A MARKOVIAN POLICY MODEL OF INTERREGIONAL MIGRATION

ANDRE ROGERS, University of California, Berkeley

Interregional migration has become a subject of increasing study in the United States during the past decades. Of all the components of population change, it is the most difficult to measure and forecast. Among the contributors to change, it is rapidly becoming the critical variable in interregional population analyses.

Geographical mobility is interrelated with a myriad of factors—socio-economic, political, psychological. Hence, any attempt to incorporate all the relevant variables in a forecasting model of migratory behavior is likely to produce an intractable abstraction of organized complexity. However, by reducing a mass of observational data on migration streams into summary form, which the mind can easily comprehend, one may observe certain regularities in movement patterns. Moreover, it appears that, at least in the short run, such summarization may provide useful indicators of current trends and their long-range implications.

This paper reports progress on the development of a simple model of interregional migration. The model is not causally structured but merely purports to describe current behavior and to indicate the distributional consequences of present mobility trends. In addition, extensions of the basic model allow one to examine the policy implications of different sets of goals with regard to long-term population distribution.

In this paper Part I defines the fundamental model of interregional migration. Part II extends the model to include population distribution goals and defines the policy requirements for the achievement of these goals. Part III focuses on California for empirical explorations. Finally, Part IV concludes with a brief consideration of how Markovian models may be used in regional data bank operations and continuously updated information systems.

I. A MARKOVIAN MODEL OF INTERREGIONAL MIGRATION

Consider an interregional system of m-1 regions. Associated with every region, i, is a population, w_i. During each unit of time, e. g., one year, it is observed that a certain fraction of the population in region i migrates to region j. Thus one may imagine an interregional flow matrix which describes the number of people, k_{ij}, who, during a specified time period, move from

---

The preparation of this report was financed in part through an urban planning grant from the Housing and Home Finance Agency under the provisions of Section 701 of the 1954 Housing Act, as amended.

region i to region j. To close out the system, introduce an additional rest-of-the-world region which collects all people who move from i to an unidentified j ($j \neq 1, 2\ldots, m$-1). That is, define an m^{th} region which, for example, collects all migrants leaving the state, all individuals who die during the unit interval, and all international movers. Thus we have:

$$K = \begin{pmatrix} k_{11}\, k_{12} \ldots k_{1m} \\ k_{21}\, k_{22} \ldots k_{2m} \\ \ldots \\ k_{m1} k_{m2} \ldots k_{mm} \end{pmatrix}.$$

Transform the m by m flow matrix into a transition matrix by dividing each element k_{ij} by its corresponding row sum. This defines a transition matrix P with elements $p_{ij} = k_{ij}/w_i$. Since $0 \leq p_{ij} \leq 1$, we may think of them as probabilities and apply Markov chain theory to what otherwise is a deterministic formulation. There are two ways to use the matrix P. One is to consider the total population of each state after n periods. Kemeny and Snell call this the "collective process."[1] The second and more common use of P is to consider each p_{ij} as the probability that an individual, selected at random from the population at i, will move to j during the next time period. This approach studies the changes of state that a single individual is likely to undergo in light of the transition structure defined by P. Kemeny and Snell define this model as the "individual process."

In the collective process, the ij^{th} entry of P^n represents the fraction of people who start in the i^{th} region and will be in the j^{th} region after n time intervals. In the individual process, $p_{ij}^{(n)}$, denotes the probability that an individual who started in the i^{th} region will be in the j^{th} region after n transition periods. If one assumes independence between movers in the latter situation, the interpretation of the latter merges with that of the former.

In this paper, we shall study interregional migration as a "collective process." However, for convenience we shall frequently use the Markovian terminology of "individual processes."

The Ergodic Model

An ergodic Markov chain makes it possible to move from state i to any other state (not necessarily in one step). Ergodic models of interregional migration, therefore, cannot include births and deaths as states since the former state cannot be reached from any other, and the latter state cannot be left once entered. Thus with our ergodic model one must accept the patently unrealistic assumption that the effects of births and deaths in the system offset one another and therefore may be ignored in the analysis. In other words, we assume that the total population in our system of regions remains constant.

As an illustration, consider the following three-region example:

---

[1] John G. Kemeny and J. Laurie Snell, *Finite Markov Chains* (Princeton, New Jersey: D. Van Nostrand Co., 1960), p. 191.

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/2 & 1/4 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{array} .$$

The long-run distributional consequence of such migratory behavior is an equilibrium situation in which 3/10 of the total population is absorbed in region A, 4/10 in region B, and the remaining 3/10 in region C. That this indeed is the "steady state" solution can be seen from the following equality:

$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ (3/10, & 4/10, & 3/10) \end{array} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/2 & 1/4 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ (3/10, & 4/10, & 3/10) \end{array} .$$

The Absorbing Model

The fundamental feature of absorbing chains is that the process ultimately reaches an absorbing state. Within our framework, this simply means that all people eventually die and are never resurrected.[2] Thus, to analyze realistically the absorbing process, births must be exogenously supplied to the system.

Define the initial distribution among the various regions by the vector $w^{(0)} = (w_1^{(0)}, w_2^{(0)}, \ldots, w_{n-1}^{(0)})$, and the equilibrium distribution by $y = (y_1, y_2, \ldots, y_{n-1})$. In both distributions, the n^{th} state, death, need not be included since its effects on the system are implicit in that the rows of the nonabsorbing submatrix of P do not sum to unity. Introduce, in addition, a vector $x = (x_1, x_2, \ldots, x_{n-1})$ where x_i is the number of births in region i during the unit interval. We now wish to investigate the interrelationships between these quantities and the transition matrix P.

