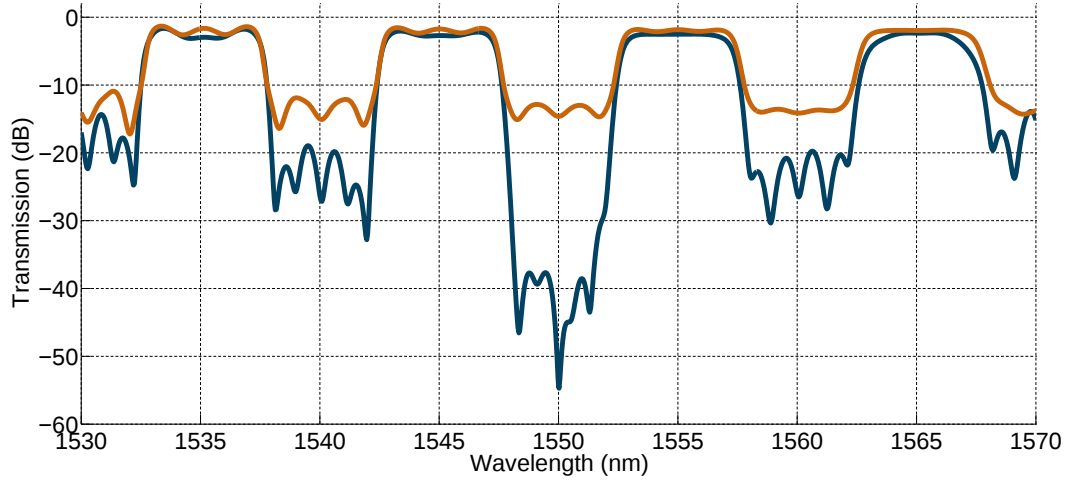


Figure 5.2: Comparison results in the wavelength domain with dispersion for the proposed and traditional minimax designs centered on stopband. The design result from the proposed method is shown in blue.

loss mechanisms present in a ring (e.g. due to absorption and defect/bend-induced scattering). The results of the optical-domain simulations are plotted in Figures 5.2 and 5.3, which show the superiority of our proposed design over the traditional design. Specifically, the traditional approach is unable to yield a filter response that has a reasonable rejection in the stopband. The proposed method, on the other hand, is able to provide 20 dB, or greater, of stopband rejection.

The spectral variation from band to band is due to the dispersions of the mode index and coupling coefficients described above; i.e. κ^2 becomes detuned from the nominal values listed in Table 5.1 as one walks away from the design wavelength of 1550 nm. The maximum amount of κ^2 error occurs at the short-wavelength end of the spectrum. At 1530 nm, κ^2 exhibits the following detuned values, corresponding to the first row of Table 5.1, for couplers 0 through 5, respectively: 0.972, 0.979, 0.935, 0.981, 0.558, and 0.974. Although the algorithm does not take these dispersion effects into account, we are still able to obtain a reasonable filter response that spans the C-band.

Additionally, Figures 5.2 and 5.3 show designs where the design wavelength (1550 nm) is centered on either a stopband or passband. These two extremes illustrate how the overall spectrum can be equalized by designing for nominal values

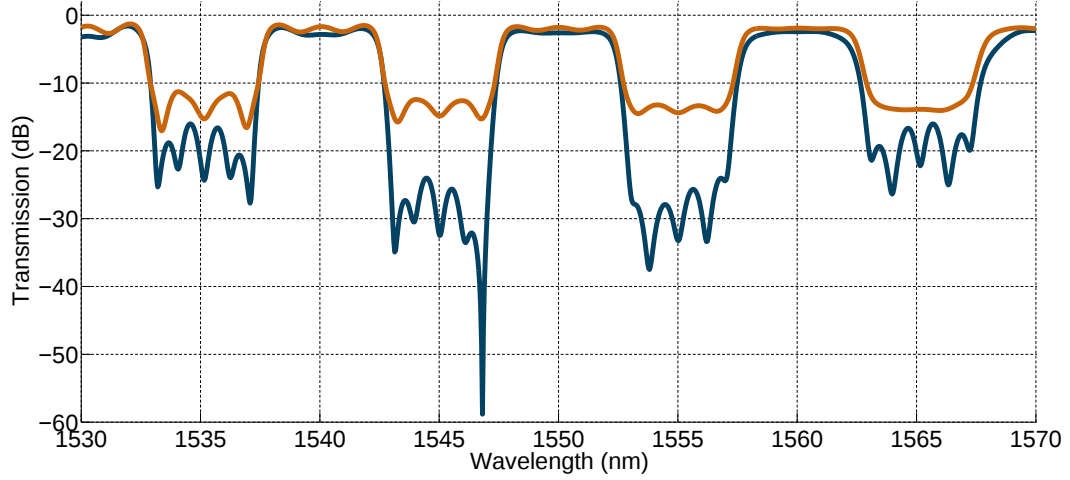


Figure 5.3: Comparison results in the wavelength domain with dispersion for the proposed and traditional minimax designs centered on passband. The design result from the proposed method is shown in blue.

in a certain spectral region. We stress that the band-to-band variation depends on the distance away from the design wavelength, and not strictly on FSR. Thus, a spectrum exhibiting a shorter FSR (i.e. a design with longer length resonators), but with all other design parameters the same, would suffer from less FSR-to-FSR variation; the variation between two specific wavelengths would remain the same.

To demonstrate the algorithm's robustness, Figure 5.4 plots the maximum absolute error between the desired lowpass response and the actual $H(e^{j\omega})$ for $p = \infty$. In this example, $N = 5$, $w_p = 0.4\pi$, $\omega_s = 0.6\pi$, and the congregated waveguide loss varies from $\gamma = 0.8$ to $\gamma = 1$. Notice that the proposed method is able to provide a constant gain over uncompensated result regardless of γ . Also note that even with a constraint of $\lambda = \frac{1}{2}$, the resulting response is still able to outperform. Figure 5.5 shows the comparison result for varying the width of the transition band. In this case, the waveguide loss is fixed at $\gamma = 0.9$, and $N = 5$. Notice that the increase in performance is exponential with narrowing transition band. Such trend is beneficial for photonic realizations because the signal processing required in the optical domain are often extremely narrowband.

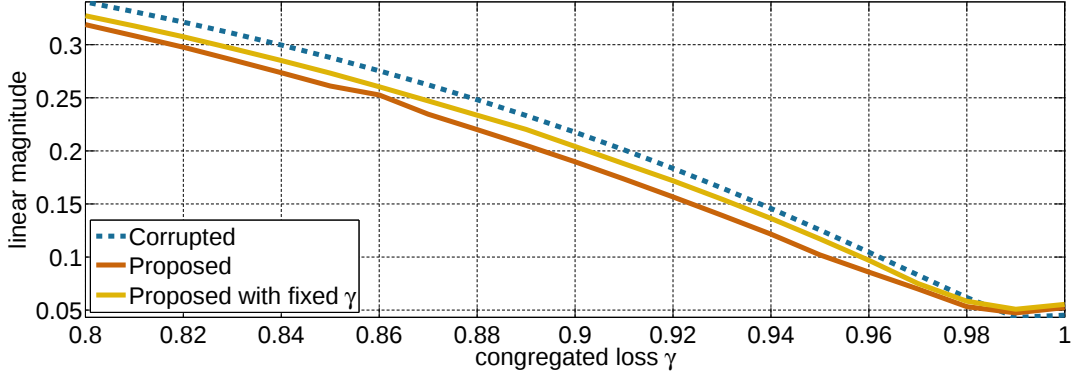


Figure 5.4: Comparison result for the maximum error between the ideal and actual responses for $H(z)$ with varying congested waveguide loss γ .

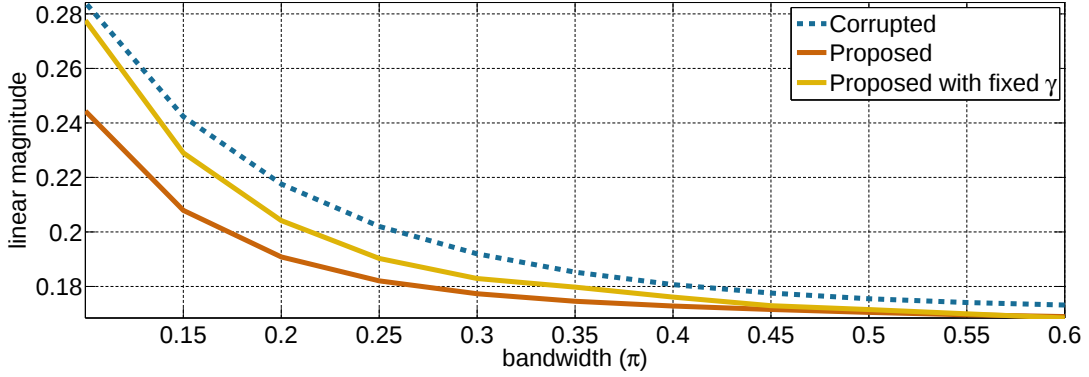


Figure 5.5: Comparison result for the maximum error between the ideal and actual responses for $H(z)$ with varying width of the transition band.

5.4 Summary

In this chapter, we propose an allpass-based lowpass design method that is specifically targeted for photonic filters. The algorithm captures the unique waveguide power loss directly into the cost function to provide filter coefficients that can compensate for such effect. Example simulation results show that the loss compensating algorithm outperforms the traditional minimax design approach under various design parameters. The proposed algorithm will aid in reducing the stringent requirements of photonic fabrication and assist the realization of future photonic signal processors. By significantly improving the performance of photonic filters under waveguide power loss, we have opened the possibility of realizing

high-performance photonic circuits using lower-cost methods where state-of-the-art materials and fabrication processes may be unavailable.

5.5 Acknowledgments

This chapter, in full, is a reprint of the published journal paper in IEEE Journal on Lightwave Technology, March 2014.

Chapter 6

DFT Allpass Filter Bank

In this chapter, we extend the allpass-based design philosophy to the design of a filter bank structure for sub-band processing. In doing so, we argue that additional loss-inherent structures such as power splitters [87, 88] and waveguide crossings [89, 90] can be utilized without detrimental effects to the results of the design algorithm. Although the idea of an allpass-based filter bank is first conceived in a signal processing context, the proposed setup is structurally dependent on additional processing to be functional [54]. We present a modified structure and design algorithm that is able to achieve the necessary filtering without extra hardware. In photonic implementation, the waveguide power loss is an inescapable source of error that is both coupled with realistic implementation setups and intrinsic to the material properties [60, 74]. Such effect also causes a photonic allpass filter to exhibit attenuation at certain controllable locations. The algorithm optimally utilizes the otherwise detrimental effect to mitigate the necessity for additional processing from the original allpass filter bank structure. The proposed structure is ideal for applications in time-stretched analog to digital converters, WDM network channelizers, and general optical signal multiplexers. The rest of this chapter explores the setup and mathematical optimization of the system in detail, and presents an algorithm based on convex and nonlinear optimization techniques.

6.1 Limitations of Existing Structure

Let us consider the design of the DFT allpass filter bank from Section 2.6. Since the features of the filter bank are structurally enforced, the design of the individual allpass filters can be decoupled and performed separately. Given a prescribed requirement $\Theta_{pre}(\omega)$, the goal of an allpass filter design algorithm would be to minimize the error between the phase response of the allpass filter and that of the prescribed response in the $\mathcal{L}_p$ -norm. Note that since the prescribed requirement is only in the phase and an allpass filter is expected to have unity response for all frequencies, we can equivalently consider the prescribed magnitude requirement

$$D_k(e^{j\omega}) = e^{j\Theta_k(\omega)} \quad (6.1)$$

The error function for the k -th allpass filter is therefore

$$\zeta_k(e^{j\omega}) = D_k(e^{j\omega}) - A_k(e^{jM\omega})e^{-jk\omega} \quad (6.2)$$

Now, the allpass frequency response can be expressed as

$$A_k(e^{jM\omega}) = \frac{\mathbf{d}^T \mathbf{J} \mathbf{e}(e^{jM\omega})}{\mathbf{d}^T \mathbf{e}(e^{jM\omega})} \quad (6.3)$$

where

$$\mathbf{d} = \begin{bmatrix} 1 & d_1^{(k)} & \cdots & d_{N-1}^{(k)} & d_N^{(k)} \end{bmatrix}^T \quad (6.4)$$

$$\mathbf{e}(e^{j\omega}) = \begin{bmatrix} 1 & e^{-j\omega} & \cdots & e^{-j(N-1)\omega} & e^{-jN\omega} \end{bmatrix}^T \quad (6.5)$$

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (6.6)$$

Note that we are forcing the allpass filters to only contain real coefficients as

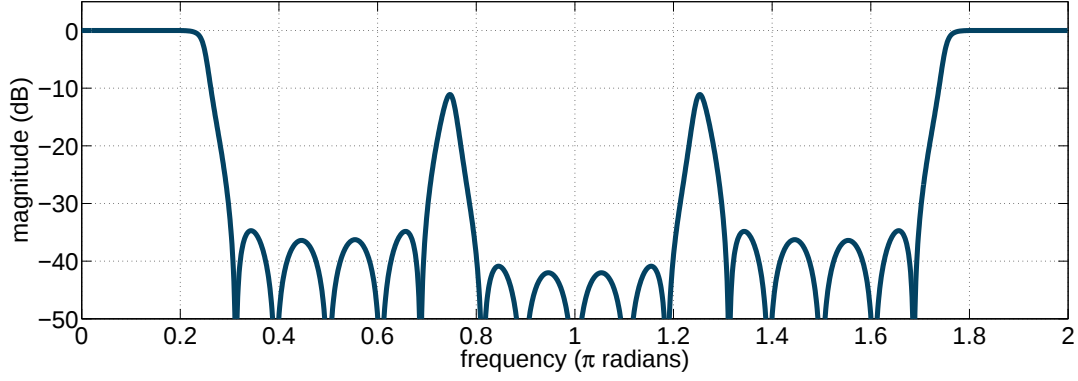


Figure 6.1: Magnitude response of $H_0(z)$ in a 4-channel DFT allpass filter bank. The baseline allpass filter $A_0(z)$ is chosen as a simple delay $A_0(z^4) = z^{-4.2}$ and $\omega_p = 0.2\pi$.

implied by the symmetry in the baseband filter $H_0(z)$. The overall error function is therefore

$$\zeta_k(e^{j\omega}) = \frac{\mathbf{d}^T (D_k(e^{j\omega})\mathbf{I} - e^{-jk\omega}\mathbf{J}) \mathbf{e}(e^{jM\omega})}{\mathbf{d}^T \mathbf{e}(e^{jM\omega})} \quad (6.7)$$

The denominator can be considered as a weight term at different frequency points [85], and can be neglected or addressed iteratively. The optimization problem is then

$$\begin{aligned} & \underset{\mathbf{d}}{\text{minimize}} \quad \|\mathbf{d}^T (D_k(e^{j\omega})\mathbf{I} - e^{-jk\omega}\mathbf{J}) \mathbf{e}(e^{jM\omega})\|_p \\ & \text{subject to} \quad \|\mathbf{d}\|_2 = 1 \end{aligned} \quad (6.8)$$

where p is typically chosen as 2 for least squares or ∞ for minimax. The setup conforms to a convex optimization form and can be readily solved using a variety of available solvers.

An example prototype filter $H_0(z)$ from a 4-channel DFT allpass structure is shown in Figure 6.1 and the corresponding filter bank is shown in Figure 6.2. Notice that the resulting $H_0(z)$ exhibits an undesirable spur at $\omega_c = \frac{3\pi}{4}$, and the effect also propagates to the rest of the channels due to symmetry. Such behavior is highly undesirable since it severely limits the overall Spurious-Free Dynamic Range (SFDR) of the overall system, and may cause cross-talk in communication structures.

The effect, however, is inherent to the structure and cannot be addressed

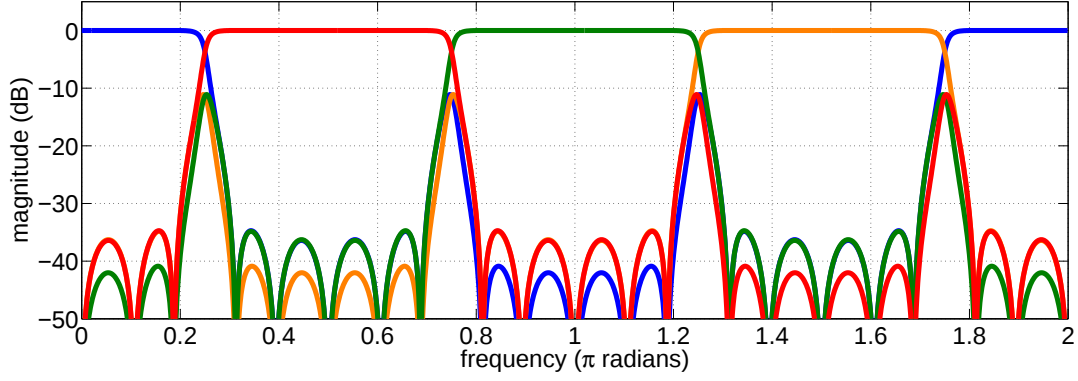


Figure 6.2: Magnitude responses of $H_i(z)$ in a 4-channel filter bank.

through design. For each allpass pair $A_k(z^M)z^{-k}$, the structure requires a phase discontinuity of $\frac{2\pi(l+1)k}{M}$ between two adjacent frequency bands of interest l and $l+1$. Such requirement would also imply that at the corner frequencies $\omega_{c,l} = \frac{(2l+1)\pi}{M}$, the phase response of the k -th allpass is approximately

$$\Theta_k\left(\frac{2\pi(l+1)}{M}\right) \approx -MN\omega - \frac{1}{2} \cdot \frac{2\pi(l+1)k}{M} \quad (6.9)$$

In other words, the frequency response $H_0(e^{j\omega})$ at the corner frequencies can be expressed as

$$H_0(e^{j\omega_{c,l}}) \approx \frac{1}{M} e^{j\Theta_0(M\omega_{c,l})} \sum_{k=0}^{M-1} e^{-j\frac{\pi(l+1)k}{M}} \quad (6.10)$$

For the 4-channel filter bank at $\omega_{c,l} = \frac{3\pi}{4}$, the frequency response of $H_0(e^{j\omega})$, regardless of design specifications, is

$$H_0(e^{j\frac{3\pi}{4}}) \approx \frac{1}{4} e^{j\Theta_0(\frac{3\pi}{4})} \sum_{k=0}^{M-1} e^{-j\frac{3k\pi}{4}} \quad (6.11)$$

which results in a magnitude response of -11dB . Since the amount of the phase change is governed by the DFT structural setup, it is a rigid requirement and cannot be modified. In electrical implementations of the DFT allpass filter bank, this drawback is addressed by introducing an additional interpolation filter to cancel the spurs [54]. Such approach, however, would imply additional hardware,

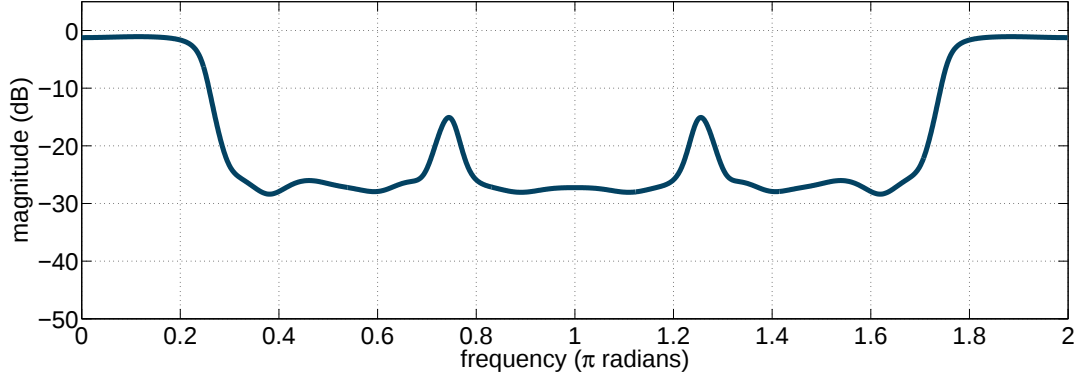


Figure 6.3: Magnitude response of $H_0(z)$ in a 4-channel DFT allpass filter bank when implemented using photonic elements with a loss of $\gamma = 0.97$. Note that the delay element is assumed to have negligible loss.

and therefore may not always be feasible.

