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Grand Initial Value Problem of High Gain Free Electron Lasers

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Initial value problem is one of the cornerstones in the framework of high gain FEL theory. It determines the startup of FEL interaction from initial signal or noise in either laser field or electron beam. Yet, this problem was solved only for the cases without emittance and betatron oscillations. I present the first solution to the initial value problem in a grand scale by expanding the startup theory into the full six-dimensional phase space, deriving both general solution valid for any beam distribution and specific solution for a Gaussian model. One of the major results of this letter is the discovery of excessively large noise power for SASE.

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It is generally believed that the most promising approach to generate intense coherent short wavelength radiation down to hard X-ray region is via the principle of single pass high gain free electron laser (FEL). A better understanding of high gain FEL requires more complete description and analysis of the physical reality, in which a radiation field in three dimensional (3D) configuration space interacts with a beam of electrons in six-dimensional (6D) phase space. From a theoretical point of view, a generic high gain amplifier can be divided into three regions along the interaction distance: initiation, growth, and saturation. Whereas the last region has to be handled by nonlinear theory and more often by simulation, the first two can be covered under the scope of linear theory. Furthermore, the linear theory can be classified into two major problems: eigenvalue problem (EVP) which is responsible for the power growth, and initial value problem (IVP) which determines the startup of FEL interaction from an initial condition.

As FEL is pushed in the short wavelength frontier, it is becoming increasingly more important to take into account the part of the reality associated with emittance and betatron oscillations in our theoretical framework. To this end, EVP has been solved to a level of great sophistication [1–4]. But for IVP, emittance and betatron oscillations have been neglected in all previous calculations [4–10]. As a result of lacking a solution to IVP in the presence of emittance and betatron oscillations, there has been no theory permitting a calculation of even the most rudimental quantities for the startup of an FEL amplifier, for instance, input coupling for coherent amplification and noise power for self-amplified spontaneous emission (SASE). The main objective of this letter is to present the first solution to IVP in a grand scale by expanding the startup theory into the full 6D phase space, deriving both general solution valid for any beam distribution and specific solution for a Gaussian model.

We specify the radiation field by a complex envelope $a_r(\mathbf{x}, z, t)$ slowly varying with respect to a carrier wave $\exp(k_r z - \omega_r t)$, where $\omega_r = ck_r$ is the carrier frequency. The envelope is normalized according to $|a_r| = eE_{rms}/mc\omega_r$, where $E_{rms}(\mathbf{x}, z, t)$ is the rms amplitude of the electric field. To properly treat beam fluctuations in SASE process, we use the Klimontovich distribution [11] to account for the discreteness of electrons

$$\mathcal{F}(X; z) = \frac{k_r}{n_0} \sum_i^{N_e} \delta[X - X_i(z)], \quad X = \{\mathbf{x}, \mathbf{p}, \theta, \eta\},$$

where n_0 is the peak volume density and N_e the total number of electrons, $\mathbf{x} = \{x, y\}$, $\mathbf{p} = d\mathbf{x}/dz$, $\theta = (k_r + k_w)z - \omega_r t$, $\eta = (\gamma - \gamma_0)/\gamma_0$, $\lambda_w = 2\pi/k_w$ is the wiggler period, γ_0 is the average value of γ related to the resonance condition by $k_r = 2\gamma_0^2 k_w/(1 + a_w^2)$, $a_w = eB_{rms}/mck_w$, and B_{rms} is the rms wiggler field assumed constant along the axis. The distribution function can be further separated into two parts $\mathcal{F}(X; z) = F(Y) + f(X; z)$, where $Y = \{\mathbf{x}, \mathbf{p}, \eta\}$, F is the ensemble-averaged, unperturbed, smooth background distribution which is assumed to be uniform in θ and independent of z, and f contains both random fluctuations and coherent modulations. Upon introducing Fourier transform by $a_\nu = \frac{1}{\sqrt{2\pi}} \int d\theta e^{-i\nu\theta}(e^{i\theta} a_r)$ and $f_\nu = \frac{1}{\sqrt{2\pi}} \int d\theta e^{-i\nu\theta} f$, where $\nu = \omega/\omega_r$, the paraxial Maxwell equation and linearized Vlasov equation can be written as

