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RESIDUAL FIELDS IN SUPERCONDUCTING DIPOLE AND QUADRUPOLE MAGNETS*

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Summary

This paper discusses the magnetic measurements of residual field in superconducting dipole and quadrupole magnets. The technique used to measure residual fields is described herein. The results of the measurement are presented and compared with a simplified theory, which predicts the basic nature of residual fields. The theory and experiment agree reasonably well despite the simplifying assumptions made in the theory. The residual fields generated in a superconducting magnet do affect the uniformity of the field generated in such a magnet. The effects of residual field are worst at low field, which is precisely where one wants the most magnet uniformity in a superconducting synchrotron. The theory suggests that one can reduce the magnitude by: (1) reducing the strand diameter in the material (smaller strands require more twist to eliminate coupling); and (2) reducing the low-field critical current of the material, which also reduces ac loss and instabilities. The theory also suggests that superconducting synchrotrons with injection inductions as low as 1 kG must have superconductor strands as small as 1 to 2 μm.

Introduction

The existence of residual fields in superconducting dipoles or quadrupoles was predictable from the theories of a number of investigators. The fact that the theory of residual fields is tied so closely to the theory of superconducting ac losses is pointed out by Smith, Hancox, Bean, and others.[1-3] What was surprising, at least to me, was the fact that these residual fields have large fractions of undesirable higher multipoles, even in magnets which are designed to eliminate these multipoles when they are excited.

We found the residual fields by accident. We had charged our 4-in. dipole, measured its field, and brought the current back to zero. We decided to measure the field at zero current in order to determine the effects of electronic noise and integrator drift on our measurements. We as a result measured the residual fields generated by superconductor circulating currents. Further investigations revealed that these fields had a number of interesting properties because of their origin.

Our magnetic measurement program was well under way when we had the opportunity to measure the 6-in. quadrupole doublet built by Los Alamos Scientific Laboratory. These measurements were made by the Laboratory Magnetic Measurements Group in August 1970. Measurements at zero current revealed the same sort of residual fields we had been measuring in our dipole magnet.

The Magnets and the Experimental Apparatus

The dipole magnet used for the residual-field measurement was built in 1969 as a test magnet to perfect our bent-winding technique. The magnet was wound with about 900 ft of twisted 0.050-in. by 0.127-in. (0.127-cm by 0.323-cm) multicore superconductor. The conductor consists of a copper niobium-titanium matrix with a ratio of copper to superconductor of 2 to 1. The superconductor is subdivided into 360 strands each with an approximate diameter of 0.0027 in. (69 μm).

The magnet windings are in four separate layers. Each layer consists of two separate coils which are wound flat and bent in a semicircle to fit the coil form. The thickness of each layer is the 0.127-in. (0.323-cm) dimension of the superconductor. The magnet has a cold bore diameter of 4 in. (10.17 cm) and an overall length of the magnet including ends of 16.6 in. (42.2 cm). As magnetic shields, the magnet has two iron shells, of which only the inner one was used during part of residual-field measurements. Its inner diameter is 5.62 in. (14.27 cm) and it is 0.500 in. thick (1.27 cm) (see Fig. 1). See Ref. 4 for more information on the magnet.

The magnetic measurement equipment used on the LRL dipole magnet consists of four primary parts: (1) the magnetic measurement coils, (2) the mechanical linkages which turn and index the measuring coils (see Fig. 2), (3) the operational amplifier integrator which provides the signal to be analyzed by the computer, and (4) the digital voltmeter and/or digital printer. The CDC 6600 computer was used to analyze the data and direct the printout of the magnetic measurement results. The theory of the magnetic measuring system and the computer program used to analyze the magnetic measurement results is discussed in Ref. 5. See Ref. 4 for detailed information on the magnetic measuring apparatus.

The Los Alamos quadrupole doublets are wound with a round 0.050-in. (0.127-cm) twisted superconductor with strands which are approximately 0.004 in. (100 μm) in diameter. These magnets have coils with an inner radius of 4.050 in. (10.287 cm). A more detailed description of the magnets is found in Ref. 6.

The Los Alamos quadrupole magnets were measured by our Magnetic Measurements Group by using a system of moving bucking coils. The signal from these coils is fed into an operational amplifier integrator and then to an x-y plotter where it is plotted for further data reduction.

The Experimental Results

The first residual magnetic induction we measured on our dipole without the iron shield had a magnitude of about 30 G. We at first thought the residual field was caused by some magnetic material within the magnet cryostat. It turned out that a portion of the measured residual field could be attributed to that cause, but by no means all of it. The first investigations of the residual field phenomena revealed that:

(1) the residual fields have long time constants, (2) the residual fields are a function of previous magnetic field in the coil, (3) the residual fields are affected by lead reversal, (4) residual fields are drastically changed by a magnet transition, and (5) the residual fields are rich in the higher symmetrical multipoles (N = 3, 5, 7, etc. for the dipole magnet).

The magnitude of the residual induction as a function of previous flux changes in the LRL dipole is illustrated in Fig. 3; this magnitude rises almost linearly until the previous flux change reaches 4 kG, where the magnitude levels off. This induction corresponds to the penetration induction of a 0.0027-in. strand of Nb-Ti. It appears then that the residual field is a function of penetration induction. Since penetration induction is a function of strand size, then one could assume that both the magnitude and leveling point of the residual field is some function of strand diameter. The increasing magnitude of the residual field is illustrated in Table 1 and Figs. 3 and 4. See Ref. 4 for more such examples.

