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Large-Eddy Simulations of heated flows in ribbed channels with spanwise rotation

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ABSTRACT

We report on the outcome of a computational study of heat and momentum transport in a heated ribbed straight channel rotated about a spanwise axis. The flow is of both practical and fundamental interest, the latter due to the simultaneous presence in this flow of a number of complicating effects. These include system rotation, mean-flow unsteadiness, large-scale flow separation, and subsequent reattachment; their interactions severely distort the turbulence structure and thus pose this flow as a challenge to engineering prediction methods. In this study, the predictions were obtained using Large-Eddy Simulations with the objectives of assessing the performance of this approach in this flow, and gaining better understanding of the factors that influence the quality of the solutions. Thus, the numerical accuracy of the simulations was determined using Grid Convergence Index (GCI) method, and additionally by comparing results obtained using discretization schemes of different orders of accuracy. The dependence of the computed results on the models for the sub-grid scale correlations in both the momentum and thermal energy equations was checked by performing computations with alternative closure assumptions. These included the Smagorinsky and the dynamic models for momentum, and both linear and non-linear models for the thermal energy fluxes. The computations, which were performed with OpenFOAM, were compared with benchmark experimental data from both heated and isothermal flows. The correspondence between predictions and measurements was generally satisfactory but some important differences remain. It is argued that these are in part due to ambiguities in the way in which temporal and spatial averages are obtained in the computations and in the measurements.

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1. Introduction

In the current practice for the cooling of rotating turbine blades, a coolant is circulated through internal cavities that resemble, in idealized form, a straight channel rotating around a spanwise axis. The resulting flow is turbulent with a structure that is severely distorted by the effects of rotation. On the pressure side (where the imposed system vorticity is in the same sense as the mean-flow vorticity), these effects are *destabilizing* in that the turbulent motions are energized leading to enhancement of the wall heat transfer rates relative to a stationary channel. On the suction side, the opposite occurs: the effects of rotation are *stabilizing* leading to reduction of the turbulence activity. At sufficiently high rotation rates, the destabilizing effects lead to the formation of Taylor-Görtler vortices, while the stabilizing effects can lead to re-laminarization of the

Nomenclature

A^+	Coefficient in van Driest wall damping model
c_p	Pressure coefficient
C_s	Smagorinsky coefficient
C_1, C_2, C_3	Coefficients in heat-flux model
D_h	Hydraulic diameter
e	Rib height
H	Channel height
L	Channel width
N	Number of computational nodes
Nu	Nusselt number
P	Periodic length
P	Pressure
Pr	Prandtl number
$\dot{Q}$	Heat transfer rate
q_{ij}	Heat flux
Re	Reynolds number
Ro	Rotation number
S_{ij}	Strain tensor
T	Temperature
t	Time, fluctuating temperature
U_b, U_b	Mean velocity, bulk velocity
u_i	Velocity fluctuations
x_i	Cartesian coordinates

Greek Symbols

β	Streamwise pressure gradient
Γ, Γ_{sgs}	Thermal diffusivity, sub-grid scale thermal diffusivity
γ	Streamwise temperature gradient
Δ	Filter width / grid size
δ_{ij}	Kronecker delta
ϵ	Dissipation of turbulent kinetic energy
λ	Thermal conductivity
μ, μ_{sgs}	Dynamic viscosity, sub-grid scale viscosity
ν	Kinematic viscosity
Ω	Rotation rate
ρ	Density
Θ	Non-dimensional temperature
τ_{ij}	Reynolds stress

flow and hence to a substantial reduction in the heat transfer rates. It is for this reason that turbulence-enhancing ribs are often installed on the suction side of the rotating passage to compensate for the adverse effects of rotation. However, the presence of these ribs, being formed from successive square or rectangular steps with axes perpendicular to the main flow direction, causes both the surface friction and the overall pressure drop to increase relative to their values in a smooth channel, the latter by as much as 15–20% [1]. There is then a clear incentive in being able to accurately predict the relationship between the strength of rotation and the disposition of the surface ribs on the stabilized side in order that these can be optimized to increase the heat transfer rates while keeping the inevitable energy losses to a minimum.

Numerous experimental and computational studies on rotating ribbed channels have been reported in the literature, mostly for isothermal flow conditions. Here, attention is confined to

the cases where the ribs are square in cross-section and are installed on only one side of the channel with the opposite side being smooth. The strength of rotation is characterized by the non-dimensional rotation number, Ro , which represents the ratio of the Coriolis to the inertia forces:

$$Ro = \frac{\Omega D_h}{U_b}, \quad (1)$$

where U_b is the bulk velocity, D_h is the hydraulic diameter, and Ω is the angular velocity. In what follows, a positive value of Ro indicates that the flow adjacent to the ribbed side of the channel is destabilized by rotation while a negative value indicates a flow that is stabilized by rotation.

