A PRIMORDIAL $^4$He CONSTRAINT ON DIRAC NEUTRINO MASSES

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The implications of the $Z^0$-width experiment for the number of neutrino flavors taken together with the $^4$He constraint on the energy density at the nucleosynthesis epoch, can give a limit on the extent to which a right-handed neutrino sea could contribute to the expansion rate of the universe during Big Bang nucleosynthesis. Since the population of the right-handed neutrino sea depends on the neutrino mass we can extract a Dirac neutrino mass limit of $m_\nu < 150$ keV. This limit could apply to $\nu_\mu$ or $\nu_\tau$ if they have purely Dirac masses.

The requirement that $^4$He not be over-produced in Big Bang nucleosynthesis, relative to the observationally-inferred primordial abundance, yields a limit on the number of light neutrino families,$^1$ which is in excellent agreement with the results from the $Z^0$ width experiments.$^2$ In this presentation we will explore the effects of a “sterile” sea of right-handed (RH) neutrinos and left-handed (LH) anti-neutrinos in addition to the expected sea of normal LH (RH) neutrinos (anti-neutrinos). We mean the degrees of freedom associated with both RH-neutrinos and LH-anti-neutrinos when we refer to “RH neutrinos” in what follows. Primordial nucleosynthesis calculations and abundance observations can be used to constrain the extent to which the RH neutrino sea is populated. Other studies have used constraints on RH-neutrino-degrees-of-freedom to delineate electromagnetic,$^3$ oscillation$^4$ and weak interaction$^5,6$ properties of neutrinos.

If neutrinos have mass then scattering processes can induce helicity-flip and, thereby, populate the RH neutrino sea. The RH-components merely correspond to the anti-neutrino when neutrinos are Majorana particles. We will assume that neutrinos are purely Dirac particles since our subsequent argument is based on the effect of added degrees of freedom during nucleosynthesis. The temperature at which the RH neutrino sea falls out of equilibrium with its LH counterpart depends on the neutrino mass $m_\nu$, since the amplitude for helicity-flip of Dirac neutrinos in a scattering process is proportional to this mass. After the RH neutrino sea falls out of equilibrium any subsequent particle annihilations or phase transitions heat the LH neutrino sea but not the RH sea, decreasing the relative energy density contribution of the RH sea at the nucleosynthesis epoch. Arguments similar to these have been used to show that a $\nu_\tau$ of mass $\sim 10$ eV has a negligible effect on $^4$He production.$^7$ Following the $Z^0$ width experiments, we assume three neutrino families and then use the above arguments to derive a limit on masses for $\nu_\mu$ and $\nu_\tau$. The requirement that the RH neutrino sea make a negligible contribution to the energy density at the nucleosynthesis epoch implies that the RH neutrinos should decouple prior to the dominant re-heating event (the confinement of quarks) which, in turn, implies an upper limit to the neutrino mass.

The RH and LH neutrino seas can be in equilibrium only if the processes that flip neutrino helicity are rapid compared to the universal expansion rate. The cross section for
helicity-flip in neutrino scattering is suppressed\(^7,8\) relative to that of the non-flip process by a factor of order \((m_\nu/E_\nu)^2\), with \(E_\nu\) the neutrino energy. The time scale for helicity flip is

\[
\tau_{\text{flip}} \approx (n_w \sigma)^{-1} \approx \left( \frac{3 \zeta(3)}{4 \pi^2} g_w G_F^2 m_\nu^2 T^3 \right)^{-1},
\]

with \(n_w\) the number density of weakly interacting particles, \(\sigma\) the cross section for helicity flip, \(\zeta(3)\) the Riemann zeta function of argument 3, \(G_F\) the Fermi constant, and \(g_w\) the weakly-interacting fermion statistical weight. The epoch of quark-confinement\(^3-7\) is at a temperature high enough that electrons, muons, neutrinos, \(u\) and \(d\) quarks, and all associated anti-particles are in equilibrium yielding \(g_\nu \approx 38\). The universal expansion rate is

\[
H = \left( \frac{4 \pi^3}{45} \right)^{1/2} m_{\text{Planck}}^{1/2} T^2,
\]

and where \(m_{\text{Planck}}\) is the Planck mass and \(g = g_b + \frac{g_f}{2}\) is the statistical weight of relativistic particles including that in bosons, \(g_b\), and that in fermions, \(g_f\). At \(T \approx 100\) MeV the statistical weight is \(g \approx 56.5\). Equilibrium between RH and LH neutrino seas obtains whenever \(\tau_{\text{flip}} \ll H^{-1}\). Thus for the RH neutrino sea to have decoupled prior to the universe reaching temperature \(T\) the neutrino mass must be bounded by

\[
m_\nu \ll \frac{2}{3(\zeta(3))^{1/2}} \left( \frac{4 \pi^7}{5} \right)^{1/4} \left( G_F^2 m_{\text{Planck}} \right)^{-1/2} \left( \frac{g_{1/4}^{1/4}}{g_{1/2}^{1/2}} \right) T^{-1/2}.
\]

