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Elon R. Close

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GENERATION OF FIRST AND SECOND ORDER TRANSFORMATION ELEMENTS
FROM A GIVEN MAGNETIC FIELD

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ABSTRACT

Equations of motion and a simple elimination scheme suitable for obtaining first and second order transformation elements by numerically integrating through a magnetic field are developed. The results are applied to ideal magnets with a constant field, a quadrupole field, and a sextupole field. The results obtained by numerical integration are compared with analytically obtained values.

I. INTRODUCTION

Over the past number of years, there have appeared numerous articles dealing with second order effects in a magnetic field. One of the most ambitious works has been that of Brown [1] in which second order equations of motion are derived and their general solution is presented. These results have been utilized in a general beam transport program TRANSPORT [2]. In all of these cases, the results have been of an analytical nature in the sense that equations are given, approximations are made that lead to new equations, and then the latter are solved for the desired quantities. There are, however, situations where it is desirable to generate the desired transformation elements from a magnetic field in a direct manner such as numerically integrating the orbits through the field. These arise quite naturally when all that is available is the measured magnetic field and they also occur in design problems where it is desired to see the effect of various trial fields on the transformation elements.

We shall consider here the problem of generating first and second order elements when an, essentially, arbitrary field is given. Our goal is to obtain a system of equations which, when integrated, will give the coordinates of a reference particle and of the nearby particles. When this is obtained, a scheme can be selected that will use these results to generate the transformation elements to first and second order. We isolate these two steps because there are numerous ways to obtain solutions for nearby particles and there are also a number of ways to obtain the transformation elements from these nearby solutions. The selection of a particular method will depend on how general a result is desired, what information is needed in the final answers, how the magnetic field is

specified, and also on the computational facilities available. We have restricted ourselves here to a rather simple approach which will, however, suffice in a number of cases.

In the development that follows, we first obtain the equations of motion for the reference particle and then those of a nearby particle with respect to the reference particle. Once this is done a simple elimination scheme is presented that uses these results to obtain the transformation elements. Some simple examples of the transformation elements for ideal magnetics that illustrate the application of these results are then presented.

II. DERIVATION OF THE EQUATIONS OF MOTION

If we let P , q , v , and B represent the momentum, charge, velocity, and magnetic field, then it is true that we can simply write down $\frac{dP}{dt} = q v \times B$ and proceed to integrate. This, however, will not lead to the desired results. We wish ultimately to use as coordinates for the nearby particle its distance from the reference particle orbit and the angles its trajectory makes with respect to the reference orbit. We shall, therefore, transform the equations of motion of the nearby particles so that they conveniently describe that trajectory with respect to the reference orbit and when integrated they will directly yield the above quantities.

2.1 Reference Orbit

From Figure 1, we see that the reference curve s_0 is established by tracing the orbit of the reference particle which has momentum $P_0 = m_0 v_0$. It is assumed that P_0 is a constant scalar value. We shall also assume that there exists a plane $\mathcal{L}$ in which the reference particle stays. Note that if the reference orbit is a straight line, then any plane containing this line will suffice for $\mathcal{L}$. By convention, we shall take $\mathcal{L}$ to be the (1,3) plane.

The equations of motion for the reference particle are:

$$\begin{aligned} \frac{d}{dt} \vec{P}_0 &= \frac{q_0}{m_0} \vec{P}_0 \times \vec{B}_0 \\ \frac{d}{dt} \vec{r}_0 &= \frac{\vec{P}_0}{m_0} \end{aligned} \quad (1)$$

We now define the vectors:

$$(z_1, z_2, z_3) = (P_1^{(o)}, P_2^{(o)}, P_3^{(o)}) = \vec{P}_o$$

$$(z_4, z_5, z_6) = (r_1^{(o)}, r_2^{(o)}, r_3^{(o)}) = \vec{r}_o$$

$$(B_1^{(o)}, B_2^{(o)}, B_3^{(o)}) = \vec{B}_o = \vec{B}(\vec{r}_o)$$

change the integration variable to the arc length s_o of the reference orbit, and note that since this orbit lies in a plane $\mathcal{L}$, we have that $z_2 = z_5 = \frac{dz_2}{ds_o} = \frac{dz_5}{ds_o} \equiv 0$. Equation (1) can then be written as

$$\begin{aligned} y_1' &= k y_2 \\ y_2' &= -k y_1 \\ y_3' &= y_1 \\ y_4' &= y_2 \end{aligned} \tag{2}$$