Detailed measurements of the isothermal flow in a ribbed rotating channel were reported by Coletti et al. [2] who used Particle Image Velocimetry to map the flow field at $Re = 15,000$ (based on bulk velocity and hydraulic diameter) and for $Ro = \pm 0.3$. Borello et al. [3] used these measurements to validate their Large-Eddy Simulations while Kubacki et al. [4] and Xun and Wang [5] employed them to check models that combined features from conventional Reynolds-averaging (RANS) closures with Large-Eddy Simulations. For the heated case, Mayo et al. [6, 7] used Liquid Crystal Thermography to map the thermal field at values of Re up to 55,000, and for rotation numbers in the range $-1 < Ro < 1$. Abdel-Wahab and Tafti [1] performed Large-Eddy Simulations with a dynamic subgrid-scale model at $Re = 20,000$ and for $Ro = 0.18, 0.36$ and 0.68 . Ahn et al. [8] used the same approach but for $Re = 30,000$. They examined the impact of the rotation on the heat transfer on the pressure and suction sides and confirmed the basic mechanism responsible for the change in the local heat transfer due to rotation. Sewall et al. [9] reported on a combined computational (LES with dynamic sub-grid scale model) and experimental (Laser Doppler Velocimetry and infrared camera) of the flow and thermal fields in a number of related geometries including one in which nine ribs were installed. Their findings confirmed the expectation that, after a suitable development length, the flow becomes periodic and is hence amenable to computation with a reduced solution domain with cyclic boundary conditions. They also pointed out the important role in the enhancement of the heat-transfer rates of the unsteady vortex structures that are formed by shear-layer separation from the ribs downstream corners.

In this study, our aim was to contribute to the still limited literature on the heated flows in rotating channels with spanwise ribs and thus provide a point of comparison that is useful in quantifying the uncertainty involved in previous predictions and measurements. To this end, Large-Eddy Simulations were performed for rotating heated asymmetric ribbed channels with due consideration given to aspects of this approach that influence the quality of the results. These aspects include the sensitivity of the computations to the computational grid used to resolve the solution domain, the choice of the discretization scheme used to represent the advection terms in the momentum and thermal energy equations, the choice of the model for the sub-grid scale correlations that appear in these equations, and the nature of the averaging process for flows with significant temporal and spatial variations.

2. Governing equations

The theoretical framework of Large-Eddy Simulations is well known and need not be expounded on in detail here. Briefly, by applying a filter of width Δ to the equations governing the conservation of mass, momentum, and thermal energy, there results (for the fixed coordinate system adopted in this work, and with the centrifugal force term absorbed in the pressure term):

$$\frac{\partial \overline{U}_i}{\partial x_i} = 0 \quad (2)$$

$$\frac{\partial \bar{U}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{U}_i \bar{U}_j) = \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{U}_i}{\partial x_j} \right) + \frac{\partial \tau_{ij}}{\partial x_j} - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} - 2\Omega \times \bar{U} \quad (3)$$

$$\frac{\partial \bar{T}}{\partial t} + \frac{\partial (\bar{U}_j \bar{T})}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\Gamma \frac{\partial \bar{T}}{\partial x_j} \right) + \frac{\partial q_{tj}}{\partial x_j}. \quad (4)$$

In Eqs. (2–4), an overbar represents a variable whose value has been averaged over the filter width. The correlations τ_{ij} and q_{tj} are the unknown sub-scale momentum and heat fluxes that require modeling.

In order to determine the extent to which the results depend on the way in which τ_{ij} is approximated, computations were performed using two different approaches. The first is the most widely-used model of Smagorinsky in which this quantity is taken to be proportional to the local (filtered) rate of strain

$$\tau_{ij} = 2\nu_{sgs} \bar{S}_{ij}, \quad (5)$$

where $S_{ij} = 1/2(\partial \bar{U}_i / \partial x_j + \partial \bar{U}_j / \partial x_i)$ and ν_{sgs} is the sub-grid scale viscosity:

$$\nu_{sgs} = (C_s \Delta)^2 |\bar{S}| \quad (6)$$

Equation (5) thus becomes:

$$\tau_{ij} = 2(C_s \Delta)^2 |\bar{S}| \bar{S}_{ij}. \quad (7)$$

Values of the coefficient C_s in the range 0.065–0.24 have been used in the past. In this work, we adopt the recommendation of [13] and set $C_s = 0.1$.

A well-known defect in the standard Smagorinsky model is its overprediction of ν_{sgs} in regions close to solid boundaries. This is often corrected for by incorporating a damping function, usually the one proposed by van Driest [10] that yields:

$$\nu_{sgs} = (C_s \Delta)^2 |\bar{S}_{ij}| (1 - e^{-y^+/A^+}), \quad (8)$$

where $y^+ = \frac{U_\tau y}{\nu}$ is the non-dimensional wall distance and $A^+ = 26$.

The second model considered was the dynamic SGS model of Germano [11] and Lilly [12] in which the coefficient C_s varies in space and time.

Similarly for the sub-grid scale fluxes q_{tj} , two alternative models were considered. The first is the familiar variation on Fourier's law wherein the fluxes are made proportional to the gradients of the filtered scalar [13]:

$$q_{tj} = \Gamma_{sgs} \frac{\partial \bar{T}}{\partial x_j}. \quad (9)$$

The thermal diffusivity Γ_{sgs} is computed from the sub-grid scale viscosity ν_{sgs} and Prandtl number Pr_{sgs} :

$$\Gamma_{sgs} = \frac{\nu_{sgs}}{Pr_{sgs}}. \quad (10)$$

In contrast to the turbulent Prandtl number Pr_t which is approximately constant in wall flows, there is little consensus on the appropriate value for Pr_{sgs} with values in $0.3 \leq Pr_{sgs} \leq 0.7$ reported in previous studies. Here, we assign to this parameter the value $Pr_{sgs} = 0.4$ suggested by [13].