Measurements with iron indicate that the residual field is increased 60% by the iron, just as the two-dimensional field was at low current excitations. The magnetization due to strained nonmagnetic stainless steel wire used to bind the magnet does not have nearly the richness of higher symmetrical multipoles found in the cases where the superconductor has been penetrated by moving flux (see Table 1).

We have found that the measured residual fields have extremely long time constants as long as the magnet is kept cold and superconducting. Our measurements indicate that within the limits of accuracy of the measuring apparatus, the time constant of the residual field is at least a year. It is most probable that the time constant for the residual fields is much longer than that.

Figure 5 illustrates the effect of magnet transitions on the residual fields generated by the magnet. A strong transition has enough energy to warm up nearly all of the superconductor in the magnet. The result is that the circulating currents in the superconductor are wiped out by the heat.

The residual field we have measured could have been caused by the following. (1) Magnetization of ferromagnetic material. This type of residual field is readily observable. (2) Eddy currents between shorted turns. This type of residual field has been observed, but the time constants are small, on the order of 30 or 40 sec. (3) Circulating current in the bulk superconductor where both the Nb-Ti and copper carry circulating currents. These time constants are quite small, say 10 to 20 sec, because the superconducting material has been twisted. (4) Circulating currents in the superconducting strands themselves. These circulating currents should have nearly infinite time constants. All of the experimental evidence points to the fourth cause as being the prominent one. The theory for residual fields caused by circulating currents in the superconducting strands predicts a residual field which agrees with measured fields reasonably well.

The Los Alamos quadrupole magnets exhibit a similar residual field behavior as the LRL dipole magnet. The residual field measured after a 1.8-kG induction change is much lower than those measured for a 12-kG or 18-kG induction change. (The penetration induction for the Los Alamos quadrupole material is estimated to be about 6 kG.) The magnitude of the residual field after a 12-kG induction change is nearly the same as after an 18-kG induction change. The quadrupoles exhibit no ferromagnetic residual field despite the presence of an iron shield. See Ref. 7 for further information on the quadrupole measurements.

Comparison of Theory with Experiment

The theory I used to calculate the residual field generated in two-dimensional dipole or quadrupole magnets is based on classical doublet theory (a doublet in hydrodynamics is a source and sink side by side). The circulating currents in a small superconducting strand can be represented by a classical doublet. One can integrate these small doublets over whole conductor areas to calculate the field generated by the superconducting strands in a coil. This field can be expanded in a power series about the origin (the

Table 1. Measured multipole components of two-dimensional residual inductions created by the LRL dipole at zero current.
(There is no previous higher current history.)
(Residual measured with no iron shell.)

Previous induction change	0 Stainless steel wire magnetization	0.803 kG	1.59 kG	3.18 kG	7.98 kG*	15.9 kG*	20.7 kG*
Dipole component of residual induction	10.8 G	12.1 G	17.3 G	28.3 G	31.8 G	29.8 G	29.5 G
				Multipole ratios**			
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0117	0.0646	0.0542	0.0229	0.0118	0.0621	0.0166
3	0.2789	0.1476	0.1341†	0.4882	0.7526	0.7482	0.7089
4	0.0068	0.0080	0.0132	0.0296	0.0224	0.0214	0.0280
5	0.0728	0.0173†	0.1212	0.2094	0.0821	0.0710	0.0449
6	0.0019	0.0008	0.0225	0.0160	0.0158	0.0278	0.0350
7	0.0269	0.0068†	0.0139	0.0513	0.0577	0.0661	0.0513

* Previous flux change exceeds the penetration induction of 4.2 kG.
** Taken at the measurement radius of 3.65 cm.
† Phase angle reversal.

center of the magnet). The only assumption needed to make the theory work is that the diameter of the strand be much smaller than the bore diameter of the magnet. Space does not permit me to derive the theory here--one should look at Ref. 8 for further details.

At the time of this writing, the full residual field theory had not been programed on the computer, so the following simplifying assumptions are made to simplify the calculations. (1) There is a dipole field distribution in the dipole coils and a quadrupole field distribution in the quadrupole coils. This is a thin-coil approximation. (2) The doublet strength is the same throughout the magnet. I use complex variable notation to calculate the two-dimensional dipole magnet with no iron shield. The following equations can be used to calculate the approximate residual field generated by a single-layer superconducting dipole or quadrupole:

$$H^*(Z) = \sum_{n=1}^{\infty} a_n Z^{n-1} ,$$

$$B^*(Z) = \mu_o H^*(Z) ,$$

$$\mu_o = 4\pi \times 10^{-7} .$$

The multipole coefficient equations are:

$$a_n = -\frac{\epsilon_o J_c \beta D}{2\pi i\,(s+1)} \sin 2\theta_o \ln\left(\frac{R_2}{R_2}\right);$$

for $n = 1$ (this can only occur when $T = 1$),

$$a_n = -\frac{T\epsilon_o J_c \beta D}{\pi i\,(s+1)} \frac{n \sin((T+n)\,\theta_o}{(n+T)(1-n)} \left[R_2^{\,1-n} - R_1^{\,1-n}\right];$$