$$\left[i\frac{\partial}{\partial z} + \frac{1}{2k_r}\frac{\partial^2}{\partial \mathbf{x}^2} - \Delta\nu k_w\right] a_\nu = -h_a \int d^2\mathbf{p} d\eta f_\nu, \qquad (1)$$

$$\left[i\frac{\partial}{\partial z} + i\left(\mathbf{p}\frac{\partial}{\partial \mathbf{x}} - k_\beta^2 \mathbf{x}\frac{\partial}{\partial \mathbf{p}}\right) - \nu\xi\right] f_\nu = h_f \frac{\partial F}{\partial \eta} a_\nu, \qquad (2)$$

where $\Delta\nu = \nu - 1$, $\xi = 2k_w\eta - (k_r/2)(\mathbf{p}^2 + k_\beta^2\mathbf{x}^2)$, $h_a = 2\pi r_e a_w f_B n_0/\gamma_0 k_r$, $h_f = k_r a_w f_B/2\gamma_0^2$, $h_a h_f = h/16k_w L_{1d}^3$, $h = (2/\sqrt{3})^3$, L_{1d} is the 1D power gain length, $f_B = 1$ for helical wiggler and $f_B = J_0[a_w^2/2(1 + a_w^2)]$ - $J_1[a_w^2/2(1 + a_w^2)]$ for planar wiggler, k_β characterizes the strength of an effectively constant-gradient betatron focusing, and r_e is the classical radius of electron. Equations (1)-(2) are essentially those of Kim [12] except the differences due to definition of the carrier wave and normalization of the distribution function.

To obtain a general solution to IVP, it is convenient to cast the coupled Maxwell-Vlasov Eqs.(1)-(2) into a vectorized form of the Schrödinger type [9,13]

$$i\frac{\partial\Psi}{\partial z} = \mathcal{H}\Psi \ , \quad \Psi = \left[\begin{array}{c} a_\nu \\ f_\nu \end{array}\right], \tag{3}$$

with the Hamiltonian operator expressed as

$$\mathcal{H} = \left[\begin{array}{cc} \left(k_w\Delta\nu - \frac{1}{2k_r}\frac{\partial^2}{\partial\mathbf{x}^2}\right), & \left(-h_a\int d^2\mathbf{p}d\eta\right) \\ \left(h_f\frac{\partial F}{\partial\eta}\right), & \left(\nu\xi - i[\mathbf{p}\frac{\partial}{\partial\mathbf{x}} - k_\beta^2\mathbf{x}\frac{\partial}{\partial\mathbf{p}}]\right) \end{array}\right].$$

Equation (3) admits eigenvectors of the form $\Psi_n = V_n e^{-i\mu_n z}$, where $\{\mu_n, V_n\}$ are determined by an eigenvalue problem $\mathcal{H}V_n = \mu_n V_n$. Since the operator $\mathcal{H}$ is neither Hermitian nor self-adjoint, its eigenvectors $\{V_n = [a_n(\mathbf{x}), f_n(\mathbf{x},\mathbf{p},\eta)]\}$ are not mutually orthogonal. However, one may construct an adjoint operator $\tilde{\mathcal{H}}$ and hence define an adjoint eigenvalue problem by $\tilde{\mathcal{H}}\tilde{V}_n = \tilde{\mu}_n\tilde{V}_n$ such that the adjoint eigenvectors $\{\tilde{V}_n = [\tilde{a}_n(\mathbf{x},\mathbf{p},\eta), \tilde{f}_n(\mathbf{x},\mathbf{p},\eta)]\}$ are orthogonal to the original set. This property is known as biorthogonality [14]. To carry out this procedure, we define the scalar product by $\left\langle V_n\tilde{V}_m\right\rangle_5 \equiv \left\langle a_n\tilde{a}_m + f_n\tilde{f}_m\right\rangle_5$, where $\langle\rangle_5 \equiv \int d^2\mathbf{x}d^2\mathbf{p}d\eta \equiv \int d^5Y$. In addition, we introduce more shorthands for later use, $\langle\rangle_2 \equiv \int d^2\mathbf{x}$ and $\langle\rangle_3 \equiv \int d^2\mathbf{p}d\eta$. The adjoint operator is found to be

