Essays on Asset Liquidity and Its Policy Implication

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Abstract

Financial market has been essential to macroeconomy and is crucial in understanding the policy transmission. Frictions in the financial market can shape agents' preference towards various financial instruments, and give extra incentives for investors to hold assets with better liquidity property. The price premium derived from such liquidity concern can further affect firms' financial decision, i.e., dividend policy. Furthermore, since real money balance, joint with dividend payment, affect agents' liquidity position and hence demand for the dividend assets, monetary policy can have important impact on firms' dividend decision and investment plan. Also, as the secondary asset market and credit become more widely available, antoher question to be asked is whether such change always improves the welfare. This dissertation focuses on the role of market friction and liquidity in understanding these issues.

In Chapter One, I provide a new theoretical resolution to the long-standing dividend puzzle. And I answer the question that why do agents have preference towards dividend assets over non-dividend assets, despite the fact the tax legislation is unfavorable towards dividend payment when comparing to the same amount of capital gain. To answer this question, I build a partial equilibrium search-theoretic model taking asset supplies as given. In this model, dividend assets are not just a store of value but also provide direct liquidity when agents have urgent consumption need. The model delivers several testable implications. First of all, in an economy where some transactions require a proper means of payment, dividend assets serve as a better liquidity instrument when comparing to non-dividend assets, and hence carry a price premium. Second, the turnover ratio of non-dividend assets is higher than the turnover ratio of dividend asset. Lastly, the price premium from dividend payment increases as the market becomes less liquid. I then provide empirical evidence to support all three propositions.

In Chapter Two, I study firms' optimal dividend decision. More specifically, I ask the question that, if paying dividend is always a better policy than not paying dividend or vice versa, then why not all firms have the same dividend policy. To answer this question, I endogenize the asset supplies which are taken as given in Chapter One. I argue that firms are facing a trade-off between having higher price, if they decided to pay dividend, versus a higher TFP, if they decided to not pay dividend but use the resource in a more productive way, i.e., R&D activities. With such trade-off, in equilibrium firms will be indifferent between paying dividend versus not paying dividend, hence have different dividend policy. For monetary policy implication, the model predicts that a contractionary monetary policy can hurt the economy not only through the traditional channel of depressing real money balance, but also through a less-explored channel of discouraging aggregate R&D activities.

In Chapter Three, a joint work with Athanasios Geromichalos, we study whether the introduction of alternative (to money) payment instruments is always welfare improving. The common belief is that the introduction of credit reduces market friction and hence should improve agents' welfare. However, we show that this is not always true. More specifically, more credit *ex post* means that transactions will not be hindered by lack of liquidity, however *ex ante*, easier access to credit means agents have less incentive to hold money, which will hurt transactions in bilateral meetings where credit is not accepted. Our model also offers a theoretical explanation to the growing empirical literature suggesting that increased access to credit is often followed recession and economic hardships.

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CHAPTER 1

A Liquidity-based Resolution to the Dividend Puzzle

1.1. Introduction

The irrelevance theorem proposed in the renowned Miller and Modigliani (1961) paper states that, in an economy characterized by perfect capital markets, rational behavior, and perfect foresight, the current value of the firm should be independent of the dividend decision. However, empirical evidence shows that assets that pay dividends command a higher price premium compared to assets that do not pay dividends. For example, Long Jr (1978) uses a case study of Citizens Utility Company to show that claims to cash dividends have commanded a premium in the market over claims to an equal amount (before taxes) of capital gains. This phenomenon remains relevant in more recent data. For instance, Hartzmark and Solomon (2013) documents that asset pricing has a positive abnormal return during the months when firms are expected to issue dividend. Also, Karpavičius and Yu (2018) show that the price premium is constantly positive for stocks that pay regular dividends. This puzzling observation becomes even harder to reconcile when one considers that dividend payments are generally taxed more heavily than (equal amounts of) capital gains.

The goal of this paper is to offer a liquidity based resolution to the aforementioned puzzle. The underlying assumptions of the Modigliani-Miller theorem require perfect capital markets, but real world secondary (asset) markets are not perfect; they are characterized by search (and other types of) frictions, intermediation fees, and trading delays. In this, more realistic setting, assets with similar cash flows may be priced differently, because of liquidity considerations. Here, I show that assets that pay regular dividends can be priced at a premium because agents can use the dividend to satisfy random liquidity needs, and, importantly, avoid having to liquidate their assets (that do not pay dividends) in secondary markets characterized by search and bargaining (or other) frictions. After establishing the main idea of the paper, i.e., that assets that pay regular dividends are effectively more liquid, I also study the firms' decision to pay dividends or not.

To answer my research question I employ a monetary-search model, as in Lagos and Wright (2005*a*), extended to incorporate an Over-the-Counter (OTC) secondary asset market, where agents can rebalance their portfolios. Agents are subject to random liquidity needs, and when such liquidity needs arrive agents must trade in a *quid pro quo* fashion using cash as a medium of exchange. Naturally, dividends that have already been delivered are virtually cash. Thus, developing a model that is explicit about the frictions that make a medium of exchange necessary in certain markets, is crucial for understanding that assets that pay (frequent) dividends provide *direct* liquidity to the agents and will be priced accordingly. It is important to highlight that assets that do not pay (frequent) dividends are also liquid, but only in an *indirect* way: if a liquidity shock arrives and the agent finds herself holding assets that do not pay dividends, she can still acquire liquidity by selling some assets for cash in the secondary market. However, if this secondary market is characterized by search or bargaining frictions, as is the case in my model, the agent faces the risk of not being able to sell at all or having to sell at a price that is lower than the fundamental market value. This is precisely why agents are willing to pay a premium for a dividend-paying asset: they can use the dividend to cover their current liquidity needs, thus, avoiding to sell assets in less-than-optimal conditions in the secondary market.

Whether agents can always avoid visiting the frictional secondary market to sell assets depends on the supply of dividend-paying assets (which in the first part of the paper is exogenous) and the value of the dividend. If the amount of 'direct liquidity' (provided by dividend-paying assets) is not enough to satisfy the liquidity needs of the economy, agents will visit the secondary asset market to sell bonds anyway. Thus, dividend-paying assets can carry both a direct liquidity premium (by paying dividend that the agent can use to purchase consumption) or an indirect liquidity premium (by being sold for cash in the secondary market). In contrast, non-dividend assets can carry only an indirect liquidity premium.

Besides offering a liquidity-based explanation for the dividend puzzle, the model also delivers two new and testable theoretical predictions. First, I show that asset trade exhibits a certain "pecking order". More precisely, when agents visit the secondary market to sell assets and meet their liquidity needs, non-dividend assets will be traded first before agents sell any dividend-paying assets. The intuition is simple: since dividend assets provide direct liquidity, selling them for money would imply that the seller (i.e., the agent in need of liquidity) is missing the opportunity to use the upcoming dividend to purchase goods. Thus, when liquidity is scarce, agents sell non-dividend assets first, and only when non-dividend assets are not enough to meet the liquidity needs of the agent, does the agent decide to sell some dividend-paying assets. The second testable prediction of the model is that the dividend premium increases in the degree of market frictions. Again, this is intuitive: as matching in the secondary market becomes less efficient, the chance of agents not being able to sell assets becomes larger, and this makes agents more willing to pay a premium in order to hold dividend-paying assets, which are more likely to help them avoid visiting the secondary asset market altogether.

To support both of these predictions, I use data from Compustat and CRSP of more than 7,000 firms between the year 1971-2016. The data shows a price premium of 10.75% if assets pay dividend. Furthermore, the turnover ratio of dividend-paying assets is lower, implying that non-dividend assets are traded more frequently comparing to dividend asset. This is consistent with the model prediction that non-dividend assets are traded first before dividend assets, thus having a higher trading volume. Finally, the data shows that the dividend alone does not provide much explanatory power in predicting asset prices, but only gives rise to the pricing premium in the presence of market friction. This result not only is consistent with the irrelevance theory that dividends should have no impact on firms' value (with the assumption of perfect financial market), but also provides direct support to the liquidity channel in my model, namely, the idea that dividends help agents avoid liquidating assets in frictional secondary market and, hence, command a premium.

1.2. Literature Review

This paper is related to two strands of literatures. The first strand is the theoretical and empirical literatures on the dividend puzzle phenomenon. This is a long-standing puzzle, so there have been theories proposed in the past trying to explain it. For example, the signaling theory first proposed in Miller and Rock (1985), in which higher dividend payout is interpreted as a positive sign of a company's future earning, and hence increases investors' preference over such stocks. Despite much supporting evidence, some of the empirical studies find evidence unfavorable to the signaling theory. For example, in Bernhardt et al. (2005), the paper provides evidence that there is no positive relation between bang-for-the-buck (positive abnormal price response per dollar of dividend) and cost of signal, which contradicts with the signaling theory prediction. Another theory tries to explain the puzzle is from the behavioral finance perspective. Proposed in Shefrin and Statman (1984), the prospect theory and self-control theory state that investors with risk-averse behavior keep dividend and principal as two separate mental accounts. Being able to use only dividend for financing their consumption allows them to leave the principal untouched, hence is valued by agents. However, the questionnaire results surveyed in Dong et al. (2005) found ambiguous evidence in supporting such theory. The transaction cost theory proposed in Allen and Michaely (2003) argues that investors prefer a dividend-paying stock due to the significantly smaller transaction cost compare to selling the portfolio. However, this is not supported by the time-series evidence on transaction cost. Due to the regulation, transaction cost decreased substantially after 1975, and according to the transaction cost explanation, the demand on dividend should be lower, but empirical evidence shows that total dividend being paid out was not reduce, and the pricing premium did not respond significantly. The difference between the mechanism I propose in this paper and the transaction cost theory is that, transaction cost explanation only considers the fee charged by brokers but does not take into account the implicit cost that asset holders might not be able to sell the assets right away or might have to sell it at a price that is lower than fundamental market value. Hence despite the evidence that is unfavorable to the transaction cost theory, it does not invalidate the discussion on search and bargaining friction in the secondary asset market. There are other theories trying to solve the puzzle, such as dividend clientele theory and uncertainty resolution, however receiving conflicting empirical testing results. Moreover, besides the mixed empirical evidence, the proposed theories do not seem to pay attention to the monetary implication. Whereas in my paper, I show that monetary policy can hurt the economy not only through reducing real money holdings as traditionally believed, but also has additional negative impact by discouraging aggregate R&D activities.

This paper also relates to the literature on asset liquidity factor and liquidity premium in pricing, such as Geromichalos et al. (2007*a*), Nosal and Rocheteau (2013), Andolfatto et al. (2014), Geromichalos et al. (2016), and Geromichalos et al. (2022). Besides the dividend puzzle I study in this paper, the liquidity service provided by assets has been used to explain some other long-standing puzzles as well, such as the on-the-run puzzle studied in Vayanos and Weill (2008), the equity premium puzzle studied in Lagos (2010), and asset home bias puzzle discussed in Geromichalos and Simonovska (2014). This paper complements this series of paper.

1.3. The Model

1.3.1. Environment. The model I employ in this paper is a monetary-search model as in Lagos and Wright (2005*a*), with an OTC secondary market which features search and bargaining as in Duffie et al. (2005). The economy has infinite horizon and time is discrete. In each period, there are three sub-markets where different economic activities take place: a secondary asset market, a decentralized market, and a centralized market. The centralized market (henceforth CM) is the settlement market of the Lagos-Wright model. Allowing agents to visit this frictionless market at the end of every period, together with quasi-linear preferences is what makes the model tractable and prevents the state space from exploding. The decentralized market (henceforth DM) captures the idea that not all trades/transactions take place in a Walrasian and frictionless market. In cases where there is lack of record keeping or commitment, trade will require a proper means of payment. The existence of the DM allows me to model these types of transactions, and gives a special role to assets that pay dividend, which is as good as money in providing liquidity services. Finally, agents may often find themselves in need of more liquidity, and may want to sell some of their assets in exchange for money. The existence of a secondary asset markets opening at the beginning of each period allows precisely these kinds of transactions to take place.

The economy has two types of agent, buyers and sellers, characterized by their role in DM, which will be permanent. The measure of buyers is normalized to 1. At the beginning of each period, a consumption shock is realized. ℓ fraction of the buyers will have consumption need in the upcoming DM and consume a special goods q, which will be produced by sellers. I will refer to them as active buyers (or A-buyers). The remaining $1-\ell$ fraction of buyers do not have such consumption need, hence I call them the inactive buyers (or I-buyers). This special goods consumption in DM can be thought of as an unusual purchase of goods, for example, an urgent buying of a house or an urgent medical bill needs to be paid quickly. Since matching friction in the DM is not the main focus of this paper, for simplicity I assume that all A-buyers will meet a seller in DM, and thus I normalize the measure of sellers to ℓ. Since all DM transactions need to be facilitated by proper medium of exchange, the idiosyncratic DM consumption opportunity realization makes buyers value their assets differently, hence gives rise to an incentive for assets trading between A-buyers and I-buyers in the secondary asset market (which opens right before the DM). A-buyers, who have liquidity need for DM consumption, will want to sell assets in exchange for money, thus they enter the secondary asset market as asset sellers. I-buyers do not have a consumption need in the DM, hence they participate in the secondary market as asset buyers.

The economy also has two types of assets. The first one is money, which has no intrinsic value, but is storable and recognizable in any type of transaction, hence it helps avoid the friction in DM created by the lack of record keeping. The market price of money in terms of general goods in the CM is φ. Its supply is controlled by a monetary authority, and it evolved according to the rule $M_{t+1} = (1+\mu)M_t$ with $\mu > \beta - 1$. The second type of assets is two sets of infinitely-lived Lucas trees, which agents can buy in the CM at market price ψ_1 and ψ_2. Both trees pay dividend d in time $t+1$ and have resale value in the CM. But the probability that they pay dividend in different sub-markets is different. The first tree (type-1 asset), with aggregate supply A_1, pays d in the DM with probability θ_1, and hence pays in the CM with probability $1 - \theta_1$. Similarly, the second tree (type-2 asset) with aggregate supply A_2, pays d in the DM with probability θ_2. Without loss of generality, $\theta_1 > \theta_2$. Since DM is the market which agents need liquidity, assuming that type-1 assets have a higher probability to pay dividend in the DM represents its superior ability to facilitate trade in that market. In a more realistic setting, type-1 assets would represent assets that pay dividends more frequently. The difference in probability of paying dividend reflects the fact that if the dividend is paid more frequently, the chance that the dividend paid at the 'right time' (i.e., when the agent needs it for liquidity) is higher. Hence type-1 asset serves the random consumption opportunities in the DM better than the type-2 asset. Later in the section, for simplicity, I take the probabilities to the extreme case in which $\theta_1 = 1$ and $\theta_2 = 0$, so that type-1 asset always pays d in the DM, while type-2 asset always pays d in the CM. This simplifying assumption does not affect the underlying mechanism of the model; it is just a stark way to capture type-1 assets' superior liquidity role in facilitating consumption compared to type-2 assets.

Next, I will describe the details of the economic activities in each sub-markets. Figure 1 summarizes the timing of the various economic activities in the model.

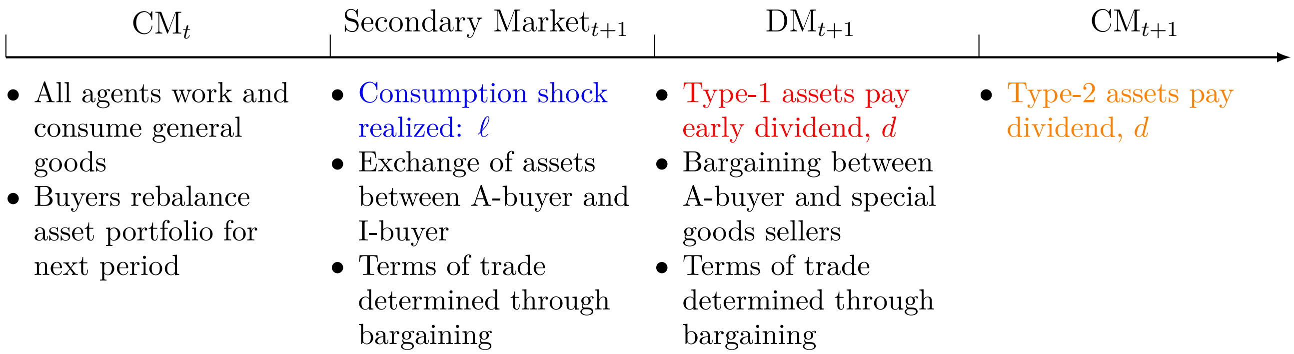


FIGURE 1.1. Timeline of economic activities.

In the CM_t, all agents work H hours and consume general goods X. I assume that one hour of work generates 1 unit of the general good, which is also the numeraire. Buyers will choose the amount of assets (i.e. money, type-1 assets, and type-2 assets) to bring into next period $t+1$, without knowing if they will have consumption needs in the upcoming DM or not. Upon entering $t+1$, the idiosyncratic consumption shock is realized, such that ℓ fraction of the buyers becomes A-buyers, while the remaining $1-\ell$ will become I-buyers. Because buyers have different consumption opportunities and, hence, value assets differently, A-buyers (who need liquidity) and I-buyers (who can provide it) will participate in the secondary asset markets to re-balance their portfolio. The terms of trade in the secondary market are determined through Kalai bargaining, with λ being A-buyers' bargaining power. More specifically, A-buyers will be the assets sellers, and sell y_1 and y_2 amount of the two types of assets, in exchange for y_m amount of money from I-buyers. The measure of meeting between A-buyers and I-buyers will be determined by a matching technology $f(\ell, 1-\ell) \leq \min\{\ell, 1-\ell\}$.

After A-buyers boost their liquidity position, they enter the DM and meet with a seller for special goods consumption. Type-1 assets pay dividend d at the beginning of this sub-markets, and agents who hold type-1 assets can use the dividend for DM consumption. Even though the dividend here is modeled as fruit of the Lucas tree, in reality it is as good as cash, and that is the idea I am aiming to capture in a simple and tractable way. The trade between an A-buyer and a seller is also determined through bargaining; more precisely, A-buyers make take-it-or leave- it (TIOLI) offer to the seller of special good. The seller produces q amount of special goods using a linear production technology that can transfer 1 hour of labor into 1 unit of special goods, and in return, A-buyers will pay an amount π of real balances, which can be either money or dividend (or a combination of both). Finally, after trade in the DM has concluded, all agents move to the CM, where type-2 assets pay dividend d. (Notice that this payment comes 'too late' to be serve the liquidity needs of agents.)

The discount rate between periods is $\beta \in (0,1)$, and there is no discounting between sub-periods. Buyers consume in CM and potentially DM, and supply labor in CM, while sellers consume only in CM but supply labor in both DM and CM. Hence buyers derive utility from consuming special goods q in DM and general goods X in CM, and disutility from working H hours in the CM. The buyers' preference is given by: $U(X,H,q) = u(q) + U(X) - H$. Sellers derive utility from consuming X in CM, and distuility from working h hours in DM and working H hours in CM. The sellers' preference is given by $V(X,H,h) = -h + U(X) - H$. Several standard properties of the utility functions are imposed here: (1) both u and U are twice continuously differentiable. (2) $u(0) = 0, u'(q) > 0, u'(\infty) = 0, u'(0) = \infty, u''(q) < 0$; $U'(X) > 0$, $U''(X) < 0$. (3) There exists an optimal level of DM consumption q^*, such that $q^* \equiv \{q : u'(q^*) = 1\}$. (4)There exists a $X^* \in (0,\infty)$ such that $U'(X^*) = 1$, $U(X^*) > X^*$.

The discussion of the model will focus on steady-state equilibrium, with focus on the asset prices ψ_1 and ψ_2 as the question this paper addresses is the pricing premium of dividend assets. In Section 3 of the discussion, the asset supplies A_1 and A_2 are given exogenously, in order to focus on the liquidity channel of dividend, and how type-1 assets can include a liquidity premium. Later in Section 6 of the paper, I also study the firms' dividend decision, that is, I determine endogenously the measure of firms who decide to pay early dividend.

Thus, the asset supplies of the various types of assets are also endogeneously determined in the equilibrium.

1.3.2. Value Functions.

1.3.2.1. *CM.* I will begin with CM value functions and work backwards. Upon entering CM, a typical buyer's state variables are the following. The first is the real balance, z, that is leftover after DM consumption of special goods. This amount of real balance could come from either the money balance the buyer carries from last period, or the dividend she received at the beginning of DM. The second state variable is the shares of type 1 asset, a_1, that she carried from last period, and has resale value in the CM. The last one is the shares of type 2 asset, a_2, that she carried from last period, which will both pay dividend and have resale value in the CM. The Bellman equation is given by

$$W(z, a_1, a_2) = \max_{X,H,\hat{a}_1,\hat{a}_2,\hat{m}} U(X) - H + \beta \mathbb{E}\{\Omega^i(\hat{m}, \hat{a}_1, \hat{a}_2)\}$$

$$\text{subject to } X + \varphi\hat{m} + \psi_1\hat{a}_1 + \psi_2\hat{a}_2 = H + z + \psi_1 a_1 + (\psi_2 + d)a_2 + \varphi\mu M$$

where φ denotes the price of money in terms of general goods, and Ω^i represents buyer-i's value function of the secondary asset market, with $i \in \{A, I\}$. Variables with hats denote choices made by agents in the current period, which will become state variables in the next period. By substituting H from the budget constraint into the value function, the buyer's CM value function becomes

$$W(z, a_1, a_2) = \Lambda^B + z + \psi_1 a_1 + (\psi_2 + d)a_2$$

$$\text{where } \Lambda^B = U(X^*) - X^* + \max_{\hat{m},\hat{a}_1,\hat{a}_2} \{-\varphi\hat{m} - \psi_1\hat{a}_1 - \psi_2\hat{a}_2 + \beta\mathbb{E}\Omega^i(\hat{m}, \hat{a}_1, \hat{a}_2)\}$$

As a standard feature of this class of model, CM value function is quasi-linear in state variables, and the optimal choice variables are not state-dependent.

Now consider a typical seller entering CM. The only state variable is z, which is the real balance the seller collects in DM by producing and selling special goods q. Sellers do not carry any assets because sellers will never have liquidity need in the DM, and it is costly to carry assets across periods. Sellers do not have the need of rebalancing portfolio in the secondary market, thus proceed directly to the DM. Hence a seller's CM value function is given by

$$W^S(z) = U(X^*) - X^* + \beta V^S \equiv \Lambda^S + z$$

where V^S is seller's value function in the next period's DM, which will be discussed in the next section.

1.3.2.2. *DM.* Next, consider the value functions in the DM. Let q be the quantity of special good produced, and π be the real balance A-buyers pay in exchange. These terms of trade will be discussed in Section 1.3.3.1. For an A-buyer entering DM with the amount of money m, type 1 asset a_1, and type 2 asset a_2, the total amount of liquidity that can be used in DM consumption is $z = \varphi m + da_1$. Thus A-buyer's value function is

$$V(\varphi m + a_1, a_1, a_2) = u(q) + W(\varphi m + da_1 - \pi, a_1, a_2)$$

Similarly, for a seller entering DM without any assets, her value function is

$$V^S = -q + W^S(\pi)$$

1.3.2.3. *OTC.* Finally, consider the value functions in the secondary asset market. At the beginning of period, the consumption shock is realized, and buyers become aware if they will have consumption opportunity in the upcoming DM (A-buyer) or not (I-buyer). This consumption opportunity will happen with probability ℓ. Hence a typical buyer's expected value function is

$$\mathbb{E}\{\Omega^i(m, a_1, a_2)\} = \ell\Omega^A(m, a_1, a_2) + (1 - \ell)\Omega^I(m, a_1, a_2)$$

After buyers become aware of their consumption type, i.e. A-buyer or I-buyer, they will value their assets differently, hence will want to exchange assets in the secondary market. A-buyers, who need more liquidity for DM consumption, are asset sellers, while I-buyers become asset buyers. Given matching efficiency parameter, γ, and the matching function $f(\ell, 1-\ell)$, the probability that A-buyers get matched with a trading counterpart in secondary market is $\alpha_A \equiv \gamma f(\ell, 1-\ell)/\ell$, and similarly the probability of I-buyers get a match in secondary market is $\alpha_I \equiv \gamma f(\ell, 1-\ell)/(1-\ell)$. Hence the probability of A-buyers not get a match is $1-\alpha_A$, while that probability for I-buyers is $1-\alpha_I$. Let y_m denote the amount of money A-buyer receives, and denote y_1 as the amount of type 1 asset, and y_2 as the amount of type 2 asset that A-buyer transfers to I-buyer. These terms of trade will be discussed in more details in Section 1.3.3.2. The expected secondary market value function for A-buyer and I-buyer are given as

$$\mathbb{E}\{\Omega^A(m, a_1, a_2)\} = \alpha_A V\Big(\varphi(m+y_m) + d(a_1-y_1), a_1-y_1, a_2-y_2\Big) + (1-\alpha_A)V\Big(\varphi m + da_1, a_1, a_2\Big)$$

$$\mathbb{E}\{\Omega^I(m, a_1, a_2)\} = \alpha_I W\Big(\varphi(m-y_m) + d(a_1+y_1), a_1+y_1, a_2+y_2\Big) + (1-\alpha_I)W\Big(\varphi m + da_1, a_1, a_2\Big)$$

Since A-buyers consume in the DM, A-buyers' secondary market value function is the weighted average of the DM value function; I-buyers do not have consumption opportunity in the DM and proceed directly to CM, thus I-buyers' secondary market value function is the weighted average of the CM value function.

1.3.3. Bargaining in Secondary Asset Market and DM. In this section, I characterize the bargaining solutions in the secondary asset market and in the DM.

1.3.3.1. *DM.* In DM, A-buyer meets with a seller. A-buyer comes into the market with total liquidity $\varphi m + da_1$ and total assets a_1, a_2, while sellers do not carry any assets as discussed in the previous section. A-buyer makes a TIOLI offer, and the two parties bargain over quantity q that seller will produce, and real balance π that A-buyer will pay. Notice that the DM meeting requires the transaction to be facilitated by proper means of payment, hence the total payment π cannot exceeds the amount of liquidity A-buyer has, i.e. $\varphi m + da_1$. The bargaining problem maximizes A-buyer's trading surplus, subject to seller's participation constraint, and A-buyer's liquidity constraint.

A-buyer's bargaining surplus is given by $u(q) + W(z - \pi, a_1, a_2) - W(z, a_1, a_2)$, in which $u(q)$ is the utility derived from consuming the special goods, and $W(z-\pi, a_1, a_2) - W(z, a_1, a_2)$ is the reduction in CM continuation value because of the payment made to sellers. Similarly, for a seller, the bargaining surplus is $-q + W^S(\pi) - W^S(0)$, in which $-q$ is the disutility from providing labor hours to produce q amount of special goods, and $W^S(\pi) - W^S(0)$ is the increase in CM continuation value because of the payment they receive from A-buyers. Hence the bargaining problem is given by

$$\max_{q,\pi} \; u(q) + W(\varphi m + da_1 - \pi, a_1, a, 2) - W(\varphi m + da_1, a_1, a_2)$$

$$\text{subject to } \; -q + W^S(\pi) = W^S(0)$$

$$\pi \leq \varphi m + da_1$$

Because of CM value function's linearity in state variables derived in previous section, DM bargaining problem can be simplified as

$$\max_{q,\pi} \; u(q) - \pi$$

$$\text{subject to } \pi = q \leq \varphi m + da_1$$

LEMMA 1.1. *The DM bargaining solution is given by* $q = \pi = \min\{q^*, \varphi m + da_1\}$.

PROOF. Proof is obvious, and hence omitted. □

The bargaining solution is straightforward and standard in the literature, hence the proof is omitted. The bargaining solution states that, if A-buyers brought abundant liquidity into

DM, then they will consume the first-best quantity q^*, and pays the same amount of liquidity in exchange. However, in the case where the liquidity is not enough to purchase q^*, i.e. $\varphi m + da_1 < q^*$, then A-buyers will give all her liquidity to the seller, and exchange for the same amount of special goods to consume according to the seller's participation constraint.