for $n = T(2p+1)$ $T=1$ and 2 $p=0, 1, 2, 3, \cdots ,$

$$a_n = 0 ;$$

for $n \neq 2T(2p+1)$ $T=1$ and 2 $p=0, 1, 2, 3, \cdots ,$

where

Z = $Re^{i\theta}$ in polar coordinates (m),
$H^*(Z)$ = the complex conjugate of magnetic field at Z (A/m),
$B^*(Z)$ = the complex conjugate of magnetic induction at z (T),
ϵ_o = doublet strength factor (for fully penetrated round conductors use ± 0.423),
J_c = superconductor critical current (A/m^2),
D = superconducting strand diameter (m),
β = the coil packing factor,
s = normal metal-to-superconductor ratio,
R_1 = inner coil radius (m),
R_2 = outer coil radius (m),
θ_o = coil angle (radians),
n = multipole pair number,
T = fundamental pole pair number ($T=1$ dipole, $T=2$ quadrupole).

The preceding equations were used to calculate the multipoles generated by the LRL dipole and the Los Alamos quadrupoles. The measured and theoretical dipole components of residual field agreed quite well. The higher multipole calculations do not agree very well because the thin-coil approximation is much less valid for the high multipoles than for the fundamental. (See Fig. 6 for the comparison of theory and experiment.) The agreement between theory and experiment on the quadrupoles was not as good as the dipole agreement. The calculated quadrupole component of magnetic induction at the magnetic measurement radius was 20.2 G as compared with a measured quadrupole component of magnetic induction of 15.5 G.

The Effect of Residual Fields on Magnet Uniformity

The residual fields measured at zero current do have an effect on the quality of fields generated by the magnet at other currents. The effect of residual fields on magnet quality is predictable from theory. Table 2 and Fig. 7 show the effects of residual field on magnet performance. Simple theory tells us that the magnitude and sign of the residual field is a function of previous flux penetrations of the material.

Figure 7 and Table 2 show the effect of charge direction and the formation of residual field on magnetic field uniformity. Data taken at 1, 10, and 100 A indicate that the field profile and field aberrations are quite different, depending on the direction of charge and the previous charge history. The cases in which the current was raised to +1, +10, and +100 A had no previous higher current history, hence there were no residual fields because of the previous charge. A fairly large residual field effect is present in the +100-A case going up, because a large portion of the superconductor has been penetrated by the 2.6-kG flux change caused by charging the magnet to 100 A. In the case in which the field was brought down from +1000 to +100 A, a total flux change of -16.7 kG was made in the coil. This flux change penetrated the superconductor fully, thus generating a residual field which is present in the measured magnetic field. The residual field is even more apparent in the 10-A case and it dominates in the 1-A case going down. Therefore one must expect the residual field to be present at low fields in a superconducting dipole or quadrupole.

Magnetization of the superconductor, which is caused by the circulating current in the superconductor, is analogous to iron magnetization. It is clear that a hysteresis loop exists for superconductor magnetization just as it does for iron magnetization. It becomes quite clear, then, that the magnitude of the residual field is related to superconductor ac loss and superconductor stability. It is expected that the residual field should be proportional to the superconductor strand diameter and the ampere meters of superconductor in the magnet. In addition, residual field is also proportional to the critical current in the strand, and is a function of the amount of superconductor used to carry circulating current. In other words, if nearly all of the available superconductor is being used to carry transport current, the residual field will be low because there is little current-carrying capacity left in the superconductor to carry circulating currents.

Residual fields are very likely to be a problem

Table 2. The effect of previous induction changes on induction uniformity measured in the LRL dipole. (With iron.)

Pole pair number	Multipole ratios[*] 1 A going up[≠]	1 A going down[§]	10 A going up[≠]	10 A going down[§]	100 A going up[≠]	100 A going down[§]
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0137	0.0414	0.0115	0.0165	0.0136	0.0111
3	0.0458	0.5689**	0.0230	0.1382**	0.0084	0.0219**
4	0.0076	0.0102	0.0016	0.0072	0.0020	0.0041
5	0.0115	0.0353	0.0011	0.0081	0.0048	0.0017
6	0.0012	0.0107	0.0008	0.0022	0.0018	0.0040
7	0.0083	0.0390**	0.0026	0.0131**	0.0022	0.0047**
Dipole component of induction	26.1 G	65.6 G	255 G	298 G	2.54 kG	2.61 kG
Previous induction change	+26.1 G	-19.3 kG	+255 G	-19.0 kG	+2.54 kG	-16.7 kG

* Taken at the measuring radius of 3.65 cm.
** Phase angle 180° off the case where the current is rising; induced circulating currents are of opposite sign.
≠ The magnet was previously charged to 19.3 kG without transition.
§ No previous flux change history--started at zero field.

that must be faced by superconducting synchrotron designers. The residual fields generated by circulating currents in the superconductor will be at their worst at injection. A reduction of strand size (which is needed for low ac losses) will, of course, reduce the magnitude of the residual fields. Experimental evidence[9] suggests that the finer stranded materials will have high critical currents (as high as 5×10^6 A/cm^2). This suggests that strand sizes as small as 1 μm are needed if we are to be able to inject into a superconducting synchrotron at fields of around 1000 Oe. Thus it is clear that residual field will be a problem to those who intend to use superconducting dipoles and quadrupoles at low fields.