$$\tilde{\mathcal{H}} = \left[\begin{array}{cc} \left(k_w\Delta\nu - \frac{1}{2k_r}\frac{\partial^2}{\partial\mathbf{x}^2}\right), & \left(h_f\frac{\partial F}{\partial\eta}\right) \\ \left(-h_a\int d^2\mathbf{p}d\eta\right), & \left(\nu\xi + i[\mathbf{p}\frac{\partial}{\partial\mathbf{x}} - k_\beta^2\mathbf{x}\frac{\partial}{\partial\mathbf{p}}]\right) \end{array}\right].$$

Indeed, it can be verified that biorthogonality holds $\left\langle V_n\tilde{V}_m\right\rangle_5 = \delta_{nm}\mathcal{N}_n$, where $\mathcal{N}_n = \left\langle V_n\tilde{V}_n\right\rangle_5$, and the two sets of eigenvalues are identical, $\{\tilde{\mu}_n\} = \{\mu_n\}$. As a result, a formal solution to IVP can be expressed as

$$\Psi(z) = \sum_n C_n V_n e^{-i\mu_n z} + ..., \quad C_n = \frac{\left\langle\Psi(0)\tilde{V}_n\right\rangle_5}{\left\langle V_n\tilde{V}_n\right\rangle_5}, \tag{4}$$

where the four components $\{a_n, f_n, \tilde{a}_n, \tilde{f}_n\}$ in two eigenvectors $\{V_n, \tilde{V}_n\}$ can be determined by first solving the eigenmode equation [3,4] for a_n

$$\left[\mu_n - k_w\Delta\nu + \frac{1}{2k_r}\frac{\partial^2}{\partial\mathbf{x}^2}\right]a_n(\mathbf{x}) = ih_a h_f \times\int_{-\infty}^{\infty}d^2\mathbf{p}\int_{-\infty}^{\infty}d\eta\frac{\partial F}{\partial\eta}\int_{-\infty}^{0}dse^{-i(\mu_n-\xi)s}a_n[\mathbf{Q}^+], \tag{5}$$

where $\mathbf{Q}^+ = \mathbf{x}\cos(k_\beta s) + (\mathbf{p}/k_\beta)\sin(k_\beta s)$. The most complete solutions of Eq.(5) were obtained for a Gaussian model [1]. Without these solutions, in particular the analytical ones, it is practically impossible to perform any specific calculation with the formal solution, Eq.(4), which is becoming drastically more complicated with the inclusion of emittance and betatron oscillations. Note that the eigenvalue of Eq.(5) is related to our earlier notation [1] by $\mu = iq/2L_{1d} + k_w\Delta\nu$. Given a_n, other components are determined by $a_n = \langle\tilde{a}_n\rangle_3$, and

$$f_n = -ih_f\frac{\partial F}{\partial\eta}\int_{-\infty}^{0}dse^{-i(\mu_n-\xi)s}a_n[\mathbf{Q}^+],$$
$$\tilde{f}_n = ih_a\int_{-\infty}^{0}dse^{-i(\mu_n-\xi)s}a_n[\mathbf{Q}^-],$$

where $\mathbf{Q}^- = \mathbf{x}\cos(k_\beta s) - (\mathbf{p}/k_\beta)\sin(k_\beta s)$. The general solution to IVP has opened the floodgate to solutions of many important problems that were simply not possible to solve before. Depending on how the initial condition, $\Psi(0) = [a_\nu(\mathbf{x},0), f_\nu(\mathbf{x},\mathbf{p},\eta,0)]$, is specified, the solution can be applied for example to the following problems: input field coupling, noise power for SASE, dynamic connection between sections of a multi-segment wiggler, cascade harmonic generation, and transverse coherence of SASE when higher order modes are included in the analysis. Let's look at some of these problems in detail.