1.3.3.2. *Secondary Asset Market.* In the secondary asset market, bargaining is between an A-buyer with asset portfolio $(m,\ a_1,\ a_2)$, and I-buyer with portfolio $(\tilde{m},\ \tilde{a}_1,\tilde{a}_2)$. The amount of money that changes hand is y_m, and A-buyers pay y_1 and y_2 amount of assets to I-buyers in exchange. In the secondary market, A-buyer's and I-buyer's bargaining surplus are given by

$$\begin{aligned}
S^A &\equiv V\Big(\varphi(m+y_m)+d(a_1-y_1), a_1-y_1, a_2-y_2\Big) - V\Big(\varphi m + da_1, a_1, a_2\Big) \\
&= u\Big(\varphi(m+y_m)+d(a_1-y_1)\Big) - u\Big(\varphi m + da_1\Big) - \psi_1 y_1 - (\psi_2+d)y_2 \\
S^I &\equiv W\Big(\varphi(\tilde{m}-y_m)+d(\tilde{a_1}+y_1), \tilde{a_1}+y_1, \tilde{a_2}+y_2\Big) - W\Big(\varphi\tilde{m}+\tilde{a_1}, d\tilde{a_1}, \tilde{a_2}\Big) \\
&= -\varphi y_m + (\psi_1+d)y_1 + (\psi_2+d)y_2
\end{aligned}$$

where the second equality follows by plugging in W and V value functions derived from previous section. Since in this bargaining, A-buyers are the asset sellers, the amount of assets she pays (in exchange for money) cannot exceeds the amount of assets she has, i.e. $y_1 \le a_1$ and $y_2 \le a_2$. I-buyers are the asset buyers and money providers, so the total money exchanged cannot exceeds I-buyers' money balance, i.e. $y_m \le \tilde{m}$. With A-buyer making TIOLI offer, the bargaining problem in secondary asset market is to maximize A-buyer's bargaining surplus, subject to I-buyer's participation constraint, A-buyer's assets constraints, and I-buyer's money constraint. Thus the secondary market bargaining problem is given by

$$\max_{y_m,y_1,y_2} \Big\{ u\Big(\varphi(m+y_m)+d(a_1-y_1)\Big) - u(\varphi m + da_1) - \psi_1 y_1 - (\psi_2+d)y_2 \Big\}$$

$$\text{s.t. } \varphi y_m = (\psi_1+d)y_1 + (\psi_2+d)y_2$$

$$0 \leq y_m \leq \tilde{m}$$

$$0 \leq y_1 \leq a_1$$

$$0 \leq y_2 \leq a_2$$

LEMMA 1.2. *Consider a secondary market meeting between an A-buyer and an I-buyer, with portfolios* (m, a_1, a_2) *and* $(\tilde{m}, \tilde{a}_1, \tilde{a}_2)$ *respectively. Define the real money balance cutoff level as*

$$\bar{z} \equiv \min\Big\{(\psi_1+d)a_1+(\psi_2+d)a_2, q^*-\varphi m-da_1+\frac{d}{d+\psi_1}\max\Big\{0, \min\{q^*-\varphi m-da_1, \varphi\tilde{m}\}-(\psi_2+d)a_2\Big\}\Big\}$$

The bargaining solution (y_m^*, y_1^*, y_2^*) *are discussed in 4 regions. The description of the region division and the corresponding bargaining solution is given as follows*

- *Region 1: If* $\varphi\tilde{m} \geq \bar{z}$ *and* $(\psi_1+d)a_1 + (\psi_2+d)a_2 \geq q^* - \varphi m$*, then*

$$(y_1^*, y_2^*) = \{(y_1, y_2) : \psi_1 y_1 + (\psi_2+d)y_2 = q^* - \varphi m - da_1\}$$

$$\varphi y_m^* = (\psi_1+d)y_1^* + (\psi_2+d)y_2^*$$

- *Region 2: If* $\varphi\tilde{m} \geq \bar{z}$ *and* $(\psi_1+d)a_1 + (\psi_2+d)a_2 < q^* - \varphi m$*, then*

$$(y_1^*, y_2^*) = (a_1, a_2)$$

$$y_m^* = \frac{1}{\varphi}[(\psi_1+d)a_1 + (\psi_2+d)a_2]$$

- *Region 3: If* $\varphi\tilde{m} < \bar{z}$ *and* $(\psi_2 + d)a_2 \geq \varphi\tilde{m}$*, then*

$$y_1^* = 0, y_2^* = \frac{\varphi\tilde{m}}{\psi_2 + d}$$
$$y_m^* = \tilde{m}$$

- *Region 4: If* $\varphi\tilde{m} < \bar{z}$ *and* $(\psi_2 + d)a_2 < \varphi\tilde{m}$*, then*

$$y_1^* = \frac{1}{\psi_1 + d}[\varphi\tilde{m} - (\psi_2 + d)a_2], y_2^* = a_2$$
$$y_m^* = \tilde{m}$$

Proof. See Appendix A.1.1.

Depending on the abundance of real balance and relative abundance/scarcity of asset balance, there exist four possible regimes, hence four sets of bargaining solutions. How are the four possible cases divided depends on the answers of the following questions. In the case where the total liquidity (after pulling together both A-buyer's and I-buyer's money balance) in the economy is abundant to allow for optimal consumption q^*, the question remains to be answered is whether the total assets (including both type-1 and type-2 assets) are enough to exchange the desired amount of money. The answer to this question divides the abundant-liquidity case into regions 1 and region 2. And in the case where the total liquidity is not enough for q^*, A-buyers would like to have all the available liquidity for consumption. Hence the desired amount of money A-buyers demand is $\varphi\tilde{m}$, which is the entire amount of money balance from I-buyers. Now, the question remains to be answered is not whether total assets, i.e. a_1 and a_2, is enough to exchange for $\varphi\tilde{m}$, but if type-2 asset alone is enough to exchange for $\varphi\tilde{m}$. And the answer to this question divides the scarce-liquidity case into region 3 and region 4.

When both real balance and assets are enough as in region 1, A-buyer is willing to give any combination of type 1 and type 2 assets in exchanged for the amount of money that allows for optimal consumption q^*. If real balance is enough for q^*, but A-buyers does not have enough assets to exchange for optimal amount of money as in region 2, then A-buyer will give up all assets she has (including both asset 1 and asset 2), in exchange for equal value of money.

However, in the case where real balance is not enough to allow for consuming q^*, i.e. in region 3 and region 4, it matters which type of asset A-buyer sells, and the reason is as follows. When liquidity is scarce, A-buyers would like to keep all liquidity she has, including money and dividend payment. By selling type-1 asset, A-buyer is missing the opportunity of using the upcoming dividend payment for consumption. Hence in case where total liquidity is scarce, the asset trading activity exhibits a certain 'pecking order'. More precisely, A-buyer will sell type-2 assets first before she sells any type-1 assets which pay dividend. If selling type-2 assets is enough to get all I-buyers' money balance, then A-buyers will not sell any of type-1 asset. But if type-2 asset alone is not enough to exchange for all of I-buyer's money balance, A-buyer will then sell some of the type 1 assets.

1.3.4. Objective Functions and Optimal Behavior. Now, all the value functions and bargaining solutions are established. In this section, I characterize a representative buyer's optimal assets holdings. Leading the secondary market value function by one period, the representative buyer's objective function is given by

$$\begin{aligned}\mathbb{E}\{\Omega^i(\hat{m},\hat{a}_1,\hat{a}_2)\} = {} & \gamma f V\Big(\hat{\varphi}(\hat{m}+y_m)+d(\hat{a}_1-y_1),\hat{a}_1-y_1,\hat{a}_2-y_2\Big)+(l-\gamma f)V\Big(\hat{\varphi}\hat{m}+d\hat{a}_1,\hat{a}_1,\hat{a}_2\Big)\\ & +\gamma f W\Big(\hat{\varphi}(\hat{m}-\tilde{y}_m)+d(\hat{a}_1+\tilde{y}_1),\hat{a}_1+\tilde{y}_1,\hat{a}_2+\tilde{y}_2\Big)+(1-l-\gamma f)W\Big(\hat{\varphi}\hat{m}+d\hat{a}_1,\hat{a}_1,\hat{a}_2\Big)\end{aligned}$$

where the four items of the expression in order represent a typical buyer's benefit by holding asset portfolio ($\hat{m}$,$\hat{a}_1$,$\hat{a}_2$) when turns out to be a matched A-buyer, unmatched A-buyer, matched I-buyer, and unmatched I-buyer, respectively. y_m, y_1, and y_2 are the bargaining solutions discussed in previous section, and variables with tilde represent the terms of trade in secondary market when the typical buyer turns out to be an I-buyer and participate in secondary market as an asset buyer. By plugging in expressions for V and W from previous sections, and inserting the obtained expression into the CM value function, I can group together all the terms that contains choice variables $\hat{m}$, $\hat{a}_1$, and $\hat{a}_2$, and call such expression $J(\hat{m}, \hat{a}_1, \hat{a}_2)$. After some rearrangement of the equation, one can verify that a representative buyer's objective function adopts the following form:

$$\begin{aligned}\beta^{-1}J(\hat{m}, \hat{a_1}, \hat{a_2}) \equiv & -\frac{\varphi}{\beta}\hat{m} - \frac{\psi_1}{\beta}\hat{a_1} - \frac{\psi_2}{\beta}\hat{a_2} \\ & +\alpha f\Big[u\Big(\hat{\varphi}(\hat{m}+y_m) + d(\hat{a_1}-y_1)\Big) + \hat{\psi_1}(\hat{a_1}-y_1) + (\hat{\psi_2}+d)(\hat{a_2}-y_2)\Big] \\ & +(l-\alpha f)\Big[u(\hat{\varphi}\hat{m}+d\hat{a_1}) + \hat{\psi_1}\hat{a_1} + (\hat{\psi_2}+d)\hat{a_2}\Big] \\ & +(1-l)[\hat{\varphi}\hat{m} + (\hat{\psi_1}+d)\hat{a_1} + (\hat{\psi_2}+d)\hat{a_2}]\end{aligned}$$

One observation is that, the objective function depends on the outcome of secondary asset market bargaining solution y_m, y_1, and y_2, which further depends on which region the economy is in. Thus the typical buyer's optimal portfolio choice should be discussed for different regions. Here I focus on symmetric equilibrium, and asset demand functions under optimal behavior of the representative buyer is summarized as follows.

LEMMA 1.3. *Taking prices $(\varphi, \psi_1, \psi_2)$, and beliefs $(\tilde{m}, \tilde{a}_1, \tilde{a}_2)$ as given, the optimal portfolio choice of a representative buyer satisfies the following condition*

Region 1:

$$\begin{aligned}\{\hat{m}\} : & \frac{1+\mu}{\beta} = 1 + (\ell - \gamma f)[u'(z+da_1) - 1] \\ \{\hat{a_1}\} : & \psi_1 = \frac{\beta}{1-\beta}\Big\{1 + (\ell-\gamma f)[u'(z+da_1)-1]\Big\} = \frac{1+\mu}{1-\beta}d \\ \{\hat{a_2}\} : & \psi_2 = \frac{\beta}{1-\beta}d\end{aligned}$$

Region 2:

$$\{\hat{m}\}: \frac{1+\mu}{\beta} = 1 + (\ell - \gamma f)[u'(z + da_1) - 1] + \gamma f\Big[u'\Big(z + (\psi_1 + d)a_1 + (\psi_2 + d)a_2\Big) - 1\Big]$$

$$\{\hat{a}_1\}: \frac{\psi_1}{\beta} = \psi_1\Big\{1 + \gamma f\Big[u'\Big(z + (\psi_1 + d)a_1 + (\psi_2 + d)a_2\Big) - 1\Big]\Big\} + \frac{1+\mu}{\beta}d$$

$$\{\hat{a}_2\}: \frac{\psi_2}{\beta} = (\psi_2 + d)\Big\{1 + \gamma f\Big[u'\Big(z + (\psi_1 + d)a_1 + (\psi_2 + d)a_2\Big) - 1\Big]\Big\}$$

Region 3:

$$\{\hat{m}\}: \frac{1+\mu}{\beta} = 1 + (\ell - \gamma f)[u'(z + da_1) - 1] + \gamma f[u'(2z + da_1) - 1]$$

$$\{\hat{a}_1\}: \psi_1 = \frac{\beta}{1-\beta}\Big\{1 + (\ell - \gamma f)[u'(z + da_1) - 1] + \gamma f[u'(2z + da_1) - 1]\Big\} = \frac{1+\mu}{1-\beta}d$$

$$\{\hat{a}_2\}: \psi_2 = \frac{\beta}{1-\beta}d$$

Region 4:

$$\{\hat{m}\}: \frac{1+\mu}{\beta} = 1 + (\ell - \gamma f)[u'(z + da_1) - 1] + \gamma f\Big[u'\Big(z + da_1 + \frac{\psi_1}{\psi_1 + d}z + \frac{\psi_2 + d}{\psi_1 + d}a_2\Big) - 1\Big]$$

$$\{\hat{a}_1\}: \psi_1 = \frac{\beta}{1-\beta}\Big\{1 + (\ell - \gamma f)[u'(z + da_1) - 1] + \gamma f\Big[u'\Big(z + da_1 + \frac{\psi_1}{\psi_1 + d}z + \frac{\psi_2 + d}{\psi_1 + d}a_2\Big) - 1\Big]\Big\} = \frac{1+\mu}{1-\beta}d$$

$$\{\hat{a}_2\}: \psi_2 = (\psi_2 + d)\Big\{1 + \frac{\gamma f d}{\psi_1 + d}\Big[u'\Big(z + da_1 + \frac{\psi_1}{\psi_1 + d}z + \frac{\psi_2 + d}{\psi_1 + d}a_2\Big) - 1\Big]\Big\}$$

Proof. See Appendix A.1.2.

1.4. Equilibrium

1.4.1. Definition of Equilibrium. I here focus on the symmetric steady state equilibrium. Since A-buyer and I-buyer are ex-ante the same, they will choose the same portfolio, i.e. $\hat{m} = \tilde{m}$, $\hat{a}_1 = \tilde{a}_1$, and $\hat{a}_2 = \tilde{a}_2$. Also, I focus on the more interesting case where the total liquidity is not too abundant, i.e. $Z + dA_1 \leq q^*$. If total liquidity supply is large enough, i.e. $Z + dA_1 > q^*$, A-buyers can rely entirely on her own liquidity to consume the first-best

q^* in the DM, and no trade will happen in the secondary asset market. Next, in describing the equilibrium, I let q_1 be the DM consumption quantity if A-buyer didn't meet an I-buyer in secondary market and thus didn't have the opportunity to boost her liquidity position, and q_2 be the quantity of DM consumption if A-buyer matched with an I-buyer in secondary market. And $z = \varphi M$ represents the real money balances.

Definition 1.1. *The symmetric steady-state equilibrium is a list of* $\{Z, \psi_1, \psi_2, y_m, y_1, y_2, q_1, q_2\}$, *which satisfy:*

(1) The representative buyer behaves optimally under the equilibrium prices φ, ψ_1, *and* ψ_2.

(2) The equilibrium quantity q_1 *satisfies* $q_1 = Z + dA_1$. *For quantity* q_2, $q_2 = q^*$ *if in case 1,* $q_2 = Z + (\psi_1 + d)A_1 + (\psi_2 + d)A_2$ *if in case 2,* $q_2 = 2Z + dA_1$ *if in case 3, and* $q_2 = Z + dA_1 + \frac{\psi_1}{\psi_1+d}A_1 + \frac{\psi_2+d}{\psi_1+d}A_2$ *if in case 4.*

(3) Secondary asset market terms of trade (y_m, y_1, y_2) *satisfies the bargaining solution when evaluated with aggregate asset supply* M, A_1 *and* A_2

(4) Market clears and expectations are rational: $\hat{a}_1 = \tilde{a}_1 = A_1$, $\hat{a}_2 = \tilde{a}_2 = A_2$, *and* $\hat{m} = \tilde{m} = (1 + \mu)M$.

Given the definition of equilibrium, the aggregate regions description is the region division equation described in Lemma 2, and the pricing functions are asset demand functions described in Lemma 3, both evaluated at the aggregate asset supplies.

1.4.2. Properties of Equilibrium. The first implication of the model under equilibrium is that, dividend assets price carries a premium when comparing to non-dividend assets, which is summarized by Proposition 1.1.

Proposition 1.1. ***Dividend Premium.*** *The equilibrium asset prices depend on aggregate asset supplies M, A_1 and A_2. Given the aggregate supplies, the equilibrium could be in one of the four cases. In each of the equilibria, dividend asset price ψ_1 is greater than non-dividend asset price ψ_2. Define the liquidity relative abundance/scarcity cutoff level as*

$$\bar{Z} \equiv \min\Big\{(\psi_1+d)A_1+(\psi_2+d)A_2, q^*-Z-dA_1+\frac{d}{d+\psi_1}\max\Big\{0, \min\{q^*-Z-dA_1, Z\}-(\psi_2+d)A_2\Big\}\Big\}$$

<u>Region 1:</u> If $Z \geq \bar{Z}$, and $(\psi_1+d)A_1+(\psi_2+d)A_2 \geq q^*-Z$, equilibrium is in Region 1 and

$$\psi_1 = \frac{\beta d}{1-\beta}\Big\{1+\Big[u'(Z+dA_1)-1\Big]\Big\} = \frac{1+\mu}{1-\beta}d$$
$$\psi_2 = \frac{\beta d}{1-\beta}$$

<u>Region 2:</u> If $Z \geq \bar{Z}$, and $(\psi_1+d)A_1+(\psi_2+d)A_2 < q^*-Z$, equilibrium is in Region 2 and

$$\psi_1 = \beta\psi_1\Big\{1+\alpha f\Big[u'\Big(Z+(\psi_1+d)A_1+(\psi_2+d)A_2\Big)-1\Big]\Big\}+(1+\mu)d$$
$$\psi_2 = \beta(\psi_2+d)\Big\{1+\alpha f\Big[u'\Big(Z+(\psi_1+d)A_1+(\psi_2+d)A_2\Big)-1\Big]\Big\}$$

<u>Region 3:</u> If $Z < \bar{Z}$, and $(\psi_2+d)A_2 \geq Z$, equilibrium is in Region 3 and

$$\psi_1 = \frac{\beta}{1-\beta}\Big\{1+(\ell-\alpha f)[u'(Z+dA_1)-1]+\alpha f[u'(2Z+dA_1)-1]\Big\} = \frac{1+\mu}{1-\beta}d$$
$$\psi_2 = \frac{\beta d}{1-\beta}$$

<u>Region 4:</u> If $Z < \bar{Z}$, and $(\psi_2+d)A_2 < Z$, equilibrium is in Region 4 and

$$\psi_1 = \frac{\beta}{1-\beta}\Big\{1+(\ell-\alpha f)[u'(Z+dA_1)-1]+\alpha f\Big[u'\Big(Z+dA_1+\frac{\psi_1}{\psi_1+d}Z+\frac{\psi_2+d}{\psi_1+d}A_2\Big)-1\Big]\Big\} = \frac{1+\mu}{1-\beta}d$$
$$\psi_2 = \beta(\psi_2+d)\Big\{1+\frac{\alpha f}{\psi_1+d}\Big[u'\Big(Z+dA_1+\frac{\psi_1}{\psi_1+d}Z+\frac{\psi_2+d}{\psi_1+d}A_2\Big)-1\Big]\Big\}$$

The pricing functions might look a bit complicated, but they're quite intuitive. First of all, type-1 asset price always carries a direct liquidity premium because dividend facilitate DM consumption directly. In the cases where the assets constraints are binding, both type-1 and type-2 assets will also carry an indirect liquidity premium because additional unit of assets can help exchange for more money in the secondary market. To help understand this idea, take Region 2 (liquidity abundant but asset scarce) and Region 3 (liquidity scarce but asset abundant) as examples, and pricing functions in region 1 and region 4 adopts the same logic.

In Region 2, total liquidity is enough, but total assets are not enough to exchange for optimal money balance. In this case, A-buyer does not get to consume optimal q^* even when she get the chance to boost her liquidity position in the secondary market. Since dividend payment is a perfect substitute for money, the value of dividend payment is always determined the same way as real money balance, by μ. In addition to the direct liquidity provided by dividend payment, the resale value of type-1 assets also allows A-buyers to exchange for more money, hence provide liquidity indirectly with probability $\ell - \gamma f$ through secondary market. Hence the resale value component of type-1 asset, ψ_1, carries a indirect liquidity premium. For type-2 assets, they do not provide liquidity directly, but provide indirect liquidity with probability γf by allowing for exchanging more money balance in the secondary market. Hence both dividend component and resale value component carry indirect liquidity premium.

In Region 3, total liquidity is scarce, thus even after pulling together all A-buyers' and I-buyers' money balance it still does not allow for optimal consumption q^*. And since type-2 asset alone is enough to exchange for all I-buyers real money balance, A-buyers will not sell any of the type-1 asset in order to keep the dividend for extra liquidity. Type-1 asset in this region only provide direct liquidity, same as money, and does not provide liquidity indirectly since agents do not sell any of the type-1 asset in secondary market. Hence the price is determined by μ, with no indirect liquidity premium. For type-2 assets, since type-2 assets is abundant in this region, additional unit of type-2 asset does not help provide liquidity either directly or indirectly, hence will always be priced at the fundamental value.

After discussing asset prices in equilibrium, next I formally describe the pecking order in trading assets, which is summarized by Proposition 1.2.

PROPOSITION 1.2. ***Turnover Ratio.*** *When total liquidity in the economy is scarce, agents first trade non-dividend (type-2) asset before selling dividend (type-1) assets. Hence non-dividend assets have higher turnover ratio than dividend assets.*

This results follow directly from the pecking order behavior of selling assets when liquidity is relatively scarce, i.e. case 3 and case 4. Since in these two cases, type-2 assets will be offloaded first before agents begin to sell type-1 assets, type-2 assets are traded more frequently and hence have a higher turnover ratio. Intuitively, this is due to the fact that type-2 assets only serve the role as store of value and provide liquidity only indirectly, while type-1 assets serve the role of both store of value and providing liquidity directly. So agents will hold onto type-1 assets longer, and turnover type-2 assets faster.

The last property of equilibrium that the model implies is that, the magnitude of dividend premium depends on how efficient the secondary market is. More precisely, if the secondary market is very efficient and matching is guaranteed, then dividend premium will be small or close to 0; when secondary market is less efficient, dividend premium would be higher. Proposition 1.3 summarizes this property.

PROPOSITION 1.3. ***Dividend Premium and Market Liquidity.*** *The dividend premium decreases in market matching efficiency, γ.*

Here, I define the dividend premium as the difference between the two asset prices $\psi_1 - \psi_2$. And it can be shown that, for any given region, $\partial(\psi_1 - \psi_2)/\partial\gamma < 0$. Intuitively, as secondary market matching becomes more efficient, agents can sell their assets to boost liquidity position right away when having liquidity needs. Hence when the market is efficient, agents' demand for dividend in facilitating urgent DM consumption is low, and are less willing to pay a premium for the liquidity service.

1.5. Empirical Analysis

In this section, I provide empirical evidence to support the model predictions. The panel data I use are from Compustat and CRSP, with more than 7,000 firms (excluding financial firms and public utlity firms) over the period 1971-2016.

1.5.1. Price Premium of Dividend Assets. To test Proposition 1 on dividend premium, I use a least-squares dummy variable model replicated from Karpavičius and Yu (2018). The specification used is given as follows

$$\textit{Market-to-Book Ratio} = a_0 + \alpha_1 DVD_{it} + \Omega'(L)Z_{it} + \lambda_t + \mu_i + \varepsilon_{it}$$

where ME/E and MA/A are the market-to-book ratio of equity and asset respectively, as a measure of price premium. For the explanatory variable, dividend status is the main variable of interest. In the regression, DVD is the dividend dummy variable that equals 1 if the stock pays dividend in that year, and equals 0 otherwise. In addition to the dividend dummy variable, the test is performed also on dividend amount as a continuous variable $DIV(Continuous)$.

Z_{it} is the set of control variables which include asset size, net income, total debt, total cash, PPE, capital expenditure, R&D, and volatility which is measured as the standard deviation of monthly stock return. All control variables are normalized either by total equity size or total asset size, depending on if the market-to-book ratio of the regression is evaluated using equity or asset. And to control for unobserved firm characteristics and year-related factors, the least-squares model also includes year fixed effects λ_t and firm fixed effects μ_i.

From Proposition 1.1 of the model, the expectation of the empirical result is that the coefficient estimates for dividend variable (dummy or continuous) should be positive, implying that dividend assets are valued more by the market comparing to non-dividend assets and carry a higher price premium. Under this empirical strategy, the results are presented in the Table 1.1, which confirm the model prediction that price of dividend assets is higher.

Variables	ME/E	ME/E	MA/A	MA/A
DVD	0.361**		0.131***	
	(0.156)		(0.0288)	
DIV (Continuous)		4.589***		0.0212
		(0.110)		(0.0314)
R-squared	0.760	0.766	0.521	0.521
Control Variables	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 1.1. Dividend as a determinant for asset market value.

The results show that dividend assets on average have a higher market-to-book ratio of equity and asset, using both dummy variable and continuous variable as indication of dividend status. The average market-to-book ratio of equity for the entire data set is 3.357, hence a coefficient estimate of 0.361 for the dividend dummy variable represents a price premium of 10.75%; market-book-ratio of asset of the data set is 1.893, the coefficient estimate of 0.131 represents a price premium of 6.92%. These results confirm that dividend assets are priced higher than non-dividend assets.

1.5.2. Pecking Order in Selling Assets and Assets Turnover Ratio. Next to test Proposition 1.2, that turnover ratio for non-dividend assets is higher than that of dividend assets, I use the same set of data and control variables, with normalization of control variables using either total asset or total equity. I define turnover ratio as: Turnover ratio = Trade Volume/Total Shares Outstanding. The specification is given by

$$TurnoverRatio = \alpha_0 + \alpha_1 DVD_{it} + \Omega'(L)Z_{it} + \lambda_t + \mu_i + \varepsilon_{it}$$

With Proposition 1.2, it is expected that the coefficient estimates for the dividend dummy variable should be negative, implying that the turnover ratio for a dividend asset should be lower. And this prediction is confirmed in the data.

VARIABLES	Turnover Ratio (Equity)	Turnover Ratio (Asset)
DVD	-1.935***	-1.599***
	(0.544)	(0.542)
Control Variables	Yes	Yes
Firm FE	Yes	Yes
Year FE	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 1.2. Dividend as a determinant for turnover ratio.

From the results presented in Table 1.2, dividend assets have lower turnover ratio comparing to non-dividend assets, regardless of whether the explanatory variables are normalized by asset size or equity size.

1.5.3. Market Efficiency and Dividend Premium. Finally, to test Proposition 1.3 that dividend premium decreases in market matching efficiency, I add a new dummy variable 'Illiquid', which equals to 1 if the turnover ratio of the stock is below the median. With the same matching function $f(\ell, 1-\ell)$ for all assets, the ones that are less liquid would have additional matching friction which is represented by a smaller value of γ. Thus here I use 'Illiquid' as a measure of individual stock's matching efficiency. And to test whether dividend premium is higher when market is less liquid as suggested by Proposition 1.3, I add an interaction term of dividend dummy variable and illiquid dummy variable. The specification is as the following

$$\textit{Market-to-Book Ratio} = \alpha_0 + \alpha_1 DVD_{it} + \alpha_2 Illiquid + \alpha_3 DVD \times Illiquid + \Omega'(L) Z_{it} + \lambda_t + \mu_i + \epsilon_{it}$$

According to Proposition 1.3, the coefficient estimate for the interaction term should be positive, implying that when stocks are less liquid, dividend assets carry a higher price premium. This prediction is confirmed in the data, and is summarized in Table 1.3.

Variables	ME/E	MA/A
DVD	0.0548	-0.0376
	(0.175)	(.032739)
Illiquid	-0.971***	-0.536***
	(0.118)	(.0222)
DVD× Illiquid	0.802***	0.399***
	(0.184)	(0.034)
Firm FE	Yes	Yes
Year FE	Yes	Yes
R-squared	0.769	0.5260

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 1.3. Matching efficiency and dividend premium.

This empirical exercise provides some interesting results. First of all, it is observed that now dividend dummy variable alone does not produce positive and significant prediction for price premium anymore. The coefficient estimates of 0.0548 and -0.0376 using equity and asset measures respectively have contradicting signs but are also insignificant. This result is consistent with the irrelevance theory that dividend alone should have no impact on explaining firm's value or asset prices. However the most important observation is that, the coefficient estimates for the interaction term is positive and significant, meaning that if an asset is illiquid, paying dividend now would raise the market valuation comparing to a non-dividend asset that is equally illiquid. This result provides direct evidence to support the liquidity channel of the model that, because the asset market is not always perfect as assumed in the irrelevance theory, the friction in selling assets is what gives rise to the importance of dividend payment as a superior liquidity instrument.

1.6. Conclusion

Selling assets in the frictional secondary market could be costly. Because of this, dividend assets, which provide direct liquidity when agents have consumption needs, would help avoid such friction, and hence command a price premium for its superior liquidity role. In this paper, I explore this mechanism and provide a theoretical framework to understand it. I show that through the liquidity channel, dividend assets are priced higher comparing to the non-dividend assets, with equal price only when carrying liquidity is not costly, i.e. at the Friedman Rule.

Besides offering a liquidity-based explanation for the dividend puzzle, the model delivers two new testable predictions. First of all, asset trade exhibits a certain "pecking order". More specifically, to meet an urgent liquidity needs, agents will visit the secondary asset market and sell non-dividend assets before selling any dividend assets. The intuition is that, dividend provides direct liquidity for transaction, and by selling dividend assets, agents miss the opportunity of using the upcoming dividend for consumption. Hence in the economy where liquidity is scarce, agents first sell non-dividend assets, and only when non-dividend assets alone cannot exchange for enough liquidity should agents decide to sell dividend assets. The second prediction is that, dividend premium is more pronounced when secondary market is less efficient. This is due to the fact that, less efficient secondary market means that the chance of agents not being about to sell assets becomes larger. This makes agents rely more on dividend payment for urgent consumption need, thus agents are more willing to pay a premium on dividend assets to avoid the need of visiting the frictional secondary market. I then provide empirical evidence in supporting these testable predictions.

CHAPTER 2

Firm's Optimal Dividend Policy

2.1. Introduction

Agents prefer dividend assets due to the liquidity concern. The next logical question to ask is, what about firms? How do firms choose the optimal dividend policy? More specifically, if paying dividend is always a better policy than not paying dividend or vice versa, then why not all firms have the same dividend policy.

The results described in Chapter One have been based on the assumption that the supply of dividend versus non-dividend assets are exogenously given. After carefully describing the equilibrium properties of the various asset prices, and establishing the superior liquidity of dividend-paying bonds, in this Chapter, I endogenize the firms' decision to pay dividends or not. To make things interesting and realistic I focus on the following economic trade-off: when making the dividend decision, firms realize that paying dividend can increase the valuation of their stock (because of the higher liquidity premium established in the first part of the paper) but it can also diminish the amount of resources they can invest in R&D activities, activities which could raise their productivity.

I study a game where the typical firm takes as given the fraction of other firms who pay dividend, say Σ, and chooses optimally whether to pay dividend or not. When Σ is very low, very few firms are paying dividend, and the potential *liquidity benefit* from paying dividend is extremely high, since agents are in desperate need of liquid assets. On the other hand, when Σ is very high, the liquidity needs of agents are (likely) satiated, and agents are only willing to buy assets at their fundamental value (which is another way of saying that the stocks will not carry any liquidity premium). In this case, the obvious optimal choice for the firm is to not pay dividend but instead use their resources for investment in R&D. In all, I show that there exists a unique *interior* equilibrium Σ^*, i.e., the model predicts that a fraction of firms will choose to pay dividend while the remaining firms will engage in R&D activities. I also show that the fraction of firms who choose to pay dividends is increasing in inflation. Thus, my model suggests that higher inflation can hurt the economy not only through the traditional channel, i.e., by reduction agents' real money balances, but also through the reduction in aggregate R&D activities.

