Title

Computing systemic risk using multiple behavioral and keystone networks: The emergence of a crisis in primate societies and banks

Journal
International Journal of Forecasting, 30(3)

ISSN
0169-2070

Authors
Fushing, Hsieh
Jordà, Òscar
Beisner, Brianne
et al.

Publication Date
2014-07-01

DOI
10.1016/j.ijforecast.2013.11.001

Peer reviewed

HHS Public Access

Author manuscript

Int J Forecast. Author manuscript; available in PMC 2015 July 01.

Published in final edited form as:

Int J Forecast. 2014 ; 30(3): 797–806. doi:10.1016/j.ijforecast.2013.11.001.

Computing Systemic Risk Using Multiple Behavioral and Keystone Networks: The Emergence of a Crisis in Primate Societies and Banks*

Hsieh Fushing†,
Department of Statistics, University of California, Davis

Òscar Jordà†,
Federal Reserve Bank of San Francisco, Department of Economics, University of California, Davis

Brianne Beisner, and
International Institute for Human-Animal Networks, Department. of Population Health & Reproduction, School of Veterinary Medicine, California National Primate Research Center, University of California, Davis

Brenda McCowan
International Institute for Human-Animal Networks, Department. of Population Health & Reproduction, School of Veterinary Medicine, California National Primate Research Center, University of California, Davis

Abstract

What do the behavior of monkeys in captivity and the financial system have in common? The nodes in such social systems relate to each other through multiple and keystone networks, not just one network. Each network in the system has its own topology, and the interactions among the system's networks change over time. In such systems, the lead into a crisis appears to be characterized by a decoupling of the networks from the keystone network. This decoupling can also be seen in the crumbling of the keystone's power structure toward a more horizontal hierarchy. This paper develops nonparametric methods for describing the joint model of the latent architecture of interconnected networks in order to describe this process of decoupling, and hence provide an early warning system of an impending crisis.

*This paper was prepared for the 9th International Institute of Forecasters' Workshop, Sept. 28–29, 2012, held at the Federal Reserve Bank of San Francisco. The views expressed herein are the sole responsibility of the authors, and should not be interpreted as reflecting the views of either the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

†Correspondence: Hsieh Fushing (fhsieh@ucdavis.edu) and Òscar Jordà (Oscar.Jorda@sf.frb.org).

Keywords

global collection of local information; decoupling; network; keystone; primate society; power; macaque; monkey

1. Introduction

Humanity is becoming increasingly interconnected. This interconnectivity has clear benefits, such as food security, technological innovation, and rapid information exchange. However, an increased connectivity can introduce vulnerabilities of its own — a common feature of complex systems. These vulnerabilities are such that they can sometimes threaten the integrity of the entire network directly. Examples include catastrophic failures, such as stock market crashes; the rapid propagation and dissemination of adverse entities, such as disease outbreaks or internet viruses; and the concentration of key resources around central hubs or clusters, such as oil in OPEC countries, or rare earth minerals in China. This paper introduces non-parametric methods for detecting the build-up of these vulnerabilities empirically. The objective is to provide an early warning system that can be used to prevent a crisis from breaking out.

Social network analysis has become a natural tool for modeling a variety of complex dynamic systems (see, e.g., the special issue on "Complex Systems and Networks" in *Science*, 2009). Although there is no generally accepted definition, it seems fair to say that a system is complex when there are emergent phenomena that are the spontaneous outcome of the interactions of many constituent elements (see for example Amaral & Barthélemy, 2003; Amaral & Ottino, 2004; and Barabási, 2005). Network theory is designed to reveal the hidden architecture of complex systems (Simon, 1962) and the candidate sources of a network’s instability. When paired with computational statistics, this theory can be used to detect the early formation of network vulnerabilities empirically. Ultimately, the objective is not only to identify these network vulnerabilities and prevent them from materializing, but also to design more resilient structures that maximize social welfare and minimize the costs of increased connectivity. Therefore, the concept of a network’s *resilience* has received a considerable amount of attention and is related to the literature on *percolation* in complex directed networks (see e.g. Newman, Strogatz, & Watts, 2001; Schwartz, Cohen, ben Avraham, Barabási, & Havlin, 2002; Dorogovtsev & Mendes, 2001; and Boguñá & Serrano, 2005).

The recent Global Financial Crisis laid bare some of these vulnerabilities, and in its aftermath, considerable research effort has been dedicated to understanding its causes. Research focusing on network theory has appeared particularly promising (see for example May & Arinaminpathy, 2010; and Haldane & May, 2011). Initially, financial crises were characterized as being the result of exogenous shocks that propagate through a static network. More recently, the literature has evolved to incorporate endogenous tuning factors that allow for richer and more realistic dynamics of network propagation (see for example Arinaminpathy, Kapadia & May, 2012).

The structures of the social systems that we investigate are characterized by nodes that relate to each other through multiple networks. Each network has its own topology, and the networks are related to each other in varying degrees. Moreover, we distinguish between the *keystone* network and the subsidiary networks. The keystone network is most closely associated with the hierarchy prevailing among the nodes in the social system.

It is important to recognize that a network's topology is often endogenous, and, as a consequence, is dynamic rather than static. The connections across networks are dynamic as well. Importantly, a network's global characteristics can be very sensitive to local perturbations, especially in directed networks. A theoretical model of a social system with these characteristics is difficult to construct. In this paper, we argue that certain features of social systems can be determined using data-driven, nonparametric-based methods — a natural complement to existing methods which are based on a more structural approach.

One basic assumption of our analysis is that information is available to all nodes in a social system equally. One important reason to entertain such an assumption is in order to give more weight to endogenous mechanisms that generate phase transition dynamics. Assuming that nodes have a heterogeneous access to information tends to place more weight on exogenous factors as an explanation for phase transition dynamics, thus inherently explaining these transition dynamics outside the system.

The assumption that information is available to all nodes equally is certainly justifiable in the context of the two systems investigated here. However, although we assume that all nodes have an equal access to information, we allow the nodes to have heterogeneous information processing capabilities. This heterogeneity leads to the formation of asymmetric and diverse hierarchical structures, and this diversity in turn becomes an endogenous source of tension and a natural source of instability.