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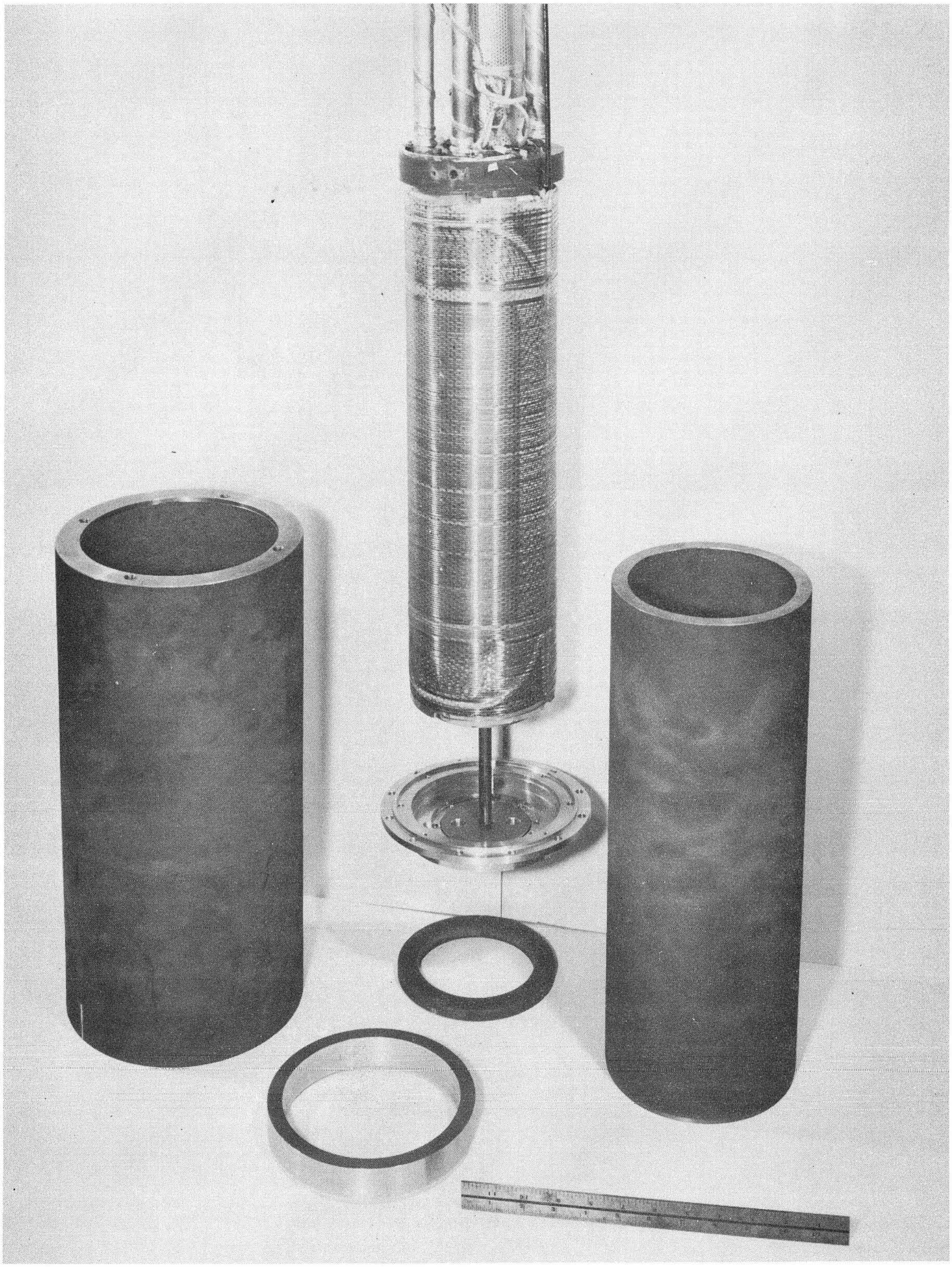

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Fig. 1. 4-in. Superconducting dipole magnet.