The strand of literature this paper is related to is firms' optimal dividend policy. There are many factors being proposed as determinants of firms' dividend policy. For example, Redding (1997) proposed the firm size as a key determinant of dividend policy. In DeAngelo et al. (2006), the authors argue that earned/contributed capital mix is important in determining dividend payout, more specifically firms are more likely to pay dividend if retained earning contributes to a larger share of equity. My paper complements this strand of literature by showing that firms face trade-off between paying dividend and raising future TFP, and also study the monetary implication on firms' dividend decision. The channel that firms issue dividend shares in order to benefit from the premium is also studied in Caramp and Singh (2020), that when Modigliani-Miller theorem does not hold and hence bond carries a premium, firms issue safe bonds to benefit from the bond premium. The result from my paper, that aggregate dividend increases as interest rate rises, is consistent with the prediction in Akyildirim et al. (2014)). Basse and Reddemann (2011) shows a positive relation between inflation and dividend payments, which provide empirical evidence for my model prediction as well.

2.2. The Model

2.2.1. Environment. To better understand firms' behavior, especially what activities do dividend firms and non-dividend firms do differently, I look into the balance sheets of the firms in the dataset. Because in order for the balance sheet to balance, i.e. Asset =

Liability + Equity, the action of paying dividend (which affects the equity category) must be accompanied by a change in other balance sheet account(s). I use the balance sheet data with the same 7,000 firms, and normalize all balance sheet items with asset size. Then in order to control for the impact of firm size on firms' strategic planning, i.e. small firms might have different priorities than big firms in allocating the limited funding resource, I further divide the data set into three groups based on firm size. One balance sheet item that is significantly different between dividend firms and non-dividend firms is the R&D expenditures. More specifically, R&D expenditures are significantly higher for non-dividend firms than for dividend firms, and this is true across all three size groups, which the result is summarized in Table 2.1.

	R&D Expenditures		
Firm size	Dividend	Non-dividend	Difference
Small	0.023	0.345	0.322***
Medium	0.014	0.092	0.078***
Large	0.013	0.035	0.022***

TABLE 2.1. Normalized R&D expenditure from balance sheet

This observation provides a direction about the trade-off that firms are facing when making dividend decisions, hence in modeling firms' optimal dividend choice, I study the trade-off between paying dividend and investing in R&D activities. By paying dividend, firms benefit from having a higher share price because of the dividend premium. And by not paying dividend but invest into R&D instead, firms can make use of the resource in a more productive way by having higher TFP in production.

The structure of this general equilibrium model is the same as in the partial equilibrium model in previous section, with several differences to accommodate a more non-trivial optimal behavior from the supply side of the economy. And since this section focuses on firms' optimal decision, I first impose a few simplifying assumptions, without loss of generality. First of all, there will be no secondary asset market. This assumption should not affect firms' decision because participants of the secondary asset market are A-buyers and I-buyers, but not the firms. Second, to simplify the math expression, assets in this environment will have no resale value. And this again should have no impact on firms making optimal decision, since with or without resale value, dividend assets carry a price premium hence firms still benefit from the dividend premium and facing the same trade-off between higher share price and higher TFP.

There are firms with measure of 1, which will replace the sellers' role in the DM and produce the special goods. In the CM, all firms are endowed with k amount of capital to work with in next period $t+1$. For simplicity, I assume k is large enough to produce optimal quantity q^*. This assumption is not necessary for the results to go through, but simplifies the discussion. In addition to the endowed capital, firms can issue stock shares to raise additional resource for next period. The goal of firms is to maximize firms' value, with commitment to either pay dividend or engage in R&D activities in next period. Among the measure of 1 firms, Σ fraction of the firms are the dividend firms, while the remaining $1-\Sigma$ firms are the non-dividend firms. This fraction Σ will be uniquely determined in equilibrium.

In the CM, firms need to make decision on whether to enter next period as a dividend firm or a non-dividend firm. And when making such entry decision, all firms take asset prices ψ_1, ψ_2 and all other firms' entry decision, Σ, as given. If firms decide to enter as the dividend type, they will issue type-1 stocks, pay early dividend d in DM_{t+1}, and distribute the remaining firm's value, Δ_1^{CM}, back to shareholders in CM_{t+1}. If the firms decide to enter as the non-dividend type, they issue type-2 stocks, and invest e amount in R&D activities, which will translate into higher TFP factor, $A(e)$, for producing intermediate goods for DM production. Since type-2 firms do not pay dividend, they distribute back the entire firm's value back to shareholders in CM_{t+1}.

The objective of the firms is to maximize the total amount of 'value' they can give to the shareholders. For type-1 (dividend) firms, this 'value' includes the early dividend d paid in the DM, and the remaining firms value shareholders are entitled to at CM, i.e. $d + \Delta_1^{CM}$. And for type-2 (non-dividend) firms, this 'value' is only the firm's value shareholders entitled to at CM, i.e. Δ_2^{CM}. Given this, if paying dividend produces a higher value, then firms will choose to enter the market as a dividend type, and vice versa. The model predicts that, in equilibrium, there exist a unique and interior fraction, $\Sigma^* \in (0, 1)$, such that firms are indifferent between entering the market as a dividend type or as a non-dividend type.

2.2.2. Value Functions and Bargaining Solution. In this section, I will describe the value functions in each sub-markets. The main differences comparing to the exogenous asset supply model is that, the objective of a firm (replacing the seller's role in exogenous supply model) is to maximize the value of the firm, instead of utility. In addition, since firms' role is non-trivial now, I allow a more general bargaining protocol in DM such that the bargaining surplus is shared between A-buyers and firms. Thus comparing to A-buyer making a TIOLI offer as in exogenous model, now firm's DM surplus is not 0 anymore.

2.2.2.1. *CM.* Upon entering the CM, firms' state variables are: remaining capital after DM production z_k, and profit p from DM production. Firms maximize the remaining value to distribute to the shareholders:

$$W^F(z_i^k, p_i) = \max_{\Delta_i^{CM}} \Delta_i^{CM}$$

$$\text{s.t. } \Delta_i^{CM} = z_i^k + p_i$$

Hence firms' CM value function is given by $W^F(z_i^k, p_i) = z_i^k + p_i$, where $i \in \{1, 2\}$ denotes the type-i firms.

Buyers' CM value functions adopts the same form as in the previous section but simpler because of the simplifying assumptions of no resale value and no secondary asset market: $W^B(z, a_1, a_2) = \Lambda^B + z + da_2$.

2.2.2.2. *DM.* DM bargaining is between A-buyer buyers and firms. Firms produce special goods using intermediate goods they brought into DM as input. The production technology transform one unit of intermediate goods into one unit of special goods. A-buyer's bargaining surplus remains unchanged from previous section: $u(q) - \pi$. Firm's bargaining surplus adopts from sellers' bargaining surplus, $\pi - q$, from previous section as well. Terms of trade is determined by Kalai bargaining, with θ being A-buyer's bargaining power. Besides the bargaining constraint and liquidity constraint as in the exogenous model, now the production faces additional input capital constraint, i.e. the amount of special goods being produced, q, cannot exceeds the amount of intermediate goods that firm brings into the DM. Hence the bargaining problem is given by

$$\max_{q,\pi} u(q) - \pi$$

$$\text{s.t. } u(q) - \pi = \frac{\theta}{1-\theta}(\pi - q)$$

$$\pi \leq da_1$$

$$q \leq z_i^k$$

With Kalai bargaining, the bargaining constraint implies that $\pi = (1-\theta)u(q) + \theta q$. I denote $\nu(q) \equiv \pi = (1-\theta)u(q) + \theta q$ as the total payment A-buyer makes to the firm in consuming q unit of special goods.

Now, since there are two types of firms, with potentially different amount of input capital, the bargaining solution would depend on which type of firm A-buyer meets with. For a type-1 firm entering $t+1$, they carry k amount of endowed capital and ψ_1 amount of additional capital raised from issuing stocks. However, because they are the dividend type and promised to pay dividend d at the beginning of DM, the amount of working capital they have that can be used in DM production is $k+\psi_1-d$, which can be transformed one-to-one into input capital for DM production. If the firm pays too much dividend in DM, it is possible that the firms do not have enough input capital to produce what A-buyer demands. So the quantity of special goods being produced when meeting with a type-1 firm is limited by either the input capital amount firms have, or the amount of liquidity A-buyers carry, whichever side is more constrained. The bargaining solution is hence given by

$$\begin{cases} q_1 = \min\{\psi_1 + k - d, \nu^{-1}(da_1)\} \\ \pi_1 = \nu(q_1) = (1-\theta)u(q_1) + \theta q_1 \end{cases}$$

For a type-2 firm entering time $t+1$, with e amount invested into R&D, total TFP is higher and can transform resources into input capital more efficiently. For such TFP factor, $A(e)$, I impose the following properties: $A(0) = 0$, $A'(0) \to \infty$, $A'(e) > 0$, and $A''(e) < 0$. If firms decide to invest $e = 0$, then the production technology is just one-to-one, which is the same technology faced by type-1 firm with no R&D investment, i.e. $e = 0$.

Upon entering $t+1$, type-2 firm has k amount of endowed capital and ψ_2 amount of additional capital raised from issuing stocks. After spending e amount in R&D activities, firm's TFP is raised to $A(e) > 1$, and the remaining working capital is hence $k+\psi_2-e$. The TFP factor $A(e)$ can thus transform the working capital $k+\psi_2-e$ in to $A(e)(k+\psi_2-e)$ amount of input capital to be used in DM production. Different from type-1 firms that dividend policy could potentially lower its capital to a sub-optimal level for production, investing in R&D can only raise the amount of input capital beyond the level of special goods production that firms is originally capable of producing, q^*. Thus the input capital constraint when meeting with a type-2 firm is never binding. The bargaining solution is given by

$$\begin{cases} q_2 = \nu^{-1}(da_1) \\ \pi_2 = \nu(q_2) = (1-\theta)u(q_2) + \theta q_2 \end{cases}$$

2.2.3. Objective Functions and Optimal Behavior. After describing the value functions and bargaining solution, I proceed to the objective functions, and analyze the optimal behavior of buyers and firms.

2.2.3.1. *Buyers.* By plugging in expression for V and W into the CM value function, buyers' objective function adopts a similar form

$$\beta^{-1}J(\hat{a}_1, \hat{a}_2) = -\psi_1 a_1 - \psi_2 a_2 + \beta(1-\ell)W^C(d\hat{a}_1, \hat{a}_1, \hat{a}_2) + \beta\ell\Big\{u(\nu^{-1}(d\hat{a}_1) - d\hat{a}_1 + W^C(d\hat{a}_1, \hat{a}_1, \hat{a}_2)\Big\}$$

where the first two terms represent the cost of carrying assets, and the remaining terms represent the benefit of carrying assets. Hence by taking derivative to the objective function with respect to $\hat{a}_1$ and $\hat{a}_2$, the pricing functions are given by

$$\frac{\psi_1}{\beta} = (d + \Delta_1^{CM}) + \ell d\left\{\frac{u'[\nu^{-1}(d\hat{a}_1)]}{\nu'[\nu^{-1}(d\hat{a}_1)]} - 1\right\}$$
$$\frac{\psi_2}{\beta} = \Delta_2^{CM}$$

2.2.3.2. *Firms.* In the CM, type-1 firms will choose and announce the amount of dividend d to be paid in DM_{t+1} in order to maximize the total value of the firm. Hence type-1 firms solve the following problem:

$$\max_d \Delta_1 = d + \Delta_1^{CM} = d + [(\psi_1 + k - d - q) + \pi] = \psi_1 + (1-\theta)[u(q) - q]$$

where the value Δ_1^{CM} consists of remaining capital after production and dividend payment, $\psi_1 + k - d - q_1$, and the DM profit $(1-\theta)u(q_1) + \theta q_1$. One observation is that, dividend d does not directly show up in the expression. This is because for firms, the timing of paying dividend (either in the DM or in the CM) does not affect the total payment they make to the shareholders, hence does not affect firms' objective directly. However, dividend d could potentially affect the objective through DM bargaining surplus $(1-\theta)[u(q_1)-q_1]$. If the dividend payment is too high, such that after paying dividend, firms do not have enough input capital to produce the quantity of special goods that buyers demand, then the total profit would be lower. Thus the dividend d would be optimal only if firms preserve enough input capital to produce the demanded quantity of special goods, and distributing the residual capital as dividend.

As for type-2 firms, they will choose the optimal e amount to invest into R&D and get a higher TFP. Hence the problem for type-2 firm is:

$$\max_{e} \Delta_2 = \Delta_2^{CM} = [A(e)(\psi_2 + k - e) - q_2] + (1-\theta)[u(q_2) - q_2]$$

where items in the first bracket represents the remaining capital after produce q_2 amount of special goods, and the second item represents the bargaining surplus from DM production. With the properties of the TFP factor $A(e)$, there exists a unique $e^* \in (0, \psi_2 + k)$ such that Δ_2 is maximized.

LEMMA 2.1. *Given firms' expectation of A-buyers liquidity position* $\tilde{d\tilde{a}}_1$*, a type-1 (dividend) firm's optimal dividend payment is* $d^* \in [0, \psi_1 + k - \nu^{-1}(\tilde{d\tilde{a}}_1)]$*; and for a type-2 (R&D) firm, there exists a unique and interior* $e^* \in (0, \psi_2 + k)$ *such that firm's value is maximized.*

PROOF. See Appendix A.2.1. □

2.2.4. Equilibrium. In this section, I define the equilibrium of the endogenous asset supply model, and describe the policy implications in equilibrium.

DEFINITION 2.1. *The symmetric steady-state equilibrium is a list of DM bargaining solution* $\{q_1,\ q_2\}$*, firms' dividend and R&D decision* $\{d,\ e\}$*, firms' entry decision* Σ*, and prices* $\{\psi_1,\ \psi_2\}$ *such that:*

(1) The representative buyers and firms behave optimally under the equilibrium price ψ_1, ψ_2, and equilibrium entry decision Σ

(2) Σ satisfies $\Delta_1(\Sigma) = \Delta_2$, so that firms are indifferent between entering as a dividend firm and non-dividend firm

(3) $\{q_1, q_2\}$ satisfies the bargaining solution evaluated at the aggregate asset supply, Σ and $1 - \Sigma$

(4) Market clears: $\hat{a}_1 = \Sigma$, $\hat{a}_2 = 1 - \Sigma$

Given the definition of equilibrium, the pricing functions in equilibrium are given as

$$\frac{\psi_1}{\beta} = (d + \Delta_1^{CM}) + \ell d \left\{ \frac{u'[\nu^{-1}(d\Sigma)]}{\nu'[\nu^{-1}(d\Sigma)]} - 1 \right\}$$
$$\frac{\psi_2}{\beta} = \Delta_2^{CM}$$

And firms' values are given by

$$\Delta_1 = \psi_1(\Sigma) + (1-\theta)[u(q) - q]$$
$$\Delta_2 = A(e^*)(\psi_2 + k - e^*) + (1-\theta)[u(q) - q]$$

One direct observation is that, in equilibrium, dividend asset price and hence firms' value Δ_1 depends on aggregate dividend amount $d\Sigma$, while non-dividend firms share price and hence Δ_2 does not depend on Σ. Given this, if $\Delta_1(\Sigma) > \Delta_2$, then all firms should be entering the market as a type-1 firm, and vice versa.

PROPOSITION 2.1. *In equilibrium, there exists a unique fraction of dividend firms, Σ^*, such that the values of dividend firms and non-dividend firms are the same, i.e. $\Delta_1(\Sigma^*) = \Delta_2$.*

Intuitively, Σ determines the total amount of dividend in the economy, and hence how much premium buyers are willing to pay for dividend asset. When all other firms decide to pay dividend, i.e. $\Sigma = 1$, aggregate liquidity is enough to allow buyers to consume optimal amount of special goods, q^* in the DM. The liquidity need is satiated, and buyers are not willing to pay premium on additional dividend anymore. This is the case where the dividend premium, which is also the benefit of firms entering as a dividend firm, is completely exploited. Firms do not wish to enter as the dividend firm anymore, and the optimal entry decision would be to enter as a non-dividend firm and invest in R&D for higher TFP. Oppositely, if all other firms choose to not pay dividend, i.e. $\Sigma = 0$, the aggregate dividend is scarce, and buyers would be willing to pay an extremely high premium to get dividend. The optimal entry decision now would be to enter as a dividend firm to take advantage of such high dividend premium. And since the dividend premium is monotonically decreasing in aggregate dividend $d\Sigma$, there exist a unique fraction Σ^* such that the dividend premium and the higher TFP from R&D investment contribute the same to firms' value, and hence $\Delta_1(\Sigma^*) = \Delta_2$.

2.2.5. Policy Implication. From previous analysis, firms' entry decision is uniquely determined by aggregate liquidity and hence dividend premium. But besides dividend, real money balance plays the same role in determining the aggregate liquidity and hence dividend premium. Hence I next analyze how monetary policy and aggregate dividend jointly determine the dividend premium, which further affects firms' entry decision and aggregate R&D decision.

Notice that real money balance Z enters the pricing function through DM utility the same way as dividend. And when real money balance and aggregate dividend jointly determine the total liquidity, the indifference condition for firms to enter as a dividend firm and a non-dividend firm, $\Delta_1 = \Delta_2$, now becomes

$$(d + \Delta_1^{CM}) + \ell d \left\{ \frac{u'[\nu^{-1}(z + d\Sigma^*)]}{\nu'[\nu^{-1}(z + d\Sigma^*)]} - 1 \right\} + \pi = A(e^*)(\psi_2 + k - e^*) + \pi$$

where $\pi = (1-\theta)[u(q) - q]$ is the DM bargaining surplus. This surplus is the same for both types of firms since DM production for both firms are determined by A-buyers' liquidity holding.

By taking total differentiation on both side with respect to interest rate i, it can be shown that $\frac{\partial \Sigma^*}{\partial i} = -d\frac{\partial z}{\partial i} > 0$. This implies that, if interest rate increases, there will be more firms enter the market as the dividend type, and less firms invest in R&D. Proposition 2.2 summarizes this result.

Proposition 2.2. *A contractionary monetary policy raises aggregate dividend payment, and lowers aggregate R&D activities, i.e.* $\frac{\partial \Sigma^*}{\partial i} > 0$.

The intuition for this result is simple as the following. An increase in interest rate depresses real money balance, which makes buyers rely more on dividend payment for DM consumption. The higher demand for dividend raises the premium for dividend assets, which makes the option of paying dividend more attractive for firms when comparing to investing in R&D. Thus more firms will pay dividend and take advantage of the high share price, while at the same time discourage firms to invest in R&D activities. This prediction shows that, besides the common belief that a contractionary monetary policy hurts the economy through reducing real money balance and hence consumption, it can have additional negative impact on the economy through discouraging R&D activities.

2.3. Conclusion

In this paper, I use a search-theoretic model to study firms' dividend decision and endogenize the aggregate asset supplies. I show that when firms face trade-off between higher share price and higher production TFP, in equilibrium, there exists a unique fraction of firms, Σ^* that will pay dividend, while the remaining firms will engage in R&D activities. Furthermore, I show that the fraction of firms who decide to pay dividend (the fraction of R&D

firms) increases (decreases) in inflation. Thus my model suggests that higher inflation hurts the economy not only through the traditional channel of depressing real money balance, but also through a less-explored channel by discouraging aggregate R&D activities.

CHAPTER 3

Money and Competing Means of Payment

3.1. Introduction

In monetary theory, money is typically introduced as an object that can help agents carry out transactions in markets characterized by frictions, such as anonymity and lack of commitment, which preclude unsecured credit (see Kiyotaki and Wright (1989) and Kocherlakota (1998)). For example, if trade is bilateral and a consumer cannot commit to repaying her/his debt, then the transaction has to be set in a *quid pro quo* manner, and money is usually the means that allows such a transaction to take place, by helping bypass the friction. Consequently, common wisdom suggests that if agents had access to more unsecured credit (i.e., to a commitment device that allows them to credibly promise repayment of a debt), the frictions in the economy would become less severe and welfare would improve. Similarly, common wisdom suggests that as societies get access to more payment instruments/systems, i.e., more ways to *bypass* the aforementioned frictions, welfare would also increase.

The goal of this paper is to examine whether the introduction of alternative means of payment, like credit, financial assets, or secondary markets where agents can boost their liquidity, is (always) welfare improving. We show that for a large variety of settings and market structures, the common wisdom described in the previous paragraph does not turn out to be accurate. While our paper establishes this surprising result for four different settings (or alternative payment systems), the intuition can perhaps be best described in a simple environment with money and (unsecured) credit. If every agent in the economy has access to perfect credit, indeed the economy will reach maximum welfare, since this would be world without any frictions. However, if access to credit is low to begin with, increasing it can actually hurt the economy's welfare, i.e., increasing the friction in the economy makes people better off.

What gives rise to this counter-intuitive result? Our model exhibits the following interesting feature: agents need to pay a cost to carry money/liquidity (that cost is no other than inflation), and they decide how much money to carry before they know whether they will actually need it for transactions or whether they can use credit. *Ex post*, more credit is certainly good for welfare because it means that transactions will not be hindered by the lack of liquidity. But *ex ante*, easier access to credit diminishes the demand for money and hinders trade in bilateral meetings where credit is *not available*. Obviously, this describes a situation with two opposing forces fighting each other. In the paper, we analyze the dynamic general equilibrium model and describe precisely the set of parameter values for which the second, negative force can dominate, so that ultimately an increase in credit availability (a decrease in frictions) can be welfare improving.

We then generalize the result by considering a specification of the model, where the alternative (to money) system/means of payment can be a financial asset, as opposed to credit. We also consider the case where money is the only direct medium of exchange, but agents have access to a secondary market where they can boost their liquidity either by obtaining an unsecured loan or by selling assets for cash. In each case, we are able to show that there exists a set of parameters for which increased access to the respective alternative payment method can be welfare decreasing. Therefore, we conclude that access to more (and more advanced) payment systems alternative to money is not always welfare improving. Our model offers a simple and intuitive explanation to the recent empirical literature suggesting that increased access to credit is often followed by recessions and decline in economic activity; see for example Schularick and Taylor (2012) and Jordà et al. (2013).

Our paper is related to a large literature that studies the coexistence of money and alternative means of payment. Papers such as Telyukova and Wright (2008), Gu et al. (2013), and Gu et al. (2016) study the coexistence of money and various types of credit (secured or unsecured). However, none of these papers examines whether higher availability of credit can hurt welfare. Our paper is also related to the growing literature that studies the coexistence of money and other financial assets as means of payment. Examples include Geromichalos et al. (2007*b*), Lagos (2011), Lester et al. (2012), Nosal and Rocheteau (2012), Andolfatto et al. (2013), and Hu and Rocheteau (2015). In these papers the liquidity properties of assets are 'direct', in the sense that assets serve as a media of exchange or collateral, helping agents (directly) facilitate trade in decentralized goods markets.

In Section 3.4 we consider the case where agents, who receive an idiosyncratic liquidity shock, can boost their liquidity in a secondary market. This idea builds on the work of Berentsen et al. (2007), where agents with different liquidity needs visit a competitive banking sector to rebalance their positions. In our model agents can visit a secondary market and boost their liquidity holdings either by obtaining unsecured loans (Section 3.4.1) or by selling assets (Section 3.4.2). Thus, money does not have a 'direct' competitor as a medium of exchange (all transactions in the goods market must be settled with money), but it has an 'indirect' competitor, in the sense that assets (in Section 3.4.2) can be sold for money in the secondary market, and so they are indirectly liquid. This empirically relevant approach to asset liquidity has also been explored in Berentsen et al. (2014, 2016), Mattesini and Nosal (2016), Geromichalos and Herrenbrueck (2016), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019), and Madison (2019).

3.2. The Model

3.2.1. Environment. The economy has infinite horizon and time is discrete. In each period, there are two sub-markets that open for different economics activities: a decentralized market (henceforth DM), and a centralized market (henceforth CM). The CM is the settlement market of Lagos and Wright (2005*b*). Access to this market together with quasi-linear preferences helps keep the model tractable. In the DM, agents meet and trade a special good in anonymous bilateral meetings where perfect commitment might not be available. This gives rise to a need for a medium of exchange, and we will discuss various cases in which one or more payment methods are recognizable/acceptable in DM trades. In later sections, we will extend the model to include an additional sub-market, a secondary over-the-counter (henceforth OTC) asset market, where agents with different liquidity needs in the upcoming DM can trade with each other to rebalance their portfolio. More precisely, agents who have an urgent need for cash in the DM can boost their money holdings either by obtaining a loan or by selling assets.

Agents discount future at a rate of $\beta \in (0, 1)$, but there is no discounting between sub-markets. There are two types of agents, consumers and producers, characterized by their roles in the DM which remain permanent. The measure of each type of agents is normalized to 1.

In the DM, consumers will consume a special goods q, and producers will produce it. The quantity of goods that the producer produces, and payment that consumer pays in exchange, will be determined through bargaining. More precisely, the terms of trade are determined by proportional bargaining, following Kalai (1977). We will let θ denote the bargaining power of consumers and $1 - \theta$ be the bargaining power of producers. To consume the special goods, there are three objects that could potentially serve as a proper means of payment: money, a real asset, and credit. Money is fiat, storable, and perfectly divisible, with supply of $M_{t+1} = (1 + \mu)M_t$ controlled by the monetary authority through lump-sum transfer in the CM. When $\mu > 0$, new money is introduced into the economy; when $\mu < 0$, money is withdrew. Consumers can obtain money in the CM at the ongoing price of φ. (The CM is a Walrasian market, hence, all market participants take its price as given.) The second payment method is a one-period physical asset with fixed supply A. Each share of the assets can be purchased at a price of ψ in CM_t. It will pay 1 unit of numeraire good as dividend in DM_{t+1}, and then the asset dies and gets replaced by an identical set of assets.

The other payment method is credit, with which consumers in the DM can purchase special goods from the producer by promising to pay back in the following CM. Thus, credit here is unsecured.

In the CM, both consumers and producers supply labor and consume a general good. The technology transforms 1 unit of labor input into 1 unit of the general good. Consumers' and producers' utility in a given period are given by $\mathcal{U}(X, H, x) = U(X) - H + u(q)$ and $\mathcal{V}(X, H, q) = U(X) - H - q$ respectively, where X is the consumption of numeraire good in the CM, H is the labor supply in the CM, and q is the special good consumed/produced in the DM. We assume that u and U are twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, and $u'' < 0$. In later sections, we will also consider the special case of a quadratic utility function, which allows us to sharply characterize some of our results by deriving closed form solutions for the key equilibrium variables. In this case the Inada condition will be relaxed. Let q^* be the optimal level of output in the DM, i.e. $u'(q^*) = 1$. Also assume that there exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$.

In the DM, there are three objects that could potentially serve the role of a medium of exchange: money, a real asset, and credit. With this setting, we will discuss four different environments. In the first two, credit (Section 3.3.1) and then assets (Section 3.3.2) will serve as direct means of payment, in the sense that a subset of producers will be able to accept these alternative payment methods. Then, we extend the model (in Section 3.4) to incorporate an OTC secondary market in which we allow consumers with different liquidity needs to trade with each other in order to rebalance their liquidity holdings. In this section, money is the ultimate medium of exchange in the DM, but it has 'indirect' competition from loans (Section 3.4.1) or assets (Section 3.4.2). Considering all these cases allows us to conclude that our finding, namely, the idea that increased access to new alternative payment methods can sometimes be welfare decreasing, is not a coincidence, but a robust result in this class of models.

3.3. Money and competing media of exchange

We start the analysis with two versions of the model in which money has a direct competitor as a medium of exchange. That competitor is first unsecured credit and then a real asset.

3.3.1. Money and Credit. In this section, the two forms of payment that could potentially be used in the DM bilateral trade are money and credit. We will let σ be the probability that a producer recognize credit and has the ability to enforce a payment from consumers in the CM. We will refer to these types of producers as type-0 producers. Then, $1-\sigma$ is the probability that a producer does not have the ability of identifying or accepting unsecured credit; these will be the type-1 producers.[1]

3.3.1.1. *Value Functions and Bargaining Solutions.* We start by describing the value functions in the CM. For a typical consumer entering the CM, the state variables are d and m. d is the amount of debt that agents took in the preceding DM for special good consumption, and m is the amount of money agents brought into the CM. The value function is given by

$$W(m,d) = \max_{X,H,\hat{m}} \left\{ U(X) - H + \beta V(\hat{m}) \right\} \tag{3.1}$$

$$\text{s.t. } X + \varphi \hat{m} = \varphi m + H - d + T$$

where $V(\hat{m})$ is the value function of next DM, and $\hat{m}$ is the amount of money that the consumer chooses to bring into next period. Substituting H from the budget constraint allows us to rewrite the value function as

$$W(m,d) = \varphi m - d + \Lambda \tag{3.2}$$

where $\Lambda = U(X^*) - X^* + T + \max_{\hat{m}}\{-\varphi\hat{m} + \beta V(\hat{m})\}$.

[1] As a mnemonic rule, a type-i, $i = 0, 1$, producer is a producer who requires i "assets" or "objects" as media of exchange in order to trade in the DM; of course, in the case of type-0 producers no medium of exchange is required, since that producer accepts unsecured credit.