where $' = \frac{d}{ds_o}$ and

$$\begin{aligned} -k &= \frac{q_o}{P_o} B_2^{(o)}(r_1^{(o)}, 0, r_2^{(o)}) \\ y_1 &= z_1/P_o = P_1^{(o)}/P_o \\ y_2 &= z_3/P_o = P_3^{(o)}/P_o \\ y_3 &= z_4 \\ y_4 &= z_6 \\ P_o &= |\vec{P}_o| = \sqrt{(P_1^{(o)})^2 + (P_3^{(o)})^2} \end{aligned} \tag{3}$$

As a check that the reference orbit is remaining in the plane $\mathcal{L}(1,3)$ we have that

$$y_2 B_1^{(0)} - y_1 B_3^{(0)} = 0$$

should be true with $B_i^{(0)} = B_i(y_3, 0, y_4)$, $i = 1, 3$.

2.2 Nearby Orbit

Let $\vec{P}$, q , m , $\vec{B}$, $\vec{r}$ refer respectively to the momentum, charge, mass magnetic field, and radius vector of the nearby particle. Then, the equations of motion for this particle are given by (1), provided we omit the subscript.

We shall now resolve these quantities on to the moving trihedral with the unit vectors $\vec{\eta}$, $\vec{\beta}$, $\vec{\xi}$, in the $1'$, $2'$, $3'$ directions. See Figures 1, 2, and 3. The field, momentum, and radius vector become respectively

$$\begin{aligned}\vec{B} &= B_1 \vec{\eta} + B_2 \vec{\beta} + B_3 \vec{\xi} \\ \vec{P} &= P_1 \vec{\eta} + P_2 \vec{\beta} + P_3 \vec{\xi}\end{aligned}\tag{4}$$

$$\vec{r} = \vec{r}_0 + \delta r_1 \vec{\eta} + \delta r_2 \vec{\beta} + \delta r_3 \vec{\xi}.$$

The independent variable is again changed to the arc length s_0 of the reference curve using

$$\frac{d}{dt} = \frac{ds_0}{ds} \frac{ds}{dt} \frac{d}{ds_0}.\tag{5}$$

If we substitute (4) into (1) and utilize (5), then the equations of motion for the nearby particle can be written as

$$\begin{aligned}P_1' &= \alpha q \{P_2 B_3 - P_3 B_2\} + \tau P_2 - k P_3 \\ P_2' &= \alpha q \{P_3 B_1 - P_1 B_3\} - \tau P_1 \\ P_3' &\equiv \alpha q \{P_1 B_2 - P_2 B_1\} + k P_1\end{aligned}\tag{6}$$

$$\delta r_1' = \alpha P_1 + \tau \delta r_2 - k \delta r_3$$

$$\delta r_2' = \alpha P_2 - \tau \delta r_1$$

$$\delta r_3' = \alpha P_3 + k \delta r_1$$

where

$$' \equiv \frac{d}{ds_0}$$

$$\alpha^{-1} = P \frac{ds_0}{ds}$$

$$P = \sqrt{P_1^2 + P_2^2 + P_3^2} = |\vec{P}| \quad (7)$$

and k and τ are the curvature and torsion defined by

$$\begin{aligned} \vec{\eta}' &= -k \vec{\xi} + \tau \vec{\beta} \\ \vec{\beta}' &= -\tau \vec{\eta} \\ \vec{\xi}' &= k \vec{\eta} \end{aligned} \quad (8)$$

The quantity ds_0/ds can be determined by requiring that $\delta r_3 = 0$ identically. That is, the moving trihedral is always positioned in such a manner that the reference particle lies in the $(1', 2') = (\vec{\eta}, \vec{\beta})$ plane. This is the reference frame in which Brown [1] has developed his results and any of our transfer elements will be directly comparable to his. A disadvantage of this reference frame is that all time dependence is completely lost.

The fact that δr_3 is zero identically yields

$$\alpha P_3 + k \delta r_1 = 0$$

from which $\frac{ds_0}{ds}$ can be obtained.

We shall also find it convenient to replace P_i by $\delta P_i = P_i/P_0$,
 $i = 1, 2, 3$ in all our equations.