The non-parametric methods that we discuss are applied to a primate social system (a large captive group of rhesus macaques), which are observed under both stable and unstable states or phases. This set-up has many points of commonality with the architecture of a banking system. We argue that the mechanics of systemic risk propagation in a monkey social system which is on the brink of *social collapse* are comparable to those in a banking system on the brink of a *financial crisis*.

Admittedly, comparing rhesus macaques to a financial system is unconventional. However, we believe that readers will find this comparison compelling. The fundamental mechanisms underlying the instabilities in these two systems are, in fact, quite similar. Small-scale models of social systems can be quite effective when thinking about models which are applicable to larger human systems (in both scope and scale). Our methods show one approach that could be scaled up in order to model the vulnerabilities of the financial system.

2. Systemic risk propagation: When multilayered networks decouple under primary network collapse

The broad outline of the methods that we describe below can be sketched in a few sentences. We assume that the nodes in the system under consideration all have an equal access to information. However, we also assume that the nodes each have different information processing abilities. This way of introducing heterogeneity is computationally convenient and leads to the endogenous formation of a power structure within the system. The nodes are related to each other in a variety of different ways. Each results in a different network within the system. In normal times, the hierarchical structure of the system results in a natural pattern of network interactions. However, a degradation of the power structure results in a degradation of the relationships across networks, or decoupling. During a crisis, the power structure collapses and the system's networks decouple. The endogenous dynamics of the system in the aftermath of the crisis are quite different from those before the crisis strikes. However, it is difficult to predict when a crisis will arise based solely on observations of the behaviors of the individual networks in the system in isolation. Instead, we argue that one can detect when a crisis is likely to set in by modeling the evolution of the degree of decoupling across the system's networks.

Consider two features of a dynamic system which are relevant for our analysis: (1) the *power structure* of the primary network; and (2) the assumption of *global collection of local* (GCL) information. The power structure characterizes the major flow of information through network connectivity characteristics. It is determined endogenously as the system evolves over time. The GCL information assumption means that each node in the system has an equal access to information that may be local to any given node. The heterogeneity in node-specific information processing abilities leads to the formation of vertical hierarchies within the system — a power structure.

The nodes within a social system interact in many different ways, and each type of interaction generates a different network. Thus a system will typically exhibit a variety of networks that probably interface with each other. In a banking system, examples include: interbank lending, syndication of loans, bond issuance advisory services, insurance, etc. In a monkey system, monkeys groom each other, fight with each other, offer coalitionary support during fights, and display status signals. Behavior-based connectivity generates a directed node-to-node relation. The collection of directed relationships typically constitutes a directed network.

However, not all behavioral networks are created equal — some networks within a given social system have more fundamental roles than others. The reason for this is that they govern or influence the manner in which the remaining networks in the system interact. In the field of primatology, evidence is mounting that the subordination signaling network is the fundamental network of monkey society, as these signals (a) influence the long-term affiliation and aggression (Beisner & McCowan, in press), (b) demonstrate decoupling from other behavioral networks when the system becomes unstable (Chan, Fushing, Beisner, & McCowan, 2013), and (c) have a perfectly transitive network structure, the rigidity of which appears to structure other behaviors (Fujii, Fushing, Beisner, & McCowan, submitted). We refer to this fundamental behavior as a *keystone network* in social dynamics. The social hierarchy that arises from this keystone network is expected to be the most relevant for understanding the underlying system dynamics.

Different networks generate different hierarchical arrangements. Some networks contain many connections, such as grooming and aggression networks in the monkey system, and corporative bond holdings in a banking network. In the parlance of network theory, they are assortative. In contrast, other networks contain very few or no upstream connections and consist of a strict downward flow of dominance, such as the status network in a monkey society or inter-bank lending in a banking network — that is, they are disassortative. The rigid flow structure of these last two networks makes them good candidates for being the keystone networks of their respective systems, particularly because power structures are characterized by having a primarily downward flow of connectivity (Fujii et al., submitted).

Consider how this power structure develops and arises in a monkey system. Each monkey (node) has a different ability to gather and exploit resources, such as food, mates, alliance partners, and social information. Monkeys that are more intelligent or more socially adept will be more successful in gathering resources than less intelligent or less socially adept individuals. The same is true of a banking system — some banks have a greater ability to process information, thus allowing them to make better lending and investment decisions, and therefore grow larger. This diversity in ability, which produces heterogeneity in "size" and "dominance," generates a "ranking" structure. This structural pattern is particularly ubiquitous in the stable phase of many systems.

In the monkey system that we analyze, there are no visual barriers within the cage in which the monkeys are kept, and therefore any monkey can observe the behaviors displayed by any subgroup of his peers. The type of information that a monkey in a cage can collect includes those with whom his or her siblings and family members are cooperating or fighting; which families are harassing or being harassed by other families; who is challenging the dominance rank; who is grooming or being groomed; and many other positive and negative behaviors. Thus, the captive cage environment in which the monkeys live allows each monkey to have unlimited access to GCL information regarding all events within the group.

It is important to note that neither humans nor monkeys evolved in an environment in which they had access to the same types of information as other group members, nor having access to such large volumes of information. In today's world of social media, cell phone and internet technologies, humans find themselves in an unnatural environment where they have economic, financial, social, and political information at their fingertips. Similarly, in captivity, monkeys find themselves in an unnatural environment without visual barriers such as dense foliage, where they are able to observe the interactions of all their group mates freely. Further details of these unnatural environments are provided below.

A network with GCL information tends to generate two distinct power structures. In the stable phase, heterogeneity in information processing ability results in a more unequal distribution of resources, and therefore, a well-defined vertical hierarchy. In the unstable phase, nodes revert to their survival instincts and largely disregard the extant hierarchy.

Informational advantages no longer play a role, and the hierarchical arrangement becomes virtually horizontal.

The transition from a subtle and complex vertical power structure to an unsophisticated horizontal power structure is called *social collapse*. In many ways, this collapse is equivalent to the concept of a tipping point in the dynamic systems literature — a point in time when the system suddenly shifts from one phase to a drastically different phase. A good example from thermodynamics is when matter changes from a solid to a liquid. This paper illustrates just such a collapse in the context of a captive monkey system. The recent financial crisis also exhibited many of these traits. Importantly, the sudden wholesale shutdown of connectivity does not appear to be the result of contagion. Instead, the apparent synchronicity observed is better explained as being the result of a simultaneous response by all nodes to the same constraints under GCL information.