Next, consider a producer's CM value function. Notice that producers will never leave the CM with a positive amount of money because money is costly to hold, in equilibrium, and producers will never have the need to use it (precisely due to their permanent identity as producers in the DM). Of course, producers may enter the CM with some money that they received as means fo payment in the preceding DM. Hence for a type-0 producer who accepts credit, the value function is

$$W^{S0}(d) = \max_{X,H}\{U(X) - H\} \tag{3.3}$$

$$\text{s.t. } X = H + d$$

By substituting H from the budget constraint into the value function, it can be rewritten as

$$W^{S0}(d) = \Lambda^{S0} + d \tag{3.4}$$

where $\Lambda^S = U(X^*) - X^*$.

For a type-1 producer, the value function is

$$W^{S1}(m) = \max_{X,H}\{U(X) - H\} \tag{3.5}$$

$$\text{s.t. } X = H + \varphi m$$

which can again be rewritten as

$$W^{S1}(m) = \Lambda^S + \varphi m \tag{3.6}$$

Having established the CM value functions, we can now discuss the bargaining between a consumer and a producer in the DM. Consumers and producers negotiate over the quantity q to be produced by the producer and the payment to be made to the producer, conditional on which payment(s) are being accepted in this particular meeting. In a type-0 meeting (where producers accept credit), the bargaining problem is given by

$$\max_{q,d}\ u(q) + W(m,d) - W(m,0) \tag{3.7}$$
$$\text{s.t. } u(q) + W(m,d) - W(m,0) = \frac{\theta}{1-\theta}\{-q + W^{S0}(d) - W^{S0}(0)\}$$

Substituting equations (3.2) and (3.4) into equation (3.7), the bargaining problem can be simplified as

$$\max_{q,d}\ u(q) - d \tag{3.8}$$
$$\text{s.t. } u(q) - d = \frac{\theta}{1-\theta}(d - q)$$

LEMMA 3.1. *Define* $z(q) = (1-\theta)u(q) + \theta q$. *The solution to the bargaining problem is:*

$$q = q^* \tag{3.9}$$

$$d = z(q^*) \tag{3.10}$$

PROOF. The proof is obvious, and hence is omitted. □

The bargaining solution is straightforward, which states that in meetings where producers accept unsecured credit, the first-best quantity q^* should be exchanged, and the producer should be promised a payment d (to take place in the CM) that satisfies the Kalai surplus-splitting constraint.

Now, consider a type-1 meeting. Without credit, consumers will face an additional budget constraint. Whether they will be able to achieve the first-best level of DM consumption now depends on the money they have carried into the DM. Hence in a type-1 meeting, let x be the amount of money that changes hands, the bargaining problem is given by:

$$\max_{q,x} \; u(q) + W(m-x,0) - W(m,0) \tag{3.11}$$

$$\text{s.t.} \; -q + W^{S1}(x) - W^{S1}(0) = \frac{\theta}{1-\theta}[u(q) + W(m-x,0) - W(m,0)]$$

$$x \leq m$$

From the Kalai constraint, we have $\varphi x = (1-\theta)u(q) + \theta q \equiv z(q)$. By substituting the simplified bargaining constraint, as well as equation (3.6) into the objective function, we can rewrite the bargaining problem as follows:

$$\max_{q,x} \; \theta[u(q) - q] \tag{3.12}$$

$$\text{s.t.} \; \varphi x = (1-\theta)u(q) + \theta q$$

$$x \leq m$$

LEMMA 3.2. *Define $m^* \equiv \{m : \varphi m = z(q^*)\}$, which is the amount of money that allows the consumer to afford the optimal consumption q^*. Then the bargaining solution is given by:*

$$q = \begin{cases} q^*, \text{ if } m \geq m^* \\ \tilde{q}(m) \equiv \{q : \varphi m = z(q)\}, \text{ if } m < m^* \end{cases} \tag{3.13}$$

$$x = \begin{cases} m^*, & \text{if } m \geq m^* \\ m, & \text{if } m < m^* \end{cases} \tag{3.14}$$

PROOF. The proof is obvious, and hence is omitted. □

Now the expected DM value function of a consumer with money holdings m at the beginning of the DM is given by

$$\begin{aligned} V(m) &= (1-\sigma)\Big[u\Big(q(m)\Big) + W(m - x(m), 0)\Big] + \sigma\Big[u(q^*) + W(m, z(q^*))\Big] \\ &= (1-\sigma)\Big[u\Big(q(m)\Big) + \varphi x(m)\Big] + \sigma\Big[u(q^*) - z(q^*)\Big] + W(m, 0) \end{aligned}$$

where the second equality is obtained by substituting the bargaining solutions (3.9), (3.10), (3.13), (3.14), and CM value functions (3.4), and (3.6) into the first equality.

The first part of the value function represents that consumer trade with a type-1 producer, which happens with probability $1-\sigma$, and the second part of the value function captures the trade with a type-0 producer, with probability σ.

3.3.1.2. *Objective Function and Optimal Money Choice.* To describe the consumer's choice of $\hat{m}$ in the CM, we first derive the consumer's objective function in the CM. To do this, first lead the DM value function by one period, and then substitute it into the CM value function. The maximization problem over money choice $\hat{m}$ becomes:

$$\begin{aligned} (3.15) \qquad & \max_{\hat{m}} \Big\{ -\varphi\hat{m} + \beta V(\hat{m}) \Big\} \\ &= \max_{\hat{m}} \Big\{ -\varphi\hat{m} + (1-\sigma)\beta\Big[u\Big(q(\hat{m})\Big) - \hat{\varphi}x(\hat{m})\Big] + \sigma\beta[u(q^*) - z(q^*)] + \beta W(\hat{m}, 0) \Big\} \end{aligned}$$

We collect all items that contain $\hat{m}$, and call the resulting expression $J(\hat{m})$, or the agent's "objective function". After simplifying the expression, one can verify that $J(\hat{m})$ adopts the following form:

$$(3.16) \qquad J(\hat{m}) = (\varphi + \beta\hat{\varphi})\hat{m} + (1-\sigma)\beta\Big[u\Big(q(\hat{m})\Big) - \hat{\varphi}x(\hat{m})\Big]$$

Notice that since it is costly to carry money when the economy is away from the Friedman rule, the consumer will never carry $\hat{m} \geq m^*$. So we can substitue that $q(\hat{m}) = \tilde{q}(\hat{m})$ and the objective function can be rewritten as:

$$J(\hat{m}) = (\varphi + \beta\hat{\varphi})\hat{m} + (1-\sigma)\beta[u\Big(\tilde{q}(\hat{m})\Big) - \hat{\varphi}x(\hat{m})] \tag{3.17}$$

In order to simplify things, we focus on the special case where $\theta = 1$, or equivalently, consumer makes TIOLI offer in DM bargaining. The bargaining solution thus can be simplified as $z(q) = q$ and $\tilde{q}(m) \equiv \{q : \varphi m = q\}$.

Obtaining the first-order condition from the objective function $J(\hat{m})$ yields:

$$\varphi = \beta\hat{\varphi} + \beta(1-\sigma)[\hat{\varphi}u'(\hat{\varphi}\hat{m}) - \hat{\varphi}] = \beta\hat{\varphi}\Big\{1 + (1-\sigma)\Big[u'(\hat{\varphi}\hat{m}) - 1]\Big\} \tag{3.18}$$

The discussion of the model will focus on steady-state equilibrium, hence the equilibrium condition is given by:

$$\frac{1+\mu-\beta}{\beta} = i = (1-\sigma)[u'(q_1) - 1] \tag{3.19}$$

where i is the interest rate on a perfectly safe, yet illiquid asset.[2]

Definition 3.1. *Let q_i stands for the quantity of special good traded in a type-i meeting, with $i = \{0, 1\}$. A steady state equilibrium can be summarized by a pair (q_0, q_1), where in any equilibrium $q_0 = q^*$, and q_1 is given by the solution to equation (3.19).*

We have $q_0 = q^*$, since in every type-0 meeting that accepts credit, consumers can always consume first-best quantity q^*. In turn, q_1 is determined by equation (3.19), so it depends on parameters of the model, including the policy parameter i.

3.3.1.3. *Welfare Analysis.* We are now ready to move to the welfare analysis. We define welfare as:

$$\mathcal{W} = \sigma[u(q_0) - q_0] + (1-\sigma)[u(q_1) - q_1] = \sigma[u(q^*) - q^*] + (1-\sigma)[u(q_1) - q_1] \tag{3.20}$$

[2] See Geromichalos and Herrenbrueck (2017) for more details.

One observation from Equation (3.19) is that, q_1 decreases in i. This is intuitive. A higher interest rate depresses real money balance, which further constrains DM transaction in meetings where credit is not available. Thus for the welfare equation, this implies that that $\frac{\partial \mathcal{W}}{\partial i} \leq 0$. This is the traditional channel that rising interest rate hurts welfare by depressing real money holdings, and hence consumption.

But what about $\frac{\partial \mathcal{W}}{\partial \sigma}$? How would a change in σ, i.e. the probability that a producer accepts credit, affect the welfare? One may expect that, higher acceptability of credit should better facilitate DM trade, hence raise welfare. But in what follows, we will show that this is not always true.

First notice that

$$\frac{\partial \mathcal{W}}{\partial \sigma} = u(q^*) - q^* - [u(q_1) - q_1] + (1-\sigma)[u'(q_1) - 1]\frac{dq_1}{d\sigma} \tag{3.21}$$

and with $q_1 \equiv \{q : i = (1-\sigma)[u'(q) - 1]\}$, we can easily verify that $dq_1/d\sigma = i/(1-\sigma)^2 u''(q_1)$

Hence, the derivative of welfare with respect to σ becomes:

$$\frac{\partial \mathcal{W}}{\partial \sigma} = u(q^*) - q^* - [u(q_1) - q_1] + (\frac{i}{1-\sigma})^2 \frac{1}{u''(q_1)} \tag{3.22}$$

The first terms of equation (3.22) $u(q^*) - q^* - [u(q_1) - q_1]$ is clearly positive, since q^* is the unique maximizer of the surplus $u(q) - q$. And with the economy away from the Friedman rule (i.e., for any $i > 0$), $q_1 < q^*$. However, the last term in equation (3.22) will be negative since the agent's utility is strictly concave, i.e., $u'' < 0$. This is already providing some intuition about the channels at work. A higher σ benefits welfare as it increases the fraction of meetings in which unsecured credit is accepted and, therefore, the first-best quantity is traded. However, a higher σ also gives agents the (accurate) impression that they will not be needing money as often, which reduces their equilibrium real balances and hurts welfare in meetings where credit is not available. Which of these two forces prevails depends on parameters, but we can provide a sharper characterization of equilibrium if we focus on a specific functional form.

Thus, in what follows, we focus on the quadratic utility function, and assume that $u(q) = -q^2/2 + (1+\gamma)q$. With this quadratic utility function, we can easily verify that $q^* = \gamma$, and $u(q^*) - q^* = \gamma^2/2$. Also, $q_1 = \gamma - i/(1-\sigma)$ and $dq_1/d\sigma = i/(1-\sigma)^2$.

Now we define $\bar{\sigma} \equiv 1 - i/\gamma$ as the cutoff level of probability that the seller accepts credit. Then for all $\sigma \in [0, \bar{\sigma})$, equilibrium is monetary, such that consumers will still carry money to consume in the type-1 meeting. When $\sigma \geq \bar{\sigma}$, the cost of carrying money is too high. DM consumption $q_1 \leq 0$, and consumers will not consume in meetings where only money is accepted. We carry out the welfare analysis in both monetary and non-monetary equilibrium.

- Monetary equilibrium: i.e. $\sigma \in [0, \bar{\sigma})$. In this parameter range, $\partial \mathcal{W}/\partial\sigma < 0$ and $\partial^2 \mathcal{W}/\partial\sigma^2 < 0$, so the welfare function is decreasing and concave in σ for all $\sigma \in [0, \bar{\sigma})$. Hence, we verify that in the monetary equilibrium, increasing in the acceptability of credit can actually hurt the welfare.
- Non-monetary equilibrium: i.e, $\sigma \in [\bar{\sigma}, 1]$. If the equilibrium is non-monetary, there will be no trade in type-1 meeting anymore, and the welfare function can be reduced to

$$\mathcal{W}(\sigma) = \sigma[u(\gamma) - \gamma] = \sigma\frac{\gamma^2}{2} \tag{3.23}$$

  And in such equilibrium, welfare is linear and increasing in σ. As $\sigma \to 1$, $\mathcal{W}(1) = \gamma^2/2 \geq \mathcal{W}(0)$, with equality only at the Friedman Rule.

We summarize the findings in Proposition 3.1 and Figure 3.1.

PROPOSITION 3.1. *Define $\bar{\sigma} \equiv 1 - \frac{i}{\gamma}$. For $\sigma \in [0, \bar{\sigma})$, the equilibrium is monetary, and welfare is decreasing and concave in σ (the acceptability of credit in the economy); for $\sigma \in [\bar{\sigma}, 1]$, the equilibrium is non-monetary, and welfare is increasing and linear in σ.*

Inspection of the figure makes it obvious that an increase in σ need not be welfare increasing, as common wisdom may suggest. Indeed, if this economy could achieve unsecured credit in every DM meeting (i.e., $\sigma = 1$), welfare would be maximized. However, if the economy starts with a small measure of producers who accept credit (any $\sigma < \bar{\sigma}$), an increase in credit availability (i.e., a small increase in σ) would certainly hurt welfare.

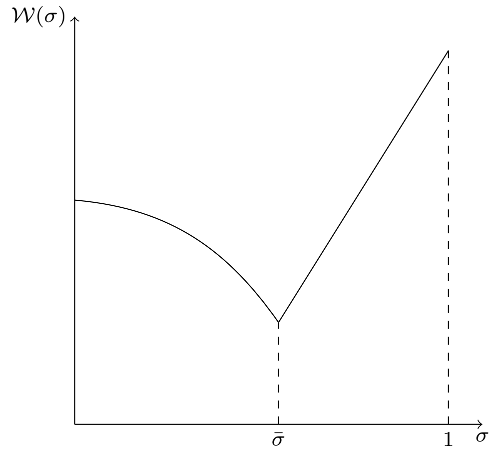


FIGURE 3.1. Welfare as a function of σ

3.3.2. Money and Assets.

3.3.2.1. *Value Functions and Bargaining Solution.* We now move to the second type of competing media of exchange, i.e. assets. We assume that there exists an asset, with fixed supply A, that pays 1 unit of numeraire good in period $t+1$. Consumers can purchase such asset in the CM of period t at the given price ψ. We will maintain the σ notation, but this time it will stand for the fraction of producers who accept both money and assets as media of exchange. Then, $1 - \sigma$ will be the number of producers that accept only money.[3]

The CM value functions for both consumers and producers are similar to the previous section, in the sense that they are linear in all arguments. In this version of the model there are two types of DM meetings: in type-1 meetings, producers accept only money, while

[3] This version of the model coincides with the model of Lester et al. (2012). However the authors in that paper do not examine how welfare is affected by the fraction of producers who accept assets, which the central question for us.

in type-2 meetings, producers accept both money and assets.[4] With similar set up as in previous section, the bargaining solutions for the two types of meetings also adopt similar forms.

LEMMA 3.3. *In type-1 meeting, where only money is accepted as the proper media of exchange, the bargaining solution are as follows*

$$q = \begin{cases} q*, & \text{if } m \geq \frac{q^*}{\varphi} \\ \varphi m, & \text{if } m < \frac{q^*}{\varphi} \end{cases} \tag{3.24}$$

$$d = \begin{cases} m^*, & \text{if } m \geq \frac{q^*}{\varphi} \\ m, & \text{if } m < \frac{q^*}{\varphi} \end{cases} \tag{3.25}$$

In type-2 meetings, both money and assets can be used as medium of exchange. Let d_a and d_m be the amount of assets and money changed hands in the DM meeting, respectively. The bargaining solution is given by

$$\text{If } a + \varphi m \geq q^*, \begin{cases} q = q* \\ d_a + \varphi d_m = q^* \end{cases} \tag{3.26}$$

$$\text{If } a + \varphi m < q^*, \begin{cases} q = a + \varphi m \\ d_m = m \\ d_a = a \end{cases} \tag{3.27}$$

PROOF. Proof is obvious, hence is omitted. □

[4]Thus, the mnemonic rule remains the same as the one described in footnote 1: the index 1 or 2 stands for the number of assets traded in this type of meeting.

Notice that in a type-2 meeting, since both money and assets can be used to purchase DM goods, what matters is whether the total liquidity (money and asset together) is enough to allow for the first-best level of consumption.

3.3.2.2. *Objective Function and Optimal Behavior.* Following the same steps as in section 3.3.1.2, the "objective function" of the typical agent/consumer is:

$$J(\hat{m}, \hat{a}) = (-\varphi + \beta\hat{\varphi})\hat{m} + (-\psi + \beta)\hat{a} + \beta(1-\sigma)[u(q_1(\hat{m})) - \hat{\varphi}d(\hat{m})] + \beta\sigma[u(q_2(\hat{m}, \hat{a})) - d_a(\hat{m}, \hat{a}) - \hat{\varphi}d_m(\hat{m}, \hat{a})] \tag{3.28}$$

where $q_1(\hat{m})$ is the amount of DM consumption when meeting a type-1 producer, and $q_2(\hat{m}, \hat{a})$ is the DM consumption when meeting a type-2 producer.

As before, it is sub-optimal to bring $\hat{m} > q^*/\hat{\varphi}$, i.e. more money than what is needed to achieve first-best consumption q^*, so we always have $\hat{\varphi}\hat{m} \leq q^*$. But it is possible that total liquidity from both money and asset together is greater than q^*, i.e., $\hat{a} + \hat{\varphi}\hat{m} > q^*$. So for now, we discuss 2 cases, when total liquidity is plentiful, i.e., $\hat{\varphi}\hat{m} + \hat{a} \geq q^*$, and when total liquidity is scarce, i.e., $\hat{\varphi}\hat{m} + \hat{a} < q^*$. (Eventually, which of the two cases is relevant will depend on parameters of the model, and we provide more details below.)

<u>Case 1: $\hat{\varphi}\hat{m} + \hat{a} < q^*$</u>. Let subscript 1 denote the objective function under case 1, and subscript m (and a) denote the derivative of J with respect to money (asset). Since total liquidity is scarce, according to equation (3.27), $q_2 = \hat{a} + \hat{\varphi}\hat{m}$. Hence equation (3.28) becomes

$$J_1(\hat{m}, \hat{a}) = (-\varphi + \beta\hat{\varphi})\hat{m} + (-\psi + \beta)\hat{a} + \beta(1-\sigma)[u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m}] + \beta\sigma[u(\hat{a} + \hat{\varphi}\hat{m}) - \hat{a} - \hat{\varphi}\hat{m}]$$

The FOCs with respect to $\hat{m}$ and $\hat{a}$ are as follows

$$J_{1m} = 0 \Rightarrow \varphi = \beta\hat{\varphi}\Big\{1 + (1-\sigma)[u'(\hat{\varphi}\hat{m}) - 1] + \sigma[u'(\hat{a} + \hat{\varphi}\hat{m}) - 1]\Big\} \tag{3.29}$$

$$J_{1a} = 0 \Rightarrow \psi = \beta\Big\{1 + \sigma[u'(\hat{a} + \hat{\varphi}\hat{m}) - 1]\Big\} \tag{3.30}$$

We again focus on the steady state equilibrium. Same as in the previous sections, we define $z \equiv \psi m$ as the real money balances. So in a type-1 meeting which only accepts money as the medium of exchange, total special good consumption is determined as $q_1 = z$; in type-2 meeting which accepts both money and assets, special goods consumption is determined as $q_2 = z + A$.

Case 2: $\hat{\varphi}\hat{m} + \hat{a} \geq q^*$. Under case 2, the total liquidity is plentiful for consumers to consume the first-best quantity in the DM in type-2 meetings, i.e. $q_2 = q^*$. So equation (3.28) becomes

$$J_2(\hat{m}, \hat{a}) = (-\varphi + \beta\hat{\varphi})\hat{m} + (-\psi + \beta)\hat{a} + \beta(1 - \sigma)[u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{a}] + \beta\sigma[u(q^*) - q^*]$$

The FOCs with respect to $\hat{m}$ and $\hat{a}$ are as follows:

$$J_{2m} = 0 \Rightarrow \varphi = \beta\hat{\varphi}\Big\{1 + (1 - \sigma)[u'(\hat{\varphi}\hat{m}) - 1]\Big\} \tag{3.31}$$

$$J_{2a} = 0 \Rightarrow \psi = \beta \tag{3.32}$$

Definition 3.2. *A steady state equilibrium is a pair* (z,ψ)*. If* $z + A < q^*$*,* (z,ψ) *solves:*

$$i = (1 - \sigma)[u'(z) - 1] \tag{3.33}$$

$$\psi = \beta \tag{3.34}$$

If $z + A \geq q^*$*,* (z,ψ) *solves:*

$$i = (1 - \sigma)[u'(z) - 1] + \sigma[u'(A + z) - 1] \tag{3.35}$$

$$\psi = \beta\Big\{1 + \sigma[u'(A + z) - 1]\Big\} \tag{3.36}$$

Let us focus on the interesting case where $A < q^*$, so that asset alone is not enough to allow consumers to consume q^*. Now the question is, for what parameter values are we in each of the cases described above? For any given level of A, the critical point is where

$q_2 = A + z \to q^*$, so that $u'(A+z) \to 1$. To find such z, we look at steady state case of J_{2m}. Let us define $\tilde{i} \equiv (1-\sigma)[u'(q^* - A) - 1]$, which is the cutoff value of interest rate i such that the real money balance z, together with assets, is just enough to allow for optimal consumption q^*.

For any given $A < q^*$, if $i \leq \tilde{i}$, then cost of holding money is relatively low, and consumers' total liquidity (real money balance and assets) is enough for consuming q^*. So if $i < \tilde{i}$, we are in case 2, where $\psi = \beta$ and z is determined by equation (3.33). Oppositely, if $i > \tilde{i}$, the real money balance is too low, and the economy will be in case 1. Now (z, ψ) are determined by equations (3.35) and (3.36).

Before we check for a general theorem, we will start with the quadratic utility form $u(q) = -q^2/2 + (1+\gamma)q$ that allows us to derive a nice closed-form solution. This utility function yields some immediate results: $u'(q) = 1 + \gamma - q$, $q^* = \gamma$, and $u'(q) - 1 = \gamma - 1$. Also, we can find the cutoff interest rate $\tilde{i} = (1-\sigma)(\gamma - \gamma + A) = A(1-\sigma)$.

We start with finding $\bar{i}$, the upper bound of i for which monetary equilibrium exists, i.e. $z > 0$. In other word, we want to find $\bar{i}$, such that if $i \geq \bar{i}$, $z = 0$. To find the boundary case such that $z = 0$, we use Equation (3.35) as this is the relevant condition for when z is small. Then with quadratic utlity, Equation (3.35) becomes $i = (1-\sigma)(\gamma - z) + \sigma(\gamma - A - z)$. Thus the corresponding $\bar{i}$ such that $z = 0$ is, $\bar{i} = \gamma - \sigma A$. And for all $A < \gamma$, we have $\tilde{i} < \bar{i}$ as $A - A\sigma < \gamma - A\sigma$. Following such parameter restriction, we have 2 cases:

Case 1: $i \in (0, \tilde{i}]$, the equilibrium is monetary, and also total liquidity allows consumers to consume first-best q^* in the DM. Then

$$\psi = \beta \tag{3.37}$$

$$z = \gamma - \frac{i}{1-\sigma} \tag{3.38}$$

Case 2: $i \in (\tilde{i}, \bar{i})$, the equilibrium is monetary, but total liquidity is scarce so that consumers consume less than q^* in the DM. Then

$$\psi = \beta[1 + \sigma(\gamma - A - z)] \tag{3.39}$$

$$i = (1 - \sigma)(\gamma - z) + \sigma(\gamma - z - A) \tag{3.40}$$

The last equation implies that $z = \gamma - i - \sigma A$, so that $\psi = \beta\Big\{1 + \sigma[i - A(1 - \sigma)]\Big\}$. In what follows, we assume that $i < \gamma$, or otherwise there is no monetary equilibrium even with $A = 0$, which is pointless for analysis.

3.3.2.3. *Welfare Analysis.* Now we are ready to discuss the impact of σ on welfare. With the definition of q_1 and q_2 the same as in the previous section, the welfare function is

(3.41)

$$\mathcal{W} = (1 - \sigma)\Big[u(q_1) - q_1\Big] + \sigma\Big[u(q_2) - q_2\Big] \Rightarrow \mathcal{W}(\sigma) = (1 - \sigma)\Big[u(z) - z\Big] + \sigma\Big[u(z + A) - (z + A)\Big]$$

and we are interested in $\partial\mathcal{W}/\partial\sigma$. First of all, when σ changes, there is a direct effect on welfare through the numbers of various meetings. But there is also an indirect effect, through money demand. Formally,

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial \sigma} &= -[u(z) - z] + (1 - \sigma)[u'(z) - 1]\frac{dz}{d\sigma} + [u(z + A) - (z + A)] + \sigma[u'(z + A) - 1]\frac{dz}{d\sigma} \\ &= \underbrace{\Big[u(z + A) - (z + A)\Big] - \Big[u(z) - z\Big]}_{+} + \underbrace{\Big\{(1 - \sigma)\Big[u'(z) - 1\Big] + \sigma\Big[u'(z + A) - 1\Big]\Big\}}_{+}\underbrace{\frac{dz}{d\sigma}}_{-} \end{aligned}$$

Similar to Section 3.3.1 with money and credit, the impact on welfare from the increasing in σ might not be positive as many would predict. To see the behavior of $\partial\mathcal{W}/\partial\sigma$, we again focus on quadratic utility form, and discuss it for different range of σ value.

We start with the value of $\sigma = 0$. In this case, asset cannot serve the liquidity role in DM at all, and $\bar{i} = \gamma$. Here z is determined by $i = u'(z) - 1$, hence with the quadratic utility form, $z = \gamma - i$. So we have:

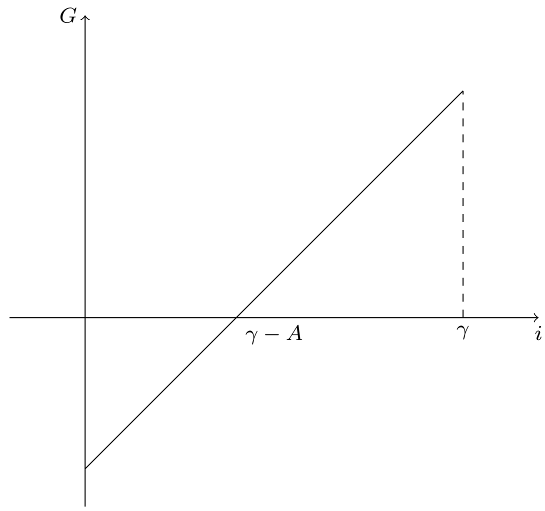
$$\mathcal{W}(0) = u(\gamma - i) - (\gamma - i) = \gamma(\gamma - i) - \frac{(\gamma - 1)^2}{2} = \frac{\gamma^2 - i^2}{2} \tag{3.42}$$

Now if σ starts to increase, asset will start to play a liquidity role. In the cases where σ takes positive values, we first examine the behavior at the other extreme, i.e. $\sigma = 1$, and then come back to the interior value of $\sigma \in (0, 1)$. With $\sigma = 1$, $\mathcal{W}(1) = u(z + A) - (z + A)$, $\tilde{i} = 0$, and $\bar{i} = \gamma - A$. So in order to calculate $\mathcal{W}(1)$, we need to know the value of real money balance, z, and hence we distinguish between two cases: a) $i \leq \bar{i}$ and b) $i > \bar{i}$. In case a), we do not have to worry about non-monetary equilibrium even if all meetings accept assets as payment, i.e. $\sigma = 1$. Then as σ increases, the non-monetary cutoff point $\bar{i}$ becomes smaller. Thus there exists a unique $\bar{\sigma}$, such that if $i > \bar{i}$ and $\sigma > \bar{\sigma}$, monetary equilibrium stops to exist.

<u>Case a:</u> If $i \leq \bar{i}$ (and $\sigma = 1$), monetary equilibrium exists for all $i \in (0, \bar{i}]$. In this case, $z = \gamma - i - A$, and $\mathcal{W}(1) = u(\gamma - i) - (\gamma - i) = (\gamma^2 - i^2)/2 = \mathcal{W}(0)$.

<u>Case b:</u> If $i > \bar{i}$, we are in the non-monetary equilibrium. There exists a unique $\bar{\sigma} \in (0, 1)$, such that for all $\sigma > \bar{\sigma}$, the equilibrium is non-monetary and $\mathcal{W}(\sigma) = \sigma[u(A) - A]$, i.e. $\mathcal{W}$ is linear and increasing in $\sigma \in [\bar{\sigma}, 1]$. At $\sigma = 1$, $\mathcal{W}(1) = u(A) - A = A(\gamma - A/2)$. And we claim that $\mathcal{W}(1) > \mathcal{W}(0)$.

Proof. $\mathcal{W}(1) > \mathcal{W}(0) \iff G(i) \equiv A(\gamma - \frac{A}{2}) - \frac{\gamma^2 - i^2}{2} > 0$. Now, G is clearly continuous and increasing in i, and $G(\gamma - A) = A\gamma - \frac{A^2}{2} - \frac{\gamma^2 - (\gamma - A)^2}{2} = 0$. So $G(i) > 0$ for all $i > \gamma - A$, and $\mathcal{W}(1)|_{NME} > \mathcal{W}(0)|_{NME}$. $\square$



The lesson is that, if i is small enough, the cost of carrying money is relatively small. Thus even when all producers accept assets, i.e. $\sigma = 1$, consumers could still want to hold some money, and $\mathcal{W}(0) = \mathcal{W}(1)$. Then as i increases, both $\mathcal{W}(1)$ and $\mathcal{W}(0)$ decreases. However, for $\mathcal{W}(1)$ there is a lower bound on how much it can decrease as i increases: if i becomes so high that a monetary equilibrium ceases to exist (for $\sigma = 1$), consumers can still use assets for consumption, hence $\mathcal{W}(1)$ cannot go below $u(A) - A$. But when assets are not accepted at all, i.e. $\sigma = 0$, consumers rely solely on money for DM consumption. Hence $\mathcal{W}(0)$ will keep on going down. And this is precisely why $\mathcal{W}(1) > \mathcal{W}(0)$ when $i > \bar{i}$.