Recall that for an absorbing chain we generally partition P into four submatrices:

$$P = \left(\begin{array}{c|c} Q & R \\ \hline O & I \end{array} \right) .$$

Thus, given a $w^{(0)}$ and P we can find $\bar{w}^{(0)}$, by the relation:

$$\bar{w}^{(0)} = w^{(0)}Q .$$

This results in a new population distribution, $\bar{w}^{(0)}$, which in the aggregate is smaller than $w^{(0)}$, since a number of deaths have occurred during the transition period. If now we add the number of births, x, to $\bar{w}^{(0)}$ we have:

$$w^{(1)} = \bar{w}^{(0)} + x = w^{(0)}Q + x$$

---

[2] In short, the equilibrium distribution is:

$$\begin{array}{cccc} A & B & C & D \\ (0, & 0, & 0, & 1) . \end{array}$$

$$w^{(2)} = (w^{(0)}Q + x)Q + x$$

and

$$w^{(n)} = w^{(0)}Q^n + x(I + Q + \ldots + Q^{n-1}) .$$

Notice that since $Q^n \to 0$ as n increases, $w^{(0)}Q^n \to 0$ also and we are left with $w^{(n)} \to x(I + Q + \ldots + Q^{n-1})$. However, since as n increases, $I + Q + \ldots + Q^n \to (I - Q)^{-1}$, we have the result that

$$y = \lim_{n\to\infty} w^{(n)} = x(I - Q)^{-1} .$$

To clarify a few of the above ideas, let us now turn to a three-region, four-state example:

$$P = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/5 & 2/5 & 1/5 & 1/5 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} .$$

For this chain

$$(I - Q)^{-1} = \begin{pmatrix} 2.00 & 1.25 & 1.00 \\ 1.00 & 2.50 & 1.00 \\ 1.00 & 1.25 & 2.00 \end{pmatrix} .$$

Thus, given a vector of births, x, one immediately can solve for the equilibrium distribution vector, y, by using the relation $y = x(I - Q)^{-1}$. For example, if the initial population

$$w^{(0)} = (16,\ 11,\ 9) \text{ and } x = (3,\ 2,\ 1) \text{ then}$$

$$w^{(1)} = (11.45,\ 12.65,\ 9.45) ,$$

$$w^{(2)} = (10.76,\ 12.29,\ 8.76) ,$$

$$w^{(3)} = (10.34,\ 11.80,\ 8.34) ,$$

and the limiting distribution is

$$y = (3,\ 2,\ 1) \begin{pmatrix} 2.00 & 1.25 & 1.00 \\ 1.00 & 2.50 & 1.00 \\ 1.00 & 1.25 & 2.00 \end{pmatrix} = (9,\ 10,\ 7) .$$

Notice that the equilibrium solution is completely independent of the initial distribution of population. Also notice that the system upon reaching the "steady state" retains that distribution forever after, i. e.:

$$(9,\ 10,\ 7) \begin{pmatrix} 1/4 & 1/4 & 1/4 \\ 1/5 & 2/5 & 1/5 \\ 1/4 & 1/4 & 1/4 \end{pmatrix} = \begin{array}{r} (6,\quad 8,\quad 6) \\ \underline{+(3,\quad 2,\quad 1)} \\ (9,\ 10,\ 7) . \end{array}$$

II. A POLICY MODEL OF INTERREGIONAL MIGRATION

In the absence of governmental intervention, the ergodic Markovian migration model converges to an equilibrium distribution.[3] The distribution, however, may be incompatible with long-range planning goals. Thus, intervention to effect a desired allocation of population to regions may be in order. The intervention could be in the form of a policy which sought to attract a certain minimum number of people, each year, from one set of regions to another, thereby redirecting the system toward the desired equilibrium goal.

Two considerations immediately emerge: (1) not all distributional goals are feasible given a particular flow structure, and (2) if the goal is feasible, the right number of people must be transferred during each time period.

The Ergodic Policy Model

Let us define the number of people to be supplied or withdrawn from each region by a vector f which may have both positive and negative components. A positive f_i indicates the number of people which must be attracted into a region during each unit interval of time; a negative f_i denotes the population that has to be periodically withdrawn from region i. As before, the vector w describes the current population distribution, g defines the desired equilibrium goal, P is the transition matrix, and A is the matrix composed of identical steady state probability vectors a.

It was shown in Part I that if w is the initial distribution of people over the m states then wP is the distribution after one time period, wP^2 after two, and wP^n after n time periods. We now introduce the effects of governmental intervention. The number of people transferred from region to region during the initial transition period changes the total in each region by fP^{n-1} after n periods. The addition of the vector f after the next transition alters the total in each region by fP^{n-2} after n periods, etc. Thus, the total population after n periods is

$$wP^n + \sum_{k=0}^{n-1} fP^k \,. \tag{1}$$

Since the goal of the policy making body is represented by g, clearly what is desired is that (1) should, in the limit, converge to g. For such a goal to be feasible, however, we must include the restriction that during this limiting process, the population of each region never becomes negative, i.e.,

$$wP^n + \sum_{k=0}^{n-1} fP^k \geqq 0 \text{ for all } k = 0,\ 1,\ \ldots,\ n-1\,.$$

Consider the second term in (1), the series $f + fP + fP^2 + \ldots$. For this series to coverge, fP^k must tend to zero as k increases. But, fP^k tends to $(f1)a$, where 1 is the unit vector, and $a \neq 0$. Hence, a necessary condition for

[3] This section borrows heavily from the models developed by John G. Kemeny and J. Laurie Snell in chap. 6 of their book: *Mathematical Models in the Social Sciences.* The author acknowledges his debt to that seminal work.