The identification and detection of the process leading to tipping-points is still in its infancy (see Scheffer et al., 2009 for a review). The major obstacle lies in capturing its onset — an extremely difficult task. Our approach differs from that which has been customary hitherto. The onset of a tipping point is hardly predictable when using only one dimension (i.e., network) of the dynamic system, which has been the focus of past efforts. Instead, we propose a multidimensional (i.e., many networks) approach. In particular, we introduce a new way of monitoring when a social system may be approaching a tipping point.

As the system approaches the tipping point, its multiple behavioral networks gradually decouple from the keystone network, as its power structure crumbles. In the stable phase of the dynamic system, rules, norms, or even cultures and traditions, govern and regulate members' behaviors. Behavioral dynamics are interconnected or tightly coupled, probably through the underlying power structure, expressed by the keystone network of the system. Fujii et al. (submitted) provide evidence on this very issue. However, such rules and norms can be disrupted, and hence the system's networks decouple from one another during a system's collapse. In particular, determining when the keystone network decouples from the subsidiary networks could be used to identify the onset of the tipping point.

We propose to model the evolution of the interaction of multiple networks in order to determine when decoupling begins to occur. The computational technique required to model this evolution relies on a joint model of a system's networks (see e.g. Chan et al., 2013). Decoupling is measured using the variation in the degree of inter-network dependence. In this paper, we implement these ideas using a captive monkey social system as a model of a case in which a real social collapse was observed. We explain the details of our approach below.

3. Contagion versus synchronicity, statics versus dynamics

When it comes to characterizing network vulnerability, the state-of-the-art follows a well-worn path. Models of electric grid networks (Amin & Schewe, 2007), ecosystems (Haldane & May, 2011; May, Levin, & Sugihara, 2008; May & Arinaminpathy, 2010) and flow networks have inspired network-based explanations of the most recent financial crisis. These applications share a common theme: they are based on approximating the complex dynamics of the system considered using a *single* behavioral network.

The literature views each node as only taking in information from local interactions and local connections. Similarly, each node is thought to affect only its local neighborhood. This is the primary method of shock propagation and risk contagion considered. Such a localized perspective of risk contagion is reflected further by the definition of systemic risk used by the Bank for International Settlements: “the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties.”

The view that local information flows propagate through contagion is at odds with the wealth and speed of local information transfer at a global level. Information disseminates at a much faster pace than the slow, chain-reaction effects delivered by contagion mechanisms. Instead, a more sensible position is to characterize the information-processing heterogeneity of commonly available information, even if this information refers to the local level outside the node’s neighborhood. In recognizing such effects, Arinaminpathy et al. (2012) include “confidence” and “individual health status” components in their systemic risk modeling in order to go beyond the usual rigid network propagation constraint.

Moreover, another critical assumption which is made in the literature is that the network remains static as shocks propagate. This is the implicit assumption of the ecological, power-grid, and flow-theory modeling approaches, see for example Gai and Kapadia (2010). This assumption, however, ignores the “feedback and mutuality” mechanisms between a node and the banking system. Instead, network connectivity is likely to evolve dynamically.

Our premise is that, collectively, nonlinear feedback loops can change the network topology very quickly, that such changes occur continuously over time, and that they are endogenous. Specifically, under GCL information, both large and small nodes interact with the system as a whole directly and closely, producing mutually reinforcing feedback loops. This feedback mechanism and the resultant changes in the network’s topology present a challenge to the traditional analysis of systemic risk in a banking system.

Justification for this endogenous, multidimensional and dynamic view of banking networks is easy to find. Diminishing inter-bank loan availability, including “liquidity hoarding” and “funding liquidity shocks”, is a natural consequence of feedback mechanisms and the resultant re-wiring in the network. This sort of outcome is a manifestation of the systemic risk being studied. It should not be designated as the initial exogenous shock, as is typical of studies of systemic risk contagion.

Finding an early-warning signal using a static, one-dimensional network driven by exogenous shocks is likely to provide an incomplete signal. At the same time, we are well aware of the complications that modeling a dynamic, multidimensional, endogenous network would entail. The solution we offer is to have a more modest goal: that of providing a computationally tractable summary of multidimensional network connectivity that can be used to detect the onset of a tipping point before the next crisis strikes.

4. Banks and monkeys are two sides of the same coin

The way in which individuals respond to GCL information depends on the endogenous stable/unstable state of the network at any given time, as well as the individual's information processing ability. In the stable phase, individuals can engage in riskier behavior under the protection of the hierarchy, but are also constrained by the boundaries of a social-norm. In an unstable phase, the breakdown of the hierarchy and the need for self-preservation can quickly modify risk-taking behavior as a new social norm emerges (for a recent experiment reflecting disruption-induced local changes in risk-taking behavior by monkeys, see Flack, Girvan, de Waal, & Krakauer, 2006).

The topology of a banking system has a clear hierarchy. Trillion dollar global players interact in the same ecosystem as hundred million dollar small community players. The network relationships among these bank nodes are defined by a variety of modes of connection, ranging from formal ongoing contractual relations, and inter-bank lending in the reserves market, to syndication through common assets, and various other financial products and banking arrangements. The key concept here is that each type of inter-bank relationship constitutes one banking network (for example, lending networks describe inter-bank lending relationships), and each banking behavior network can be used to approximate a single dynamic aspect of the banking system under study. That is, versatile relationships are represented by multiple networks pertaining to the same collection of banks. It is the dynamic interactions across these different network layers that best characterize the state of the system as a whole.

Monkey society is also a complex system consisting of many layers of inter-behavioral relationships. Monkeys groom each other, fight with each other, offer coalitionary support during some of these fights, and exchange status signals that communicate social power and dominance (Sade, 1972; Datta, 1986; Beisner, Jackson, Cameron, & McCowan, 2011; McCowan et al., 2011; Beisner and McCowan, in press). Each type of behavior can be used to construct a network for a single aspect of monkey society. The interactions of these behaviors offer the clearest picture of the stability and hierarchy of the society.