Now that the extremes are discussed, we study the interior value of σ. Of course, what happens in the middle depends on the regions/cases, which further depends on parameters. Hence we first define $\bar{\sigma} \equiv \min\{1, (\gamma - i)/A\}$, $\tilde{\sigma} \equiv \max\{0, (A - i)/A\}$. Notice that $\bar{\sigma} > \tilde{\sigma}$ for all i, and the equilibrium will be monetary if and only if $\sigma \leq \bar{\sigma}$, and the equilibrium will be plentiful if and only if $\sigma \leq \tilde{\sigma}$. Given the definitions of $\bar{\sigma}$ and $\tilde{\sigma}$, there are 4 possible cases that satisfy $\bar{\sigma} > \tilde{\sigma}$ with $\bar{\sigma} \in [0, 1]$ and $\tilde{\sigma} \in [0, 1]$.

<u>Case 1</u>: $0 < \tilde{\sigma} < \bar{\sigma} < 1$. This is true when: (1) $\tilde{\sigma} > 0$ or $i < A$, and (2) $\bar{\sigma} < 1$ or $i > \gamma - A$. For this to be possible, we need $A > \gamma - A$ or equivalently $A > \gamma/2$. When these

conditions are satisfied, we have $0 < \tilde{\sigma} < \bar{\sigma} < 1$, and the equilibrium is

$$\begin{cases} \text{Plentiful } \forall \sigma \in [0, \tilde{\sigma}] \\ \text{Scarce } \forall \sigma \in (\tilde{\sigma}, \bar{\sigma}) \\ \text{Non-monetary } \forall \sigma \in [\bar{\sigma}, 1] \end{cases}$$

<u>Case 2</u>: $0 < \tilde{\sigma} < \bar{\sigma} = 1$. This is true when: (1) $\tilde{\sigma} > 0$ or $i < A$, and (2) $\bar{\sigma} = 1$ or $i \leq \gamma - A$. This can happen under two circumstances:

a) If $A > \gamma/2$ (or just $A > \gamma - A$), then we need $i \leq \gamma - A$

b) If $A < \gamma/2$ (or $A < \gamma - A$), we need $i < A$

If either of these two conditions happens, then $0 < \tilde{\sigma} < \bar{\sigma} = 1$, and the equilibrium is

$$\begin{cases} \text{Plentiful } \forall \sigma \in [0, \tilde{\sigma}] \\ \text{Scarce } \forall \sigma \in (\tilde{\sigma}, 1] \end{cases}$$

<u>Case 3</u>: $0 = \tilde{\sigma} < \bar{\sigma} < 1$. This is true when: (1) $\tilde{\sigma} = 0$ or $i \geq A$, and (2) $\bar{\sigma} < 1$ or $i > \gamma - A$. This can happen under 2 circumstances:

a) If $A > \gamma/2$ (or $A > \gamma - A$), we need $i \geq A$

b) If $A < \gamma/2$ (or $A < \gamma - A$), we need $i > \gamma - A$

When one of these circumstances happen, we have $0 = \tilde{\sigma} < \bar{\sigma} < 1$ and the equilibrium is

$$\begin{cases} \text{Scarce } \forall \sigma \in [0, \bar{\sigma}) \\ \text{Non-monetary } \forall \sigma \in [\bar{\sigma}, 1] \end{cases}$$

<u>Case 4</u>: $0 = \tilde{\sigma} < \bar{\sigma} = 1$. This is true when: (1) $i \geq A$, and (2) $i \leq \gamma - A$, which requires $A \leq \gamma/2$. So the economy is in case 4 when $A \leq \gamma/2$ and $i \in [A, \gamma - A]$. The equilibrium is scarce (but still monetary) for all $\sigma \in [0, 1]$.

Figure (3.2) and Figure (3.3) give a visual illustration of the parameter range of the four cases. And from the figures, we verify that the discussion covers all possible parameter values.

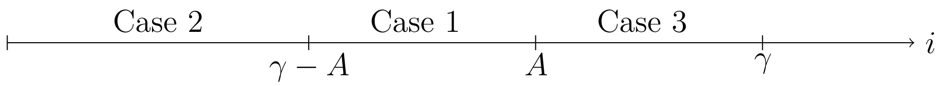


FIGURE 3.2. $A > \frac{\gamma}{2}$

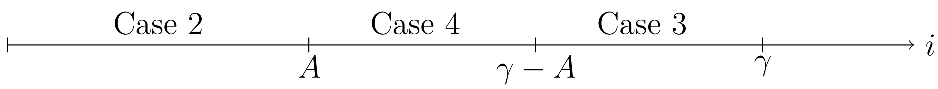


FIGURE 3.3. $A \leq \frac{\gamma}{2}$

Now that all the cases are clear, we study the behavior of $\mathcal{W}(\sigma)$ for all $\sigma \in [0,1]$. The impact of improved asset acceptability on welfare is summarized in Proposition 3.2, which again shows that a better asset acceptability does not necessarily improve welfare.

PROPOSITION 3.2. *Define* $\bar{\sigma} \equiv \min\{1, (\gamma - i)/A\}$, $\tilde{\sigma} \equiv \max\{0, (A-i)/A\}$. *The equilibrium can be summarized as the following four cases*

- ***Case 1:*** *For all* $A > \gamma/2$ *and* $i \in (\gamma - A, A)$, $\mathcal{W}(0) < \mathcal{W}(1)$; *and*
  - *if* $\sigma \in [0, \tilde{\sigma}]$, *equilibrium is plentiful;* $\mathcal{W}(\sigma)$ *is decreasing and concave in* σ;
  - *if* $\sigma \in (\tilde{\sigma}, \bar{\sigma})$, *equilibrium is scarce; furthermore, if*
    - *(i)* $\tilde{\sigma} < 1/2 < \bar{\sigma}$, $\mathcal{W}$ *has a unique minimizer at* $\sigma = 1/2$;
    - *(ii)* $1/2 < \tilde{\sigma} < \bar{\sigma}$, $\mathcal{W}$ *has a unique minimizer at* $\sigma = \tilde{\sigma}$;
    - *(iii)* $\tilde{\sigma} < \bar{\sigma} < 1/2$, $\mathcal{W}$ *has a unique minimizer at* $\sigma = \bar{\sigma}$.
  - *if* $\sigma \in [\bar{\sigma}, 1]$, $\mathcal{W}$ *is linear and increasing in* σ.
- ***Case 2:*** *For all* $A > \gamma/2$ *&* $i \leq \gamma - A$, *or* $A < \gamma/2$ *&* $i < A$, *and if*
  - $\sigma \in [0, \tilde{\sigma}]$, *equilibrium is plentiful;* $\mathcal{W}$ *is decreasing and concave in* σ;
  - $\sigma \in (\tilde{\sigma}, 0]$, *equilibrium is scarce;* $\mathcal{W}$ *is convex in* σ, *and if*
    - *(i)* $\tilde{\sigma} < 1/2$, $\mathcal{W}$ *has a unique minimizer at* $\sigma = 1/2$;

(ii) $\tilde{\sigma} > 1/2$, *$\mathcal{W}$ has a unique minimizer at* $\sigma = \tilde{\sigma}$

- ***Case 3:*** *For all* $A > \gamma/2$ *& $i \geq A$, or* $A < \gamma/2$ *& $i > \gamma - A$, and if*

  - $\sigma \in [0, \bar{\sigma})$, *equilibrium is scarce; $\mathcal{W}$ is convex in σ, and if*

    (i) $\bar{\sigma} > 1/2$, *$\mathcal{W}$ has a unique minimizer at* $\sigma = 1/2$*;*

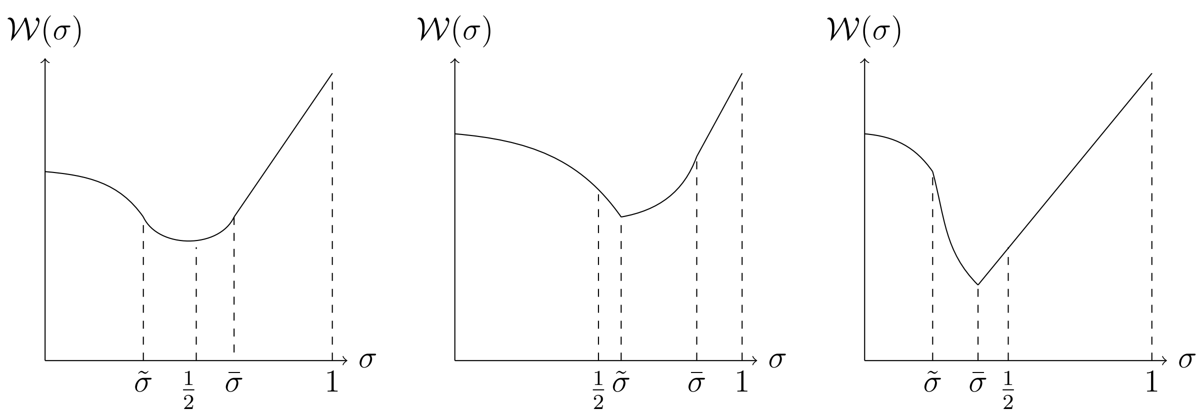
    (ii) $\bar{\sigma} < 1/2$, *$\mathcal{W}$ has a unique minimizer at* $\sigma = \bar{\sigma}$*;*

  - $\sigma \in [\bar{\sigma}, 1]$, *equilibrium is non-monetary; $\mathcal{W}$ is increasing and linear in σ.*

- ***Case 4:*** *For all* $A \leq \gamma/2$ *and* $i \in [A, \gamma - A]$, *and for all* $\sigma \in [0, 1]$, *equilibrium is scarce; $\mathcal{W}$ is convex in σ, with a unique minimizer at* $\sigma = 1/2$.

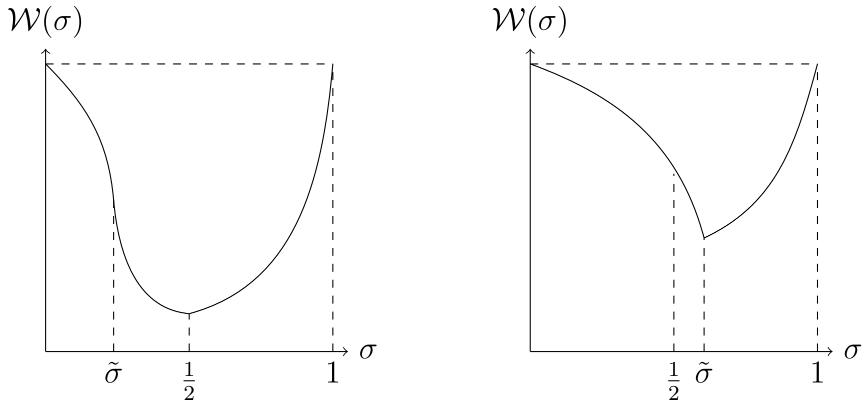
PROOF. See the appendix A.3.1. □

Welfare as a function of asset acceptability in the four different regions are illustrated by Figure 3.4, 3.5, 3.6, and 3.7 respectively.



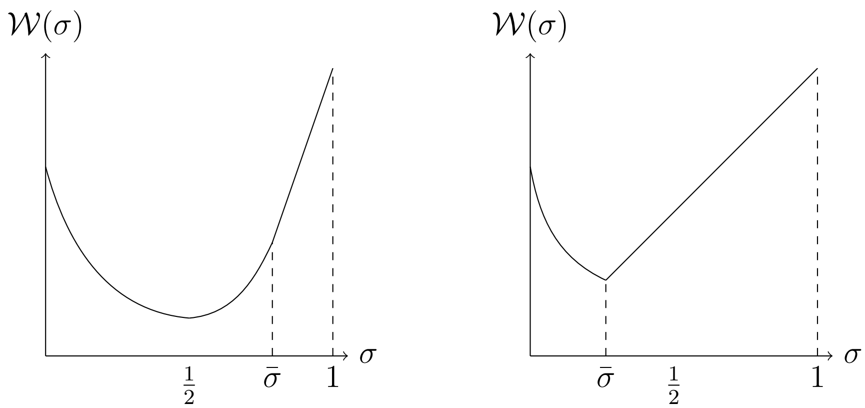
(a) Case 1(i) (b) Case 1(ii) (c) Case 1(iii)

FIGURE 3.4. Case 1 Summary



(a) Case 2(i) (b) Case 2(ii)

FIGURE 3.5. Case 2 Summary



(a) Case 3(i) (b) Case 3(ii)

FIGURE 3.6. Case 3 Summary

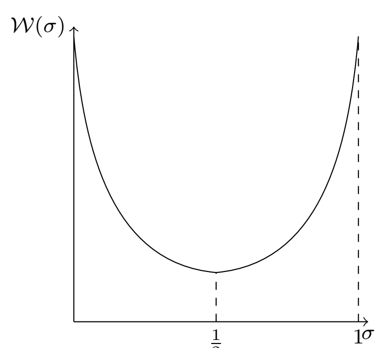


FIGURE 3.7. Case 4 Summary

3.4. Model with a Secondary Market

In this section, we extend the model to include a secondary asset market. We consider the scenario where a consumption shock is realized at the beginning of each period. With the realization of this shock, ℓ fraction of consumers will have consumption opportunity (henceforth called C-type) in the DM, while the remaining $1-\ell$ fraction (henceforth called the N-type) do not have such opportunity. The consumers find out about the consumption opportunity after they choose their portfolio. In this setting, we consider two types of secondary markets: secondary market for assets, and secondary market for loans. In the first specification, C-type sells bonds for cash; in the second specification, C-type would receive an unsecured loan. Both types of secondary markets will be over-the-counter market (henceforth OTC market) with bilateral meetings. We conduct welfare analysis with respect to the probability of matching in the secondary market.

Let $f(\ell, 1-\ell)$ be the matching function between C-types and N-types, hence also the number of matches in the OTC market. Thus the probability of a typical C-type getting a match in the OTC is $f(\ell, 1-\ell)/\ell$.

3.4.1. Uncollateralized Loans.

3.4.1.1. *Value Functions and Bargaining Solutions.* We first consider the scenario where C-type can obtain uncollateralized loans in the OTC market. Let d be the amount of debt that C-type takes in the OTC, which need to be paid back in the following CM. Then the Bellman equation of a typical consumer in the CM is given by:

$$W(m, d) = \max_{\hat{m}, X, H} \{X - H + \beta \mathbb{E}\Omega(\hat{m})\}$$

$$\text{s.t. } X + \varphi\hat{m} + d = \varphi m + T + H$$

As is standard in the literature, the choice variables will be independent of the current state variables, and the CM value function adopts the form:

$$W(m,d) = \varphi m - d + T + \max_{\hat{m}}\{-\varphi \hat{m} + \beta \mathbb{E}\Omega(\hat{m})\} \tag{3.43}$$

First consider the case where the producer in the DM meetings only accept money as the medium of exchange. Let q be the amount of special goods changed hands, and let p be the amount of money that C-type pays to the producer.

LEMMA 3.4. *Define $m^* = q^*/\varphi$ as the amount of real money balance that is required for optimal DM consumption q^*. The DM bargaining solution is as follows*

$$q = \begin{cases} q^*, & \text{if } m \geq m^* = \frac{q*}{\varphi} \\ \varphi m, & \text{if } m < m^* \end{cases}$$

$$p = \begin{cases} m^*, & \text{if } m \geq m^* \\ m, & \text{if } m < m^* \end{cases}$$

PROOF. The proof is obvious, hence, omitted. □

After discussing the DM bargaining solution, we proceed to the bilateral meetings between a C-type buyer and an N-type buyer in the OTC market. Denote C-type's money holding as m, and N-type's money holding as $\tilde{m}$. In the bilateral meeting, C-type takes an unsecured loan from N-type, and promise to pay back in the upcoming CM. Hence in the OTC market, C-type and N-type bargain over the amount of loan (money), x, that C-type takes from N-type, and the quantity, d, to be paid back to N-type in the CM. Thus the value function of a typical consumer entering OTC with money holding m is:

$$\mathbb{E}\Omega(m) = \ell\Big[\frac{f}{\ell}V(m+x,d)+(1-\frac{f}{\ell})V(m,0)\Big]+(1-\ell)\Big[\frac{f}{1-\ell}W(m-\tilde{x},\tilde{d})+(1-\frac{f}{1-\ell}W(m,0))\Big]$$

where variables with tilde denote the quantities being exchanged when the consumer turns out to be an N-type, supplying liquidity in the OTC market. Since it is never optimal for consumers to bring more than enough money into next period, i.e. $m \leq m^*$, DM value function $V(m,d) = u(\varphi m) + W(0,d)$. By substituting in the expressions of V and W, the expected OTC value function becomes:

$$
\begin{aligned}
\mathbb{E}\Omega(m) = & f\Big[u\Big(\varphi(m + x(m,\tilde{m}))\Big) - d(m,\tilde{m})\Big] + (\ell - f)u(\varphi m) \\
& + f\Big[\varphi(m - x(\tilde{m},m)) + d(\tilde{m},m)\Big] + (1 - \ell - f)\varphi m + \Lambda
\end{aligned}
$$

We assume that the bargaining solution in OTC market is determined by C-type making take-it-or-leave-it offer. The OTC bargaining problem is to maximize C-type's bargaining surplus, subject to N-type's participation constraint, and N-type's money constraint:

$$
\begin{aligned}
& \max_{x,d} V(m+x,d) - V(m,0) \\
\text{s.t.} \ & W(\tilde{m} - x, -d) - W(\tilde{m}, 0) = 0 \\
& x \leq \tilde{m}
\end{aligned}
$$

Using expressions of V and W derived from previous sections, C-type's surplus (CS) and N-type's surplus (NS) are $CS = u(\varphi(m+x)) - u(\varphi m) - d$ and $NS = d - \varphi x$. Thus the bargaining problem becomes

$$
\begin{aligned}
& \max_{x,d} u(\varphi(m+x)) - u(\varphi m) - d \\
\text{s.t.} \ & d = \varphi x \\
& x \leq \tilde{m}
\end{aligned}
$$

The bargaining solution depends on whether N-type's money constraint binds or not, hence can be discussed in two cases.

Lemma 3.5. *Given the same definition of m^* and q^*, the OTC bargaining solution is as follows*

$$x = \begin{cases} m^* - m, & \textit{if } m + \tilde{m} \geq m^* \\ \tilde{m}, & \textit{if } m + \tilde{m} < m^* \end{cases}$$

$$d = \begin{cases} q^* - \varphi m, & \textit{if } m + \tilde{m} \geq m^* \\ \varphi \tilde{m}, & \textit{if } m + \tilde{m} < m^* \end{cases}$$

Proof. Proof is obvious, and hence omitted. □

The bargaining solution is intuitive. If money is abundant, i.e., $\tilde{m} + m \geq m^*$, money constraint is not binding. C-type in this case would borrow just enough money from N-type to consume the first-best in DM, and the solution is $x = m^* - m$ and $d = q^* - \varphi m$. However, if money is scarce, i.e., $\tilde{m} \leq m^* - m$, N-type does not have enough money to lend to C-type for first-best consumption. Then C-type would want to acquire all money that N-type carries, so $x = \tilde{m}$, and in exchange, C-type will pay back the same amount of real balance $d = \varphi \tilde{m}$ in the upcoming CM.

Given the bargaining solutions, these two cases are not hard to analyze. However, later when we study the model with secondary *asset* market, the analysis would become more complicated. Thus to make further analysis simpler, and to keep symmetry between secondary credit market and secondary asset market, we assume that money constraint never binds, i.e. $m + \tilde{m} \geq m^*$. Given this assumption, the bargaining solution would be restrained to the abundant case only. So in all secondary credit market meeting, first-best amount of money will change hands, and C-type will be able to consume first-best, q^*, in all DM meeting. But we will show in later section that this is not always true if the only available secondary market is for assets trading instead of uncollateralized loan. More specifically, C-type might not be able to consume first-best quantity q^* even with plentiful money in the economy. The quantity of special goods consumption would depend also on the asset supply.

The expected OTC value function of a consumer with money holding m is then

$$\begin{aligned}\mathbb{E}\Omega(m) =& f\Big[u(q^*) - q^* + \varphi m\Big] + (\ell - f)u(\varphi m) + \Lambda \\ &+ f\Big[\varphi(m - m^* + \tilde{m}) + q^* - \varphi\tilde{m}\Big] + (1 - \ell - f)\varphi m\end{aligned}$$

where the four terms in order represent the benefit of holding money m if the consumer turns out to be a matched C-type, an unmatched C-type, a matched N-type, and an unmatched N-type. By substituting this expression into equation (3.43), and grouping all terms that contain choice variable $\hat{m}$, we call this term $J(\hat{m})$, which adopts the following form:

$$J(\hat{m}) = -\varphi\hat{m} + \beta f[u(q^*) - q^* + \hat{\varphi}\hat{m}] + \beta(\ell - f)u(\hat{\varphi}\hat{m}) + \beta(1 - \ell)\hat{\varphi}\hat{m}$$

The optimal money choice should satisfy the first order condition:

$$\varphi = \beta\hat{\varphi}[f + 1 - \ell] + \beta(\ell - f)u'(\hat{\varphi}\hat{m})\hat{\varphi} = \beta\hat{\varphi}\Big\{1 + (\ell - f)[u'(\hat{\varphi}\hat{m} - 1)]\Big\}$$

And if we focus on steady state equilibrium, then

$$\frac{\varphi M(1 + \mu)}{\beta\hat{\varphi}\hat{M}} - 1 = i = (\ell - f)[u'(z) - 1]$$

Before further discussion on the impact of matching probability on welfare, there are some key issues we need to address. We assumed that money constraint is not a concern for consumption, i.e. $\hat{m} + \tilde{m} \geq m^*$. In the symmetric equilibrium, we then have $z \geq q^*/2$ or $i \leq \gamma(\ell - f)/2$, which is an implicit assumption in the model. But if this is true, then not only will we never reach a non-monetary equilibrium, but we will never even get close to it. And this might be a problem given how crucial the non-monetary equilibrium was in the previous section. So to make sure that we do not overlook the welfare implication in the non-monetary equilibrium, we first analyze the model without imposing the quadratic utility function.

Let m be C-type's money position, and $\tilde{m}$ be C-type's belief of N-type trading partner's money position. Then $x = \min\{m^* - m, \tilde{m}\}$, and $d = \min\{\varphi\tilde{m}, q^* - \varphi m\}$, and the expected OTC value function is

$$\begin{aligned}\mathbb{E}\Omega(m) =& f[u(\varphi(m + x(m, \tilde{m}))) - d(m, \tilde{m})] + (\ell - f)u(\varphi m) \\ &+ f[\varphi(m - x(\tilde{m}, m)) + d(\tilde{m}, m)] + (1 - \ell - f)\varphi m + \Lambda\end{aligned}$$

With the same definition of the $J(\hat{m})$ as we defined in previous section, we have

$$\begin{aligned}J(\hat{m}) =& -\varphi\hat{m} + \beta f\Big[u\Big(\hat{\varphi}(\hat{m} + x(\hat{m}, \tilde{m}))\Big) - d(\hat{m}, \tilde{m})\Big] + \beta(\ell - f)u(\hat{\varphi}\hat{m}) \\ &+ \beta f\Big[\hat{\varphi}\Big(\hat{m} - x(\tilde{m}, \hat{m})\Big) + d(\tilde{m}, \hat{m})\Big] + \beta(1 - \ell - f)\hat{\varphi}\hat{m} \\ =& -\varphi\hat{m} + \beta f\Big[u\Big(\hat{\varphi}(\hat{m} + x(\hat{m}, \tilde{m}))\Big) - d(\hat{m}, \tilde{m})\Big] + \beta(\ell - f)u(\hat{\varphi}\hat{m}) + \beta(1 - \ell)\hat{\varphi}\hat{m}\end{aligned}$$

Notice that the first term in the second line, $-\hat{\varphi}x(\tilde{m}, \hat{m})$ and $d(\tilde{m}, \hat{m})$, will cancel out with each other. This is due to the fact that with TIOLI bargaining protocol, N-type's bargaining surplus is zero. Thus even if N-type's money holding binds, it will not affect the $J(\hat{m})$. Given the belief $\tilde{m}$ and price $\hat{\varphi}$, we discuss the objective function and pricing function in two cases, i.e. money constraint binds, and money constraint does not bind.

Region 1: If $\hat{m} + \tilde{m} \geq m^*$, and call this region 1, then the J term and pricing function take the form

$$J_1(\hat{m}) = -\varphi\hat{m} + \beta f[u(q^*) - q^* + \hat{\varphi}\hat{m}] + \beta(\ell - f)u(\hat{\varphi}\hat{m}) + \beta(1 - \ell)\hat{\varphi}\hat{m}$$

$$\varphi = \beta\hat{\varphi}\Big\{1 + (\ell - f)[u'(\hat{\varphi}\hat{m}) - 1]\Big\}$$

Region 2: If $\hat{m} + \tilde{m} < m^*$, and call this region 2, then J term and pricing functions are

$$J_2(\hat{m} =) - \varphi\hat{m} + \beta f[u(\hat{\varphi}(\hat{m} + \tilde{m})) - \hat{\varphi}\tilde{m}] + \beta(\ell - f)u(\hat{\varphi}\hat{m}) + \beta(1 - \ell)\hat{\varphi}\hat{m}$$

$$\varphi = \beta\hat{\varphi}\Big\{1 + f[u'(\hat{\varphi}(\hat{m} + \tilde{m})) - 1] + (\ell - f)[u'(\hat{\varphi}\hat{m}) - 1]\Big\}$$

Definition 3.3. *In the symmetric steady state equilibrium, we have* $\hat{\varphi}\hat{m} = \hat{\varphi}\tilde{m} = z$*. An equilibrium is a list* $\{z, q_1, q_2\}$ *such that :*

(1) $q_1 = z$, $q_2 = \min\{2z, q^*\}$

(2) *In region 1, where money constraint does not bind,* z *solves* $i = (\ell - f)[u'(z) - 1]$

(3) *In region 2, where money constraint binds,* z *solves* $i = f[u'(q_2) - 1] + (\ell - f)[u'(z) - 1]$

What is the interest rate i that will bring us to the border of the 2 regions? More specifically, what is the i that satisfies $q_2 = 2z = q^*$? We define this cutoff level of interest rate as $\bar{i}$, and this interest rate would satisfy $\bar{i} = (\ell - f)[u'(q^*/2) - 1]$. At $i = \bar{i}$, real money balance is just enough to allow for optimal consumption q^*. If the interest rate is below $\bar{i}$, then it is not very costly to carry money, hence agents would bring enough money to consume first-best q^* in the DM; and if interest rate goes beyond $\bar{i}$, then agents would not bring enough money to consume q^*. Hence

- $\forall i \in (0, \bar{i}]$, we have $q_2 = q^*$, where z solves $i = (\ell - f)[u'(z) - 1]$
- $\forall i > \bar{i}$, we have $q_2 = 2z$, where z solves $i = f[u'(2z) - 1] + (\ell - f)[u'(z) - 1]$

If we again focus on the same quadratic utility function $u(q) = (1 + \gamma)q - q^2/2$, we have $\bar{i} = \gamma(\ell - f)/2$, thus

- $\forall i \in (0, \frac{\gamma(\ell-f)}{2}]$, $q_2 = \gamma$ and $i = (\ell - f)(\gamma - z) \Rightarrow z = \gamma - \frac{i}{\ell-f}$
- $\forall i > \frac{\gamma(\ell-f)}{2}$, $q_2 = 2z$, where z solves $i = f[\gamma - 2z] + (\ell - f)(\gamma - z) \Rightarrow z = \frac{\gamma\ell-i}{\ell+f}$

and notice that as $i \to \bar{i}, z \to \gamma/2$ as it should.

Notice that with Inada condition, we will always have $z > 0$ even when i is huge. However, this is not true under quadratic utility function. Hence in order for the equilibrium to be monetary, there also exists an upper bound for i such that if i exceeds such upper bound, the equilibrium becomes non-monetary. And this upper bound would be $i = \gamma\ell$, so the second region would be categorized by all $i \in (\gamma(\ell - f)/2, \gamma\ell]$.

After specifying each regions, we are ready to analyze the welfare impact. And since we are interested in $\mathcal{W}(f)$, we need to check that for any given i, how the value of f might get the equilibrium into different regions as we define above. The idea is that, z decreases in f. So if f becomes too large (above a certain threshold $\bar{f}$), equilibrium will switch from the "plentiful" to "scarce". And this threshold $\bar{f}$ depends on i, which is given by $(\ell - \bar{f})\gamma/2 = i$. Hence $\bar{f} = \ell - 2i/\gamma$, which decreases in i. This result is intuitive. When $i = 0$, $\bar{f} = \ell$. Then for all admissible f, we have $f \leq \bar{f} = \ell$, and equilibrium is always the plentiful case. As i increases, $\bar{f}$ decreases, and the equilibrium starts to shift to the "scarce" case. Also notice that, if $i \geq \ell\gamma/2$ or $\bar{f} \leq 0$, then all admissible values of f satisfy $f \geq \bar{f}$, and equilibrium is always in the the scarce case.

Define $\mathcal{W}(f) = (\ell - f)[u(z) - z] + f[u(q_2) - q_2]$. Then we have

$$\frac{\partial \mathcal{W}}{\partial f} = u(q_2) - q_2 - [u(z) - z] + \left\{ f\Big[u'(q_2) - 1\Big]\frac{dq_2}{dz} + (\ell - f)\Big[u'(z) - 1\Big]\right\}\frac{dz}{df}$$

and we check that given different values of i, what value does this threshold $\bar{f}$ take, and hence what type of equilibrium, i.e. "plentiful" or "scarce", the economy is in.