The second model considered is one that differs from Fourier's law in allowing for q_{tj} to depend on the gradients of both the filtered scalar and the filtered velocity. Such dependence is suggested by the exact equations that govern the conservation of the turbulent scalar fluxes irrespective of the filter size [14]. The model is given by:

Figure 1. The coordinates system, the computational domain, and a close-up view of the grid.

$$\begin{aligned}
 q_{ij} = & C_1 \frac{k^2}{\epsilon} \frac{\partial \bar{T}}{\partial x_j} \\
 & + C_2 \frac{k}{\epsilon} \overline{u_i u_j} \frac{\partial \bar{T}}{\partial x_i} \\
 & + C_3 \frac{k^2}{\epsilon^2} \left(\overline{u_i u_k} \frac{\partial \bar{U}_j}{\partial x_k} + \overline{u_j u_k} \frac{\partial \bar{U}_i}{\partial x_k} \right) \frac{\partial \bar{T}}{\partial x_i}.
 \end{aligned} \tag{11}$$

The coefficients of this model are assigned the values suggested by Müller et al. [15] from the study of the heated flow in a smooth channel with spanwise rotation viz. $C_1 = 0.03$, $C_2 = 0.21$, $C_3 = -0.105$.

3. Computational details

The governing equations were solved by means of the OpenFOAM finite-volume solver. The time derivatives were discretized by a second-order accurate scheme [16] with the Courant number set equal to 0.6 to ensure stability [17]. The diffusion terms were discretised using second-order accurate central-differencing. The convective terms were discretized using three different schemes to obtain a measure of the numerical accuracy of the computations. These were the upwind scheme which is first-order accurate, the bounded central-differencing scheme which is second-order accurate, and cubic scheme which is third-order accurate when used on structured grids. Comparisons of results obtained with these schemes are presented in the following section.

Colletti et al. [2] and Sewall et al. [9] have shown that the flow over the spanwise ribs becomes periodic after a suitable development length and hence the choice of solution domain shown in Figure 1. The values assigned to the various geometric dimensions are the same as in Coletti et al. [2] and Mayo et al. [7] i.e., a channel height $H = 83 \text{ mm}$, width $L = 75 \text{ mm}$, and length $P = 80 \text{ mm}$. The ribs are square in cross section with height $e = 8 \text{ mm}$. In terms of the proportions of the domain size to e , the total length $P/e = 10$, the height $H/e = 10.375$, and the spanwise extent $L/e = 9.375$. The origin of the coordinates system is at the lower corner of the upstream rib and a positive value of the rotation speed Ω represents rotation in the counter-clockwise direction.

The computations were performed on a structured mesh formed of an O-type mesh surrounding the ribs and an H-type rib elsewhere (Figure 1). This combination was found to yield smooth transition across the entire solution domain with sensible grid-expansion ratios throughout. In constructing the meshes used in the next section, consideration was given to the recommendations of Georgiadis et al. [18] regarding the mesh spacings in near-wall regions. Thus, in terms of wall units, Δ_x^+ turned out to be equal to 24 which is within the recommended range of $50 \leq \Delta_x^+ \leq 150$, Δ_y^+ was equal to 0.4 which conforms to the recommended range of $\Delta y_{wall}^+ < 1$, and Δ_z^+ equal to 25 which is within the recommended range of $15 \leq \Delta_z^+ \leq 40$.

The boundary conditions used were as follows. For the momentum equations, the no-slip condition was applied at all walls. For the temperature equation, at the heated wall where the ribs are located, a uniform heat flux boundary condition was specified. For the opposite, smooth wall,

the adiabatic wall condition was applied. The two side boundaries were modeled as symmetry planes for all the variables solved.

At inlet and outlet, cyclic boundary conditions were implemented for both the momentum and the thermal energy consistent with the periodic nature of the flow. For the momentum equations, the methodology used is that of Patankar [19]. Thus, for periodic boundaries located at distance P apart, the pressure gradient is expressed as:

$$\frac{p(x, y) - p(x + P, y)}{P} = \beta, \quad (12)$$

and thus:

$$p(x, y) = -\beta x + \tilde{p}(x, y), \quad (13)$$

where βx is related to the pressure drop and the global mass flow rate and $\tilde{p}$ expresses the periodicity of the flow. Thus the pressure gradient term in Eq. (3) becomes:

$$\frac{\partial \tilde{p}}{\partial x_i} = \frac{\partial \tilde{p}}{\partial x_i} - \beta \delta_{i1} \quad (14)$$

Similarly, for the thermal field, a streamwise global temperature gradient γ is defined as

$$\frac{T(x + P, y) - T(x, y)}{P} = \gamma \quad (15)$$

with γ being

$$\gamma = \frac{\dot{Q}}{\dot{m} c_p P}, \quad (16)$$

with $\dot{Q}$ being the rate of added heat to the fluid over the period length P . Thus the temperature can be decomposed in two parts:

$$T(x, y) = \gamma x + \tilde{T}(x, y). \quad (17)$$

with $\tilde{T}(x, y)$ being the periodicity of the temperature field and $T(x, y)$ representing the increase of the global temperature. Substitution into the filtered temperature equation yields:

$$\frac{\partial \tilde{T}}{\partial t} + \frac{\partial (\bar{U}_j \tilde{T})}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\Gamma \frac{\partial \tilde{T}}{\partial x_j} \right) - \gamma \bar{U}_i \delta_{i1} + \frac{\partial \tau_{ij}}{\partial x_j}. \quad (18)$$

Further details of the derivation and implementation of the periodic boundary conditions for momentum and temperature in OpenFOAM may be found in [20].