A banking crisis can be best described by a sudden, unintended, system-wide loss of inter-banking interactions. Monkey societies in captivity suffer from analogous crises, known as cage wars or social collapses, in which serious fighting erupts because group members no longer agree on the dominance hierarchy (Oates-O'Brien, Farver, Anderson-Vicino, McCowan, & Lerche, 2010). Typically, these societal collapses involve lower-ranking monkeys attacking and killing the highest ranking family, which completely disrupts the dominance hierarchy. These tragic events, while relatively infrequent, are extremely costly and create many management problems, as the entire group must be disbanded and relocated elsewhere.

Such behaviors, whether in a monkey society or a banking system, give rise to a (weighted) directed network. A directed path between two nodes is called a "flow", and multiple flows may exist between a pair of nodes. Collectively, these network flows constitute a computable flow-chart. Some behaviors (flows) are more informative and important than others. Here, by relying on expert knowledge (e.g., a primatologist or an economist, depending on the problem), both systems are equipped with one fundamental or keystone behavior, the flow-chart of which constitutes a *power structure* of the system.

5. Visualizing the power structure through flow topology

As the keystone behavior, status interactions are governed by dominance — they are signals given by a subordinate animal to a dominant animal. Most important among them is the “silent-bared-teeth” (SBT) display, as a peaceful communication of subordination. Because the SBT is a unidirectional submission signal (always given by a subordinate to a dominant animal), unlike other dominance communications, whose direction can vary depending upon social context, it conveys a higher degree of dominance certainty than other submission signals (de Waal, 1986; Flack and de Waal, 2007; Beisner and McCowan, in press).

SBTs express true dominance relationships. Once dominance is understood, it governs aggression, grooming, and alliances (Beisner & McCowan, in press). Aggression is mostly from dominant to subordinate; grooming is often from subordinate to dominant, unless dominants initiate grooming as reconciliation after a fight; and alliances are most often made between kin, which rank near each other. In this way, the status network provides the basis for all other behavioral relationships.

The power structure of a monkey society can be visualized through flow topology. Basically, flow topology summarizes the status relations between nodes, and, as these relations evolve over time, so do the topological features of the power structure. A pairwise comparison of flow topologies for two different behaviors can also be usefull, while changes in the interactions between two networks over time can reveal (social) stress particularly well.

Fujii et al. (submitted) provides a detailed description of the construction of the power structure of a network using flow topology. We provide a brief summary of the steps involved in Appendix A. Below, we show flow topologies of SBT networks using data from one monkey group both at a stable time point (in 2009) and at an unstable time point (in 2011), four months before a social collapse.

The trickling-down percolation algorithm allows one to visualize the power structure of the monkey society at two points in time. In the 2009 flow topology, there is a clear hierarchical structure linking a large membership, with female n. 1 and male n. 35 at the top of the power pyramid. This corresponds to a stable period of time. By 2011, the flow-topology strongly indicates that the previous power structure has almost entirely disintegrated. Many previous relationships have disappeared. The remaining power structure is much more horizontal and fragmented, with female n. 22 now leading a small group that appears disconnected from the remainder. This topology predates the social collapse that ensued four months later.

What caused this social collapse? Was it driven by the breakdown in a dyadic relationship that propagates through the entire society? Or did endogenous changes in GCL information prompt a synchronous adaption by society members, each according to their information processing abilities? These are important causal questions, but extend beyond the scope of this paper. Instead, we focus on designing structural methods that can help us uncover the shift in group dynamics displayed in Figure 1.

6. Early-warning patterns: Cross-network dependence through maximum entropy

The calculation of the flow topology of the society's power structure through trickling-down percolation is a useful descriptive device. The variation in the flow-topology over time helps in visualizing the disintegration of the power structure before a complete social collapse. However, a single behavioral network (such as the fundamental network upon which the power structure is based) is not sufficient to detect the collapse in power structure or the tipping point. This section introduces methods of multidimensional network analysis which can quantify the inter-network dependence. Thus, we can monitor a society approaching collapse using the variation over time in this type of dependence.

In a stable monkey society, primatologists have discovered that social dynamics are governed by a set of general rules and constraints. For example, females form close alliances with kin in order to defend their resources and their family rank against other families in the group. This means that aggression, status, and alliance interactions all have interdependencies. Dyads that form alliances tend not to fight much (and are likely to be kin). Aggression and status both follow dominance relationships and are primarily unidirectional. We holistically term the overall interrelationships *behavioral subtlety*.

As the society migrates to a more unstable phase, behavioral subtlety gradually decreases. For example, if dominance no longer governs aggressive and status interactions, then the inter-dependencies between these two networks will gradually be lost, and they will become increasingly independent.

However, the dependencies between two or more behaviors are not observable directly. This section shows how such dynamic features can be evaluated by coupling multiple network data. Note that a monkey can only interact with another through one behavior at a time. That is, the data are not multivariate for any given point in time. As a result, the classical Pearson correlation and its variants are not applicable directly. Instead, behavior-specific networks are constructed across a temporal span. The unique features of these data require new methods of analysis. We have developed one evaluation technique based on the maximum entropy principle found in statistical mechanics (Chan et al., 2013). This technique is one way to provide essential early-warning pattern information.

The maximum entropy-based, joint modeling approach is described briefly below (for full details about this methodology, see Chan et al., 2013). Recall that SBT status behavior is taken as the closest marker of the power structure. We therefore focus on coupling this keystone network with the grooming, aggression, and alliance networks. Here, we only discuss pairwise coupling, for simplicity. In the case of the banking system, one would need to determine the equivalent primary and subsidiary networks based on the available data and the specifics of that problem.

As an illustration, our basic idea is to model two binary (unweighted) networks jointly, corresponding to two types of social behaviors. The specific goal is to model the probabilistic distribution of a link in one network being associated with a link in the other network. For a given network, each directed link between two nodes is classified with a binary code: 00 if no relationship exists, 01 if there is a unidirectional relationship (notice that, since we do not identify the nodes, the 10 code is equivalent), or 11 if the relationship is bidirectional. Therefore, a two-behavior directional network can be catalogued using a 4-dimensional binary code, with the first two figures belonging to one network and the latter two belonging to the other network.