Defining

$$\omega = \delta P \frac{ds_0}{ds}$$

we can write

$$\omega^{-1} = \frac{1 - k \delta r_1}{\sqrt{\delta P^2 - \delta P_1^2 - \delta P_2^2}}$$

where

$$\delta P^2 = \delta P_1^2 + \delta P_2^2 + \delta P_3^2 = (P/P_0)^2$$

The instantaneous radius of curvature k is given by

$$k = -\frac{1}{\rho} = -\frac{P_0}{q_0 B_0}$$

where $B_0 = |\vec{B}(\vec{r}_0)|$ is the field on the reference particle orbit. We also have that $\tau = 0$ since the reference orbit stays in a plane. Since it is, in general, rather difficult to accurately calculate the torsion, this restriction that $\tau = 0$ has been imposed. It is possible to do an arbitrary reference orbit, but the approach should be such that it is not necessary to calculate τ .

We now define

$$y_5 = \delta r_1$$

$$y_6 = \delta P_1$$

$$y_7 = \delta r_2$$

$$y_8 = \delta P_2$$

(9)

The nearby orbit equations of motion then become

$$\begin{aligned}
 y_5' &= \frac{1}{\omega} y_6 \\
 y_6' &= \frac{q}{P_0 \omega} \{y_8 B_3 - \delta P_3 B_2\} - k \delta P_3 \\
 y_7' &= \frac{1}{\omega} y_8 \\
 y_8' &= \frac{q}{P_0 \omega} \{\delta P_3 B_1 - y_6 B_3\}
 \end{aligned} \tag{10}$$

where

$$\begin{aligned}
 k &= - \frac{q}{P_0} B_2^{(0)} \\
 \omega &= \frac{\delta P_3}{1 - k y_5} \\
 \delta P_3 &= \sqrt{\delta P^2 - \delta P_1^2 - \delta P_2^2} .
 \end{aligned} \tag{11}$$

The system $y' = f(y)$ defined by (2), (3), (10), (11) will, when integrated, give the coordinates of the reference particle in the global reference frame and of one nearby particle in the local reference frame. The generalization to n nearby particles is obvious. The origin of the nearby particle can be considered the reference particle provided we note that

$$\frac{\Delta P_i}{P_0} = \delta P_i - 1$$

gives the fractional momentum increment.

We note in passing that we can, if desired, find the path length of the nearby particle by integrating

$$s' = \frac{\delta P}{\omega} . \tag{12}$$

III. TRANSFORMATION ELEMENTS

We now wish to find the transformation elements for a particle moving in a specified magnetic field. What is meant here is that the orbit of a particle depends upon its initial conditions. Thus, given a suitable coordinate system and a reference orbit, we wish to expand the solution with respect to the initial conditions, using the reference orbit as the origin, and then calculate the first derivative (obtaining first order transformation elements) and the second derivative (obtaining second order transformation elements).

We do this in a straight forward fashion. Let $\{\delta x, \delta \theta, \delta y, \delta \psi, \frac{dP}{P_0}\}$ specify the coordinates of a nearby particle with respect to the reference orbit. These quantities are illustrated in Figure 4. For ease in carrying out the expansions, write the initial condition vector as

$$t = \{t_1, t_2, t_3, t_4, t_5\} = \{\delta x, \delta \theta, \delta y, \delta \psi, dP/P_0\} \quad (13)$$

The Taylor's expansion for the solution vector y can be written as

$$y_i(t|s_0) = F_{ij} t_j + S_{ijk} t_j t_k + \dots \quad (14)$$

where the convention of summing on repeated indices has been employed, the partial derivatives are written as

$$\begin{aligned} F_{ij} &= \frac{\partial y_i}{\partial t_j} (0|s_0) \\ S_{ijk} &= \frac{1}{2!} \frac{\partial^2 y_i(0|s_0)}{\partial t_j \partial t_k} \end{aligned} \quad (15)$$

and we have that $y_i(0|s_0) = 0$. We shall truncate (14) at the desired order when calculating the partial derivatives. Since F and S are partial

derivatives, various methods can be used to calculate their values. We shall, however, restrict ourselves here to what might be considered the simplest approach.

Define the following vectors

$$\begin{aligned} t^{(j)} &= (0, \dots, a_j, \dots, 0) \\ t^{(j,k)} &= (0, \dots, a_j, \dots, a_k, \dots, 0) \end{aligned} \quad (16)$$

which have all zero components except in the j or j and k position. Then, the solution $y(t^{(k)})$ will denote the solution vector at s_0 corresponding to the initial conditions $t^{(k)}$ and similarly for $y(t^{(j,k)})$.