convergence is that $f1 = 0$. This means that to achieve an equilibrium state, the net total population transfer at each transition must be zero. That is, the number of people attracted to regions in the system must equal the number which are removed from others. Hence, we have

$$f + fP + fP^2 + \ldots = f + f \sum_{n=1}^{\infty} P^n ,$$

and, since if $f1 = 0$, $fA = 0$, we have a sufficient condition for convergence with

$$f + f \sum_{n=1}^{\infty} P^n = f + f \sum_{n=1}^{\infty} (P^n - A) = fZ .$$

Thus to achieve the goal g, we require

$$g = (w1)a + fZ . \tag{2}$$

Now noting that $aP = a$, $aA = a$, and $Z = (I - P + A)^{-1}$, we can solve (2) by multiplying through by Z^{-1} which yields

$$g(I - P + A) = (w1)(a - a + a) + f . \tag{3}$$

Since $Z1 = 1$, and because the net population transfer in each period is 0, the total population at any period must remain constant. Thus,

$$g1 = (w1)(a1) + f1 = w1 . \tag{4}$$

Let us now combine the results of equations (3) and (4) to derive

$$f = g(I - P) . \tag{5}$$

To check that the total population transfer is still 0, note that $P1 = 1$ and $(I - P)1 = 0$ thus $f1 = 0$, and our requirement is satisfied.

In summary, we have shown that $g1 = w1$ is the necessary and sufficient condition for the existence of an f such that

$$wP^n + \sum_{k=0}^{n-1} fP^k$$

converges to g, in the limit, and we have established that the unique f which accomplishes this is given by

$$f = g(I - P) .$$

We must now check the nonnegativity constraint to establish feasibility. Replacing f by (5) in (1), we have

$$wP^n + g(I - P) \sum_{k=0}^{n-1} P^k \geqq 0 ,$$

and noting that $(I - P)(I + P + P^2 + \ldots) = (I - P^n)$ we reduce the constraint to

$$wP^n + g(I - P^n) \geqq 0 ,$$

or equivalently,

$$(g - w)P^n \leqq g , \quad \text{for } n \geqq 0 . \tag{6}$$

This constraint may appear, at first, to be impossible to enforce since an infinite number of conditions are specified. However, a theorem by Kemeny and Snell provides an algorithm for establishing feasibility in a finite number of steps.[4]

The Kemeny-Snell Theorem

(i) If for some k the sum of the absolute values of the components of $(g - w)P^k$ is no greater than the least entry of g, then (6) holds for all $n \geqq k$; and this must occur for some k.

(ii) If $(g - w)P^{k+1} \leqq (g - w)P^k$ and (6) holds for k, then (6) holds for $n \geqq k$.

(iii) If $gP \leqq g$, then (6) holds for all n, for all w.

Proof: Let $h = (g - w)P^k$. Then $(g - w)P^n = hP^{n-k}$, and the components of P to any power never exceed unity. Hence a component of hP^{n-k} is bounded by $\sum |h_i|$. Thus if

$$\sum_i |h_i| \leqq \min_i g_i ,$$

then (6) holds for $n \geqq k$, and since $h = (g - w)P^k$ converges to $gA - wA$, which is zero by (2), h also tends to 0. Hence, $\sum_i |h_i| \leqq \min_i g_i$ for sufficiently high k. (Recall that the i elements of g are by definition nonnegative.)

To establish (ii), note that if $hP \leqq h$ then we can multiply both sides by the nonnegative matrix P^s to obtain $hP^{s+1} \leqq hP^s$ and thereby show that hP^s is monotone decreasing.

Finally, if $gP \leqq g$, then $gP^{n+1} \leqq g$ for all n; hence (6) holds for all n, even without the term wP^n. Thus, (iii) holds.

With the Kemeny-Snell theorem one has an effective procedure for checking (6) in a finite number of steps. That is, we compute $(g - w)P^n$ for succesive n, beginning with $n = 0$. If (6) is violated for any of these, then g is not feasible. Ultimately a vector satisfying condition (i) is reached, and if (6) has not been violated up to this point, it will never be violated and, therefore, g is feasible.

Parts (ii) and (iii) of the theorem offer us shortcuts. Namely, if a computed vector does not exceed the previous one, one may stop. Alternatively, if $gP \leqq g$ (i. e., $f \geqq 0$), g is feasible, and the entire computation is unnecessary.

The Absorbing Policy Model

The mathematical analysis of the absorbing chain is almost identical to that of the ergodic model, with Q playing the role of P. Thus, the total population after n periods is

$$wQ^n + \sum_{k=0}^{n-1} fQ^k . \tag{7}$$

For an absorbing chain, Q^n tends to zero as n increases; therefore, the first term of (7) tends to 0. Thus, to achieve a desired goal, $\sum_{k=0}^{n-1} fQ^k$ must tend to g as $n \to \infty$.

[4] Kemeny and Snell, *op. cit.*, p. 68.

Since

$$I + Q + Q^2 + \dots = (I - Q)^{-1}\,,$$

we want

$$f = g(I - Q)\,. \tag{8}$$

If g is feasible, the unique f given by (8) will achieve the goal. The nonnegativity constraint becomes

$$wQ^n + g(I - Q)\sum_{k=0}^{n-1} Q^k = wQ^n + g(I - Q^n) \geqq 0$$

or

$$(g - w)Q^n \leqq g \text{ for } n \geqq 0\,. \tag{9}$$

As in the ergodic model, the Kemeny-Snell theorem provides an algorithm for establishing whether a particular g is feasible. The theorem and the proof for the absorbing case are almost identical with those for the ergodic model. The only modification is the replacement of P by Q.[5]

Examples

The Ergodic Model: let us return to our earlier example of a three-region ergodic model of interregional migration:

$$P = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/2 & 1/4 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{array}\,.$$

For this ergodic model, we found a "steady state" solution of $a = (3/10, 4/10, 3/10)$. Thus, for example, an initial population distribution of $w^{(0)} = (16, 11, 9)$ will yield an equilibrium distribution of $g = (10.8, 14.4, 10.8)$.