For example, consider two behaviors: grooming and aggression. A monkey dyad with mutual grooming but no aggression can be represented by the 4-dimensional code vector (11, 00). An example of this case can be seen between nodes 2 and 3 in Figure 2. A pair of monkeys with opposite directional grooming and aggression is represented by the vector (10,01). An example of this case can be seen between nodes 3 and 4 in Figure 2. In principle, there are 16 possible 4-dimensional linkage vectors, although there are only 10 biologically-distinct vectors (again, since the network-specific 10 and 01 codes are equivalent). The empirical distributions of these 10 categories of linkage vectors represent the empirical information association between these two behaviors of interest. Figure 2 provides an example of two small grooming and aggression networks (Panel (a) of Figure 2) and how they can be combined (Panel (b) of Figure 2).

The maximum entropy-based, joint modeling approach proposed here involves constructing a 4-dimensional distribution model via the following iterative updating procedure:

1. Choose a known baseline (null) distribution, such as one that assumes component-wise independence, and compute the expected counts (E_i) of the 10 categories under the baseline distribution.
2. Calculate the Chi-square value, $(O_i - E_i)^2 / E_i$, as a measurement of discrepancy between the empirical and expected counts, that is, O_i vs E_i, for each of the 10 categories. Here, O_i denotes the *observed* count.
3. Remove the largest Chi-square value, then add a new structural constraint into the baseline distribution via the maximum entropy approach (see the Appendix).
4. Take the modified distribution as the new baseline distribution and repeat steps 1–3 until the sum of 10 Chi-square values is less than a threshold value chosen according to the Chi-square distribution with nine degrees of freedom.

Step 3 is the key step in this iterative updating procedure. A large Chi-square value indicates a significant discrepancy between the baseline and empirical distributions. This discrepancy leads us to modify the baseline distribution so as to fit the data better. The choice of structural constraint in the form of a correlation is established between two component dimensions or two behaviors. The final distribution model is then taken as the data-driven joint modeling of the empirical distribution. Each structural (inter-relational) constraint is taken as one piece of learned knowledge that is supported by the data. The set of structural constraint components reveals all key association information embedded within the empirical distribution. That is, scientists need to accommodate all of these aspects of learned knowledge in order to create a realistic parametric probability model that fits the empirical data. The advantage of using maximum entropy is that no extra or artificial assumptions are imposed on the modeling.

We illustrate this approach using three models of the status network against each of the following three behaviors: grooming, alliance and aggression. Next, we consider four different types of constraint. The network data refer to data collected in 2009 and 2011. The notation $(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12})$ stands for the 4-dimensional vector and $\mathbb{p}_0(x_2^{12})$ stands for the null marginal distribution, which is evaluated as the empirical count proportions,

$$f_1(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = (x_1^{12} - \mathbb{p}_0(x_1^{12}))(x_2^{12} - \mathbb{p}_0(x_2^{12}); \quad (1)$$

$$f_2(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = (x_3^{12} - \mathbb{p}_0(x_3^{12}))(x_4^{12} - \mathbb{p}_0(x_4^{12})); \quad (2)$$

$$f_3(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = \begin{cases} 1 & if (x_1^{12} = x_4^{12}) and (x_2^{12} = x_3^{12}) \\ -1 & if (x_1^{12} = x_3^{12}) and (x_2^{12} = x_4^{12}); \\ 0 & else \end{cases} \quad (3)$$

$$f_4(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = ((x_1^{12} \quad x_2^{12}) - \mathbb{p}_3(x_1^{12} \quad x_2^{12} = 1))(x_3^{12} \quad x_4^{12}) - \mathbb{p}_3(x_3^{12} \quad x_4^{12} = 1)). \quad (4)$$

For a more detailed description of the methods described here, the reader is referred to Chan et al. (2013).

Our expectation is that, if each of the three behavioral networks is jointly linked or coupled to the keystone status network under the stable condition but not the unstable condition, then, as constraints are added at each additional step, the Chi-square values should fall to significant levels at a greater rate under the unstable condition than under the stable condition.

Table 1 summarizes the main results. The iterative procedure via the four structural constraints described in Eqs. (1)–(4) does improve the model's fit at each step. Some network dependence pairs are modeled very well from the outset, while others require more structural constraints to be discovered and incorporated. For instance, the 2009 inter-relationship between status and aggression behaviors is rather complicated. The Chi-square values are reduced at each step, though they never reach critical levels. In other words, these four constraint functions are not sufficient to model the 2009 network data. However, in sharp contrast, the same set of structural constraints works well for the 2011 data. As expected, the Chi-square values fall to significant levels at a greater rate in the unstable than in the stable conditions. This pattern reflects behavioral subtlety in the stable phase of the 2009 dynamics, while the same inter-behavioral relationship loses a large degree of subtlety in the unstable phase of the 2011 dynamics.

In fact, this loss in pattern is rather consistent across all pairwise comparisons between 2009 and 2011. These results strongly suggest that changes in the pairwise inter-network dependence could be used as an early warning signal of social collapse in captive monkey societies. It is natural to think that similar computations could potentially lead to an early warning signal for impending financial crises in a banking system.

7. Final thoughts

Our study of a captive monkey society is characterized by a particular set of features. Critically, data are available for several different behaviors (thus generating multiple layered networks), and are observed during both a stable phase and an unstable phase immediately preceding a social collapse. That is, there are observable and quantifiable characteristics predating a complete social collapse — the equivalent of a financial crisis in our banking system analogy. This dynamic feature of the data serves to inform our approach. This approach is based on two assumptions which are likely to characterize a banking system as well: that local information is available globally to all nodes, but that this information can be processed differently by each node.

These assumptions lead us to characterize the social system otherwise than has been commonplace in the network literature, especially considering the manner in which this literature has been adapted to characterize the banking system. Examples include the "social network" topology of the Austrian interbank market by Boss, Elsinger, Summer, and Thurner (2004) and of the federal funds market by Bech and Atalay (2008). The social network analyses in these studies rely on a single network and its characteristics. Moreover, these studies only report network summary statistics. Rather than thinking of the crisis as originating from the propagation of an exogenous shock or shocks through a static network, we believe that it is more fruitful to consider a banking system's network topology to be endogenous, varying over time, and multilayered.