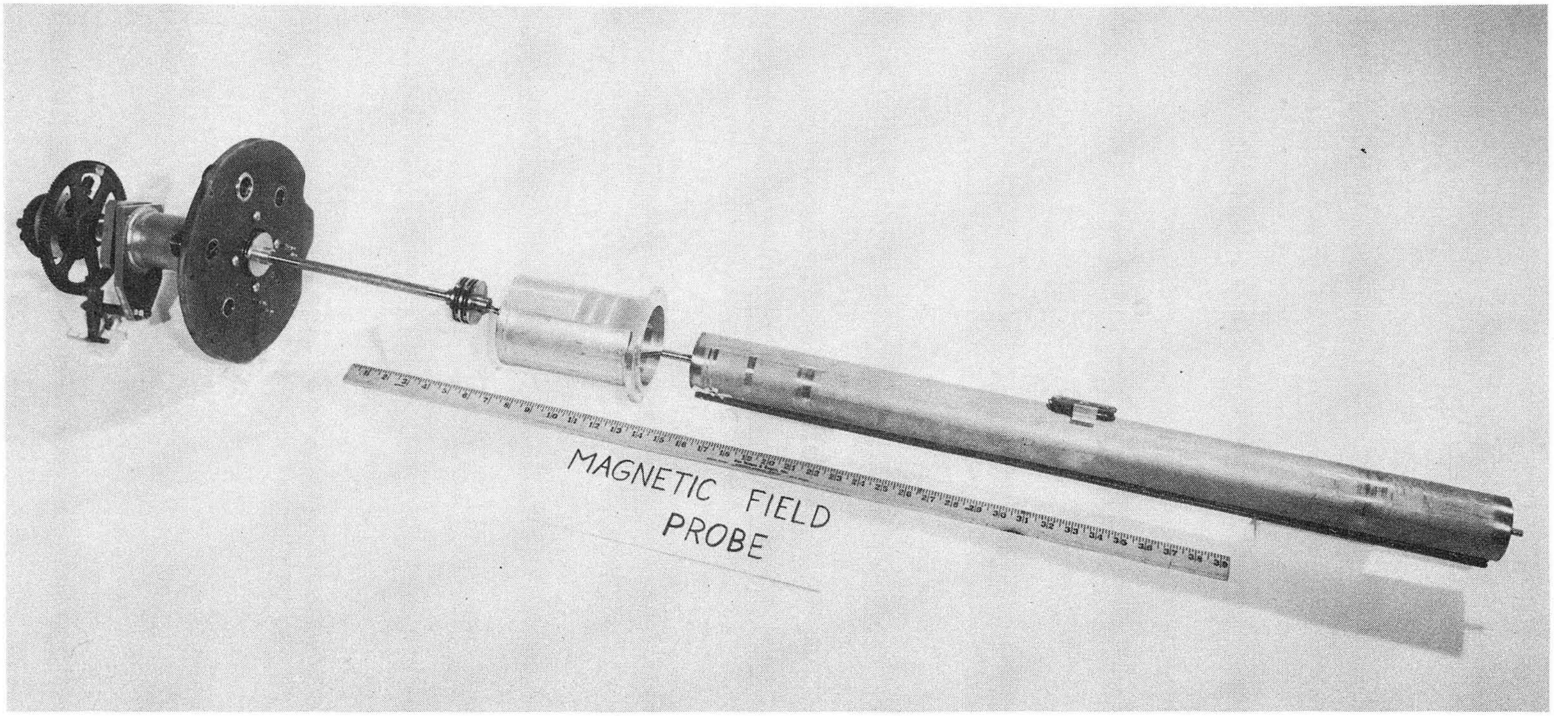

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Fig. 2. Magnetic measurement coils and indexing system.

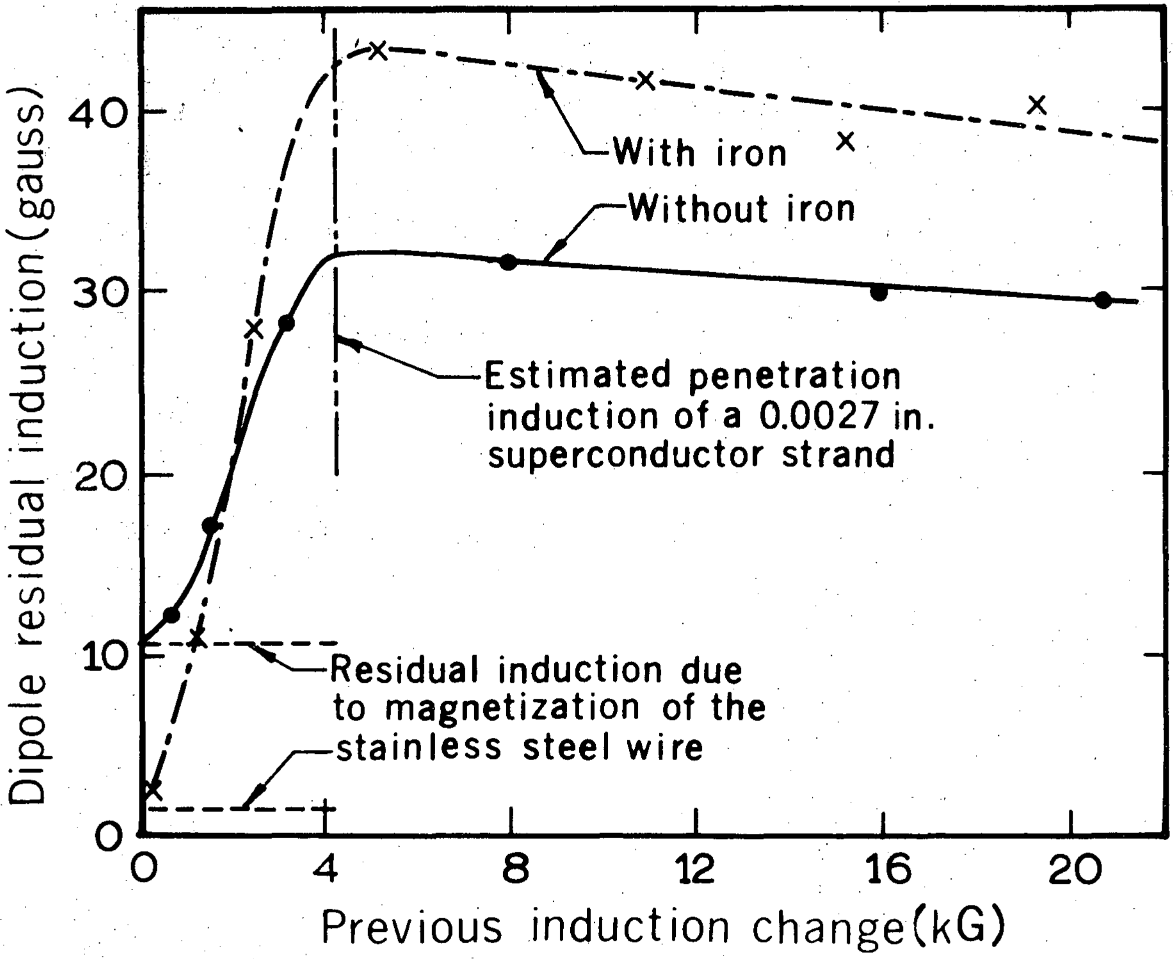

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Fig. 3. Dipole component of residual induction vs previous induction change.