3.1 First Order Elements

These are simple to obtain. If we settle for an error of $\mathcal{O}(t^2)$, then

$$F_{ij} = \frac{y_i(t^{(j)})}{a_j}, \quad i, j = 1, \dots, 5 \quad (17)$$

and, thus, by tracing six rays, one reference, and five nearby rays, we can obtain all the first order elements. This result will not, however, suffice when obtaining second order terms that are to be used in (14).

We can, however, write

$$y_i(\pm t^{(j)}) = \pm F_{ij} a_j + S_{ijj} a_j^2 \quad (18)$$

which yields

$$\begin{aligned} F_{ij} &= [y_i(t^{(j)}) - y_i(-t^{(j)})]/2 a_j \\ S_{ijj} &= [y_i(t^{(j)}) + y_i(-t^{(j)})]/2 a_j^2. \end{aligned} \quad (19)$$

Thus, the ten vectors $\pm t^{(j)}$, $j = 1, \dots, 5$, along with one central

ray, give all first order elements and the second order 'trace' of S_{ijk} . The truncation error in F is on the order of $\mathcal{O}(t^2)$ and for S it is on the order of $\mathcal{O}(t)$ and their combined use in (14) has an error of $\mathcal{O}(t^3)$.

3.2 Second Order Elements

We already have from (19) the diagonal elements S_{ijj} . To obtain the off diagonal elements, write

$$S_{ijk} = [y_i(\pm t^{(j,k)}) \pm F_{ij} a_j \pm F_{ik} a_k - S_{ijj} a_j^2 - S_{ikk} a_k^2] / 2 a_j a_k \quad (20)$$

which is easily arrived at from (14) and (19). From (20), we can obtain

$$S_{ijk} = [y_i(t^{(j,k)}) + y_i(-t^{(j,k)}) - 2(S_{ijj} a_j^2 + S_{ikk} a_k^2)] / 4 a_j a_k \quad (21)$$

We shall refer to (20) as the asymmetric solution and (21) as the symmetric solution. Thus, for ten more nearby rays, we can obtain the off diagonal elements from (20) and for twenty more rays, the off diagonal elements from (21). The second order elements can, therefore, be generated from a total of twenty one or thirty one rays depending on whether the asymmetric (20) or symmetric case (21) is used.

By choosing a suitable set of nearby rays, we thus arrive at the first and second order transfer elements. If the truncation error is sufficiently small and the roundoff errors are also sufficiently small, then all the results calculated using (17), (19), (20), (21) should be the same. This, then furnishes a means of checking whether the calculated results have any numerical significance when interpreted as the derivatives of the solution with respect to the initial conditions. This is illustrated in the examples given.

IV. SIMPLE ERROR CRITERIA

We intend to arrive at our solution vectors y by integrating through a given magnetic field. Thus, the results obtained for the solution vectors will not be exact; in fact, they will contain errors arising from the numerical integration and from the field inaccuracies.

We shall assume that the perturbations a_i are exact and that the solution vector y is calculated with an error dy . We can write the error in F as

$$dF_{ij} = [dy_i(t^{(j)}) - dy_i(-t^{(j)})]/2 a_j.$$

If we now let δy represent the absolute value of the maximum error in all components of the solution vectors, then we can write that

$$|dF_{ij}| \leq \frac{1}{2|a_j|} 2\delta y = \frac{\delta y}{|a_j|}.$$

The same reasoning can be applied to (20) and (21) and we thus have for an elementary error bound that

$$\begin{aligned} |dF_{ij}| &\leq \delta y/|a_j| \\ |dS_{ijj}| &\leq \delta y/a_j^2 \end{aligned} \quad (22)$$

$$|dS_{ijk}| \leq 3 \delta y/2 |a_i a_j|.$$

In order to reduce the truncation error introduced when the infinite series is replaced by a finite sum, it is necessary to make the perturbations a_i small. This, however, amplifies the errors δy of the solution vector y and thus causes a loss of accuracy in the calculation of the transfer elements. One obvious solution is to use multiple precision; this may, however, not be justified when dealing with experimentally determined fields.