6.2 Approach

While the effect of the spurs cannot be avoided, photonic implementation may present itself as the optimal platform for the DFT allpass filter bank since optical setups naturally mitigates the effect. Recall that in photonic signal processors, the effect of waveguide power loss must always be considered. The magnitude response of the baseband filter $H_0(z)$ when implemented using photonic allpass filters with a loss of $\gamma = 0.97$ is shown in Figure 6.3. While the power loss seems to simply result in a degradation of the response, its effect can be harnessed and used in place of any additional post-processing filter.

The benefit of the waveguide loss in a DFT filter bank can be understood from examining the poles and zeros of the allpass filters. The poles and zeros of the allpass filters in the 4-channel filter bank example are shown in Figure 6.5. Notice that in each $A_k(z^M)$, there exists a pole-zero pair in close proximity to the unit circle at $\omega = \frac{(2l+1)\pi}{M}$.

Proposition 1. In the design of a M -channel allpass filter bank, each of the allpass filters $A_k(z)$ (i.e. post downsample) will contain a pole-zero pair at $\omega = \frac{\pi}{M}$.

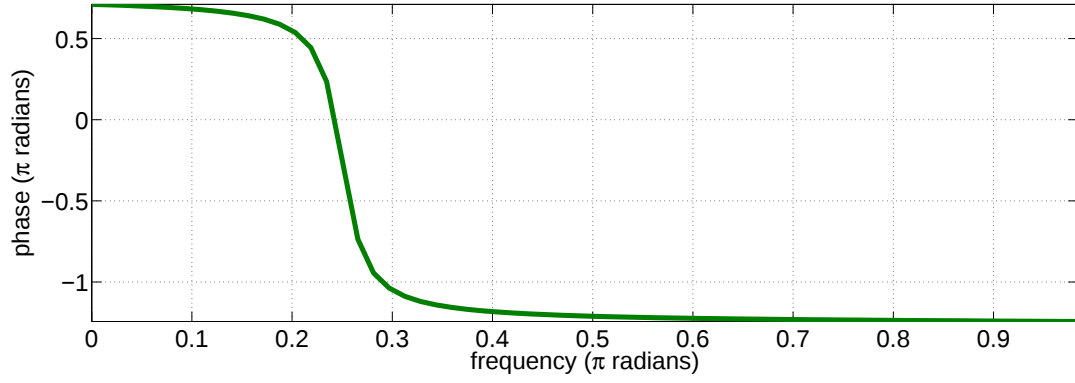


Figure 6.4: Example phase response of a single pole zero pair at $\omega_n = \frac{\pi}{4}$ in an allpass filter response.

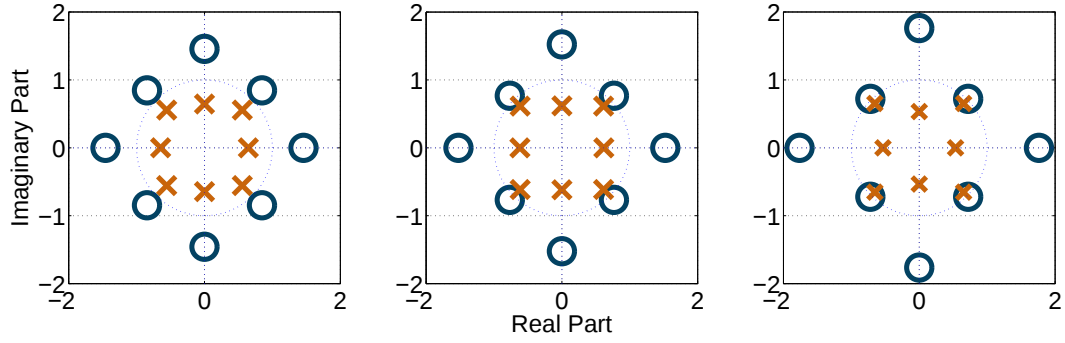


Figure 6.5: Poles and zeros of the allpass filters $A_k(z^4)$ in a 4-channel filter bank prior to considering the effect of waveguide power loss.

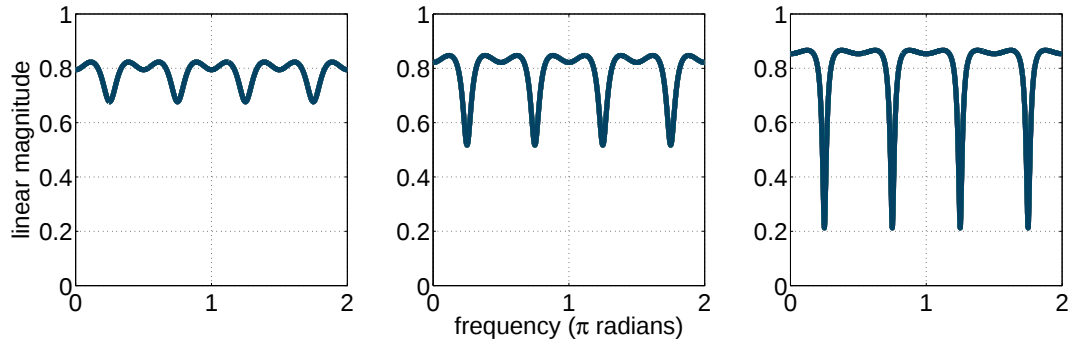


Figure 6.6: Magnitude responses of the allpass filters under waveguide loss $\gamma = 0.97$ in a 4-channel filter bank.

Proof. The phase response of each $A_k(z)$ takes the form

$$\Theta_k(\omega) = -N\omega + 2 \sum_{n=0}^{N-1} \arctan \frac{r_n \sin(\omega - \theta_n)}{1 - r_n \cos(\omega - \theta_n)} \quad (6.12)$$

where r_n and θ_n are the radius and angle of the n -th pole in $A_k(z)$. Figure 6.4 shows an example phase response for $r_n = 0.95$, $\theta_n = \frac{\pi}{4}$. The prescribed phase requires a discontinuity of $\frac{k\pi}{M}$ between $\omega = \frac{\pi}{M} \pm \Delta \omega$ for each $A_k(z)$, where $\Delta \omega = \frac{\pi}{M} - \omega_p$. In terms of design, this can only be achieved by the arctan terms. The design of the allpass filter $A_k(z)$ can therefore be viewed as the best approximated decomposition of $\Theta_{pre}(\omega)$ into N arctan bases of different centers and curvature degrees. Similar to concepts such as Fourier decomposition, the optimal representation would require a dominant component that best matches the signal at the first order. In terms of the phase response of $A_k(z)$, this condition can only occur when a pole-zero pair is exactly at $\omega_n = \frac{\pi}{M}$ as seen in (6.12). $\square$

The waveguide loss changes the frequency response of an allpass filter to $A_k(\gamma^{-1}z)$, which can be viewed as shifting the roots of the ideal allpass towards the origin. Such effect causes the allpass to lose the precise balance of having each pole zero pair at radii r_n and $\frac{1}{r_n}$. As a zero approaches the unity circle in the z -plane, the magnitude response would then exhibit a concavity. The effect of $\gamma = 0.97$ on the allpass filters' magnitude responses in the 4-channel example is shown in Figure 6.6. Notice that the loss causes each of the allpass filters $A_k(z^M)$ to exhibit a strong null at $\frac{(2l+1)\pi}{4}$. This is because each allpass will have a pole zero pair close to the unit circle as indicated by Proposition 1. The locations of these nulls exactly align with the spurs in the analysis filters $H_i(z)$. A photonic implementation therefore naturally offers a means to mitigating the undesirable effects in a DFT allpass filter bank. However, the effect of γ is not uniform for all of the allpass filters, and must be properly optimized.

In the original structure setup, each of the allpass filters is given a weight of $\frac{1}{M}$ due to equal magnitude contributions. Under loss, the nominal levels of the allpass magnitude responses are no longer equal, as seen in Figure 6.6. A first step in designing the photonic filter bank is therefore to redistribute the weights. The

analysis filter responses for the proposed structure takes the form

$$H_i(z) = \sum_{k=0}^{M-1} W_k e^{-j2\pi \frac{i \cdot k}{M}} A_k(\gamma^{-M} z^M) z^{-k} \quad (6.13)$$

where W_k 's are weights that are optimized for a given loss parameter γ . Since the M analysis filters are identical, it is sufficient to optimize W_k 's for $H_0(z)$. Using the designed allpass coefficients from the lossless setup, let us first formulate

$$\mathbf{a}(e^{j\omega}) = \begin{bmatrix} e^{-jNM\omega} \\ A_1(\gamma^{-M} e^{jM\omega}) e^{-j\omega} \\ \vdots \\ A_{M-2}(\gamma^{-M} e^{jM\omega}) e^{-j(M-2)\omega} \\ A_{M-1}(\gamma^{-M} e^{jM\omega}) e^{-j(M-1)\omega} \end{bmatrix} \quad (6.14)$$

where $A_k(\gamma^{-M} e^{jM\omega})$ is the allpass response under loss obtained using the coefficients derived from the ideal allpass filter bank setup.

The goal of the W_k 's is to counterbalance the uneven nominal magnitude responses of the allpass filters, we therefore formulate the following optimization setup

$$\begin{aligned} & \underset{\mathbf{w}}{\text{minimize}} && \|\mathbf{w}^T \mathbf{a}(e^{j\omega})\|_2^2 \\ & \text{subject to} && \mathbf{w}^T \mathbf{1} = 1 \\ & && \mathbf{w} > 0 \end{aligned} \quad (6.15)$$

for $\omega_s \leq \omega \leq \pi$, where $\mathbf{w} = [W_0 \ W_1 \ \dots \ W_{M-1}]^T$. The optimization setup is equivalent to maximizing the stopband attenuation for $H_0(z)$. Note that the passband is neglected because the general shape of the response is already determined by the allpass coefficients. The stopband profile, however, depends on magnitude cancellation through phase matching, and therefore must be readjusted. The formulation conforms to a least squares setup and can be easily solved. The resulting response is shown in Figure 6.7.

By recalculating the weights, we are able to enhance the resolution between the passband and stopband of $H_0(z)$. However, the spurs still exist and limit the

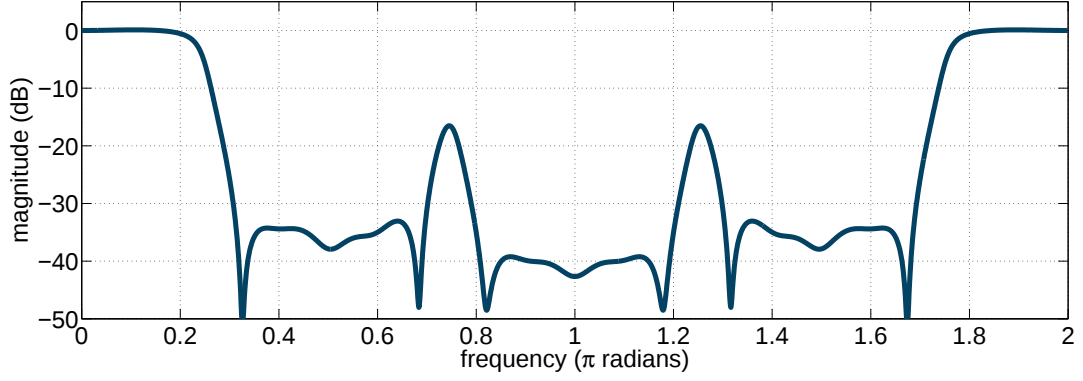


Figure 6.7: Magnitude response of $H_0(z)$ with of $\gamma = 0.97$ after optimizing the channel weights to account for the waveguide loss.

filter performance. To remove the spurs, we must optimize the filter coefficients specifically for photonic implementation. Under waveguide power loss γ , the frequency response of $H_0(z)$ is

$$H_0(e^{j\omega}) = W_0 e^{-jNM\omega} + \sum_{k=1}^{M-1} W_k \frac{\mathbf{d}^T \mathbf{J} \mathbf{e}(e^{jMw})}{\mathbf{d}^T \mathbf{e}(e^{jMw})} e^{-jk\omega} \quad (6.16)$$

Let

$$\mathbf{d} = [\mathbf{d}_1^T \quad \mathbf{d}_2^T \quad \cdots \quad \mathbf{d}_{M-2}^T \quad \mathbf{d}_{M-1}^T \quad \mathbf{w}^T]^T \quad (6.17)$$

$$\mathbf{I}_k = \begin{bmatrix} \mathbf{0}_{(k-1)N \times N} \\ \mathbf{I} \\ \mathbf{0}_{(M-2-k)N \times N} \\ \mathbf{0}_{M \times N} \end{bmatrix}, \quad \mathbf{M}_k = \begin{bmatrix} \mathbf{0}_{(M-1) \times N} \\ \mathbf{0}_{k \times N} \\ 1 \\ \mathbf{0}_{(M-1-k) \times N} \end{bmatrix} \quad (6.18)$$

The frequency response $H_0(e^{j\omega})$ can be expressed in vector matrix notations as

$$H_0(e^{j\omega}) = \mathbf{d}^T \mathbf{M}_0 e^{-jNM\omega} + \sum_{k=1}^{M-1} \mathbf{d}^T \mathbf{M}_k \frac{\mathbf{d}^T \mathbf{I}_k \mathbf{J} \mathbf{e}(e^{jMw})}{\mathbf{d}^T \mathbf{I}_k \mathbf{e}(e^{jMw})} e^{-jk\omega} \quad (6.19)$$

And the overall design goal is

$$\underset{\mathbf{d}}{\text{minimize}} \quad \left\| |H_0(\gamma^{-1} e^{j\omega})|^2 - D(\omega) \right\|_{\infty} \quad (6.20)$$

where

$$D(\omega) = \begin{cases} 1 & 0 \leq \omega \leq \omega_p \\ 0 & \omega_s \leq \omega \leq \pi \end{cases} \quad (6.21)$$

The $\mathcal{L}_\infty$ -norm is necessary because it ensures that the maximum error within the entire stop band region is minimized. In other words, it puts heavy emphasis on the undesirable spur from baseline design. Notice, however, the optimization setup is highly nonlinear and involved a M th-order polynomial in the optimization variable $\mathbf{d}$.

Although such problem formulation cannot be solved using any standard convex optimization solver, we can reach a reasonable optimal through gradient descent approaches given a reasonable starting point [80]. Notice that $\gamma = 1$ converts the formulation back into the original signal process allpass filter bank setup. The effect of γ can therefore be viewed as a perturbation on the optimal solution obtained from the DSP approach, and the coefficients from the original design can be used as a starting point to the iterative algorithm. The overall design algorithm is presented in Algorithm 4.

An example signal flow diagram of a 4-channel DFT allpass filter bank is shown in Figure 6.8. The implementation of Figure 6.8 as an optical integrated circuit is straight forward from the diagram. The delays z^{-1} 's are simple specific-length sections of waveguides. The allpass sections are ring resonators. The splitters and combiners can be implemented as y-junctions, which have an intrinsic attenuation of 3 dB per path plus an excess loss of $0.2 \sim 2$ dB [87, 90]. The crossings in the diagram, while not usually taken literally in a signal flow diagram, are implemented as actual waveguide crossings, which have been shown to exhibit an insertion loss as low as 0.02 dB [90]. More moderate values of 1 dB can be handled by our algorithm [89]. Figure 6.9 shows an example realization using ring resonators. The signal in the photonic implementation can be time stretched prior to entering the filters for efficient processing. Also note that more advanced FFT schemes [91, 92, 93] can be applied to reduce the implementation complexity.

Algorithm 4 Photonic Allpass Filter Bank

Require: Filter order N , number of channels M , waveguide power loss γ , pass-band and stopband frequencies ω_p, ω_s .

Formulate prescribed phase and frequency responses shown in (2.16) and (6.1).

Compute initial allpass filter coefficients according to (6.8), and initial weights according to (6.15).