The salient features of our analysis include a primary or keystone network that summarizes the overall relationship status across nodes, and a set of subsidiary networks. Each subsidiary network is related to the keystone network, and in fact may be partially governed or influenced by this network, but it also conveys independent information. Therefore, by examining the evolution of dependence patterns between the keystone network and its subsidiaries over time, we provide a natural metric of stress or tipping points that can be used as an early-warning indicator of a network collapse.

The flow topology of interbank lending is a natural candidate for the implementation of this nonhuman primate model-informed approach and its methods, as introduced in this paper. The informational assumptions we make are more natural than those characterizing networks for power grids or ecosystems. We recognize that a dynamic, multi-layered network specification in which crises can develop endogenously is inherently difficult to characterize fully. However, our approach is not about crafting a detailed model of the banking system; rather, it is about uncovering the empirical features of such a system that help describe and characterize its critical slowing, so as to identify the emergence of stress or tipping points. The nature of the information needed to apply this approach to interbank lending or other financial (human) systems successfully is up to the experts, such as policymakers and financial analysts (as was true for the expert primatologists), and thus conveniently falls outside the scope of this paper.

References

Amaral, LAN.; Barthelémy, M. Complex systems: challenges, successes and tools. In: Korutcheva, E.; Cuerno, R., editors. Advances in condensed matter and statistical physics. Nova Science Publishers; 2003.

Amaral LAN, Ottino JM. Complex networks: augmenting the framework for the study of complex systems. The European Physical Journal B. 2004; 38:147–162.

Amin, M.; Schewe, PF. Scientific American. 2007 May. Preventing blackouts; p. 60-76.

Arinaminpathy N, Kapadia S, May RM. Size and complexity in model financial systems. Proceedings of the National Academy of Sciences. 2012; 109(45):18338–18343.

Bank for International Settlement. 64th annual report. 1994. Technical report.

Barabási A-L. Taming complexity. Nature Physics. 2005; 1:68–70.

Bech, ML.; Atalay, E. The topology of the Federal fund market. Federal Reserve Bank of New York: Staff Reports; 2008.

Beisner BA, McCowan B. Signaling context modulates social function of silent-bared teeth displays in rhesus macaques. American Journal of Primatology. (in press). in press.

Beisner BA, Jackson ME, Cameron A, McCowan B. Detecting instability in animal social networks: genetic fragmentation is associated with social instability in rhesus macaques. PLoS One. 2011; 6(1):e16365. [PubMed: 21298105]

Boguñá M, Serrano MA. Generalized percolation in random directed networks. Physical Review E. 2005; 72:016106.

Boss M, Elsinger H, Summer M, Thurner S. Network topology of the interbank market. Quantitative Finance. 2004; 4:677–684.

Chan S, Fushing H, Beisner B, McCowan B. Joint modeling of multiple social networks to elucidate primate social dynamics: I. Maximum entropy principle and network-based interactions. PLoS One. 2013; 8(2):e51903. [PubMed: 23468833]

Datta, SB. The role of alliances in the acquisition of rank. In: Else, JG.; Lee, PC., editors. Primate ontogeny, cognition, and social behaviour. New York: Cambridge University Press; 1986. p. 219-225.

de Waal FBM. The integration of dominance and social bonding in primates. Quarterly Review of Biology. 1986; 61:459–479. [PubMed: 3543991]

Dorogovtsev SN, Mendes JFF. Comment on "Breakdown of the Internet under intentional attack". Physical Review Letters. 2001; 87:219801. [PubMed: 11736386]

Flack JC, de Waal F. Context modulates signal meaning in primate communication. Proceedings of the National Academy of Sciences. 2007; 104:1581–1586.

Flack JC, Girvan M, de Waal F, Krakauer DC. Policing stabilizes construction of social niches. Nature. 2006; 439:426–429. [PubMed: 16437106]

Fujii K, Fushing H, Beisner B, McCowan B. Computing power structure in directed networks: flows, percolation and imputed conductance. Journal of the Royal Society Interface. 2013 (submitted).

Fushing H, McAssey MP, Beisner B, McCowan B. Ranking network of captive Rhesus Macaque society: a sophisticated corporative kingdom. PLOS ONE. 2011; 6:e17817. [PubMed: 21423627]

Fushing H, McAssey MP, McCowan B. Computing a ranking network with confidence bounds from a graph-based Beta random field. Proceedings of the Royal Statistical Society, Series A. 2011; 467(2136):3590–3612.

Gai P, Kapadia S. Contagion in financial networks. Proceedings of the Royal Statistical Society, Series A. 2010; 466:2401–2423.

Haldane AG, May RM. Systemic risk in banking ecosystems. Nature. 2011; 469:351–355. [PubMed: 21248842]

May R, Levin S, Sugihara G. Ecology for bankers. Nature. 2008; 451:893–895. [PubMed: 18288170]

May R, Arinaminpathy N. Systemic risk: the dynamics of model banking systems. Journal of the Royal Society Interface. 2010; 46:823–838.

McCowan B, Beisner BA, Capitanio JP, Jackson ME, Cameron A, Seil SK, et al. Network stability is a balancing act of personality, power, and conflict dynamics in rhesus macaque societies. PLoS One. 2011; 6(8):e22350. [PubMed: 21857922]

Newman MEJ, Strogatz SH, Watts DJ. Random graphs with arbitrary degree distributions and their applications. Physical Review E. 2001; 69:026113.

Oates-O’Brien RS, Farver TB, Anderson-Vicino KC, McCowan B, Lerche N. Predictors of matrilineal overthrows in large captive breeding groups of rhesus macaques (Macaca mulatta). Journal of the American Association for Laboratory Animal Science. 2010; 49:196–201. [PubMed: 20353695]

Sade, DS. A longitudinal study of social behavior of rhesus monkeys. In: Tuttle, R., editor. The functional and evolutionary biology of primates. New York: Aldine Atherton; 1972. p. 378-398.