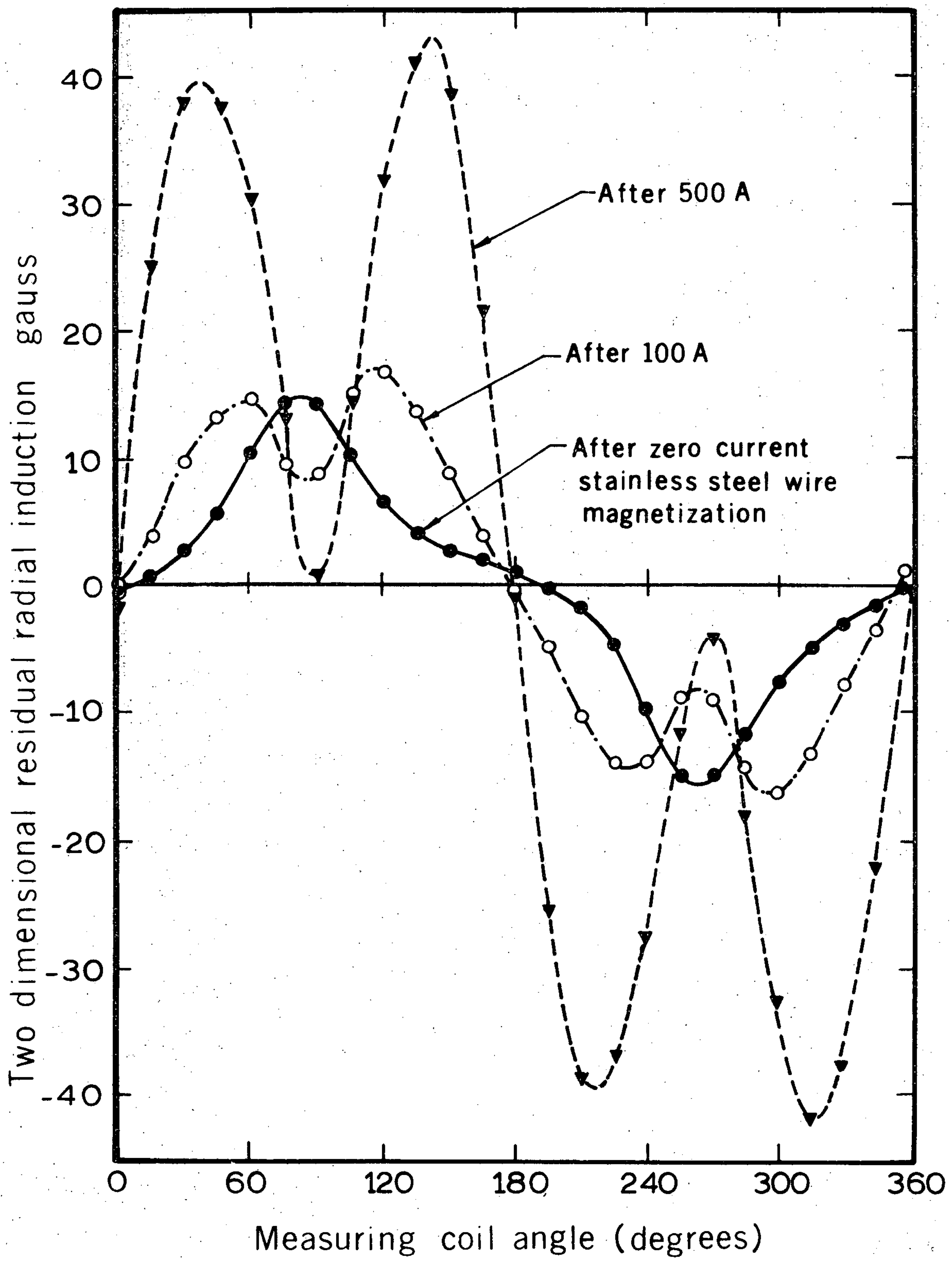

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Fig. 4. Radial component of induction vs measuring coil angle (taken at a radius of 3.65 cm).

