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ASYMPTOTIC OSCILLATION HYPOTHESIS

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ASYMPTOTIC OSCILLATION HYPOTHESIS*

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ABSTRACT

The observed persistence of threshold effects into the Regge asymptotic region of single-particle inclusive cross sections can be accommodated in a natural fashion by complex Regge poles which, if factorizable, will produce asymptotic damped oscillations in a wide variety of other osbservable quantities. It is shown that a single pair of such poles, the imaginary part of whose position in the complex J plane is of the order ± 1, can qualitatively explain various anomalies that have been observed in high energy phenomena. Predictions are made for future measurements.

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The initial tendency of single-particle inclusive cross sections outside the fragmentation regions to rise with energy has been convincingly attributed to threshold kinematics [1], [2], [3], and since the rise persists up to energies where Regge asymptotic expansions have enjoyed success, one is led to ask how Regge parameters manage to accommodate threshold effects. A natural mechanism is through complex Regge poles that generate damped oscillations in high energy cross sections, ref. [4] having shown through a simple multiperipheral example how even a single pair of complex poles can approximate the kinematic consequences of thresholds. (The initial rising tendency of single-particle inclusive cross sections, for instance, corresponds to the increasing phase of the first oscillation.) References [2, 3] do not associate the threshold effect with Regge poles, even though they also employ a multiperipheral model, because these papers include threshold kinematics for only a single link of the multiperipheral chain. Treating all links equivalently leads to an asymptotic expansion entirely in terms of factorizable Regge poles--in accord with Mueller's fundamental postulate for inclusive cross sections [5]. The approximation employed in ref. [2, 3], although adequate for an early stage of the threshold increase, is inconsistent with an asymptotic expansion.

Now, if factorizable complex poles indeed constitute the Regge mechanism for accommodating threshold kinematics, a number of striking qualitative effects are to be sought in the experimental data: (1) After the initial rise, single-particle inclusive cross sections should reach a maximum and then fall--the initial rise being the first cycle of an indefinite damped oscillation. (2) Not only single-particle inclusive cross sections but, because of the factorizability of complex Regge poles, also total cross sections, elastic amplitudes, average multiplicities, etc., should exhibit oscillations. (3) To the extent that a single leading pair of complex poles may dominate the remainder of the complex Regge spectrum, one expects the <u>period</u> of all asymptotic oscillations to be simply related, although the phase and the magnitude of the oscillation will differ from case to case. For example all total cross sections and average multiplicities should exhibit a common asymptotic period in the rapidity difference Y (roughly $\propto \ln s$) of the colliding particles, while all nonfragmentation inclusive cross sections should exhibit twice this period (the factor 2 will be explained later).

In the example studied in ref. [4] it was found that, although there exists an infinite sequence of complex Regge poles, a substantial spacing separates the different poles and the real parts of the pole positions in J decrease as the imaginary parts increase. We are led to conjecture that in general the leading complex pole, which by virtue of being farthest to the right in J is the most persistent as Y increases, has the smallest imaginary displacement from the real J-axis. Since the oscillation period associated with a complex pole at $J = \alpha_c$ is $2\pi(\mathrm{Im}\ \alpha_c)^{-1}$, the leading complex pole would then correspond to the longest wavelength oscillation--shorter wavelength components of the oscillation being assumed to damp out more rapidly. There remains the important practical consideration of how large a value of Y is required in order to neglect secondary complex poles without disastrously misrepresenting the physics. Again leaning on ref. [4] for guidance, it is reasonable to hope that an asymptotic expansion terminating after a single pair of complex poles will have qualitative validity all the way down to the threshold for significant inelastic scattering--that is, down to $Y \approx 1$.

Since the complex Regge-pole mechanism is not an alternative to, but rather a reexpression of, threshold phase-space considerations, one expects the first maximum in an inclusive cross section to be reached earlier for the less-massive produced particles and later for the more massive. ISR measurements support this expectation: In pp collisions over an interval where the pp rapidity difference Y increases from 7.0 to 8.0 the 90 degree antibaryon inclusive cross section rises by about 25% while the π inclusive cross section is rising by less than 5% [6]. Thus at the ISR we may have reached (or almost reached) the first maximum in the least-massive single-particle inclusive cross section. We shall see, however, that the single-particle inclusive period of oscillation is expected to be so long that it will be difficult soon to confirm a subsequent decreasing tendency in the π inclusive cross section.

The situation in this respect is more favorable for total cross sections. The pp total cross section, in particular, shows a maximum at $Y \approx 1$ (near the effective threshold for π production), a minimum near $Y = 5$ and a subsequent increase that is still continuing at $Y = 8$. We are thus led to guess at 4 units of Y as a rough estimate of the half period, corresponding to $\mathrm{Im}\,\alpha_c \approx \pi/4$. A systematic Regge-pole fit to σ_{pp}^{tot} for $Y > 2$ has led to $\mathrm{Im}\,\alpha_c \approx 0.6$ but with a large uncertainty [7].* If in the pp total cross section we already have seen both a maximum and a minimum in the oscillation associated with the leading pair of complex Regge poles, we know roughly what period to anticipate in other experimental quantities.

We have remarked that the oscillation period for (non-fragmentation) single-particle inclusive cross sections should be twice that for total cross sections--which we have roughly estimated to be 8 units of rapidity. Is it then reasonable that the ISR range $(Y \approx 7)$ should be near a maximum in the single-pion inclusive cross section. Using the crude idea that the distance from threshold to first maximum should be roughly $1/4$ of a wavelength,** one might guess the position of the first maximum to be near

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* A quantitative Regge representation that extends to low energy is sensitive to real poles such as the ω. Such contributions drastically affect the height of maxima and minima but have less effect on the location.