Scheffer M, Bascompte J, Brock WA, Brovkin V, Carpenter SR, Dakos V, et al. Early-warning signals for critical transitions. Nature. 2009; 461:53–59. [PubMed: 19727193]

Schrödinger, E. What is life?. Cambridge University Press; 1944.

Schwartz N, Cohen R, ben Avraham D, Barabási A-L, Havlin S. Percolation in directed scale-free networks. Physical Review E. 2002; 66:015104.

Simon HA. The architecture of complexity. Proceedings of the American Philosophical Society. 1962; 106:467–482.

Special issue on "Complex Systems and Networks". Science. 2009; 325:406–432. [PubMed: 19628850]

Appendix A. Trickling-down percolation

This appendix relies on the work of Fushing, McAssey, Beisner, and McCowan (2011) and Fushing, McAssey, and McCowan (2011). These papers describe the trickling-down percolation algorithm in more detail, together with its advantages over similar algorithms which are available in the literature. We provide the basic steps here.

Percolation relies on two components: the empirical (status) relational data matrix $C = [c_{ij}]$ and the Beta random field $\{\text{Beta}(ac_{ij}+b, ac_{ji}+b)\}$ which is built onto it. Using these components, the algorithm’s steps are:

- [P-0] Consider a potential dominance action initiated by the i^{th} subject toward a randomly selected immediate neighbor, say the j^{th} subject, that is, $c_{ij} > 0$. The probability of this action being successful is s_{ij} which is a random simulated strength from $\text{Beta}(ac_{ij} + b, ac_{ji} + b)$.
- [P-1] Generate a Bernoulli random variable $B(1, s_{ij})$ with probability $q^{(0)}(i,j)$ for the outcome “success (= 1)”. If it turns out to be a “failure (= 0)”, this trickling down process stops.
- [P-2] Repeat steps [P-0]–[P-1] and cycle until it stops. Then record the trickling down path in a progressive fashion into a matrix format as follows:
  1. Let the trickling-down path be $< i - (i_1, \ldots. i_k) - i_{k+1} >$ with $i = i_0$ and only the ending action from i_k to i_{k+1} being a failure;
  2. The percolation matrix is denoted by $E_m = [e_{hl}]$, which was initially set to zeros for all its entries, and then the entries on the i^{th} row and $\{i_1, \ldots. i_k\}$

columns are added by 1; the entries on the i_1^{th} row and $\{i_2, \ldots.i_k\}$ are added by 1. Proceed with this recording until the entry (i_{k-1}, i_k) is added by 1.

3. Record the path-ending action by adding 1 to the entry (i_{k+1}, i_k), since it is a failure.

- [P-3] Repeat step [P-2] M times in order to construct an ensemble of trickling down paths and record them in the ensemble $E_M = \sum_{m=1}^{M} E_m$.
- [P-4] Convert the ensemble matrix E_M into an action transmission matrix $A_M = [a_{ij}]$ with $a_{ij} = \frac{E_{M,ij}}{E_{M,ij} + E_{M,ji}}$.
- [P-5] Finally, perform the rescaling step: $DA_M = diag\,(\ldots., \Sigma_{j=1}\, c_{ij,\ldots..})[aij]$ as the final conductance matrix.

Appendix B: Derivation of the maximum entropy procedure by Chan et al. (2013)

To simplify the notation, we let $\boldsymbol{x}$ be the four-dimensional vector $(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12})$. We maximize the relative entropy for $\mathbb{p}_{\mathrm{m}}$ by maximizing

$$\Sigma \mathbb{p}_{\mathrm{m}}(\boldsymbol{x}) \log\left(\frac{\mathbb{p}\mathrm{m}(\boldsymbol{x})}{\mathbb{p}0(\boldsymbol{x})}\right),$$

summed over all probabilities, where $\mathbb{p}_0$ is the null probability distribution and $\mathbb{p}_{\mathrm{m}}$ refers to the probability distribution with maximum entropy, subject to the constraints of the data. These constraints are

$$\hat{E}_1[f_1(\boldsymbol{x})] = \Sigma f_1(\boldsymbol{x}) \frac{d(\boldsymbol{x})}{n_{12}},$$

where the expectation is determined by

$$\hat{E}_1[f_1(\boldsymbol{x})] = \Sigma_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) f_1(\boldsymbol{x}).$$

Thus, we have two constraints:

$$\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) f_1(\boldsymbol{x}) = \hat{E}_1[f_1(\boldsymbol{x})],$$

and the sum of all probabilities is one:

$$\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) = 1.$$

Therefore, we can maximize the entropy using the Lagrange operation

$$L(\mathbb{p}_1, \lambda_1, \mu) = \sum \mathbb{p}_1(\boldsymbol{x}) \log\left(\frac{\mathbb{p}_1(\boldsymbol{x})}{\mathbb{p}_0(\boldsymbol{x})}\right) - \lambda_1 \left(\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) f_1(\boldsymbol{x}) - \hat{E}_1[f_1(\boldsymbol{x})]\right) - \mu \left(\sum_{\boldsymbol{x}} \mathbb{p}_1(\boldsymbol{x}) - 1\right).$$

We take the derivative of the Lagrange operation to get

$$\frac{\partial}{\mathbb{p}_1} L(\mathbb{p}_1, \lambda_1, \mu) = \log\left(\frac{\mathbb{p}_1(\boldsymbol{x})}{\mathbb{p}_0(\boldsymbol{x})}\right) + 1 - \lambda_1 f_1(\boldsymbol{x}) - \mu = 0.$$

By solving this equation for $\mathbb{p}_1$, we get

$$\mathbb{p}_1(\boldsymbol{x}) = \mathbb{p}_0(\boldsymbol{x}) \exp\left(-\lambda_1 f_1(\boldsymbol{x})\right) \exp(-\mu + 1).$$

Let $Z(\lambda_1) = \Sigma_{\boldsymbol{x}} \mathbb{p}_0(\boldsymbol{x}) \exp(-\lambda_1 f_1(\boldsymbol{x}))$, which is called the partition function. Applying the constraint that all probabilities must sum to 1, we determine that

$$\exp(-\mu + 1) = \frac{1}{Z(\lambda_1)}.$$

Then, applying the first constraint, we get

$$\sum_{\boldsymbol{x}} \frac{1}{Z(\lambda_1)} \mathbb{p}_0(\boldsymbol{x}) \exp\left(-\lambda_1 f_1(\boldsymbol{x})\right) f_1(\boldsymbol{x}) = \hat{E}_1[f_1(\boldsymbol{x})],$$

which is equivalent to

$$\frac{\partial}{\partial \lambda_1} \log Z(\lambda_1) = \hat{E}_1[f_1(\boldsymbol{x})].$$

In order to find $\mathbb{p}_1(x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = \mathbb{p}_1(\boldsymbol{x}) = \frac{1}{Z(\lambda_1)} \mathbb{p}_0(\boldsymbol{x}) \exp\left(-\lambda_1 f_1(\boldsymbol{x})\right)$, we solve for λ_1 by the previous equation. This process can be repeated iteratively for each f_k and $\mathbb{p}_k$.