In any particular application, it will be necessary to devise some criteria that can be used to evaluate the significance of the numbers generated for the transfer elements. Our examples illustrate that in an ideal case, it is possible to find a best set of values for the a_i and that with this best set, the error bounds (22) are conservative. For example, if $\delta y \sim 10^{-8}$ and $\delta P/P_0 \sim 10^{-4}$, then $|dS_{ijj}| \sim 1.0$ which indicate no significance to the numerical values. This is not the case, but (22) does give an indication of the problem.

V. EXAMPLES

We illustrate the use of these results on three simple magnetic fields: a uniform field, an ideal quadrupole, and an ideal sextupole. In each case, we present what we shall refer to as the exact answer and the elements obtained from a program [3] that integrates the equations of motion (2) and (6) and performs the elimination indicated (19) and (21). The results that we call exact are calculated from the formulae furnished in [1]. These are analytical expressions for the first and second order elements that are correct to second order and should provide a reference check to within the precision of their evaluation. For our purposes, we shall consider them as exact.

The results of our calculations are presented in Tables I through VII. The first example is a uniform field such that a proton of $pc = 510.72$ MeV has a radius of curvature of one meter. The angle of bend is 45 degrees. The generated results used a perturbation vector $\{\delta x, \delta\theta, \delta y, \delta\psi, dP/P_0\} = \{.01 \text{ [cm]}, .1 \text{ [mr]}, .01 \text{ [cm]}, .1 \text{ [mr]}, .0001\}$. The results obtained in Table II, when compared to the exact results of Table I, are seen to agree quite well. The calculations were carried out in single precision on a CDC 6600 and thus we have that an optimistic estimate for zero is on the order of 10^{-14} . It is interesting to note that, for this field, a choice of smaller perturbations leads to worse results; that is, round-off errors enter the calculations using smaller perturbations.

The next case is that of an ideal quadrupole. Again, the reference particle is a proton of $pc = 510.72$ MeV. The quadrupole has an operating radius of two inches and length of ten inches. The pole top field corresponds to a radius of curvature of two meters for the reference particle;

that is, just half that of the previous case. The perturbation vector used to generate the results of Table IV is the same as for the uniform field and the results show quite adequate agreement.

Our last case is an ideal sextupole. The physical dimensions are the same as for the quadrupole. The pole tip field has been reduced by a factor of ten from that of the uniform field case. Thus, the reference particle has, at the pole tips, a radius of curvature of ten meters.

The generated elements presented in Table VI have the same perturbation vector as was used for the first two cases. Although these results agree fairly well with the exact results of Table V, we can do better with smaller perturbations. The results given in Table VII were generated using the perturbations $\{.0001 \text{ [cm]}, .01 \text{ [mr]}, .0001 \text{ [cm]}, .01 \text{ [mr]}, .0001\}$.

The sextupole illustrates a point that should be kept in mind. The results given here are ideal and, in a sense, represent the best that we can do. The fields are analytically defined, as contrasted to measured field data, and the error limits set in the integration routine are about as small as is practical for our single precision calculations.

Finally, we have used fields for which we know the answer and can thus adjust our perturbations to produce optimal results within the limits of roundoff error.

Of course, most real life problems do not present themselves in such a fashion. This means that, as is usual with numerical calculations, some independent criteria must be found to judge the significance of the elements generated.