Use computed results as an initial guess for a gradient descent algorithm on (6.20).

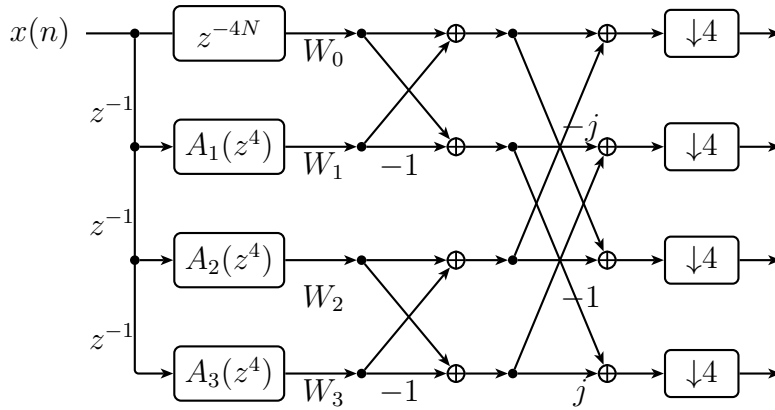


Figure 6.8: Example signal flow for a four-channel allpass-based photonic filter.

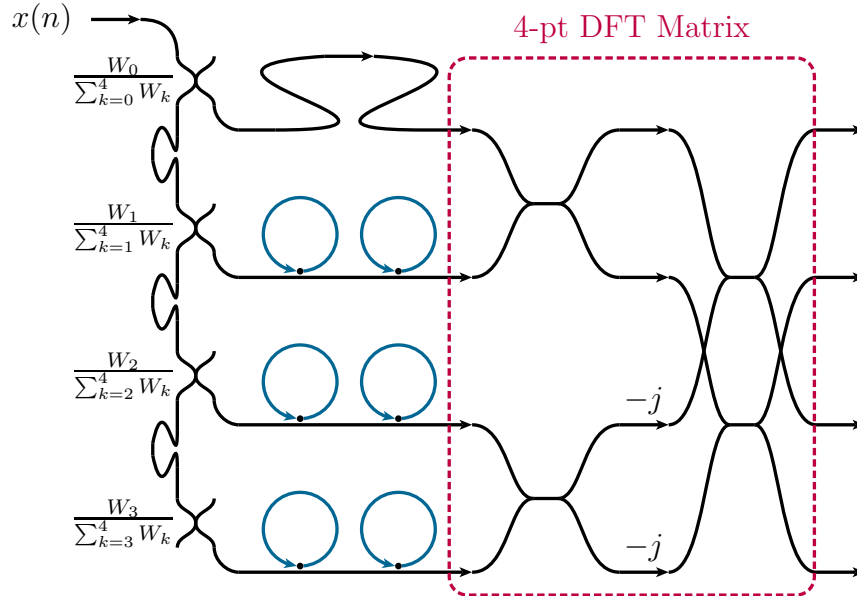


Figure 6.9: Example diagram for a ring resonator based implementation for a 4-channel allpass filter bank. Note that the downsample operations are not explicitly captured in this figure.

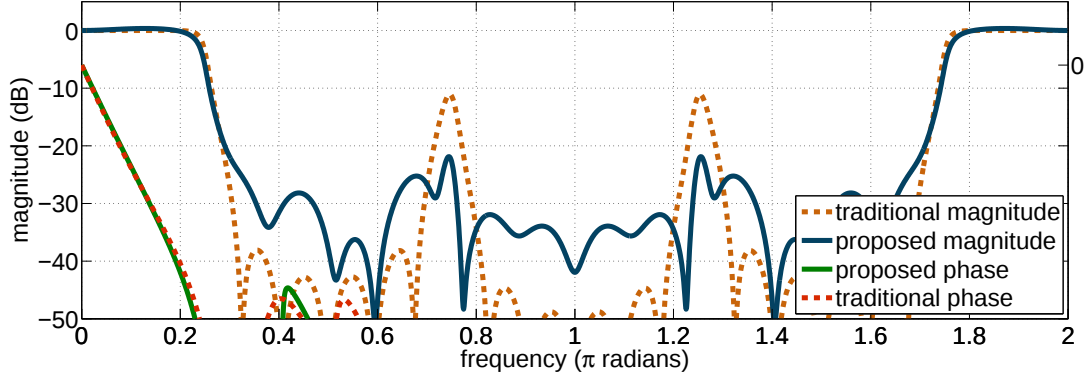


Figure 6.10: Magnitude response of $H_0(z)$ in a 4-channel DFT allpass filter bank when implemented using photonic elements with a loss of $\gamma = 0.97$ using the proposed algorithm.

6.3 Simulation Results

To demonstrate the performance of the proposed algorithm, we first compare our results to that of the classic allpass filter bank design. Figure 6.10 shows the comparison results for the magnitude response of the baseband analysis filter $H_0(z)$ for a 4-channel filter bank. In this example, $N = 2$, $\omega_p = 0.2\pi$, and $\gamma = 0.97$, which corresponds to -0.11dB per 90 degree turn in a ring resonator based setup. The proposed design algorithm is able to decrease the spur level by $> 10\text{dB}$ while maintaining the specifications, enabling the allpass-based filter bank structure for photonic implementation. The allpass-based structure is also able to provide linear-phase, which is crucial in higher-order modulation schemes such as MPSK and QAM.

The proposed algorithm is able to provide high-performance filters with strong stopband rejection while the traditional methods are limited by the effect of the spurs. Figure 6.11 shows the comparison results for the baseband filters in an 8-channel filter bank. Notice that the maximum cancellation from the traditional design is $\sim 11\text{dB}$ due to structural limitations, regardless of the number of channels and filter order. The proposed method is able to optimally utilize the waveguide loss effect to provide $\sim 47\text{dB}$ of attenuation in the stopband. In WDM applications, it is often necessary to have a large number of channels in the filter bank setup for high density packing. Figures 6.12 and 6.13 show the baseband

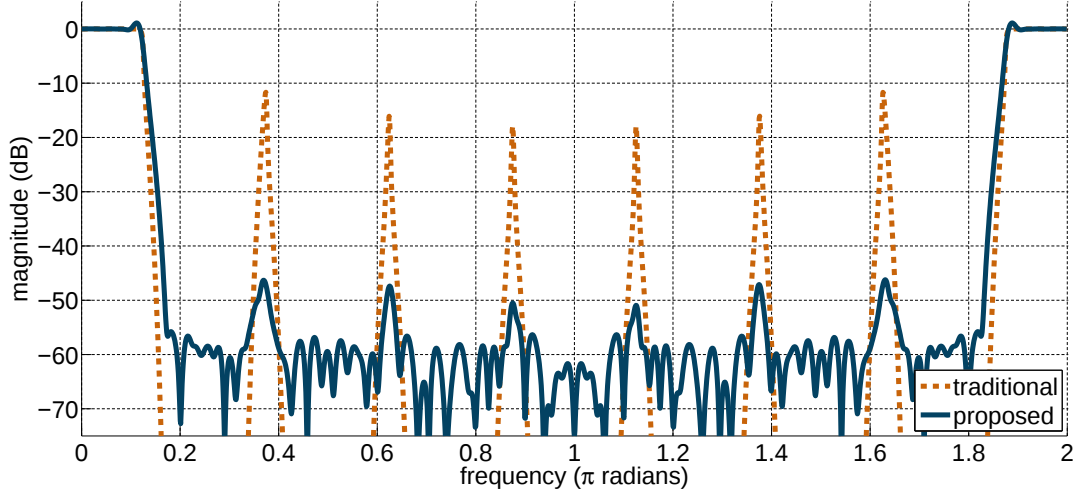


Figure 6.11: Magnitude response of $H_0(z)$ in an 8-channel DFT allpass filter bank with $N = 6$, $\omega_p = 0.125\pi$, $\omega_s = 0.175\pi$, $\gamma = 0.97$.

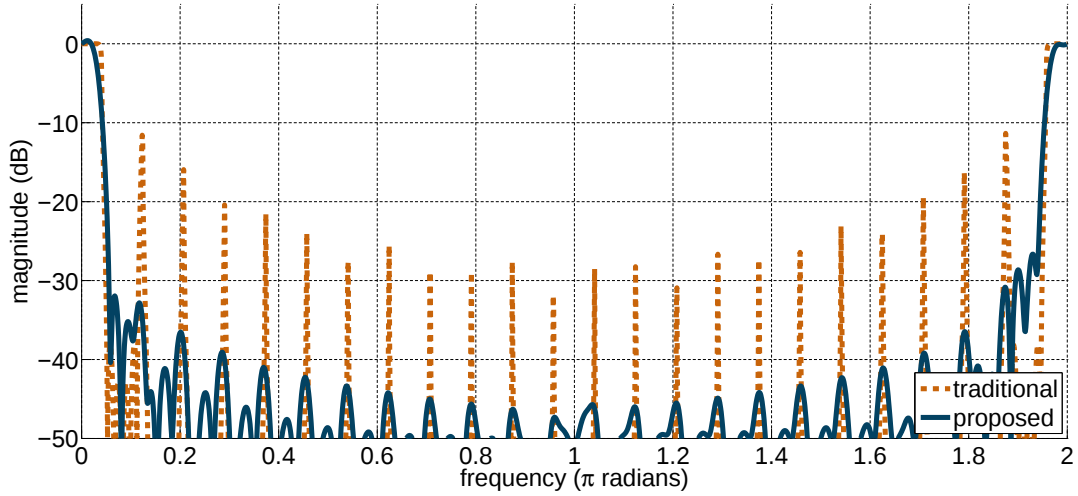


Figure 6.12: Magnitude response of $H_0(z)$ in a 24-channel DFT allpass filter bank with $\gamma = 0.95$.

response for a 24-channel allpass filter bank and its corresponding spectra for all channels for a waveguide loss of $\gamma = 0.95$ and $N = 2$. As shown by the figures, the proposed design algorithm is able to remove the spurs without any additional followup filtering. The results suggest that an attenuation of more than 30dB can be achieved using only 2 rings for each allpass filter. Also notice that unlike a sequential filtering approach, all 24 channels have identical responses due to the enforced constraint from the structural setup.

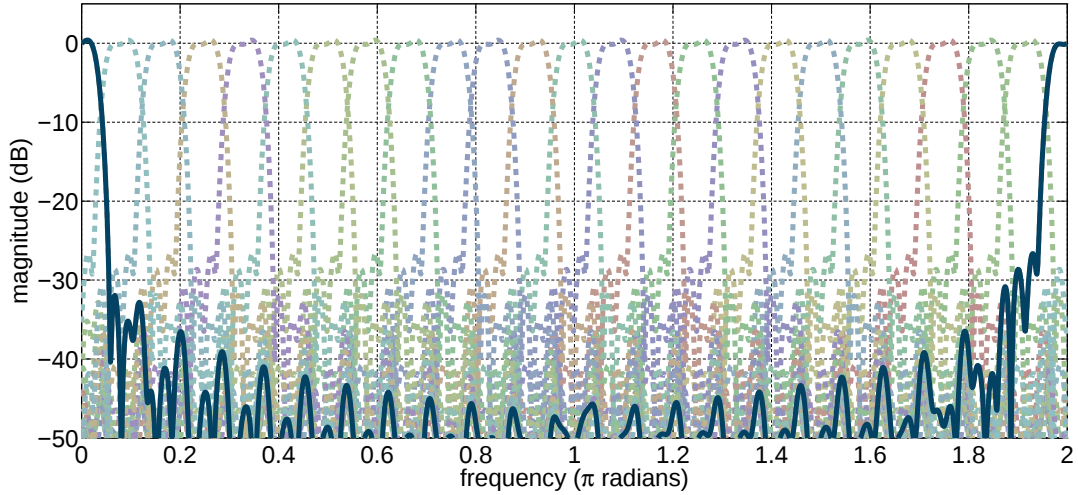


Figure 6.13: Spectra for all analysis filters in a 24-channel filter bank.

6.4 Summary

This chapter presents the structure and design of a filter bank setup specifically for photonic implementation. The proposed structure is completely based on allpass elements, which are naturally observed in a variety of nanoscale photonic dielectric components. The system is also optimized such that it can be readily combined with techniques such as time-stretching to reduce the implementation complexity and improve the filtering performance. While similar design exists in the digital signal processing domain, the structure requires further filtering due to the presence of undesirable spurs. We present a novel design technique that optimally utilizes the waveguide loss to mitigate the inherent challenges of the structure. The proposed algorithm based on both convex and nonlinear optimization techniques shows that the resulting allpass-based filter bank can be derived without any additional hardware. The presented structure is ideal for higher order modulation schemes and high speed time stretch ADCs that require superior filtering needs with large number of channels and constant group delay.

6.5 Acknowledgments

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Chapter 7

Realizable Component Values

Aside from corrupting the filter response, capabilities of contemporary fabrication techniques also limits the parameter values of each individual photonic allpass section within system. The reflection coefficient ρ governs the first-order filter's root location, but cannot be made to arbitrary precision. Therefore, from a signal processing perspective, we must constrain the poles to be within a certain maximum pole radius $r_{\max}$ for the cascade setup. The focus of this chapter is to provide a constraint for the design algorithm so that the resulting filter is guaranteed to be realizable using conventional technology. Although previous *ad-hoc* approach demonstrates a design technique that is targeted for a specific application [22], we need a generalized algorithm that can be applied to a wide range of photonic applications. Our research focuses on deriving a constraint that can be easily incorporated into the design for the allpass coefficients. We therefore focus on the cascade realization because there is a linear mapping between the poles and zeroes of the filter to the parameter values of the individual stages. Because the zeroes and poles of the filter are indirectly related to the lattice coefficients through nonlinear frequency transform, the cascade approach also serves as a basis for future lattice-based research.

7.1 Constraint for Cascade Form

The photonic parameter values such as the reflection coefficient ρ cannot be fabricated to arbitrary precision. This is important because it directly controls the pole location in a first-order photonic allpass. A realizable limit on the value of ρ directly translates to a limitation on the filter performance. An algorithm that incorporates the limitation in the design phase is therefore critical. To properly formulate the constraint on the maximum pole magnitudes, we use the formulation based on the Argument Principle [94] following the approaches shown in [68, 72]. The Argument Principle states that for a function $\hat{D}(z)$ that is differentiable inside a contour $\mathcal{C}$ except at a number of singularities, the following condition is satisfied

$$N_z - N_p = \frac{1}{2\pi j} \oint_{\mathcal{C}} \frac{\hat{D}'(z)}{\hat{D}(z)} dz \quad (7.1)$$

where N_z is the number of zeros of the function inside region $\mathcal{C}$, and N_p is the number of poles. The contour $\mathcal{C}$ in our setup is a circle defined by the maximum allowable pole radius $r_{\max}$. The maximum radius is directly related to the quantity $\gamma\rho$ from the manufacturable component values of a photonic allpass filter. Notice that as long as the roots of $D(z) = \sum_{k=0}^N d_k z^{-k}$ in (2.4) are within a circle prescribed by $r_{\max}$, the entire allpass filter will be realizable using photonic components such as FBG or rings. We can directly translate this information into the Argument Principle by relating the function $\hat{D}(z)$ to the backward path transfer function of the allpass filter

$$\hat{D}(z) = z^N D(z) = \sum_{k=0}^N d_k z^{N-k} \quad (7.2)$$

This formulation of $\hat{D}(z)$ contains the exact same root locations as $D(z)$, but will help in managing the constants in the Argument Principle setup. To properly derive the constraint, we start by rearranging the contour integral

$$\oint_{\mathcal{C}} \frac{\hat{D}'(z)}{\hat{D}(z)} dz = \oint_{\mathcal{C}} d \ln \hat{D}(re^{j\omega}) \quad (7.3)$$

Decomposing into magnitude and phase yields

$$\oint_{\mathcal{C}} d \ln \hat{D}(re^{j\omega}) = \oint_{\mathcal{C}} d \ln |\hat{D}(re^{j\omega})| + j \oint_{\mathcal{C}} d \arg \hat{D}(re^{j\omega}) \quad (7.4)$$

Since we are interested in filters with real coefficients, the magnitude response of $\hat{D}(z)$ is even, making $\oint_{\mathcal{C}} d \ln |\hat{D}(re^{j\omega})|$ a closed contour integral of an odd function. The first term vanishes to zero, and we have

$$\oint_{\mathcal{C}} \frac{\hat{D}'(z)}{\hat{D}(z)} dz = j \int_0^{2\pi} \frac{d}{d\omega} \arg \hat{D}(re^{j\omega}) d\omega \quad (7.5)$$

Based on the setup, we require that $N_z = N$, $N_p = 0$ inside the contour $\mathcal{C}$. The constraint thus becomes

$$2\pi N = \int_0^{2\pi} \frac{d}{d\omega} \arg \hat{D}(re^{j\omega}) d\omega \quad (7.6)$$

Now,

$$\frac{d}{d\omega} \arg D(re^{j\omega}) = N - \frac{d}{d\omega} \arctan \frac{\mathbf{d}^T \mathbf{R} \mathbf{s}(\omega)}{\mathbf{d}^T \mathbf{R} \mathbf{c}(\omega)} \quad (7.7)$$

where $\mathbf{R} = \text{diag}(1, r^{-1}, \dots, r^{-N})$. The constraint thus simplifies to

$$\int_0^{2\pi} \frac{d}{d\omega} \arctan \frac{\mathbf{d}^T \mathbf{R} \mathbf{s}(\omega)}{\mathbf{d}^T \mathbf{R} \mathbf{c}(\omega)} d\omega = 0 \quad (7.8)$$

Taking the derivative, we get

$$\int_0^{2\pi} \frac{\mathbf{d}^T \mathbf{R} (\mathbf{S}(\omega) + \mathbf{C}(\omega)) \mathbf{R} \mathbf{N} \mathbf{d}}{\mathbf{d}^T \mathbf{R} (\mathbf{S}(\omega) + \mathbf{C}(\omega)) \mathbf{R} \mathbf{d}} d\omega = 0 \quad (7.9)$$

where $\mathbf{N} = \text{diag}\{0, 1, \dots, N\}$.