Assume that an equal distribution of population has been set as a goal by the policy-making body. What are the required transfer levels? That is, what f added after each transition will provide in the limit a $g = (12, 12, 12)$?

$$f = g(I - P) = (12,\ 12,\ 12)\begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/4 & 1/2 & -1/4 \\ -1/3 & -1/3 & 2/3 \end{pmatrix} = (1,\ -2,\ 1)\,,$$

$$Z = (I - P + 1a)^{-1} = \begin{pmatrix} .967 & .067 & -.033 \\ .050 & .900 & .050 \\ -.033 & .067 & .967 \end{pmatrix}^{-1} = \begin{pmatrix} 1.04 & -.08 & .04 \\ -.06 & 1.12 & -.06 \\ .04 & -.08 & 1.04 \end{pmatrix}.$$

Checking condition (2):

$$g = (w1)a + fZ$$

[5] To show that h tends to zero in the absorbing case, one only needs to recall that $Q^n \to 0$ as n increases and thus $h = (g - w)Q^n$ also converges to zero.

$$= 36(3/10,\ 4/10,\ 3/10) + (1.2,\ -2.4,\ 1.2)$$
$$= (12,\ 12,\ 12)\,.$$

Checking condition (6):

$$(g - w) = (-4,\ 1,\ 3)$$
$$\sum |\, h_i \,| = 8 < \min_i g_i\,.$$

Hence (6) is satisfied, and $g = (12,\ 12,\ 12)$ is feasible.

The Absorbing Model: for the absorbing example recall:

$$P = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/5 & 2/5 & 1/5 & 1/5 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

and

$$(I - Q)^{-1} = \begin{pmatrix} 2.00 & 1.25 & 1.00 \\ 1.00 & 2.50 & 1.00 \\ 1.00 & 1.25 & 2.00 \end{pmatrix}.$$

Thus, if $g = (12,\ 12,\ 12)$, then $f = (g - gQ) = (3.6,\ 1.2,\ 3.6)$. We verify that $f(I - Q)^{-1} = g$, and since $gQ \leqq Q$, condition (iii) of the Kemeny-Snell theorem is satisfied. Hence, g is feasible.

The Policy Model with Incomplete Control

Consider the problem of partial control. Suppose the policy-making body can influence population transfers only in a subset of regions. Without loss of generality, we may assume that these regions are the first k of the m that constitute the system. This means that $f_i = 0$ for $i > k$.

The results of the previous two sections are still applicable. However, we now must further restrict the set of feasible goals. The restriction will ensure that f has nonzero components only in regions $i = 1,\ \ldots,\ k$.

The Ergodic Model: let us partition the matrix P into four submatrices:

$$P = \begin{array}{c} \\ \text{controlled regions} \\ \text{uncontrolled regions} \end{array} \begin{array}{c} \begin{array}{cc} \text{controlled} & \text{uncontrolled} \end{array} \\ \left(\begin{array}{c|c} T & U \\ \hline V & W \end{array}\right) \end{array}.$$

Let $g = (g^*,\ g^{**})$, with g^* containing the goals for the controlled regions.

$$g(I - P) = \begin{cases} g^* - g^*T - g^{**}V \text{ in controlled regions} \\ g^{**} - g^*U - g^{**}W \text{ in all others.} \end{cases} \tag{10}$$

Thus, our new constraint is

$$g^{**} - g^*U - g^{**}W = 0$$

or

$$g^{**}(I - W)\, g^* U\,.$$

Since W is the Q matrix of an absorbing chain, $(I - W)$ has an inverse, and we have for our constraint

$$g^{**} = g^* U(I - W)^{-1}\,. \tag{11}$$

Therefore, the policy-making body has no choice in values of g for the uncontrolled states since once g^* is selected, g^{**} is strictly determined by (11). To compute the nonzero components of f from (11), we note that

$$f^* = g^*[I - T - U(I - W)^{-1}V]\,,$$

and defining $P^* = T + U(I - W)^{-1}V$, we have

$$f^* = g^*(I - P^*)\,. \tag{12}$$

Notice that if $P1 = 1$, $P^*1 = 1$. Hence, (12) guarantees that $f^*1 = 0$. Moreover, since $f_i = 0$ for uncontrolled states, $f1 = 0$. Thus, for the ergodic policy model with incomplete control, g is feasible if and only if (4), (6), and (11) are satisfied, and in these instances (12) provides the solution.

The Absorbing Model: in the absorbing case Q^* is formed in an analogous manner to P^* above. The only difference is that row sums do not sum to unity, as before. For the absorbing model, g is feasible if and only if (9) and (11) hold, and in such instances the unique solution is

$$f^* = g^*(I - Q^*)\,. \tag{13}$$

Examples

The Ergodic Model: let us suppose that region C, in our ergodic example, is beyond the control of the policy-making body, e. g., let A and B be Northern and Southern California, and assume C is the rest of the United States.

$$T = \begin{pmatrix} 1/3 & 1/3 \\ 1/4 & 1/2 \end{pmatrix} \qquad U = \begin{pmatrix} 1/3 \\ 1/4 \end{pmatrix}$$

$$V = (1/3 \quad 1/3) \qquad W = (1/3)$$

$$(I - W)^{-1} = 3/2 \qquad U(I - W)^{-1} = \begin{pmatrix} 1/2 \\ 3/8 \end{pmatrix}$$

$$P^* = \begin{pmatrix} 1/2 & 1/2 \\ 3/8 & 5/8 \end{pmatrix} \qquad U(I - W)^{-1}V = \begin{pmatrix} 1/6 & 1/6 \\ 1/8 & 1/8 \end{pmatrix}.$$

If $w^{(0)} = (16, 11, 9)$, as before, then the restriction on $g = (g_a, g_b, g_c)$ are

$$g_a + g_b + g_c = 36$$

and

$$g_c = 1/2\, g_a + 3/8\, g_b\,.$$

Or equivalently,

$$g_b = \frac{288}{11} - \frac{12}{11} g_a$$

$$g_c = \frac{108}{11} + \frac{1}{11} g_a . \qquad (14)$$

To keep all elements of g greater than zero, one is restricted in selecting g_a by the inequality $0 < g_a < 24$. For any g_i within this range, g_b and g_c are determined by (14). The solution is

$$f^* = g^*(I - P^*)$$

or

$$f_a = 1/2\, g_a - 3/8\, g_b = 10/11\, g_a - 108/11$$

and

$$f_b = - f_a .$$

However, a further restriction in the choice of g_a is given by (6). Thus, for $g_a = 12$, we have

$$g = (12,\ 13.09,\ 10.91)$$

and

$$f = (1.09,\ -1.09,\ 0) .$$

It is easily established that $g = (w1)a + fZ$, as before, and that (6) is satisfied. Hence, g is feasible.