Proposition 3.3. *Define $\bar{f} = \frac{l-2i}{\gamma}$ as the cut-off level of matching probability that divides the economy into "scarce" and "plentiful" equilibrium.*

- *For all $i \in (0, \frac{\gamma\ell}{2})$, $\bar{f} \in (0, \ell)$.*
  - *$\forall\ f \in [0, \bar{f})$, the economy is in "plentiful" equilibrium, and $\mathcal{W}$ is decreasing and concave in f*
  - *$\forall\ f \in [\bar{f}, l]$, the economy is in "scarce" equilibrium; moreover, if*
    - *$i \leq \frac{\gamma\ell}{3}$, then $\mathcal{W}$ is increasing in f*

* $i \in (\frac{\gamma\ell}{2}, \frac{\gamma\ell}{2})$, *then* $\mathcal{W}$ *is decreasing in* f *for all* $f \in [\bar{f}, \frac{\ell}{2}$*; and* $\mathcal{W}$ *is increasing in* f *for all* $f \in [\frac{\ell}{3}, \ell]$

- *For all* $i \in (\frac{\gamma\ell}{2}, \gamma\ell)$, $\bar{f} \leq 0$. *The economy is in "scarce" equilibrium, and* $\mathcal{W}$ *is convex in* f*, with a unique minimizer at* $f = \frac{l}{3}$.
- $\mathcal{W}(0) = \mathcal{W}(l)$ *in both "plentiful" and "scarce" equilibrium.*

PROOF. See the appendix A.3.2. □

3.4.2. Secondary asset market.

3.4.2.1. *Value Functions and Bargaining Solution.* In this section, we consider the case where the secondary market opens for assets trading, instead of secondary market for un-collateralized loan. In this case, the state variables for a typical consumer entering the CM would be: m, which is the amount of money leftover from previous DM, and a, the amount of asset they consumer carries from last period. And consumers choose the quantity of money and asset, $\hat{m}$ and $\hat{a}$ to bring into next period. So the Bellman equation in the CM is given by

$$W(m,a) = \max_{\hat{m},\hat{a},X,H} \{X - H + \beta \mathbb{E}\Omega(\hat{m})\}$$

$$\text{s.t. } X + \varphi\hat{m} + \varphi\hat{a} = \varphi m + a + T + H$$

and as is standard in the literature, choice variables are independent of current state variables. And the CM value function is given by

$$W(m,a) = \varphi m + a + T + \max_{\hat{a},\hat{m}} \{-\varphi\hat{m} - \psi\hat{a} + \beta\mathbb{E}\Omega(\hat{m},\hat{a})\}$$

DM bargaining is the same as in previous section, and the solution is given by $d = \min\{m, m^*\}$ and $q = \min\{\varphi m, q^*\}$, where d is the amount of money paid to the producer. For the OTC bargaining problem, let x and χ be the amount of cash and assets exchanged in the OTC respectively. The expected OTC value function is given by

$$
\begin{aligned}
\mathbb{E}\Omega(m,a) =& \ell[\frac{f}{\ell}V(m+x,a-\chi)+(1-\frac{f}{\ell})V(m,a)] \\
&+(1-\ell)[\frac{f}{1-\ell}W(m-\tilde{x},a+\tilde{\chi})+(1-\frac{f}{1-\ell})W(m,a)] \\
&=f[u(\varphi(m+x))-\chi]+(\ell-f)u(\varphi m)+f(\tilde{\chi}-\varphi\tilde{x})+(1-\ell)\varphi m+\Lambda+a
\end{aligned}
$$

The OTC bargaining take places between a C-type with portfolio (m,a) and an N-type with portfolio $(\tilde{m},\tilde{a})$.

$$
\begin{aligned}
&\max_{x,\chi} V(m+x,a-\chi)-V(m,a) \\
\text{s.t. } & W(\tilde{m}-x,\tilde{a}+\chi)=W(\tilde{m},\tilde{a})
\end{aligned}
$$

After substituting in the DM and CM value functions into the bargaining problem, C-type's bargaining surplus is given by $u(\varphi(m+x))-u(\varphi m)-\chi$, and N-type's bargaining surplus is given by $\chi-\varphi x$. So the bargaining problem is reduced to

$$
\begin{aligned}
&\max_{x,\chi} u(\varphi(m+x))-u(\varphi m)-\chi \\
\text{s.t. } & \chi=\varphi x \\
& x\leq\tilde{m},\ \chi\leq a
\end{aligned}
$$

And depending on whether the constraints bind or not, we can end up with different sets of bargaining solutions.

Lemma 3.6. *Consider a meeting in the OTC market between a C-type and an N-type with portfolios (m,a) and (m̃,ã), respectively, and define the cutoff level of asset holdings as* $\bar{a}\equiv\min\{\varphi\tilde{m},q^*-\varphi m\}$. *Then the solution to the bargaining problem in the OTC market is given by*

$$x = \begin{cases} \min\{\tilde{m}, m^* - m\}, & \textit{if } a \geq \bar{a} \\ \frac{a}{\varphi}, & \textit{if } a < \bar{a} \end{cases}$$

$$\chi = \begin{cases} \bar{a}, & \textit{if } a \geq \bar{a} \\ a, & \textit{if } a < \bar{a} \end{cases}$$

PROOF. See the appendix A.3.3. □

What matters for the determination of x and χ are $m, \tilde{m}, a$. More specifically, N-type's asset position does not matter. So both x and χ are potentially functions of $m, \tilde{m}, a$, i.e. $x = x(m, \tilde{m}, a)$, and $\chi = \chi(m, \tilde{m}, a)$. Using these notations, the expected OTC value function is given by:

$$\begin{aligned} \mathbb{E}\Omega(m, a) =& f[u(\varphi(m + x(m, \tilde{m}, a))) - \chi(m, \tilde{m}, a)] + (\ell - f)u(\varphi m) \\ &+ f[\chi(m, \tilde{m}, a) - \varphi x(m, \tilde{m}, a)] + (1 - \ell)\varphi m + \Lambda + a \end{aligned}$$

and the first term in line 2 equals 0 regardless of which region the equilibrium is in. And hence the objective function $J(\hat{m}, \hat{a})$ is

$$\begin{aligned} J(\hat{m}, \hat{a}) =& -\varphi\hat{m} - \psi\hat{a} + \beta\hat{a} + \beta f[u(\hat{\varphi}(\hat{m} + x(\hat{m}, \tilde{m}, \hat{a}))) - \chi(\hat{m}, \tilde{m}, \hat{a})] \\ &+ \beta(\ell - f)u(\hat{\varphi}\hat{m}) + \beta(1 - \ell)\hat{\varphi}\hat{m} \end{aligned}$$

The bargaining solution depends on whether the asset constraints bind or not, which has three potential outcomes. Hence the objective function and the pricing functions should also be discussed for each region, and we derive them as follows.

Notice that the bargaining solutions in region 2 and region 4 are the same, and so are the the objective functions. Hence we derive the optimal portfolio choice of the Representative agent for each of the three regions, and summarize the optimal behavior in Lemma 3.7.

LEMMA 3.7. *Taking prices* (φ,$\hat{\varphi}$,ψ,$\hat{\psi}$) *and belief* $\tilde{m}$ *as given, the optimal choice of a representative agent with portfolio* (m,a) *satisfies:*

$$\beta^{-1}J^1_m(\hat{m},\hat{a}) = -\frac{\varphi}{\beta} + \hat{\varphi}\Big\{1 + (\ell - f)\Big[u'(\hat{\varphi}\hat{m}) - 1\Big]\Big\} \tag{3.44}$$

$$\beta^{-1}J^1_a(\hat{m},\hat{a}) = -\frac{\psi}{\beta} + 1 \tag{3.45}$$

$$\beta^{-1}J^2_m(\hat{m},\hat{a}) = -\frac{\varphi}{\beta} + \hat{\varphi}\Big\{1 + f\Big[u'(\hat{\varphi}\hat{m} + \hat{a}) - 1\Big] + (\ell - f)\Big[u'(\hat{\varphi}\hat{m}) - 1\Big]\Big\} \tag{3.46}$$

$$\beta^{-1}J^2_a(\hat{m},\hat{a}) = -\frac{\psi}{\beta} + \Big\{1 + f\Big[u'(\hat{\varphi}\hat{m} + \hat{a}) - 1\Big]\Big\} \tag{3.47}$$

$$\beta^{-1}J^3_m(\hat{m},\hat{a}) = -\frac{\varphi}{\beta} + \hat{\varphi}\Big\{1 + f\Big[u'(\hat{\varphi}(\hat{m} + \tilde{m})) - 1\Big] + (l - f)\Big[u'(\hat{\varphi}\hat{m}) - 1\Big]\Big\} \tag{3.48}$$

$$\beta^{-1}J^3_a(\hat{m},\hat{a}) = -\frac{\psi}{\beta} + 1 \tag{3.49}$$

PROOF. See the appendix A.3.4. □

Again, we focus on symmetric steady state equilibrium, i.e. C-type and N-type carry same amount of assets since they are ex-ante identical. Hence in equilibrium, $\hat{\varphi}\hat{m} = \hat{\varphi}\tilde{m} = z$ and $\hat{a} = \tilde{a} = A$, and asset pricing functions in equilibrium are given as follows.

Region 1: $i = (\ell - f)[u'(z) - 1], \psi = \beta$.

Region 2: $i = f[u'(z + A) - 1] + (\ell - f)[u'(z) - 1], \psi = \beta\Big\{1 + f[u'(z + A) - 1]\Big\}$.

Region 3: $i = f[u'(2z) - 1] + (\ell - f)[u'(z) - 1], \psi = \beta$.

And the aggregate regions can be summarized by Figure 3.8.

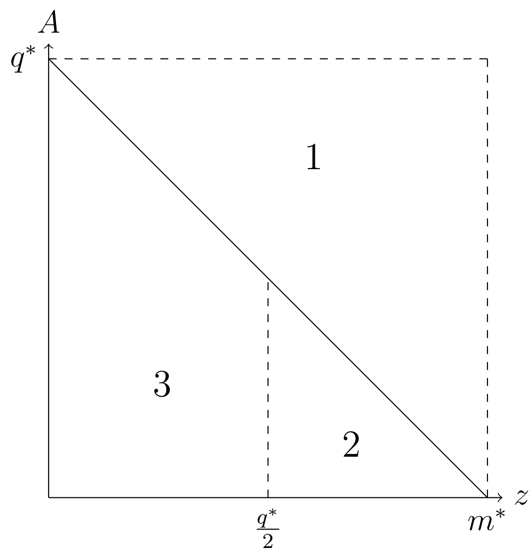


FIGURE 3.8. Aggregate Regions by A and z

Figure 3.8 shows how are the aggregate regions divided given different combinations of real money balance z and asset supply A. If $z+A \geq q^*$, then the asset constraint never binds and C-type gets optimal consumption in the DM, hence the equilibrium is in region 1. If $z+A < q^*$ and $z > q^*/2$, then total money balance is enough to afford q^*, but C-type's assets are not enough to purchase the desired amount of money from N-type, hence equilibrium is in region 2. If $z + A < q^*$ and $z < q^*/2$, assets constraint binds and money is scarce, so $q_2 < q^*$, hence equilibrium is in region 3.

Next, we describe these regions in terms of i instead of z. Given the definitition of $\tilde{i}$, i.e. $\tilde{i} \equiv \Big\{ i : (u')^{-1}\Big[1 + i/(\ell - f)\Big] = q^*/2 \Big\}$, we can describe the regions in terms of A and i with Figure 3.9.

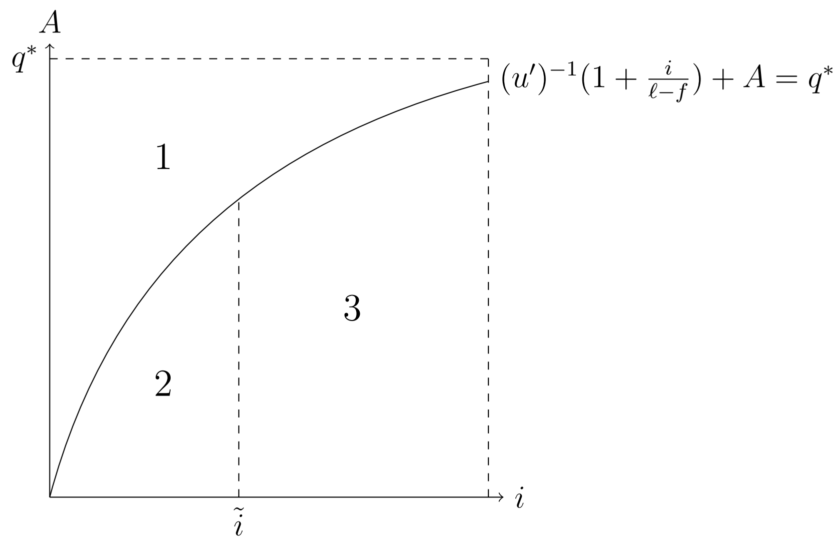


FIGURE 3.9. Aggregate Regions

The border that divides region 1 and region 2 is characterized by $z + A \to q^*$. Since z solves $i = (\ell - f)[u'(z) - 1]$, the boundary can also be expressed as $(u')^{-1}\Big[1 + i/(\ell - f)\Big] + A = q^*$, which has a positive slope. The intuition is straightforward. Increasing in interest rate i depresses real money balance z, hence it requires a higher A to consume optimal q^*. When $i = 0$, the required amount of asset to consume q^* is $A = 0$. Because at Friedman Rule, it is costless to carry money, hence agents always hold enough money to consume q^* and does not rely on asset for DM consumption. When $A = q^*$, regardless of the value of i, total balance is always enough to consume q^*, i.e. $z + A = q^*$.

Besides the boundary between region 1 and region 2, we now discuss the boundary between region 2 and region 3, which is defined by the critical point where $z = (u')^{-1}\Big[1 + i/(\ell - f)\Big] \to q^*/2$. This boundary is a perpendicular line since A does not show up here, and the corresponding i that defines such boundary is $\tilde{i}$ as defined in previous discussion.

DEFINITION 3.4. *A symmetric steady state equilibrium is a list of* $\{z, q_1, q_2, \psi\}$ *with* $q_1 = z$. *And*

- *If* $A + (u')^{-1}(1 + \frac{i}{\ell - f}) \geq q^*$, *then* $q_2 = q^*$, $\psi = \beta$, z *solves* $i = (\ell - f)[u'(z) - 1]$

- *If* $A + (u')^{-1}(1 + \frac{i}{\ell - f}) < q^*$ *and* $i \leq \tilde{i}$*, then* $q_2 = z + A$*,* $\psi = \beta\{1 + f[u'(z + A) - 1]\}$*,* z *solves* $i = f[u'(z + A) - 1] + (\ell - f)[u'(z) - 1]$
- *If* $A + (u')^{-1}(1 + \frac{i}{\ell - f}) < q^*$ *and* $i > \tilde{i}$*, then* $q_2 = 2z$*,* $\psi = \beta$*,* z *solves* $i = f[u'(2z) - 1] + (\ell - f)[u'(z) - 1]$

Given the definition of the three regions, we again focus on the quadratic utility function. With quadratic utility, $z = \gamma - i/(\ell - f)$, and the condition $A + z \geq q^* = \gamma$ is satisfied if $A \geq i/(\ell - f)$. Figure 3.10 shows the aggregate regions with quadratic utility function, and the equilibrium is defined as follows.

DEFINITION 3.5. *A symmetric steady state equilibrium is a list of* $\{z, q_1, q_2, \psi\}$ *with* $q_1 = z$*. And*

- *If* $A \geq \frac{i}{\ell - f}$*, then* $q_2 = \gamma$*,* $\psi = \beta$*,* $z = \gamma - \frac{i}{\ell - f}$*.*
- *If* $A < \frac{i}{\ell - f}$ *and* $i \leq \tilde{i} \equiv \frac{\gamma(\ell - f)}{2}$*, then* $z = i - \frac{i + fA}{\ell}$*,* $q_2 = \gamma - \frac{i}{\ell} + A(\frac{\ell - f}{\ell})$*,* $\psi = \beta[1 + \frac{f}{\ell}(i - (\ell - f)A)]$*.*
- *If* $A < \frac{i}{\ell - f}$ *and* $i > \frac{\gamma(\ell - f)}{2}$*, then* $z = \frac{\ell\gamma - i}{f + \ell}$*,* $q_2 = 2z$*,* $\psi = \beta$*.*

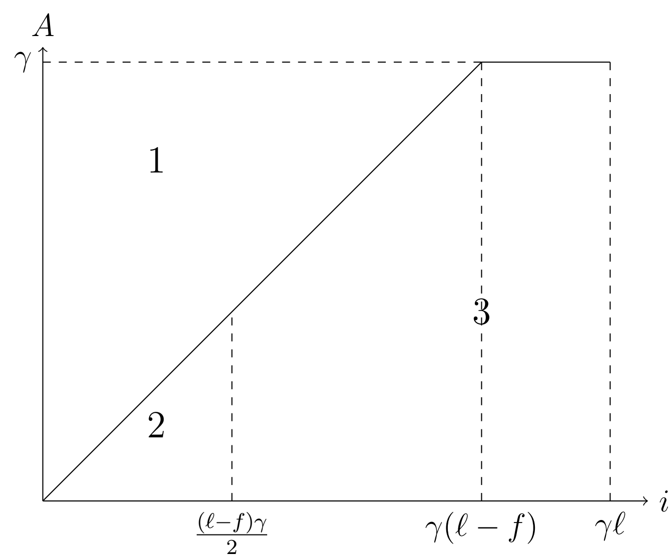


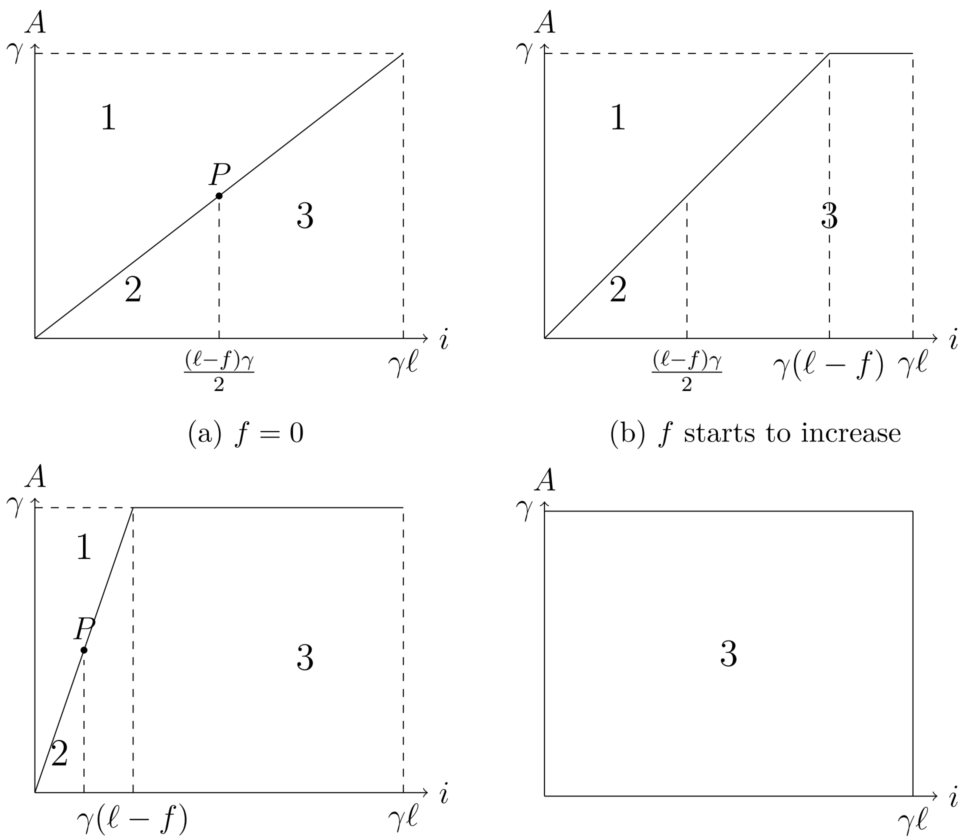
FIGURE 3.10. Aggregate Regions with Quadratic Utility

3.4.2.2. *Welfare Analysis.* After specifying the aggregate regions, now we proceed to analyze welfare. Same as in the previous section, the impact of f on welfare is given by

$$\frac{\partial \mathcal{W}}{\partial f} = u(q_2) - q_2 - \Big[u(z) - z\Big] + \Big\{ f\Big[u'(q_2) - 1\Big]\frac{dq_2}{dz} + (\ell - f)\Big[u'(z) - 1\Big]\Big\}\frac{dz}{df}$$

The analysis we have done so far is with respect to i and A, taking other parameters as given. But since now we are interested in seeing how change in f affects welfare, we need to figure out how the different regions are affected by the change in f.

Before detailed discussion, we make a couple of clarifying notes. z decreases in f for all regions, hence it is more likely that the economy is in "scarce" equilibrium as f increases. The idea is that, for any given A and i, an increase in f will move the regions to the left (i.e. any point now is more likely to be scarce). This is illustrated by Figure 3.11. As f increases, we get the figure on the right. And depending on the value of (A, i) in panel (a), an increase in f can typically get the economy into a different region(s).



(a) $f = 0$

(b) f starts to increase

(c) $f \approx \ell$

(d) $f = \ell$

FIGURE 3.11. Regions division as f keeps increasing

Notice that the point P in Figure 3.11 panel (a) corresponds to the asset level A when interest $i = (\ell - f)/2$, and hence $A = \gamma/2$. It is clear that this value depends on γ only but not on f. Now, we describe each region with parameters A and i, and analyze the welfare.

PROPOSITION 3.4. *The equilibrium welfare depends on the value of A and i. We have four cases:*

- *Case 1: If $A > \frac{\gamma}{2}$ and $A > \frac{i}{l}$, then there exists a unique cutoff level of matching probability $f_{13} = l - \frac{2i}{\gamma}$ such that if $f \in [0, f_{13})$, the equilibrium is in Region 1; if $f \in [f_{13}, l]$, equilibrium is in Region 3. Furthermore,*
  - *If $i > \frac{\gamma l}{3}$, then $\partial \frac{\mathcal{W}}{\partial f} < 0$ for all $f \in [0, f_{13})$; $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in [f_{13}, l]$.*

- If $i < \frac{\gamma l}{3}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, \frac{l}{3})$; $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in [\frac{l}{3}, l]$.

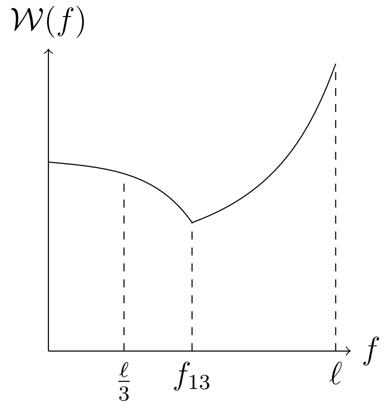
- *Case 2: If $A < \frac{\gamma}{2}$, $A > \frac{i}{\gamma}$, and $i < \frac{\gamma l}{2}$, then there exist cutoff levels of matching probability $f_{12} = l - \frac{i}{A}$ and $f_{23} = \frac{\gamma l - i}{A} - l$ such that if $f \in [0, f_{12})$, equilibrium is in Region 1; if $f \in [f_{12}, f_{23})$, equilibrium is in Region 2; if $f \in [f_{23}.l]$, equilibrium is in Region 3. Furthermore,*
  - If $\frac{l}{2} < f_{12}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, f_{12}]$; and $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in (f_{12}, l]$.
  - If $\frac{l}{2} \in (f_{12}, f_{23})$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, \frac{l}{2}]$; $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in (\frac{l}{2}, l]$.
  - If $f_{12} < \frac{l}{3} < f_{23} < \frac{l}{2}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, f23]$; $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in (f_{23}, l]$.
  - If $f_{12} < f_{23} < \frac{l}{3} < \frac{l}{2}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, \frac{l}{3}$; $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in (\frac{l}{3}, l]$.

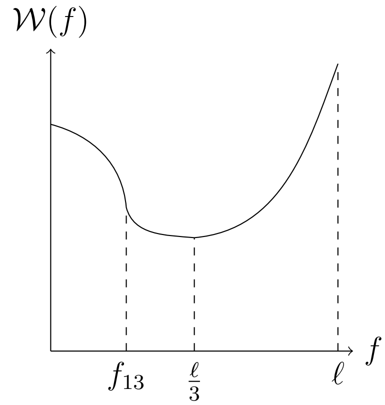
- *Case 3: If $A < \frac{i}{l}$ and $i < \frac{\gamma l}{2}$, there exists a cutoff level of matching probability $f_{23} = \frac{\gamma l - i}{A} - l$ such that if $f \in [0, f_{23})$, equilibrium is in Region 2; if $f \in [f_{23}, l]$, equilibrium is in Region 3. Furthermore,*
  - If $\frac{l}{2} < f_{23}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, \frac{l}{2}]$; and $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in (\frac{l}{2}, l]$.
  - If $\frac{l}{3} < f_{23} < \frac{l}{2}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, f_{23})$; and $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in [f_{23}, l]$.
  - If $f_{23} < \frac{l}{3}$, then $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, \frac{l}{3})$; and $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in [\frac{l}{3}, l]$.

- *Case 4: If $A \leq \frac{i}{l}$ and $i \geq \frac{\gamma l}{2}$, then equilibrium is always in Region 3. Furthermore, $\frac{\partial \mathcal{W}}{\partial f} < 0$ for all $f \in [0, \frac{l}{3})$, and $\frac{\partial \mathcal{W}}{\partial f} > 0$ for all $f \in [\frac{l}{3}, l]$.*

PROOF. See the appendix A.3.5. □

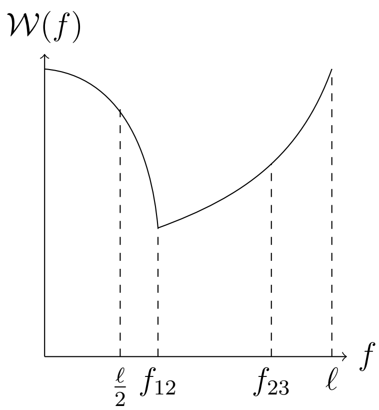
Welfare as a function of OTC matching probability in the four different regions are illustrated by Figure 3.12, 3.13, 3.14, and 3.15.



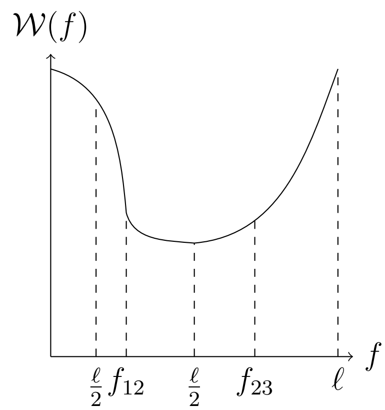


(a) Case 1.1: $\frac{\ell}{3} \leq f_{13}$ (b) Case 1.2: $\frac{\ell}{3} > f_{13}$

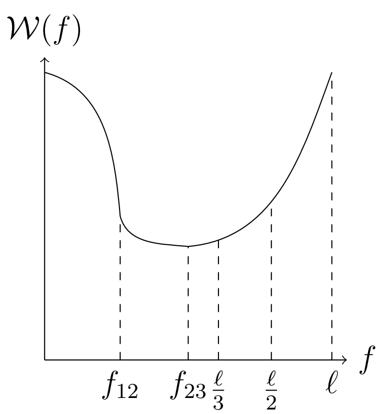
FIGURE 3.12. Case 1 Summary



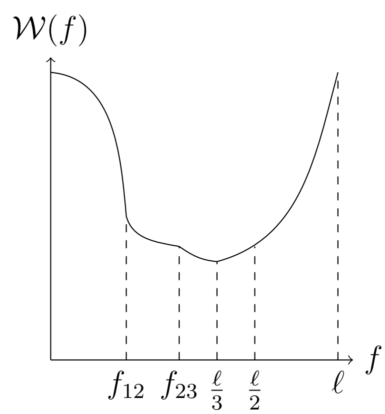
(a) Case 2.1



(b) Case 2.2

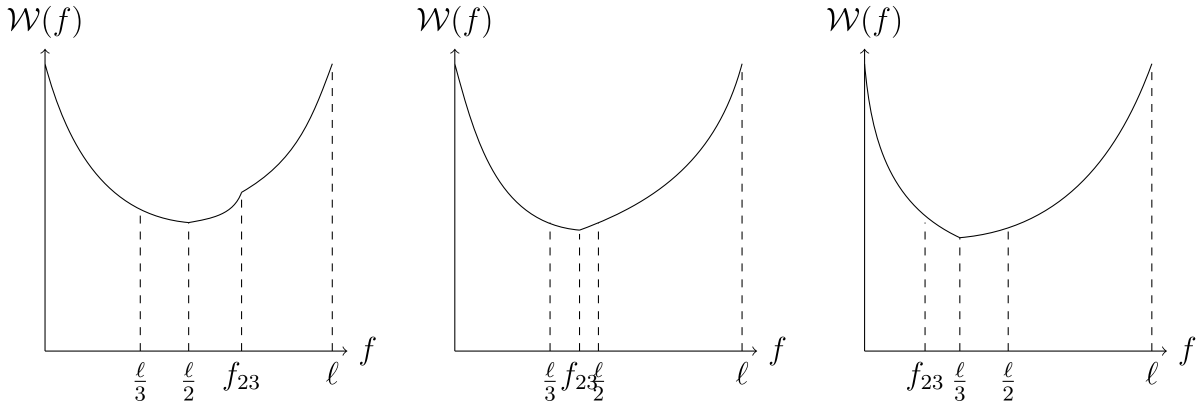


(c) Case 2.3



(d) Case 2.4

FIGURE 3.13. Case 2 Summary



(a) Case 3.1 (b) Case 3.2 (c) Case 3.3

FIGURE 3.14. Case 3 Summary

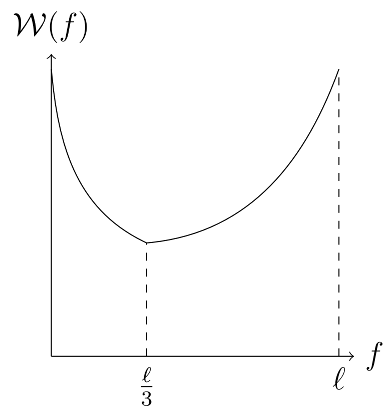


FIGURE 3.15. Case 4 Summary

3.5. Conclusion

In this paper, we build a search-theoretic model to show that the introduction of alternative means of payment (to money) is not always welfare improving as many have believed. In an economy where every agent accept these alternative payment, i.e., credit, financial asset, and secondary market, welfare is indeed maximized because the economy has no friction to begin with. However, if the acceptability of these payment is low to start with, the improved acceptability can actually hurt the economy's welfare. More credit *ex post* is good for the welfare because it facilitates more transactions. However, *ex ante*, more credit discourage agents from carrying money, which hurts welfare in transactions that take place in a *quid pro quo* manner. This result offers a theoretical explanation to a recent empirical literature which suggest that increased access to credit is often followed by economic hardship.