The fractional form can be reduced to an iterative approach following similar concepts as the waveguide loss compensating designs we present in Chapter 4. The constraint can be satisfied iteratively with the denominator from the previous iteration used as a weight for the current iteration. For notation simplicity, let $\mathbf{\Lambda}(\omega) = \mathbf{R}(\mathbf{S}(\omega) + \mathbf{C}(\omega))\mathbf{R}$, then the constraint can be best satisfied iteratively by

minimizing

$$\begin{aligned}\kappa_L^{(i)} &= \mathbf{d}^{(i)\top} \left(\int_0^{2\pi} \frac{\mathbf{\Lambda}(\omega)\mathbf{N}}{\mathbf{d}^{(i-1)\top}\mathbf{\Lambda}(\omega)\mathbf{d}^{(i-1)}} d\omega \right) \mathbf{d}^{(i)} \\ &= \mathbf{d}^{(i)\top} \hat{\mathbf{K}}^{(i-1)} \mathbf{d}^{(i)}\end{aligned}\tag{7.10}$$

Note that the term $\mathbf{\Lambda}(\omega)\mathbf{N}$ causes $\kappa_L^{(i)}$ to lose its symmetry, making it difficult to be applied as a general constraint for both convex and nonlinear objectives. To improve its utility, we relax the constraint by introducing an additional error term $\kappa_R^{(i)} = \mathbf{d}^{(i)\top} \hat{\mathbf{K}}^{\top(i-1)} \mathbf{d}^{(i)}$ to form the overall constraint

$$\begin{aligned}\kappa^{(i)} &= \frac{1}{2}(\kappa_L + \kappa_R) \\ &= \mathbf{d}^{(i)\top} \left(\int_0^{2\pi} \frac{\frac{1}{2}(\mathbf{\Lambda}(\omega)\mathbf{N} + \mathbf{N}^\top \mathbf{\Lambda}^\top(\omega))}{\mathbf{d}^{(i-1)\top}\mathbf{\Lambda}(\omega)\mathbf{d}^{(i-1)}} d\omega \right) \mathbf{d}^{(i)} \\ &= \mathbf{d}^{(i)\top} \mathbf{K}^{(i-1)} \mathbf{d}^{(i)}\end{aligned}\tag{7.11}$$

Note that the additional κ_R is a valid constraint by itself, since it is formulated by simply taking the transpose of the original κ_L . While more sophisticated methods to relax the constraint such as semidefinite relaxation [78] exist, a simple grouping of the terms is sufficient for the eigenfilter approach. Based on our formulation of the constraint, $\mathbf{K}^{(i)}$ is symmetric, real valued, and positive definite. The constraint can therefore be easily combined with the waveguide loss compensating designs presented in Chapter 4. The constraint can also be combined with traditional allpass approaches such as the eigenfilter design [63] when the effect of waveguide loss is negligible. By introducing a fabrication based constraint into the eigenfilter method, we can ensure that the design is realizable while still having the benefit of a fast design algorithm. When the loss effect can be ignored, combining the constraint with the eigenfilter approach results in a Quadratic Constrained Quadratic Programming (QCQP) optimization problem, which can be efficiently solved using Rayleigh Quotients.

7.2 Simulation Results

To isolate the performance gain from considering a constraint on the maximum pole radii, we combine the proposed constraint with the least squares based eigenfilter approach [63]. The eigenfilter cost function is

$$\begin{aligned} E_{\text{EF}}^{(i)} &\approx \mathbf{d}^{(i)\top} \left(4 \int_R W(\omega) \frac{\mathbf{S}_\beta(\omega)}{\mathbf{d}^{(i-1)\top} \mathbf{C}_\beta(\omega) \mathbf{a}^{(i-1)}} d\omega \right) \mathbf{d}^{(i)} \\ &= \mathbf{d}^{(i)\top} \mathbf{P}^{(i-1)} \mathbf{d}^{(i)} \end{aligned} \quad (7.12)$$

where $\mathbf{S}_\beta(\omega) = \mathbf{s}_\beta(\omega) \mathbf{s}_\beta(\omega)^\top$, $\mathbf{C}_\beta(\omega) = \mathbf{c}_\beta(\omega) \mathbf{c}_\beta(\omega)^\top$, and

$$\mathbf{s}_\beta(\omega) = \begin{bmatrix} \sin \beta(\omega) & \cdots & \sin(\beta(\omega) - N\omega) \end{bmatrix}^\top \quad (7.13)$$

$$\mathbf{c}_\beta(\omega) = \begin{bmatrix} \cos \beta(\omega) & \cdots & \cos(\beta(\omega) - N\omega) \end{bmatrix}^\top \quad (7.14)$$

where we defined $\beta(\omega) = \frac{1}{2} (\Theta_{\text{pre}}(\omega) + N\omega)$. The modified eigenfilter design algorithm with the constraint is shown in Algorithm 5. The constrained filter design

Algorithm 5 Constrained Allpass Filter Design

Require: Prescribed phase response $\Theta_{\text{pre}}(\omega)$, filter order N , weight function $W(\omega)$, λ ;
 $\mathbf{d}^{(0)}$ = eigenvector corresponding to the minimum eigenvalue of $\int_R W(\omega) \mathbf{S}_\beta(\omega)$;
 $i=1$;
repeat
 Formulate $\lambda \mathbf{P}^{(i-1)} + (1 - \lambda) \mathbf{K}^{(i-1)}$;
 Compute $\mathbf{d}^{(i)}$ = eigenvector corresponding to the minimum eigenvalue;
 $i = i + 1$;
until $\|\mathbf{d}^{(i)} - \mathbf{d}^{(i-1)}\| < \delta$

algorithm allows for frequency selective realizations using allpass substructures in case of very narrowband operations. Let us consider the design of an allpass filter with prescribed phase response

$$\Theta_{\text{pre}}(\omega) = \begin{cases} -(N-1)\omega & 0 \leq \omega \leq w_p \\ -(N-1)\omega + \pi & \omega_s \leq \omega \leq \pi \end{cases} \quad (7.15)$$

Table 7.1: Comparison results showing the passband attenuation A_p , stopband attenuation A_s and the maximum pole magnitude $r_{\max}$ for various allpass designs.

Requirement	$r_{\max}$	A_p (dB)	A_s (dB)
unconstrained	0.9181	-0.002717	-35.1
$r_{\max} < 0.9$	0.8879	-0.02984	-21.73
$r_{\max} < 0.8$	0.7822	-0.426	-11.67
$r_{\max} < 0.7$	0.6797	-1.287	-9.321

We can then use the resulting allpass filter to form a lowpass filter (2.11). We first consider a benign example with $\omega_p = 0.1\pi$, $\omega_s = 0.2\pi$, and $N = 15$. Figure 7.1 shows the design results for the unconstrained approach in contrast to limiting the largest pole magnitudes 0.9, 0.8 and 0.7. The characteristics of the various designs are summarized in Table 1. Notice that while the algorithm successfully yields an allpass design with the required maximum pole magnitude, it does so at the cost of reducing passband and stopband attenuations. To compare with other least squares designs, Figure 7.2 shows the result from simply setting thresholds at 0.9, 0.8 and 0.7. In this setup, the unconstrained eigenfilter method is first used to solve for the allpass filter coefficients. The poles with magnitudes higher than the threshold are then scaled down to be within the constraint. It is evident that a systematic design algorithm is necessary since simple scaling of the poles greatly deteriorates the filter response.

In the second example, we consider a very narrowband lowpass filter design with $N = 15$, $\omega_p = 0.0025\pi$, $\omega_s = 0.01\pi$. These design requirements closely resemble the expected operation frequency ranges of a DTCOP setup. Figure 7.3 shows the overall design result of the constrained approach with maximum radius set to 0.97. The resulting maximum pole radius is 0.969, with $A_p = -0.3141\text{dB}$, and $A_s = -4.096\text{dB}$. Note that similar design requirements with the same pole magnitude constraint were targeted in [22], but was only achieved through an *ad-hoc* method that involved a $N = 24$ filter as well as design complications such as interpolation and post filtering. The constrained eigenfilter approach is able to produce a realizable design through a generalized algorithm at lower filter order and lower complexity.

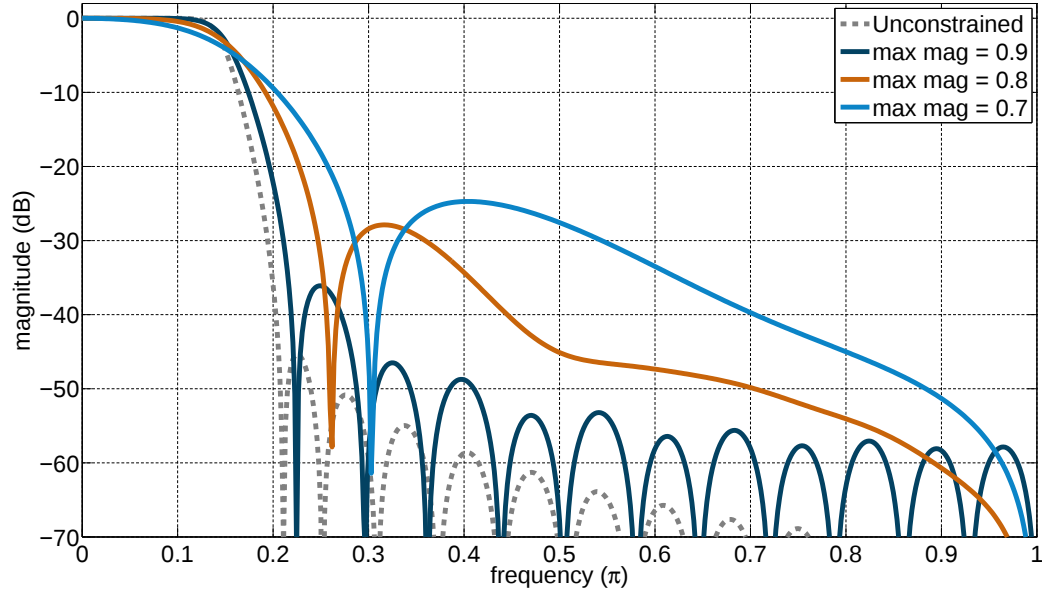


Figure 7.1: Lowpass design using allpass subsections showing comparison results of unconstrained approach versus constraining the maximum pole magnitudes to 0.9, 0.8 and 0.7.

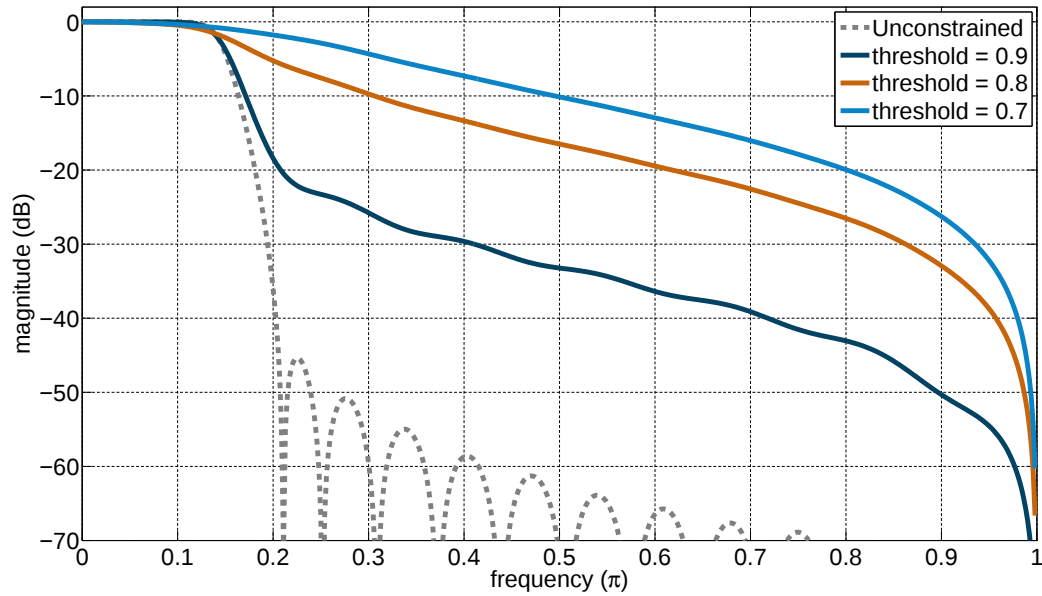


Figure 7.2: Results from setting a hard threshold of 0.9, 0.8 and 0.7 on the maximum pole magnitudes. Poles with magnitudes outside of the threshold are down scaled.

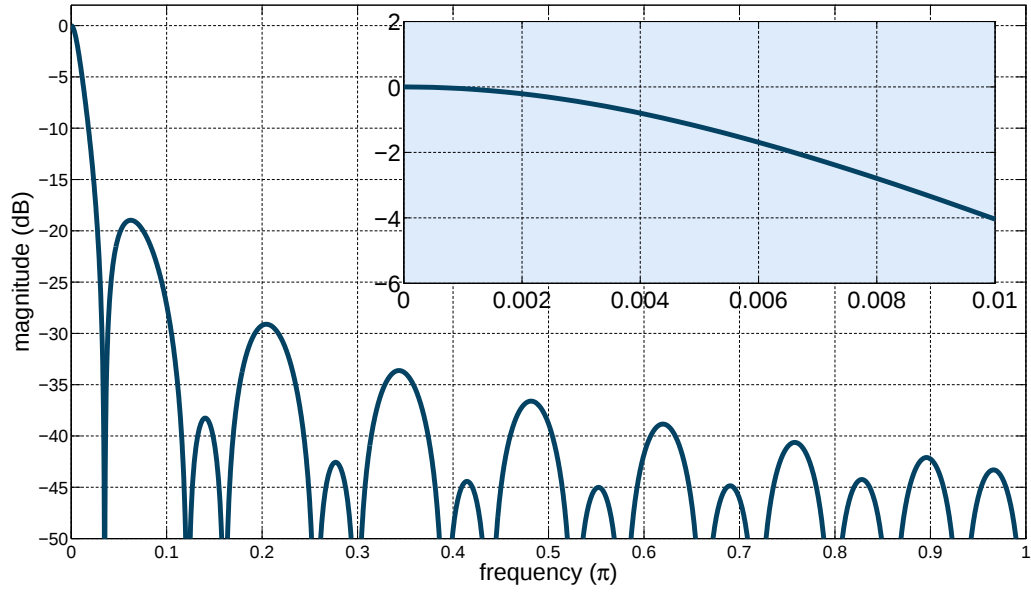


Figure 7.3: Narrowband design using allpass subsections from the constrained approach with maximum pole magnitude of 0.97. A magnification of the passband and stopband region is also shown.

7.3 Summary

We outline the formulation of a constraint that captures a prescribed requirement on the maximum pole magnitude. The constraint is first formulated through Argument Principle, and then relaxed to yield a symmetric matrix that can be easily combined with the cost function of an iterative allpass design algorithm. The proposed constraint will be excellent in parametrizing allpass filters realized using photonic components in the cascade form since it directly incorporates limitations that result from capabilities of conventional fabrication processes. The derivation for the cascade structures will also be able to serve as a basis for future lattice based constraint developments.

7.4 Acknowledgments

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Chapter 8

Model for Fabrication Inaccuracy

In previous chapters, we addressed the imperfections of photonic fabrication that are deterministic and known *a-priori*. An issue that has not yet been considered is the inaccuracy that are inherent to the process of realizing photonic waveguides. The effect is stochastic in nature and is difficult to compensate. However, a proper statistical analysis is critical in understanding the effect of the resulting random phase error. This chapter provides a study on the random effect, and provides a set of models that capture the statistical characteristics of the photonic allpass filters post fabrication. These stochastic measures are critical for yield and performance analysis, and can be utilized to derive filter design algorithms that can improve the overall performance of a suboptimal fabrication process. In particular, we are interested in deriving the average performance of photonic allpass filters that underwent the same processing.

8.1 Physical Origins of Error

The process of creating a sample via electron beam lithography may be divided into the following general steps [95]. To begin, the wafer on which the pattern will be etched must be spin coated with a layer of resist. The wafer is then secured to a mount and inserted into the lithography apparatus. Prior to writing, the electron beam must be aligned, focused, and corrected for aberrations such as astigmatism. The desired pattern is then transferred to the resist by scanning

the beam across the surface of the wafer. For small areas, this is accomplished by deflecting the beam, while larger areas require the wafer to be physically repositioned. The wafer is then removed from the apparatus and the resist is chemically developed. For negative resists, the development process removes the portion of resist that was not exposed to the electron beam (vice versa for positive resists). The wafer is then subjected to an etching process which selectively removes the portions that are not protected by resist.

Error can be introduced to the sample during each of the above steps, and are random in nature. The alignment and focusing of the electron beam are performed at a limited number of points on the wafer. Any mounting irregularity or imperfections in the wafer and resist that cause the sample to deviate from being perfectly planar will introduce distortion into the written pattern. The beam itself is subject to drift that interferes with beam alignment and resist exposure. Uncertainty associated with deflecting the beam during writing can lead to geometrical distortion of the pattern. Physically repositioning the sample during stitching can lead to error where adjacent write fields are stitched together. Finally, roughness introduced during the etching process introduces additional error.

Although there are many distinct sources of error during the fabrication of a photonic filter, they are all ultimately manifested as error in the physical dimensions of the waveguide. Error in dimension orthogonal to the direction of propagation of the optical signal translates into error in the waveguide effective index. Error in dimension parallel to the direction of signal propagation translates directly into optical path length error. Since repeatable sources of error can be precompensated for at the design stage, the expression of this type of error in an actual device may be considered to have zero mean. Likewise, this type of error is arguably uncorrelated over long sections of waveguide. In other words, the δ_i 's that capture the random phase errors of the individual sections can be assumed to be independent and identically distributed (i.i.d.).