Radiation power spectrum can be expressed from Eq.(4) as a summation of diagonal and cross terms

$$\frac{dP(z)}{d\omega} = \sum_n\sum_m\frac{dP_{nm}(z)}{d\omega}, \tag{6}$$

because the eigenmodes $\{a_n\}$ are not power-orthogonal. Taking the initial condition as $\Psi(0) = [a_\nu(\mathbf{x},0), 0]$ for an input field, we have for each term above

$$\frac{dP_{nm}(z)}{d\omega} = G_{nm}\frac{dP_{in}}{d\omega}\ e^{-i\mu_n z + i\mu_m^* z}, \tag{7}$$

$$G_{nm} = \frac{\langle a_\nu(0)a_n\rangle_2\langle a_\nu(0)a_m\rangle_2^*\langle a_n a_m^*\rangle_2}{\mathcal{N}_n\mathcal{N}_m^*\langle|a_\nu(0)|^2\rangle_2}, \tag{8}$$

$$\frac{dP_{in}}{d\omega} = \frac{mc^2}{4\pi r_e l_r}\left\langle|a_\nu(0)|^2\right\rangle_2, \tag{9}$$

where G_{nm} is input coupling coefficient, l_r is pulse length and $dP_{in}/d\omega$ is power spectrum of the input field. If input bandwidth is narrower than FEL gain bandwidth, one may use the form $a_\nu(\mathbf{x},0) = a_{in}(\mathbf{x})g_{in}(\nu)$ and express frequency integrated power as $P_{nm}(z) = G_{nm}P_{in}\exp(-i\mu_n z + i\mu_m^* z)$, where P_{in} is the total input power. For diagonal terms, the coefficient $G_n \equiv G_{nn}$ is a positive quantity which can be maximized by varying input field profile $a_{in}(\mathbf{x})$, yielding $a_{in}(\mathbf{x}) = a_n^*(\mathbf{x})$, where an asterisk indicates complex conjugate. This condition is known as conjugate input mode coupling [7], under which maximum power coupling to mode a_n is reached at $G_n = \left\langle|a_n|^2\right\rangle_2^2/|\mathcal{N}_n|^2$. In 1D limit where all transverse modes become degenerate, we have from Eq.(8) the well-known result $G_{1d} = 1/9$ [10].

Initial condition for SASE is specified by $\Psi(0) = [0, f_\nu(\mathbf{x},\mathbf{p},\eta,0)]$. The term describing beam fluctuation, $f_\nu(\mathbf{x},\mathbf{p},\eta,0)$, is governed by the statistics of noise that is completely uncorrelated in 6D phase space [11], $\langle\mathcal{F}(X;0)\mathcal{F}(X';0)\rangle_{en} = (k_r/n_0)\delta(X-X')F(Y)$, where the angle bracket $\langle\rangle_{en}$ indicates ensemble average. Hence, SASE power spectrum after ensemble average is

$$\frac{dP_{nm}(z)}{d\omega} = P_{\omega nm} e^{-i\mu_n z + i\mu_m^* z}, \tag{10}$$

$$P_{\omega nm} = \frac{mc^2 k_r^2}{8\pi^2 r_e n_0} \frac{\left\langle F \tilde{f}_n \tilde{f}_m^* \right\rangle_5 \langle a_n a_m^* \rangle_2}{\mathcal{N}_n \mathcal{N}_m^*}. \tag{11}$$