4. Assessment of numerical accuracy

This section presents a summary of the extensive tests that were performed to quantify the uncertainty in the computed results.

4.1. Grid density

In order to quantify the numerical accuracy, we adopted the Grid Convergence Index (GCI) method [21]. This required performing computations on three different grids (coarse, medium, and fine) that differ by a nearly-constant refinement factor. The parameter chosen to determine the sensitivity of the results to grid effects was the domain-averaged Nusselt number Nu which was non-dimensionalized by the value given by the Dittus-Boelter and Blasius correlation Nu_0 :

Table 1. Parameters of the Grid Convergence Index tests.

$N1$	$N2$	$N3$
3.808×10^6	2.6499×10^6	1.9565×10^6
ϕ_1	ϕ_2	ϕ_3
1.551	1.493	1.357
$\phi_{ext}^{21} = 1.578$		
$e_{ext}^{21} = 1.72\%$		
$GCI_{fine}^{21} = 2.19\%$		

$$Nu = \frac{D_h \dot{q}_w}{\lambda(T_w - T_b)} \quad (19)$$

$$Nu_0 = 0.023 Re_b^{0.8} Pr^{0.4}. \quad (20)$$

The outcome of the GCI analysis is presented in **Table 1** (where $\phi = Nu/Nu_0$) and is shown in **Figure 2**. Significant differences are apparent between the coarse and medium mesh results manifested by the steep gradient in **Figure 2**. These differences diminish between the results of the medium and fine grids. In **Table 1**, the parameter ϕ_{ext}^{21} represents the Richardson extrapolation value of Nu/Nu_0 , i.e., the value that would be obtained from an infinitely fine mesh. The parameter e_{ext}^{21} represents the extrapolation error. The parameter GCI_{fine}^{21} quantifies the numerical uncertainty in the fine-mesh result for the globally-averaged Nusselt number. For the present grid, this uncertainty is estimated to be 2.19% which is well within the limits of uncertainty in the measurements. **Figure 2** shows that the gradient between the present fine mesh and the theoretically infinitely fine mesh is flat meaning that the further refinement of the grid will not lead to worthwhile improvements in accuracy and hence all subsequent results were obtained with the fine mesh.

4.2. Choice of discretization scheme

Three alternative schemes were used to discretize the convective fluxes: the first-order accurate Upwind scheme, the second-order accurate Gauss linear scheme, and the third-order accurate cubic scheme. Details of these schemes can be found in [16]. The computations were performed for several flow conditions with only some representative results (for $Ro=0.3$) presented here. **Figure 3** compares the predicted cross-stream profiles of axial velocity at $x/e = 4$. The second- and third-order accurate schemes yield results that are indistinguishable from each other across the entire cross section. The first-order schemes shows clear differences from the others, most notably in the region $0 < y/e < 3$ which is where the velocity gradients are steepest. Far greater differences between the the first-order scheme and the higher-order ones are apparent in the thermal field results such as in the distribution of Nu shown in **Figure 4**. This is again attributed to the significant errors that are inherent in the use of first-order accurate schemes in flows where

Figure 2. Grid Convergence Index results for the global Nu . ▲: Results from different computational meshes, ■: Richardson extrapolation.

Figure 3. Streamwise velocity profile at $x/e = 4$ as obtained with different discretization schemes: (---) first order, (—) second order, (...) third order. Experimental data [2]: $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Figure 4. Predicted average Nu as obtained with different discretization schemes: (---) first order, (—) second order, (...) third order. Experimental data [6]: $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

steep spatial gradients of velocity and temperature are present. In contrast, the results of the second- and third-order accurate schemes are indistinguishable for this parameter as well.

4.3. Model for SGS stresses

The sensitivity of the computed results to the choice of a model for τ_{ij} depends on the choice of parameter used to quantify it. Thus, in **Figure 5**, the computed streamwise velocity profiles at $x/e = 4$ and for the case of $Ro = 0.3$ show virtually no differences between the results of the standard Smagorinsky model with its assumption of constant C_s , and the more sophisticated dynamic model in which this coefficient can vary through the field. However, the differences between the two modeling approaches become more apparent when details of the turbulence structure are considered, specifically in the distributions of the sub-grid stresses $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$.

Figure 5. Dependence of computed streamwise velocity on choice of subgrid-scale models: Smagorinsky (—), dynamic model (...). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Figure 6. Dependence of computed turbulent stresses on choice of subgrid-scale models: Smagorinsky (—), dynamic model (....). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

These are shown in [Figure 6](#), also at $x/e = 4$ and for $Ro = 0.3$. Compared to the dynamic model, the standard Smagorinsky model predicts slightly higher levels of turbulence activity in the outer regions of the flow. Closer to the wall, the two approaches yield closely matched results.

Since the velocity and thermal fields are coupled, it is to be expected that the differences between the two approaches to modeling the SGS stresses will translate analogous to differences in the prediction of the thermal field. Specifically, the generally higher levels of turbulent activity predicted with the standard model should lead to the prediction of higher rates of heat transfer to the wall. This is indeed the case as can be seen from [Figure 7](#) where the predicted streamwise distribution of Nusselt number are plotted. The comparison there is with the experimental data of [7] for $Ro = -0.3, 0, 0.3$. Overall, there seems to be little benefit to justify the use of the dynamic model over the simpler and more robust Smagorinsky closure.