As a preliminary characterization of the uncertainty associated with our fabrication process, we fabricated sets of waveguide Bragg mirror resonators with both 100 μm and 200 μm cavity lengths as shown in Figure 8.1. The observed

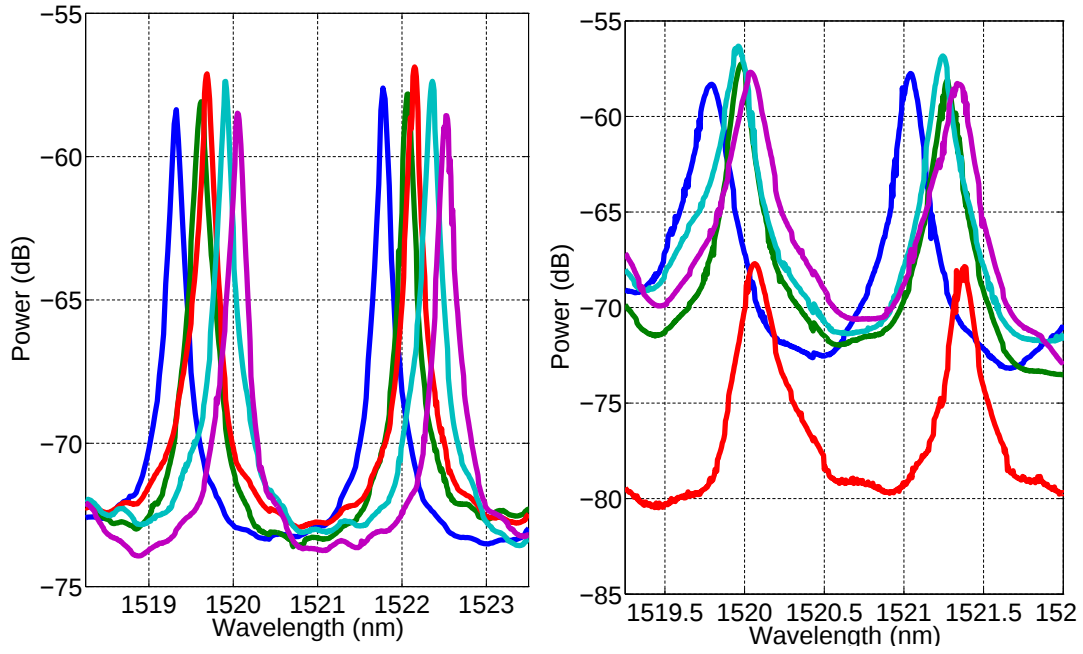


Figure 8.1: Transmission spectral responses of 5 nominally identical waveguide Bragg mirror Fabry-Pérot resonators with 100 μm (left) and 200 μm (right) resonance cavities. Note that the transmission of the red colored waveguide in (right) is lower than the others due to a defect in its input coupler.

standard deviation in relative optical path length for the samples with 100 μm cavity length is 0.030 μm . The observed standard deviation in relative optical path length for the samples with 200 μm cavity length is 0.031 μm , which can be readily converted into phase measures. As demonstrated by the Figure, the dimensional error introduces a shift in the locations of the poles of the filter. The following sections provide a detailed analysis and models for the random phase error.

8.2 First-Order Effect

Recall that the ideal first-order photonic allpass transfer function is

$$\hat{A}(z) = -e^{js} \frac{-\rho e^{-js} + z^{-1}}{1 - \rho e^{js} z^{-1}} \quad (8.1)$$

The phase delay s in the structure is a product of engineering and may contain errors that are stochastic by nature. Small optical path errors lead to large errors in the phase factor in the form of $e^{j(s+\delta)}$ according to

$$\delta = \frac{2\pi\Delta L \cdot n_{\text{eff}}}{\lambda} \quad (8.2)$$

where ΔL is the optical path length error, n_{eff} is the waveguide effective index and λ is the free space wavelength. For a real coefficient first-order photonic allpass, the fabrication imperfection on the device's length will therefore yield a transfer function

$$\hat{A}_e(z) = -e^{j(s+\delta)} \frac{-\rho e^{-j(s+\delta)} + z^{-1}}{1 - \rho e^{j(s+\delta)} z^{-1}} = \hat{A}(e^{-j\delta} z) \quad (8.3)$$

Note that the constant sign and phase term can be neglected without loss of generality. To simplify our analysis, let us instead consider the effect for the canonical form

$$A_e(z) = A(e^{-j\delta} z) = \frac{k_i^* + e^{j\delta} z^{-1}}{1 + k_i e^{j\delta} z^{-1}} \quad (8.4)$$

where k_i is a composite parameter representing the fabricated device values. The error δ therefore exists in both the feed-forward and feed-back signal paths within the waveguide shown in Figure 8.2.

For post fabrication yield and performance analysis, the average performance of photonic allpass filters that are fabricated within the same batch is critical. The expected value of $A_e(z)$ is

$$\mathbb{E}[A(e^{-j\delta} z)] = \mathbb{E} \left[\sum_{n=0}^{\infty} a(n) e^{jn\delta} z^{-n} \right] \quad (8.5)$$

where $a(n) \triangleq \mathcal{Z}^{-1}\{A(z)\}$ is the time domain response of the allpass filter obtained through inverse z -transform. Since the expectation is taken only on the random variable δ , all other parameters in (8.5) become deterministic. Therefore

$$\mathbb{E}[A(e^{-j\delta} z)] = \sum_{n=0}^{\infty} a(n) \mathbb{E}[e^{jn\delta}] z^{-n} = \sum_{n=0}^{\infty} \phi(n) a(n) z^{-n} \quad (8.6)$$

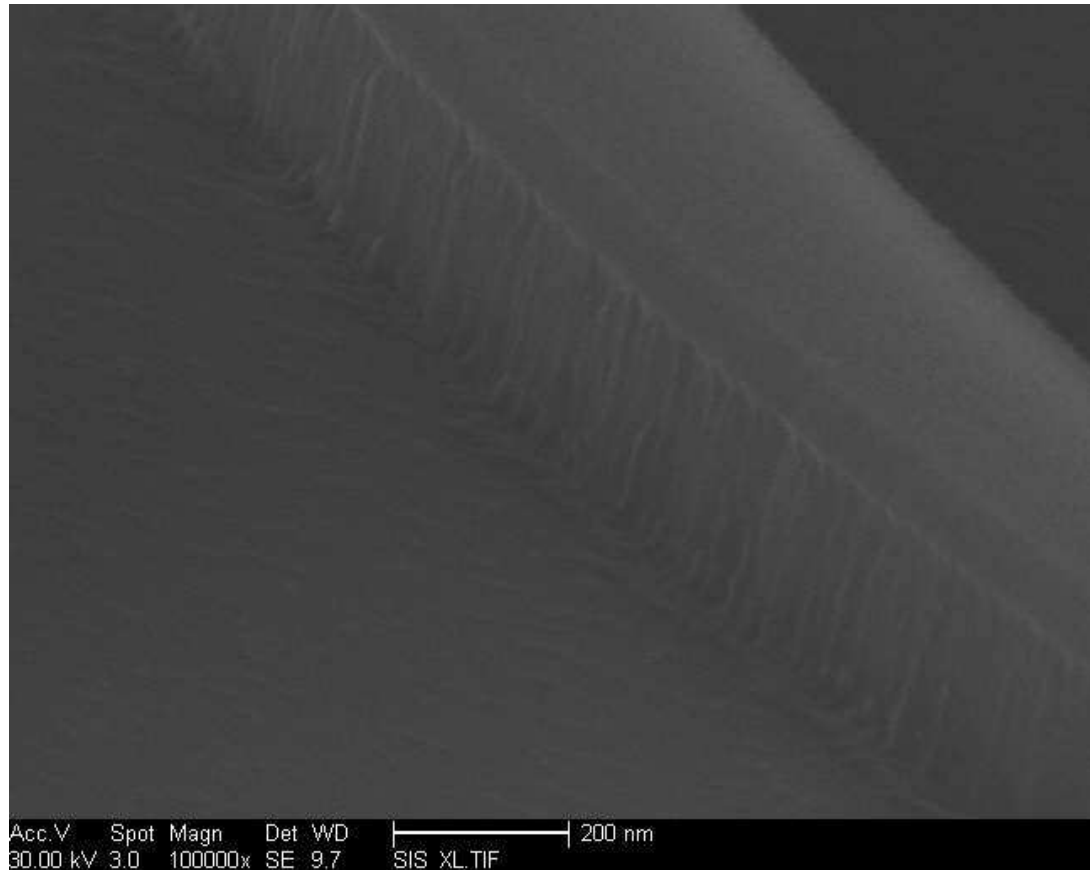


Figure 8.2: Perspective image of an uncladded silicon waveguide produced by RIE etching. The roughness in the waveguide sidewall is the source of scattering loss.

where

$$\phi(n) \triangleq \mathbb{E}[e^{jn\delta}] \quad (8.7)$$

can be recognized as the characteristic function of δ with parameter n . The right hand side of (8.6) is z -transform of the product of two sequences, $a(n)$ and $\phi(n)$, which can be re-expressed as the convolution of the respective z -transforms,

$$\mathbb{E}[A(e^{-j\delta}z)] = A(z) * \Phi(z) \quad (8.8)$$

where $\Phi(z) \triangleq \mathcal{Z}\{\phi(n)\}$ is the bilateral z -transform of the characteristic function of δ .

8.3 Cascade Form

We now apply the result of a single stage allpass to find the expected magnitude response of a N th-order cascade allpass filter. An N th-order cascade allpass filter is constructed by fabricating N first-order photonic allpasses, and then coupling the output of one section to the input port of the next. Since each section is independently created, the phase errors can be assumed to be independent. Let $A_e^{\text{cas}}(z)$ denote the error corrupted response of an N th-order cascade allpass filter, δ_i be the phase error associated with the i -th stage, and $A_i(z)$ be the error free response of the i th section. The expectation is

$$\mathbb{E}[A_e^{\text{cas}}(z)] = \prod_{i=1}^N \mathbb{E}[A_i(e^{-j\delta_i}z)] = \prod_{i=1}^N \Phi(z) * A_i(z) \quad (8.9)$$

since the phase error δ_i 's are assumed to be independent and identically distributed (i.i.d.) to yield $\Phi_i(z) = \Phi(z)$.

8.4 Lattice Form

Since the N th-order lattice allpass is constructed recursively, we will only examine the error corrupted response of the second stage. The analysis can then

be iteratively applied for higher-order lattice structures. An ideal second-order lattice allpass frequency response can be written as

$$A_{\text{lat}}^{(2)}(z) = S_1(zS_2^{-1}(z)) = \frac{k_1^* + z^{-1}S_2(z)}{1 + k_1z^{-1}S_2(z)} \quad (8.10)$$

Let $A_e^{\text{lat}}(z)$ denote the error corrupted response, we therefore have

$$A_e^{\text{lat}}(z) = \frac{k_1^* + e^{j\delta_1}z^{-1}S_2(e^{-j\delta_2}z)}{1 + k_1e^{j\delta_1}z^{-1}S_2(e^{-j\delta_2}z)} \quad (8.11)$$

We can again assume the δ_i 's to be i.i.d because the inaccuracy in the tooling is uncorrelated from one section to the next. We can further decompose the second-order response into two parts as

$$A_e^{\text{lat}}(z) = A_l(z) + A_r(z) \quad (8.12)$$

where

$$\begin{aligned} A_l(z) &= \frac{k_1^*}{1 + k_1e^{j\delta_1}z^{-1}S_2(e^{-j\delta_2}z)} \\ A_r(z) &= \frac{e^{j\delta_1}z^{-1}S_2(e^{-j\delta_2}z)}{1 + k_1e^{j\delta_1}z^{-1}S_2(e^{-j\delta_2}z)} \end{aligned} \quad (8.13)$$

To find the expected performance on each part of $A_e^{\text{lat}}(z)$, we introduce a mixed domain approach. First note that $|S_2(e^{-j\delta_2}z)| = 1$, and we can write $A_l(z)$ as

$$A_l(z) = k_1^* \sum_{n=0}^{\infty} (-e^{j\delta_1}S_2(e^{-j\delta_2}z)k_1)^n z^{-n} \quad (8.14)$$

The expectation can therefore be found to be

$$\mathbb{E}[A_l(z)] = k_1^* \sum_{n=0}^{\infty} \phi(n) \mathbb{E}[S_2^n(e^{-j\delta_2}z)] (-k_1)^n z^{-n} \quad (8.15)$$

Through discrete-time domain analysis, we see that

$$\mathbb{E}[S_2(e^{-j\delta_2}z)^n] = \sum_{m=0}^{\infty} \mathbb{E}[a'(m) * a'(m) * \dots] z^{-m} \quad (8.16)$$

where $a'(m) = e^{jm\delta_2}a_2(m)$, and there are a total of n terms in the chain convolution. Note that

$$\begin{aligned} a'(m) * a'(m) &= \sum_{l=-\infty}^{\infty} e^{jl\delta_2}a_2(l)e^{j(m-l)\delta_2}a_2(m-l) \\ &= e^{jm\delta_2}a_2(m) * a_2(m) \end{aligned} \quad (8.17)$$

Inserting the term back into the expectation, we can find the overall result

$$\mathbb{E}[S_2(e^{-j\delta_2}z)^n] = \Phi(z) * S_2^n(z) \quad (8.18)$$

The expectation on the first term $A_l(z)$ is therefore

$$\mathbb{E}[A_l(z)] = k_1^* \sum_{n=0}^{\infty} \phi(n)\Omega(n, z)z^{-n} \quad (8.19)$$

where $\Omega(n, z) = \Phi(z) * (-k_1 S_2(z))^n$. While the result is a mixed domain response, $\mathbb{E}[A_l(z)]$ is consisted only of non-stochastic terms that can be readily evaluated. The expectation on the second term $A_r(z)$ can be found using a similar approach to be

$$\mathbb{E}[A_r(z)] = \frac{1}{k_1} \sum_{n=1}^{\infty} \phi(n)\Omega(n, z)z^{-n} \quad (8.20)$$

The overall expectation on a second-order lattice allpass is therefore

$$\mathbb{E}[A_e^{\text{lat}}(z)] = k_1^* \sum_{n=0}^{\infty} \phi(n)\Omega(n, z)z^{-n} + \frac{1}{k_1} \sum_{n=1}^{\infty} \phi(n)\Omega(n, z)z^{-n} \quad (8.21)$$

Depending on the realization, the phase error that occurs during first-order photonic allpass fabrication has drastically different effect on the overall system. A definitive mathematical model for both the cascade and the lattice realizations allows for more in-depth understanding of how to use a phase error corrupted allpass

in a photonic signal processor.

8.5 Frequency Selective Filters

To understand a photonic signal processor's stochastic behavior, we can examine the expected performance of the frequency selective filter's magnitude response under error. Recall that (2.11) is ideal for photonic realization of frequency selective filters. Let $H_e(z)$ denote the filter response of (2.11) under fabrication error, the magnitude response is

$$|H_e(z)|^2 = \frac{1}{4} \left[1 + e^{-j\tau} z^{N-1} A_e(z) + e^{j\tau} z^{-(N-1)} A_e^*(1/z^*) + |A_e(z)|^2 \right] \quad (8.22)$$

where $A_e(z)$ is the response under error for the N th-order photonic allpass and τ is the phase error from the plain waveguide. The magnitude of the allpass response $|A_e(z)|^2$ is unity regardless of error. We therefore have

$$|H_e(z)|^2 = \frac{1}{4} \left[2 + e^{-j\tau} z^{N-1} A_e(z) + e^{j\tau} z^{-(N-1)} A_e^*(1/z^*) \right] \quad (8.23)$$

The plain waveguide for realizing the delay element $z^{-(N-1)}$ is of a different process from fabricating the photonic allpasses, and therefore the associated phase delay τ is uncorrelated with the error corrupted allpass response $A_e(z)$. The expectation on the error corrupted magnitude response is therefore

$$\mathbb{E}[|H_e(z)|^2] = \frac{1}{4} \left[2 + \mathbb{E}[e^{-j\tau}] z^{N-1} \mathbb{E}[A_e(z)] + \mathbb{E}[e^{j\tau}] z^{-(N-1)} \mathbb{E}[A_e^*(1/z^*)] \right] \quad (8.24)$$

Notice that $\mathbb{E}[e^{-j\tau}] = \psi(-1)$ is the characteristic function of τ with a parameter -1 , and similarly $\mathbb{E}[e^{j\tau}] = \psi(1)$. Therefore,

$$\mathbb{E}[|H_e(z)|^2] = \frac{1}{4} \left[2 + \psi(-1) z^{N-1} \mathbb{E}[A_e(z)] + \psi(1) z^{-(N-1)} \mathbb{E}[A_e^*(1/z^*)] \right] \quad (8.25)$$

Finding the expected performance of a frequency selective photonic filter therefore critically depends on the performance of the N th-order photonic allpass

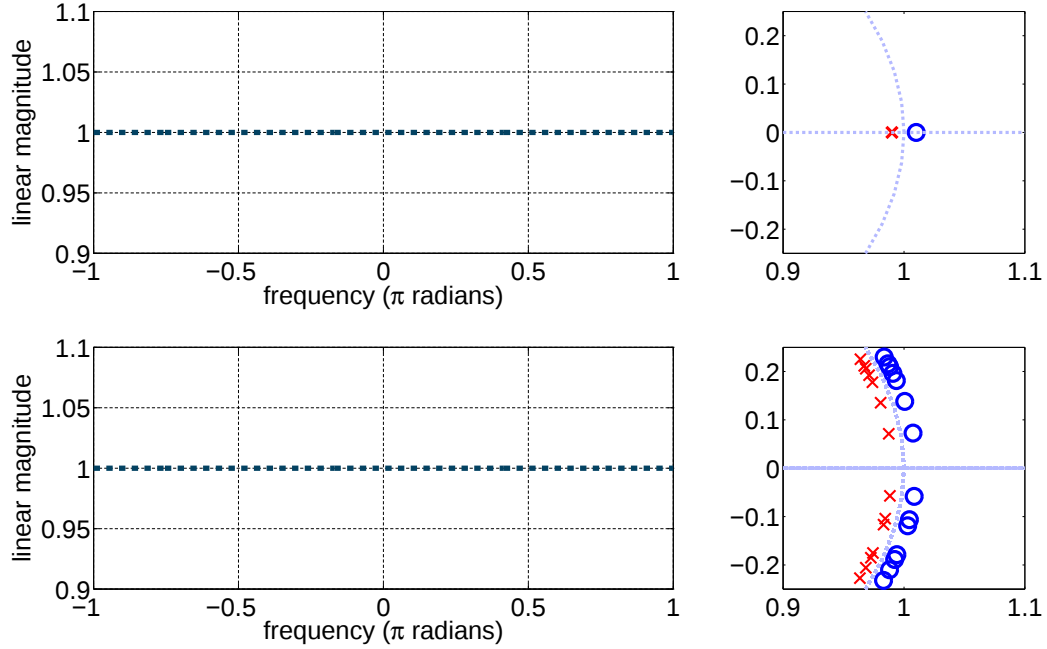


Figure 8.3: Ideal response of a first-order allpass filter $A(z)$ and the effect of fabrication error with $\delta \sim \mathcal{N}(0, \frac{\pi}{3})$. The pole zero pair displaced by the same degree under each trial.