APPENDIX A

A.1. Proofs of Chapter One

A.1.1. Proof of Lemma 1.2.

Proof. For the secondary market bargaining between an A-buyer and an I-buyer, the Lagrangian function becomes:

$$
\begin{aligned}
\mathcal{L} =& u\Big(\varphi(m+y_m)+d(a_1-y_1)\Big)-u(\varphi m+da_1)-\psi_1 y_1-(\psi_2+d)y_2 \\
&+\tau_B\Big[(\psi_1+d)y_1+(\psi_2+d)y_2-\varphi y_m\Big]+\tau_m(\tilde{m}-y_m)+\lambda_m y_m \\
&+\tau_1(a_1-y_1)+\lambda_1 y_1+\tau_2(a_2-y_2)+\lambda_2 y_2
\end{aligned}
$$

where τ_b is the Lagrangian multiplier on the bargaining constraint, τ_m, τ_1, τ_2 are the Lagrangian multipliers on the feasibility constraints, and λ_m, λ_1, λ_2 are the Lagrangian multipliers on the non-negativity constraints. The corresponding first order conditions are given by

$$u'\Big(\varphi(m+y_m)+d(a_1-y_1)\Big)\varphi-\tau_B\varphi-\tau_m+\lambda_m=0 \tag{aA.1}$$

$$-u'\Big(\varphi(m+y_m)+d(a_1-y_1)\Big)d-\psi_1+\tau_B(\psi_1+d)-\tau_1+\lambda_1=0 \tag{aA.2}$$

$$-(\psi_2+d)+\tau_B(\psi_2+d)-\tau_2+\lambda_2=0 \tag{aA.3}$$

$$\varphi y_m = (\psi_1 + d)y_1 + (\psi_2 + d)y_2 \tag{aA.4}$$

I describe the cases that yield the bargaining solutions, and omitted the cases which reach a contradiction.

Case 1: $\tau_m = 0, \lambda_m = 0; \tau_1 = 0, \lambda_1 = 0; \tau_2 = 0, \lambda_2 = 0 \Rightarrow 0 < y_m < \tilde{m}, 0 < y_1 < a_1, 0 < y_2 < a_2$.

Equation (aA.3) implies that $\tau_B = 1$, which further implies from equation (aA.1) and (aA.2) that $\varphi(m+y_m)+d(a_1-y_1) = q^*$. Equation (aA.4) implies that $\varphi y_m = (d+\psi_1)a_1+(d+\psi_2)a_2$. Hence with these two conditions, it is implied that the $\varphi y_m - dy_1 = q^* - \varphi m - da_1 \leq \psi_1 a_1 + (d+\psi_2)a_2$. And also, $\varphi \tilde{m} \geq \varphi y_m = q^* - \varphi m - da_1 + \frac{d}{d+\psi_1}[q^* - \varphi m - da_1 - (d+\psi_2)a_2]$ as the liquidity constraint.

Case 2: $\tau_m = 0, \lambda_m = 0; \tau_1 > 0, \lambda_1 = 0; \tau_2 > 0, \lambda_2 = 0 \Rightarrow 0 < y_m < \tilde{m}, y_1 = a_1, y_2 = a_2$.

Equation (aA.4) implies that $\varphi \tilde{m} > \varphi \bar{y}_m = (d + \psi_1)a_1 + (d + \psi_2)a_2$. Equation (aA.3) implies that $\tau_B > 1$, which further implies from equation (aA.1) and (aA.2) that $\varphi(m+\bar{y}_m) < q^*$.

Case 3: $\tau_m > 0,\ \lambda_m = 0;\ \tau_1 > 0, \lambda_1 = 0;\ \tau_2 > 0, \lambda_2 = 0 \Rightarrow y_m = \tilde{m}, y_1 = a_1, y_2 = a_2$.

In this case, equation (aA.3) implies that $\tau_2 = (\tau_B - 1)(d+\psi_2) > 0$, or $\tau_B > 1$. And using this in equation (aA.1) shows that $u'\Big(\varphi(m + \tilde{m})\Big) - \tau_B = \frac{\tau_m}{\varphi} > 0$, hence $u'\Big(\varphi(m + \tilde{m})\Big) > \tau_B > 1$. By plugging $\tau_B > 1$ into (aA.2), there exists positive τ_1. The corresponding total liquidity then must satisfy $\varphi(m + \tilde{m}) < q^*$. The corresponding asset constraint is given by equation (aA.4): $\varphi m = (\psi_1 + d)a_1 + (\psi_2 + d)a_2$.

Case 4: $\tau_m > 0, \lambda = 0; \tau_1 = 0, \lambda_1 = 0; \tau_2 > 0, \lambda_2 = 0 \Rightarrow y_m = \tilde{m}, 0 < y_1 < a_1, y_2 = a_2$.

Equation (aA.4) implies that $(d+\psi_1)y_1^* = \varphi\tilde{m}-(d+\psi_2)a_2$. And similar to case 1, equation (aA.3) implies $\tau_B > 1$, which satisfies equation (aA.2) as well. This further implies that

$\varphi(m+\tilde{m}+d(a_1-y_1^*)) < q^*$. Hence for this case, the asset constraint satisfies $(d+\psi_1)a_1+(d+\psi_2)a_2 > \varphi\tilde{m}$, and the liquidity constraint satisfies $\varphi\tilde{m} < q^*-\varphi m-da_1+\frac{d}{d+\psi_1}[\varphi\tilde{m}-(d+\psi_2)a_2]$.

Case 5: $\tau_m > 0, \lambda_m = 0; \tau_1 = 0, \lambda_1 > 0; \tau_2 = 0, \lambda_2 = 0 \Rightarrow y_m = \tilde{m}, y_1 = 0, 0 < y_2 < a_2$.

Equation (aA.4) implies that $\varphi\tilde{m} = (d+\psi_2)y_2 < (d+\psi_2)a_2$ as the asset constraint. Equation (aA.3) shows that $\tau_B = 1$, which further implies from equation (aA.1) that $\varphi(m+\tilde{m}+da_1) < q^*$ as the liquidity constraint, which is also confirmed from equation (aA.2).

Case 6: $\tau_m > 0, \lambda_m = 0; \tau_1 = 0, \lambda_1 > 0; \tau_2 > 0, \lambda_2 = 0 \Rightarrow y_m = \tilde{m}, y_1 = 0, y_2 = a_2$.

Equation (aA.4) implies that $\varphi\tilde{m} = (d+\psi_2)a_2$ as the asset constraint. Equation (aA.3) shows that $\tau_B > 1$, which further implies from equation (aA.1) that $\varphi(m+\tilde{m}+da_1) < q^*$ as the liquidity constraint, which is also confirmed from equation (a.2).

The other combinations of Lagrangian multipliers yield contradicting results among equation (aA.1)-(aA.4), and hence cannot be the solution to the bargaining problem. Case 1 corresponds to the bargaining outcome for region 1, while case 2 is the bargaining solution for region 2. Case 3 & 4 jointly determine the bargaining solution for region 3, and case 5 & 6 jointly determine the bargaining solution for region 4. □

A.1.2. Proof of Lemma 1.3. Since the bargaining solution depends on which region the economy is in, hence different sets of bargaining solution will yield different objective functions for each region.

Region 1: The bargaining solution (y_m^*, y_1^*, y_2^*) satisfy the asset condition derived in Lemma 2, $(d+\hat{\psi}_1)y_1^* + (d+\hat{\psi}_2)y_2 = q^* - \hat{\varphi}\hat{m} - d\hat{a}_1$. Hence the objective function for region 1 adopts the following form

$$\beta^{-1}J^1(\hat{m},\hat{a_1},\hat{a_2}) = -\frac{\varphi}{\beta}\hat{m} - \frac{\psi_1}{\beta}\hat{a}_1 - \frac{\psi_2}{\beta}\hat{a}_2 + \gamma f u(q^*) - \gamma f(q^* - \hat{\varphi}\hat{m} - d\hat{a}_1)$$
$$+(\ell-\gamma f)u\Big(\hat{\varphi}\hat{m} + d\hat{a}_1\Big) - \ell(\hat{\varphi}\hat{m} + d\hat{a}_1) + \Big[\hat{\varphi}\hat{m} + (d+\hat{\psi}_1)\hat{a}_1 + (d+\hat{\psi}_2)a\hat{a}_2\Big]$$

The superscript 1 denotes it is the objective function in region 1, and $\in (1,2,3,4)$ for the 4 regions. The FOCs with respect to the three variables are given as following, with the subscript denotes which variable the derivative is taken with respect to, e.g. $j = 1$ means FOC with respect to the first variable $\hat{m}$.

$$\{\hat{m}\} : \beta^{-1} J^1_1(\hat{m}, \hat{a}_1, \hat{a}_2) = -\frac{\varphi}{\beta} + \hat{\varphi}\Big\{1 + (\ell - \gamma f)\Big[u'\Big(\hat{\varphi}\hat{m} + d\hat{a}_1\Big) - 1\Big]\Big\}$$

$$\{\hat{a}_1\} : \beta^{-1} J^1_2(\hat{m}, \hat{a}_1, \hat{a}_2) = -\frac{\psi_1}{\beta} + d\Big\{1 + (\ell - \gamma f)\Big[u'\Big(\hat{\varphi}\hat{m} + d\hat{a}_1\Big) - 1\Big]\Big\} + \hat{\psi}_1$$

$$\{\hat{a}_2\} : \beta^{-1} J^1_3(\hat{m}, \hat{a}_1, \hat{a}_2) = -\frac{\psi_2}{\beta} + (d + \hat{\psi}_2)$$

By setting the FOCs = 0, the results imply the following pricing functions.

$$\frac{1+\mu}{\beta} = 1 + (\ell - \gamma f)\Big[u'(\hat{\varphi}\hat{m} + d\hat{a}_1) - 1\Big]$$

$$\frac{\psi_1}{\beta} = d\Big\{1 + (\ell - \gamma f)\Big[u'(\hat{\varphi}\hat{m} + d\hat{a}_1)\Big]\Big\} + \hat{\psi}_1 \Rightarrow \psi_1 = \frac{1+\mu}{1-\beta} d$$

$$\frac{\psi_2}{\beta} = d + \hat{\psi}_2 \Rightarrow \psi_2 = \frac{\beta d}{1-\beta}$$

Region 2: The bargaining solution in region 2 is $y^*_1 = a_1, y^*_2 = a_2, y^*_m = (d + \hat{\psi}_1)\hat{a}_1 + (d + \hat{\psi}_2)\hat{a}_2$. Hence the objective function adopts the form

$$\beta^{-1} J^2(\hat{m}, \hat{a_1}, \hat{a_2}) = -\frac{\varphi}{\beta}\hat{m} - \frac{\psi_1}{\beta}\hat{a_1} - \frac{\psi_2}{\beta}\hat{a_2} + \gamma f u\Big[\hat{\varphi}\hat{m} + (d + \hat{\psi}_1)\hat{a}_1 + (d + \hat{\psi}_2)\hat{a}_2\Big]$$
$$- \gamma f[\hat{\psi}_1\hat{a}_1 + (\hat{\psi}_2 + d)\hat{a}_2] + (\ell - \gamma f)u\Big(\hat{\varphi}\hat{m} + d\hat{a}_1\Big) - \ell(\hat{\varphi}\hat{m} + d\hat{a}_1)$$
$$+ \Big[\hat{\varphi}\hat{m} + (d + \hat{\psi}_1)\hat{a}_1 + (d + \hat{\psi}_2)\hat{a}_2\Big]$$

$$\{\hat{m}\}:\beta^{-1}J^2_1(\hat{m},\hat{a}_1,\hat{a}_2)=-\frac{\varphi}{\beta}+\hat{\varphi}\Big\{1+(\ell-\gamma f)\Big[u'\Big(\hat{\varphi}\hat{m}+d\hat{a}_1\Big)-1\Big]$$

$$+\gamma f u'\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]\Big\}$$

$$\{\hat{a}_1\}:\beta^{-1}J^2_2(\hat{m},\hat{a}_1,\hat{a}_2)=-\frac{\psi_1}{\beta}+d\Big\{1+(\ell-\gamma f)\Big[u'\Big(\hat{\varphi}\hat{m}+d\hat{a}_1\Big)-1\Big]$$

$$+\gamma f u'\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]\Big\}$$

$$+\hat{\psi}_1\Big\{1+\gamma f\Big[u'\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]-1\Big]\Big\}$$

$$\{\hat{a}_2\}:\beta^{-1}J^2_3(\hat{m},\hat{a}_1,\hat{a}_2)=-\frac{\psi_2}{\beta}+(d+\hat{\psi}_2)\Big\{1+\gamma f\Big[u'\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]-1\Big]\Big\}$$

By setting the FOCs = 0, the results imply that pricing functions in region 2 are

$$\frac{1+\mu}{\beta}=1+(\ell-\gamma f)\Big[u'\Big(\hat{\varphi}\hat{m}+d\hat{a}_1\Big)-1\Big]+\gamma f\Big[u'\Big(\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big)-1\Big]$$

$$\frac{\psi_1}{\beta}=d\frac{1+\mu}{\beta}+\hat{\psi}_1\Big\{1+\gamma f\Big[u'\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]-1\Big]\Big\}$$

$$\frac{\psi_2}{\beta}=(d+\hat{\psi}_2)\Big\{1+\gamma f\Big[u'\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]-1\Big]\Big\}$$

Region 3: The bargaining solution in this region is $y^*_m=\tilde{m}, y^*_1=0, y^*_2=\frac{\hat{\varphi}\hat{m}}{\hat{\psi}_2+d}$. Hence the objective function is

$$\beta^{-1}J^3(\hat{m},\hat{a}_1,\hat{a}_2)=-\frac{\varphi}{\beta}\hat{m}-\frac{\psi_1}{\beta}\hat{a}_1-\frac{\psi_2}{\beta}\hat{a}_2+\gamma f u\Big[\hat{\varphi}(\hat{m}+\tilde{m})+d\hat{a}_1\Big]-\gamma f\hat{\varphi}\tilde{m}$$

$$+(\ell-\gamma f)u\Big(\hat{\varphi}\hat{m}+d\hat{a}_1\Big)-\ell(\hat{\varphi}\hat{m}+d\hat{a}_1)$$

$$+\Big[\hat{\varphi}\hat{m}+(d+\hat{\psi}_1)\hat{a}_1+(d+\hat{\psi}_2)\hat{a}_2\Big]$$

$$\{\hat{m}\}: \beta^{-1}J_1^3(\hat{m},\hat{a}_1,\hat{a}_2) = -\frac{\varphi}{\beta} + \hat{\varphi}\Big\{1 + (\ell - \gamma f)u'(\hat{\varphi}\hat{m} + d\hat{a}_1) + \gamma f\Big[u'\Big(\hat{\varphi}(\hat{m}+\tilde{m}) + d\hat{a}_1\Big) - 1\Big]\Big\}$$

$$\{\hat{a}_1\}: \beta^{-1}J_2^3(\hat{m},\hat{a}_1,\hat{a}_2) = -\frac{\psi_1}{\beta} + d\Big\{1 + (\ell - \gamma f)u'(\hat{\varphi}\hat{m} + d\hat{a}_1) + \gamma f\Big[u'\Big(\hat{\varphi}(\hat{m}+\tilde{m}) + d\hat{a}_1\Big) - 1\Big]\Big\} + \hat{\psi}_1$$

$$\{\hat{a}_2\}: \beta^{-1}J_3^3(\hat{m},\hat{a}_1,\hat{a}_2) = -\frac{\psi_2}{\beta} + (d + \hat{\psi}_2)$$

By setting FOCs $= 0$, the pricing functions in region 3 are

$$\begin{aligned}
\frac{1+\mu}{\beta} =& 1 + (\ell - \gamma f)\Big[u'\Big(\hat{\varphi}\hat{m} + d\hat{a}_1\Big) - 1\Big] + \gamma f u'\Big[\hat{\varphi}(\hat{m}+\tilde{m}) + d\hat{a}_1) - 1\Big] \\
\frac{\psi_1}{\beta} =& d\Big\{1 + (\ell - \gamma f)u'(\hat{\varphi}\hat{m} + d\hat{a}_1) + \gamma f\Big[u'\Big(\hat{\varphi}(\hat{m}+\tilde{m}) + d\hat{a}_1\Big) - 1\Big]\Big\} + \hat{\psi}_1 \Rightarrow \psi_1 = \frac{1+\mu}{1-\beta}d \\
\frac{\psi_2}{\beta} =& d + \hat{\psi}_2 \Rightarrow \psi_2 = \frac{\beta}{1-\beta}d
\end{aligned}$$

Region 4: The bargaining solution in region 4 is $y_m^* = \tilde{m}, y_1^* = \frac{\hat{\varphi}\tilde{m} - (d+\hat{\psi}_2)a_2}{d+\hat{\psi}_2}, y_2^* = a_2$. To save on math expression, I define $q_2 = \hat{\varphi}(\hat{m}+\tilde{m}) + d\hat{a}_1 - \frac{d}{d+\hat{\psi}_1}\hat{\varphi}\tilde{m} + \frac{d}{d+\hat{\psi}_1}(d+\hat{\psi}_2)\hat{a}_2$ as the DM consumption quantity when A-buyers matched with an I-buyer and was able to boost their liquidity position. So the objective function in region 4 adopts the following form

$$\begin{aligned}
\beta^{-1}J^4(\hat{m},\hat{a}_1,\hat{a}_2) =& -\frac{\varphi}{\beta}\hat{m} - \frac{\psi_1}{\beta}\hat{a}_1 - \frac{\psi_2}{\beta}\hat{a}_2 \\
&+ \gamma f u(q_2) - \gamma f\Big[\frac{\hat{\psi}_1}{d+\hat{\psi}_1}\hat{\varphi}\tilde{m} + \frac{d}{d+\hat{\psi}_1}(d+\hat{\psi}_2)\hat{a}_2\Big] \\
&+ (\ell - \gamma f)u\Big(\hat{\varphi}\hat{m} + d\hat{a}_1\Big) - \ell(\hat{\varphi}\hat{m} + d\hat{a}_1) \\
&+ \Big[\hat{\varphi}\hat{m} + (d+\hat{\psi}_1)\hat{a}_1 + (d+\hat{\psi}_2)\hat{a}_2\Big]
\end{aligned}$$

$$\{\hat{m}\}: \beta^{-1}J_1^4(\hat{m},\hat{a}_1,\hat{a}_2) = -\frac{\varphi}{\beta} + \hat{\varphi}\Big\{1 + (\ell-\gamma f)u'(\hat{\varphi}\hat{m}+d\hat{a}_1) + \gamma f\Big[u'(q_2)-1\Big]\Big\}$$

$$\{\hat{a}_1\}: \beta^{-1}J_2^4(\hat{m},\hat{a}_1,\hat{a}_2) = -\frac{\psi_1}{\beta} + d\Big\{1 + (\ell-\gamma f)u'(\hat{\varphi}\hat{m}+d\hat{a}_1) + \gamma f\Big[u'(q_2)-1\Big]\Big\} + \hat{\psi}_1$$

$$\{\hat{a}_2\}: \beta^{-1}J_3^4(\hat{m},\hat{a}_1,\hat{a}_2) = -\frac{\psi_2}{\beta} + \gamma f\frac{d+\hat{\psi}_2}{d+\hat{\psi}_1}[u'(q_2)-1]d + \hat{\psi}_2$$

By setting FOCs = 0, the pricing functions in region 4 are

$$\begin{aligned}
\frac{1+\mu}{\beta} =& 1 + (\ell-\gamma f)\Big[u'\Big(\hat{\varphi}\hat{m}+d\hat{a}_1\Big)-1\Big] + \gamma f\Big[u'(q_2)-1\Big] \\
\frac{\psi_1}{\beta} =& d\Big\{1 + (\ell-\gamma f)\Big[u'\Big(\hat{\varphi}\hat{m}+d\hat{a}_1\Big)-1\Big] + \gamma f\Big[u'(q_2)-1\Big]\Big\} + \hat{\psi}_1 \Rightarrow \psi_1 = \frac{1+\mu}{1-\beta}d \\
\frac{\psi_2}{\beta} =& (d+\hat{\psi}_2)\Big\{1 + \gamma f\frac{d}{d+\hat{\psi}_1}[u'(q_2)-1]\Big\}
\end{aligned}$$

A.2. Proofs of Chapter Two

A.2.1. Proof of Lemma 2.1.

PROOF. For a dividend firm facing objective function $\psi_1 + (1-\theta)[u(q_1)-q_1]$, notice that $(1-\theta)[u(q_1)-q_1]$ increases in q_1 before reaching $q_1 = q^*$. The bargaining solution between an A-buyer with a type-1 firm is $q_1 = \min\{\psi_1+k-d, \nu^{-1}(da_1)\}$ derived in the main text. So given the expectation of A-buyers' liquidity position, if firms pay out too much dividend, i.e. $\psi_1+k-d < \nu^{-1}(da_1)$, then total value of the firm is $\psi_1+(1-\theta)[u(\psi_1+k-d)-(\psi_1+k-d)]$, which is less than the firm's value $\psi_1 + (1-\theta)[u(\nu^{-1}(\tilde{d}\tilde{a}_1)) - (\tilde{d}\tilde{a}_1]$ when paying slightly less dividend so that the input capital is enough to produce the demanded quantity. Since endowed capital k alone is enough to produce q^*, so as long as d is not too high, firms are always able to satisfy A-buyer's DM demand. Hence the optimal dividend policy is to reserve enough input capital, $\nu^{-1}(\tilde{d}\tilde{a}_1)$, to meet A-buyers' DM demand, and distribute the residual capital $\psi_1 + k - \nu^{-1}(\tilde{d}\tilde{a}_1)$ as dividend.

For a non-dividend R&D firm facing objective $[A(e)(\psi_2+k-e)-q_2]+(1-\theta)[u(q_2)-q_2]$, first notice that with the properties of $A(e)$, $A(e)(\psi_2+k-e) \geq \psi_2+k > q^*$ regardless of the value of e. Hence q_2 is determined by A-buyer's liquidity position that $q_2 = \nu^{-1}(\tilde{d}\tilde{a}_1)$. So now the objective is to maximize $K = A(e)(\psi_2+k-e)$. When $e \to 0$, $\frac{\partial K}{\partial e} \to \infty$; when $e \to \psi_2 + k$, $\frac{\partial K}{\partial e} \to -\infty$. And $\frac{\partial^2 K}{\partial e^2} < 0$, so there exists a unique and interior solution of e^* such that $\frac{\partial K}{\partial e} = 0$, and firm's value is maximized. □

A.3. Proofs of Chapter Three

A.3.1. Proof of Proposition 3.2.

PROOF. **Case 1:** $A > \gamma/2$ and $i \in (\gamma - A, A)$. We already know that, $\mathcal{W}(0) = (\gamma^2 - i^2)/2 < A(\gamma - A/2) = \mathcal{W}(1)$. Now, for all $\sigma \in [0, \tilde{\sigma}]$ (plentiful equilibrium), we have

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \sigma} &= u(\gamma) - \gamma - \Big[u\Big(\gamma - \frac{i}{1-\sigma}\Big) - (\gamma - \frac{i}{1-\sigma})\Big] + \Big\{(1-\sigma)[u'(\gamma - \frac{i}{1-\sigma}) - 1] + \sigma\Big\}[-\frac{i}{(1-\sigma)^2}] \\ &= -\frac{1}{2}(\frac{i}{1-\sigma})^2\end{aligned}$$

Notice that for all $\sigma \in [0, \tilde{\sigma}]$, $\partial \mathcal{W}/\partial \sigma < 0$, and $\partial^2 \mathcal{W}/\partial \sigma^2 = -i^2/(1-\sigma)^3 < 0$, so the welfare is decreasing and concave in σ.

Next, for all $\sigma \in (\tilde{\sigma}, \bar{\sigma})$ (scarce equilibrium), we have:

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \sigma} &= u(\gamma - i + A(1-\sigma)) - (\gamma - i + A(1-\sigma)) - \Big[u(\gamma - i - \sigma A) - (\gamma - i - \sigma A)\Big](-A) \\ &= [\gamma - i + A(1-\sigma)]\Big[\gamma - \frac{\gamma - i + A(1-\sigma)}{2}\Big] - [\gamma - i - A\sigma]\Big[\gamma - \frac{\gamma - i - A\sigma}{2}\Big] \\ &+ \Big\{(1-\sigma)(i + \sigma A) + \sigma(i + A(1-\sigma))\Big\}(-A) \\ &= (\sigma - \frac{1}{2})A^2\end{aligned}$$

This result could be positive or negative, depending on the value of σ relative to $\frac{1}{2}$. And $\partial^2 \mathcal{W}/\partial \sigma^2 = A^2 > 0$ implies that welfare is convex in this region.

Finally, for all $\sigma \in [\bar{\sigma}, 1]$ (non-monetary equilibrium), we have $\mathcal{W}(\sigma) = \sigma[u(A) - A]$, so that $\mathcal{W}$ is increasing and linear in σ, with $\mathcal{W}(1) > \mathcal{W}(0)$.

To sum up case 1, we have that $\mathcal{W}$ is:

- decreasing and concave in σ for $\sigma \in [0, \tilde{\sigma}]$
- convex in σ for $\sigma \in (\tilde{\sigma}, \bar{\sigma})$, and
- increasing and linear in σ for $\sigma \in [\bar{\sigma}, 1]$

In order to plot it, we still need to figure out the sign of $\partial \mathcal{W}/\partial \sigma$ for $\sigma \in (\tilde{\sigma}, \bar{\sigma})$, which depends on the value of $\bar{\sigma}$ and $\tilde{\sigma}$ relative to the stationary point $\sigma = \frac{1}{2}$, and there are three possibilities:

Case 1.1: $\tilde{\sigma} < 1/2 < \bar{\sigma}$. The corresponding i is in the range $A/2 < i < \gamma - A/2$. Hence if $i \in \Big(\max\{\gamma - A, A/2\}, \min\{\gamma - A/2, A\}\Big)$, and the minimizer of $\mathcal{W}$ is $\sigma = 1/2$.

Case 1.2: $\tilde{\sigma} > \frac{1}{2}$ (and necessarily $\bar{\sigma} > \frac{1}{2}$) The corresponding i must be that $i < A/2$, which can only happen if $A/2 > \gamma - A$ or $A \in (\frac{2}{3}\gamma, \gamma)$ and $i \in (\gamma - A, \frac{A}{2})$. The minimizer of $\mathcal{W}$ when $\frac{1}{2} < \tilde{\sigma} < \bar{\sigma}$ is $\sigma = \tilde{\sigma}$.

Case 1.3: $\bar{\sigma} < \frac{1}{2}$ (and necessarily $\tilde{\sigma} < 1/2$). Hence $i > \gamma - A/2$. This can only happen if $A \in (2\gamma/3, \gamma)$ and $i \in (\gamma - A/2, A)$. The minimizer in this case would be $\sigma = \bar{\sigma}$.

Case 2: $A > \gamma/2$ & $i \leq \gamma - A$, or $A < \gamma/2$ & $i < A$.

- For all $\sigma \in [0, \tilde{\sigma}]$ (plentiful equilibrium), $\partial \mathcal{W}/\partial \sigma = -i^2/2(1-\sigma)^2 < 0$.
- For all $\sigma \in (\tilde{\sigma}, 1]$ (scarce equilibrium), $\partial \mathcal{W}/\partial \sigma = (\sigma - 1/2)A^2$, which is convex but the sign can be positive or negative, depending on the value of σ relative to $1/2$. So we discuss this in two sub-cases.

  Case 2.1: $\tilde{\sigma} < 1/2$. The corresponding i must be that $i > A/2$. With the parameter range for case 2, this means that we can potentially be in the following two cases.

  - $A > \gamma/2$ and $i < \gamma - A$. Hence $A \in (\gamma/2, 2\gamma/3)$ and $i \in (A/2, \gamma - A)$
  - $A < \gamma/2$ and $i < A$. Hence $A < \gamma/2$ and $i \in (A/2, A)$

Case 2.2: $\tilde{\sigma} > 1/2$. The corresponding i must be that $i < A/2$. Again, for parameter range of case 2, we have two cases.