4.4. Model for SGS heat fluxes

In [Figure 8](#), the effects of the two different models for the SGS heat fluxes are examined. Plotted there are their predictions of the Nusselt number distribution along the wall for the three values of Ro reported in the experiments of [7]. Overall, the differences between the simpler Fourier's law and the more complete model are quite small despite their very different formulations. Coupled with the results for the SGS stress models, the absence of clear dependence on the model

Figure 7. Comparison of space averaged and time averaged Nusselt number: Smagorinsky (—), dynamic Smagorinsky (...). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

for the scalar fluxes suggests that the fine grid that was used in the present tests was sufficient to resolve the flow to the extent that the precise details of the SGS models were no longer significant in the predictions.

4.5. Temporal and spatial averaging

The unsteady nature of the flow is evident in Figure 9 where contours of the instantaneous velocity along the channel centerline are plotted for $Ro = \pm 0.3$. As expected, the shear layer that develops downstream of the rib exhibits the presence of regular, large-scale unsteady separation in the form of vortices of a fairly regular frequency. These vortices cause corresponding oscillations in both the free stream and in the cavity flow. The effects of stabilized and destabilized rotation are quite evident: the enhanced mixing in the latter case leads to rapid re-attachment of the separated flow and thus an increase in the flow velocity within the cavity.

The effects of the free shear-layer oscillations on the near-wall flow are also evident in Figure 10 where the wall static pressure coefficient c_p at a representative point along the channel centerline is plotted against the Flow Through Time ($FTT = P/U_b$). This plot gives an indication of the time it takes for the flow at a specific location in the flow field to evolve sufficiently to lose memory of the arbitrary initial conditions used to start the computations. This corresponds to the minimum time that must elapse before meaningful temporal averaging can commence. Thus, for example, when the effects of rotation are stabilizing ($Ro = -0.3$), it takes about 20 FTTs before averaging can commence. In contrast, in the destabilized case, it would appear that the corresponding FTT is only about 10, noting that the more intense oscillations in this case are due to the expected enhancement of turbulent fluctuations due to rotational effects rather than purely to start-up effects. It should be borne in mind that the results in Figure 10 refer to a single parameter (pressure), at a single point - it cannot be taken with any confidence as being representative of the behavior of other parameters, at other locations in the flow. Thus, while it is clear that the averaging process cannot begin from the start of the computations, the extent of the time interval that must elapse before averaging can begin is not known a priori.

In this study, the time interval over which the averaging was performed was found to depend quite strongly on the sense of rotation, that is, whether stabilizing or destabilizing. Figure 11 shows the dependence of the profiles of mean velocity (along the centerline, at $x/e = 4$) and of the centerline distribution of Nusselt number on the length of the averaging process expressed in terms of the number of Flow Through Time (FTT). The case of stabilized rotation required the least number of FTTs to converge. The opposite was true for the destabilized case, as can be seen from Figure 11.

Another aspect of the averaging process that is of interest is whether it is more appropriate, when comparing with time-averaged experimental results along the channel centerline, to consider only the LES results along the centerline and average these over an appropriate number of

Figure 8. Comparison of space averaged and time averaged Nusselt number: Fourier's law (—), YSC (....). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Figure 9. Instantaneous velocity field at the centerline for $Ro = -0.3$ and $Ro = 0.3$.

Figure 10. Variation of wall static pressure coefficient with Flow Through Time.

FTTs, or to first obtain spanwise averages of the computed results and then time-average these. In this particular flow, with the side boundaries being planes of symmetry, the differences between the two approaches should be fairly small reflecting the presence of somewhat weak three-dimensional effects along the span. This would not be the case at much higher values of Ro where the Görtler vortices due to rotation would make the order of the averaging process important in determining the final shape of the profiles. In Figure 12, the differences between time-averaging the centerline results at $x/e = 4$ and first performing spanwise averaging followed by time averaging at the same location are shown for $Ro = -0.3, 0$ and 0.3 . The differences are small but not insignificant which indicates that the three-dimensionality of the flow in the spanwise direction, if not properly accounted for in averaging the computations or the measurements, can lead to ambiguities in the processed results.

The sensitivity of the predicted Nusselt number to the averaging process is shown in Figure 13 where the centerline distributions of Nu are compared with the data of Mayo et al. It can be seen that the time averaged Nu values are generally higher than the time and spanwise averaged

Figure 11. Plots of axial velocity (top) and Nu as a function of FTT and Ro . For $Ro = -0.3$: 10 FTT (---), 25 FTT (—). For $Ro = 0$: 40 FTT (---), 60 FTT (—). For $Ro = 0.3$: 90 FTT (---), 125 FTT (—). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Figure 12. Time averaged vs. time and spanwise averaged streamwise velocity profiles at $x/e = 4$. Present results: Time averaging (---), spanwise and time averaging (—). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

values. A quantitative assessment of the differences between predictions obtained with different averaging approaches and the measurements is given in [Table 2](#).

5. Results and discussion

In this section, we present and discuss the results that are obtained with the best practices identified in the previous section. Contours of the streamwise component of velocity, and of the spanwise vorticity are presented in [Figure 14a and b](#). Shown there are the time- and spanwise-averaged results for $Ro = -0.3$, 0 and 0.3. Three recirculation regions are observed between the ribs for each of the cases. The largest is formed from the separation of the flow from the top of the upstream rib leading to the formation of a detached free shear layer and a recirculation zone below it. For all three cases, the shear layer reattaches in the region between the two ribs. The reattachment length (x_R) is determined by the level of turbulence activity in the shear layer since it is that serves to transport the high momentum flow from the central core of the channel towards the ribs. Consequently, for the destabilized rotation case ($Ro = 0.3$), the reattachment

Figure 13. Results for time-averaging and time-and-space averaging: time averaged (---), time and space averaged (—). Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Table 2. Predicted maximum Nu as a function of the averaging method.