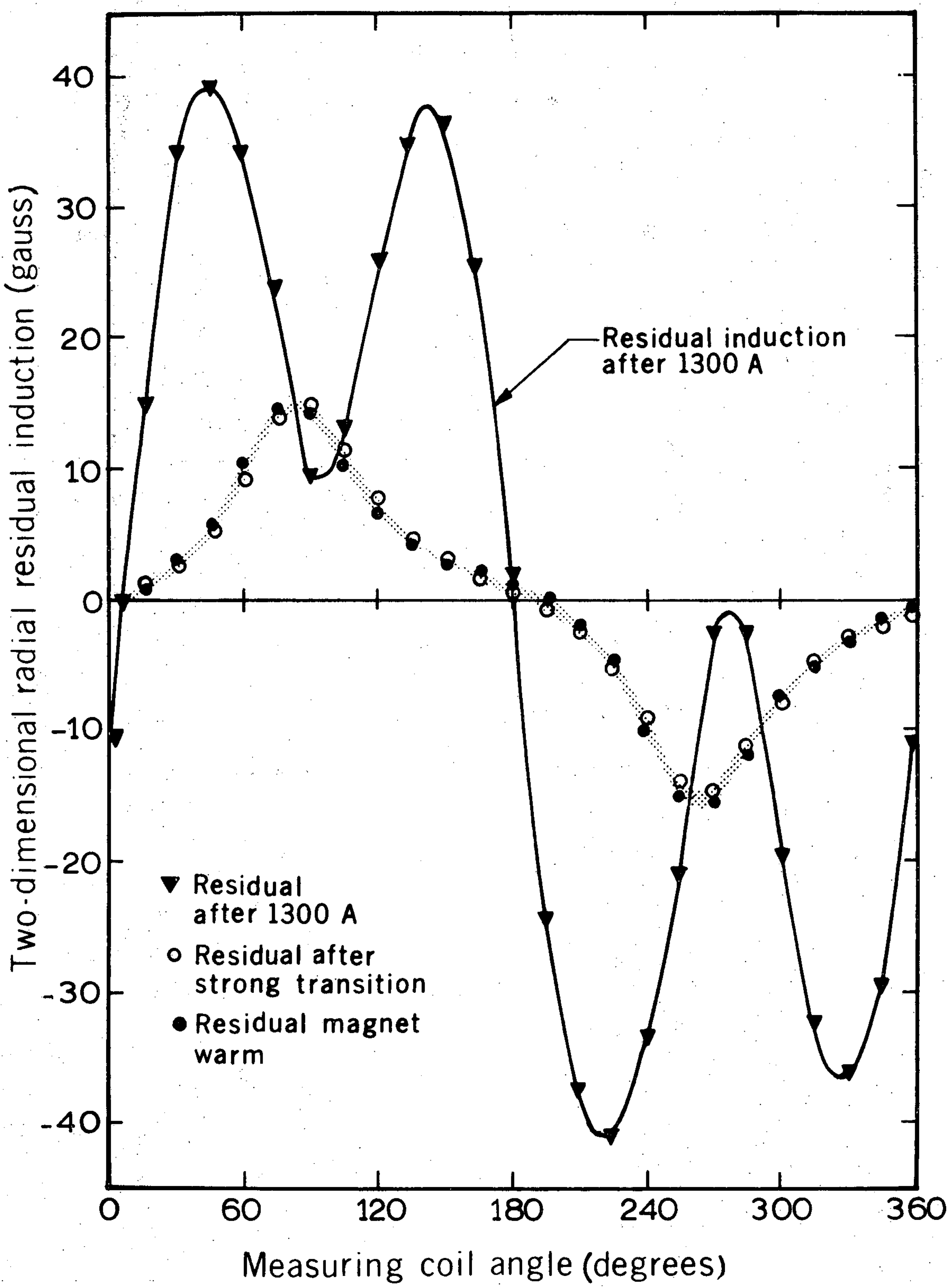

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Fig. 5. Effect of magnet transition on residual induction-radial component of induction vs measuring coil angle (taken at a radius of 3.65 cm).