Fig. 1. Reference Particle

XBL707-3283

Fig. 2. Nearby Particle and Reference Particle

XBL707-3282

Fig. 3. Moving Trihedral

XBL 707-3285

Fig. 4. Local Coordinates

TABLE I

X(CM),TX(MR),Y(CM),TY(MR),DELTAP/PO MOMENTUM MEV, P = 510.7200000000 SIGNED NO. OF CHARGES, Q = 1.0000000000 RADIUS OF CURVATURE,METERS R = 1.0000000000 ANGLE OF BEND,DEG THETA = 45.0000000000 FIELD,KG B = 17.03573774 ARC LENGTH,METERS S0 = .78539816						
UNIFORM FIELD BENDING MAGNET						
R(J,K)	J / K =	1	2	3	4	5
		7.0710678E-01 -7.0710678E+00 0.0000000E 00 0.0000000E 00 0.0000000E 00	7.0710678E-02 7.0710678E-01 0.0000000E 00 0.0000000E 00 0.0000000E 00	C.0000000E 00 0.0000000E 00 1.0000000E+00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 7.8539816E-02 1.0000000E+00 0.0000000E 00	2.9289322E+01 7.0710678E+02 0.0000000E 00 0.0000000E 00 1.0000000E+00
T(I,J,K)	I / J/ K =	1	2	3	4	5
1 1		-2.5000000E-03	2.5000000E-04 1.0355339E-05	C.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 -1.4644661E-05	2.5000000E-01 1.0355339E-02 0.0000000E 00 0.0000000E 00 -2.5000000E+01
2 1		0.0000000E 00	0.0000000E 00 -3.5355339E-04	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 -3.5355339E-04	3.5355339E+00 0.0000000E 00 0.0000000E 00 0.0000000E 00 -7.0710678E+02
3 1		0.0000000E 00	0.0000000E 00	C.0000000E 00	3.5355339E-04	0.0000000E 00
3 2			0.0000000E 00	0.0000000E 00	1.4644661E-05	0.0000000E 00
3 3				0.0000000E 00	0.0000000E 00	0.0000000E 00
3 4					0.0000000E 00	3.9145691E-03
3 5						0.0000000E 00
4 1		0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
4 2			0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
4 3				0.0000000E 00	0.0000000E 00	0.0000000E 00
4 4					0.0000000E 00	0.0000000E 00
4 5						0.0000000E 00
5 1		0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
5 2			0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
5 3				0.0000000E 00	0.0000000E 00	0.0000000E 00
5 4					0.0000000E 00	0.0000000E 00
5 5						0.0000000E 00

TABLE II

UNIFORM FIELD 45 DEG. BEND, 1(M) RADIUS. GENERATED ELEMENTS					
R(J,K) J / K =	1	2	3	4	5
	7.0710678E-01 -7.0710678E+00 0.0000000E 00 0.0000000E 00 0.0000000E 00	7.0710678E-02 7.0710678E-01 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 1.0000000E+00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 7.8539817E-02 1.0000000E+00 0.0000000E 00	2.9289322E+01 7.0710679E+02 0.0000000E 00 0.0000000E 00 1.0000000E+00
T(I,J,K) I/ J/ K =	1	2	3	4	5
1 1 1 2 1 3 1 4 1 5	-2.4999979E-03	2.4999991E-04 1.0355356E-05	-8.4703295E-18 4.2351647E-19 0.0000000E 00	-4.3512594E-11 -2.4701251E-12 -4.2351647E-19 -1.4644658E-05	2.4999991E-01 1.0355328E-02 -8.4703295E-16 -3.0758907E-09 -2.5000013E+01
2 1 2 2 2 3 2 4 2 5	4.6447290E-08	-1.7405893E-09 -3.5355301E-04	1.2924697E-22 -1.3552527E-17 0.0000000E 00	-8.8496523E-10 -7.5073929E-11 -6.7762636E-18 -3.5355329E-04	3.5355317E+00 -2.2711833E-07 -2.7105054E-14 -8.5854197E-08 -7.0710708E+02
3 1 3 2 3 3 3 4 3 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	3.5355339E-04 1.4644661E-05 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 3.9145690E-03 0.0000000E 00
4 1 4 2 4 3 4 4 4 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00
5 1 5 2 5 3 5 4 5 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	-1.2621774E-23 -1.2621774E-24 -1.2621774E-23 -1.2621774E-24 -3.5527137E-07

TABLE III

X(CM),TX(MR),Y(CM),TY(MR),DELTAP/PO FIELD IN GAUSS,BO = 8517.8688700000 APERTURE IN CM.,A = 5.0800000000 LENGTH,METERS S = .2540000000 MOMENTUM MEV, P = 510.7200000000 SIGNED NO. OF CHARGES, Q = 1.0000000000 GRADIENT (KG/CM) = 1.67674584						
IDEAL QUADRUPOLE						
R(J,K)	J / K =	1	2	3	4	5
		6.9894942E-01 -2.2436913E+01 0.0000000E 00 0.0000000E 00 0.0000000E 00	2.2795904E-02 6.9894942E-01 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 1.3346607E+00 2.7731120E+01 0.0000000E 00	0.0000000E 00 0.0000000E 00 2.8174818E-02 1.3346607E+00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 1.0000000E+00
T(I,J,K)	I/ J/ K =	1	2	3	4	5
1 1		0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	1.4247440E-01 1.2606470E-C3 0.0000000E 00 0.0000000E 00 0.0000000E 00
1 2						
1 3						
1 4						
1 5						
2 1		0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	9.9776621E+00 1.4247440E-01 0.0000000E 00 0.0000000E 00 0.0000000E 00
2 2						
2 3						
2 4						
2 5						
3 1		0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 -1.7609261E-C1 -1.4313912E-03 0.0000000E 00
3 2						
3 3						
3 4						
3 5						
4 1		0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 -1.5274409E+01 -1.7609261E-01 0.0000000E 00
4 2						
4 3						
4 4						
4 5						
5 1		0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00
5 2						
5 3						
5 4						
5 5						