The Absorbing Model: let us now consider the absorbing model, and again suppose that the policy-making body has no control over region C.

$$T = \begin{pmatrix} 1/4 & 1/4 \\ 1/5 & 2/5 \end{pmatrix} \qquad U = \begin{pmatrix} 1/4 \\ 1/5 \end{pmatrix}$$

$$V = (1/4 \quad 1/4) \qquad W = (1/4)$$

$$(I - W)^{-1} = 4/3 \qquad U(I - W)^{-1} = \begin{pmatrix} 1/3 \\ 4/15 \end{pmatrix} \qquad Q^* = \begin{pmatrix} 1/3 & 1/3 \\ 4/15 & 7/15 \end{pmatrix} .$$

Our restrictions over g now require

$$g_c = 1/3\, g_a + 4/15\, g_b .$$

Since $f_c = 0$ and $f^* = g^*(I - Q^*)$, we have

$$f_a = 2/3\, g_a - 4/15\, g_b\, ; \; f_b = 8/15\, g_b - 1/3\, g_a .$$

Thus, for example, with a $g = (12,\ 12,\ 7.2)$ we have

$$f = (4.8,\ 2.4,\ 0) .$$

To clarify the interrelated policy models, we summarize the numerical results of this section in Tables 1 through 6.

TABLE 1

ERGODIC EXAMPLE: NO INTERVENTION

$$(16.00,\ 11.00,\ 9.00)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = (11.08,\ 13.83,\ 11.08)$$

$$(11.08,\ 13.83,\ 11.08)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = (10.84,\ 14.34,\ 10.84)$$

$$(10.84,\ 14.34,\ 10.84)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = (10.80,\ 14.39,\ 10.80)$$

$$(10.80,\ 14.39,\ 10.80)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = (10.80,\ 14.40,\ 10.80)$$

TABLE 2

ERGODIC EXAMPLE: INTERVENTION WITH FULL CONTROL

$$(16.00,\ 11.00,\ 9.00)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.08,\ 13.83,\ 11.08)\\ \underline{+(1\quad,-2\quad,\ 1\quad)}\\ (12.08,\ 11.83,\ 12.08)\end{array}$$

$$(12.08,\ 11{,}83,\ 12.08)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.02,\ 13.97,\ 11.02)\\ \underline{+(1\quad,-2\quad,\ 1\quad)}\\ (12.02,\ 11.97,\ 12.02)\end{array}$$

$$(12.02,\ 11.97,\ 12.02)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.00,\ 14.00,\ 11.00)\\ \underline{+(1\quad,-2\quad,\ 1\quad)}\\ (12.00,\ 12.00,\ 12.00)\end{array}$$

$$(12.00,\ 12.00,\ 12.00)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.00,\ 14.00,\ 11.00)\\ \underline{+(1\quad,-2\quad,\ 1\quad)}\\ (12.00,\ 12.00,\ 12.00)\end{array}$$

TABLE 3

ERGODIC EXAMPLE: INTERVENTION WITH PARTIAL CONTROL

$$(16.00,\ 11.00,\ 9.00)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.08,\ 13.83,\ 11.08)\\ \underline{+(\ 1.09,-1.09,\ \ 0\quad)}\\ (12.17,\ 12.74,\ 11.08)\end{array}$$

$$(12.17,\ 12.74,\ 11.08)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(10.94,\ 14.12,\ 10.94)\\ \underline{+(\ 1.09,-1.09,\ \ 0\quad)}\\ (12.03,\ 13.03,\ 10.94)\end{array}$$

$$(12.03,\ 13.03,\ 10.94)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.91,\ 14.18,\ 10.91)\\ \underline{+(\ 1.09,-1.09,\ \ 0\quad)}\\ (12.00,\ 13.09,\ 10.91)\end{array}$$

$$(12.00,\ 13.09,\ 10.91)\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/4 & 1/2 & 1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} = \begin{array}{r}(11.91,\ 14.18,\ 10.91)\\ \underline{+(\ 1.09,-1.09,\ \ 0\quad)}\\ (12.00,\ 13.09,\ 10.91)\end{array}$$

TABLE 4

ABSORBING EXAMPLE: NO INTERVENTION

$$
\begin{aligned}
(16.00,\ 11.00,\ 9.00)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = (8.45,\ 10.65,\ 8.45)\\
(8.45,\ 10.65,\ 8.45)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = (6.36,\ 8.46,\ 6.36)\\
(6.36,\ 8.46,\ 6.36)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = (4.87,\ 6.56,\ 4.87)\\
&\vdots\\
(0.00,\ 0.00,\ 0.00)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = (0.00,\ 0.00,\ 0.00)
\end{aligned}
$$