From now on we will consider the case when SASE is dominated before saturation by the fundamental mode, thus Eq.(6) is simplified to $dP(z)/d\omega = P_\omega \exp[2Im(\mu)z]$, where P_ω can be identified from Eq.(11) with $n = m = 0$. To obtain frequency integrated power, the factor with dominant frequency dependence, $\exp[2Im(\mu)z]$, can be approximated by an expansion near the peak of the gain spectrum at ν_p, $2Im(\mu)z = z/L_g - \delta\nu^2/2\sigma_\nu^2 + ...$, where L_g is the power gain length corresponding to the peak gain, $\delta\nu = \nu - \nu_p$ and $\sigma_\nu = 1/\sqrt{\alpha_\omega (k_w L_{1d})(k_w z)}$. The factor α_ω can be determined by either a perturbation calculation or a fit to the gain curve near the peak. In 1D limit without energy spread, $\alpha_{\omega 1d} = 2/3$. Defining effective SASE bandwidth by $\Sigma_\omega = \sqrt{2\pi}\omega_r \sigma_\nu$, we may express frequency integrated power as $P(z) = P_\omega \Sigma_\omega \exp(z/L_g)$. It is useful to introduce a quantity known as effective startup noise power, P_{sn}, by expressing the SASE power in the same form as that for coherent amplification from an input field, thus $P_{sn} = P_\omega \Sigma_\omega / G_0$. All earlier results on SASE power [4–6,8] can be obtained trivially as limiting cases of Eq.(11). Particularly in 1D limit [4–6], we have $P_{\omega 1d} = (1/9)(\rho E_0/2\pi)$, where $\rho = 1/2\sqrt{3} k_w L_{1d}$ is the Pierce parameter and $E_0 = \gamma_0 mc^2$.

To perform specific calculations, we choose a Gaussian model, $F(\mathbf{x}, \mathbf{p}, \eta) = F_\perp(\mathbf{x}, \mathbf{p}) F_\parallel(\eta)$ with $F_\perp = (1/2\pi\sigma_x^2 k_\beta^2) \exp[-(k_\beta^2 \mathbf{x}^2 + \mathbf{p}^2)/2k_\beta^2\sigma_x^2]$, $F_\parallel = (1/\sqrt{2\pi}\sigma_\eta) \exp(-\eta^2/2\sigma_\eta^2)$, where σ_η is relative rms energy spread, $\sigma_x = \sqrt{\varepsilon/k_\beta}$ is rms beam size matched to the betatron focusing channel, and ε is rms emittance. Three lowest order modes in analytical form have been obtained for the Gaussian model by variational approximation, compared with the exact numerical solutions, and found highly accurate in the short wavelength region [1]. For the fundamental mode with a profile given by $a_0(\mathbf{x}) = \exp[-\alpha(\mathbf{x}/\sigma_x)^2]$ [1], we obtain the results

$$G_0 = \frac{4\alpha\alpha^*}{(\alpha + \alpha^*)^2} \frac{1}{|1 + i4h\alpha I|^2}, \tag{12}$$

$$\frac{P_\omega}{P_{\omega 1d}^g} = \frac{72\alpha\alpha^*}{(\alpha + \alpha^*)} \frac{J}{|1 + i4h\alpha I|^2}, \tag{13}$$

where α is a complex quantity known as the mode parameter, G_0 is the coefficient under conjugate input coupling, P_ω is normalized by the Gaussian asymptotic 1D limit, $P_{\omega 1d}^g = (4/3)P_{\omega 1d}$, to be discussed later, I and J, originally each a 7-dimensional integral, are reduced to 1-dimensional integrals as follows

$$I = \int_0^\infty d\tau f_i, \quad J = \frac{1}{2\kappa_i} \left[\int_0^\infty d\tau f_j + c.c. \right],$$

$$f_i = \frac{\tau^2 e^{i\kappa\tau - 2\eta_\gamma^2 \tau^2}}{(1 - i\eta_\varepsilon \tau)^2 + 4\alpha(1 - i\eta_\varepsilon \tau) + 4\alpha^2 \sin^2(2\sqrt{\eta_d \eta_\varepsilon} \tau)},$$

$$f_j = \frac{e^{i\kappa\tau - 2\eta_\gamma^2 \tau^2}}{(1 - i\eta_\varepsilon \tau)^2 + 4\alpha_r(1 - i\eta_\varepsilon \tau) + 4|\alpha|^2 \sin^2(2\sqrt{\eta_d \eta_\varepsilon} \tau)},$$