$E[A_e(z)]$. A higher-order system is constructed from single stage allpass sections in cascade of lattice form, making the N th-order error performance structurally dependent. Note that $E[A_e^*(1/z^*)]$ can be easily derived from $E[A_e(z)]$ by a change of variables.

8.6 Simulation Results

We first demonstrate that the allpass magnitude response is immune to the effects of the phase error δ , since $E[|A(e^{-j\delta}z)|^2] = 1$ regardless of the pole and zero displacements. Figure 8.3 shows the magnitude response and pole-zero map for a first-order allpass filter corrupted by random error of Gaussian distribution $\delta \sim \mathcal{N}(0, \frac{\pi}{3})$, and the corresponding error-free responses.

While the magnitude response of the allpass filter is not affected, the effect becomes much more prominent in a frequency selective architecture. The displacements of the poles and zeros of the allpass filter directly translate to an offset in

the transition band of the frequency selective filter. An examination in the z -plane on the average of these displacements would result in the probability density function (p.d.f.) of the random variables δ_i 's. From here it can be deduced that the transition bands of a frequency selective filter would reflect the p.d.f of the random error, and is readily demonstrated by our model. The effect can be immediately observed in a $N = 1$ frequency selective filter

$$H(z) = \frac{1}{2} (1 + A(z)) \quad (8.26)$$

where $A(z)$ is a first-order allpass filter. Notice that there is no fabrication error coupled with the delay path, since the source signal is available. Let $H_e(z)$ denote the performance under error, the expectation is

$$E[|H_e(z)|^2] = \frac{1}{4} (2 + \Phi(z) * A_i(z) + \Phi(z) * A_i^*(1/z^*)) \quad (8.27)$$

The smoothing kernel $\Phi(z)$ has unit area, or $\int_{-\infty}^{\infty} \Phi(z) dz = 1$ because it is the transform of a distribution for a random variable. We can then readily express the constant 1 as $1 * \Phi(z)$. Therefore the expected performance of the filter can be observed as

$$E[|H_e(z)|^2] = \Phi(z) * \left| \frac{1}{2} (1 + A(z)) \right|^2 \quad (8.28)$$

It now becomes obvious that the kernel $\Phi(z)$ has a smoothing effect on the frequency selective filters. Figure 8.4 shows the effect on the location of the transition band on a first-order system. Figures 8.5 and 8.6 subsequently demonstrate the performance of our error model for the extreme cases of $\delta \sim \mathcal{N}(0, \frac{\pi}{3})$ and $\delta \sim \mathcal{N}(0, \frac{\pi}{32})$. In each result, the ensemble average of 1000 independent trials is calculated and plotted along with the theoretical model.

To evaluate the effect in a cascade realization, consider a second-order system

$$H(z) = \frac{1}{2} (z^{-1} + A_{\text{cas}}^{(2)}(z)) \quad (8.29)$$

where $A_{\text{cas}}^{(2)}(z)$ is a second-order allpass filter with coefficients c_1, c_2 arbitrarily chosen as -0.5291 and 0.546 . Note that we are ignoring the error γ on the delay

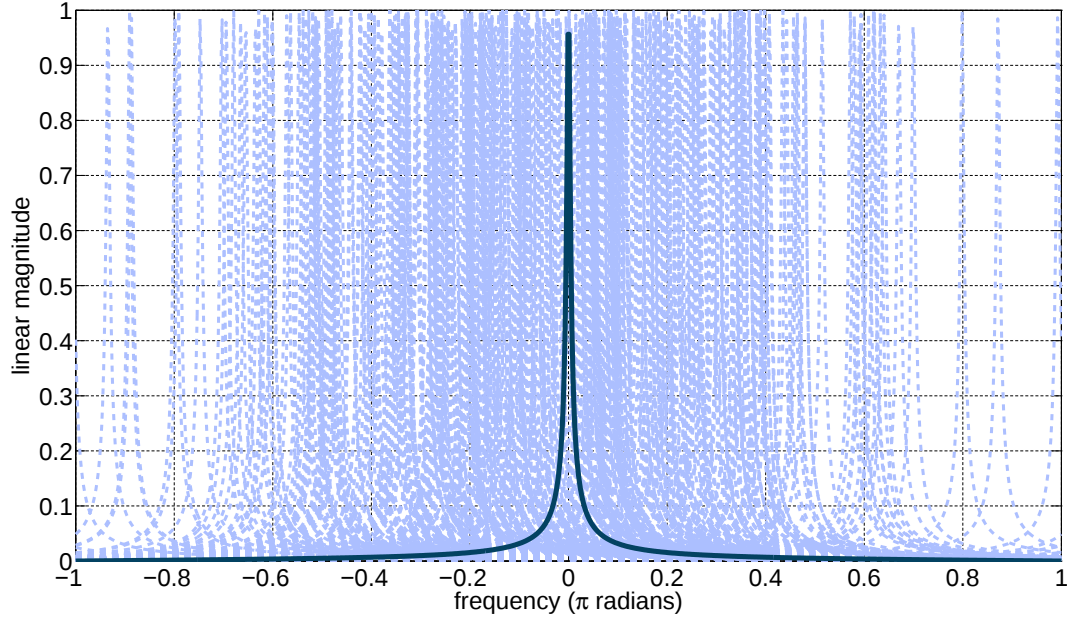


Figure 8.4: Ideal performance of $H(z) = \frac{1}{2}(1 + A(z))$ and the effect of fabrication error. $A(z)$ is a first-order allpass filter with filter coefficient arbitrarily chosen at -0.99 .

term z^{-n} to emphasize the effect from fabricating the photonic allpass filter. The ideal magnitude response and the effect of the error are shown in Figure 8.7. Figure 8.8 subsequently demonstrates the performance of our error model in comparison to the ensemble average of 1000 independent trails with $\delta_i \sim \mathcal{N}(0, \frac{\pi}{16})$. The results for a lattice filter is obtained from

$$H(z) = \frac{1}{2} \left(z^{-1} + A_{\text{lat}}^{(2)}(z) \right) \quad (8.30)$$

where $A_{\text{lat}}^{(2)}(z)$ is a second-order allpass filter with reflection coefficients derived from the cascade system as $k_1 = -0.2889$ and $k_2 = 0.0238$. The behavior of the filter under error and simulation showing the validity of the lattice error model are shown in Figures 8.9 and 8.10.

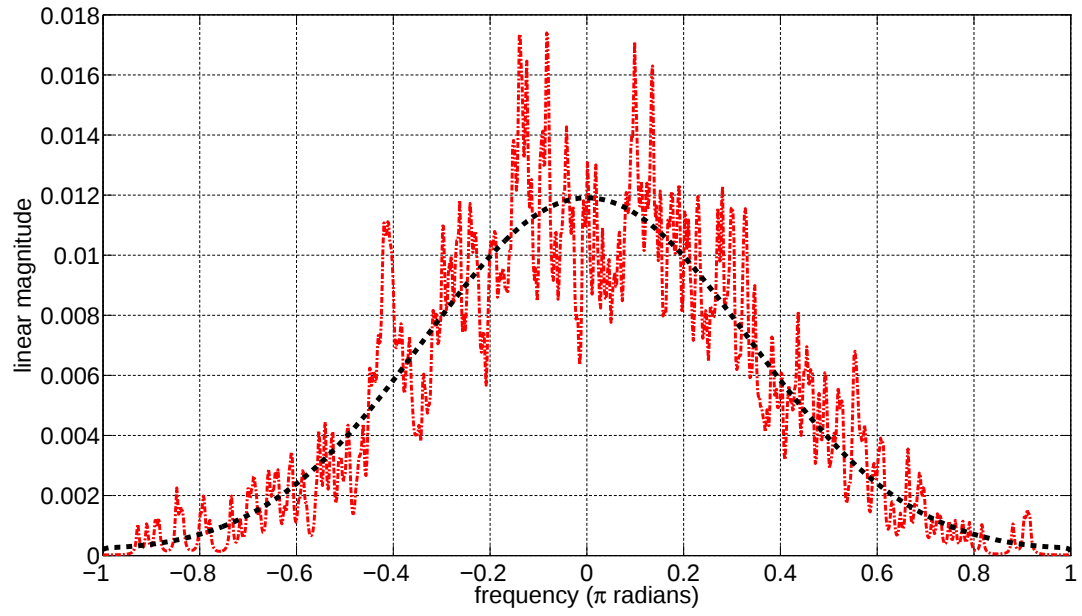


Figure 8.5: Calculated model and the ensemble average of 1000 independent trials with $\mathbf{d} \sim \mathcal{N}(0, \frac{\pi}{3})$.

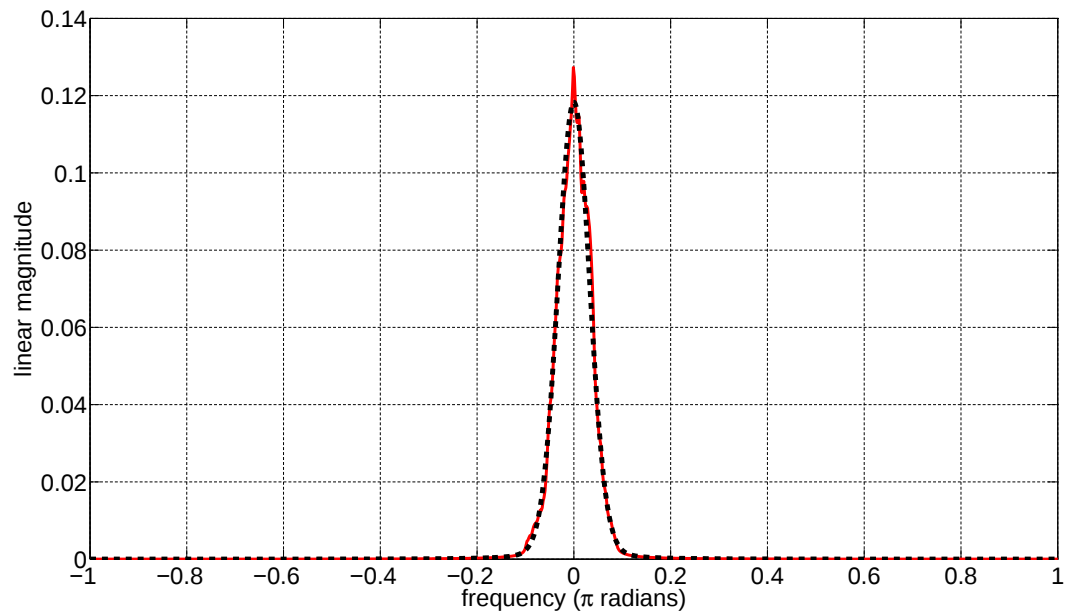


Figure 8.6: Calculated model and the ensemble average of 1000 independent trials with $\mathbf{d} \sim \mathcal{N}(0, \frac{\pi}{32})$.

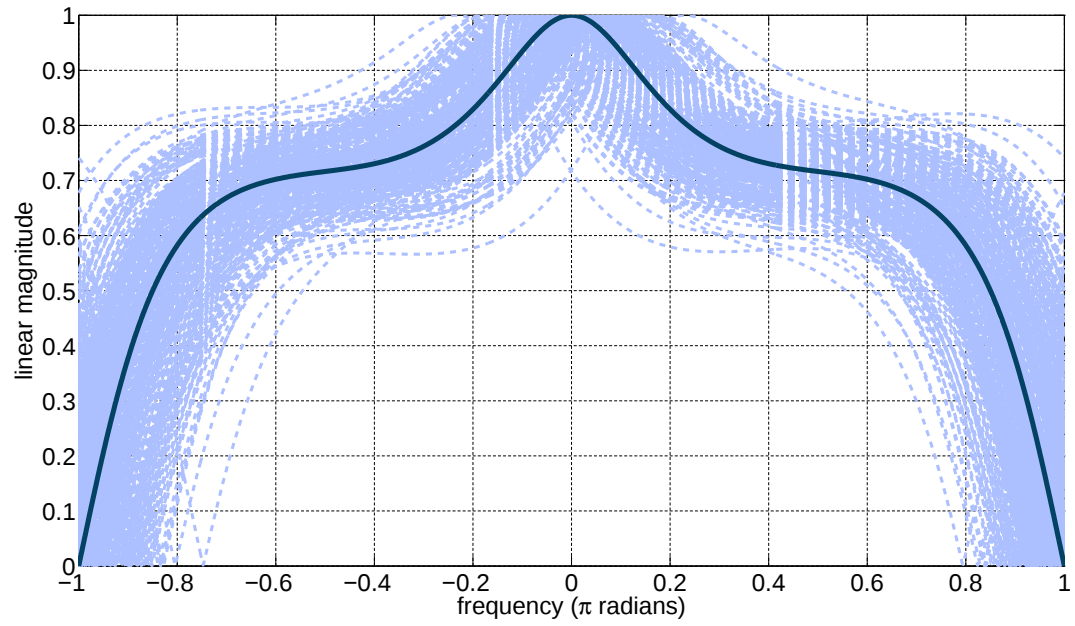


Figure 8.7: Ideal performance of $H(z) = \frac{1}{2} \left(z^{-1} + A_{\text{cas}}^{(2)}(z) \right)$ and the effect of fabrication error.

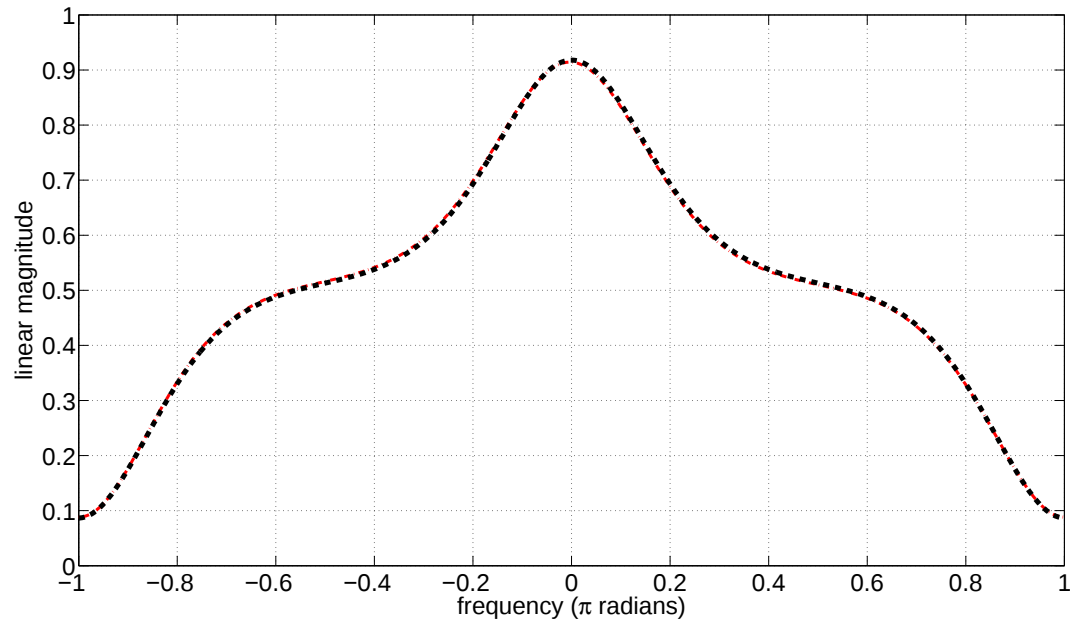


Figure 8.8: Calculated model and the ensemble average of 1000 independent trials with $d_i \sim \mathcal{N}(0, \frac{\pi}{16})$.

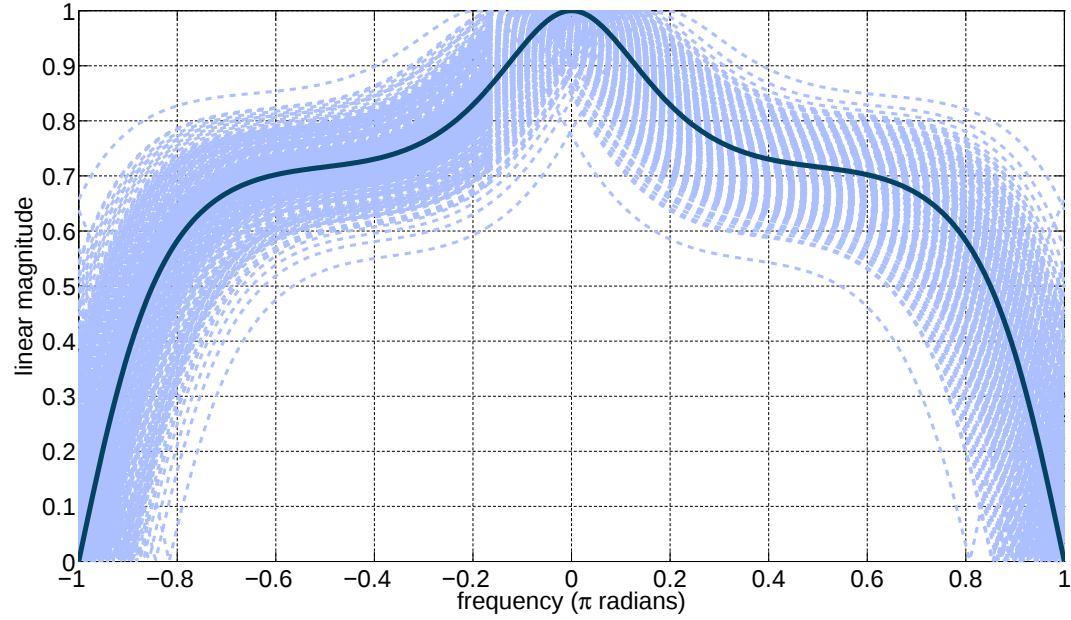


Figure 8.9: Ideal performance of $H(z) = \frac{1}{2} (z^{-1} + A_{\text{lat}}^{(2)}(z))$ and the effect of fabrication error.