TABLE 5

ABSORBING EXAMPLE: INTERVENTION WITH FULL CONTROL

$$
\begin{aligned}
(16.00,\ 11.00,\ 9.00)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(8.45,\ 10.65,\ 8.45)\\ \underline{+(3.6,\ 1.2,\ 3.6)}\\ (12.05,\ 11.85,\ 12.05)\end{array}\\
(12.05,\ 11.85,\ 12.05)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(8.39,\ 10.76,\ 8.39)\\ \underline{+(3.6,\ 1.2,\ 3.6)}\\ (11.99,\ 11.96,\ 11.99)\end{array}\\
(11.99,\ 11.96,\ 11.99)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(8.39,\ 10.78,\ 8.39)\\ \underline{+(3.6,\ 1.2,\ 3.6)}\\ (11.99,\ 11.98,\ 11.99)\end{array}\\
&\vdots\\
(12.00,\ 12.00,\ 12.00)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(8.40,\ 10.80,\ 8.40)\\ \underline{+(3.6,\ 1.2,\ 3.6)}\\ (12.00,\ 12.00,\ 12.00)\end{array}
\end{aligned}
$$

TABLE 6

ABSORBING EXAMPLE: INTERVENTION WITH PARTIAL CONTROL

$$
\begin{aligned}
(16.00,\ 11.00,\ 9.00)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(8.45,\ 10.65,\ 8.45)\\ \underline{+(4.8,\ 2.4,\ 0)}\\ (13.25,\ 13.05,\ 8.45)\end{array}\\
(13.25,\ 13.05,\ 8.45)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(8.04,\ 10.65,\ 8.04)\\ \underline{+(4.8,\ 2.4,\ 0)}\\ (12.84,\ 13.05,\ 8.04)\end{array}\\
(12.84,\ 13.05,\ 8.04)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(7.84,\ 10.45,\ 7.84)\\ \underline{+(4.8,\ 2.4,\ 0)}\\ (12.64,\ 12.85,\ 7.84)\end{array}\\
&\vdots\\
(12.00,\ 12.00,\ 7.20)&\begin{pmatrix}1/4&1/4&1/4\\1/5&2/5&1/5\\1/4&1/4&1/4\end{pmatrix} = \begin{array}{r}(7.20,\ 9.60,\ 7.20)\\ \underline{+(4.8,\ 2.4,\ 0)}\\ (12.00,\ 12.00,\ 7.20)\end{array}
\end{aligned}
$$

III. ALTERNATIVE DISTRIBUTIONAL GOALS FOR CALIFORNIA REGIONS

The application of the Markovian policy model to interregional flows in California is relatively straightforward. For ease of exposition, let us focus on the reduced ergodic and absorbing transition matrices of Table 7. Here the state has been divided into three regions: the San Francisco-Oakland SMSA, the Los Angeles-Long Beach SMSA, and the rest of the state. The rest of the United States and the absorbing state of death complete the model. For the ergodic chain, let us assume a partial control situation with control available only over movement into and out of the three California regions. In

TABLE 7
TRANSITION MATRICES, EQUILIBRIUM DISTRIBUTIONS AND THE 1955 POPULATION DISTRIBUTION FOR CALIFORNIA: ERGODIC AND ABSORBING FOUR-REGION MODEL

A. Ergodic Model

	A	F	CAL.	U.S.
A	.8544	.0172	.0688	.0596
F	.0076	.8907	.0475	.0542
CAL.	.0219	.0306	.8737	.0738
U.S.	.0017	.0056	.0054	.9873

Equilibrium Vector:

$a =$ (.0244 .0674 .0743 .8339)

California's Equilibrium Distribution:

$b =$ (.1469 .4058 .4473)

B. Absorbing Model

	A	F	CAL.	U.S.	DEATH
A	.8072	.0172	.0688	.0596	.0472
F	.0076	.8407	.0475	.0542	.0500
CAL.	.0219	.0306	.8296	.0738	.0441
U.S.	.0017	.0056	.0054	.9385	.0488
DEATH	0	0	0	0	1.0

Equilibrium Vector*:

$a =$ (.0201 .0545 .0600 .8654)

California's Equilibrium Distribution:

$b =$ (.1493 .4049 .4458)

C. 1955 Population Distribution

A	F	CAL.	U.S.
(2,547,000	5,332,000	5,109,000	152,082,000)
(.0154)	(.0323)	(.0310)	(.9213)
(.1961)	(.4105)	(.3934)	

* The equilibrium distribution was found by assuming a constant birth vector and using the relation $y = x(I - Q)^{-1}$, see Part I.

the absorbing model, assume that full control over interregional movement is possible.

The Ergodic Model

Consider the equilibrium distribution in the absence of intervention. This suggests that the four-region system is heading toward a long-run distribution with 2.44 per cent of the national population residing in the San Francisco-Oakland SMSA, 6.74 per cent in the Los Angeles-Long Beach SMSA, 7.43 per cent in the rest of California, and 83.39 per cent in the rest of the United States. Assume that we wish to limit the expected equilibrium distributions of the two SMSA's to 3 and 6 per cent of the national total, respectively. Is such a goal feasible? What transfer levels are required to direct the system toward this desired equilibrium?

Proceeding according to the steps outlined previously, we first find:

$$T = \begin{pmatrix} .8544 & .0172 & .0688 \\ .0076 & .8907 & .0475 \\ .0219 & .0306 & .8737 \end{pmatrix} \qquad U = \begin{pmatrix} .0596 \\ .0542 \\ .0738 \end{pmatrix}$$

$$V = (.0017 \quad .0056 \quad .0054) \qquad W = (.9873)$$

$$P^* = \begin{pmatrix} .8624 & .0435 & .0941 \\ .0149 & .9146 & .0706 \\ .0318 & .0631 & .9051 \end{pmatrix} .$$