where $\kappa = 2L_{1d}\mu$, $\kappa_i = Im(\kappa)$ and $\alpha_r = Re(\alpha)$. The scaling parameters are defined by [1] $\eta_d = L_{1d}/2k_r\sigma_x^2$, $\eta_\varepsilon = 4\pi\,(L_{1d}/\lambda_\beta)\,k_r\varepsilon$, $\eta_\gamma = 4\pi\,(L_{1d}/\lambda_w)\,\sigma_\eta$, where $\lambda_\beta = 2\pi/k_\beta$. For LCLS nominal case [15], the scaling parameters take the values [1] $\eta_d = 0.0367$, $\eta_\varepsilon = 0.739$ and $\eta_\gamma = 0.248$. Another scaling parameter for frequency detuning, $\eta_\omega = 4\pi\,(L_{1d}/\lambda_w)\,\Delta\nu$, will be optimized for peak growth rate in all subsequent calculations. The input coupling coefficient for the fundamental mode, G_0 from Eq.(12), is plotted in Fig.1, the normalized SASE spectral power, $P_\omega/P_{\omega 1d}^g$ from Eq.(13), is plotted in Fig.2, all as functions of η_ε and η_γ with $\eta_d = 0.0367$.

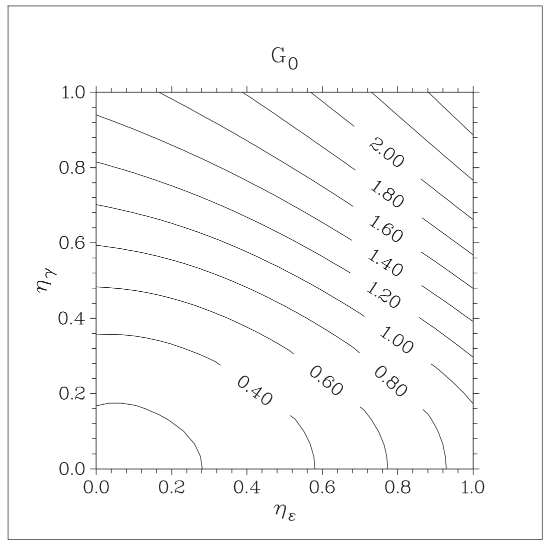

FIG. 1. Coutour plot of $G_0 \in \{\eta_\varepsilon, \eta_\gamma\}$ with η_d=0.0367

It is shown in Fig.1 that G_0 increases monotonically to magnitude even beyond unity for larger η_ε and η_γ. This result, suggesting that partial can be larger than the whole, seems to be in apparent violation with energy conservation. Since power in the fundamental mode at $z = 0$ is given by $P_0(0) = G_0 P_{in}$, thus $G_0 > 1$ would necessarily imply that initial power coupled into one mode out of many is larger than there is in the whole input field to start with. It turns out that this phenomenon is a direct consequence of the non-power-orthogonal nature of the eigenmodes. In this situation, conservation of energy is maintained by the nonvanishing cross terms, of which some are negative. This intriguing paradox is shown next to be responsible, at least partially, also for the excessively large noise power.

One of the most surprising and important results of this paper is the discovery of excessively large noise power for SASE. It is shown in Fig.2 that the noise power can be much larger than what previously known and the ratio increases monotonically without bound for larger η_ε and η_γ. Specific calculations of SASE noise power were performed previously only in two cases. In the case of 1D limit ($\eta_\varepsilon = \eta_d = 0$) specified by $F(\mathbf{x}, \mathbf{p}, \eta) = \delta(\mathbf{p})F_{\|}(\eta)$, where $F_{\|}(\eta)$ is a tophat profile, it was found that $P_\omega/P_{\omega 1d} \leq 1.7$ [5]. In the case of parallel beam limit ($\eta_\varepsilon = \eta_\gamma = 0$) specified by $F(\mathbf{x}, \mathbf{p}, \eta) = u_\perp(r)\delta(\mathbf{p})\delta(\eta)$, where $u_\perp(r)$ is again a tophat profile, it was found that $P_\omega/P_{\omega 1d} \leq 1$ [8]. Thus, one may take $P_{\omega 1d}$ as the reference magnitude for SASE noise power known from previous explorations of severely limited parameter space and models.