- $A > \gamma/2$ and $i < \gamma - A$. If $A/2 > \gamma - A$ or $A > 2\gamma/3$, then $i < A/2$ is automatically satisfied. If $A \in (\gamma/2, 2\gamma/3)$, then $A/2 < \gamma - A$, and $i < A/2$ becomes meaningful.
- $A < \gamma/2$ and $i < A$. We need $A < \gamma/2$ and $i < A/2$

Case 3: $A > \gamma/2$ & $i \geq A$, or $A < \gamma/2$ & $i > \gamma - A$.

- For all $\sigma \in [0, \bar{\sigma})$ (scarce equilibrium), $\partial \mathcal{W}/\partial\sigma = (\sigma - 1/2)A^2$, which is convex but can be positive or negative, depending on the value of σ relative to 1/2. This will be discussed in the following two sub-cases.
  - Case 3.1: $\bar{\sigma} > 1/2$. Hence $i < \gamma - A/2$. Given the parameter values in case 3, we potentially have the following two cases:
    - $A > \gamma/2$ and $i \geq A$, which means that $A \in (\gamma/2, 2\gamma/3)$ and $i \in [A, \gamma - A/2)$.
    - $A < \gamma/2$ and $i > \gamma - A$. which means that $A < \gamma/2$ and $i \in (\gamma - A, \gamma - A/2)$
  - Case 3.2: when $i \geq \gamma - A/2$. And we potentially have the following two cases:
    - $A > \gamma/2$ and $i \geq A$. If $A > \gamma - A/2$, then $i \geq \gamma - A/2$ is automatically satisfied, which implies that all we need is $A > 2\gamma/3$ and $i \geq A$. If $A \in (\gamma/2, 2\gamma/3)$, then $\gamma - A/2 > A$, and all we need is $A \in (\gamma/2, 2\gamma/3)$ and $i \geq \gamma - A/2$
    - $A < \gamma/2$ and $i > \gamma - A$. Of course, here it is guaranteed that $\gamma - A/2 > \gamma - A$, which means that all we need is $A < \gamma/2$ and $i \geq \gamma - A/2$.
- For all $\sigma \in [\bar{\sigma}, 1]$ (non-monetary equilibrium), $\mathcal{W}(\sigma) = \sigma[u(A)]$, which is linear and increasing in σ.

Case 4: $A \leq \gamma/2$ and $i \in [A, \gamma - A]$. For all $\sigma \in [0,1)$ (scarce equilibrium), $\partial\mathcal{W}/\partial\sigma = (\sigma - 1/2)A^2$, and $\mathcal{W}$ has a unique minimizer at $\sigma = 1/2$. □

A.3.2. Proof of Proposition 3.3.

Proof. **Case 1:** $i \in (0, \frac{\ell\gamma}{2})$, then $\bar{f} \in (0, \ell)$.

- For all $f \in [0, \bar{f})$, which is the plentiful equilibrium,

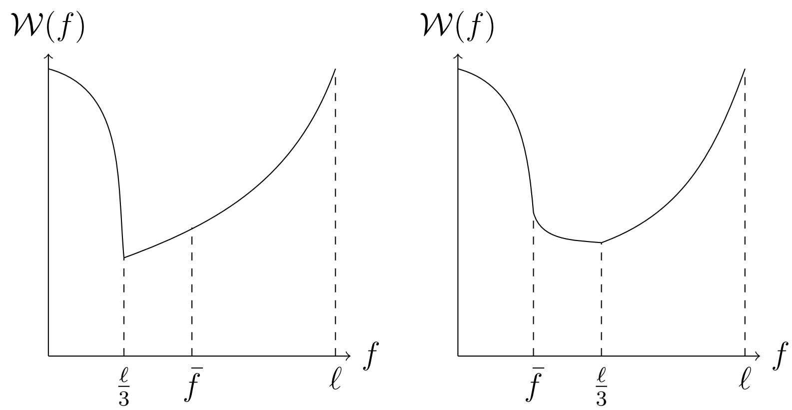
$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial f} =& u(\gamma) - \gamma - [u(\gamma - \frac{i}{\ell - f}) - (\gamma - \frac{i}{\ell - f})] + (\ell - f)[u'(\gamma - \frac{i}{\ell - f}) - 1]\frac{d[\gamma - \frac{i}{\ell - f}]}{dz} \\ =& -\frac{1}{2}(\frac{i}{\ell - f})^2 < 0\end{aligned}$$

  So in this range of matching probability, increasing in f reduces welfare. Also, the function is concave in such parameter range.

- For all $f \in [\bar{f}, \ell]$, which is the scarce equilibrium, we have $z = (\gamma\ell - i)/(\ell + f)$ and $q_2 = 2z$,

$$\frac{\partial\mathcal{W}}{\partial f} = u(2z) - 2z - [u(z) - z] - \{2f(\gamma - 2z) + (\ell - f)(\gamma - z)\}\frac{\gamma\ell - i}{(\ell + f)^2} = \frac{3f - \ell}{\ell + f}\frac{z^2}{2}$$

Welfare is convex in f. But the sign of $\partial\mathcal{W}/\partial f$ depends on $3f - \ell$. If $f > \ell/3$, then $\partial\mathcal{W}/\partial f > 0$. Also recall that, here we are discussing the case where $f \geq \bar{f} \equiv \ell - 2i/\gamma$, and $i < \ell\gamma/2$. Depending on how the value of $\ell/3$ compare with $\bar{f}$, we can have two cases, which are summarized by Figure A.1. If $\ell/3 < \bar{f}$, then $f > \bar{f} > \ell/3$, and welfare increases in f. If $\ell/3 > \bar{f}$, then for all $f \in (\ell/3, \bar{f})$, welfare decreases in f; and for all $f \in [\ell/3, \ell]$, welfare increases with matching probability f.



(a) Case 1.1: $\frac{\ell}{3} < \bar{f}$ (b) Case 1.2: $\frac{\ell}{3} > \bar{f}$

FIGURE A.1. Case 1 Summary

The figure on the left is the scenario when $\bar{f} > \frac{\ell}{3}$ or $i < \ell\gamma/3$. We already assumed that $i < \ell\gamma/2$, but this does not necessarily mean that we automatically have $i < \frac{\ell\gamma}{3}$. So we need to discuss the sign in terms of i in two cases:

Case 1.1: $i < \ell\gamma/3$, hence $\bar{f} > \frac{\ell}{3}$. Then $\partial\mathcal{W}/\partial f > 0\ \forall f \in [\bar{f}, \ell]$, which is illustrated by the figure on the left panel of Figure A.1.

Case 1.2: $i \in (\ell\gamma/3, \ell\gamma/2)$, hence $\bar{f} < \ell/3$. Then $\partial\mathcal{W}/\partial f < 0\ \forall f \in [\bar{f}, \ell/3)$ and $\partial\mathcal{W}/\partial f > 0\ \forall f \in (\ell/3, \ell]$, which corresponds to the figure on the right panel. In this case, $\mathcal{W}$ has a unique minimum at $f = \ell/3$.

Case 2: $i \in (\ell\gamma/2, \ell\gamma)$, which means $\bar{f} \leq 0$, or in other words, all possible fs will satisfy $f > \bar{f}$, and so all equilibirum will be in the scarce case. In "scarce" equilibrium, we have $q_2 = 2z$, and $z = (\ell\gamma - i)/(\ell + f)$. and as already shown, $\partial\mathcal{W}/\partial f = z^2(3f - \ell)/2(\ell + f)$ or $\partial\mathcal{W}/\partial f = (3f - \ell)(\ell\gamma - i)^2/2(\ell + f)^3$. Case 2 is summarized by Figure A.2.

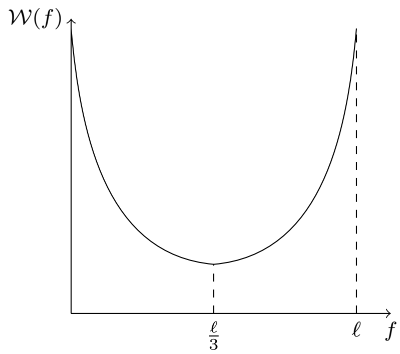


FIGURE A.2. Case 2 Summary

Notice that we know about the convexity, but we never really compare $\mathcal{W}(0)$ and $\mathcal{W}(\ell)$, so let's do a quick comparison. First, at $f = \ell$, we are always at the scarce case, and hence $\mathcal{W}(\ell) = \left[(\ell\gamma)^2 - i^2\right]/2\ell$.

What about $\mathcal{W}(0)$? If we are in the scarce equilibrium (case 2), then $z = (\gamma\ell - i)/\ell$, and $\mathcal{W}(0) = \left[(\ell\gamma)^2 - i^2\right]/2\ell = \mathcal{W}(\ell)$. So in the scarce case, we have $\mathcal{W}(0) = \mathcal{W}(\ell)$. If the equilibrium is plentiful as in case 1, we have $z = \gamma - \frac{i}{\ell - f} = \gamma - \frac{i}{\ell}$, and hence $\mathcal{W}(0) = \ell[u(z) - z] = \ell z(\gamma - z/2) = \left[(\ell\gamma)^2 - i^2\right]/2\ell = \mathcal{W}(\ell)$. So we always have $\mathcal{W}(0) = \mathcal{W}(\ell)$, regardless of whether the economy is in plentiful equilibrium or scarce equilibrium. □

A.3.3. Proof of Lemma 3.6.

PROOF. Case 1: a and $\tilde{m}$ are huge, such that the asset constraints do not bind, i.e. $m + \tilde{m} \geq m^*$ and $a \geq q^* - \varphi m$. Then if we plug in $\chi = \varphi x$, the bargaining problem becomes

$$\max_x u(\varphi(m + x)) - u(\varphi m) - \varphi x$$

and the bargaining solution is $x = m^* - m$ and $\chi = q^* - \varphi m$.

Case 2: $m + \tilde{m} \geq m^*$ (so a C-type would like to have $x = m^* - m$), but $a \leq q^* - \varphi m$. In this case, a C-type woud be willing to give all her assets to N-type, in exchange for the right amount of money. So the bargaining solution is given by $\chi = a$ and $x = \frac{a}{\varphi}$.

Case 3: $a \geq q^* - \varphi\tilde{m}$, but money is limited, i.e. $m + \tilde{m} < m^*$. In this case, C-type wants to obtain all of N-type's money, so $x = \tilde{m}$, and pay just enough asset, $\chi = \varphi\tilde{m}$, in exchange.

Case 4: $m + \tilde{m} < m^*$ (so C-type wants to have all of N-type's money $\tilde{m}$), but C-type does not have enough assets to buy this amount, i.e. $a \leq \varphi\tilde{m}$. In this case, the bargaining solution is given by $\chi = a$ and $x = \frac{a}{\varphi}$, same as in case 2.

Figure A.3 summarizes these four cases.

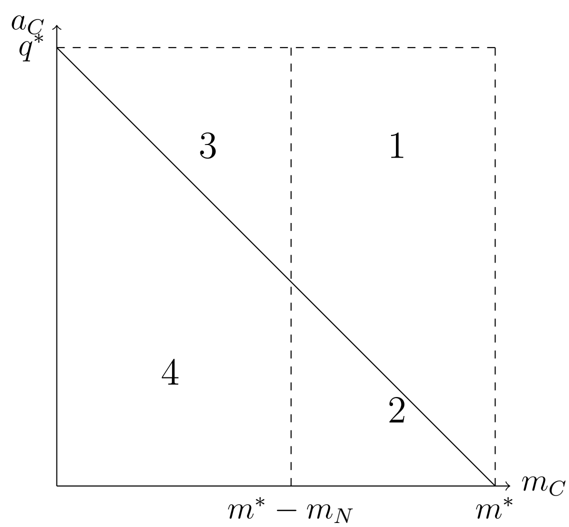


FIGURE A.3. Bargaining Solution

□

A.3.4. Proof of Lemma 3.7.

PROOF. Region 1: Both money and asset are plentiful, thus the objective functions adopts the following form:

$$J^1(\hat{m}, \hat{a}) = -\varphi\hat{m} - \psi\hat{a} + \beta\hat{a} + \beta f[u(q^*) - q^* + \hat{\varphi}\hat{m}] + \beta(\ell - f)u(\hat{\varphi}\hat{m}) + \beta(1 - \ell)\hat{\varphi}\hat{m}$$

And the asset pricing functions are:

$$J_m^1 = 0 \Rightarrow \varphi = \beta\hat{\varphi}(f+1-\ell) + \beta(\ell-f)\hat{\varphi}u'(\hat{\varphi}\hat{m}) = \beta\hat{\varphi}\Big\{1+(\ell-f)\Big[u'(\hat{\varphi}\hat{m}-1)\Big]\Big\}$$

$$J_a^1 = 0 \Rightarrow \psi = \beta$$

Region 2: Total money allows for first-best consumption q^*, but total assets are not enough for C-type to exchange for optimal amount of money.

$$J^2(\hat{m},\hat{a}) = -\varphi\hat{m} - \psi\hat{a} + \beta\hat{a} + \beta f\Big[u(\hat{\varphi}\hat{m}+\hat{a}) - \hat{a}\Big] + \beta(\ell-f)u(\hat{\varphi}\hat{m}) + \beta(1-\ell)\hat{\varphi}\hat{m}$$

Hence asset pricing functions are:

$$J_m^2 = 0 \Rightarrow \varphi = \beta\hat{\varphi}\Big\{1+f\Big[u'(\hat{\varphi}\hat{m}+\hat{a})-1\Big]+(\ell-f)\Big[u'(\hat{\varphi}\hat{m})-1\Big]\Big\}$$

$$J_a^2 = 0 \Rightarrow \psi = \beta\Big\{1+f\Big[u'(\hat{\varphi}\hat{m}+\hat{a})-1\Big]\Big\}$$

Region 3: C-type's asset is enough to buy all of N-type's money, but total money is not enough to allow for optimal consumption q^*.

$$J^3(\hat{m},\hat{a}) = -\varphi\hat{m} - \psi\hat{a} + \beta\hat{a} + \beta f[u(\hat{\varphi}(\hat{m}+\tilde{m})) - \hat{\varphi}\tilde{m}] + \beta(\ell-f)u(\hat{\varphi}\hat{m}) + \beta(1-\ell)\hat{\varphi}\hat{m}$$

the pricing functions in this regions is given by

$$J_m^3 = 0 \Rightarrow \varphi = \beta\hat{\varphi}\Big\{1+f\Big[u'(\hat{\varphi}(\hat{m}+\tilde{m}))-1\Big]+(\ell-f)\Big[u'(\hat{\varphi}\hat{m})-1\Big]\Big\}$$

$$J_a^3 = 0 \Rightarrow \psi = \beta a$$

□

A.3.5. Proof of Proposition 3.4.

PROOF. **Case 1:** $A > \gamma/2$ and $A > i/\ell$. There exists a $f_{13} \in (0, \ell)$ such that given (A, i), equilibrium is in region 1 if $f \in [0, f_{13})$, and in region 3 if $f \in [f_{13}, \ell]$, where f_{13} solves $\gamma - i/(\ell - f_{13}) = \gamma/2$. Hence $f_{13} = \ell - 2i/\gamma$.

- If equilibrium is in region 1, $q_2 = \gamma, z = \gamma - i/(\ell - f)$, and

$$\frac{\partial \mathcal{W}}{\partial f} = u(\gamma) - \gamma - [u(z) - z] + (\ell - f)(\gamma - z)\frac{d\left[\gamma - i(\ell - f)^{-1}\right]}{df} = -\frac{1}{2}(\frac{i}{\ell - f})^2 < 0$$

- If equilibrium is in region 3, $z = (\ell\gamma - i)/(\ell + f), q_2 = 2z$, and

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial f} =&[u(2z) - 2z] - [u(z) - z] + [2f(\gamma - 2z) + (\ell - f)(\gamma - z)]\frac{d\left[(\ell\gamma - i)(f + \ell)^{-1}\right]}{df}\\ =&\frac{3f - \ell}{f + \ell}\frac{1}{2}(\frac{\gamma f - i}{\ell + f})^2\end{aligned}$$

Whether this expression is positive or negative depends on the value of $3f - \ell$. Since in region 3 we have $f \in [\ell - 2i/\gamma, \ell]$, if the lower bound of f is greater than $\ell/3$, then all f in this case would yield a positive result. Hence the sign depends on how $\ell - 2i/\gamma$ compares with $\ell/3$, or if $i > \ell\gamma/3$.

Case 1.1: If $i \geq \gamma\ell/3$, then $f_{13} \geq \ell/3$. Hence $\partial\mathcal{W}/\partial f > 0 \; \forall f \in [f_{13}, \ell]$.

Case 1.2: If $i < \gamma\ell/3$, then $f_{13} < \ell/3$. Hence $\partial\mathcal{W}/\partial f < 0 \; \forall \; f \in [0, f_{13})$; and $\partial\mathcal{W}/\partial f > 0 \; \forall \; f \in [\frac{\ell}{3}, \ell]$. $\mathcal{W}$ has a unique minimum at $f = \ell/3$.

Case 2: $A < \gamma/2$, $A > i/\ell$, and $i < \ell\gamma/2$. This corresponds to the lower half of the aggregate regions, in which the interest rate is not too large, and the equilibrium could be in any of the 3 regions. Hence there exist $f_{12} = \ell - i/A$ and $f_{23} = (\gamma\ell - i)/A - \ell$ with $0 < f_{12} < f_{23} < \ell$, such that equilibrium is in region 1, if $f \in [0, f_{12})$; in region 2, if $f \in [f_{12}, f_{23})$; and in region 3, if $f \in [f_{23}, \ell]$.

- If equilibrium is in region 1, $z = \gamma - i/(\ell - f), q_2 = z$, and

$$\frac{\partial \mathcal{W}}{\partial f} = u(\gamma) - \gamma - [u(z) - z] + (\ell - f)(\gamma - z)\frac{d(i - \frac{i}{\ell - f})}{df} = -\frac{1}{2}(\frac{i}{\ell - f})^2 < 0$$

- If equilibrium is in region 2, $z = \gamma - (i + \gamma A)/\ell, q_2 = \gamma - i/\ell + A(\ell - f)/\ell$.

$$\frac{\partial \mathcal{W}}{\partial f} = u(q_2) - q_2 - \Big[u(z) - z\Big] + \Big[f(\gamma - q_2)\frac{dq_2}{dz} + (\ell - f)(\gamma - z)\Big]\frac{d\Big[\gamma - \frac{i+fA}{\ell}\Big]}{df} = A^2(\frac{f}{\ell} - \frac{1}{2})$$

and the sign depends on how f/ℓ compares to $1/2$. Recall that in region 2, $f \in [f_{12}, f_{23})$, so there are 3 possibilities:

(1) $f_{12} > \ell/2$. In this case, $i/A < \ell/2$, hence $A > 2i/\ell$, and any admissible f will be greater than $\ell/2$. $\partial\mathcal{W}/\partial f > 0$ for all fs in this range.

(2) $\ell/2 > f_{23}$. In this case, $A > 2(\gamma\ell - i)/3\ell$. Then any admissible fs will be smaller than $\ell/2$, and $\partial\mathcal{W}/\partial f < 0$ for all fs.

(3) $f_{12} < \ell/2 < f_{23}$. Given the parameter range specified for case 2, $A < 2i/\ell$ and $A < 2(\gamma\ell - i)/3\ell$, hence $\partial\mathcal{W}/\partial f < 0$ for all $f \in [f_{12}, \frac{\ell}{2})$; $\partial\mathcal{W}/\partial f > 0$ for all $f \in [\frac{\ell}{2}, f_{23})$.

- If equilibrium is in region 3, and we have $z = (\ell\gamma - i)/(\ell + f), q_2 = 2z$, and

$$\frac{\partial \mathcal{W}}{\partial f} = \frac{3f - \ell}{f + \ell}\frac{1}{2}(\frac{\gamma\ell - i}{\ell + f})^2$$

The sign depends on the value of $3f - \ell$, or how the value of f_{23} compares to $\ell/3$.

(1) $f_{23} > \ell/3$, then $A < 3(\gamma\ell - i)/4\ell$, and all admissible fs are greater than $\ell/3$, so $\partial\mathcal{W}/\partial f > 0$ for all fs.

(2) $f_{23} < \ell/3$, then $A > 3(\gamma\ell - i)/4\ell$, and $\partial\mathcal{W}/\partial f < 0$ for all $f \in [f_{23}, \ell/3)$; $\partial\mathcal{W}/\partial f > 0$ for all $f \in [\ell/3, \ell]$.

Thus taking parameters f and ℓ, and asset supplies (A, i) as given, the equilibrium could potentially have 6 cases as region 2 and region 3 each has 3 and 2 subcases respectively. But 2 cases will be ruled out: (1) $\ell/2 < f_{12}$ and $\ell/3 > f_{23}$; (2) $\ell/2 \in (f_{12}, f_{23})$ and $\ell/3 > f_{23}$. Next we put together everything we learnt about case 2:

Case 2.1: $\ell/2 < f_{12}(< f_{23})$ hence $A > \frac{2i}{\ell}$. Then for all $f \in [0, f_{12}]$ $\partial\mathcal{W}/\partial f < 0$; for all $f \in (f_{12}, f_{23})$, $\partial\mathcal{W}/\partial f > 0$; for all $f \in [f_{23}, \ell]$, $\partial\mathcal{W}/\partial f > 0$

Case 2.2: $\ell/3 < \ell/2 \in (f_{12}, f_{23})$. Then for all $f \in [f_{12}, \ell/2]$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in [\ell/2, f_{23})$, $\partial\mathcal{W}/\partial f > 0$; for all $f \in [f_{23}, \ell]$, $\partial\mathcal{W}/\partial f > 0$. Besides the general parameter specification of case 2, this subcase also requires $A < 2i/\ell$, $A < 2(\gamma\ell - i)/3\ell$.

Case 2.3: $f_{12} < \ell/3 < f_{23} < \ell/2$. Then for all $f \in [f_{12}, f_{23}]$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in (f_{23}, \ell]$, $\partial\mathcal{W}/\partial f > 0$. This requires $A > 2(\gamma\ell - i)/3\ell$ and $A < 3(\gamma\ell - i)/4\ell$, which can be satisfied with $3(\gamma\ell - i)/4\ell > 2(\gamma\ell - i)/3\ell$.

Case 2.4: $f_{12} < f_{23} < \ell/3 < \ell/2$. Then for all $f \in [f_{12}, f_{23}]$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in (f_{23}, \ell/3)$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in (\ell/3, \ell]$, $\partial\mathcal{W}/\partial f > 0$, with a (smooth) minimum at $\ell/3$. This region requires $A > 3(\gamma\ell - i)/4\ell$, which also guarantees that $\ell/2 > f_{23}$.

Figure A.4 shows how the aggregate regions are dividend given the values of (A,i).

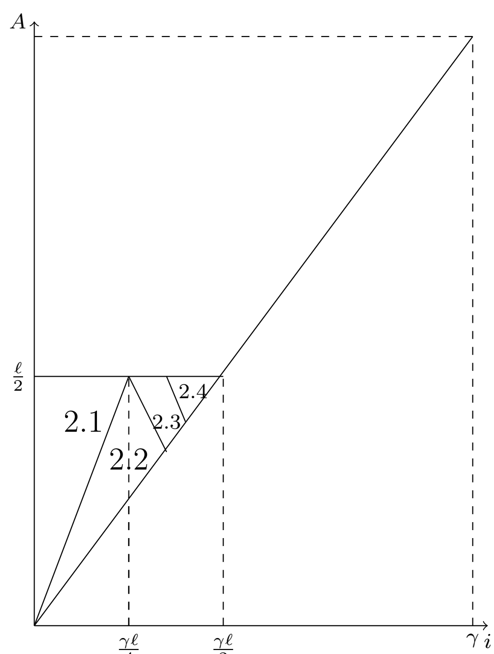


Figure A.4. Aggregate Regions of Case 2

<u>Case 3:</u> $A \leq i/\ell$, $i < (\gamma\ell)/2$. This is the case where the asset supply A is scarce, hence equilibrium could be in either region 2 or region 3. Given the definition of f_{23}, if $f \in [0, f_{23})$, equilibrium is in region 2; if $f \in [f_{23}, \ell]$, equilibrium is in region 3.

- If the equilibrium is in region 2, $z = \gamma - (i + \gamma A)/\ell$, $q_2 = z + A$. Then

$$\frac{\partial \mathcal{W}}{\partial f} = A^2(\frac{f}{\ell} - \frac{1}{2})$$

The sign depends on how the value of f compares with $\ell/2$, hence we discuss this with the following two cases.

(1) $\ell/2 > f_{23}$. Then $A > 2(\gamma\ell - i)/3\ell$, and all admissible fs are smaller than $\ell/2$, hence $\partial\mathcal{W}/\partial f < 0$ for all fs in region 2.

(2) $\ell/2 < f_{23}$. Then $A < 2(\gamma\ell - i)/3\ell$. $\partial\mathcal{W}/\partial f < 0$ for all $f \in [0, \frac{\ell}{2})$; and $\partial\mathcal{W}/\partial f > 0$ for all $f \in [\ell/2, f_{23})$.

- For all $f \in [f_{23}, \ell]$, the equilibrium is in region 3 and

$$\frac{\partial \mathcal{W}}{\partial f} = \frac{3f - \ell}{f + \ell}\frac{1}{2}(\frac{\gamma\ell - i}{f + \ell})^2$$

  and the sign depends on whether how the value of $\ell/3$ compare with f_{23}, hence we discuss this with the following two cases.

(1) $\ell/3 < f_{23}$. Then $A < 3(\gamma\ell - i)/4\ell$, and all admissible fs are greater than $\ell/3$, hence $\partial\mathcal{W}/\partial f > 0$ for all fs in region 3.

(2) $\ell/3 > f_{23}$. Then $A > 3(\gamma\ell - i)/4\ell$. $\partial\mathcal{W}/\partial f < 0$ for all $f \in [f_{23}, \ell/3)$; $\partial\mathcal{W}/\partial f > 0$ for all $f \in [\ell/3, \ell]$.

Hence given the parameter range, there could potentially be four cases in total. But one case can be ruled out since it is impossible to have $\ell/2 < f_{23}$ and $\ell/3 > f_{23}$ at the same time. We summarize the remaining three cases as follows.

<u>Case 3.1</u>: $\ell/3 < \ell/2 < f_{23}$. Then for all $f \in [0, \ell/2)$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in [\ell/2, f_{23})$, $\partial\mathcal{W}/\partial f > 0$; for all $f \in [f_{23}, \ell]$, $\partial\mathcal{W}/\partial f > 0$.

Case 3.2: $\ell/3 < f_{23} < \ell/2$. Then for all $f \in [0, f_{23})$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in [f_{23}, \ell]$, $\partial\mathcal{W}/\partial f > 0$. Also, besides the general parameter specification of case 3, this subcase also requires $\ell/2 > f_{23}$ or $A < 3(\gamma\ell - i)/4\ell$.

Case 3.3: $f_{23} < \ell/3 < \ell/2$. Then for all $f \in [0, f_{23})$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in [f_{23}, \ell/3)$, $\partial\mathcal{W}/\partial f < 0$; for all $f \in [\ell/3, \ell]$, $\partial\mathcal{W}/\partial f > 0$. This subcase again requires additional parameter restriction that $\ell/3 > f_{23}$ or $A > 3(\gamma\ell - i)/4\ell$.

Figure A.5 shows how the aggregate regions are dividend given the values of (A,i) for case 3.

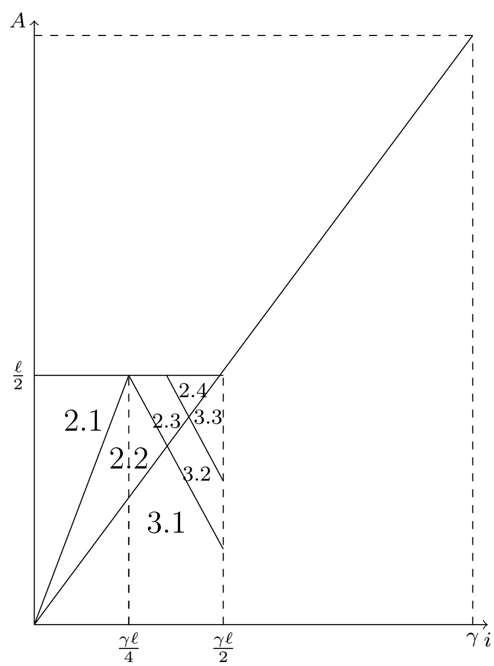


FIGURE A.5. Regions Specification of Case 3

Case 4: $A \leq i/\ell, i \geq \gamma\ell/2$. Given this parameter values, the equilibrium is always in region 3. Thus for all $f \in [0, \ell]$,

$$\frac{\partial\mathcal{W}}{\partial f} = \frac{1}{2}\frac{3f - \ell}{f + \ell}(\frac{\gamma\ell - i}{f + \ell})^2$$

which is positive if $f > \ell/3$, and is negative if $f < \ell/3$.

We use the Figure A.6 to summarize the parameter values of all the regions:

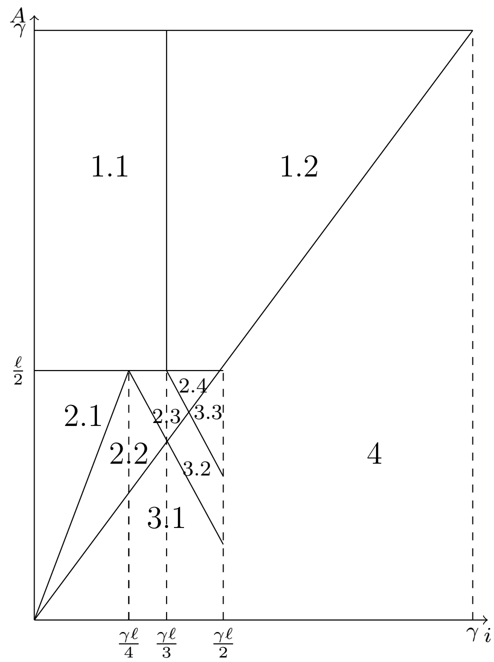


FIGURE A.6. Summary of all regions

□

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