Now recalling that in the ergodic case, we lose one degree of freedom in the selection of goals because of the constraint $g1 = w1$, we derive our "goal" for the third region, i. e., the rest of California:

$$g_a + g_b + g_c + g_d = 165{,}070{,}000$$
$$4.6929\, g_a + 4.2677\, g_b + 5.8110\, g_c - g_d = 0$$

where

$$g_a = .03(165{,}070{,}000) = 4{,}952{,}000 = 5{,}000{,}000$$

and

$$g_b = .06(165{,}070{,}000) = 9{,}904{,}000 = 10{,}000{,}000 .$$

Solving the two equations in two unknowns we find g_c and hence our full set of goals is

$$g^* = (5{,}000{,}000;\ 10{,}000{,}000;\ 12{,}320{,}000) .$$

Recall that once g^* is specified, g^{**} is strictly determined by the relation (equation 11):

$$g^{**} = g^* U(I - W)^{-1} .$$

Thus, the population of the noncontrollable region, i. e., the rest of the United States, which follows from our choice of g^* is

$$g^{**} = 137{,}750{,}000 .$$

TABLE 8

ERGODIC POLICY MODEL FOR CALIFORNIA: PARTIAL CONTROL

$$
\begin{array}{cccc}
\text{A} & \text{F} & \text{CAL.} & \text{U.S.} \\
(2{,}547; & 5{,}332\ ; & 5{,}109; & 152{,}082)^{*}
\end{array}
\begin{pmatrix}
.8544 & .0172 & .0688 & .0596 \\
.0076 & .8907 & .0475 & .0542 \\
.0219 & .0306 & .8737 & .0738 \\
.0017 & .0056 & .0054 & .9873
\end{pmatrix}
=
\begin{array}{cccc}
\text{A} & \text{F} & \text{CAL.} & \text{U.S.} \\
(2{,}587; & 5{,}801\ ; & 5{,}713.5; & 150{,}968) \\
+(\ 148; & -141.5; & -6.5; & 0) \\
\hline
(2{,}735; & 5{,}659.5; & 5{,}707\ ; & 150{,}968)
\end{array}
$$

$$
(2{,}735;\ 5{,}659.5;\ 5{,}707;\ 150{,}968)
\begin{pmatrix}
.8544 & .0172 & .0688 & .0596 \\
.0076 & .8907 & .0475 & .0542 \\
.0219 & .0306 & .8737 & .0738 \\
.0017 & .0056 & .0054 & .9873
\end{pmatrix}
=
\begin{array}{cccc}
(2{,}761; & 6{,}108\ ; & 6{,}258.5; & 149{,}942) \\
+(\ 148; & -141.5; & -6.5; & 0) \\
\hline
(2{,}909; & 5{,}966.5; & 6{,}252\ ; & 149{,}942)
\end{array}
$$

$$\vdots$$

$$
(5{,}000;\ 10{,}000\ ;\ 12{,}320;\ 137{,}750)
\begin{pmatrix}
.8544 & .0172 & .0688 & .0596 \\
.0076 & .8907 & .0475 & .0542 \\
.0219 & .0306 & .8737 & .0738 \\
.0017 & .0056 & .0054 & .9873
\end{pmatrix}
=
\begin{array}{cccc}
(4{,}852; & 10{,}141.5; & 12{,}326.5; & 137{,}750) \\
+(\ 148; & -141.5; & -6.5; & 0) \\
\hline
(5{,}000; & 10{,}000\ ; & 12{,}320\ ; & 137{,}750)^{\dagger}
\end{array}
$$

* Population figures are in thousands.

† Errors due to rounding preclude the strict convergence shown in this table.

After first establishing the feasibility of our choice of g^* by means of the Kemeny-Snell theorem, we calculate the intervention vector f:

$$f^* = g^* (I - P^*)$$
$$= (148{,}000;\ -141{,}500;\ -6{,}500)\,.$$

This computation indicates that to redirect the system away from an equilibrium distribution of (.0244/.0674/.0743/.8339) toward one of (.030/.060/.075/.835) we must, during each interval of time transfer 141,500 people from the Los Angeles-Long Beach SMSA and 6,500 from the rest of California region to the San Francisco-Oakland SMSA. Table 8 illustrates the convergence. Table 9 presents alternate goals and their intervention vectors.

TABLE 9
ALTERNATIVE DISTRIBUTIONAL GOALS FOR CALIFORNIA REGIONS AND THEIR POLICY IMPLICATIONS: THE ERGODIC MODEL

EQUILIBRIUM DISTRIBUTION (in thousands)

	A	F	CAL.	U.S.
$a =$	(4,028;	11,126;	12,265;	137,652)
	(.0244;	.0674;	.0743;	.8339)

	GOAL (in thousands) g				INTERVENTION VECTOR (in thousands) f			
	A	F	CAL.	U.S.	A	F	CAL.	U.S.
1. $a =$	(5,000;	10,000;	12,320;	137,750)	(148;	−141.5;	−6.5;	0)
	(.0303;	.0606;	.0746;	.8345)				
2. $a =$	(6,000;	9,000;	12,260;	137,810)	(302;	−266 ;	−36 ;	0)
	(.0363;	.0545;	.0743;	.8349)				
3. $a =$	(4,000;	12,000;	11,612;	137,458)	(3;	118 ;	−121 ;	0)
	(.0242;	.0727;	.0703;	.8328)				
4. $a =$	(8,000;	8,000;	11,362;	137,708)	(621;	−382 ;	−239 ;	0)
	(.0485;	.0485;	.0688;	.8342)				
5. $a =$	(3,000;	9,000;	14,768;	137,302)	(−190;	−294 ;	484 ;	0)
	(.0182;	.0545;	.0895;	.8318)				

The Absorbing Model

For the absorbing model, the computational procedure is identical to that outlined above with Q^* replacing P^*. In the full control situation, the asterisk is removed, and the fundamental relation is $f = g(I - Q)$.

Consider, for example, the equilibrium distribution for the absorbing transition matrix in Table 7. Here the projected long-run distribution forecasts that ultimately 2.01 per cent of the national population will reside in the San Francisco-Oakland SMSA, 5.45 per cent in the Los Angeles-Long Beach SMSA, 6.00 per cent in the rest of California, and 86.54 per cent in the rest of the

United States. Assume that this allocation is undesirable. Assume, further, that the desired distribution over the four regions is a (.03/.06/.08/.83) division. Is such a goal feasible? What level of intervention will redirect the system to this equilibrium distribution?