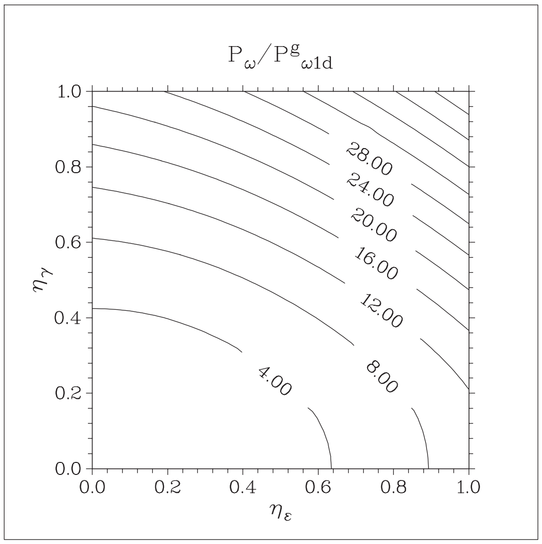

FIG. 2. Coutour plot of $P_\omega/P^g_{\omega 1d} \in \{\eta_\varepsilon, \eta_\gamma\}$ with η_d=0.0367

Although the normalized noise power increases with η_ε and η_γ, this effect is accompanied by the drop in the growth rate [1]. Nevertheless, the surprising effect may translate into real benefits for existing designs of SASE FELs. Let's define a factor of surprise by the ratio of frequency integrated noise power relative to the corresponding 1D value, $f_S = P(0)/P_{1d}(0) = (P_\omega/P_{\omega 1d})(\Sigma_\omega/\Sigma_{\omega 1d})$. Taking LCLS nominal case as an example, from Fig.2 we have $P_\omega/P^g_{\omega 1d} = 7.2$, which gives $P_\omega/P_{\omega 1d} = 9.6$, and with $\Sigma_\omega/\Sigma_{\omega 1d} = 0.9$, the factor is $f_S = 8.6$. A reduction in saturation length is then given by $\Delta L_{sat} = -L_g \ln(f_S)$ $= -13$m, where $L_g = 6$m [1]. Alternatively, the factor of surprise can be expressed as $f_S = (G_0/G_{1d})(P_{sn}/P_{sn1d})$. Thus, noting $G_0/G_{1d} = 6.7$ from Fig.1, we see that most of the increase in noise power is due to the enhancement in input coupling for this case. As explained before, the dramatic increase in the coupling coefficient, leading consequently to the paradox that partial can be larger than the whole, is a direct manifestation of non-power-orthogonal nature of the eigenmodes.

Until recently, there has been only one 1D limit. Let's call it the fictitious 1D limit. It is generally believed that this limit can be approached asymptotically by increasing the transverse beam size from a finite distribution. However, it is found recently for Gaussian distribution that asymptotic behavior of the eigenmodes are different from what expected [1]. As a result, two more differences are discovered here. The two quantities from Gaussian asymptotic 1D limit, G^g_{1d} and $P^g_{\omega 1d}$, are related to that of the fictitious 1D limit by $G^g_{1d} = (4/3)\, G_{1d}$ and $P^g_{\omega 1d}$ $= (4/3)\, P_{\omega 1d}$. For model dependent self-consistency, we have used the Gaussian asymptotic 1D limit as a normalization factor in Eq.(13).

In summary, I have presented the first solution to the grand initial value problem. It is the first time for the startup theory to reach full 6D in climbing the mountain of dimensions. The solution has provided a rigorous foundation for the analysis of initiation and subsequent evolution of power and coherence in a high gain FEL amplifier before saturation. The specific solution developed for Gaussian model has been made so efficient that it is now possible to map out SASE noise power, input coupling and many other important quantities in the entire parameter space of interest to short wavelength FELs. In particular, the discovery of excessively large SASE noise power may have significant impact on current development of high gain FELs. Furthermore, the solution has provided a unique benchmark in noise power and other startup properties for modeling and simulation of SASE process in 6D phase space. Last but not least, this work has opened the floodgate to solutions of many more important pending problems. This work was supported by the US DOE under contract No.DE-AC03-76SF00098.

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