	$Ro = -0.3$	$Ro = 0$	$Ro = 0.3$
Time averaged $Nu/Nu_{0,max}$	1.145 at $x/e = 4.893$	1.965 at $x/e = 3.093$	2.970 at $x/e = 3.356$
Time and space averaged $Nu/Nu_{0,max}$	1.054 at $x/e = 4.706$	1.886 at $x/e = 3.468$	2.932 at $x/e = 3.206$
Experimental $Nu/Nu_{0,max}$	1.073 at $x/e = 3.666$	1.949 at $x/e = 3.333$	2.808 at $x/e = 3.333$
Relative Error (Time averaging)	6.71%	0.82%	5.77%
Relative Error (Time and space averaging)	1.77%	3.23%	4.41%

Figure 14. Time and spanwise averaged contours of axial velocity (left) and vorticity for $Ro = 0$, $Ro = 0.3$ and $Ro = -0.3$.

length is expected to be significantly shorter than for the other two cases. This can be seen in Table 3 where the present results are compared with the measurements of [2]. The agreement is quite satisfactory considering that the flow in the cavity is unsteady and is thus difficult to

Table 3. Predicted and measured reattachment lengths as a function of Ro .

	$Ro = -0.3$	x_R/e $Ro = 0$	$Ro = 0.3$
Measurements [2]	5.65	3.85	3.45
Present study	6.1	3.9	3.65

measure to a high degree of accuracy. Two other recirculation zones are generated in the gap between the ribs, much smaller than the first, but are nevertheless important as will be seen later when the heat transfer case is considered. The contours of mean vorticity in [Figure 14b](#) show the effects of the imposition of the global vorticity due to system rotation on to the flow vorticity. Thus, for the case of $Ro = -0.3$, the sense of global rotation, being clockwise, is opposite to that of flow vorticity and hence the reduction of the latter downstream of the rib.

[Figure 15](#) shows the distribution of the skin friction coefficient ($c_f = \tau_w/0.5\rho U_b^2$) along the wall in between two ribs. The wall shear stress was evaluated by extrapolating the normal gradient of streamwise velocity to the wall. There are no measurements of this parameter. The effects of rotation on this parameter are immediately apparent: the stabilizing rotation leads to significant drop in the magnitude of this parameter relative to the stationary case. A reduced rate of momentum transfer to the wall, while advantageous in normal circumstances, suggests a proportionate reduction in the rate of heat transfer which is distinctly undesirable in this case. In contrast, the destabilizing effects lead to an increase in this parameter everywhere along the wall, especially in the region $1.0 < x/e < 3.0$ which is directly below the large recirculating flow observed in [Figure 14](#).

The global static pressure drop Δp , caused by friction and flow reversal, represents an energy loss and hence the need to minimize it in the interest of overall efficiency. [Figure 16](#) shows the variation of this parameter (non-dimensionalized with respect to the no rotation value) with Ro . It is evident that destabilizing rotation effects initially lead to substantial increase in the global pressure drop corresponding to 45% at $Ro = 0.15$ followed by a modest subsequent increase to 50% at $Ro = 0.3$. In contrast, for stabilized rotation, the reduction in pressure drop appears to be monotonic over the range of Ro considered suggesting a trend towards reverse transition to a laminar-like state.

We turn now to consideration of the secondary flows that are generated in the channel's cross section. These motions are formed by a combination of the pressure gradients that arise from rotation (Prandtl's first kind of secondary flow), and from anisotropy of the normal stresses (Prandtl's second kind). For the case of flow in a square-sectioned channel with $Ro = 0$, these motions would be symmetric but this would not be the case in which ribs are located on one wall. This is what is obtained here as can be seen from [Figure 17](#). The superimposition of rotation further complicates the pattern of secondary flows as is discussed in detail in [\[22\]](#). For the case of $Ro = -0.3$, the two large vortices generated at the upper half of the channel split into four smaller ones. The opposite occurs for the case of $Ro = 0.3$ where the effects of rotation appear to significantly strengthen the vortical flow there. These changes affect the Nusselt number distribution on the ribbed surface, as will be discussed below.

The predicted and measured cross-stream profiles of time- and spanwise-averaged streamwise velocity are compared in [Figure 18](#). The results shown there are presented for the three rotation numbers, and are plotted at four streamwise location. At $x/e = 0$, which is at the corner of the upstream rib, the profile shapes are shaped by the effects of adverse pressure gradients and, for the case of $Ro = 0.3$, by enhanced rate of transfer of momentum from the core. The region of separated flow is apparent in the two subsequent profiles, at $x/e = 2$ and 4, and the subsequent recovery at $x/e = 8$. For the case of $Ro = 0.3$, the emergence of a region of momentum deficit at the channel core is a clear indication of the high rate of momentum transfer towards the ribbed surface due to destabilizing rotation.

Figure 15. Time-averaged skin friction distribution along channel centerline: (···) $Ro = -0.3$, (—) $Ro = 0$, (---) $Ro = 0.3$.