** Because of the exponential dependence on Y, associated with the supposition that the real part of the complex pole position lies below $J = 1$ (i.e. that there is a damping factor), one expects that for Y small the complex pole will probably contribute a larger share of the cross section than the leading real pole. Thus the vanishing at threshold of the single-pion inclusive cross section implies an approximate vanishing of the oscillatory component thereof.

$$Y \approx 1 + \frac{1}{4}(2 \times 8) = 5 ,$$

lower than the ISR range although only by a small fraction of the full period involved. There is nevertheless a suggestion here that we have somewhat underestimated the period in the total cross section. Such an error is easy to make because the location of the minimum in σ_{pp}^{tot} is not sharply defined.

What about total cross sections other than pp ? The K^+p total cross section has a threshold-associated maximum at $Y \approx 1.6$ followed by an extremely shallow minimum. The minimum is so shallow in the K^+p case that lower-lying real poles may substantially shift the minimum location with respect to the neighboring maxima. Assuming that the spacing between K^+p maxima is not greatly shifted with respect to the pp case a maximum is predicted for the K^+p total cross section near $Y \approx 10$, unfortunately above the NAL range. Although a high-energy minimum in other total cross sections has not yet been observed, perhaps because of the important role played by lower-lying real trajectories, our hypothesis predicts that such minima (and subsequent maxima) will be found to be universal phenomena.

A severe test of the oscillation hypothesis will be given by ISR studies of the real part of the forward pp elastic amplitude. As remarked in ref. [7] the phase of the oscillation in the real part is advanced by a quarter cycle with respect to the total cross section (imaginary part), leading to the expectation of a maximum in the former quantity near $Y \approx 7$, within the ISR range. If the real part is found to rise vigorously throughout this range, with no sign of approach to a maximum, the oscillation hypothesis will be in trouble.

What about oscillations in average multiplicity? These might be expected to have a larger magnitude for individual particle types than for overall charged multiplicity, but may be difficult to establish because the oscillations would be superimposed on a persistent rising tendency. There are expected to be no maxima or minima in $\langle n_i \rangle$ -- only periodic changes in the slope of this quantity when plotted against Y. As noted previously, Mueller-Regge theory predicts the oscillation period in $\langle n_i \rangle$ to be the same as in the total cross section but with a different phase.

Let us now consider in more detail the single-particle inclusive cross section near 90 degrees in the c.m. system of a pp collision. Assuming a real pomeron pole at $J = 1$, the leading contribution from a pair of complex Regge poles located at $J = \alpha_c, \alpha_c^*$ can be shown by standard Mueller-Regge analysis to be proportional to the real part of

$$r_c^{\,i}\, e^{(\alpha_c-1)Y/2} \cosh(\alpha_c - 1)y_i \,, \tag{1}$$

where y_i is the rapidity of the produced particle in the c.m. system. The complex coefficient $r_c^{\,i}$ depends on the nature and transverse momentum of the produced particle. It is because of the factor 1/2 in the exponent that the period of Y oscillation is twice that of the total cross section.

Although Formula (1) predicts at $y_i = 0$ that the second derivative with respect to y_i is four times as large as the second derivative with respect to Y,* the first derivative with respect to

* Thus a maximum in the Y dependence implies a maximum in the y_i dependence.

Y cannot be directly inferred from a measurement of y curvature. Additional information about the phase of the oscillation is required. In particular it is perfectly possible for the Y first derivative to be small while the y second derivative (curvature of the "central plateau") is substantial--a situation that is observed in the ISR range [6] and that cannot be understood if the dominant secondary Regge pole is real.

The observed magnitude of the y_π second derivative near $Y \approx 7$ [6], together with our estimate of $\mathrm{Im}\,\alpha_c$ indicates that the magnitude of the oscillatory component must in the ISR range be comparable to the pomeron component. Thus when the energy is increased sufficiently far we anticipate a substantial decrease from the ISR measured value of the 90 degree single-pion inclusive cross section.

A tacit assumption made in this paper is that

$$1 - \mathrm{Re}\,\alpha_c < |\mathrm{Im}\,\alpha_c| ,$$

i.e. that the oscillations are not so rapidly damped as to be invisible. The fit of ref. [7], for example, led to

$$(1 - \mathrm{Re}\,\alpha_c)/|\mathrm{Im}\,\alpha_c| \approx 1/3 .$$

Multiperipheral models, such as that of ref. [4], tend to give this ratio a similar value. The damping over a full period may nevertheless be very important. With a 1:3 ratio, for instance, two successive peak heights in the oscillating component differ by a factor $\exp(2\pi/3) \approx 8$.

The greatest puzzle associated with the asymptotic oscillation hypothesis is the origin of the large period suggested by the data.*

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* The period in the model of ref. [4] was about 2 units of rapidity.

A possible mechanism for a large period will be discussed by the authors in a separate paper. In this letter we have confined our efforts to emphasizing the simplest and most general aspects of the asymptotic oscillation hypothesis--aspects that can be tested in the same sense that the real Regge-pole hypothesis has been subjected to test--independently of detailed dynamical mechanisms for "generating" the poles. We have not attempted to be exhaustive in our list of experimental tests. With expansion of the accuracy and variety of high energy measurements, the range of checks on the oscillation hypothesis will also expand.

In closing we draw attention to the possibility that the leading complex pole-pair may, in addition to generating asymptotic oscillations, be responsible at lower energies for the physical effects heretofore associated with a real P'.* It is conceivable, in other words, that a Regge asymptotic expansion may not require a real P' in addition to a pair of complex poles.

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* We shall show in a separate paper how the physical effects associated with exchange degeneracy between P' and ω can be approximately maintained over a moderate energy interval by a real ω and a complex P' pair, even though the imaginary part of the latter's position is of order ± 1.

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