TABLE IV

GENERATE IDEAL QUADRUPOLE EMEENTS					
R(J,K) J / K =	1	2	3	4	5
	6.9894940E-01 -2.2436914E+01 0.0000000E 00 0.0000000E 00 0.0000000E 00	2.2795905E-02 6.9894940E-01 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 1.3346608E+00 2.7731122E+01 0.0000000E 00	0.0000000E 00 0.0000000E 00 2.8174819E-02 1.3346608E+00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 1.0000000E+00
T(I,J,K) I / J / K =	1	2	3	4	5
1 1 1 2 1 3 1 4 1 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	1.4247444E-01 1.2606460E-03 0.0000000E 00 0.0000000E 00 0.0000000E 00
2 1 2 2 2 3 2 4 2 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	9.9776641E+00 1.4247443E-01 0.0000000E 00 0.0000000E 00 0.0000000E 00
3 1 3 2 3 3 3 4 3 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 -1.7609266E-01 -1.4313928E-03 0.0000000E 00
4 1 4 2 4 3 4 4 4 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 -1.5274413E+C1 -1.7609265E-01 0.0000000E 00
5 1 5 2 5 3 5 4 5 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	-1.2621774E-23 -1.2621774E-24 -1.2621774E-23 -1.2621774E-24 -3.5527137E-07

TABLE V

X(CM), TX(MR), Y(CM), TY(MR), DELTAP/PO FIELD IN GAUSS, BO = 1703.5737740000 APERTURE IN CM., A = 5.0800000000 LENGTH, METERS S = .2540000000 MOMENTUM MEV, P = 510.7200000000 SIGNED NO. OF CHARGES, Q = 1.0000000000 GRADIENT (KG/CM**2) = .13202723					
IDEAL SEXTUPOLE					
R(J,K) J / K =	1	2	3	4	5
1 1	1.0000000E+00	2.5400000E-02	0.0000000E 00	0.0000000E 00	0.0000000E 00
1 2	0.0000000E 00	1.0000000E+00	0.0000000E 00	0.0000000E 00	0.0000000E 00
1 3	0.0000000E 00	0.0000000E 00	1.0000000E+00	2.5400000E-02	0.0000000E 00
1 4	0.0000000E 00	0.0000000E 00	0.0000000E 00	1.0000000E+00	0.0000000E 00
1 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	1.0000000E+00
T(I,J,K)					
I / J / K =	1	2	3	4	5
1 1	-1.2500000E-02	-1.0583333E-04	0.0000000E 00	0.0000000E 00	0.0000000E 00
1 2		-1.3440833E-06	0.0000000E 00	0.0000000E 00	0.0000000E 00
1 3			1.2500000E-02	1.0583333E-04	0.0000000E 00
1 4				1.3440833E-06	0.0000000E 00
1 5					0.0000000E 00
2 1	-9.8425197E-01	-1.2500000E-02	0.0000000E 00	0.0000000E 00	0.0000000E 00
2 2		-2.1166667E-04	0.0000000E 00	0.0000000E 00	0.0000000E 00
2 3			9.8425197E-01	1.2500000E-02	0.0000000E 00
2 4				2.1166667E-04	0.0000000E 00
2 5					0.0000000E 00
3 1	0.0000000E 00	0.0000000E 00	1.2500000E-02	1.0583333E-04	0.0000000E 00
3 2		0.0000000E 00	1.0583333E-04	1.3440833E-06	0.0000000E 00
3 3			0.0000000E 00	0.0000000E 00	0.0000000E 00
3 4				0.0000000E 00	0.0000000E 00
3 5					0.0000000E 00
4 1	0.0000000E 00	0.0000000E 00	9.8425197E-01	1.2500000E-02	0.0000000E 00
4 2		0.0000000E 00	1.2500000E-02	2.1166667E-04	0.0000000E 00
4 3			0.0000000E 00	0.0000000E 00	0.0000000E 00
4 4				0.0000000E 00	0.0000000E 00
4 5					0.0000000E 00
5 1	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
5 2		0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
5 3			0.0000000E 00	0.0000000E 00	0.0000000E 00
5 4				0.0000000E 00	0.0000000E 00
5 5					0.0000000E 00