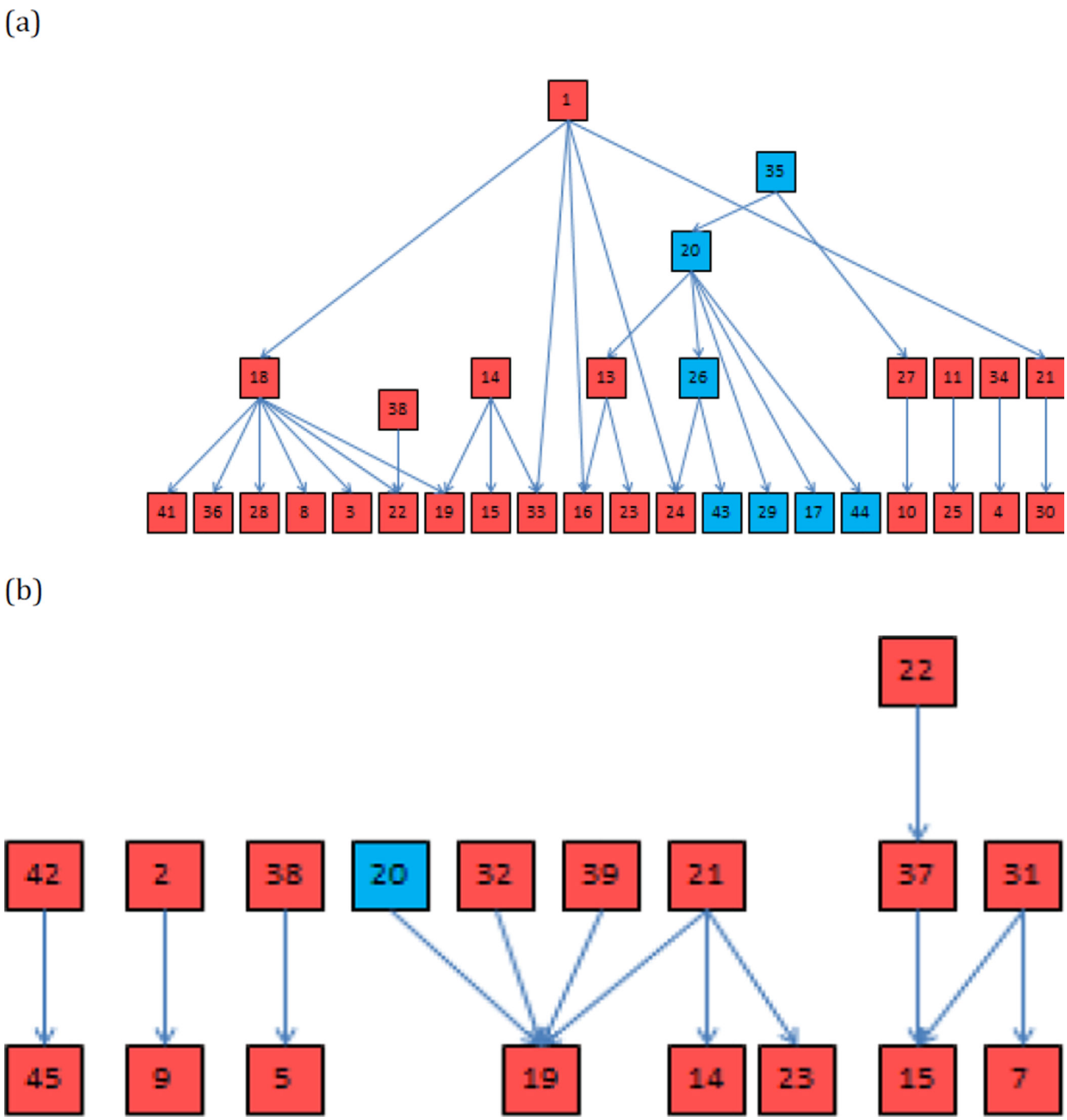

Figure 1.

Status power structure over time using trickling-down percolation: panel (a) 2009; panel (b) 2011. Note the differences in structural subtlety and complexity between (a) stable and (b) unstable phases, with the unstable phase appearing bare and featureless in comparison to the stable phase.

Notes: each box indicates a monkey. Females are represented in red and males in blue. Each number denotes a different individual. Arrows point away from the dominant animal to the subordinate animal, indicating that the subordinate animal gives status signals to the dominant animal.

Figure 2.

An example of network coupling of two directional networks (grooming and agression).

Notes: each network in panel (a) has four nodes. Directional links are represented by an arrow. Panel (b) shows how the information from the two networks can be combined using four binary element vectors.

Table 1

Total Chi-square values of iterative joint modeling: 2009 vs 2011.

2009 vs. 2011	**indep**	**f1**	**f2**	**f3**	**f4**
Groom/status-2009	297.07	81.04	49.71	38.75	23.72
Groom/status-2011	170.47	19.56*	16.78**	6.95**	4.81**
Aggression/status-2009	537.94	458.30	383.15	106.35	84.88
Aggression/status-2011	243.52	217.10	210.87	31.75	25.63
Alliance /status-2009	238.13	67.86	36.82	27.28	18.85**
Alliance/status-2011	142.46	23.53	20.59*	4.07**	3.87**

Note: The 95% and 99% critical values of the Chi-square distribution for this problem are ** 19.02, and * 21.67 respectively.