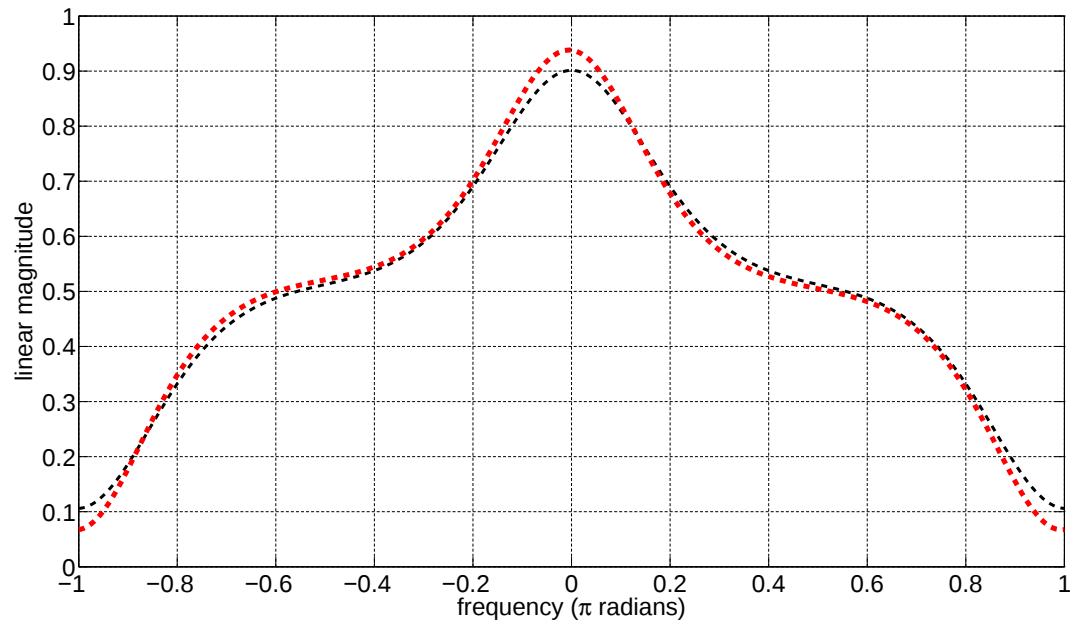


Figure 8.10: Calculated model and the ensemble average of 1000 independent trials with $d_i \sim \mathcal{N}(0, \frac{\pi}{16})$.

8.7 Summary

There are two primary sources of error that distort a photonic system response and prevent it from providing an ideal allpass response. This chapter examines the stochastic random phase error that arises from dimensional inaccuracy in fabricating the waveguides. While the waveguide loss can be known *a-priori* and compensated for in the design phase, the phase error is random in nature and cannot be easily addressed before fabricating the actual device. We can, however, analyze the effect for post fabrication performance and yield analysis. This chapter provides a detailed mathematical model on the phase error for photonic allpass filters that are fabricated within the same batch, regardless of the realization structure used.

8.8 Acknowledgments

This chapter, in full, is a reprint of a published conference paper in IEEE Global Communications Conference 2011.

Chapter 9

Future Work

To support the increasing amount of data being transmitted in future telecommunication infrastructures, all-optical networks are optimal and necessary. In order to further facilitate the development of digital filter design algorithms for photonic signal processors, we recommend the following topics for future research

- Design algorithm that is not restricted to real coefficients. Unlike digital signal processors, photonic first-order allpass sections can be easily fabricated to provide complex coefficient. From a design perspective, removing the restriction on real coefficients may both reduce filter complexity and improve overall performance.
- Improve the allpass-based filter bank design algorithm. While the proposed method is able to provide high performance filter banks based on allpass filters, the algorithm contains room for improvement through high complexity optimization techniques. The current method uses a simple gradient descent technique to arrive at the optimal solution. A future research direction could be to examine the exact optimization setup and derive an *ad-hoc* techniques specifically for the highly non-linear filter design cost function.
- Study the effect of randomness on the waveguide power loss. The proposed algorithms assume that the waveguide power loss can be expressed through its nominal value. While this assumption is justified for large scale fabrication

settings, the research community could benefit from a detailed analysis on how the randomness affects the filter design results.

- Propose a filter design algorithm that can mitigate the random phase error from fabrication. This dissertation provides a mathematical model for understanding the effect of dimensional offsets in the waveguide. An extension work could be to derive filter design algorithms that consider these effects. Such design techniques would improve yield and increase throughput in a realistic fabrication setting.
- Analyze the performance of the filters designed using the proposed methods in high bitrate optical communication networks. To confirm the superiority of the proposed design algorithms, it would be beneficial to deploy the design filters in optical networks that can support telecommunication protocols such as MPSK and QAM. The resulting analysis would be able to aid the seamless integration of the photonic signal processors in optical networks.
- Consider other sources of error that affect photonic integrated circuits. Effects such as temperature grating also alter the performance of photonic devices. A close examination of how errors other than the waveguide power loss affect the resulting filter performance is necessary.
- Derive alternative filter bank structures based on allpass elements. Other allpass-based filter banks, such as the cosine modulated filter bank, have also been proven to be useful in digital communications. It would be worthwhile to study which filter bank structure best suits the need of optical signal processing.
- Derive a constraint for the lattice form that governs the maximum coefficient value of the individual sections. Although being able to constrain the maximum pole radius allows for readily realizable cascade forms of N th-order allpass filters, an equivalent form for the lattice structure has yet to be derived. The lattice realization can often be the preferred form due to their

small footprint. A direct consideration of the realistic component values for the particular structure will therefore be invaluable.

Chapter 10

Conclusion

This dissertation presents a digital signal processing based investigation of realistic photonic allpass filters before and after fabrication. While traditional design methods can be directly applied, artifacts that are specific to photonic signal processors cannot be readily addressed. We present allpass design techniques that can mitigate the deterministic waveguide power loss effect, which can be readily accomplished prior to fabrication. For post fabrication, we derive a model that can characterize the stochastic dimensional inaccuracy for yield and performance analysis.

To combat the waveguide power loss, we present a set of design techniques that directly absorbs the effect into the filter design cost function. In order to satisfy the different needs of various photonic applications, we present a class of filter designs based on allpass filters for phase compensators, bandpass filters, and filter banks. For phase compensators, we ignore the effect of loss on the magnitude response of the photonic allpass filter, and maximize the performance gain from the corrupted structure purely in terms of phase. This setup is ideal for applications where the phase behavior dominates the overall system performance, such as using allpass structures for dispersion compensation in WDM. In this setup, the filter design optimization problem becomes highly non-linear and NP-hard due to the biased weighing effect from the waveguide loss. To provide a solution, we instead consider a relaxed problem that can be solved using a global search approach Branch and Bound. The resulting filter coefficients can also be further improved

by using the proposed iterative refinement algorithm. Simulation results show that the proposed method yields significant performance gain in terms of the maximum phase error.

An extension from the design of pure allpass filters is the application of allpass in bandpass structures. An arbitrary frequency selective filter can be constructed simply using an allpass and a delay through proper phase matching of the two branches. The waveguide power loss corrupts the photonic allpass and causes it to lose the unity magnitude behavior, which in turn causes the overall system to violate the phase matching conditions. We propose a lowpass filter design technique that examines the loss effect in its entirety for the specific magnitude and phase requirements under a given application. The resulting algorithm significantly outperforms traditional design techniques such as minimax, which are not catered towards photonic implementations. Simulation results demonstrate that the proposed algorithm is able to provide exponential performance gain with shrinking transition bandwidth while maintaining constant group delay in the passband.

An advanced application of the allpass filter is the DFT allpass filter bank. While the concept of sub-band processing exists in the photonic domain, the proper design of the filters that are crucial to the operation is absent. The allpass-based filter bank is first proposed for digital and electrical implementation, but saw limited application due to artifacts that are inherent to the structure. We demonstrate in this dissertation how the waveguide loss effect can be optimally used to mitigate the undesirable effects, which in turn proves the allpass-based filter bank structure to be an ideal setup for photonic implementation. We present a modified structure along with a design algorithm to solving the nonlinear optimization that arises when formulating the problem for photonic allpass-based filter bank. Simulation results demonstrate the the proposed architecture is ideal for deployment in high density packing optical communication architectures.

A parallel consideration when deriving filter designs that are practical for photonic implementations is the parameter values of the individual allpasses. Conventional fabrication technology limits the maximum allowable pole magnitude of a first-order photonic allpass section. In this dissertation, we demonstrate how

such limitation can be directly formulated into a quadratic constraint on the N th-order filter coefficients in the cascade setup. The constraint can then be readily combined with the proposed waveguide loss compensating algorithms.

While deterministic effects such as waveguide power loss and maximum allowable parameter values can be considered prior to fabrication, stochastic artifacts such as inaccuracy in the waveguide dimensions are difficult to address. Although the challenge lies largely within process engineering, a signal processing technique can be taken to provide models for post fabrication yield analysis. We present a stochastic model for the random phase error resulting from tooling inaccuracies. Both cascade and lattice forms are carefully examined, and their propagating effects to lowpass based on allpass substructures are demonstrated.

With a set of loss compensating designs that can be applied prior to fabrication and a filter performance model for post fabrication analysis, we have integrated digital signal processing techniques to physical realizations of the photonic structures. Through a class of allpass-based designs that carefully considers the waveguide power loss effect specific to photonic implementation, we enable the realistic fabrication of high performance integrated optics. The resulting algorithms and models will facilitate the widespread of photonic filter applications and aid in the development of next generation communication networks.

This chapter, in part, is a reprint of the published journal papers in OSA Optics Express, December 2013, IEEE Journal on Lightwave Technology, March 2014, the published conference paper in IEEE International Conference on Acoustics, Speech, and Signal Processing 2011, IEEE Global Communications Conference 2011, European Signal Processing Conference 2012, and a submitted paper to IEEE Journal on Lightwave Technology.

Appendix A

Narrowband Photonic Filters

While the focus of the dissertation is on the design of various photonic filters based on allpass structures that can compensate for the effect of waveguide power loss, it is also important to consider the general design challenge faced by photonic devices. The carrier frequency of photonic signal processors can be 100 GHz to 30 THz [50], while the bandwidth of the inputs are often within the megahertz range of RF signals. Such specifications translate to the need for designing filters with extremely narrow bandwidth, and cause classical algorithms to fail. In this section, we consider the design approach to a narrowband IIR digital filter that achieves 0.0025π passband edge and 60dB stopband attenuation in order to demonstrate how signal processing techniques can be framed into an approach that fits the need of photonic devices. To meet the given requirements, a Finite Impulse Response (FIR) design would require filter orders higher than 200. A realizable photonic structure requires low complexity, therefore we aim to obtain a filter design with an order less than 64.

Classical IIR filters are only able to achieve these requirements with pole magnitudes larger than 0.999, marginally satisfying the system stability criterion in the traditional DSP implementation. In a photonic system, however, design parameters are further restricted as a result of fabrication error and waveguide loss [77]. Our challenge is therefore to derive a filter design method that yields a narrow passband with high stopband attenuation, while maintaining small pole magnitudes. Traditional filter design approaches are unable to yield results with

pole magnitudes less than 0.97 for the given requirements, which are necessary for realistic fabrication. Our design method yields a transfer function in the form:

$$H_f(z) = (H_l(z^M))^L I(z) \quad (\text{A.1})$$

Through interpolation by a factor of M , we ease the design constraint on passband edge and thus reduce the complexity of the problem. We first design a prototype filter $H_l(z)$ with a 0.01π passband edge and 15dB attenuation. The prototype filter can then be cascaded to reach high attenuation, and interpolated to meet the required passband frequency. The images that arise from interpolation are removed by a subsequent lowpass filter $I(z)$. For our specific design presented in this appendix, the interpolation factor M and the cascade factor L are $M = L = 4$. The design method is detailed in the following subsections.

A.1 Prototype Lowpass Filter

Relaxing the design specifications allows for a slower roll off in the passband and transition band, thus allowing for lower magnitude poles. We increase the cutoff from 0.0025π to 0.01π , and reduce the attenuation from 60dB to 15dB. Since photonic filters are best realized as allpass sections, we use the method described in [34] to obtain the lowpass filter as sum of two allpasses:

$$H_l(z) = \frac{1}{2} (z^{-(N-1)} + A_l(z)) \quad (\text{A.2})$$

where $A_l(z)$ is an allpass filter of order N . The design of the lowpass prototype filter thus reduces to designing the allpass filter $A_l(z)$. It can be shown that a 7th-order allpass filter will be able to achieve the reduced specifications. To obtain the allpass section, we employ the iterative method presented in [63]. As outlined in [63], we first calculate the eigenvector corresponding to the smallest eigenvalue of $\mathbf{P}_2$ in the error measure

$$E_{LS}^{(2)} = \mathbf{a}^t \mathbf{P}_2 \mathbf{a} \quad (\text{A.3})$$

where

$$\begin{aligned} \mathbf{P}_2 = & \alpha \sum_{k=1}^K \int_{\omega_{k,1}}^{\omega_{k,2}} \mathbf{s}_\beta(\omega) \mathbf{s}_\beta^t(\omega) d\omega \\ & + (1 - \alpha) \sum_{k=1}^K \int_{\omega_{k,1}}^{\omega_{k,2}} [\mathbf{c}(\omega_k) - \mathbf{c}(\omega)] [\mathbf{c}(\omega_k) - \mathbf{c}(\omega)]^t d\omega \end{aligned} \quad (\text{A.4})$$

and

$$\mathbf{c}(\omega) = \begin{bmatrix} 1 & \cos\omega & \cdots & \cos N\omega \end{bmatrix}^t \quad (\text{A.5})$$

$$\mathbf{s}_\beta(\omega) = \begin{bmatrix} \sin\beta(\omega) & \cdots & \sin(\beta(\omega) - N\omega) \end{bmatrix}^t \quad (\text{A.6})$$

$$\beta(\omega) = \frac{1}{2} (\Theta_{pre}(\omega) + N\omega) \quad (\text{A.7})$$

Θ_{pre} is the desired response, and in our case corresponds to two flat sections in the passband and stopband. N is order of the filter, which we have determined to be 7. α is the control parameter that distributes the weighing to each of the error terms, and is set at 0.999 to give large weight to the phase error term. $K = 2$ is the number of frequency bands of interest, signifying the passband and stopband regions. The $\omega_{k,1}$ and $\omega_{k,2}$ terms are the start and stop frequencies of band k ; ω_k is the reference frequency when the magnitude response of $H(z)$ is expected to reach maximum.

The resulting eigenvector $\mathbf{a}$ contains the weights for the allpass filter that approximately minimize the least square error. $\mathbf{a}$ is then used as the initial value for the cost function

$$E_{LS}^{(3)} = 4 \int_R \frac{\mathbf{a}^{(q)t} \mathbf{s}_\beta(\omega) \mathbf{s}_\beta^t(\omega) \mathbf{a}^{(q)}}{\mathbf{a}^{(q-1)t} \mathbf{c}_\beta(\omega) \mathbf{c}_\beta^t(\omega) \mathbf{a}^{(q-1)}} d\omega \quad (\text{A.8})$$

where $\mathbf{c}_\beta(\omega) = \begin{bmatrix} \cos\beta(\omega) & \cdots & \cos(\beta(\omega) - N\omega) \end{bmatrix}^t$, with q as the iteration variable. The resulting filter is then summed with a 6th-order delay to form the prototype lowpass $H_l(z)$. The section has a maximum pole magnitude of 0.882, and can be used as the building block to obtaining the desired narrowband filter.

A.2 Interpolation

The passband edge is 4 times that of the design requirement. To obtain a final filter with 0.0025π passband edge, we upsample the prototype filter by a factor of 4. The poles of the interpolated filter $H_l(z^4)$ are the solutions to

$$\prod_i^M (1 - k_i z^{-4}) \prod_j^{\frac{N-M}{2}} (1 - k_j z^{-4})(1 - k_j^* z^{-4}) = 0 \quad (\text{A.9})$$

where M corresponds to the number of real valued poles and k_i, k_j are the filter taps of the original prototype filter. From (A.9), it can be seen that upsampling by a factor of 4 also magnifies the pole magnitudes by the same amount. The prototype lowpass filter is designed to counter this effect, allowing for a resulting filter with pole magnitudes less than 0.97. Upsampling also creates images of the desired filter throughout the spectrum, which can be suppressed using a lowpass filter $I(z)$. Since the first image occurs at $\frac{\pi}{2}$, the interpolation lowpass can be designed with lenient specifications and poses no concerns for pole magnitudes. The final designs for (A.10) are $H_l(z) = \frac{1}{2}(z^{-6} + A_l(z))$ and $I(z) = \frac{1}{2}(z^{-3} + A_i(z))$.

Here we illustrate it with a digital filter that can achieve a 0.0025π passband edge and 60dB stopband attenuation using the FBG structure. Traditional filter design approaches are unable to yield results with pole magnitudes less than 0.97 for the given requirements, rendering them impossible to fabricate. Our *ad-hoc* design method yields a transfer function in the form:

$$H_f(z) = (H_l(z^M))^L I(z) \quad (\text{A.10})$$

We first design a prototype filter $H_l(z)$ with a 0.01π passband edge and 15dB attenuation. The prototype filter can then be cascaded to achieve high attenuation, and interpolated to meet the required passband frequency. The images that arise from interpolation are removed by a subsequent lowpass filter $I(z)$. For our specific design, the interpolation factor M and the cascade factor L are $M = L = 4$.