The Kemeny-Snell theorem quickly establishes the feasibility of this goal and we go on to find:

$$f = g(I - Q)$$

$$f = (12{,}000{,}000;\ 18{,}000{,}000;\ 30{,}000{,}000;\ 376{,}241{,}000) \begin{pmatrix} .1928 & .9828 & .9312 & .9404 \\ .9924 & .1593 & .9525 & .9458 \\ .9781 & .9694 & .1704 & .9262 \\ .9983 & .9944 & .9946 & .0615 \end{pmatrix}$$

$$= (880{,}000;\ -364{,}000;\ 1{,}400{,}000;\ 19{,}234{,}000)\,.$$

Since, in the absorbing model, births are introduced into the system at each iteration, the intervention vector includes the birth vector. That is, it is the sum of births and net migration. For example, the negative element of f in the above derivation indicates a very high outmigration for that region, i.e., net migration exceeds the number of births in the region by 364,000.

Table 10 presents alternate goals and their intervention vectors for the

TABLE 10
ALTERNATIVE DISTRIBUTIONAL GOALS FOR CALIFORNIA REGIONS AND THEIR POLICY IMPLICATIONS: THE ABSORBING MODEL

EQUILIBRIUM DISTRIBUTION (in thousands)

	A	F	CAL.	U. S.
$a =$	(8,786;	23,750;	26,171;	377,525)
	(.0201;	.0545;	.0600;	.8654)

		GOAL (in thousands) g				INTERVENTION VECTOR (in thousands) f			
		A	F	CAL.	U. S.	A	F	CAL.	U. S.
1.	$a =$	(12,000;	18,000;	30,000;	376,241)	(880;	−364;	1,400;	19,234)
		(.0275;	.0412;	.0688;	.8625)				
2.	$a =$	(10,000;	20,000;	30,000;	376,241)	(479;	−11;	1,442;	19,245)
		(.0229;	.0458;	.0688;	.8625)				
3.	$a =$	(9,000;	25,000;	26,000;	376,241)	(336;	925;	592;	19,329)
		(.0206;	.0573;	.0596;	.8625)				
4.	$a =$	(8,000;	8,000;	44,000;	376,241)	(−122;	−2,317;	4,535;	18,981)
		(.0183;	.0183;	.1009;	.8625)				
5.	$a =$	(22,000;	28,000;	10,000;	376,241)	Not Feasible*			
		(.0504;	.0642;	.0229;	.8625)				

* Produces a negative population in the third region.

absorbing model. Notice that goal 5 is infeasible in light of the current transition structure.

IV. CONCLUSION

The models described in this paper are conceptually simple and have been derived on the basis of very strict assumptions which clearly do not hold in the real world. In this respect, they are analytic systems. That is, they do not claim descriptive accuracy only a logical consistency which requires that all laws be derived from given initial premises according to a rigorous chain of deductive reasoning. Thus, their use for long-term forecasting of interregional flows is limited. However, in an era of state and metropolitan data banks and continuously updated information systems, these models may prove to be powerful instruments for monitoring intraregional and interregional flow systems and for generating short-range forecasts of the kind provided by projections of economic time series.[6]

Consider, for example, a state information system which receives continuous data concerning interstate and intrastate movements. (Perhaps this information is estimated on the basis of reported changes of address in drivers' license registrations, provided by the State Department of Motor Vehicles.[7]) At any point in time, an updated interregional transition matrix is readily available and analyses of the kind outlined in this paper become very useful indicators of the current migration pattern. Changes in the flow structure are immediately recorded, and the implications of revised equilibrium distributions are quickly assessed. Policy makers are alerted to any sudden shifts in the geographical disposition of the state's population. The degree of policy intervention required to redirect the system back toward a desired "target path" may be studied and the required annual transfer levels computed. In short, within the context of a real-time information system, the Markovian model of interregional migration becomes a valuable tool for system surveillance and control.

The application of Markovian models to the study of geographical mobility is of relatively recent origin. Despite its infancy, however, several interesting directions for further research are emerging. An extension of the policy model to include gradual withdrawal of system intervention is currently being developed by the author. The notion that some people are more mobile and tend to move more frequently is a refinement which may be included into the basic model. Such a "mover-stayer" hypothesis was successfully applied in a

---

[6] For a more detailed exposition of this point of view see Andrei Rogers, "The Research Implications of Real-Time Urban Information Systems," *Proceedings of the Third Annual Conference on Urban Planning Information Systems and Programs* (Chicago, 1965), pp. 11-22.

[7] A 2 per cent sample of reported changes of address in license registrations was analyzed in a recently completed study of interregional migration in California. See Andrei Rogers, *An Analysis of Interregional Migration in California*, Preprint No. 13, Center for Planning and Development Research (Berkeley: University of California, December, 1965).

study of the industrial mobility of labor.[8] An extension of the mover-stayer dichotomy to include degrees of attachment has given rise to the axiom of cumulative inertia.[9] Finally, the introduction of latency or response uncertainty into migration analysis is a very suggestive direction for further research which needs to be explored.[10]

[8] Isadore Blumen, Marvin Kogan, and Philip J. McCarthy, *The Industrial Mobility of Labor as a Probability Process*, VI, Cornell Studies of Industrial and Labor Relations (Ithaca, New York: The New York State School of Industrial and Labor Relations, Cornell University, 1955).

[9] Robert McGinnis and John E. Pilger, "On a Model for Temporal Analysis," paper presented at the 58th Annual Meeting of the American Sociological Association, Los Angeles, August 29, 1963.

[10] James S. Coleman, *Models of Change and Response Uncertainty* (Englewood Cliffs, New Jersey: Prentice-Hall, 1964).