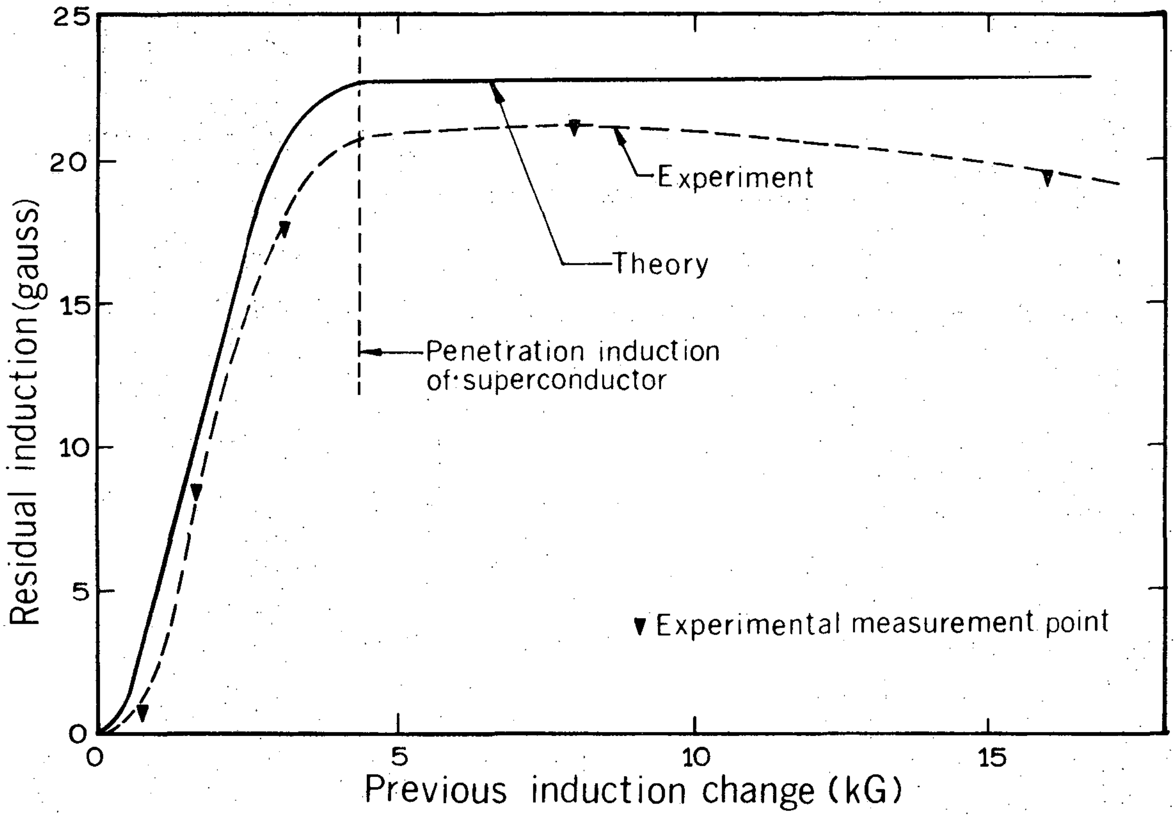

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Fig. 6. Comparison of theory and experiment--residual induction vs previous induction change.

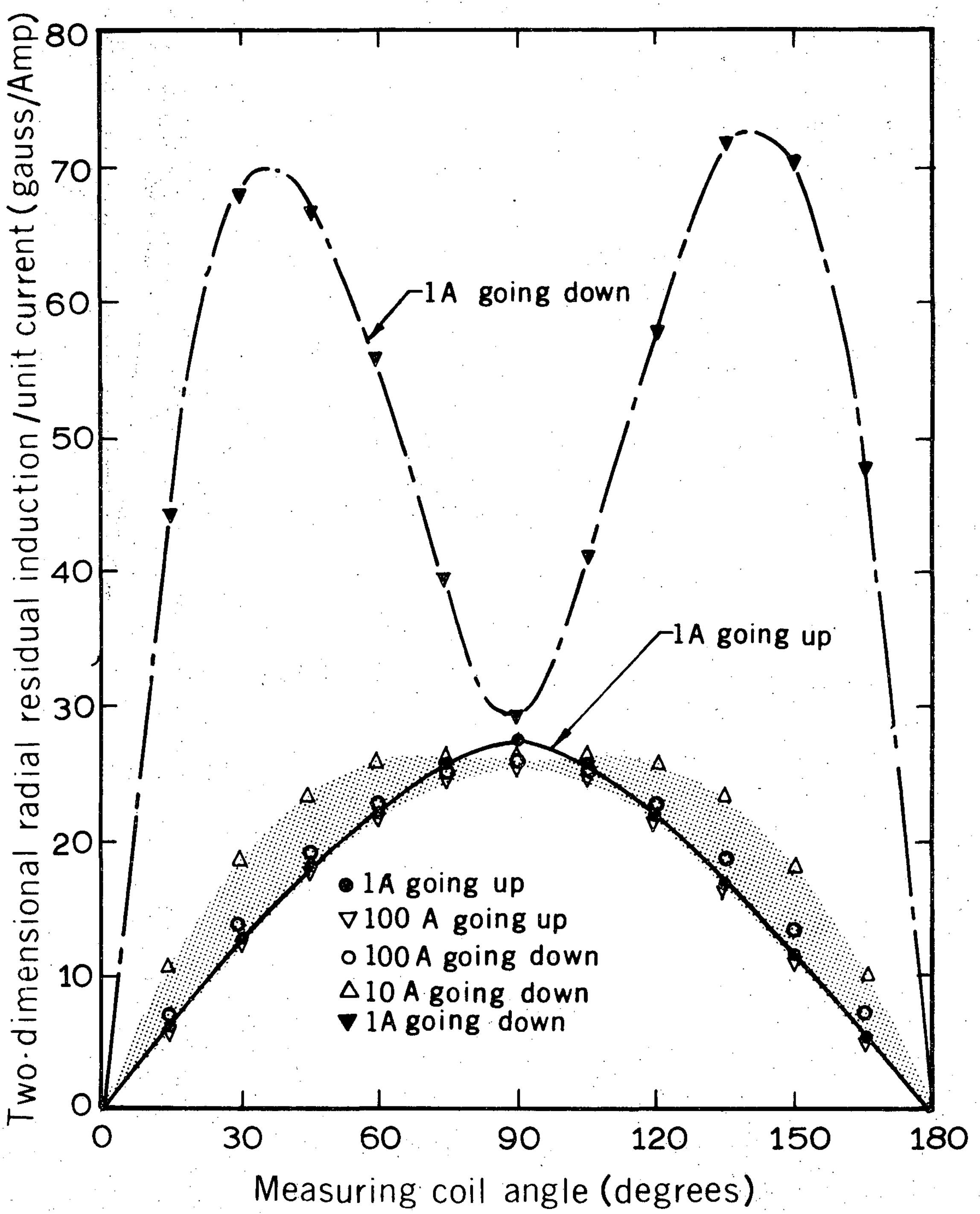

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Fig. 7. Effect of residual field on magnet uniformity--radial induction per unit current vs coil angle (taken at a radius of 3.65 cm, with the iron shield).

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