At conditions relevant to the QCD epoch this becomes,

\[
m_\nu \ll 150\text{ keV} \left( \frac{100\text{ MeV}}{T} \right)^{1/2}.
\]

The comoving entropy density of the universe remains constant, so that the ratio of the temperatures of the LH and decoupled-RH neutrino seas after a phase transition or particle-annihilation epoch is \(T_{\text{LH}}/T_{\text{RH}} = (g_1/g_2)^{1/3}\), where the subscripts 1 and 2 refer to before and after the reheating event respectively. For our purposes the only significant reheating event is the annihilation of the quarks and gluons, where the statistical weight changes by a factor of about 3. The largest upper limit on \(m_\nu\) corresponds to the case where the quarks annihilate at the lowest possible temperature, which would be about 100 MeV.\(^9,10\)

Before quark-annihilation the statistical weight is \(g_1 \approx 56.5\); whereas, afterward \(g_2 \approx 17.25\) and \(T_{\text{LH}}/T_{\text{RH}} \approx 1.44\). Therefore, the relative energy densities in a LH and RH neutrino species will be \(\rho_{\text{LH}}/\rho_{\text{RH}} \approx 4.3\) at the nucleosynthesis epoch, and thus the RII degrees of freedom count as less than 1/4 of an additional neutrino flavor. Muons and pions will drop out of equilibrium when \(T < 100\) MeV, and if their entropy density is absorbed in the differential heating of the LH and RH seas then a RH neutrino species would count \(\approx 0.1\) of the LH components of an additional neutrino generation.

Standard Big Bang nucleosynthesis (SBBN) calculations predict a primordial \(^4\)He mass fraction\(^11\)

\[
Y_\text{p} \approx 0.228 + 0.010 \ln \eta_{10} + 0.012 (N_\nu - 3) + 0.185 \left( \frac{\tau_\nu - 889.8}{889.8} \right),
\]

with \(\eta_{10}\) the photon-to-baryon ratio in units of \(10^{10}\), \(N_\nu\) the number of neutrino families and \(\tau_\nu\) the neutron mean life in seconds. With a lower limit of \(\eta_{10} > 2.6\) from observations\(^11\)
of $D+^{3}\text{He}$, and using\textsuperscript{12} $\tau_{n} < 894.2$ s, Eq. (4a) can be recast as a limit on the number of relativistic neutrino species for a given $Y_{p}$,

$$N_{\nu} \leq 3.4 + 20 \left( \frac{Y_{p} - 0.240}{0.240} \right).$$

(4b)

$Y_{p} = 0.23 \pm 0.01$ from observation\textsuperscript{11,13} then implies $N_{\nu} \leq 3.4$. The $Z^{0}$-width experiments\textsuperscript{2} now show, independently, that $N_{\nu} = 2.98 \pm 0.06$. If the RH components of one of these three neutrino flavors did not decouple until after quark-annihilation then they would count for 0.7 neutrino flavor over and above the LH contributions of three light neutrino flavors. This would not be consistent with the limit on the relativistic degrees of freedom at the nucleosynthesis epoch for the observed primordial $^{4}\text{He}$, which is, equivalently, $N_{\nu} \leq 3.4$. We conclude that the RH components of ($\nu_{\mu}$ or $\nu_{\tau}$) must decouple prior to the QCD epoch, giving the limit in Eq. (3b).

This limit on the mass of $\nu_{\mu}$ or $\nu_{\tau}$ extends existing experimental and astrophysical limits, since it can apply to unstable neutrinos with nonradiative decay modes. This mass limit applies if neutrinos have purely Dirac masses and interactions, they are light, and they have lifetimes exceeding the nucleosynthesis timescale ($\sim 100$ s). For this limit to apply the LH and RH neutrino components must have the same mass. We caution that a small Majorana mass would nullify our limit. If the decay mode of the neutrino involves a final state with a lighter neutrino and a relativistic weakly-interacting particle then the limit will apply so long as the neutrino-lifetime exceeds the weak decoupling timescale ($\sim 1$ s). For neutrino decay after weak decoupling the decay products would not thermalize with the plasma, locking-in the extra degrees of freedom. The neutrino mass limit described here would most likely apply to the simplest extensions of the Standard Model which include Dirac masses for neutrinos. We have given an in depth discussion of this mass constraint elsewhere.\textsuperscript{14}

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REFERENCES