Figure 16. Variation of pressure drop with rotation number Ro .

Figure 17. Time averaged secondary flow in the $y-z$ plane at $x = L/2$.

Contours of the turbulence kinetic energy k are presented in [Figure 19](#). This parameter is used in a wide range of turbulence closures of the type used in practical engineering calculations (e.g. the $k-\epsilon$ and $k-\omega$ models). Its governing equation is obtained by taking half the trace of the equations for the Reynolds stresses ($\overline{u_i u_j}$). In these equations, the effects of system rotation enter through a rate of generation term [23]:

$$G_{ij} = -2\Omega_k (\overline{u_j u_m} \epsilon_{ikm} + \overline{u_i u_m} \epsilon_{jkm}). \quad (21)$$

For the present flow, where the components of the rotation vector Ω_k are $(0, 0, \Omega)$, it turns out that $G_{\bar{u}^2} = 4\Omega \overline{u\bar{v}}$, $G_{\bar{v}^2} = -4\Omega \overline{u\bar{v}}$ and $G_{\bar{w}^2} = 0$ with the result that the rate of production of k due explicitly to rotation is zero and hence the equation for k would suggest that this parameter is insensitive to the effects of rotation. In reality, the effects of rotation on k are very substantial and enter into its equation indirectly via changes to the Reynolds stresses and hence to the rate

Figure 18. Predicted and measured mean streamwise velocity profiles along the centerline: (. . .) $Ro = -0.3$, (—) $Ro = 0$, (- - -) $Ro = 0.3$. Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Figure 19. Contours of k for $Ro = -0.3, 0, 0.3$.

of production of k due to their interaction with the mean shear. The contours of Figure 19 show this most clearly. Note that the stabilizing effects of rotation on k drastically diminished this quantity to the extent that, for clarity, the color scale for $Ro = -0.3$ was adjusted relative to the other two cases. The enhancement of k due to rotation is apparent in both the magnitude of this parameter which is everywhere greater than for the stationary case, and in the extent of the rotation-enhanced region.

Figure 20. Predicted and measured turbulent stresses: (· · ·) $Ro = -0.3$, (—) $Ro = 0$, (---) $Ro = 0.3$. Experimental data [2]: $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Comparisons with the Reynolds-stress components reported in the experiment are presented in Figure 20. The maximum value of each stress component occurs at a location that is approximately level with the rib height, i.e., in the regions where the mean shear is greatest. The substantial differences between the levels of $\overline{u^2}$ and $\overline{v^2}$ indicate significant turbulence anisotropy due to the action of the wall in damping the fluctuating velocity component normal to it. The shear stress $\overline{uv}$ changes sign which is to be expected in flows with embedded recirculation. Overall, the correspondence between predictions and measurements is fairly satisfactory especially when the difficulty in obtaining high resolution in PIV measurements is taken into account [3].

Turning now to the thermal field, instantaneous contours of Nu for the three rotation numbers are presented in Figure 21. The patterns of heat-transfer rates to the wall correlate very closely with the velocity contours presented in Figure 14. Thus, for example, the relatively low values of Nu immediately downstream of the first rib which is also where the small, secondary, vortex was located. This is to be expected due to the low velocities there and hence heat transfer from the surface is largely by the less effective process of conduction. Further downstream, the values of Nu increase though it should again be noted that the contour scales for $Ro = -0.3$ have been adjusted to more clearly show the results. Overall, the unsteady nature of the flow is clearly evident from these plots. By time-averaging the instantaneous results, much of the variations in the spanwise direction disappear (as can be seen in Figure 22) giving rise to a pattern of heat transfer rates that is approximately two dimensional across the larger part of the span, with small

Figure 21. Instantaneous (top) and time-averaged (bottom) Nu distribution between the ribs for $Ro = -0.3, 0., 0.3$.

Figure 22. Time- and space-averaged vs. instantaneous Nu distribution for $Ro = -0.3, 0., 0.3$.

Figure 23. Nu distribution between the ribs for $Re = 15000$: (...) $Ro = -0.3$, (—) $Ro = 0$, (-.-) $Ro = 0.3$. Experimental data (Mayo, private communication): $\square$ $Ro = -0.3$, $\circ$ $Ro = 0$, $\triangleright$ $Ro = 0.3$.

Figure 24. Surface averaged Nu number as function of Ro: $\blacktriangle$ present LES, $\circ$ experiments [7].

Figure 25. Cross-stream variation of non-dimensional temperature: (....) $Ro = -0.3$, (—) $Ro = 0$, (- - -) $Ro = 0.3$.

Figure 26. Space-averaged non-dimensional temperature distribution between the ribs: (....) $Ro = -0.3$, (—) $Ro = 0$, (- - -) $Ro = 0.3$.

departures evident due to the effects of the side walls and the strong secondary flows set-up by rotation.

On the face of it, the results shown in [Figure 21](#) might suggest that steady-state, two-dimensional computations might be sufficient to obtain Nu distributions that are acceptable for engineering design. This is not the case, as can be seen from [Figure 22](#) which compares the time- and space- averaged Nu distributions with instantaneous values that were averaged in space only. Of particular interest are the results for $Ro = 0.3$ where it is evident that the intermittent behavior of the destabilized flow leads to instantaneous variations that can be vastly different from the time-averaged values.