TABLE VI

GENERATE IDEAL SEXTUPOLE ELEMENTS					
R(J,K) J / K =	1	2	3	4	5
	1.0000000E+00 8.2020998E-07 0.0000000E 00 0.0000000E 00 0.0000000E 00	2.5400000E-02 1.0000000E+00 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 1.0000000E+00 8.2020998E-07 0.0000000E 00	0.0000000E 00 0.0000000E 00 2.5400000E-02 1.0000000E+00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 1.0000000E+00
T(I,J,K) I / J / K =					
1 1 1 2 1 3 1 4 1 5	-1.2500000E-02	-1.0583334E-04 -1.3440834E-06	3.9265229E-11 9.3943747E-12 1.2500000E-02	-3.1028342E-12 2.6600530E-14 1.0583334E-04 1.3440833E-06	-1.1435301E-08 -7.2858439E-12 6.2500054E-09 6.7204176E-12 0.0000000E 00
2 1 2 2 2 3 2 4 2 5	-9.8425197E-01	-1.2500000E-02 -2.1166667E-04	9.2275243E-09 2.5373779E-10 9.8425197E-01	-2.4244018E-10 3.2030632E-12 1.2500000E-02 2.1166667E-04	-1.3123361E-06 -1.1546326E-09 4.9212652E-07 1.0583345E-09 0.0000000E 00
3 1 3 2 3 3 3 4 3 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	1.2500000E-02 1.0583333E-04 0.0000000E 00	1.0583334E-04 1.3440834E-06 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 -5.2041704E-09 0.0000000E 00 0.0000000E 00
4 1 4 2 4 3 4 4 4 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	9.8425197E-01 1.2500000E-02 0.0000000E 00	1.2500000E-02 2.1166667E-04 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 -8.2020999E-07 -1.2212453E-10 0.0000000E 00
5 1 5 2 5 3 5 4 5 5	0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00	-1.2621774E-23 -1.2621774E-24 -1.2621774E-23 -1.2621774E-24 -3.5527137E-07

TABLE VII

GENERATE IDEAL SEXTUPOLE ELEMENTS						
R(J,K)	J / K =	1	2	3	4	5
		1.0000000E+00 8.2020997E-11 0.0000000E 00 0.0000000E 00 0.0000000E 00	2.5400000E-02 1.0000000E+00 0.0000000E 00 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 1.0000000E+00 8.2020997E-11 0.0000000E 00	0.0000000E 00 0.0000000E 00 2.5400000E-02 1.0000000E+00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 0.0000000E 00 1.0000000E+00
T(I,J,K)	I / J / K =	1	2	3	4	5
1 1	-1.2500000E-02		-1.0583333E-04 -1.3440833E-06	-1.4131264E-11 -4.6657329E-13 1.2500000E-02	-7.2180194E-14 3.6240023E-16 1.0583333E-04 1.3440833E-06	-7.5894193E-11 -4.3368128E-13 6.2499989E-11 6.7204248E-13 0.0000000E 00
2 1	-9.8425197E-01		-1.2500000E-02 -2.1166667E-04	9.2417476E-13 1.5440564E-11 9.8425197E-01	-2.4233083E-13 1.3399002E-15 1.2500000E-02 2.1166667E-04	-5.0032807E-09 -1.1102236E-10 4.9212588E-09 1.0583343E-10 0.0000000E 00
3 1	0.0000000E 00		0.0000000E 00 0.0000000E 00	1.2500000E-02 1.0583333E-04 0.0000000E 00	1.0583333E-04 1.3440833E-06 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 0.0000000E 00 8.6736174E-13 0.0000000E 00
4 1	0.0000000E 00		0.0000000E 00 0.0000000E 00	9.8425197E-01 1.2500000E-02 0.0000000E 00	1.2500000E-02 2.1166667E-04 0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00 -8.2020999E-11 0.0000000E 00 0.0000000E 00
5 1	0.0000000E 00		0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00	0.0000000E 00 0.0000000E 00	-1.2621774E-21 -1.2621774E-23 -1.2621774E-21 -1.2621774E-23 -3.5527137E-07

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