The lowpass filter is constructed using a delay and an allpass. It can be shown that a 7th-order allpass filter will be able to achieve the reduced specifica-

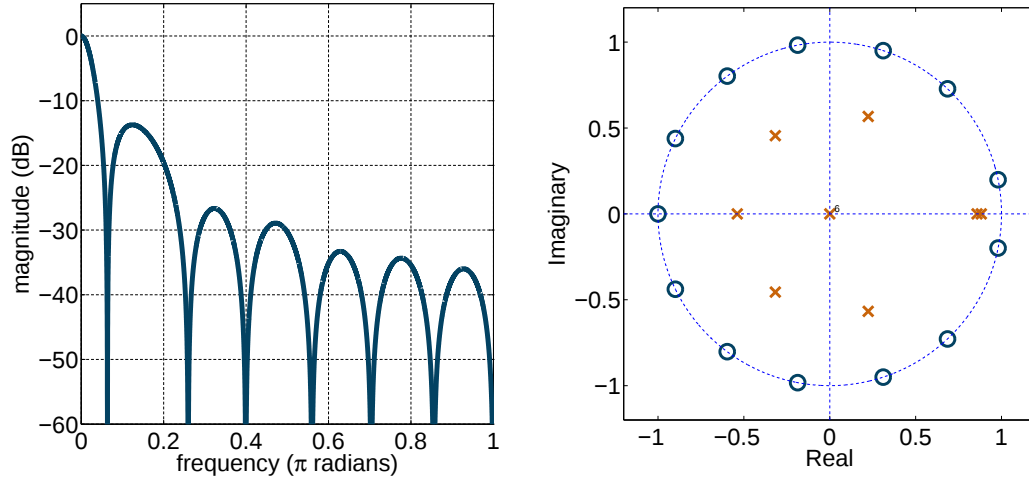


Figure A.1: Magnitude Response of $H_l(z)$ (left) and its corresponding pole and zero map (right).

tions. To obtain the allpass section, we use the eigenfilter approach, which will be examined in detail in later section. The resulting filter is then summed with a 6th-order delay to form the prototype lowpass $H_l(z)$. Upsampling by a factor of 4 translates the passband to the desired frequency, and a cascaded lowpass filter $I(z)$ removes the images from interpolation. The final designs for (A.10) are

$$H_l(z) = \frac{1}{2}(z^{-6} + A_l(z)) \quad (\text{A.11})$$

and

$$I(z) = \frac{1}{2}(z^{-3} + A_i(z)) \quad (\text{A.12})$$

The prototype lowpass filter $H_l(z)$ has a maximum pole magnitude of 0.882 and an attenuation of -13.73dB . The resulting narrowband filter $H_f(z)$ has a passband of 0.0026π and an attenuation of -57.762dB with maximum pole magnitude of 0.962. Figure A.1 displays the magnitude response of $H_l(z)$ and the corresponding pole and zero placements. The interpolated version of $H_l(z)$ and the filter used to remove images, $I(z)$, are shown in Figure A.2. A magnified version of the magnitude and phase response for the final design is presented in Figure A.3.

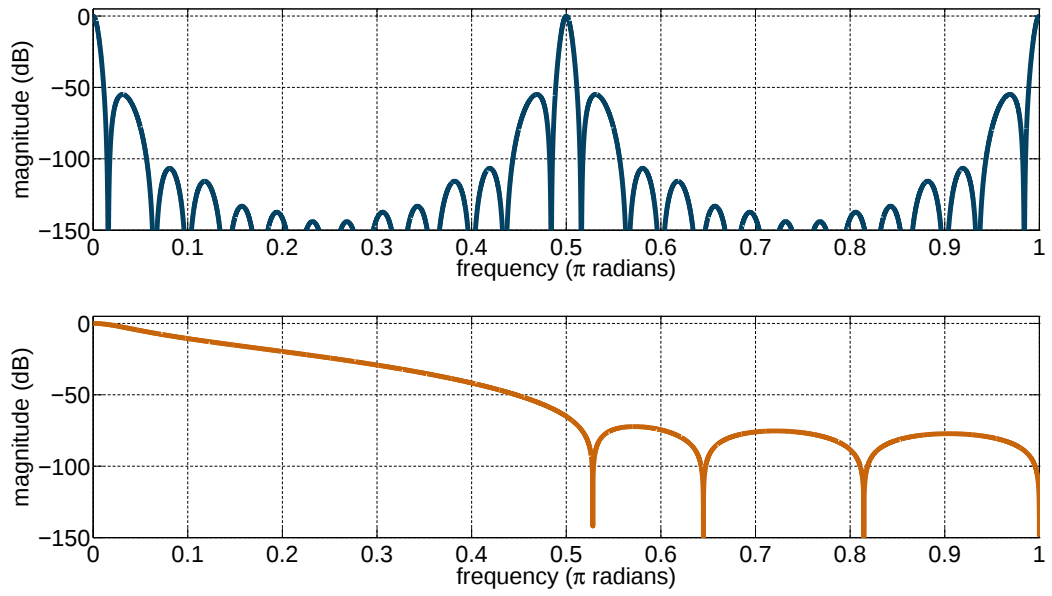


Figure A.2: Magnitude Response of $H_l(z^4)$ (top) and $I(z)$ (bottom).

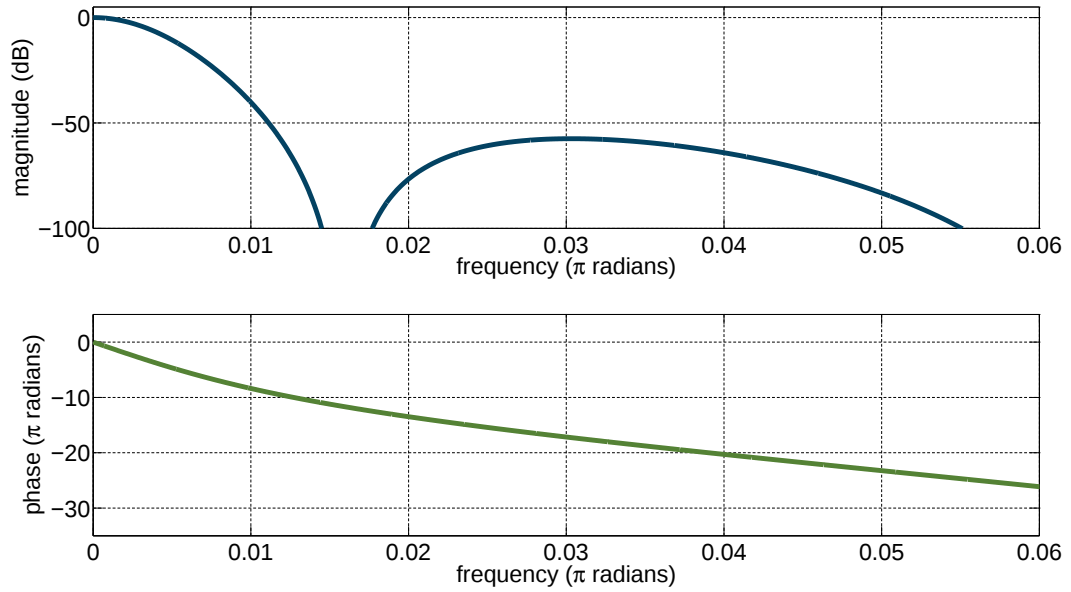


Figure A.3: Magnitude Response of $H_f(z)$ (top) and its corresponding phase (bottom).

A.3 Acknowledgments

This appendix, in full, is a reprint of the published conference paper in IEEE International Conference on Acoustics, Speech, and Signal Processing 2011.

Appendix B

Phase Compensator Cost Function

This appendix contains the details to the derivation of the cost function for the waveguide loss compensating phase compensator design. Given that the filters available are of the form

$$A(z) = \frac{d_N + d_{N-1}\gamma z^{-1} + \dots d_1\gamma^{N-1}z^{-(N-1)} + d_0\gamma^N z^{-N}}{d_0 + d_1\gamma z^{-1} + \dots d_{N-1}\gamma^{N-1}z^{-(N-1)} + d_N\gamma^N z^{-N}} \quad (\text{B.1})$$

where γ is real and known, we wish to design the filter such that it matches a prescribed phase response $\Theta_{pre}(\omega)$. We first write the transfer function as

$$A(z) = \gamma^N z^{-N} \frac{D(\gamma z^{-1})}{D(\gamma^{-1}z)} \quad (\text{B.2})$$

where $D(z) = d_0 + d_1 z^{-1} + \dots d_{N-1} z^{-(N-1)} + d_N z^{-N}$. Now the phase response of this filter is

$$\Theta_A(\omega) = -N\omega + \arctan \frac{\sum_{k=0}^N d_k \gamma^{-k} \sin k\omega}{\sum_{k=0}^N d_k \gamma^{-k} \cos k\omega} + \arctan \frac{\sum_{k=0}^N d_k \gamma^k \sin k\omega}{\sum_{k=0}^N d_k \gamma^k \cos k\omega} \quad (\text{B.3})$$

or in vector notations

$$\Theta_A(\omega) = -N\omega + \arctan \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{s}(\omega)}{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{c}(\omega)} + \arctan \frac{\mathbf{d}^T \mathbf{A} \mathbf{s}(\omega)}{\mathbf{d}^T \mathbf{A} \mathbf{c}(\omega)} \quad (\text{B.4})$$

where

$$\mathbf{d} = \begin{bmatrix} d_0 & d_1 & \cdots & d_N \end{bmatrix} \quad (\text{B.5})$$

$$\mathbf{s}(\omega) = \begin{bmatrix} 0 & \sin \omega & \cdots & \sin N\omega \end{bmatrix} \quad (\text{B.6})$$

$$\mathbf{c}(\omega) = \begin{bmatrix} 1 & \cos \omega & \cdots & \cos N\omega \end{bmatrix} \quad (\text{B.7})$$

$$\mathbf{A} = \text{diag} \left\{ 1 \quad \gamma \quad \cdots \quad \gamma^N \right\} \quad (\text{B.8})$$

we know that to combine the arctan terms, we need to use

$$\arctan u + \arctan v = \arctan \frac{u + v}{1 - uv} \quad (\text{B.9})$$

from here, we have

$$\Theta_A(\omega) = -N\omega + \arctan \frac{\frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{s}(\omega)}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{c}(\omega)} + \frac{\mathbf{d}^\top \mathbf{A} \mathbf{s}(\omega)}{\mathbf{d}^\top \mathbf{A} \mathbf{c}(\omega)}}{1 - \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{s}(\omega)}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{c}(\omega)} \cdot \frac{\mathbf{d}^\top \mathbf{A} \mathbf{s}(\omega)}{\mathbf{d}^\top \mathbf{A} \mathbf{c}(\omega)}} \quad (\text{B.10})$$

$$= -N\omega + \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{s}(\omega) \mathbf{c}^\top(\omega) \mathbf{A} \mathbf{d} + \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{c}(\omega) \mathbf{s}^\top(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{c}(\omega) \mathbf{c}^\top(\omega) \mathbf{A} \mathbf{d} - \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{s}(\omega) \mathbf{s}^\top(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.11})$$

$$= -N\omega + \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{s}(\omega) \mathbf{c}^\top(\omega) + \mathbf{c}(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{c}(\omega) \mathbf{c}^\top(\omega) - \mathbf{s}(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d}} \quad (\text{B.12})$$

We now need to formulate the difference between this response and the desired response

$$\Delta\Theta(\omega) = \Theta_{pre}(\omega) + N\omega - \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{s}(\omega) \mathbf{c}^\top(\omega) + \mathbf{c}(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{c}(\omega) \mathbf{c}^\top(\omega) - \mathbf{s}(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d}} \quad (\text{B.13})$$

For notation simplicity, let us define $\beta(\omega) = \Theta_{pre}(\omega) + N\omega$, then

$$\Delta\Theta(\omega) = \beta(\omega) - \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.14})$$

$$= \arctan \tan \beta(\omega) - \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.15})$$

$$= \arctan \frac{\tan \beta(\omega) - \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d}}}{1 + \beta(\omega) \frac{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d}}} \quad (\text{B.16})$$

$$= \arctan \frac{\tan \beta(\omega) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d} - \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d} + \tan \beta(\omega) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.17})$$

$$= \arctan \frac{\sin \beta(\omega) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d} - \cos \beta(\omega) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}}{\cos \beta(\omega) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{X}(\omega) \mathbf{A} \mathbf{d} + \sin \beta(\omega) \mathbf{d}^\top \mathbf{A}^{-1} \mathbf{Y}(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.18})$$

where

$$\mathbf{X}(\omega) = \mathbf{c}(\omega) \mathbf{c}^\top(\omega) - \mathbf{s}(\omega) \mathbf{s}^\top(\omega) \quad (\text{B.19})$$

$$\mathbf{Y}(\omega) = \mathbf{s}(\omega) \mathbf{c}^\top(\omega) + \mathbf{c}(\omega) \mathbf{s}^\top(\omega) \quad (\text{B.20})$$

the terms can be grouped as

$$\Delta\Theta(\omega) = \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{C}_1(\omega) - \mathbf{S}_1(\omega) - \mathbf{L}_2(\omega) - \mathbf{L}_2^\top(\omega)) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{C}_2(\omega) - \mathbf{S}_2(\omega) + \mathbf{L}_1(\omega) + \mathbf{L}_1^\top(\omega)) \mathbf{A} \mathbf{d}} \quad (\text{B.21})$$

where

$$\mathbf{C}_1(\omega) = \sin \beta(\omega) \mathbf{c}(\omega) \mathbf{c}^\top(\omega) \quad \mathbf{C}_2(\omega) = \cos \beta(\omega) \mathbf{c}(\omega) \mathbf{c}^\top(\omega) \quad (\text{B.22})$$

$$\mathbf{S}_1(\omega) = \sin \beta(\omega) \mathbf{s}(\omega) \mathbf{s}^\top(\omega) \quad \mathbf{S}_2(\omega) = \cos \beta(\omega) \mathbf{s}(\omega) \mathbf{s}^\top(\omega) \quad (\text{B.23})$$

$$\mathbf{L}_1(\omega) = \sin \beta(\omega) \mathbf{s}(\omega) \mathbf{c}^\top(\omega) \quad \mathbf{L}_2(\omega) = \cos \beta(\omega) \mathbf{s}(\omega) \mathbf{c}^\top(\omega) \quad (\text{B.24})$$

Now, in the numerator, we have the term

$$((\mathbf{t}_1(\omega) - \mathbf{t}_2(\omega)) \mathbf{c}^\top(\omega) - (\mathbf{t}_3(\omega) + \mathbf{t}_4(\omega)) \mathbf{s}^\top(\omega)) \quad (\text{B.25})$$

where

$$\mathbf{t}_1(\omega) = \sin \beta(\omega) \mathbf{c}(\omega) \quad \mathbf{t}_2(\omega) = \cos \beta(\omega) \mathbf{s}(\omega) \quad (\text{B.26})$$

$$\mathbf{t}_3(\omega) = \sin \beta(\omega) \mathbf{s}(\omega) \quad \mathbf{t}_4(\omega) = \cos \beta(\omega) \mathbf{c}(\omega) \quad (\text{B.27})$$

we know that

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v \quad (\text{B.28})$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v \quad (\text{B.29})$$

and let us define

$$\mathbf{s}_\beta(\omega) = \begin{bmatrix} \sin \beta(\omega) & \sin(\beta(\omega) - \omega) & \cdots & \sin(\beta(\omega) - N\omega) \end{bmatrix} \quad (\text{B.30})$$

$$\mathbf{c}_\beta(\omega) = \begin{bmatrix} \cos \beta(\omega) & \cos(\beta(\omega) - \omega) & \cdots & \cos(\beta(\omega) - N\omega) \end{bmatrix} \quad (\text{B.31})$$

Then the numerator is

$$\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{s}_\beta(\omega) \mathbf{c}^\top(\omega) - \mathbf{c}_\beta(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d} \quad (\text{B.32})$$

Similarly, in the denominator we have

$$((\mathbf{t}_1(\omega) + \mathbf{t}_2(\omega)) \mathbf{c}^\top(\omega) - (\mathbf{t}_3(\omega) - \mathbf{t}_4(\omega)) \mathbf{s}^\top(\omega)) \quad (\text{B.33})$$

which means the entire denominator term is

$$\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{c}_\beta(\omega) \mathbf{c}^\top(\omega) + \mathbf{s}_\beta(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d} \quad (\text{B.34})$$

and the phase difference is

$$\Delta\Theta(\omega) = \arctan \frac{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{s}_\beta(\omega) \mathbf{c}^\top(\omega) - \mathbf{c}_\beta(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d}}{\mathbf{d}^\top \mathbf{A}^{-1} (\mathbf{c}_\beta(\omega) \mathbf{c}^\top(\omega) + \mathbf{s}_\beta(\omega) \mathbf{s}^\top(\omega)) \mathbf{A} \mathbf{d}} \quad (\text{B.35})$$

We can use the trigonometry identities again to get

$$\Delta\Theta(\omega) = \arctan \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.36})$$

where the n, m element in matrices $\mathbf{P}(\omega)$ and $\mathbf{Q}(\omega)$ are

$$\mathbf{P}_{n,m}(\omega) = \sin(\beta(\omega) - n\omega - m\omega) \quad (\text{B.37})$$

$$\mathbf{Q}_{n,m}(\omega) = \cos(\beta(\omega) - n\omega - m\omega) \quad (\text{B.38})$$

The next thing to do is use Taylor series expansion on $\arctan$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad (\text{B.39})$$

so we can discard all the non-linear terms for small approximation errors

$$\Delta\Theta(\omega) \approx \frac{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d}}{\mathbf{d}^T \mathbf{A}^{-1} \mathbf{Q}(\omega) \mathbf{A} \mathbf{d}} \quad (\text{B.40})$$

Let us first ignore the denominator, we can re-introduce this later as an iterative weighing term, then we have

$$\Delta\Theta(\omega) \approx \mathbf{d}^T \mathbf{A}^{-1} \mathbf{P}(\omega) \mathbf{A} \mathbf{d} \quad (\text{B.41})$$

which is the optimization cost function we use in the allpass phase equalizer design with waveguide loss compensation.

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