[Figure 23](#) illustrates the distribution of space and time averaged Nusselt number Nu between the ribs for the stationary and the rotating cases. The comparisons are with the experimental data of Mayo (private communication). The LES results at $Ro = 0.3$ are in satisfactory correspondence with the measurements considering that the maximum relative uncertainty in the experiments (represented by the error bars) is about $\pm 9.8\%$. A slight augmentation of the heat transfer rate is apparent in the rib wake region ($0 < x/e < 0.8$) for the $Ro = 0$ and 0.3 cases due to flow reattachment. In both cases, the heat transfer rate increases very rapidly in the main recirculation region where the maximum value reached for the $Ro = 0.3$ case is approximately $Nu_{max}/Nu_0 = 2.94$ at $x/e = 3.35$.

Mayo et al. [7] reported experimental results showing the variation of the globally averaged Nusselt number $\overline{Nu}$ with rotation number. The global Nusselt number is evaluated by integrating of the time-averaged Nusselt number over the surface between the ribs. The present predictions are compared with the experiment in [Figure 24](#). The agreement is generally good with a maximum discrepancy of 8% at $Ro = -0.38$. It is noted that the rotation dependency of the global Nusselt number is approximately linear for moderate rotation, i.e., in the range $-0.15 < Ro < 0.15$ thereafter, Nu increases at a much reduced rate and appears to reach a state of saturation wherein further increases in Ro produce very little additional benefits.

The cross-stream variation of the time- and spanwise-averaged temperature are presented in [Figure 25](#). Plotted there is the non-dimensional temperature (θ) which is calculated from:

$$\theta = \frac{\bar{T} - T_0}{-\left.\frac{\partial T}{\partial y}\right|_w D_h}, \quad (22)$$

where T_0 is the averaged inlet temperature and $\left.\frac{\partial T}{\partial y}\right|_w$ is the constant normal temperature gradient along the heated wall. The non-dimensional temperature profiles are presented at the same locations as the velocity profiles ([Figure 18](#)), and for the same values of Ro . The profiles clearly show the substantial changes in both the magnitude and profile shapes wrought by rotation. Specifically, the temperature is lowest in the destabilizing case ($Ro = 0.3$) and highest in the stabilizing case ($Ro = -0.3$) where the normal gradients of temperature are flat consistent with much lower heat-transfer rates. [Figure 26](#) demonstrates clearly the effects of rotation on the spatially-averaged wall temperature between the ribs. Compared to the case of a stationary channel, the temperature is much reduced by destabilizing rotation, and increased by stabilizing rotation. The profiles show the presence of two maxima whose position correlates very closely with the location of the secondary recirculation zones that are formed at the rib corners ([Figure. 14a and b](#)). The highest peak appears below the upstream rib where the velocity gradients are quite small.

6. Conclusions

Our purpose in performing the research reported herein was to contribute to the limited published literature on the computation of heat transfer in an asymmetrically ribbed channel subject to spanwise rotation. Considering the complexity of this flow due to the presence of large-scale flow separation and subsequent re-attachment, system rotation and mean-flow unsteadiness, the

computations were performed using large-eddy simulations with considerable effort expended to reduce the uncertainties inherent in the computation of modeled, unsteady, three-dimensional non-linear equations. Thus a systematic grid-independence study was performed utilizing three different meshes, the results of which provided input to the Grid Convergence Index method for quantifying the numerical errors in the computations. The outcome of this method indicated that the finest mesh used, which consisted of 3.8 Million active cells, was sufficient to reduce the error in the target parameter to around 2% of the theoretically extrapolated value. By adopting this fine grid in subsequent tests, it was found that the results obtained with second- and third-order accurate discretization schemes were virtually indistinguishable. This is a significant finding since, in the LES framework, refining the mesh simultaneously reduces the discretization errors but also reduces the contributions of the model for the sub-grid scale correlations. The fine-mesh computations eliminated another source of uncertainty in LES; specifically, the dependence of the computations on the manner in which the sub-grid scale fluxes of momentum and thermal energy are modeled. For the momentum equations, the differences between simulations with the standard Smagorinsky model and the more complex dynamic model were insignificant suggesting that the contribution of the sub-grid scale fluxes to the overall balance of terms in the filtered equations was quite limited. The same proved to be the case for the thermal fluxes where the linear and non-linear models yielded closely-matched predictions. The uncertainty associated with obtaining temporal and spatial averaged was also explored with respect to both the mean-flow and the thermal fields. It was found that the period over which averages are obtained depends very strongly on whether the sense of rotation is stabilizing or destabilizing, with the latter requiring considerably longer averaging intervals. The best practices to emerge from the above were then used to obtain results that serve to quantify the effects of rotation in this flow. In agreement with the limited experiments available, the effects of rotation were found to be quite significant most notably on the Nu distribution along the ribbed wall. Evidence of the enhancement of turbulent activity by destabilizing rotation was also clear in the overall patterns of flow such as the significant shortening of the length of the reversed flow region. Examination of the secondary flow patterns at a cross section normal to the main flow indicates significant changes can occur due to rotation most notably in the enhancement, in the destabilized case, of the magnitude of these motions relative to the main flow velocity. Finally, the evaluation of the pressure drop and the global Nusselt number for a certain interval of the Rotation number $-0.38 < Ro < 0.38$ shows that, for the case of destabilized rotation that is of primary interest here, both parameters increase quite significantly over the range $0 < Ro < 0.15$, thereafter at a much lower rate suggesting the attainment of a saturation-like state where further increases in the rotation rate produce only marginal improvements in the efficiency of the surface ribs.

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